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9583 2022

February 2022

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
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CESifo *Working Paper No. 9583*

Non-Paternalistic Foundation of Sugar Taxation and Missing Markets for Sugar Content

Abstract

This paper provides a new foundation of soft drink taxation. We abstract from externalities and internalities previously used to justify such taxation and, instead, emphasize that neither explicit nor implicit markets and prices for sugar content can be expected to emerge. Hence, in the absence of any regulation, the sugar content of sugar-sweetened beverages (SSBs) would be inefficiently high. This market failure can be corrected by a tax on the sugar content per unit of the SSB. However, a sugar content tax alone leads to an unintended downward distortion of the quantity of SSBs, which has to be corrected by an additional revenue-neutral subsidy on each unit of the SSB.

JEL-Codes: D500, H210, I180.

Keywords: obesity, sugar-sweetened beverages, sugar content, sugar content tax, soft drink subsidy.

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14th February 2022

1 Introduction

Obesity has reached epidemic proportions globally, with at least 2.8 million people dying each year as a result of being overweight or obese (World Health Organization, 2022). Overweight and obesity is caused to a large extent by overconsumption of sugar, in particular sugar-sweetened beverages (SSBs). The World Health Organization (2017) therefore recommends taxing SSBs and, according to Allcott et al. (2019a), by 2019 already 39 countries worldwide have introduced soda taxes, in addition to a number of cities and counties in the US. Many governments tax the SSB itself by a tax rate applied to each volume unit (e.g. ounce or litre) of the SSB. But very recently, taxes on the sugar content of SSBs have become more and more popular. In 2018, for instance, the UK introduced a soft drink tax with a rate of 18p per litre on SSBs with 5-8g sugar per 100ml and 24p per litre on SSBs with more than 8g sugar per 100ml. Soda taxes with a similar rate structure are implemented in Chile since 2014 and Portugal since 2017 (see Griffith et al. 2019), and in 2018 also France reformed its soft drink tax such that the tax rate is differentiated according to the sugar content of the SSBs (see Kurz and König, 2021).

The present paper provides a new economic foundation for taxes on SSBs, in particular taxes on the sugar content of SSBs, and additionally shows that such taxes have to be supplemented by a revenue-neutral (!) subsidy (!) on each volume unit of the SSBs.[1] We abstract from externalities and internalities previously used to justify sugar taxation (for a survey see Allcott et al., 2019a, and Griffith et al., 2020). Instead, we emphasize that in practice neither explicit nor implicit markets and prices for sugar content can be expected to emerge. This argument is key to our analysis and empirically supported by the evidence on the so-called 'uniform product pricing', according to which an SSB and the corresponding diet or zero-sugar variant of the SSB are usually sold at the same price (e.g. Bollinger and Sexton, 2019). As consequence of missing markets for sugar content, in the absence of any regulation (laissez-faire), the sugar content would be inefficiently high, since the social marginal costs of the sugar content are not included in the price system. The market failure can be corrected by a tax on the sugar content per unit of the SSB. However, as the sugar content tax applies to each unit of the SSB, tax payments

[1] Although our analysis is tailored to SSBs and their sugar content, it is also applicable and transferable to other sin goods like, for example, consumption goods containg fat or alcohol.

increase with the number of SSBs. This leads to an unintended downward distortion of the quantity of SSBs, which has to be corrected by an additional subsidy on each volume unit of the SSB. Interestingly, such a tax-subsidy solution can be implemented in a revenue-neutral way, i.e. SSB producers' net tax-payments become zero, which may reduce the opposition of the SSB industry against soft drink taxation.

These results are derived in a general equilibrium model with sugar production, SSB production and SSB consumption. Each unit of the SSB contains a certain amount of sugar that explicitly enters the analysis as production factor. The embodied sugar per unit of SSB output, called sugar content, is a product characteristic that can be varied in the SSB production process. Since it is the same for each unit of the SSB sold to consumers, the sugar content set by the SSB producer is a public good (if it causes social benefits) or a public bad (if it causes social costs).[2] Consumers have heterogeneous preferences regarding the quantity and the sugar content of the SSB they consume, and they are harmed by their sugar intake stemming from SSBs. Within this framework, we derive the Pareto efficient allocation as benchmark and then compare the efficient allocation with the equilibrium outcome under different market concepts.

Due to the public good/bad property of the sugar content we begin with a competitive economy that encompasses a Lindahl market for sugar content, where each consumer pays a personalized price for sugar content. The Lindahl market scheme turns out to be efficient, but it lacks incentives to emerge in the real world, since consumers do not have an incentive to truthfully reveal their preferences, as required for personalized pricing. An alternative approach is the concept of an indirect market for sugar content, where the price of the SSB is a function of the embodied sugar content. However, with heterogenous consumers a price function does not possess the necessary flexibility to reflect the different consumers' willingness-to-pay for the (public good/bad) sugar content. We therefore show that such markets work only in the very specific case of identical willingness-to-pay for

[2]For many products, modern production technologies allow to individualize product characteristics to consumers' preferences. However, for mass products like SSBs such an individualization is too costly and, thus, each consumer has to consume the same sugar content, which makes the sugar content a public good/bad. Producers may offer variants of a SSB that differ in the sugar content, indeed, but then the sugar content of each variant is a public good/bad as well. Our basic results would thus remain unchanged if we consider more than one SSB in our model. We briefly discuss this extension in the Conclusion.

sugar content. In addition, even if Lindahl or indirect markets were feasible, there is another problem. Lindahl or indirect prices of sugar content reflect the difference between the consumers' marginal utility of sugar content and the consumers' marginal health costs of sugar content. Since this difference is likely to be negative due to the high health costs that we observe in reality and that have triggered the whole debate about obesity and sin taxation, a market price for sugar content has to be negative. A negative price contradicts the free disposal assumption in general equilibrium models first introduced by Arrow and Debreu (1954) and Debreu (1959): Since the SSB producer actually would have to pay for selling the sugar content to consumers, it would not supply the sugar content implying that sugar content is not traded on markets and its market price is zero.

This insight may help to explain the above-mentioned uniform pricing phenomenon of Bollinger and Sexton (2019). It induces us to investigate the case of missing markets for sugar content. We first investigate the laissez-faire economy without any regulation and show that the resulting market allocation is inefficient. More specific, under mild qualifications regarding production technologies and consumption preferences and with the assumption that sugar content is a public bad that reduces the consumers' utility net of health costs, a comparative static analysis reveals that the sugar content of the SSB in the market economy is inefficiently high, while the SSB price is inefficiently low, since it does not reflect the social marginal costs of the sugar content. Whether the quantity of the SSB and the amount of sugar are inefficiently low or high depends on the complex interplay of substitution and income effects which the low SSB price and the high sugar content exert on the consumers' decision. Interestingly, we identify cases where these effects are such that the equilibrium SSB consumption and the sugar use are *inefficiently low*, for instance, if the SSB and its sugar content are close substitutes in consumption and, thus, an increase in sugar content induces a large reduction in SSB consumption.

Finally, we turn to the question how to correct the market failure in the competitive economy with missing markets for sugar content. We identify three tax policies that restore efficiency. First, a tax on sugar content in the formulation of the SSB directly replaces the missing producer price for sugar content and therefore gives the SSB producer the incentive for choosing the efficient sugar content. The problem of this tax on sugar content is that the SSB producer has to pay a 'one-time' amount on the sugar content in its

SSB formulation and that it has to reveal the sugar content of its SSB. Second, efficiency is attained by a tax on sugar content per unit of the SSB combined with a revenue-neutral subsidy on each unit of the SSB. This is the policy already discussed at the beginning of the Introduction. It encompasses a sugar content tax like those implemented in the UK, Chile or Portugal, but also an – in practice so far not implemented – subsidy on the SSB itself, since the sugar content tax alone would distort SSB consumption downward. If both instruments are levied on the SSB producer, then this tax-subsidy policy is revenue-neutral for the SSB producer which may increase its political acceptability. Third, the sugar content tax per unit of the SSB in the second policy option may also be replaced by a tax on the sugar input in the production of the SSB, still combined with the revenue-neutral subsidy on each unit of the SSB. The additional advantage of this policy is that policymakers do not need to know the sugar content of the SSB.

The present paper contributes to the literature on sin taxation, in general, and sugar taxation, in particular. In this literature, sin or SSB taxes are usually justified by standard externality and internality arguments. The externality argument is mainly associated with the moral hazard problem in health insurance markets that leads to excessive sugar consumption, while the internality problem refers to behavioral consumers which may have self-control problems and present-biased preferences that also induces a too high sugar intake. Excellent surveys on this literature are given in Allcott et al. (2019a), Grummon et al. (2019) and Griffith et al. (2020). Original contributions can be found in Gruber and Koszegi (2001), O'Donoghue and Rabin (2003, 2006), Cremer et al. (2012), Allcott et al. (2019b), Fahri and Gabaix (2020), Kalamov and Runkel (2020, 2022) and Arnabal (2021), to name only a few.[3] There are several important differences of our approach to this literature. First, moral hazard effects on consumption due to health care insurance represent 'only' a second-best argument for sugar taxation, since such effects would vanish if we abolish health insurance or if insurers have full information. In contrast, the tax-subsidy policy derived in our paper implements the Pareto optimum and, thus, is a first-best policy. Second, the internality approach is paternalistic, while the optimal policy

[3]We here only refer to theoretical papers related to the justificiation of sin taxes, while ignoring the works on the incidence of sin taxes (e.g. Kotarkorpi, 2008; Bonnet and Réquillart, 2013; Dragone et al. 2016) and empirical studies on the effects of SSB taxes (see Allcott et al., 2019a, and Griffith et al., 2020, for surveys and Dubois et al., 2020, as an example for an important recent contribution).

derived in our paper presupposes a non-paternalistic social planner who takes into account the revealed preferences of consumers, even if these preferences are characterized by some behavioral distortions.[4] Third, and perhaps most important, all above mentioned articles do not explicitly model the sugar content of SSBs and, thus, do not derive the results which we obtain in our general equilibrium model of SSB production and consumption with an endogenously determined sugar content. An important exception is Cremer et al. (2019) who show that the first-best solution is achieved by a Pigouvian tax on each unit of SSB which is proportional to the sugar content. They also assume an endogenous sugar content chosen by the SSB producers, indeed, but in contrast to our analysis they consider present-biased and homogeneous consumers, and a SSB monopoly or oligopoly in a partial equilibrium model. More importantly, they assume that an indirect market with an implicit price for sugar content exists. Hence, they do not derive any of the results which we obtain in case of missing markets for sugar content.

