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Simon Martin, Alexander Rasch

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Collusion by Algorithm: The Role of Unobserved Actions

Abstract

We analyze the effects of better algorithmic demand forecasting on collusive profits. We show that the comparative statics crucially depend on the whether actions are observable. Thus, the optimal antitrust policy needs to take into account the institutional settings of the industry in question. Moreover, our analysis reveals a dual role of improving forecasting ability when actions are not observable. Deviations become more tempting, reducing profits, but also uncertainty concerning deviations is increasingly eliminated. This results in a u-shaped relationship between profits and prediction ability. When prediction ability is perfect, the 'observable actions' case emerges.

JEL-Codes: L410, L130, D430.

Keywords: algorithm, collusion, demand forecasting, unobservable actions, secret price cutting.

Simon Martin
Duesseldorf Institute for Competition Economics, (DICE), Duesseldorf University
Germany – 40225 Duesseldorf
simon.martin@dice.hhu.de

Alexander Rasch
Duesseldorf Institute for Competition Economics, (DICE), Duesseldorf University
Germany – 40225 Duesseldorf
rasch@dice.hhu.de

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1 Introduction

The digitization of the economy in recent years has led to an ever-growing massive accumulation of data and the development of tools to analyze big data. Today firms' decision-making is often based on autonomous algorithms, or supported by machine learning and artificial intelligence. The use of such digitally advanced tools has raised the suspicion of antitrust and competition authorities all over the world because it may give rise to anti-competitive conduct (see, for example, OECD, 2017). One of the key concerns is that firms can make use of algorithms, artificial intelligence, and machine learning to facilitate collusion, leading to a substantial loss in customer welfare (Calvano et al., 2019, Calvano et al., 2021, Calvano et al., 2020a, Assad et al., 2020, Assad et al., 2021, Miklós-Thal and Tucker, 2019, Johnson et al., 2020).[1]

We contribute to the ongoing and intensifying debate on this topic by setting up a theoretical model that focuses on one essential aspect of the use of algorithms: more precise demand forecasting through better data analysis to improve pricing. Indeed, Ferreira et al. (2016) stress the importance of machine-learning techniques to estimate historical lost sales and predict future demand of new products to optimize pricing decisions. This idea is mirrored by recent developments in the industry (Chase, 2013, Feng and Shanthikumar, 2018).

We use these observations as a starting point to investigate how improvements in forecasting technologies impact firms' ability to tacitly coordinate their pricing in an infinitely repeated game. While Miklós-Thal and Tucker (2019) and O'Connor and Wilson (2021) have already investigated similar questions in frameworks with observable and unobservable actions this is the first paper that provides a unified framework and, hence, allows to analyze the *interactive effect* of prediction ability and action observability. Unobserved price-setting is relevant in many important and large markets.

We build on the model by Baye and Morgan (2001) to extend and reinterpret their set-up by introducing two states of the world and prediction ability as in Miklós-Thal and Tucker (2019). In each period, there are two states of the world in which two firms potentially face high demand or low (zero) demand. Algorithms allow firms to forecast more accurately whether demand will be high or low. Firms receive an imperfect common signal about the state of the world. If demand is high, a share of the customers are loyal to one firm, whereas the remaining share are flexible; flexible customers buy from the cheapest firm they know of. To inform flexible customers, firms can promote their prices after learning about the state of the world. Possible interpretations of promotions include building up a sales force and advertising. It is assumed that firms cannot engage in price

[1] An example in real-life markets is the *US v. Topkins* case. Allegedly, sellers of posters and similar wall décor used pricing algorithms to coordinate their price-setting on the Amazon Marketplace (see Mehra, 2016 for an in-depth account of this case).

discrimination.[2] Because promotions are costly, firms face a trade-off between attracting a small loyal customer base at high prices and also appealing to flexible customers at lower prices and spending resources on promotional activities. The latter is more attractive when a favorable signal about the state was received, and the more so the more precise the signal is. Promotions can expand a given firms' demand, but only at the expense of the demand of the other firm; total demand is assumed to be fixed throughout.

In efficient collusive arrangements, the firms seek to sustain an equilibrium in which no resources are wasted on promotions, and monopoly prices are charged. When actions are observable, such collusive arrangements can be supported by grim-trigger strategies when firms are sufficiently patient. We present the simplest possible model in which prediction ability has no effect when actions are observable.

The crucial role of imperfect monitoring, that is, whether actions are publicly observable or not, has been recognised at least since the seminal work of Green and Porter (1984). Ceteris paribus, imperfect monitoring makes collusion harder to sustain because firms can no longer respond to deviations immediately. Instead, they need to infer possible deviations from own sales, which is an imperfect signal, that is, firms face a difficult signal extraction problem.

Starting from these basic building blocks, we show that there are novel interactive effects. In particular, we show that in a situation in which collusion can be sustained under both observable and unobservable actions, collusive profits with observable actions are always higher (and do not change in signal precision in our setting). By contrast, collusive profits under unobservable actions follow a u-shaped pattern as signal precision increases. Colluding firms always fully extract customers' valuation and never promote their products. The u-shaped pattern under unobservability can then be explained by the relative strength of two opposing effects. First, there is a poaching effect: As signal precision gets better, deviating to a lower price and promoting it becomes more profitable. As a result, it tends to be more difficult to sustain collusion. Second, there is a monitoring effect: As the signal becomes more precise, entering an uncalled-for punishment phase becomes less likely. As a consequence, collusion becomes more profitable, and collusion tends to be facilitated. For a less precise signal, the first effect dominates, and collusive profits decrease with higher precision levels; otherwise, collusive profits increase.

Moreover, we find that higher prediction accuracy can lead to higher customer surplus. More precisely, there is an inverse u-shaped relationship between signal precision and customer surplus. This puts some of the aforementioned concerns of competition authorities about algorithms into question. Moreover, our model provides an explanation for why we see considerable firm investments into better machine learning tools in certain industries, but less so in other industries. Additionally, our model delivers novel

[2] The idea is that even online, capabilities for targeted advertising are still limited, and a rationale for a fixed, uniform investment remains.

predictions about the frequency and trigger of price wars (see Green and Porter, 1984, Rotemberg and Saloner, 1986, Ellison, 1994).

Related literature

We contribute to the literature as follows. We add to the emerging literature that analyzes the relationship between collusion and the use of algorithms. One strand of this literature has mainly addressed the question how prices develop in an experimental setting when simple learning algorithms are used (Calvano et al., 2020b, Klein, 2021, Normann and Sternberg, 2021, Johnson et al., 2020). These studies find that although supra-competitive prices can sometimes be sustained, algorithms may take very long to learn to coordinate. Moreover, as demonstrated by Asker et al. (2021), outcomes crucially depend on the learning protocol used. Calvano et al. (2021) consider the case of imperfect monitoring.[3]

The paper most closely related to our paper is Miklós-Thal and Tucker (2019). In contrast to our paper, Miklós-Thal and Tucker (2019) build on the classic set-up of Rotemberg and Saloner (1986) with perfect monitoring. They analyze the impact of better demand predictions on the sustainability of collusion in a situation in which rivals' price-setting can be observed. Their findings are less skeptical than the experimental results in Calvano et al. (2020b) and Klein (2021): In the model by Miklós-Thal and Tucker (2019), better predictive power can lead to higher customers surplus. In some cases, better predictive power can also lead to lower profits. We complement Miklós-Thal and Tucker (2019) by considering the role of unobserved actions, and show that in that case, lower profits as a result of better algorithms arise. Additionally, our model delivers novel predictions about the occurrence and length of price wars. Martin and Schmal (2021) extend the model of Miklós-Thal and Tucker (2019) by allowing for different collusive compensation schemes and analyzing the interactive effect with prediction ability. They find that higher prediction ability can make collusive agreements without side payments more attractive, calling for novel regulative measures.

In O'Connor and Wilson (2021), actions are unobservable. There are four states of the world. Better prediction ability only improves the signal precision in one dimension, and the other dimension is assumed to be entirely orthogonal. Similar to our findings, they show that better prediction ability can have ambiguous effects on collusive prices. In contrast to our setting in which we allow for arbitrary degrees of prediction ability, they only consider the extreme cases of no or perfect ability in one dimension. Additionally, our framework is more tractable, and we obtain closed-form expressions throughout, whereas some of their results can only be derived by means of numerical examples. In addition, we focus on the contrasting effects under observability within a unified framework.

[3]Gautier et al. (2020) discuss the technical challenges with regard to tacit collusion (and price discrimination). Harrington (2018), Schwalbe (2018), and Ezrachi and Stucke (2020), among others, focus on legal aspects and issues with regard to a change in competition policy.

The capacity constraints in the collusion model of Compte et al. (2002) play a role to the market expansion due to promotional activities in our model. We endogenize the decision to promote and embed it in a framework with prediction ability and action (un-)observability. In contrast to Harrington (2022), we do not, however, endogenize the decision to use algorithms for demand forecasting. In our environment, there is always a unilateral incentive to use better algorithms.

