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Searching for a "Golden Rule" of Economic Regulation of an Infectious Disease

Abstract

This paper investigates whether a "Golden Rule" of regulation of an infectious disease may be elicited that balances the economic control and disease costs when the arrival of a future vaccine or a cure is uncertain. Formulating an optimal control problem applied to standard compartment models of infection, an optimality rule is derived. This rule is more complex than other similar Golden Rules related to optimal economic growth or extraction of natural resources. The paper contains interpretation of the derived rule and numeric examples of how the rule functions under the compartment models (i.e., the SI, the SIS, and the SIR models).

JEL-Codes: H510, I180.

Keywords: infectious disease, economic regulation, Golden Rule, compartment models.

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1. Introduction

While waiting for the arrival of an efficient vaccine or an efficient cure of a serious infectious disease such as Covid-19, a challenge is to manage the level of the disease so as to balance the cost of controlling the disease against the cost of the disease itself. In this paper we investigate how control costs should develop over time until a vaccine or a cure arrives with a view to minimizing the sum of control and disease costs. As an epidemic without a vaccine may extend over several years, a dynamic analysis, involving some degree of discounting, is called for. Furthermore, the uncertainty related to how long the epidemic will last before a vaccine or a cure is made available on a large scale, should also be reflected in the management of the disease. Ignoring the major complexities of the disease (e.g., seasonal features, recurring waves of aggressive contamination, age related features, new variants of the viruses) we seek to elicit what economic theory has to say about the basic economic principles that should govern the time profile of resources spent on infection control measures. As a base for the analysis, we apply simple standard compartment models (Kermack and McKendrick, 1927; Hechcote, 2000; Weiss, 2013), and add economic elements.

The cost of the disease includes costs such as hospitalization costs (including intensive care), imputed cost of suffering of the infected individuals, value of lost statistical lives, and social costs faced by mourning relatives left behind. Infection control measures are many and varied and encompass activities such as frequent hand washing, use of face masks, quarantine and other kinds of social distancing (e.g., home office arrangements, regulation of travel and leisure activities, temporarily closing down educational institutions, bars and restaurants, closing borders with temporary complete lock down with strict curfew rules). Consequently, much of this will end up as reduced economic activity.

In general, the literature on infectious diseases is abundant covering mathematical and biological analyses in the tradition of Kermack and McKendrick (2027), (e.g., Hechcote, 2000; Miller, 2012 and 2017; Berger et al. 2020). Also, economic analyses have increased in number in recent years, especially since the outbreak of Covid-19. Surveys of economic epidemiology include Philipson (2000), Gersovitz and Hammer (2004), Klein et al. (2007), Manfredi and d´Onofrio (2013). Many of the economic analyses deal with matching mechanisms regarding how the disease is transmitted and how measures such as prevention and/or treatment optimally may affect transmission rates and development of infected individuals. (e.g., Rowthorn and

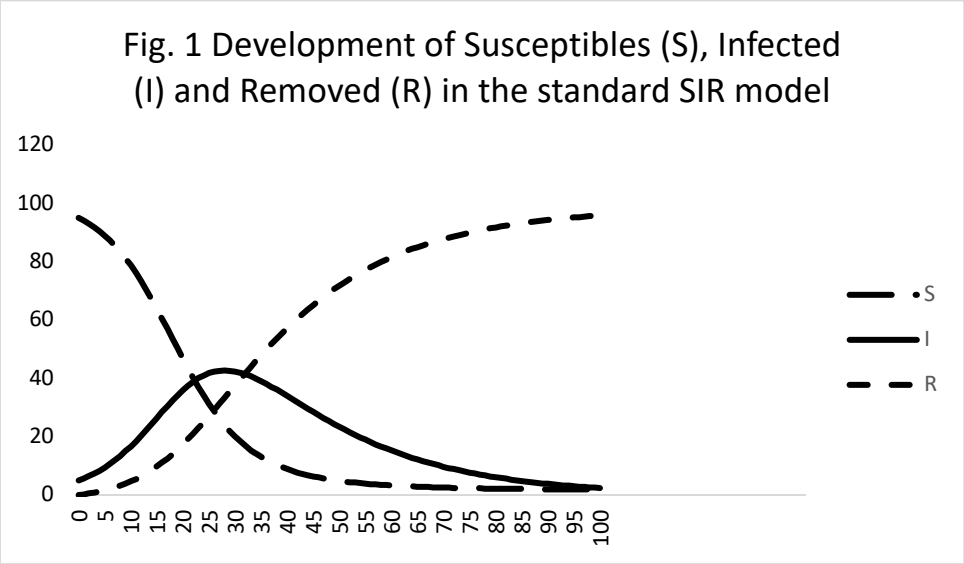
Toxvaerd, 2020; Eichenbaum, et al. 2020; Alvarez et al., 2020; Jones et al., 2020, Farboodi et al., 2020; Garriga et al., 2020; Garibaldi et al.2020 and Pissarides, 2020). Some of the economic literature also deal with macro-oriented empirical analyses (e.g., Weiss, 2013; Atkeson, 2020; Eichenbaum et al., 2020; Toda 2020). Contrary to these, this paper seeks answer to a basic economic question: which principle should govern the control measures taken in basic models of infectious deceases. Once detected, additional more realistic assumptions may be included. In particular, we set out to investigate whether some general "golden rule" of regulation exists for the simpler compartment models (SI, SIS and SIR) that is comparable to the Hotelling rules (Hotelling, 1931) for extraction of a non-renewable resource (e.g., oil) or a renewable natural resource (e.g., biomass) or similarly for optimal growth investments in an economy (Solow (1956) and Phelps (1961). To my knowledge none has taken this particular view earlier. Hence, to fill in this void, I proceed with the following model.