The paper is organized as follows. Section 2 outlines the model. In Section 3, we derive the properties of the efficient allocation. Section 4 and 5 analyze competitive economies with Lindahl and indirect markets for sugar content, respectively. In Section 6, we characterize the laissez-faire economy with missing market for sugar content, and Section 7 proceeds by investigating corrective tax-subsidy schemes. Section 8 concludes.

2 Model

We consider an economy with the whole supply and demand chain of a sin good, from the production of the sin good's input and the production of the sin good itself, over the consumption of the sin good by households up to the health effects of sin good consumption. In order to ease exposition, we refer to a sugar-sweetened beverage (SSB) as the sin

[4] There is a controversial debate in the literature about the suitability of the internality foundation of sin taxation. For example, Whitman and Rizzo (2015) criticize that the rationality axioms adopted by behavioral paternalism are not justified and that there is no evidence that policymakers make better decisions than consumers, while the idea of libertarian paternalism or nudging discussed in, e.g., Loewenstein and Chater (2017) basically allows an internality foundation of sin taxation. Our analysis is not indented to make the case for the one or the other position in this discussion. Instead, we would like to emphasize the non-paternalistic nature of our sin tax foundation, but at the same time view our analysis as complementary to other kinds of foundations, including the paternalistic internality approach.

good and sugar as the relevant ingredient, but the analysis also applies to other kinds of sin goods, for instance, fatty food or alcoholic beverages.

On the production side of the economy, the intermediate good sugar is produced in quantity z^s with the help of labor input in quantity ℓ_z^d according to the convex technology[5]

$$z^s \leq Z(\ell_z^d), \tag{1}$$

with $Z_\ell > 0$. The sin good, called SSB, is produced in quantity x^s with two types of ingredients which are embodied in the output. One ingredient is the intermediate good sugar which is employed in quantity z^d. The other ingredient is health-neutral and itself generated from ℓ_x^d units of labor. For simplicity and in order to focus on sugar as the unhealthy ingredient, we do not explicitly model the production of the health-neutral ingredient and assume that it is already integrated into the production function of the SSB. Formally, we consider the convex SSB production technology[6]

$$x^s \leq X(\ell_x^d, z^d), \tag{2}$$

with $X_\ell, X_z > 0$. The technology (2) allows for varying the input mix or, more generally, the formulation of the SSB, as measured by the share of sugar per unit of the SSB, i.e.

$$q^s := \frac{z^d}{x^s}. \tag{3}$$

The SSB's sugar content defined in (3) is an 'intrinsic' product characteristic chosen by the SSB producer. In the following, we refer to q^s as the *supplied* sugar content.

On the consumption side of the economy there are $n > 1$ heterogenous consumers indexed by subscripts $i, j = 1, \ldots, n$. Consumer i supplies ℓ_i^s units of labor and consumes

[5]We use the convention that lower-case letters represent variables or parameters. The superscript s and d indicate quantities supplied and demanded, respectively. Upper-case letters are reserved to denote functions and subscripts attached to them indicate first derivatives.

[6]An explicit modeling of the second ingredient would require to introduce the technology $w = W(\ell_x^d)$ with $W_\ell > 0$, where w may be interpreted as water input. The SSB production function can then be written as $\widetilde{X}(w, z^d)$. Replacing water input w by the water production function $W(\ell_x^d)$ then gives the integrated production function $\widetilde{X}[W(\ell_x^d), z^d] =: X(\ell_x^d, z^d)$. Our main results will remain unchanged if we assume a separate water sector producing water from labor according to the function W and delivering its output to the SSB production sector. We can also generalize our results to the case where the SSB production needs a further separate labor input that is not used for water generation. Such generalizations would increase the complexity of our formal analysis without providing any further insights.

x_i^d units of the SSB with the *demanded* sugar content q_i^d. Moreover, consumer i's health status is decreasing in sugar intake $z_i := x_i^d q_i^d$ and represented by the health function $h_i = H^i(z_i)$ with $H_z^i < 0$. Utility of consumer i is given by the quasi-concave function

$$u_i = U^i(x_i^d, q_i^d, \ell_i^s, h_i) \tag{4}$$

with $U_x^i, U_q^i, U_h^i > 0$ and $U_\ell^i < 0$. Notice that we do not explicitly model a behavioral distortion of consumers. Nevertheless, such a behavioral distortion may be implicitly captured by our approach. For example, preferences of present-biased consumers are represented by the additively separable utility function $U^i(x_i^d, q_i^d, \ell_i^s, h_i) = \breve{U}(x_i^d, q_i^d, \ell_i^s) + \beta_i V^i(h_i)$, where V^i is consumer i's true health utility and $\beta_i V^i$ with $\beta_i \in]0,1[$ represents health utility perceived by consumer i. However, since we focus on a non-paternalistic foundation of sin taxation, the efficient allocation will be derived under the assumption that the social planner calculates with the perceived health utility $\beta_i V^i$ instead of the true health utility V^i of consumer i, in contrast to the approach typically used for a behavioral foundation of sin taxation. Notice also that in the largest part of our analysis we consider the general functional form of consumer i's utility function given in (4).

The model is closed by the resource constraints

$$z^s \geq z^d, \tag{5}$$

$$x^s \geq \sum_{j=1}^{n} x_j^d, \tag{6}$$

$$\sum_{j=1}^{n} \ell_j^s \geq \ell_x^d + \ell_z^d. \tag{7}$$

$$q^s = q_i^d, \qquad \text{for all } i, \tag{8}$$

Inequalities (5), (6) and (7) present the resource constraints for sugar, the sin good and labor, respectively. In each inequality the LHS captures the quantity supplied and the RHS the quantity demanded. The constraints (6) and (8) show the transactions between the SSB producer and the consumers. The SSB producer offers (x^s, q^s) to the consumers and consumer i demands (x_i^d, q_i^d). Equation (8) elucidates the public good property of the sugar content embodied in the SSB. It requires that each consumer's demanded sugar content has to be equal to the sugar content supplied by the SSB producer.

3 Pareto efficiency

In this section we focus on the Pareto-efficient allocation in the economy described above. Consider a non-paternalistic social planner who takes the revealed preferences represented by U^j for all j as the relevant utility levels. The planner then maximizes consumer i's utility (4), subject to the technologies (1)–(3), the resource constraints (5)–(8) and

$$U^j[x_j^d, q_j^d, \ell_j^s, H^j(x_j^d q_j^d)] \geq \bar{u}_j, \qquad \text{for all } j \neq i, \tag{9}$$

where $\bar{u}_j$ is consumer j's exogenously given utility level. The Lagrangean and the full set of first-order conditions to this maximization problem are given in the appendix. In the Pareto optimum, we have $z^s = z^d =: z$, $q^s = q_i^d =: q$, $x_i^d =: x_i$ and $x^s = \sum_{j=1}^n x_j^d =: x$. With these properties, the first-order conditions of the Pareto optimum can be rearranged to obtain the conditions listed in the first column of Table 1.

	row	Pareto efficiency	Lindahl market	Indirect market	Regulated market
column		1	2	3	4
sugar production	1	$\frac{1}{Z_\ell} = \frac{\lambda_z}{\lambda_\ell}$	$\frac{1}{Z_\ell} = \frac{p_z}{p_\ell}$	$\frac{1}{Z_\ell} = \frac{p_z}{p_\ell}$	$\frac{1}{Z_\ell} = \frac{p_z - \tau_z^s}{p_\ell}$
SSB production	2	$\frac{1}{X_\ell} = \frac{\lambda_x}{\lambda_\ell} - \frac{q}{x}\frac{\lambda_q}{\lambda_\ell}$	$\frac{1}{X_\ell} = \frac{p_x}{p_\ell} - \frac{q}{x}\frac{p_q}{p_\ell}$	$\frac{1}{X_\ell} = \frac{P^x}{p_\ell} - \frac{q}{x}\frac{P_q^x x}{p_\ell}$	$\frac{1}{X_\ell} = \frac{p_x - \tau_x^s}{p_\ell} + \frac{q}{x}\frac{\tau_q}{p_\ell}$
	3	$\frac{X_z}{X_\ell} = \frac{\lambda_z}{\lambda_\ell} - \frac{1}{x}\frac{\lambda_q}{\lambda_\ell}$	$\frac{X_z}{X_\ell} = \frac{p_z}{p_\ell} - \frac{1}{x}\frac{p_q}{p_\ell}$	$\frac{X_z}{X_\ell} = \frac{p_z}{p_\ell} - \frac{1}{x}\frac{P_q^x x}{p_\ell}$	$\frac{X_z}{X_\ell} = \frac{p_z + \tau_z^d}{p_\ell} + \frac{1}{x}\frac{\tau_q + \tau_{xq} x}{p_\ell}$
	4	$\frac{\lambda_q}{\lambda_\ell} = \sum_{j=1}^{n} \frac{\lambda_{qj}}{\lambda_\ell}$	$\frac{p_q}{p_\ell} = \sum_{j=1}^{n} \frac{p_{qj}}{p_\ell}$	$\frac{P_q^x x}{p_\ell} = \sum_{j=1}^{n} \frac{P_q^x x_j}{p_\ell}$	$--$
consumer i	5	$-\frac{U_x^i + qU_h^i H_z^i}{U_\ell^i} = \frac{\lambda_x}{\lambda_\ell}$	$-\frac{U_x^i + qU_h^i H_z^i}{U_\ell^i} = \frac{p_x}{p_\ell}$	$-\frac{U_x^i + qU_h^i H_z^i}{U_\ell^i} = \frac{P^x}{p_\ell}$	$-\frac{U_x^i + qU_h^i H_z^i}{U_\ell^i} = \frac{p_x + \tau_x^d}{p_\ell}$
(for all i)	6	$-\frac{U_q^i + x_i U_h^i H_z^i}{U_\ell^i} = \frac{\lambda_{qi}}{\lambda_\ell}$	$-\frac{U_q^i + x_i U_h^i H_z^i}{U_\ell^i} = \frac{p_{qi}}{p_\ell}$	$-\frac{U_q^i + x_i U_h^i H_z^i}{U_\ell^i} = \frac{P_q^x x_i}{p_\ell}$	$--$

Table 1: Conditions for Pareto efficiency and market equilibria

The Lagrange multipliers λ_z, λ_x, λ_{qi}, λ_q and λ_ℓ represent shadow prices of sugar, the SSB, the demanded sugar content of consumer i, the supplied sugar content and labor, respectively. Since the Pareto optimum determines only relative shadow prices, we choose labor as numéraire and set $\lambda_\ell \equiv 1$. From column 1 of Table 1 we infer

$$\lambda_z > 0, \quad \lambda_x \gtreqless 0, \quad \lambda_q \gtreqless 0, \quad \lambda_{qi} \gtreqless 0, \quad \text{for all } i. \tag{10}$$

While the shadow price of sugar is unambiguously positive, the shadow price of the SSB as well as of the sugar content may be positive or negative. According to column 1, row 5 of Table 1, the intuition for the ambiguous sign of the SSB's shadow price λ_x is that the marginal social value of one unit of the SSB may be positive or negative, depending on the relation between the marginal consumption utility $U_x^i > 0$ and the marginal health costs $-qU_h^i H_z^i > 0$ of the SSB. The same is true for the shadow price λ_{qi} of the sugar content demanded by consumer i, as shown by the marginal utility $U_q^i > 0$ and the marginal health costs $-xU_h^i H_z^i > 0$ of sugar content, see column 1, row 6 of Table 1. Since the SSB producer's supplied sugar content is a public good, the shadow price λ_q of this sugar content equals the sum of shadow prices λ_{qi} of the consumers' demanded sugar content as shown in column 1, row 4 of Table 1, and, thus, may also take any sign.