Similar to our finding, also in Peiseler et al. (2021), the firms' ability to collude decreases once algorithms are sufficiently precise. However, in contrast to our paper, in their model, better algorithms facilitate third-degree price discrimination. In our model, the discrimination aspect is absent, and prediction accuracy is about the entire industry environment. In Liu and Serfes (2007), collusion becomes harder to sustain when firms become better in segmenting markets.

A quickly emerging strand of the literature also establishes the empirical relevance of algorithmic collusion (see, for example, Brown and MacKay, 2020, Assad et al., 2020, and Wieting and Sapi, 2021).

The remainder of the paper is structured as follows. We present our model in Section 2. We analyze the stage game and the infinitely repeated game with tacit collusion in Sections 3 and 4. Section 5 concludes.

2 Model

We set up an infinitely repeated game based on Baye and Morgan (2001). We extend and reinterpret their model for our application by introducing two states of the world and prediction ability as in Miklós-Thal and Tucker (2019).[4] There are two firms that have a common discount factor δ. In each period of the infinitely repeated game, there are two states of the world, denoted by H and L; both states are ex ante equally likely. Before taking actions, both firms receive a common signal $s \in \{h, l\}$ with precision $\rho \in [1/2, 1]$ about the state of the world. Thus, the posterior is also given by $\Pr(H|h) = \Pr(L|l) = \rho$.

In state L, there is no demand at all, irrespective of the prices (there are no customers, or, equivalently, all customers have a valuation of 0). In state H, there is a total mass 1 of customers. A fraction λ of customers are *flexible*, which means that they have valuation v for both products. The customers are potentially attracted by promotional activities of the firms and purchase from the cheapest firm they are aware of. If no prices are promoted, they visit a firm randomly and purchase as long as the price does not exceed v. The remaining $1 - \lambda$ customers are *loyal* to one firm, split equally, and have valuation v for this firm only. All customers are short-lived and leave the market after one period.

After receiving the common signal s about the state of the world, firms simultane-

[4]Simpler versions of the model are discussed in a handbook chapter in Baye et al. (2006) and in the textbook by Belleflamme and Peitz (2015).

ously set their prices p_s and also decide whether to promote their prices at fixed cost F.[5] Firms cannot identify customer segments ex ante; hence, firms can neither target their promotional efforts nor charge discriminatory prices. Throughout, we assume that marginal costs are zero.

As a benchmark, note that a monopolist would simply always set a price v and never promote its price. Promotional activities in this setting are socially inefficient. When firms compete, they face the following trade-off. First, they need to consider whether promotional activities at cost F pay off. This is more likely to be the case when the good signal h was received. Second, conditional on promoting, the pricing decision entails a trade-off between charging high prices and appropriating high rents from the loyal customer segment, and charging low prices that potentially also attract flexible customers.

We make the following assumptions:

Assumption 1.

$$\frac{\lambda v}{4} \leq F \leq \frac{\lambda v}{2}$$

and

$$\rho \geq \bar{\rho} := \frac{2F}{\lambda v}.$$

As the subsequent analysis will make clear, $F \geq \lambda v/4$ implies that in case the signal is entirely uninformative ($\rho = 1/2$), firms never promote and simply set the monopoly price v after both signals in the static Nash equilibrium. $F \leq \lambda v/2$ implies that when the signal is fully informative ($\rho = 1$), firms have an incentive to promote and potentially attract flexible customers. These conditions imply that prediction ability plays a relevant and interesting role. If this condition failed, fixed costs F for promotional activities could not even be recovered when firms are certain that demand conditions are favorable, so prediction ability would never play any role.

From these considerations it follows that there exists a cutoff point $\bar{\rho}$ such that promoting becomes attractive once $\rho > \bar{\rho}$. Note that the first part of Assumption 1 implies that $\bar{\rho} \in (1/2, 1)$.

This completes the description of the stage game. For the infinitely repeated version of the model, we first consider tacit collusion with observable actions (perfect monitoring). We are interested in the most profitable outcome sustainable in a subgame-perfect equilibrium supported by grim-trigger strategies.[6] Additionally, we consider the case with

[5]Note that *simultaneity* refers to the informational status of the competing firm and of customers. The firm can condition its own price on its own promotional choice when the promotional choice is in mixed strategies.

[6]We could easily incorporate optimal punishments in the setting with perfect monitoring, but such an adjustment is less straightforward under imperfect monitoring. Thus, we present our main results using

unobservable actions (imperfect monitoring) that entails finite-duration punishment on the equilibrium path as in Green and Porter (1984) or in the secret price-cutting version by Tirole (1988).

Model discussion

A few comments with regard to the interpretation of the model are in order. In the original setting of Baye and Morgan (2001), promotional activities take place on a central 'clearinghouse', accessible by a fraction of 'shopping' customers. In our environment with tacit collusion under imperfect monitoring, we prefer the interpretation that advertising does not take place through a central institution but is rather towards customers directly. Thus, we use the term 'promotion' throughout, with the implicit understanding that this subsumes all activities related to increasing sales. This could, for example, include approaching customers directly, targeted coupons and advertising (which are particularly relevant in online environments), building up a stronger local sales force, or investing into the logistic network. We thereby preserve the incentive to secretly deviate from a possible collusive agreement while leaving the key mechanics of the model unchanged.

3 Stage game

In this section, we present an analysis of the Nash equilibrium of the stage game, which also serves to illustrate the main forces of the model.

As a starting point, consider symmetric candidate equilibrium prices $p_s = v$ after signal s, $s \in \{h, l\}$, assuming that neither firm pays the promotion costs F.[7] The signal structure induces a posterior $\Pr(H|s)$. If these prices constitute an equilibrium, then both loyal and flexible customers split equally across firms, resulting in expected per-firm profits

$$\Pr(H|s)\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\right)v = \Pr(H|s)\frac{v}{2}.$$

Given this behaviour of the other firm, the most profitable deviation is to promote and charge a slightly lower price that attracts all the flexible customers. Such a deviation is not profitable as long as the following inequality holds:

$$\Pr(H|s)\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\right)v \geq \Pr(H|s)\left(\frac{1-\lambda}{2}+\lambda\right)v - F. \tag{1}$$

Our assumptions on v and F imply that such a deviation is never profitable when signal

grim-trigger strategies for better comparability of different observability environments.

[7] As pointed out by Baye and Morgan (2001), there are no asymmetric equilibria in the two-firm model.

$s = l$ was received because we can simplify expression (1) as

$$(1-\rho)\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\right)v \geq (1-\rho)\left(\frac{1-\lambda}{2}+\lambda\right)v - F$$
$$\Leftrightarrow F \geq \frac{(1-\rho)\lambda}{2}v,$$

which holds for all admissible ρ. Thus, $p_l = v$ is sustainable in a Nash equilibrium.

After signal h, however, condition (1) becomes

$$\rho\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\right)v \geq \rho\left(\frac{1-\lambda}{2}+\lambda\right)v - F$$
$$\Leftrightarrow F \geq \frac{\lambda\rho}{2}v,$$

which fails for all $\rho \geq \bar{\rho}$.[8] Thus, $p_h = v$ is not sustainable in a Nash equilibrium.

Moreover, any other pure-strategy candidate price $p'_h < v$ without promotion cannot be an equilibrium. If this was an equilibrium, only loyal customers would purchase from each firm. But since these customers have a willingness to pay of v, there would be profitable upward deviation to charging a price $p_h = v$ instead of p'_h. Thus, there is no pure-strategy equilibrium after signal h. Instead, the Nash equilibrium that is played after signal h was received entails mixing, both in the choice of prices and the promotion activity.

The exact characterization of the Nash equilibrium and Nash equilibrium profits is presented in the following proposition:

Proposition 1. *The equilibrium prices depend on the signal received:*

(i) If firms receive signal l, they set $p_l = v$.

(ii) If firms receive signal h, there is mixing in prices and promotion: Firms promote with probability $\alpha_N = 1 - 2F/\lambda\rho v$. When firms promote, they draw prices p_h from a continuous and atomless price distribution $G(p)$ with upper bound v; otherwise, they set a deterministic price $p_h = v$.

The resulting per-firm profit is given by

$$\pi_N = \frac{(1-\lambda\rho)v + 2F}{4},$$

which decreases in prediction ability ρ.

[8]The condition holds for all $\rho < \bar{\rho}$. Thus, in that case, $p_l = p_h = v$ represents a Nash equilibrium, implying that prediction ability does matter at all. We, hence, focus on the more interesting case in which $\rho \geq \bar{\rho}$, and prediction ability matters.

Proof. (i) Conditional on signal l, prices are $p_l = v$, and expected profits are given by $(1-\rho)v/2$. Given Assumption 1, there is no profitable deviation.