2. Model

Without regulation, the typical development of an epidemic in the so-called SIR model with herd immunity, is that the number of new infected individuals at first increases, then reaches a maximum[1] and, thereafter, fades off (See Fig. 1.). By controlling the number of infected individuals this typical development may be modified.

The standard SIR model contains three state variables: S, I and R. Hence, S_t is the number of individuals susceptible to be infected at date t, I_t is the number of infected and infectious individuals at date t while R_t is the number of removed (recovered or dead) individuals at date t. The total population, N, is taken to be constant for the period considered. Hence, at all dates we have $N = S_t + I_t + R_t$. Furthermore, there are initial levels of the various states. These are denoted $S_0 = N - \varepsilon,\ I_0 = \varepsilon,\ R_0 = 0$, where ε is a small number. The transmission rate from susceptible individuals to infectives and the transmission rate from infectives to removed individuals, are denoted α and β, respectively.

[1] In the SIR model explained below, the maximum is given by $I_{max} = I_0 + S_0 - \frac{\beta}{\alpha}(1 + \ln\left(\frac{\alpha}{\beta}S_0\right)$, provided that $S_0 > \frac{\beta}{\alpha}$. The fraction: $\frac{\alpha}{\beta}$ is called the Contact Ratio and the fraction: $R_0 = \frac{\alpha S_0}{\beta}$ is called the Basic Reproduction Rate.



The equation of motion for each of state variables are

1) $\frac{dS_t}{dt} = \dot{S}_t = -\alpha S_t I_t$
2) $\frac{dI_t}{dt} = \dot{I}_t = \alpha S_t I_t - \beta I_t$
3) $\frac{dR_t}{dt} = \dot{R}_t = \beta I_t$

As N, is a constant, we must have $\dot{S}_t + \dot{I}_t + \dot{R}_t = 0$. Clearly, with herd immunity, the number of susceptible individuals will fall over time while the number of recovered individuals will increase. Hence, in this simple model the disease will disappear sooner or later. However, with an epidemic such as Covid-19, it is not yet clear to what extent the disease gives rise to total herd immunity. In the extreme case where it does not, the number of susceptible individuals will fall over time with a corresponding increase of the number of infected individuals, until the total population is infected (without any recovered individuals). This is reflected in the so-called SI-model, where $\dot{S}_t = -\alpha S_t I_t$ and $\dot{I}_t = \alpha S_t I_t$. In an intermediate case, immunity may last for a period until the recovered individuals again get susceptible. For this so-called SIS-model, we have $\dot{S}_t = -\alpha S_t I_t + \beta I_t$ and $\dot{I}_t = \alpha S_t I_t - \beta I_t$, where an infection eventually results in a steady state characterized by $I = \left(1 - \frac{\beta}{\alpha}\right) N$.