In order to further characterize the Pareto optimum, we eliminate the shadow prices from the conditions in column 1 of Table 1. From rows 1, 3, 4 and 6 we obtain

$$\frac{1}{Z_\ell} = \frac{X_z}{X_\ell} - \frac{1}{x} \sum_{j=1}^{n} \frac{U_q^j + x_j U_h^j H_z^j}{U_\ell^j}, \tag{11}$$

while rows 2, 4, 5 and 6 imply

$$\frac{1}{X_\ell} - \frac{q}{x} \sum_{j=1}^{n} \frac{U_q^j + x_j U_h^j H_z^j}{U_\ell^j} = -\frac{U_x^i + qU_h^i H_z^i}{U_\ell^i}, \qquad \text{for all } i. \tag{12}$$

Equations (11) and (12) represent the efficient allocation rules for sugar and the SSB, respectively. In order to understand the efficient allocation rule for sugar, suppose the social planner keeps constant SSB output x and marginally increases the amount of sugar z. The rule (11) then requires that, in the Pareto optimum, the additional costs of sugar production ($1/Z_\ell = \mathrm{d}\ell_z^d/\mathrm{d}z^s$) equals the SSB production benefits in terms of saved labor input ($X_z/X_\ell = -(\mathrm{d}\ell_x^d/\mathrm{d}z^d)|_{x^s=const.}$) and the sum of the consumers' net changes in utility, measured by the marginal willingness-to-pay for sugar content $q = z/x$ in terms of labor.

The latter marginal-willingness-pay is composed of two partial effects which are opposite in sign. The term $-\frac{1}{x}\sum_{j=1}^{n} U_q^j/U_\ell^j > 0$ represents the consumers' consumption benefits of increasing q via increasing z, whereas $-\frac{1}{x}\sum_{j=1}^{n} x_j U_h^j H_z^j/U_\ell^j < 0$ reflects the consumers' health costs of increasing q via increasing z. For interpreting the efficient allocation rule of the SSB output, suppose the social planner keeps constant the amount of sugar z and marginally increases the SSB output x. Equation (12) then requires that, in the Pareto optimum, the additional SSB production costs ($1/X_\ell = \mathrm{d}\ell_x^d/\mathrm{d}x^s$) and the consumers' net utility loss due to a fall in sugar content $q = z/x$, measured again by the sum of consumers' marginal willingness to pay for sugar content $(-\frac{q}{x}\sum_{j=1}^{n}(U_q^j + x_j U_h^j H_z^j)/U_\ell^j)$, equal consumer i's additional utility of the SSB, measured by her marginal willingness-to-pay for the SSB. The latter marginal willingness-to-pay also composes of two opposing effects. The term $-U_x^i/U_\ell^i > 0$ is consumer i's additional utility from the increased SSB consumption, whereas $-qU_h^i H_z^i/U_\ell^i < 0$ reflects consumer i's additional health costs from a higher sugar intake. Observe that the allocation rule (12) holds for every consumer, so that the marginal willingness-to-pay for the SSB has to be equal across consumers.

4 Lindahl market for sugar content

Next we consider various market concepts and investigate their potential to decentralize the efficient allocation by prices. In this section we begin with a full set of perfectly competitive markets. Since the sugar content of SSB is a public good, the concept of perfectly competitive markets for private goods must be appropriately adjusted to include the public good. In an idealized way this is done by introducing a Lindahl market (see Foley 1970, Roberts 1974) with a 'personalized' price for the sugar content. To be more specific, labor is supplied by consumers and demanded by the sugar sector and SSB sectors at price p_ℓ. Sugar is sold by the sugar sector to the SSB sector at price p_z. SSB is traded between the SSB sector and consumers at price p_x. Finally, the sugar content is supplied by the SSB sector at price p_q, whereas consumer i demands the sugar content at the personalized (Lindahl) price p_{qi}. Due to the public good property of sugar content, consumer i's personalized price p_{qi} deviates from the producer price p_q.

The sugar sector and the SSB sector maximize their profit, whereas consumers maximize their utility. The sugar sector chooses labor input and output in order to maximize its profit subject to the sugar technology. The associated profit maximization problem is

$$\max_{\ell_z^d, z^s} \pi_z = p_z z^s - p_\ell \ell_z^d \quad \text{s.t.} \quad (1). \tag{13}$$

The SSB sector sets its inputs and outputs in order to maximize its profit subject to the SSB production technology and the definition of the supplied sugar content, i.e

$$\max_{\ell_x^d, z^d, x^s, q^s} \pi_x = p_x x^s + p_q q^s - p_\ell \ell_x^d - p_z z^d \text{ s.t. } (2),(3). \tag{14}$$

Finally, consumer i chooses consumption and labor supply in order to maximize utility subject to her budget constraint. The maximization problem reads

$$\max_{x_i^d, q_i^d, \ell_i^s} (4) \quad \text{s.t.} \quad p_\ell \ell_i^s + \psi_i \Pi \geq p_x x_i^d + p_{qi} q_i^d. \tag{15}$$

Consumer i's income consists of labor income and profit income, where $\Pi := \pi_z + \pi_x$ is total profit in the economy and $\psi_i \in [0, 1]$ with $\sum_{j=1}^n \psi_j = 1$ represents consumer i's share of total profit. Consumer i takes the profit income as given and uses her total income to finance expenditures for the SSB and its sugar content. The market economy is closed by the market clearing conditions (5)–(7) and equation (8) which requires the supplied sugar content to be equal to each consumer's demanded sugar content.

The Lindahl market for sugar content attains an equilibrium if the consumers' personalized prices for demanded sugar content sum up to the price which the SSB producer receives for the supplied sugar content. Formally, we obtain the equilibrium condition in column 2, row 4 of Table 1. Solving the maximization problems (13), (14) and (15) yields the remaining conditions in column 2 of Table 1. The Lagrangeans to these problems and the full set of first-order conditions are given in the appendix. Eliminating the Lagrange multipliers from the full set of conditions and using $z^s = z^d =: z$, $q^s = q_i^d =: q$, $x^s = \sum_{j=1}^n x_j^d =: x$ and $x_i^d =: x_i$ implies the conditions in column 2, rows 1-3, 5 and 6.

As the market equilibrium determines only relative prices, we choose labor as numéraire and set $p_\ell = 1$. By comparing columns 1 and 2 of Table 1 we then immediately obtain

Proposition 1. (*Efficient Lindahl market for sugar content*)
If $p_\ell = \lambda_\ell = 1$, $p_z = \lambda_z$, $p_x = \lambda_x$, $p_q = \lambda_q$ and $p_{qi} = \lambda_{qi}$ for all i, then the competitive equilibrium with a Lindahl market for sugar content is efficient.

In the competitive equilibrium with a Lindahl market for sugar content characterized by Proposition 1 the prices of sugar and SSB output equal the corresponding shadow prices. Moreover, the price of the supplied sugar content equals the shadow price of the supplied sugar content and each consumer pays the shadow price for the sugar content which it demands. Hence, all shadow prices are reflected by the competitive price system and the market equilibrium becomes efficient without any intervention.

However, the above characterized market solution requires personalized Lindahl prices p_{qi} for each consumer i and it is well known that such consumer-specific prices are unrealistic. They would only be feasible if the consumers have an incentive to truthfully reveal their preferences for sugar content embodied in the demanded SSBs. But if the sugar content is a public good (public bad), consumers have an incentive to understate (overstate) their willingness-to-pay (willingness-to-accept) for sugar content. It follows that personalized prices for sugar content do not emerge in the real world and we cannot expect that perfectly competitive markets encompass a Lindahl market for sugar content.

5 Indirect market for sugar content

As an alternative to the Lindahl market, in this section we consider an indirect market for sugar content with a hedonic price function. The concept of indirect markets goes back to Lancaster (1966) and Rosen (1974). In our context, the sugar content is an 'intrinsic' attribute of the SSB and we may assume that producers and consumers are aware that changes in this attribute have an impact on the SSB price. More precisely, imagine a Walrasian auctioneer who announces the price of the SSB as a function $P^x(q)$ of the sugar content q supplied or demanded by the market participants. In determining their decisions, the SSB producer and the consumers take this functional relationship into consideration. A market equilibrium is attained if the price function is such that the market for the SSB is cleared and the sugar content demanded by consumers equals the sugar content supplied by the SSB producer.

With such an indirect market for sugar content, the profit maximization problem of the sugar sector remains unchanged and is still given by equation (13). The profit maximization problem of the SSB production sector turns into

$$\max_{\ell_x^d, z^d, x^s, q^s} \pi_x = P^x(q^s)x^s - p_\ell \ell_x^d - p_z z^d \text{ s.t. (2),(3)} \tag{16}$$

Comparing (14) and (16) reveals that in (16) the SSB producer's revenues for the supplied sugar content, $p_q q^s$, are replaced by an indirect payment for the supplied sugar content contained in the SSB's price $P^x(q^s)$. Consumer i's maximization problem changes to

$$\max_{x_i^d, q_i^d, \ell_i^s} \text{(4)} \quad \text{s.t.} \quad p_\ell \ell_i^s + \psi_i \Pi \geq P^x(q_i^d)x_i^d. \tag{17}$$

Similarly to the SSB sector's maximization problem, also consumer i does no longer face a direct payment for the demanded sugar content, $p_{qi}q_i^d$, but is only indirectly charged for its sugar content according to the price function $P^x(q_i^d)$.

Prices p_ℓ, p_z, the price function $P^x(\cdot)$ and the allocation resulting from solving (13), (16) and (17) constitute a competitive equilibrium with an indirect market for sugar content if the constraints (5)-(8) are satisfied as equalities. The full set of first-order conditions to the maximization problems (13), (16) and (17) are derived in the appendix. Eliminating the respective Lagrange multipliers from the first-order conditions and denoting again $z^s = z^d =: z$, $q^s = q_i^d =: q$, $x^s = \sum_{j=1}^n x_j^d =: x$ and $x_i^d =: x_i$ for all i as well as $P^x(q^s) = P^x(q_i^d) =: P^x$ and $P_q^x(q^s) = P_q^x(q_i^d) =: P_q^x$ for all i, we obtain the equilibrium conditions listed in column 3 of Table 1. Comparing column 1 and column 3 of Table 1, it is straightforward to prove the following proposition.