(ii) The analysis after signal h is similar to the proof in Baye and Morgan (2001), adjusted for state uncertainty. As the above considerations show, there is no pure-strategy equilibrium after signal h. When firms mix, they must be indifferent between charging v and not promoting on the one hand, and charging v but promoting on the other hand (Baye and Morgan, 2001). We obtain the equilibrium promotion probability α through the indifference condition

$$\rho\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\underbrace{(1-\alpha)}_{\text{other firm does not promote}}\right)v = \rho\left(\frac{1-\lambda}{2}+\lambda(1-\alpha)\right)v - F$$

which yields

$$\alpha_N = 1-\frac{2F}{\lambda\rho v}.$$

We can plug this α into the equilibrium profits and obtain that conditional on signal h, expected profits are given by

$$\rho\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}(1-\alpha)\right)v = \frac{1-\lambda}{2}\rho v + F.$$

Since ex-ante both signals are equally likely, we obtain that

$$\pi_N = \frac{\frac{(1-\rho)v}{2}+\frac{(1-\lambda)\rho v}{2}+F}{2} = \frac{(1-\lambda\rho)v+2F}{4}.$$

Furthermore, it must hold that firms obtain the same profit for all prices on the equilibrium support if they promote. We can readily solve for the equilibrium price distribution $G(p)$ as the solution to:

$$\underbrace{\rho\left(\frac{1-\lambda}{2}+\lambda(1-\alpha)\right)v-F}_{\text{promote and charge } v} = \underbrace{\rho\left(\frac{1-\lambda}{2}+\lambda(1-\overbrace{\alpha G(p)}^{\text{other firm promotes and charges } p'\leq p})\right)p-F}_{\text{promote and charge } p\leq v}$$

so we obtain

$$G(p) = \frac{2\alpha\lambda v-(1-\lambda)(v-p)}{2\alpha\lambda p}. \tag{2}$$

Setting $G(p)=0$, we obtain the lower bound of the price distribution $\underline{p}$ as the solution to

$$G(\underline{p}) = 0$$

which yields

$$\underline{p} = \frac{1+\lambda(1-2\alpha)}{1+\lambda} v = \frac{\rho v(1-\lambda)+4F}{\rho(1+\lambda)}.$$

The argument that the price distribution G is continuous and atomless follows directly from Baye and Morgan (2001) and, hence, is not repeated. □

Note that Nash equilibrium profits π_N decrease in prediction ability ρ. The incentive to engage in promotional activities after signal h increases in ρ because favorable market conditions become increasingly likely. Thus, firms increasingly invest in promotions, and when they do, they price more aggressively. Thus, equilibrium profits are reduced.

Given equilibrium pricing, we can also compute customer surplus and total welfare, defined as the sum of total profits and customer surplus. Customer surplus is 0 whenever the state is L, irrespective of firm pricing. In state H, all customers purchase. If firms additionally received signal h, which happens with probability ρ, firms promote with probability $\alpha > 0$ and set a stochastic price $p \leq v$. In that case, flexible customers purchase at the expected minimum price of these two stochastic prices (denoted by $E^{mix}_{min}(p)$), whereas loyal customers purchase at the expected price from one firm (denoted by $E^{mix}(p)$).

We are now ready to characterize customer surplus and total welfare in the following proposition:

Proposition 2. *In Nash equilibrium, surplus of flexible and loyal customers given by*

$$\begin{aligned} CS_{flexible} &= \rho \frac{2\alpha(1-\alpha)(v-E^{mix}(p)) + \alpha^2(v-E^{mix}_{min}(p))}{2} \quad \textit{and} \\ CS_{loyal} &= \rho \frac{\alpha(v-E^{mix}(p))}{2}, \end{aligned}$$

respectively, resulting in total customer surplus

$$CS_N = \frac{(\rho\lambda v - 2F)^2}{2\rho\lambda v}$$

and total welfare

$$TS_N = \frac{v-2F}{2} + \frac{2F^2}{\rho\lambda v}.$$

Customer surplus increases in prediction ability ρ, but total surplus decreases in prediction ability ρ due to socially wasteful promotions.

Proof. We first characterize the expected price and the expected minimum price conditional on mixing.

The density g associated with the cumulative distribution function G from Proposition 1 is given by

$$g(p) = \frac{1+\lambda(1-2\alpha)}{2\alpha\lambda p^2}v = \frac{\rho v(1-\lambda)+4F}{2\lambda\rho v - 4F}\frac{v}{p^2},$$

where we use $\alpha = 1 - 2F/\lambda\rho v$ from Proposition 1. Then we can compute

$$\begin{aligned}
E^{mix}(p) &= \int_{\underline{p}}^{v} pg(p)dp = v\frac{\rho v(1-\lambda)+4F}{2\lambda\rho v - 4F}\int_{\underline{p}}^{v}\frac{1}{p}dp \\
&= v\frac{\rho v(1-\lambda)+4F}{2\lambda\rho v - 4F}\left(\log(p)\Big|_{\underline{p}}^{v}\right) \\
&= v\frac{\rho v(1-\lambda)+4F}{2\lambda\rho v - 4F}\left(\log(v) - \log\left(\frac{4F+\rho v(1-\lambda)}{\rho(1+\lambda)}\right)\right).
\end{aligned}$$

The density of the expected minimum price by $g_{min}(p) = 2(1-G(p))g(p)$ since $G_{min}(p) = 1-(1-G(p))^2$. Evaluating these expressions leads to

$$\begin{aligned}
E^{mix}_{min}(p) &= \int_{\underline{p}}^{v} pg_{min}(p)dp \\
&= v\frac{((\lambda-1)\rho v - 4F)\left((4F-(\lambda-1)\rho v)\left(\log(v)-\log\left(\frac{4F+\rho v(1-\lambda)}{\rho(1+\lambda)}\right)\right)+4F-2\lambda\rho v\right)}{2(\lambda\rho v - 2F)^2}.
\end{aligned}$$

For all customer segments, positive surplus is only possible whenever the state is H, and signal h was received at the same time, which happens with ex ante probability $\rho/2$. Flexible customers purchase from the cheapest of the two firms in case they promoted their products, whereas loyal customers always buy from one firm at random. Thus, customer surplus for flexible and loyal customers is given by

$$\begin{aligned}
CS_{flexible} &= \rho\frac{(1-\alpha)^2\cdot 0 + 2\alpha(1-\alpha)(v-E^{mix}(p)) + \alpha^2(v-E^{mix}_{min}(p))}{2} \quad \text{and} \\
CS_{loyal} &= \rho\frac{\alpha(v-E^{mix}(p)) + (1-\alpha)\cdot 0}{2}.
\end{aligned}$$

As a result, customer surplus in the equilibrium amounts to

$$CS_N = \lambda CS_{flexible} + (1-\lambda)CS_{loyal} = \frac{(\rho\lambda v - 2F)^2}{2\rho\lambda v}.$$

Customer surplus increases in ρ because

$$\begin{aligned}\frac{\partial CS_N}{\partial \rho} &= \frac{2\lambda v(\rho\lambda v - 2F)(2\rho\lambda v) - 2\lambda v(\rho\lambda v - 2F)^2}{(2\rho\lambda v)^2} \\ &= \frac{(\rho\lambda v - 2F)(2\rho\lambda v) - (\rho\lambda v - 2F)^2}{2\rho^2\lambda v} \\ &= (\rho\lambda v - 2F)\frac{2\rho\lambda v - \rho\lambda v + 2F}{2\rho^2\lambda v} \\ &= (\rho\lambda v - 2F)\frac{\rho\lambda v + 2F}{2\rho^2\lambda v}\end{aligned}$$

which is always positive since the fraction is strictly positive and $\rho \geq \bar{\rho}$ implies that

$$\rho\lambda v - 2F \geq \bar{\rho}\lambda v - 2F = 0.$$

Total surplus as the sum of firms' profits and customer surplus is equal to

$$TS_N = 2\pi_N + CS_N = \frac{v - 2F}{2} + \frac{2F^2}{\rho\lambda v}, \tag{3}$$