In the SIR-model, we assume that there is a social cost of infected individuals (e.g., imputed utility loss of being infected and value of lost statistical life) equal to $c(I)$ where $\frac{dc}{dI} = c'(I) >$

0 and $\frac{d^2c}{dI^2} = c''(I) \geq 0$. Furthermore, we assume that the level of infected individuals can be controlled through the transmission rate, $\alpha(z_t)$, where the infection control measures, z_t, are taken to reduce the transmission rate from susceptible individuals to infected individuals, i.e. where $\frac{d\alpha}{dz} = \alpha'(z) < 0$ and $\frac{d^2\alpha}{dz^2} = \alpha''(z) \geq 0$. The cost of these activities is assumed to show up as a reduction of the economy´s production and consumption of (non-epidemic) goods and services, x_t. This relationship is captured by the function $f(x_t) = f(\bar{y} - z_t)$, where $\bar{y}$ is the normal level of economic activity without the disease. For simplicity, we assume $\frac{df}{dx} = f'(x_t) > 0$ and $\frac{d^2f}{dx^2} = f''(x_t) = 0$. Otherwise, we denote the (constant) social discount rate by δ. Further, it should be noted that the imputed infection costs are not assumed do not show up as a reduction of the economy´s production capacity, as these costs are mainly in the form of utility loss of non-marketed goods.

In the formulation of the economic optimization problem to follow, we assume that the unknown date of the arrival of the vaccine, τ, follows a Poisson process. The essential assumption is that the conditional probability of the event to occur, provided that the event has not already happened, remains constant as time proceeds. Hence, the length of the time period before the vaccine arrives does not affect the probability that the vaccine will arrive at a given date. Under these assumptions, Barlow and Proschan (1975) show that the distribution function is given by $F(t) = \Pr(\rho \leq t) = (1\text{-}e^{-\rho t})$, while the probability density function is equal to $f(t) = \Pr(\tau) = \rho e^{-\rho t}$. Here ρ represents the "failure rate," which is the frequency of the event to occur within a given time horizon. Expected waiting time until the arrival of the event is equal to $1/\rho$. Hence, an increase of the failure rate leads to a shorter expected waiting time. Once the vaccine or cure has arrived at date τ, the economy reverts to the same state as prior to the outbreak of the epidemic. The net present value of future normal economic activity evaluated at date τ , is denoted $F(\bar{y})$.

Under the assumptions of the model, we consider the following optimization problem[2]

$$Max \int_0^{\infty} \rho e^{-\delta\tau} \left\{ \int_0^{\tau} [f(\bar{y} - z_t) - c(I_t)] e^{-\delta t} dt + F(\bar{y}) e^{-\delta\tau} \right\} d\tau$$

[2] Observe, that $F(\bar{y})$ functions as a constant scrap value.

Subject to[3]

$$\dot{I}_t = \alpha(z_t)S_tI_t - \beta I_t$$
$$S_0 = N$$
$$I_0 = \varepsilon$$
$$R_0 = 0$$
$$z_t \geq 0$$

Upon integrating by parts (see Amundsen and Bjørndal, 1999; Tsur and Zemel, 2004), the objective function may be reformulated as

$$Max \int_0^{\infty} [f(\bar{y} - z_t) - c(I_t) + \rho F(\bar{y})]e^{-(\delta+\rho)t}dt$$

Denoting the co-state variable by λ_t[4] and assuming that $z_t > 0$ at all dates, the corresponding Hamiltonian reads

$$H_t = [f(x_t) - c(I_t) + \rho F(\bar{y})]e^{-(\delta+\rho)t} + \lambda_t[\alpha(z_t)S_tI_t - \beta I_t]$$

First order conditions of this problem are

4) $\frac{dH_t}{dz_t} = -f'(x_t)e^{-(\delta+\rho)t} + \lambda_t\alpha'(z_t)S_tI_t - \gamma_t = 0$

5) $\frac{dH_t}{dI_t} = -c'(I_t)e^{-(\delta+\rho)t} + \lambda_t(\alpha(z_t)S_t - \beta) + \lambda_t\alpha(z_t)I_t\frac{dS_t}{dI_t} = -\dot{\lambda}_t$

We seek to eliminate $\lambda_t$and $\dot{\lambda}_t$ from these conditions. Hence, using 4) to solve for λ_t, taking the total time differential of this to obtain an expression of $\dot{\lambda}_t$, inserting this into 2), and further recognizing that $\frac{\dot{I}_t}{I_t} = \alpha(z_t)S_t - \beta$, and that $\frac{\partial f(x_t)}{\partial z_t} \equiv f'(z_t) = -f'(x_t)$, the following optimality condition emerges

[3] We assume free terminal states and time.