Proposition 2. (*Efficient indirect market for sugar content*)
If $p_\ell = \lambda_\ell = 1$, $p_z = \lambda_z$, $P^x = \lambda_x$, $P_q^x = \lambda_q/x$ and $P_q^x = \lambda_{qi}/x_i$, where x and x_i are evaluated at the efficient allocation, then the competitive equilibrium with an indirect market for sugar content is efficient.

According to Proposition 2, the competitive equilibrium with an indirect market for sugar content is efficient if the markt prices of labor, sugar and the SSB, i.e. p_ℓ, p_z and P^x, equal the respective shadow prices, i.e. $\lambda_\ell = 1$, λ_z and λ_x, and if the implicit price of the sugar content, P_q^x, equals both the SSB-weighted shadow price of the supplied sugar content, i.e. λ_q/x, and all consumers' SSB-weighted shadow prices of demanded sugar content, i.e. λ_{qi}/x_i for all i. Under such a price system, all consumers and producers receive the right price signals in order to implement the efficient allocation.

Unfortunately, the efficiency result of the indirect market approach in Proposition 2 is more limited than it appears at first glance. The decisive point is that the implicit price of the sugar content, P_q^x, has to be the same for all consumers and the SSB producer and, thus, efficiency is obtained only if $P_q^x = \lambda_q/x = \lambda_{qi}/x_i$ for all i. Dividing this condition by λ_x and using the conditions in column 1, rows 5 and 6, yields

$$\frac{\lambda_q}{x\lambda_x} = \frac{\lambda_{qi}}{x_i\lambda_x} = \frac{\mathrm{MRS}^i}{x_i} \quad \text{for all } i, \tag{18}$$

where $\mathrm{MRS}^i := (U_q^i + x_i U_h^i H_z^i)/(U_x^i + qU_h^i H_z^i)$ is consumer i's marginal rate of substitution between sugar content and the SSB. Equation (18) is a necessary condition for the efficiency of a competitive equilibrium with an indirect market for sugar content. It requires that all consumers have the same SSB-weighted marginal rate of substitution between sugar content and the SSB, i.e. MRS^i/x_i has to be the same for all consumers. This requirement is rather specific and puts severe limitations on the efficiency of the indirect market for sugar content. It holds for identical consumers, indeed, but for the realistic case of heterogeneous consumers it is usually not satisfied.

In order to illustrate the restrictiveness of (18), suppose health is inversely related to total sugar intake, i.e. $h_i = H^i(z_i) = 1/z_i$ with $z_i = x_i^d q_i^d$, and the utility of consumer i is specified by the Cobb-Douglas function $U^i\left(x_i^d, q_i^d, \ell_i^s, h_i\right) = \left(x_i^d\right)^{\alpha_i}\left(q_i^d\right)^{\nu_i}(1-\ell_i^s)^{\kappa_i}(h_i)^{\varepsilon_i}$ with $\alpha_i, \nu_i, \kappa_i, \varepsilon_i > 0$ and $\alpha_i + \nu_i + \kappa_i + \varepsilon_i \leq 1$. Under these specifications, (18) turns into

$$\frac{\lambda_q}{x\lambda_x} = \frac{\lambda_{qi}}{x_i\lambda_x} = \frac{\mathrm{MRS}^i}{x_i} = \frac{\nu_i - \varepsilon_i}{(\alpha_i - \varepsilon_i)q} \quad \text{for all } i, \tag{19}$$

where we have used $q_i^d = q$ from the Pareto optimum. It is obvious that (19) can only be satisfied, if $(\nu_i - \varepsilon_i)/(\alpha_i - \varepsilon_i) = (\nu_j - \varepsilon_j)/(\alpha_j - \varepsilon_j)$ for all i, j and $i \neq j$, which is a generic case. In all other cases we have $(\nu_i - \varepsilon_i)/(\alpha_i - \varepsilon_i) \neq (\nu_j - \varepsilon_j)/(\alpha_j - \varepsilon_j)$ for at least one $i \neq j$ and, thus, $\lambda_q/x = \lambda_{qi}/x_i$ cannot be satisfied for each consumer i as required in equation (18). The efficiency result identified in Proposition 2 therefore holds only in rather specific cases and the competitive economy with an indirect market for sugar content cannot be expected to yield the efficient allocation in general.

The intuition for the limited validity of efficiency of an indirect market for sugar content becomes obvious if we again consider the above specification of the health and utility functions and divide column 3, row 6 by column 3, row 5 of Table 1 to obtain

$$\frac{P_q^x}{P^x} = \frac{\mathrm{MRS}^i}{x_i} = \frac{\nu_i - \varepsilon_i}{(\alpha_i - \varepsilon_i)q}. \tag{20}$$

If $(\nu_i - \varepsilon_i)/(\alpha_i - \varepsilon_i) \neq (\nu_j - \varepsilon_j)/(\alpha_j - \varepsilon_j)$ for at least one $i \neq j$, then the SSB-weighted marginal rates of substitution are different between consumers, and condition (20) is violated. Hence, a competitive economy with an indirect market for sugar content fails to attain an equilibrium at all and, thus, cannot implement the efficient allocation. The intuition is that the price function $P^x(q)$ does not allow for the necessary flexibility to reflect the willingness-to-pay of different consumers for the public good sugar content.

6 Missing market for sugar content

The analyses in the previous sections have shown that a Lindahl market for sugar content does not work because consumers do not have an incentive to truthfully reveal their preferences and that, in general, an indirect market for sugar content does not work either, because the price function cannot reflect heterogeneous consumer preferences for sugar content. In addition to these arguments, there is a further reason why we cannot expect Lindahl and indirect markets to provide the right price signals for sugar content. Even if consumers would truthfully reveal their preferences and even if the price function would reflect preference heterogeneity, the high health costs of sugar consumption indicate that sugar content is likely to be a public bad instead of a public good. In our formal model, if consumer i's marginal health costs $-x_i U_h^i H_z^i / U_\ell^i < 0$ of sugar content overcompensate the marginal utility $-U_q^i / U_\ell^i > 0$ of sugar content, then the net effect of sugar content on consumer i, measured by the marginal-willingness-to-pay $-(U_q^i + x_i U_h^i H_z^i)/U_\ell^i < 0$, is negative. If this is true for the majority of consumers, then the term $-\sum_{j=1}^{n}(U_q^j + x_j U_h^j H_z^j)/U_\ell^j$ is likely to be negative as well, implying that sugar content chosen by the SSB producer is a public bad. Due to rows 4 and 6 in column 2 or 3 of Table 1, the consequence is a negative sugar content price $p_q < 0$ (in the Lindahl market) or $P_q^x < 0$ (in the indirect market). But a negative price means that the SSB producer has to pay for the sugar content that it supplies. When analyzing the general equilibrium in a market economy, Arrow and Debreu (1954) and Debreu (1954) have assumed that unwanted joined outputs with negative prices can be disposed of at a zero price (free disposal). Applying this idea to our framework means that the unwanted product characteristic sugar content will not be traded and, thus, not be prized in the market economy, which is supported by the evidence on uniform product pricing referred to in the Introduction.

Accordingly, a missing market for sugar content is reflected by our formal model, if we focus on the case with $-\sum_{j=1}^{n}(U_q^j + x_j U_h^j H_z^j)/U_\ell^j < 0$ and set in column 2 of Table 1 the prices $p_q \equiv p_{qi} \equiv 0$ for all i or, alternatively, in column 3 of Table 1 the price $P_q^x \equiv 0$. Both columns then coincide. Moreover, by choosing input z^d and output x^s, the SSB producer still determines the sugar content q^s, but the consumers do not receive a price signal for the sugar content and, thus, they take the sugar content as exogenously given. Hence, in columns 2 and 3 of Table 1 not only the condition in row 4 vanishes, but also does the condition in row 6. Comparing the remaining equilibrium conditions with the efficiency conditions in column 1 of Table 1 immediately proves the following result.

Proposition 3. (*Inefficient missing market for sugar content*)
Suppose there is no market for sugar content, i.e. $p_q \equiv p_{qi} \equiv P_q^x \equiv 0$ *for all* i*. Then the competitive market equilibrium is inefficient.*

In order to attain efficiency, the price that the SSB producer receives for the supplied sugar content should reflect the corresponding shadow price λ_q in the Pareto optimum, which equals the sum of shadow prices λ_{qi} for demanded sugar content. But if the market for sugar content is missing, the price for sugar content is zero and the producer does not receive the right price signal when setting the sugar content in its SSB production. This also distorts the input and output decisions of the sugar and SSB sectors as well as the labor supply decision of consumers and renders the market allocation inefficient.

We may further elaborate on the market failure identified in Proposition 3 by deriving allocation rules for sugar and the SSB, analogous to the allocation rules (11) and (12) characterizing the Pareto optimum. For this purpose, set the prices of sugar content equal to zero in column 2 or column 3 of Table 1. Inserting the condition in row 1 into the condition in row 3 and the condition in row 5 into the condition in row 2, we obtain

$$\frac{1}{Z_\ell} = \frac{X_z}{X_\ell}, \tag{21}$$

$$\frac{1}{X_\ell} = -\frac{U_x^i + qU_h^i H_z^i}{U_\ell^i}, \qquad \text{for all } i. \tag{22}$$

Comparing these allocation rules for the economy without a market of sugar content with the allocation rules (11) and (12) of the Pareto efficient allocation, it becomes visible that – due to the missing price signal for sugar content – the rules in the market economy do not reflect the effect of the sugar content on the consumers' utility, i.e. the term $-\sum_{j=1}^{n}(U_q^j + x_j U_h^j H_z^j)/U_\ell^j$ is missing in (21) and (22). As consequence, if the effect of sugar content on the consumers' utility is negative, i.e. $-\sum_{j=1}^{n}(U_q^j + x_j U_h^j H_z^j)/U_\ell^j < 0$, we expect that, in the competitive economy without a market for sugar content, the SSB price p_x is inefficiently low – since it does not reflect the negative effect of the SSB's sugar content on consumers – and, thus, the sugar content q is inefficiently high.