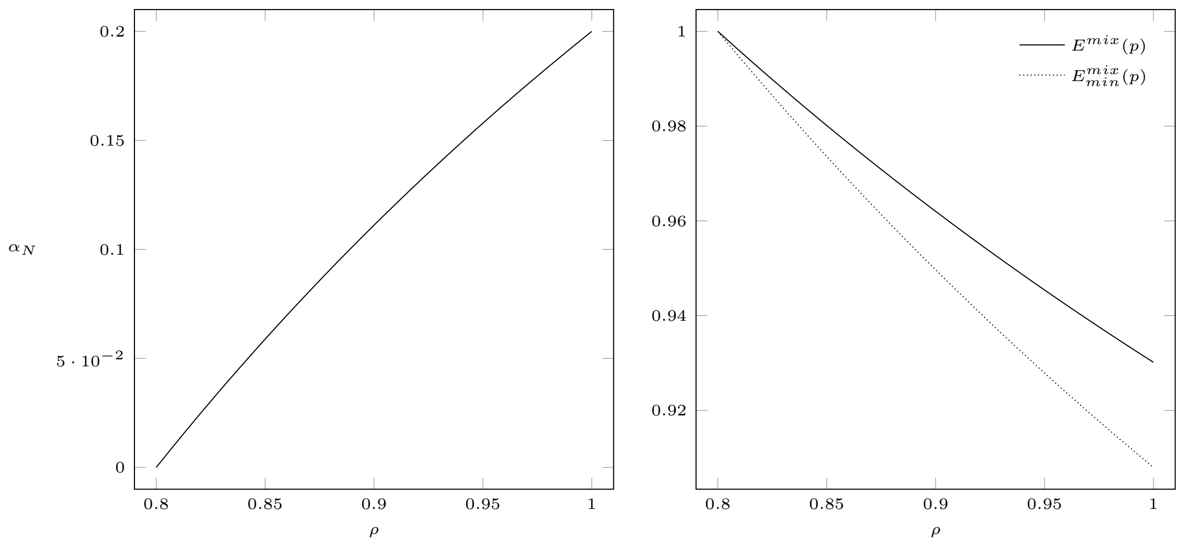
which decreases in ρ. □

Proposition 2 shows that in the Nash equilibrium of the stage game, customers gain from higher prediction ability, whereas total surplus decreases. The gain in customer surplus follows from both the firms' more aggressive promotional activities and pricing. Total surplus, however, decreases, that is, customers do not benefit as much as firms lose. Although price reductions benefit customers directly, promotional activities do not because they entail additional fixed costs F. These costs are purely wasteful from a total welfare point of view. Thus, despite the fact that customers in our model have unit demand, and, hence, there is no deadweight loss of higher prices, higher prediction ability has negative welfare consequences in the static model.

We illustrate these results by making use of a numerical example that we will use throughout the paper.

Example 1. We fix the following parameter values. Let $v = 1$, $\lambda = 1/2$ and $F = 1/5$ (note that for these values of v and λ, Assumption 1 requires that $1/4 \geq F \geq 1/8$). Then we have that $\bar{\rho} = 2F/\lambda v = 4/5 = 0.8$. In the following figures, we use this parametrization. In the left panel of Figure 1, we show the promotion probability α in Nash equilibrium, as a function of prediction ability ρ. As ρ increases, the incentive to promote becomes stronger after signal h was received, resulting in more frequent promotions. Similarly, firms price more aggressively whenever they promote, resulting in lower expected prices and expected minimum prices when ρ increases.

In Figure 2, we depict profits, customer surplus, and total welfare (left panel), as well as customer surplus per customer group (right panel), for the Nash equilibrium described



(a) Promotion probabilities.

(b) Expected prices conditional on promotion.

Figure 1: Promotion and pricing in the competitive equilibrium.

in Proposition 1, as a function of prediction ability ρ. As ρ increases, more aggressive pricing reduces profits and increases customer surplus (left panel). Because promotions also get more frequent, which is socially inefficient, total welfare decreases in ρ. The right panel shows that customer surplus of flexible customers is always higher than that of loyal customers because they get the possibility to purchase from the lower priced firm in case of promotions.

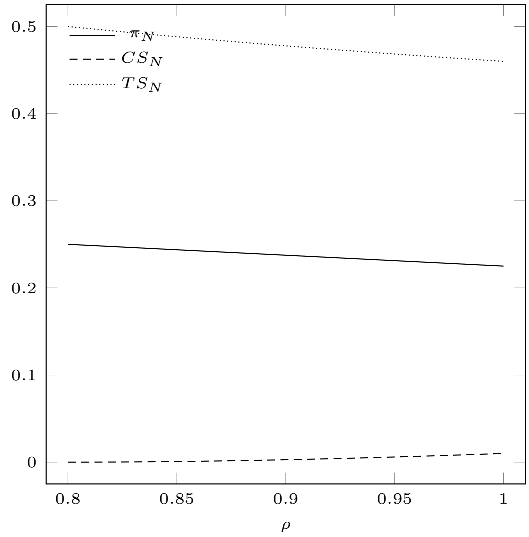
4 Tacit collusion

We now turn to the question whether tacit collusion can be sustained as a subgame-perfect equilibrium, and if so, what are the highest possible collusive profits. We distinguish between two scenarios: (i) observable actions or perfect monitoring, where firms can observe the price set by the other firm in the previous periods, and (ii) unobservable actions or imperfect monitoring, where firms cannot observe prices.

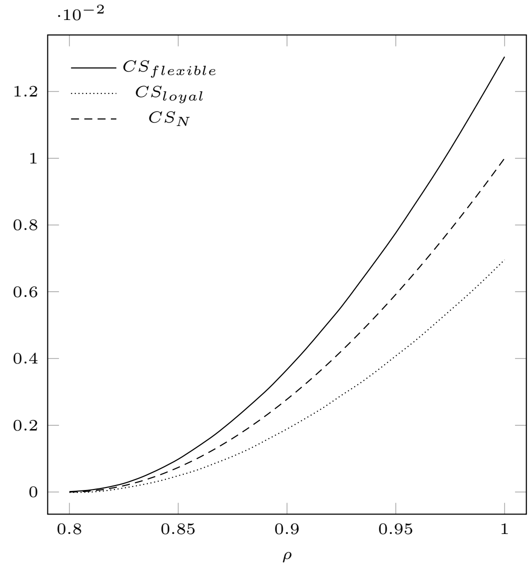
4.1 Observable actions

In this section, we investigate the most profitable equilibrium that can be supported in the infinitely repeated game with observable actions (perfect monitoring) in a subgame-perfect equilibrium with grim trigger strategies.[9] Deviations from an implicit collusive agreement are deterred through the threat of grim trigger strategies, that is, once any player deviates, both players revert to static Nash equilibrium play forever (Friedman, 1971).

[9]As discussed above, we do not consider optimal punishments for better comparability with the unobservable actions setting.



(a) Profits, customer surplus, and total welfare.



(b) Customer surplus (in total and by group).

Figure 2: Outcomes in the competitive equilibrium.

On the equilibrium path, both players set a common price p_s after signal s was received, and never promote their product because this is wasteful from a joint profit maximization point of view. Thus, an equilibrium consists of a pair (p_l, p_h) and the threat of Nash reversion. As we discussed in Section 3, there is never an incentive to deviate from prices $p_l = v$ after signal l was received. When signal h was received and $\rho \geq \bar{\rho}$, there is an incentive to deviate to promoting and undercutting the competitor. These possible deviations are deterred by the threat of Nash reversion when players are sufficiently patient. We characterize the precise conditions in the following proposition.

Proposition 3. *When actions are observable and $\delta \geq \underline{\delta}_{obs} = 2/3$, then prices $p_h = p_l = v$ and no promotions can be sustained for all ρ. The net present value of equilibrium profits is given by*

$$V_{obs} = \frac{v}{4(1-\delta)}. \tag{4}$$

Otherwise, no collusion can be sustained at all; especially, setting a lower price $p_h < v$ or opting for promotions does not help to sustain collusion.

Proof. We focus on the equilibrium candidate with $(p_l, p_h) = (v, p_h)$ and without promotions because we already established that there is never an incentive to deviate after signal l. Then, the net present value of equilibrium profits is given by

$$V_{obs} = \frac{E(\pi|h; p_h) + E(\pi|l)}{2 \cdot 2(1-\delta)} = \frac{[\rho p_h + (1-\rho) \cdot 0] + [(1-\rho)p_l + \rho \cdot 0]}{2 \cdot 2(1-\delta)} = \frac{\rho p_h + (1-\rho)v}{4(1-\delta)}$$

because ex ante both signals are equally likely, profits are shared equally in each-period, and the states are identically and independently distributed over an infinite horizon and are discounted with δ. Conditional on signal h, the most profitable deviation is promoting and slightly undercutting the competitor, which entails fixed cost F, but possibly attracts the entire demand of flexible customers if the state is H. This deviation is not profitable as long as the following incentive compatibility constraint holds:[10]

$$\rho\frac{p_h}{2} + \delta V_{obs} \geq \rho p_h \left(\frac{1-\lambda}{2} + \lambda\right) - F + \frac{\delta}{1-\delta}\pi_N \tag{5}$$

which we can write as

$$\underbrace{\rho p_h \frac{\lambda}{2} - F}_{\text{gains from deviation}} \leq \underbrace{\delta\left(V_{obs} - \frac{\pi_N}{1-\delta}\right)}_{\text{loss from deviation}}.$$

We can solve this condition for δ and obtain that the constraint holds as long as

$$\delta \geq \underline{\delta}(p_h) = \frac{4F - 2\lambda\rho p_h}{6F - \rho(p_h(1+2\lambda) - v(1-\lambda))}.$$

Note that $\underline{\delta}(p_h)$ decreases in p_h because

$$\begin{aligned}\frac{\partial \underline{\delta}(p_h)}{\partial p_h} &= \frac{(-2\lambda\rho)(6F - \rho(p_h(1+2\lambda) - v(1-\lambda))) + \rho(1+2\lambda)(4F - 2\lambda\rho p_h)}{(6F - \rho(p_h(1+2\lambda) - v(1-\lambda)))^2} \\ &= -\frac{2(1-\lambda)(\lambda\rho v - 2F)}{(6F - \rho(p_h(1+2\lambda) - v(1-\lambda)))^2} \leq 0,\end{aligned}$$

where the denominator is always positive, and the numerator is also positive since $\rho \geq \bar{\rho}$, and hence $\lambda\rho v - 2F \geq \lambda\bar{\rho}v - 2F = 0$.