[4] As noted, there are three state variables *S*, *I* and *R*. However, as there is only one control variable, z, this governs all states according to the endogenous element of the transmission rate, $\alpha(z)$ from infectives to infected and the exogenous element of transmission, β, from infected individuals to removed.

$$6)\ \frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(z_t)} + \alpha(z_t)I_t\left(\frac{dS_t}{dI_t}+1\right) - \frac{\alpha''(z_t)\dot{z}_t}{\alpha'(z_t)} = (\delta+\rho)$$

This equation expresses an optimal path of the infection control measures, z_t, that maximizes the social surplus and, thus, implies an efficient balancing of social costs. The condition is the same whether there is herd immunity ($\beta > 0$) or not ($\beta = 0$). However, for the SI and the SIS models, we have, respectively, that

$$\frac{dS_t}{dI_t} = \frac{\frac{dS_t}{dt}}{\frac{dI_t}{dt}} = \frac{-\alpha(z_t)S_tI_t}{\alpha(z_t)S_tI_t} = -1$$

and

$$\frac{dS_t}{dI_t} = \frac{\frac{dS_t}{dt}}{\frac{dI_t}{dt}} = \frac{-\alpha(z_t)S_tI_t + \beta I_t}{\alpha(z_t)S_tI_t - \beta I_t} = -1$$

Consequently, the middle term of 6) disappears for the SI and SIS models[5].

As for the SIR model, we have

$$\frac{dS_t}{dI_t} = \frac{\frac{dS_t}{dt}}{\frac{dI_t}{dt}} = -\frac{\alpha(z_t)S_tI_t}{\alpha(z_t)S_tI_t - \beta I_t} = -\frac{\alpha(z_t)S_t}{\alpha(z_t)S_t - \beta}$$

Inserting into 6) we get

$$7)\ \frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(z_t)} - \alpha(z_t)I_t\left(\frac{\beta}{\alpha(z_t)S_t - \beta}\right) - \frac{\alpha''(z_t)\dot{z}_t}{\alpha'(z_t)} = (\delta+\rho)$$

3. Analysis and discussion

It is well known that there does not exist a pure analytical solution to the system of differential equations of the SIR model, even though some solutions are often referred to as analytical solutions (see Harko, 2014 and Miller, 2015). However, even these solutions involve an integral that needs to be calculated numerically. Also, the present model is more complicated than the standard SIR model (and the corresponding SI and SIS models), since the transmission rate,

[5] See Appendix A. for an alternative derivation for the SI and the SIS models

$\alpha(z_t)$, is endogenously determined as opposed to the constant transmission rate of the standard compartment models[6]. For this reason, the following analysis will draw upon numerical simulations and illustrations (see Appendix B).

The optimality condition 7), relates to capital theory. In general, the condition expresses that the internal rate of return of the marginal regulation cost (the left -hand side) should be equal to the marginal discount rate (the right - hand side)[7]. In this case the failure rate, ρ functions as an increment to the discount rate, δ. The marginal cost of regulation, $f'(z_t)$, can be viewed as an investment that gives rise to three elements of return. Condition 7), thus, represents a Hotelling rule or a golden rule of optimal management of an epidemic. To better explain the various elements of condition 7), it may be reformulated as

$$8)\ \frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(z_t)} + \frac{\dot{S}_t}{S_t}\frac{dR_t}{dI_t} - \frac{\alpha'(\dot{z}_t)}{\alpha'(z_t)} = (\delta + \rho)$$

The first expression on the left - hand side is the percentage increase in terms of saved disease costs and the second expression is the percentage change of susceptible individuals induced by the marginal regulation cost weighted by the marginal effect on recovery $(dR_t/dI_t) = \frac{\beta}{\alpha(z_t)S_t - \beta}$. The third expression is the percentage change of the marginal transmission rate of susceptible individuals. The first element is positive, and the second and third are in general indeterminate.

To get an expression for the time path of the regulation activity, rearrange 7) to obtain

$$9)\ \dot{z}_t = \frac{\alpha'(z_t)}{\alpha''(z_t)}\left[\frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(z_t)} - \alpha(z_t)I_t\left(\frac{\beta}{\alpha(z_t)S_t - \beta}\right) - (\delta + \rho)\right]$$

Inspection of signs shows that the time path of the regulation activity is indeterminate.

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[6] For the SIS model, Rowthorn and Toxvaerd (2020) characterizes the optimal control solutions in a thorough analysis of a regulation problem that includes both prevention and treatment of infected individuals. They find that several steady state solutions are possible.