To prove this assertion and to investigate the inefficiency of the other variables in the market economy, tractability requires to introduce some mild simplifications regarding technologies and preferences. Using the additional notation $\ell_i := \ell_i^s$, the appendix proves

Proposition 4. (*Allocation in economy with missing market for sugar content*)
Suppose there is no market for sugar content, i.e. $p_q \equiv p_{qi} \equiv P_q^x \equiv 0$ *for all* i, *and* $-\sum_{j=1}^{n}(U_q^j + x_i U_h^j H_z^j)/U_\ell^j < 0$ *in the Pareto optimum. Assume the production technologies* Z *and* X *in* (1) *and* (2) *are linear homogenous. Then in the competitive market equilibrium*

(*i*) *the sugar content* q *is inefficiently high, the SSB price* p_x *is inefficiently low and the sugar prize* p_z *is efficient.*

Assume additionally that consumer i*'s utility function is specified by the CES function* $U^i(x_i^d, q_i^d, \ell_i^s, h_i) = \left[\alpha_i (x_i^d)^{-\rho_i} + \nu_i (q_i^d)^{-\rho_i} + \kappa_i (1-\ell_i^s)^{-\rho_i} + \varepsilon_i (h_i)^{-\rho_i}\right]^{-\sigma_i/\rho_i}$ *with* $\alpha_i, \nu_i, \kappa_i, \varepsilon_i > 0$, $\alpha_i + \nu_i + \kappa_i + \varepsilon_i \leq 1$, $\sigma_i \leq 1$ *and* $\rho_i \geq -1$, *whereas the health function is* $H(z_i) = 1/z_i$. *Then in the competitive market economy*

(*ii*) *with* $\rho_i = 0$ *for all* i (*Cobb-Douglas preferences*), *consumer* i*'s labor supply* ℓ_i *is efficient, whereas consumer* i*'s SSB consumption* x_i *as well as the SSB output* x *and sugar input* z *are inefficiently high,*

(*iii*) *with* $\rho_i = 1$ *for all* i, *consumer* i*'s labor supply* ℓ_i *is inefficiently low, whereas consumer* i*'s SSB consumption* x_i *as well as the SSB output* x *and sugar input* z *may be inefficiently low or high,*[7]

(*iv*) *with* $\rho_i = -1$ *for all* i, *consumer* i*'s labor supply* ℓ_i *and consumer* i*'s SSB consumption* x_i *as well as the SSB output* x *are inefficiently low, whereas sugar input* z *may be inefficiently low or high.*

[7]Part (iii) of the proposition can be generalized to all model specifications with $\rho_i > 0$.

Part (i) of Proposition 4 confirms our intuition that in the market economy the SSB price p_x is inefficiently low, while the sugar content q is inefficiently high. The reason is that the sugar content reduces utility of consumers, indeed, but is not reflected by the price system. The sugar price p_z is solely technology-driven and thereby efficient, since Proposition 4 is confined to linear homogenous technologies. While the efficiency of p_z will certainly change if we deviate from the assumption of linear homogenous technologies, intuitively the results of an inefficiently low SSB price and an inefficiently high sugar content are expected to hold also under more general assumptions regarding production technologies.

At first glance, the inefficiency of the sugar content $q = z/x$ may suggest that the sugar input z is inefficiently high and the SSB output x is inefficiently low. However, parts (ii)-(iv) show that this is not true in general. The reason is that, due to the general equilibrium nature of our model, the efficiency properties of sugar input z and SSB output x largely depend on the consumers' consumption and labor supply decisions. To see this, remember first that – according to part (i) of Proposition 4 – the transition from the efficient allocation to the market allocation is accompanied by a drop in the SSB price p_x and an increase in the sugar content q. The fall in the SSB price p_x triggers a substitution effect as well as an income effect (real income goes up), while the increase in the sugar content q causes a further substitution effect on the consumers' decision. The interaction of these substitution and income effects is rather complicated and renders the efficiency properties of sugar and the SSB in the market economy ambiguous.

To illustrate, we start with the case of complements (e.g. $\rho_i = 1$). Then all three effects – the substitution and income effects of the fall in p_x as well as the substitution effect of the increase in q – increase SSB demand x_i directly and reduce SSB demand x_i indirectly by an increase in health demand h_i, while they lower labor supply ℓ_i due to an increase in leisure demand $1 - \ell_i$. Overall, consumer i's labor supply ℓ_i unambiguously falls and becomes inefficiently low in the market equilibrium, whereas consumer i's SSB demand x_i as well as total SSB output x and the sugar input z may increase or decrease and, thus, may be inefficiently low or high in the market equilibrium, as stated in part (iii) of Proposition 4. If we turn to the case of substitutes, the direct substitution effect of p_x on the SSB demand x_i as well as the income effect of p_x remain the same, but all other effects are reversed. As consequence, under Cobb-Douglas preferences ($\rho_i = 0$), all substitution and income effects with respect to labor supply ℓ_i just compensate each other and labor supply in the market equilibrium becomes efficient. It follows directly from the budget constraint $\ell_i = p_x x_i$ and the fall in the SSB price p_x that the reversed effects lead to an increase in consumer i's SSB demand x_i as well as in SSB output x and in sugar use z, so in the market equilibrium these variables are inefficiently high, as shown in part (ii) of Proposition 4. On the other hand, if we go a step further and consider perfect substitutes ($\rho_i = -1$), the negative direct substitution effect of the increase in q on SSB demand x_i becomes so large that SSB demand x_i and SSB output x are inefficiently low in the market equilibrium, as stated in part (iv) of Proposition 4. The effect on sugar input z is ambiguous, since sugar content q is too high, while SSB output x is too low.

7 Regulated markets

Having shown that the laissez-faire market economy does not provide the right price signals for implementing the efficient allocation, we now turn to the question how the government may regulate the market participants in order to correct the market failure due to missing markets for sugar content. We focus on the appropriate tax policy and consider tax instruments levied on sugar demand and supply, the SSB demand and supply, the supplied sugar content as well as the supplied sugar content per unit of the SSB, and investigate which combination of these taxes is able to restore efficiency. In contrast to Proposition 4, we return to general technologies and preferences.

Denoting the tax rate levied on each unit of sugar supply by τ_z^s, the profit maximization problem of the sugar sector changes to

$$\max_{\ell_z^d, z^s} \pi_z = (p_z - \tau_z^s) z^s - p_\ell \ell_z^d \quad \text{s.t.} \quad (1). \tag{23}$$

In the absence of any market price for sugar content, the profit maximization problem of the SSB sector now reads

$$\max_{\ell_x^d, z^d, x^s, q^s} \pi_x = (p_x - \tau_x^s) x^s - \tau_q q^s - p_\ell \ell_x^d - (p_z + \tau_z^d) z^d - \tau_{xq} x^s q^s \text{ s.t. } (2),(3). \tag{24}$$

The SSB producer has to pay a SSB tax at rate τ_x^s, a sugar content tax at rate τ_q, a sugar input tax at rate τ_z^d and sugar content tax per unit of the SSB at rate τ_{xq}. Turning to the household sector, consumer i's utility maximization can be written as

$$\max_{x_i^d, q_i^d, \ell_i^s} \ (4) \quad \text{s.t.} \quad p_\ell \ell_i^s + \psi_i \Pi + \zeta_i T \geq (p_x + \tau_x^d) x_i^d. \tag{25}$$

Consumer i's expenditures now comprise the SSB tax payments $\tau_x^d x_i^d$. Her income additionally contains a lump-sum transfer $\zeta_i T$ from the government, where $T = \tau_z^s z^s + \tau_z^d z^d + \tau_x^s x^s + \tau_x^d \sum_{j=1}^n x_j^d + \tau_q q^s + \tau_{xq} x^s q^s$ is total tax revenue and $\zeta_i \in [0, 1]$ with $\sum_{j=1}^n \zeta_j = 1$ represents the tax revenue share received by consumer i. The consumer takes the lump-sum transfer as given. Because consumer i does not receive a price signal for sugar content, we assume that she takes as given the sugar content q_i^d of the demanded SSBs. The market economy is again closed by the clearing conditions (5)–(7).

The full set of first-order conditions to the maximization problems (23)–(25) are given in the appendix. In the regulated market equilibrium we again obtain $z^s = z^d = z$, $q^s = q_i^d = q$, $x^s = \sum_{j=1}^n x_j^d = x$ and $x_i^d = x_i$ for all i. Eliminating the Lagrange multipliers from the first-order conditions, in the appendix we derive the market equilibrium conditions listed in column 4 of Table 1. Notice that the regulated market does not encompass conditions comparable to the conditions in rows 4 and 6 in column 2 or 3 in Table 1, since consumers do not receive a price signal for sugar content and take as given the sugar content determined by the SSB producer. By comparing column 4 with the efficient solution in column 1 of Table 1, it is straightforward to prove the following result.

Proposition 5. (*Optimal Regulation*)
Suppose there is no market for sugar content, i.e. $p_q \equiv p_{qi} \equiv P_q^x \equiv 0$ *for all* i. *If* $p_\ell = \lambda_\ell = 1$, $p_x = \lambda_x - \tau_x^d$, $p_z = \lambda_z + \tau_z^s$ *and*

$$\tau_z^s + \tau_z^d + \frac{1}{x}\tau_q + \tau_{xq} = -\frac{1}{x}\sum_{j=1}^n \lambda_{qj}, \qquad -\tau_x^s - \tau_x^d + \frac{q}{x}\tau_q = -\frac{q}{x}\sum_{j=1}^n \lambda_{qj}, \tag{26}$$

then the competitive equilibrium of the regulated market economy is efficient.

The policy identified in Proposition 5 can equivalently be obtained if we derive the allocation rules for sugar and the SSB in the regulated market economy. Using $p_\ell = 1$ in the conditions listed in column 4 of Table 1, it is straightforward to show that

$$\frac{X_z}{X_\ell} = \frac{1}{Z_\ell} + \tau_z^s + \tau_z^d + \frac{1}{x}\tau_q + \tau_{xq}, \tag{27}$$

$$\frac{1}{X_\ell} = -\frac{U_x^i + qV_h^i H_z^i}{U_\ell^i} - \tau_x^s - \tau_x^d + \frac{q}{x}\tau_q, \qquad \text{for all } i. \tag{28}$$

Comparing these equations with the allocation rules (11) and (12) of the Pareto optimum reveals that the market equilibrium is efficient if

$$\tau_z^s + \tau_z^d + \frac{1}{x}\tau_q + \tau_{xq} = \frac{1}{x}\sum_{j=1}^{n} \frac{U_q^j + x_j V_h^j H_z^j}{U_\ell^j}, \quad -\tau_x^s - \tau_x^d + \frac{q}{x}\tau_q = \frac{q}{x}\sum_{j=1}^{n} \frac{U_q^j + x_j V_h^j H_z^j}{U_\ell^j}, \tag{29}$$

where all functions and variables are evaluated at the Pareto optimal allocation. Recalling from column 1, row 6 of Table 1 that $\lambda_{qj} = (U_q^j + x_j V_h^j H_z^j)/U_\ell^j$, we infer that the tax policies given in equation (26) and equation (29) are perfectly equivalent.