Thus, the incentive compatibility constraint (5) is least restrictive for $p_h = v$, and, hence, sustainability of collusion at $p_h = v$ requires

$$\begin{aligned}\delta \geq \underline{\delta}(v) &= \frac{4F - 2\lambda\rho v}{6F - \rho(v(1+2\lambda) - v(1-\lambda))} \\ &= \frac{4F - 2\lambda\rho v}{6F - 3\lambda\rho v} \\ &= \frac{2(2F - \lambda\rho v)}{3(2F - \lambda\rho v} \\ &= \frac{2}{3} = \underline{\delta}_{obs}.\end{aligned}$$

[10]Note that this immediately implies that collusion can never become sustainable if collusion entails promotions in a situation in which it is not sustainable without promotions. In this case, the left-hand side of (5) becomes $\rho p_h/2 - F + \delta V_{obs}$, whereas the right-hand side remains unaffected, which makes the condition more restrictive.

Therefore, collusion with $p_l = p_h = v$ can be sustained when $\delta \geq \underline{\delta}_{obs} = 2/3$; otherwise, collusion cannot be sustained at all. If collusion can be sustained, then the net present value of profits is given by

$$V_{obs} = \frac{\rho p_h + (1-\rho)v}{4(1-\delta)} = \frac{v}{4(1-\delta)}.$$

□

Proposition 3 shows that sustainability of the most collusive outcome with monopoly prices and without promotions is independent of the prediction quality ρ. In contrast to models in which customer valuations differ across demand states (Rotemberg and Saloner, 1986, Miklós-Thal and Tucker, 2019), equilibrium prices do not adjust to the forecasted demand. Equilibrium prices are also independent of prediction ability ρ. The reasons for this are twofold. First, monopoly prices in our model are independent of the state: In both states, customers have unit demand and willingness to pay v. Second, incentives to deviate are independent of signal precision ρ. It is true that, as ρ increases, firms are, ceteris paribus, more inclined to deviate after signal h was received because market conditions are more likely to be favorable. However, as the analysis in Section 3 indicates, Nash equilibrium profits decrease in ρ, making the threat of punishment after a deviation stronger. These two effects exactly cancel out, and, hence, incentives to deviate are independent of ρ.

For the same reason, distorting equilibrium prices after a good signal does not help in making collusion more sustainable when monopoly prices cannot be sustained. Thus, when actions are observable, collusion at monopoly prices can be sustained; or collusion cannot be sustained at all.

This equilibrium price schedule appropriates all possibles rents from customers because the same equilibrium price $p_h = p_l = v$ is charged. Irrespective of whether the signal was correct or not, customers either pay exactly their willingness to pay or do not purchase because they do not value the good at all. Thus, customer surplus is always 0. Similarly, whenever collusion can be sustained, total welfare is independent of signal precision ρ because no resources are 'wasted' on socially inefficient promotions. We summarize these considerations in the following corollary:

Corollary 1. *When actions are observable and $\delta \geq \underline{\delta}_{obs} = 2/3$ holds, then in the most profitable equilibrium, customer surplus is given by $CS_{obs} = 0$, and total surplus is given by*

$$TS_{obs} = V_{obs} = \frac{4}{4(1-\delta)}.$$

Proof. See the equilibrium prices in Proposition 3 and the above considerations. □

4.2 Unobservable actions

In this section, we analyze how sustainability of collusion, prices, and profits change when we assume that actions are not observable. In this case, adherence to collusion must be inferred from observed own sales in a given period. Collusion becomes harder to sustain because firms can no longer distinguish with certainty whether own sales were low due to a low demand state or a deviating competitor (Green and Porter, 1984, Tirole, 1988).

A well-known ingredient in collusive models of this sort is that occasional 'price wars' occur with positive probability on the equilibrium path. For high enough discount factors, firms repeatedly cycle between a 'collusive phase' with higher prices and a 'punishment phase' (price wars). The threat of price wars, induced by low sales, serves to deter deviations. In contrast to the scenario with observable actions, firms no longer threaten to revert to Nash equilibrium play forever after a deviation because a deviation cannot be inferred with certainty. These price wars instead have a length of T periods. Optimally, the punishment must be as short as possible to maximize profits, but sufficiently long to deter deviations: Firms understand that if they deviate, they will inflict zero sales on their competitor, which triggers a price war of length T. As in Tirole (1988), common knowledge of zero sales for at least one player triggers the punishment phase. In our setting, only zero sales after signal h trigger a punishment phase. After signal l, firms understand that there was no incentive to start with, and hence also no punishments are required.

Intuitively, after signal l, there is never an incentive to deviate, so a price $p_l = v$ and no promotions are always sustainable. After signal h, however, firms might want to promote their product and marginally undercut their competitor. These considerations are an immediate consequence from Assumption 1 and the analysis of the stage game.

We define the following net present values of continuing collusion as follows: We denote by V_s the net present value when firms are currently in the collusive phase, and signal $s \in \{h, l\}$ was received, by V_p the net present value when firms are currently in the punishment phase, and by V the net present value from an ex ante point of view. Throughout, we investigate efficient equilibria, where $p_l = p_h = v$ and in which no promotions take place during the collusive phase. As before, the punishment length is denoted by T.[11]

We assume that firms start in the collusive phase. Since ex ante both states and, according to our signal structure, also both signals are equally likely, the ex ante net

[11]We allow T to be any positive real number and thereby ignore integer constraints. An alternative interpretation is that firms switch to Nash equilibrium play forever with positive probability, such that the resulting profits are identical to our construction with finite length-T punishment. This is for computational convenience only. Qualitatively similar results would obtain if we imposed integer punishment of length $\bar{T}$ instead, where $\bar{T} = \lceil T \rceil$.

present value of collusion is given by

$$V = \frac{V_h + V_l}{2}. \tag{6}$$

The continuation value during the collusive phase after signal h is given by

$$V_h = \underbrace{\rho\left(\frac{v}{2} + \delta V\right)}_{\text{state H}} + \underbrace{(1-\rho)(0 + \delta \overbrace{V_p}^{\text{punishment}})}_{\text{state L}}. \tag{7}$$

After signal h, positive sales materialize with probability ρ, and firms remain in the collusive phase; with the complementary probability $1-\rho$, the state is low, and the punishment phase is triggered. Similarly to the case with observable actions, promotions during the collusive phase only reduce collusive profits without reducing the incentive to deviate; hence, promotions can never be used to help to sustain collusion. After signal l, there is no incentive to deviate according to Assumption 1 and hence also no necessity to trigger punishment. Thus, the continuation value during the collusive phase after signal l is given by

$$V_l = Pr(H|l)\frac{v}{2} + \delta V = (1-\rho)\frac{v}{2} + \delta V. \tag{8}$$

After signal h was received, the most profitable deviation entails promoting the product and marginally undercutting the competitor to attract all flexible customers. The drawback is that this triggers the punishment phase with probability one. Thus, the following incentive compatibility constraint has to hold

$$V_h = \rho\left(\frac{v}{2} + \delta V\right) + (1-\rho)\delta V_p \geq \rho\left(\frac{1-\lambda}{2} + \lambda\right) v - F + \delta V_p \tag{9}$$

which can be written as

$$\underbrace{\rho v\frac{\lambda}{2} - F}_{\text{gains from deviation}} \quad \leq \quad \underbrace{\delta\rho(V - V_p)}_{\text{loss from deviation}}$$

Inspection of the incentive compatibility constraint illustrates the two main channels through with prediction ability ρ affects collusive outcomes when actions are unobservable. First, there is the direct effect that concerns immediate gains of deviation. This attempt to attract some of the customers otherwise captive to the competitor is reminiscent of poaching (Fudenberg and Tirole, 2000, Taylor, 2004, Shaffer and Zhang, 2002), so we call this the *poaching effect*. Because $(1-\lambda)/2+\lambda = (1+\lambda)/2 > 1/2$, the incentives to deviate increase in prediction ability ρ. Thus, the poaching effect makes collusion less sustainable and, hence, leads to lower collusive prices and profits.

Additionally, there is also a *monitoring effect*. When actions are unobservable, firms face the risk that even on the equilibrium path, they enter the punishment phase with positive probability. In expression (9), this is the following term: $(1-\rho)\delta V_p$. As prediction ability ρ increases, uncertainty is increasingly eliminated, so that staying on the equilibrium path becomes more profitable and, hence, less severe punishment suffices to deter deviations. As ρ goes to 1, we are back in the case with observable actions. Thus, the monitoring effect facilitates collusion and, hence, leads to higher collusive prices and profits.