[7] A more general model assuming that $\frac{d^2f}{dx^2} > 0$, and $\frac{d^2\alpha}{dz^2} > 0$, gives rise to the following optimality condition:

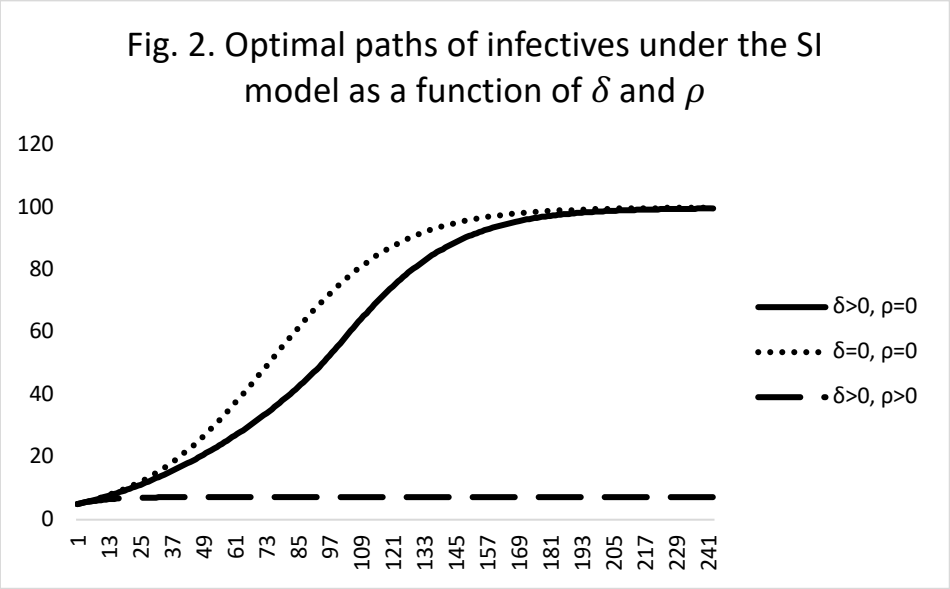
$\frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(z_t)} - \frac{f'(\dot{z}_t)}{f'(z_t)} + \alpha(z_t)I_t\left(\frac{dS_t}{dI_t} + 1\right) - \frac{\alpha'(\dot{z}_t)}{\alpha'(z_t)} = \delta + \rho.$

Considering first the SI and the SIS models, 9) reduces to

$$10)\ \dot{z}_t = \frac{\alpha'(z_t)}{\alpha''(z_t)}\left[\frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(z_t)} - (\delta+\rho)\right]$$

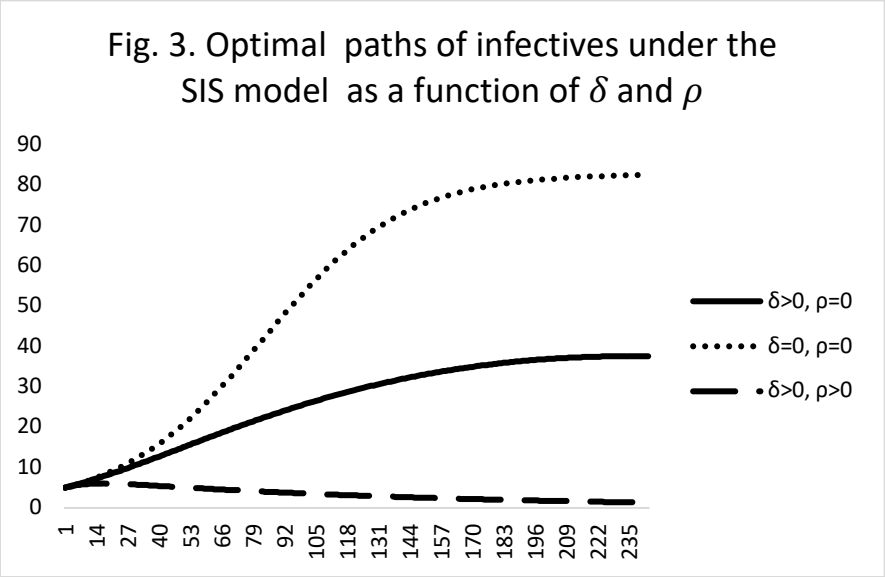
The expression is still indeterminate, but for the special case of no discounting (i.e., $(\delta+\rho)=0$) or a low degree of discounting, the time path of the regulation activity should diminish over time. To see that this is the case (i.e., that $\dot{z}_t < 0$), recall that $\alpha'(z_t) < 0$, $\alpha''(z_t) > 0$, $c'(I_t) > 0$, and that $f'(z_t) < 0$. The intuition for this result is the following: In the case of the SI model, we know that the total population will be infected sooner or later and stay infected. Hence, the costs of infection cannot be avoided. Spending resources on regulation will reduce the transmission rate from susceptible individuals to infectives and thereby reduce the speed at which people is getting infected, but without discounting and no hope of a future vaccine or cure ($\delta+\rho=0$), it does not matter at which dates the costs are incurred. The total net present value of costs will be the same irrespective of the time path of infected individuals. For this reason, it does not make any sense to use resources on affecting the transmission rate from susceptible individuals to infectives. Hence, if there are regulation costs at the outset, these should be brought down to zero. On the other hand, if resources spent on regulation could bring the transmission rate all the way to zero, then resources spent may still make sense, since the increase of the number of infected individuals could be stopped such that the number of infected individuals would be held constant at some level below the total number of susceptible individuals and thus be saving costs of infection. Whether this will be socially optimal or not depends on the costs of regulation as compared with the costs of the infection saved.