Proposition 5 offers ample opportunities for correcting the market failure caused by missing markets for sugar content. The most obvious policy would be a direct tax on the sugar content chosen by the SSB producer. To see this, set all taxes but τ_q equal to zero. The conditions in (26) or, equivalently, (29) then simplify to

$$\tau_q = \sum_{j=1}^{n} \frac{U_q^j + x_j V_h^j H_z^j}{U_\ell^j} > 0. \tag{30}$$

Hence, the government may correct the market failure by directly taxing the SSB producer's choice of the sugar content, where the tax rate in (30) reflects the consumers' net marginal costs of the sugar content. With this tax option the tax rate on the SSB producer's sugar content simply replaces the missing market price of the sugar content.

A drawback of the direct tax policy identified in (30) is that the government has to tax the SSB producer's product design, for example, by a one-time charge on the sugar content in the formulation of the SSB. While theoretical appealing, such a tax seems to be hardly implementable in practice. An alternative would be to tax the sugar content per unit of the SSB by the tax τ_{xq}. Such a tax is implemented, for example, in the UK. However, in our model, this tax alone is not sufficient to restore efficiency, since the second condition in (26) and (29) is violated, when setting all other tax rates equal to zero. Instead, the government has to regulate also the SSB quantity, either on the demand or the supply side. Formally, (26) and (29) are satisfied if $\tau_z^s = \tau_z^d = \tau_q = 0$ and

$$\tau_{xq} = \frac{1}{x}\sum_{j=1}^{n} \frac{U_q^j + x_j V_h^j H_z^j}{U_\ell^j} > 0, \qquad \tau_x^s + \tau_x^d = -\frac{q}{x}\sum_{j=1}^{n} \frac{U_q^j + x_j V_h^j H_z^j}{U_\ell^j} < 0. \tag{31}$$

The government has to supplement the tax on the sugar content per unit of SSB by a subsidy on each unit of the SSB. The policy which we observe, for instance, in the UK is therefore not sufficient to correct the market failure due to the missing market of sugar content. The intuition is that the tax base of the tax τ_{xq} equals xq, so τ_{xq} is not solely targeted at the sugar content q, but also at the SSB output x. Thus, the SSB output is falsely taxed and this distortion has to be corrected by a subsidy on the SSB output. Notice that the subsidy can either be paid to the SSB producer, via τ_x^s, or to consumers, via τ_x^d, or it can even be divided between the producer and consumers. If it is solely paid to the producer, i.e. $\tau_x^d = 0$, it has the appealing property that the SSB producer's net tax payments per unit of the SSB are zero, i.e. $\tau_x^s + q\tau_{xq} = 0$. This revenue neutrality property may reduce the opposition that many SSB producers have against SSB taxation. According to our model, the producers nevertheless receive the right incentive to choose the efficient sugar content of their SSBs, since choosing a relatively high sugar content becomes more expensive by the tax policy (τ_{xq}, τ_x^s).

Compared to the direct tax solution in (30), the tax policy identified in (31) has the advantage that it is not a one-time charge on the formulation of the SSB. Nevertheless, the government still has to know the sugar content when it applies the tax τ_{xq}. An even more convenient tax solution would be to combine the subsidy on the SSB output by a tax on the sugar input in the SSB production. Formally, equations (26) and (29) are also satisfied if the government sets $\tau_q = \tau_{xq} = 0$ and

$$\tau_z^s + \tau_z^d = \frac{1}{x} \sum_{j=1}^{n} \frac{U_q^j + x_j V_h^j H_z^j}{U_\ell^j} > 0, \qquad \tau_x^s + \tau_x^d = -\frac{q}{x} \sum_{j=1}^{n} \frac{U_q^j + x_j V_h^j H_z^j}{U_\ell^j} < 0. \tag{32}$$

Hence, the government may restore efficiency of the market equilibrium by taxing sugar input and subsidizing the SSB output. The intuition is similar to the intuition of the policy identified in (31). The tax on sugar is not solely targeted at the sugar content q, but implicitly also taxes the SSB output x. The latter distortion has to be corrected by the subsidy on the SSB output x. Such a policy seems to be particularly promising, because it does not require any information on the sugar content of the SSB. Notice also that both the tax on sugar and the subsidy on the SSB quantity can be imposed on the demand or the supply side. If they are both imposed solely on the SSB producer, i.e. $\tau_z^s = \tau_x^d = 0$, we obtain again the useful property that the SSB producer's tax payments per unit of the SSB vanishes, i.e. $\tau_x^s + q\tau_z^d = 0$, and this may reduce the resistance of SSB producers against the regulation of their production. The revenue neutrality property together with the fact that no information on the sugar content is needed, is the reason why we think that the sugar-tax-SSB-subsidy solution may be best implementable in practice.

8 Conclusion

The present paper provides a new (non-paternalistic) foundation of SSB taxation. Within a general equilibrium model in which sugar is produced and used as input in SSB production, the SSB is consumed by heterogeneous individuals and the sugar embodied in the SSB causes health costs. The sugar content of the SSB determined by SSB producers turns out to be a public good/bad. The preceding analysis reexamined the performance of markets to allocate the sugar content per unit of SSB. A Lindahl market for sugar content would implement the efficient allocation, indeed, but personalized Lindahl prices for sugar content will not emerge in reality because, on the one side, consumers do not have incentives to truthfully reveal their preferences and, on the other side, the Lindahl prices would be negative in the realistic case in which the sugar content overall harms consumers. Indirect markets for sugar content suffer from the same negative price problem and, in addition, implement the efficient allocation only in the unrealistic case in which consumers have the identical marginal willingness-to-pay for the sugar content. The efficient level of the SSB's sugar content is therefore unlikely to brought about by market forces. Due to this market failure, corrective taxation is need. It is shown that a sugar content tax per volume unit of the SSB or, equivalently, a tax on the sugar input in the production of the SSB, both combined with a revenue-neutral subsidy on each unit of the SSB, are promising policy options to restore efficiency. If all policy instruments are applied to SSB producers, the proposed tax-subsidy scheme has the charm of being tax revenue-neutral, so the SSB producers net tax payments are zero.

Of course, our stylized general equilibrium model abstracts from many important aspects and can therefore extended in several ways. One obvious extension, already mentioned in the Introduction, is to consider more than one SSB. We have already investigated such a model extension with several SSBs and shown that all our main results remain qualitatively true. However, with several SSBs the optimal policy needs SSB-specific tax and subsidy rates which may be difficult to implement in practice due to high administrative costs. Instead, policymakers may prefer to levy uniform taxes and subsidies. Such a second-best policy leads to welfare losses compared to the first-best allocation and points to an applied analysis within a computable general equilibrium model in order to quantify the welfare losses. A second obvious extension would be to deviate from the assumption of perfect competition usually made in general equilibrium models and to consider several SSB producers who act under conditions of imperfect competition. Cremer et al. (2019) already investigate such a framework, but under the assumption that markets for sugar contents work smoothly. The analysis of missing markets for sugar content under imperfect competition as well as the quantification of welfare losses under uniform sugar content taxation in the presence of several SSBs are interesting, but beyond the scope of the present paper and, thus, left for future research.

Appendix

First-order conditions for the Pareto-efficient allocation. The Lagrangean of the Pareto optimization problem characterized in Section 3 reads

$$\begin{aligned}\mathcal{L} &= U[x_i^d, q_i^d, \ell_i^s, H^i(x_i^d q_i^d)] + \sum_{j=1}^{n} \phi_{cj}\Big[U[x_j^d, q_j^d, \ell_j^s, H(x_j^d q_j^d)] - \bar{u}_j\Big] \\ &\quad + \phi_z\Big[Z(\ell_z^d) - z^s\Big] + \phi_x\Big[X(\ell_x^d, z^d) - x^s\Big] + \lambda_q\left(\frac{z^d}{x^s} - q^s\right) \\ &\quad + \lambda_z\Big(z^s - z^d\Big) + \lambda_x\left(x^s - \sum_{j=1}^{n} x_j^d\right) + \sum_{j=1}^{n}\lambda_{qj}\Big(q^s - q_j^d\Big) + \lambda_\ell\left(\sum_{j=1}^{n}\ell_j^s - \ell_x^d - \ell_z^d\right),\end{aligned}$$

where ϕ_{cj} for all $j \neq i$ are Lagrange multipliers associated with the constraint (9), ϕ_z, ϕ_x and λ_q represent Lagrange multipliers associated with the technologies (1)–(3) and λ_z, λ_x, λ_{qj} for all j and λ_ℓ are the Lagrange multipliers for the constraints in (5)–(8).

Differentiating the Lagrangean gives the first-order conditions

$$\mathcal{L}_{\ell_z^d} = \phi_z Z_\ell - \lambda_\ell = 0. \tag{33a}$$

$$\mathcal{L}_{z^s} = -\phi_z + \lambda_z = 0, \tag{33b}$$

$$\mathcal{L}_{\ell_x^d} = \phi_x X_\ell - \lambda_\ell = 0, \tag{33c}$$

$$\mathcal{L}_{z^d} = \phi_x X_z + \lambda_q \frac{1}{x^s} - \lambda_z = 0, \tag{33d}$$

$$\mathcal{L}_{x^s} = -\phi_x - \lambda_q \frac{z^d}{(x^s)^2} + \lambda_x = 0, \tag{33e}$$

$$\mathcal{L}_{q^s} = -\lambda_q + \sum_{j=1}^{n} \lambda_{qj} = 0, \tag{33f}$$

$$\mathcal{L}_{x_i^d} = U_x^i + q_i^d U_h^i H_z^i - \lambda_x = 0, \tag{33g}$$

$$\mathcal{L}_{q_i^d} = U_q^i + x_i^d U_h^i H_z^i - \lambda_{qi} = 0, \tag{33h}$$

$$\mathcal{L}_{\ell_i^s} = U_\ell^i + \lambda_\ell = 0, \tag{33i}$$

$$\mathcal{L}_{x_j^d} = \phi_{cj}\Big[U_x^j + q_j^d U_h^j H_z^j\Big] - \lambda_x = 0, \qquad \text{for all } j \neq i, \tag{33j}$$

$$\mathcal{L}_{q_j^d} = \phi_{cj}\Big[U_q^j + x_j^d U_h^j H_z^j\Big] - \lambda_{qj} = 0, \qquad \text{for all } j \neq i, \tag{33k}$$

$$\mathcal{L}_{\ell_j^s} = \phi_{cj} U_\ell^j + \lambda_\ell = 0, \qquad \text{for all } j \neq i. \tag{33l}$$

Eliminating ϕ_z from (33a) and (33b) gives the condition in column 1, row 1 of Table 1. Solving (33c) for ϕ_x and inserting into (33e), taking into account $z^d/(x^s)^2 = q/x$ yields the condition in column 1, row 2 of Table 1. The condition in column 1, row 3 of Table 1 is obtained by inserting ϕ_x from (33c) into (33d) and using $x^s = x$. The condition in column 1, row 4 of Table 1 follows immediately from (33f). Finally, the conditions in column 1, rows 5 and 6 are obtained by taking into account $q_i^d = q_j^d = q$, $x_i^d = x_i$ as well as $x_j^d = x_j$ and dividing (33g) and (33h) by (33i) or, analogously, (33j) and (33k) by (33l).