Taken together, in an environment with unobservable actions, prediction ability affects two countervailing forces, that is, the competitive *poaching effect* and the anti-competitive *monitoring effect*. The following proposition specifies how exactly those two forces operate, and which equilibrium prices result when δ is sufficiently high. We separately analyze what happens then δ falls below the threshold in Section 4.2.1. We have:

Proposition 4. *Suppose actions are not observable, and* $\delta \geq \underline{\delta}_{unobs} = 2\lambda v/(\lambda v + 4F) \geq 2/3$*. Then, collusion is sustainable for all* $\rho \geq \bar{\rho}$*. In the most collusive equilibrium, firms start in the collusive phase in which they set* $p_h = p_l = v$ *and never promote. On the equilibrium path, there are finite duration punishment phases in which firms play according to the static Nash equilibrium from Proposition 1. The net present value of equilibrium profits is given by*

$$V_{unobs} = \frac{2F(1-\rho) + \rho(1-\lambda(1-\rho))v}{4\rho(1-\delta)}. \tag{10}$$

Proof. We start by determining the net present value of the punishment phase V_p, during which firms play the Nash equilibrium for T periods. Afterwards they continue with another collusive phase. Thus, V_p is given by

$$\begin{aligned} V_p &= \sum_{t=1}^{T} \delta^{t-1}\pi_N + \delta^T V \\ &= \frac{1-\delta^T}{1-\delta}\pi_N + \delta^T V. \end{aligned}$$

Introducing the substitution $\delta^T = \hat{\delta}$, we can rewrite this value as

$$V_p = \frac{1-\hat{\delta}}{1-\delta}\pi_N + \hat{\delta}V. \tag{11}$$

Finally, we must ensure that in equilibrium, the incentive compatibility constraint (9) holds; moreover, profit maximization requires that the incentive compatibility constraint is binding. As a result, we obtain

$$V_h = \rho\left(\frac{v}{2} + \delta V\right) + (1-\rho)\delta V_p = \rho v\left(\frac{1-\lambda}{2} + \lambda\right) - F + \delta V_p. \tag{12}$$

Combining the definitions of net present values V, V_h, V_l and V_p in (6), (7), (8), and (11) with the binding incentive compatibility constraint in (12), we obtain a system of five linear equations with five unknowns (V, V_h, V_l, V_p, and $\hat{\delta}$), which can readily be solved. We obtain:

$$\begin{aligned}
V &= \frac{2F(1-\rho) + \rho v(1-\lambda(1-\rho))}{4\rho(1-\delta)}, \\
V_h &= \frac{\rho v(\delta(\lambda+2), \rho - \delta(\lambda+1) - 2(\lambda+1)\rho + 2\lambda) - 2(\delta-2)F(\rho-1)}{4(\delta-1)\rho}, \\
V_l &= \frac{\rho v(\delta(\lambda+2)\rho - \delta(\lambda+1) - 2(\lambda+1)\rho + 2\lambda) - 2(\delta-2)F(\rho-1)}{4(\delta-1)\rho}, \\
V_p &= \frac{2F(\delta\rho + \delta - 2) - \delta\rho v(\lambda\rho + \lambda + 1) + 2\lambda\rho v}{4(\delta-1)\delta\rho}, \\
\hat{\delta} &= 1 + \frac{2(1-\delta)}{\delta(1-2\rho)}.
\end{aligned}$$

As a consequence, the net present value of equilibrium profits is $V = \frac{2F(1-\rho)+\rho v(1-\lambda(1-\rho))}{4\rho(1-\delta)}$, which is exactly V_{unobs} stated in the proposition text.

Because we used the substitution $\delta^T = \hat{\delta}$ to get linear equations, we can now solve for T and obtain

$$T = \frac{\log\left(1 + \frac{2(1-\delta)}{\delta(1-2\rho)}\right)}{\log(\delta)}.$$

To make collusion with unobservable actions sustainable, we need that T is a real number. This requires that

$$\begin{aligned}
1 + \frac{2(1-\delta)}{\delta(1-2\rho)} &\geq 0 \\
\delta(2\rho - 1) &\geq 2(1-\delta) \\
\delta &\geq \frac{2}{1+2\rho}
\end{aligned}$$

which needs to hold for all ρ. (In Proposition 7 in Section 4.2.1 we analyze separately the situation where this condition fails.) This condition is binding for $\rho = \bar{\rho}$, so we need that

$$\delta \geq \underline{\delta}_{unobs} = \frac{2\lambda v}{4F + \lambda v} \geq \frac{2}{3}$$

where the last inequality follows right away from Assumption 1. □

A few remarks concerning Proposition 4 are in order. When actions are not observable, the sustainability condition $\delta \geq \underline{\delta}_{unobs} = 2\lambda v/(\lambda v + 4F)$ depends on the structural parameters of the model, which is in contrast to the constant condition for the model with observable actions. $\underline{\delta}_{unobs}$ decreases in F, so collusion is harder to sustain when F is small. This is intuitive because if F is small, deviations in the form of poaching become very attractive for firms. Note that for all admissible values of F (as given by Assumption 1), we have that $\underline{\delta}_{unobs} \geq \underline{\delta}_{obs} = 2/3$, which means that – as usual – collusion is harder to sustain when actions are not observable.

We would like to point out here already that profits non-monotonically depend on prediction ability ρ when actions are not observable, which again is in contrast to the model with observable actions. This results from the non-linear interaction of the poaching and monitoring effects whose relative strengths depend on ρ. We elaborate on this non-monotonicity in detail in Proposition 5 below.

Finally, note that the punishment duration T decreases in prediction ability ρ. On the equilibrium path, the punishment phase is initiated whenever the bad state L materialized after the signal h was received, which happens with probability $1 - \rho$. From the firms' point of view, this is a pure mistake they would rather avoid; punishment was initiated for the sole purpose of deterring deviations. As ρ increases, this is less often the case, which makes adhering to collusion more appealing. Thus, as ρ increases, even the threat of less severe punishment suffices to deter deviations, and, hence, the length of the punishment period decreases.

We characterize the impact of ρ on collusive profits in the following proposition. To this end, we define

$$\hat{\rho} := \sqrt{\frac{2F}{\lambda v}} = \sqrt{\bar{\rho}}.$$

Proposition 5. *When actions are not observable and $\delta \geq \underline{\delta}_{unobs}$, collusive profits V_{unobs} decrease in ρ for $\rho \in [\bar{\rho}, \hat{\rho}]$, but increase in ρ for $\rho \in (\hat{\rho}, 1]$. For the boundary cases in which ρ goes to $\bar{\rho}$ or to 1, profits are the same with or without observability, that is, V_{unobs} goes to V_{obs}.*

Proof. We first rewrite V_{unobs} as

$$\begin{aligned} V_{unobs} &= \frac{2F(1-\rho) + \rho v(1 - \lambda(1-\rho))}{4\rho(1-\delta)} \\ &= \frac{2F\left(\frac{1}{\rho} - 1\right) + v(1 - \lambda(1-\rho))}{4(1-\delta)} \end{aligned}$$

and then take the derivative of V_{unobs} with respect to ρ. We obtain

$$\frac{\partial V_{unobs}(\rho)}{\partial \rho} = \frac{1}{4(1-\delta)} \left(-\frac{2F}{\rho^2} + \lambda v \right).$$

Evaluating the derivative at $\rho = \bar{\rho} = \frac{2F}{\lambda v}$ gives

$$\begin{aligned} \left. \frac{\partial V_{unobs}(\rho)}{\partial \rho} \right|_{\rho=\bar{\rho}} &= \frac{1}{4(1-\delta)} \left(-\frac{2F}{(2F)^2} (\lambda v)^2 + \lambda v \right) \\ &= \frac{1}{4(1-\delta)} \left(-\frac{(\lambda v)^2}{2F} + \lambda v \right) \\ &\leq \frac{1}{4(1-\delta)} \left(-\frac{(\lambda v)^2}{2\frac{\lambda v}{2}} + \lambda v \right) = 0, \end{aligned}$$

where we used the maximal admissible F according to Assumption 1. Thus, V_{unobs} decreases in ρ around $\rho = \bar{\rho}$. Similarly, we evaluate the derivative at $\rho = 1$ and obtain

$$\begin{aligned} \left. \frac{\partial V_{unobs}(\rho)}{\partial \rho} \right|_{\rho=1} &= \frac{1}{4(1-\delta)} (-2F + \lambda v) \\ &\geq \frac{1}{4(1-\delta)} \left(-2\frac{\lambda v}{2} + \lambda v \right) = 0 \end{aligned}$$

where we used the maximum admissible F according to Assumption 1. Thus, V_{unobs} increases in ρ around $\rho = 1$.