Still considering the SI model, if the sum of the discount rate and the failure rate is strictly positive i.e., $(\delta+\rho) > 0$, then it makes perfectly good economic sense to spend resources on reducing the transmission rate. By increasing the regulation costs, and thereby lowering the transmission rate, the path of infected individuals is stretched out in time and the present value of the infection costs is lowered. As noted, the failure rate appears on the same footing as the discount rate. Hence, a higher failure rate (i.e., a higher probability of earlier arrival of a vaccine or a cure), will lead to more resources spent on lowering the transmission rate of susceptible individuals into infectives. An illustration of these cases is presented in Fig. 2.

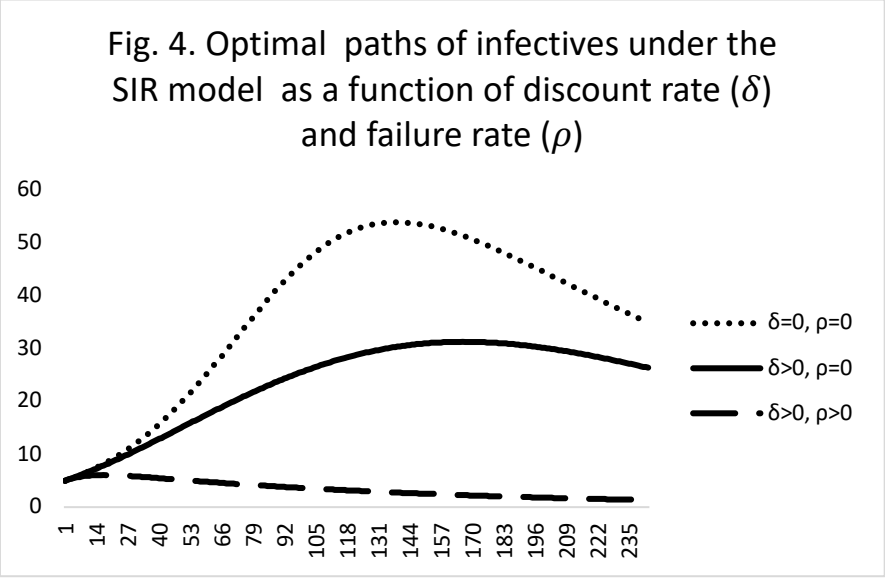


In interpreting Fig. 2., one should recognize that regulation expenses stop when infection is close to 100 percent in the SI model. Furthermore, all models have that in common, that the vaccine or the cure may be introduced at any date before the infection has reached its final state, wherefore also the regulation expenses drop to zero. Hence, the illustrated paths of infectives are not necessarily valid for all the period illustrated.

The result of the declining regulation path carries over to the SIS model when $\delta + \rho = 0$. However, one may observe that the SIS model deviates from the SI model in that the maximum number of infected individuals is not necessarily equal to the total number of susceptible individuals that exists at the outbreak of the epidemics. Hence, a steady state condition may be reached where $\dot{S}_t = -\alpha(z_t)S_tI_t + \beta I_t = 0$ and $\dot{I}_t = \alpha(z_t)S_tI_t - \beta I_t = 0$, and the level of this will be influenced by regulation. As the infection cost function may be strictly convex, cost per infected individual will be saved by lowering the maximum level of infected individuals. Otherwise, one can observe that the maximum number of infected individuals declines as the discount rate or the failure rate increases. An example of this is illustrated in Fig. 3.