Market equilibrium conditions with Lindahl market for sugar content. The Lagrangeans to the maximization problems in (13), (14) and (15) read, respectively,

$$\mathcal{L}^z = p_z z^s - p_\ell \ell_z^d + \gamma_z\Big[Z(\ell_z^d) - z^s\Big], \tag{34}$$

$$\mathcal{L}^x = p_x x^s + p_q q^s - p_\ell \ell_x^d - p_z z^d + \gamma_x\Big[X(\ell_x^d, z^d) - x^s\Big] + \gamma_q\left(\frac{z^d}{x^s} - q^s\right), \tag{35}$$

$$\mathcal{L}^{ci} = U^i[x_i^d, q_i^d, \ell_i^s, H^i(x_i^d q_i^d)] + \gamma_{ci}\Big[p_\ell \ell_i^s + \psi_i \Pi - p_x x_i^d - p_{qi} q_i^d\Big], \quad \text{for all } i, \tag{36}$$

where γ_z, γ_x, γ_q and γ_{ci} for all i are Lagrange multipliers.

Differentiating the Lagrangeans gives the first-order conditions

$$\mathcal{L}^z_{\ell_z^d} = \gamma_z Z_\ell - p_\ell = 0. \tag{37a}$$

$$\mathcal{L}^z_{z^s} = -\gamma_z + p_z = 0, \tag{37b}$$

$$\mathcal{L}^x_{\ell_x^d} = \gamma_x X_\ell - p_\ell = 0, \tag{37c}$$

$$\mathcal{L}^x_{z^d} = \gamma_x X_z + \gamma_q \frac{1}{x^s} - p_z = 0, \tag{37d}$$

$$\mathcal{L}^x_{x^s} = -\gamma_x - \gamma_q \frac{z^d}{(x^s)^2} + p_x = 0, \tag{37e}$$

$$\mathcal{L}^x_{q^s} = p_q - \gamma_q = 0, \tag{37f}$$

$$\mathcal{L}^{ci}_{x_i^d} = U^i_x + q_i^d U^i_h H^i_z - \gamma_{ci} p_x = 0, \qquad \text{for all } i, \tag{37g}$$

$$\mathcal{L}^{ci}_{q_i^d} = U^i_q + x_i^d U^i_h H^i_z - \gamma_{ci} p_{qi} = 0, \qquad \text{for all } i, \tag{37h}$$

$$\mathcal{L}^{ci}_{\ell_i^s} = U^i_\ell + \gamma_{ci} p_\ell = 0, \qquad \text{for all } i. \tag{37i}$$

Eliminating γ_z from (37a) and (37b) gives the condition in column 2, row 1 of Table 1. Solving (37c) for γ_x and (37f) for γ_q, inserting into (37e) and taking into account $x^s = x$ as well as $z^d/(x^s)^2 = q/x$ yields the the condition in column 2, row 2 of Table 1. Similarly, the condition in column 2, row 3 of Table 1 is obtained by inserting γ_x from (37c) and γ_q from (37f) into (37d) and using $x^s = x$. The conditions in column 2, row 5 and 6 follow from dividing (37g) and (37h), respectively, by (37i) and using $q_i^d = q$ and $x_i^d = x_i$.

Market equilibrium conditions with indirect market for sugar content. The Lagrangeans to the maximization problems in (13), (16) and (17) now read, respectively,

$$\mathcal{L}^z = p_z z^s - p_\ell \ell_z^d + \mu_z \Big[Z(\ell_z^d) - z^s\Big], \tag{38}$$

$$\mathcal{L}^x = P^x(q^s)x^s - p_\ell \ell_x^d - p_z z^d + \mu_x \Big[X(\ell_x^d, z^d) - x^s\Big] + \mu_q \left(\frac{z^d}{x^s} - q^s\right), \tag{39}$$

$$\mathcal{L}^{ci} = U^i[x_i^d, q_i^d, \ell_i^s, H^i(x_i^d q_i^d)] + \mu_{ci}\Big[p_\ell \ell_i^s + \psi_i \Pi - P^x(q_i^d)x_i^d\Big], \quad \text{for all } i, \tag{40}$$

where μ_z, μ_x, μ_q and μ_{ci} for all i are Lagrange multipliers.

Differentiating the Lagrangeans gives the first-order conditions

$$\mathcal{L}^z_{\ell_z^d} = \mu_z Z_\ell - p_\ell = 0. \tag{41a}$$

$$\mathcal{L}^z_{z^s} = -\mu_z + p_z = 0, \tag{41b}$$

$$\mathcal{L}^x_{\ell_x^d} = \mu_x X_\ell - p_\ell = 0, \tag{41c}$$

$$\mathcal{L}^x_{z^d} = \mu_x X_z + \mu_q \frac{1}{x^s} - p_z = 0, \tag{41d}$$

$$\mathcal{L}^x_{x^s} = -\mu_x - \mu_q \frac{z^d}{(x^s)^2} + P^x(q^s) = 0, \tag{41e}$$

$$\mathcal{L}^x_{q^s} = P^x_q(q^s)x^s - \mu_q = 0, \tag{41f}$$

$$\mathcal{L}^{ci}_{x^d_i} = U^i_x + q^d_i U^i_h H^i_z - \mu_{ci} P^x(q^d_i) = 0, \qquad \text{for all } i, \tag{41g}$$

$$\mathcal{L}^{ci}_{q^d_i} = U^i_q + x^d_i U^i_h H^i_z - \mu_{ci} P^x_q(q^d_i) x^d_i = 0, \qquad \text{for all } i, \tag{41h}$$

$$\mathcal{L}^{ci}_{\ell^s_i} = U^i_\ell + \mu_{ci} p_\ell = 0, \qquad \text{for all } i. \tag{41i}$$

Eliminating μ_z from (41a) and (41b) yields the condition in column 3, row 1 of Table 1. Determing μ_x from (41c) and μ_q from (41f), inserting into (41e) and taking into account $x^s = x$, $z^d/(x^s)^2 = q/x$, $P^x(q^s) = P^x$ and $P^x_q(q^s) = P^x_q$ yields the condition in column 3, row 2 of Table 1. In the same way, the condition in column 3, row 3 of Table 1 is obtained by inserting μ_x from (41c) and μ_q from (41f) into (41d) and using $x^s = x$ and $P^x_q(q^s) = P^x_q$. The conditions in column 3, row 5 and 6 follow from dividing (41g) by (41i) and (41h) by (41i), respectively, and making use of $q^d_i = q$, $x^d_i = x_i$, $P^x(q^d_i) = P^x$ and $P^x_q(q^d_i) = P^x_q$. Finally, the condition in column 3, row 4 of Table 1 follows from $x = \sum_{j=1}^n x^d_j$.

Proof of Proposition 4. For the proof of Proposition 4, we make use of the equilibrium conditions for the regulated market, which we derive in Section 7 and which are listed in column 4 of Table 1. If we set all tax rates equal to zero, i.e. $\tau^s_z = \tau^d_z = \tau^s_x = \tau^d_x = \tau_q = \tau_{xq} = 0$, then these conditions are equivalent to the conditions in the competitive economy without a market for sugar content (which can equivalently be obtained by setting $p_q \equiv p_{qi} \equiv P^x_q \equiv 0$ in column 2 or 3 of Table 1). Moreover, in Section 7 we show that $\tau^s_z = \tau^d_x = \tau_q = \tau_{xq} = 0$ and $\tau^d_z = -\tau^s_x/q = \tau^*_z := \frac{1}{x}\sum_{j=1}^n (U^j_q + x_j U^j_h H^j_z)/U^j_\ell > 0$ implements the efficient allocation if markets for sugar content are missing. Hence, in order to characterize the inefficiency in case of missing markets for sugar content, we assume $\tau^s_z = \tau^d_x = \tau_q = \tau_{xq} = 0$ in column 4 of Table 1 and conduct a comparative static analysis of marginal changes in τ^s_x and τ^d_z, taking advantage of the equality $\tau^s_x = -q\tau^d_z$ and restricting our attention to changes of τ^d_z in the interval $[0, \tau^*_z]$.

We start by determining the set of equations that have to be differentiated. Remember that $p_\ell = 1$ as well as $z^s = z^d = z$, $x^s = \sum_{j=1}^n x^d_j = x$, $x^d_i = x_i$, $q^d_i = q^s = q$ and $\ell^s_i = \ell_i$. The term $q^s = z^d/x^s$ can then be rewritten as

$$z = xq. \tag{42}$$

If the technologies X and Z are linear homogeneous, zero profits are necessary equilibrium conditions. In the case with taxes, profit of the sugar and SSB sectors are given in (23) and (25), respectively. Under linear homogeneity, the sugar technology can be specified as $Z(\ell_z^d) = \ell_z^d/c_z$ with the given technology parameter $c_z > 0$. Zero profit in the sugar industry and $\tau_z^s = 0$ then imply $p_z = c_z$. Taking into account $\tau_{xq} = 0$ and $\tau_z^d = -\tau_x^s/q$, the zero-profit requirement in the SSB sector yields

$$p_x = a + c_z q, \tag{43}$$

where $a := \ell_x^d/x$ is the labor-input-output coefficient of SSB production. Notice that the sugar content q represents the sugar-input-output coefficient. Cost minimization is necessary for profit maximization in the SSB sector and, together with the linear homogeneity of X, implies that a and q are functions of the after-tax factor price $p_z + \tau_z^d = c_z + \tau_z^d$, i.e.

$$a = A(c_z + \tau_z^d), \tag{44}$$

$$q = Q(c_z + \tau_z^d), \tag{45}$$

with $A'(\cdot) > 0$ and $Q'(\cdot) < 0$ due to the linear homogeneity of X. Under the CES utility function specified in Proposition 4, consumer i's first-order condition in column 4, row 5 in Table 1 can be rewritten as

$$\frac{\alpha_i x_i^{-\rho_i} - \varepsilon_i x_i^{\rho_i} q^{\rho_i}}{\kappa_i (1-\ell_i)^{-\rho_i - 1}} = p_x x_i, \tag{46}$$

Profit income $\psi_i \Pi$ of the consumer is zero, because $\Pi = \pi_z + \pi_x = 0$ due to the zero profit conditions. The same is true for consumer i's lump-sum transfer $\zeta_i T$ because $T = \tau_z^s z^s + \tau_z^d z^d + \tau_x^s x^s + \tau_x^d \sum_{j=1}^n x_j^d + \tau_q q^s + \tau_{xq} x^s q^s = 0$ due to $\tau_z^s = \tau_x^d = \tau_q = \tau_{xq} = 0$ and $\tau_x^s = -q\tau_z^d$. The consumer's budget constraint in (25) thus simplifies to

$$\ell_i = p_x x_i. \tag{47}$$

Equations (42)–(47) together with $x = \sum_{j=1}^n x_j$ represent a system of $2n + 5$ equations that determines the $2n + 5$ unknowns q, z, x, a, p_x, $x_1, \ldots, x_n$ and $\ell_1, \ldots, \ell_n$ as functions of τ_z^d. In the following we run a comparative static analysis along the lines suggested by Jones (1965) using the so-called hat calculus, where $\hat{y} := dy/y$ denotes the relative change in $y \in \{q, z, x, a, p_x, x_1, \ldots, x_n, \ell_1, \ldots, \ell_n\}$. In deviation from this convention, let $\hat{\tau}_z^d := d\tau_z^d/(c_z + \tau_z^d)$ in order to avoid that $\hat{\tau}_z^d$ is not defined if $\tau_z^d = 0$.