Because the derivative is negative at the lower bound $\rho = \bar{\rho}$, positive at the upper bound $\rho = 1$ and monotone in ρ, there has to be a unique root $\hat{\rho}$ in between, which is obtained as the solution to

$$\begin{aligned} \left. \frac{\partial V_{unobs}(\rho)}{\partial \rho} \right|_{\rho=\hat{\rho}} &= 0 \\ \frac{1}{4(1-\delta)} \left(-\frac{2F}{\hat{\rho}^2} + \lambda v \right) &= 0 \\ \hat{\rho} &= \sqrt{\frac{2F}{\lambda \rho}} = \sqrt{\bar{\rho}}. \end{aligned}$$

Our assumptions imply that $1/2 < \bar{\rho} < \hat{\rho} < 1$. Therefore, collusive profits V_{unobs} decrease in ρ for $\rho \in [\bar{\rho}, \hat{\rho}]$, but increase in ρ for $\rho \in (\hat{\rho}, 1]$.

The convergence result is obtained as follows. We evaluate V_{unobs} at $\rho = \bar{\rho} = \frac{2F}{\lambda v}$ and obtain

$$\begin{aligned}
V_{unobs}(\bar{\rho}) &= \frac{2F(1-\frac{2F}{\lambda v}) + \frac{2F}{\lambda v} v(1-\lambda(1-\frac{2F}{\lambda v}))}{4\frac{2F}{\lambda v}(1-\delta)} \\
&= \frac{2F(\frac{\lambda v}{2F}-1) + v(1-\lambda(1-\frac{2F}{\lambda v}))}{4(1-\delta)} \\
&= \frac{\lambda v - 2F + v - v\lambda + v\lambda\frac{2F}{\lambda v}}{4(1-\delta)} \\
&= \frac{\lambda v - 2F + v - v\lambda + 2F}{4(1-\delta)} \\
&= \frac{v}{4(1-\delta)} = V_{obs}.
\end{aligned}$$

Similarly, we evaluate V_{unobs} at $\rho = 1$ and obtain

$$\begin{aligned}
V_{unobs}(1) &= \frac{2F(1-1) + 1\cdot v(1-\lambda(1-1))}{4\cdot 1(1-\delta)} \\
&= \frac{v}{4(1-\delta)} = V_{obs}.
\end{aligned}$$

□

Proposition 5 shows that profits are u-shaped in prediction ability ρ when actions are not observable. When ρ is close to $\bar{\rho}$, the pro-competitive poaching effect is relatively weak because the expected gains from poaching are small. Conditional on the good state H, the wrong signal l is also obtained frequently, which ensures that no firms wants to deviate and, hence, never triggers a punishment phase either. Thus, profits similar to the observability case emerge.

Similarly, when ρ is close to 1, all uncertainty with regard to adherence to collusion is eliminated. Whenever a firm obtains zero demand when the good signal h was received, it is almost certain that this is due to a deviation, which, in turn, makes it easier to deter deviations. Thus, again, profits are similar to the observability case.

For intermediate values of ρ, profits initially decrease and then increase in ρ. When ρ increases when it is initially small ($\bar{\rho} < \rho < \hat{\rho}$), the poaching effect dominates the monitoring effect, leading to lower profits. Poaching is relatively attractive because punishment is very likely due to an imprecise signal. Precisely due to poor signal quality, the distinction between low demand and deviations is difficult, that is, monitoring is very weak.

Conversely, when ρ increases when it is already high ($\hat{\rho} < \rho < 1$), then the poaching effect is dominated by the monitoring effect.

Finally, we are interested in customer surplus and total welfare that result from tacit collusion with unobservable actions. We characterize them in the following proposition.

Proposition 6. *When actions are not observable, then in the most profitable collusive equilibrium, total customer surplus is given by*

$$CS_{unobs} = \frac{(1-\rho)(\lambda\rho v - 2F)^2}{2(1-\delta)\lambda\rho^3 v}, \tag{13}$$

and total surplus is given by

$$TS_{unobs} = \frac{4F^2(1-\rho) - 2F\lambda(2-\rho)(1-\rho)\rho v + \lambda\rho^2 v^2\left(\lambda(1-\rho)^2+\rho\right)}{2(1-\delta)\lambda\rho^3 v}. \tag{14}$$

Proof. To compute customer surplus, we proceed in a similar way as for the derivation of profits in Proposition 4. We introduce the quantities CS, CS_h, CS_l, and CS_p to denote ex ante customer surplus, customer surplus conditional on signal h and l during the collusive phase, and customer surplus in the punishment phase. Note that during the collusive phase when prices $p_h = p_l = v$ are set, customers never derive positive surplus. During the punishment phase of length T, however, customers derive surplus as in the static Nash equilibrium. Using the substitution $\hat{\delta} = \delta^T$ as introduced in the proof of Proposition 4, we get a system of four linear equations in four unknowns:

$$\begin{aligned}
CS &= \frac{CS_h + CS_l}{2}, \\
CS_h &= \rho(0 + \delta CS) + (1-\rho)\delta CS_p, \\
CS_l &= (1-\rho)0 + \delta CS, \\
CS_p &= \frac{1-\hat{\delta}}{1-\delta} CS_N + \hat{\delta} CS.
\end{aligned}$$

Solving this system gives

$$CS_{unobs} = CS = \frac{(1-\rho)(\lambda\rho v - 2F)^2}{2(1-\delta)\lambda\rho^3 v}.$$

Total welfare TS_{unobs} follows right away as

$$TS_{unobs} = 2V_{unobs} + CS_{unobs} = \frac{4F^2(1-\rho) - 2F\lambda(2-\rho)(1-\rho)\rho v + \lambda\rho^2 v^2\left(\lambda(1-\rho)^2+\rho\right)}{2(1-\delta)\lambda\rho^3 v}.$$

□

We now illustrate the tacit collusion results, allowing for both observable and unobservable actions, using the example introduced in Section 3.

Example 1 continued. Given the parameter values, $\bar{\rho} = 4/5 = 0.8$. Tacit collusion with unobservable actions is sustainable when $\delta \geq \underline{\delta}_{unobs} = 10/13 \approx 0.77$, and tacit collusion with observable actions is sustainable when $\delta \geq \underline{\delta}_{obs} = 2/3$. We let $\delta = 0.9$ such that under both observability assumptions, collusion is sustainable. Figure 3b shows industry profits ($2V$) and total welfare (left panel) and customer surplus (right panel) for observable and unobservable actions as a function of prediction ability ρ. When actions are observable, profits V_{obs} are constant in ρ: Firms make monopoly profits, irrespective of ρ, and, hence, customer surplus CS_{obs} is always 0, and total welfare TS_{obs} equals the industry profits $2V_{obs}$.

By contrast, when actions are unobservable, profits V_{unobs} are u-shaped in prediction ability ρ. For ρ close to $\bar{\rho}$ or close to 1, profits converge to the same levels as under observability. When ρ is initially small, the *poaching effect* dominates which leads to lower profits, whereas when ρ is initially high, the *monitoring effect* dominates, which leads to higher profits. Profits are lowest at $\hat{\rho} = \sqrt{\bar{\rho}} \approx 0.89$. The opposite is true for customer surplus, which is minimal when ρ is close to $\bar{\rho}$ or close to 1 because firms appropriate almost the entire monopoly rent in these cases. Customer surplus reaches a maximum at interior levels of ρ, denoted by ρ^*_{CS}, which, in general, do not coincide with $\hat{\rho}$.[12] Because punishment on the equilibrium path entails socially wasteful promotions, also total welfare is u-shaped in ρ. When ρ is on the boundary, then again, the socially efficient monopoly outcome without promotions emerges. In between, resources are 'wasted' on promotions.

Example 1 illustrates that increasing prediction ability is possibly Pareto-improving. In the example, $\rho^*_{CS} \approx 0.93$. Hence, for $\rho \in (\hat{\rho}, \rho^*_{CS})$, we have that profits, customer surplus, and, hence, also total welfare increase when ρ increases. Again, wasting resources on promotions is socially inefficient and occurs less frequently in this parameter range.