Turning to the SIR model, a similar conclusion of a declining regulation path is valid for the case of no discounting (i.e., $(\delta + \rho) = 0$) even for this model, provided that infection is in the declining phase (i.e., $\dot{I}_t = (\alpha(z_t)S_t - \beta)I_t < 0$). For an illustration of the SIR model in the more general case, where $(\delta + \rho) > 0$, see Fig. 4.



As can be seen from Fig. 4, the path of infected individuals follows in this case a typical path of an increasing number of infected individuals, then reaching a maximum number, and thereafter a diminishing number. Also, for the SIR model, one can observe that the maximum

number of infected individuals declines as the discount rate or the failure rate increases. An example of this is illustrated in Fig. 4.

4. Conclusion

In this paper we set out to investigate whether some general "Golden Rule" of regulation exists for the simpler compartment models (SI, SIS and SIR) that is comparable to the Hotelling-rules for natural resource extraction or similarly for optimal growth investments in an economy. Even though, principles from capital theory are detected, i.e., that the marginal rate of return from regulation costs should be equal to the sum of discount rate and failure rate of the arrival of a vaccine or cure, no neat and easily explicable rule emerges. Still, one element of the marginal rate of return from regulation is recognizable from the golden rule literature i.e., the percentage increase of saved disease costs following from the marginal regulation (or investment) costs. However, in addition to this element, the marginal rate of return includes the percentage change of susceptible individuals induced by the marginal regulation cost weighted by the marginal effect on recovery, and the percentage change of the marginal transmission rate of susceptible individuals.

In general, the numerical examples of the models included in the paper, show that the higher the sum of the discount rate and expected rate of the arrival of the vaccine or cure, the higher are the resources spent on reducing the transmission rate from susceptible individuals to infectives. In this way the path of infectives is delayed and the present value of disease costs is reduced through more heavy discounting.

Otherwise, the analysis shows that there is no easy guide for how the path of regulation should develop over time. An exception to this is the special (and unrealistic) case, where there is no hope for a vaccine or a cure combined with a discount rate equal to zero in the SI model. In that case, there should be no regulation (or alternatively, that regulation costs should be brought down to zero if there are regulation activities at the outset).

Appendix A

Derivation of optimality conditions for the SI model ($\beta = 0$) and SIS model ($\beta > 0$)

The problem reads

$$Max \int_0^{\infty} \rho e^{-\delta\tau} \left\{ \int_0^{\tau} [f(\bar{y} - z_t) - c(I_t)] e^{-\delta t} dt + F(\bar{y}) e^{-\delta\tau} \right\} d\tau$$

Subject to[8]

$$\dot{I}_t = \alpha(z_t) S_t I_t - \beta I_t$$
$$\dot{S}_t = -\alpha(z_t) S_t I_t + \beta I_t$$
$$S_0 = N$$
$$I_0 = \varepsilon$$
$$R_0 = 0$$
$$z_t \geq 0$$

Upon integrating by parts (see Amundsen and Bjørndal, 1999; Tsur and Zemel, 2004), the objective function may be reformulated as

$$Max \int_0^{\infty} [f(\bar{y} - z_t) - c(I_t) + \rho F(\bar{y})] e^{-(\delta+\rho)t} dt$$

Denoting the co-state variable by λ_t for the state of infection, the co-state variable by μ_t for the state of infectives and the shadow price of z_t by γ_t, the corresponding Hamiltonian reads

$$H_t = [f(x_t) - c(I_t) + \rho F(\bar{y})] e^{-(\delta+\rho)t} + \lambda_t [\alpha(z_t) S_t I_t - \beta I_t] - \mu_t [\alpha(z_t) S_t I_t - \beta I_t] - \gamma_t z_t$$

First order conditions of this problem are

1) $\frac{dH_t}{dz_t} = -f'(x_t) e^{-(\delta+\rho)t} + (\lambda_t - \mu_t) \alpha'(z_t) S_t I_t - \gamma_t = 0$
2) $\frac{dH_t}{dI_t} = -c'(I_t) e^{-(\delta+\rho)t} + (\lambda_t - \mu_t)(\alpha(z_t) S_t - \beta) = -\dot{\lambda}_t$

[8] We assume free terminal states and time.