In totally differentiating the above system of equations, we start with (42) and obtain

$$\hat{z} \;=\; \hat{x} + \hat{q}. \tag{48}$$

From the differential of (43) follows $\hat{p}_x = \theta_\ell \hat{a} + \theta_q \hat{q}$ with $\theta_\ell := a/p_x > 0$ and $\theta_q := c_z q/p_x > 0$. Notice that the Lagrangean of cost minimization is minimized and that the constraint $x \leq X(\cdot)$ is binding in the minimum. Thus, also the cost expression $\ell_x + (p_z + \tau_z^d)z$ or, equivalently, the expression $a + (c_z + \tau_z^d)q$ attains a minimum. Setting the total differential equal to zero implies $\theta_\ell \hat{a} + \theta_q \hat{q} + q\tau_z^d \hat{q}/p_x = 0$. Inserting this into $\hat{p}_x = \theta_\ell \hat{a} + \theta_q \hat{q}$ yields

$$\hat{p}_x \;=\; -\frac{q\tau_z^d}{p_x}\hat{q}, . \tag{49}$$

Differentiating (44) und (45) implies

$$\hat{a} \;=\; \eta_a \hat{\tau}_z^d \tag{50}$$

$$\hat{q} \;=\; \eta_q \hat{\tau}_z^d \tag{51}$$

with the elasticities $\eta_a := (c_z + \tau_z^d)A'/a > 0$ due to $A' > 0$ and $\eta_q := (c_z + \tau_z^d)Q'/q < 0$ due to $Q' < 0$. Even without differentiating (46) and (47) and, thus, independent of the utility function, we can now already prove part (i) of Proposition 4: If $\hat{\tau}_z^d > 0$, then $\hat{a} > 0$ due to (50) and $\eta_a > 0$, $\hat{q} < 0$ due to (51) and $\eta_q < 0$ and $\hat{p}_x > 0$ due (49) and $\hat{q} < 0$. Hence, if we move from the competitive equilibrium without markets for sugar content to the efficient solution (increase in τ_z^d), a and p_x increase, whereas q decreases. Put differently, in the competitive equilibrium a and p_x are inefficiently low, while q is inefficiently high. The efficiency of p_z follows immediately from $p_z = c_z$, where c_z is a constant.

In an analogous way, we can prove the parts (ii)-(iv) of Proposition 4. Replace $p_x x_i$ in (46) by ℓ_i from (47). Totally differentiating the resulting expression and (47) yields

$$\delta_\ell^i \, \hat{\ell}_i \;=\; -\delta_x^i \, \hat{x}_i - \delta_q^i \, \hat{q}, \tag{52}$$

$$\hat{\ell}_i \;=\; \hat{p}_x + \hat{x}_i, \tag{53}$$

with

$$\delta_\ell^i \;:=\; \kappa_i \ell_i (1-\ell_i)^{-\rho_i - 1} + (1+\rho_i)\kappa_i \ell_i^2 (1-\ell_i)^{-\rho_i-2} > 0, \tag{54}$$

$$\delta_x^i \;:=\; \rho_i(\alpha_i x_i^{-\rho_i} + \varepsilon_i x_i^{\rho_i} q^{\rho_i}) \gtreqqless 0 \qquad \Leftrightarrow \qquad \rho_i \gtreqqless 0 \tag{55}$$

$$\delta_q^i \;:=\; \rho_i \varepsilon_i x_i^{\rho_i} q^{\rho_i} \gtreqqless 0 \qquad \Leftrightarrow \qquad \rho_i \gtreqqless 0 \tag{56}$$

Solving (52) and 53 for $\hat{x}_i$ and $\hat{\ell}_i$ yields

$$\hat{x}_i = -\frac{\delta_\ell^i}{\delta_\ell^i + \delta_x^i}\,\hat{p}_x - \frac{\delta_q^i}{\delta_\ell^i + \delta_x^i}\,\hat{q} \tag{57}$$

$$\hat{\ell}_i = \frac{\delta_x^i}{\delta_\ell^i + \delta_x^i}\,\hat{p}_x - \frac{\delta_q^i}{\delta_\ell^i + \delta_x^i}\,\hat{q} \tag{58}$$

where (54) and (55) together with (46) and (47) imply

$$\delta_\ell^i + \delta_x^i = (1+\rho_i)\kappa_i \ell_i^2 (1-\ell_i)^{-\rho_i-2} + (1+\rho_i)\alpha_i x_i^{-\rho_i} + (\rho_i - 1)\varepsilon_i x_i^{\rho_i} q^{\rho_i}. \tag{59}$$

If $\rho_i = 0$ for all i, then $\delta_x^i = \delta_q^i = 0$ from (55) and (56), and (52) implies $\hat{\ell}_i = 0$, whereas (53) together with $\hat{p}_x > 0$ implies $\hat{x}_i < 0$ which, in turn, gives $\hat{x} < 0$ by $x = \sum_{j=1}^n x_j$ and $\hat{z} < 0$ by (48) and $\hat{q} < 0$. Hence, if we move from the market equilibrium to the efficient allocation (τ_z^d increases), ℓ_i remains constant and x_i, x and z decrease, which proves part (ii) of Proposition 4. If $\rho_i = 1$ for all i, then (54)-(56) and (59) yield $\delta_\ell^i > 0$, $\delta_x^i > 0$, $\delta_q^i > 0$ and $\delta_\ell^i + \delta_x^i > 0$. It follows from (57), (58), $\hat{p}_x > 0$ and $\hat{q} < 0$ that $\hat{\ell}_i > 0$ and $\hat{x}_i \gtreqless 0$ which, in turn, implies $\hat{x} \gtreqless 0$ by $x = \sum_{j=1}^n x_j$ and $\hat{z} \gtreqless 0$ by (48). This proves part (iii) of Proposition 4. Finally, if $\rho_i = -1$ for all i, then (54)-(56) and (59) give $\delta_\ell^i > 0$, $\delta_x^i < 0$, $\delta_q^i < 0$ and $\delta_\ell^i + \delta_x^i < 0$. From (57),(58), $\hat{p}_x > 0$ and $\hat{q} < 0$ we obtain $\hat{\ell}_i > 0$ and $\hat{x}_i > 0$ which, in turn, implies $\hat{x} > 0$ by $x = \sum_{j=1}^n x_j$ and $\hat{z} \gtreqless 0$ by (48) and $\hat{q} < 0$. This shows part (iv) and completes the proof of Proposition 4.

Equilibrium conditions for regulated markets. The Lagrangeans to the maximization problems in (23), (24) and (25) read, respectively,

$$\mathcal{L}^z = (p_z - \tau_z^s)z^s - p_\ell \ell_z^d + \omega_z\Big[Z(\ell_z^d) - z^s\Big], \tag{60}$$

$$\mathcal{L}^x = (p_x - \tau_x^s)x^s - \tau_q q^s - p_\ell \ell_x^d - (p_z + \tau_z^d)z^d - \tau_{xq}x^s q^s \tag{61}$$

$$+ \omega_x\Big[X(\ell_x^d, z^d) - x^s\Big] + \omega_q\left(\frac{z^d}{x^s} - q^s\right), \tag{62}$$

$$\mathcal{L}^{ci} = U^i[x_i^d, q_i^d, \ell_i^s, H^i(x_i^d q_i^d)] + \omega_{ci}\Big[p_\ell \ell_i^s + \psi_i \Pi + \zeta_i T - (p_x + \tau_x^d)x_i^d\Big], \quad \text{for all } i, \tag{63}$$

where ω_z, ω_x, ω_q and ω_{ci} for all i are Lagrange multipliers.

Differentiating the Lagrangeans gives the first-order conditions

$$\mathcal{L}_{\ell_z^d}^z = \omega_z Z_\ell - p_\ell = 0. \tag{64a}$$

$$\mathcal{L}^z_{z^s} = -\omega_z + p_z - \tau^s_z = 0, \tag{64b}$$

$$\mathcal{L}^x_{\ell^d_x} = \omega_x X_\ell - p_\ell = 0, \tag{64c}$$

$$\mathcal{L}^x_{z^d} = \omega_x X_z + \omega_q \frac{1}{x^s} - p_z - \tau^d_z = 0, \tag{64d}$$

$$\mathcal{L}^x_{x^s} = -\omega_x - \omega_q \frac{z^d}{(x^s)^2} + p_x - \tau^s_x - \tau_{xq} q^s = 0, \tag{64e}$$

$$\mathcal{L}^x_{q^s} = -\tau_q - \tau_{xq} x^s - \omega_q = 0, \tag{64f}$$

$$\mathcal{L}^{ci}_{x^d_i} = U^i_x + q^d_i U^i_h H^i_z - \omega_{ci}(p_x + \tau^d_x) = 0, \qquad \text{for all } i, \tag{64g}$$

$$\mathcal{L}^{ci}_{\ell^s_i} = U^i_\ell + \omega_{ci} p_\ell = 0, \qquad \text{for all } i. \tag{64h}$$

Combining (64a) and (64b) gives the condition in column 4, row 1 of Table 1. Inserting (64c) and (64f) into (64e) and (64d) implies the conditions in column 4, rows 2 and 3, respectively. Finally, dividing (64g) by (64h) yields the condition in column 4, row 5.

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