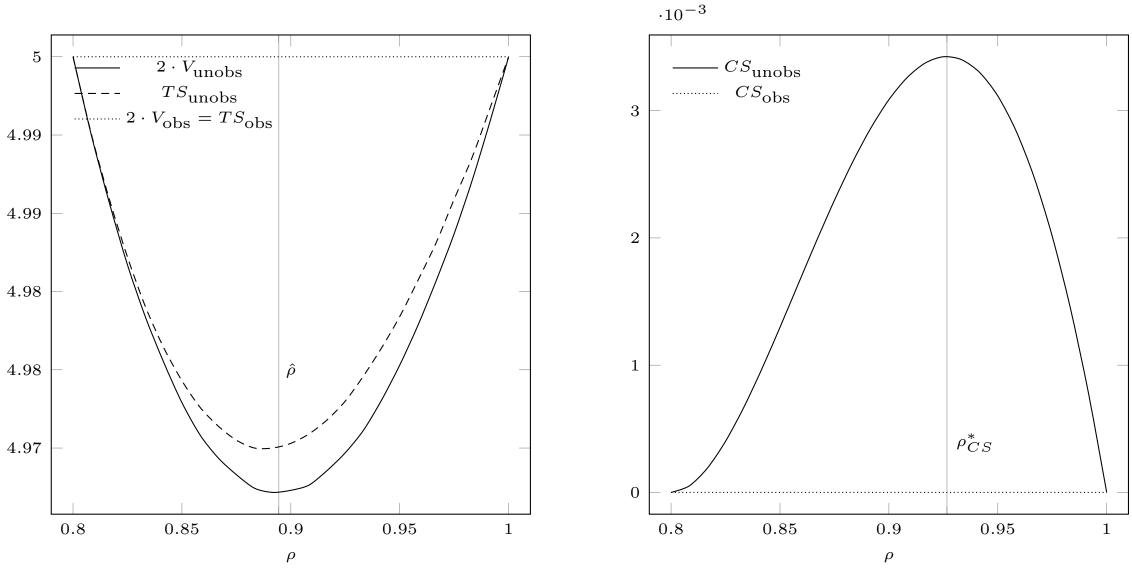
4.2.1 Collusion not always stable

In the previous subsection, we analyzed the effect of prediction ability on collusive profits when firms are sufficiently patient such that collusion is sustainable for all level of prediction ability ρ. As we show in this subsection, higher prediction ability may also serves an additional role, namely by making collusion with unobservable actions sustainable where it otherwise is not.

Proposition 7. *Suppose actions are not observable and that $\delta \in (\underline{\delta}_{obs}, \underline{\delta}_{unobs})$. Then collusion is sustainable if and only if $\rho \geq \tilde{\rho}(\delta) = (2-\delta)/2\delta$, where $\tilde{\rho}(\delta)$ is always between 1/2 and 1 and decreasing in δ.*

Proof. The overall structure of the proof is identical to the proof of Proposition 4 and hence omitted.

[12] Note that the customer-surplus-maximizing level of signal precision ρ^*_{CS} may, in general, be above or below $\hat{\rho}$. In Example 1, $\rho^*_{CS} > \hat{\rho}$.



(a) Collusive profits and total welfare.

(b) Customer surplus.

Figure 3: Impact of signal precision on collusive profits, customer surplus, and total welfare for $\delta = 0.9$.

However, we now rewrite the sustainability condition (13) from Proposition 4

$$\delta \geq \frac{2}{1 + 2\rho}$$

as a function on ρ and obtain

$$\rho \geq \tilde{\rho}(\delta) = \frac{2 - \delta}{2\delta}.$$

By definition of $\underline{\delta}_{obs}$ and $\underline{\delta}_{unobs}$, we have that $\tilde{\rho}(\delta) \in [1/2, 1]$ for all $\delta \in (\underline{\delta}_{obs}, \underline{\delta}_{unobs})$. Note that we can rewrite this as

$$\tilde{\rho}(\delta) = \frac{2 - \delta}{2\delta} = \frac{2/\delta - 1}{2}$$

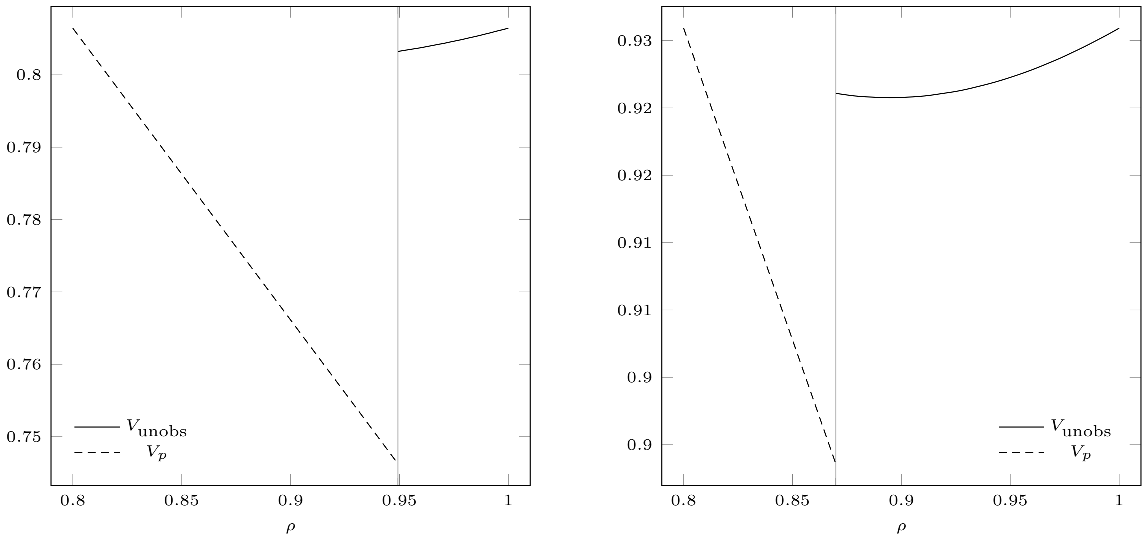
which clearly decreases in ρ.

Thus, collusion is sustainable if $\rho \geq \tilde{\rho}(\delta)$. □

Combining the results of Proposition 4 and Proposition 7, we can readily characterize the effect of prediction ability ρ on profits even if collusion is not sustainable on the entire domain. To this avail, we define

$$\delta_1 = \frac{2}{2\hat{\rho} + 1},$$

which is always strictly between $\underline{\delta}_{obs}$ and $\underline{\delta}_{unobs}$.



(a) $\delta = 0.69$.

(b) $\delta = 0.73$.

Figure 4: Impact of signal precision on collusive profits with unobserved actions.

Corollary 2. *Let $\delta \in (\underline{\delta}_{obs}, \delta_1)$. Then profits are decreasing in ρ for $\rho \leq \tilde{\rho}(\delta)$ (Nash equilibrium), but increasing in ρ otherwise (collusion). Conversely, when $\delta \in (\delta_1, \underline{\delta}_{unobs})$, then again profits are decreasing in ρ for $\rho \leq \tilde{\rho}(\delta)$ (Nash equilibrium), but strictly above Nash equilibrium levels and u-shaped in ρ otherwise (collusion).*

Proof. In both cases, we have from Proposition 7 that no collusion is sustainable when $\rho < \tilde{\rho}(\delta)$, so the Nash equilibrium emerges, where profits are decreasing in ρ according to Proposition 1.

Now consider $\delta \in (\underline{\delta}_{obs}, \delta_1)$. By definition of δ_1, we have that $\tilde{\rho}(\delta) > \hat{\rho}$, so once collusion becomes sustainable, we are in the range of ρ where profits are increasing according to Proposition 4 and Proposition 5.

Conversely, for $\delta \in (\delta_1, \underline{\delta}_{unobs})$, we have that $\tilde{\rho}(\delta) < \hat{\rho}$, so once collusion becomes sustainable, we are in the range of ρ where profits are u-shaped according to Proposition 4 and Proposition 5. □

Example 1 continued. For our parameter values, we have that $\delta_1 \approx 0.71$. So we let $\delta' = 0.69$ and $\delta'' = 0.73$, such that $\underline{\delta}_{obs} < \delta' < \delta_1 < \delta'' < \underline{\delta}_{unobs}$. Then we have $\hat{\rho}(\delta') \approx 0.95$ and $\hat{\rho}(\delta'') \approx 0.87$, such that $\hat{\rho}(\delta'') < \hat{\rho} < \hat{\rho}(\delta')$. Consider Figure 4 as an illustration. Both for δ' and δ'', collusion is not sustainable when δ is very small, but becomes sustainable once ρ is sufficiently high, which induces an upward jump in profits. For δ', this jump is into the range of ρ where collusive. profits increase in ρ. For δ'', however, profits initially decrease again within the collusive range. Thus, there is a double non-monotonicity in profits for δ''.

5 Conclusion

Algorithms, artificial intelligence, and machine learning have become essential tools for market analysis, pricing, and other strategic dimensions in many industries. This trend is very likely to continue in the future. Competition authorities have started to have a closer look at the implications for collusion when firms make use of these tools. In out set-up, we focus on firm's ability to collude when such tools improve demand forecasts. To this end, we build on classic theoretical models from the industrial organization literature.

We find that the sustainability of collusion is crucially affected by market characteristics: Whereas better prediction accuracy has no effect when actions (that is, prices) are observable, there is a u-shaped relationship between prediction quality and collusive profits.

Our results contribute to a better understanding of how the use of algorithms affects firms' incentives and ability to collude. To conclude, we point out a few limitations of our approach. Our signal structure is very simple, where firms receive an identical signal. Clearly, in real-life industries, firms can and do invest significant amounts to improve their demand predictions. As a consequence, firms' signals might not be identical, but they may be correlated to a certain degree. On a related note, we have not analyzed firms' incentives to invest in better prediction ability, but assumed a common industry standard. Furthermore, whereas we use grim-trigger strategies, other punishment strategies, such as optimal punishment, can be contemplated as well. The problem with optimal punishment in the present set-up is that it is unclear what such a punishment looks like with unobservable actions. We leave these aspects for future research.

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