3) $\frac{dH_t}{dS_t} = (\lambda_t - \mu_t)\alpha(z_t)I_t = -\dot{\mu}_t$

Upon taking the total time differential of 1), while considering the case where $z_t > 0$ (i.e. $\gamma_t = 0$), arrive at

1)′ $(\delta + \rho)f'(x_t)e^{-(\delta+\rho)t} + (\dot{\lambda}_t - \dot{\mu}_t)\alpha'(z_t)S_tI_t + (\lambda_t - \mu_t)(\alpha''(z_t)\dot{z}_tS_tI_t + \alpha'(z_t)\dot{S}_tI_t + \alpha'(z_t)S_t\dot{I_t}) = 0$

Next, combine 2) and 3) to obtain

$$c'(I_t)e^{-(\delta+\rho)t} - (\lambda_t - \mu_t)(\alpha(z_t)S_t - \beta + \alpha(z_t)I_t) = \dot{\lambda}_t - \dot{\mu}_t$$

Then, observe from 1) that

$$\lambda_t - \mu_t = \frac{f'(x_t)e^{-(\delta+\rho)t}}{\alpha'(z_t)S_tI_t}$$

Inserting the two last expressions into 1)′, arrive at

$$(\delta + \rho) + \frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(x_t)} + \alpha(z_t)I_t - \alpha(z_t)S_t + \beta + \frac{\alpha''(z_t)\dot{z}_t}{\alpha'(z_t)} + \frac{\dot{S}_t}{S_t} + \frac{\dot{I}_t}{I_t} = 0$$

Recognizing that $\frac{\dot{S}_t}{S_t} = -\alpha(z_t)I_t$ and that $\frac{\dot{I}_t}{I_t} = \alpha(z_t)S_t - \beta$, the above expression reduces to

$$(\delta + \rho) + \frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(x_t)} + \frac{\alpha''(z_t)\dot{z}_t}{\alpha'(z_t)} = 0$$

Rearranging terms and defining $\frac{\partial f(x_t)}{\partial z_t} \equiv f'(z_t) = -f'(x_t)$, arrive at

$$4)\ \frac{c'(I_t)\alpha'(z_t)S_tI_t}{f'(z_t)} - \frac{\alpha''(z_t)\dot{z}_t}{\alpha'(z_t)} = (\delta + \rho)$$

For periods where $z_t = 0$, we have that $\gamma_t > 0$.

Appendix B

Functional forms and optimality conditions of the numerical model

Cost of infected individuals: $c = \bar{c}I$, where $\bar{c}$ is a positive constant

Cost of control: $\bar{f}z$, derived from the gross production function: $f = \bar{f}(\bar{y} - z)$. The marginal cost $\bar{f}$ is a positive constant.

Transmission rate function from susceptible individuals to infective individuals: $\alpha(z) = e^{-(g+hz)}$, where g and h are positive scalars

Transmission rate from susceptible individuals to removed individuals: $\beta \geq 0$

The SI model:

Optimality conditions

$$\frac{\bar{c}he^{-(g+hz_t)}S_tI_t}{\bar{f}} - h\dot{z}_t = (\delta + \rho)$$

Equations of motion

$$\dot{I}_t = e^{-(g+hz_t)}S_tI_t$$
$$\dot{S}_t = -e^{-(g+hz_t)}S_tI_t$$

The SIS model:

Optimality conditions

$$\frac{\bar{c}he^{-(g+hz_t)}S_tI_t}{\bar{f}} - h\dot{z}_t = (\delta + \rho)$$

Equations of motion

$$\dot{I}_t = e^{-(g+hz_t)}S_tI_t - \beta I_t$$
$$\dot{S}_t = -e^{-(g+hz_t)}S_tI_t + \beta I_t$$

The SIR model:

$$\frac{\bar{c}he^{-(g+hz_t)}S_tI_t}{\bar{f}} + \frac{I_te^{-(g+hz_t)}\beta}{S_te^{-(g+hz_t)} - \beta} - h\dot{z}_t = (\delta + \rho)$$

Equations of motion

$$\dot{I}_t = e^{-(g+hz_t)} S_t I_t - \beta I_t$$
$$\dot{S}_t = -e^{-(g+hz_t)} S_t I_t$$
$$\dot{R}_t = \beta I_t$$

Parameter values applied for calculations in the numerical examples as presented in Fig. 2, Fig 3. and Fig. 4: $S_0 = 95, I_0 = 5, R_0 = 0, \bar{c} = 1, \bar{f} = 2000$, $\beta = 0.007, g = 7.8, h$ =1.3, $(\delta + \rho)$ take on different values.

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