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Insurance for Catastrophes – Indemnity vs. Parametric Insurance with Imperfect Information

Abstract

Insurance for natural hazards - earthquakes, hurricanes, or pandemics - is rarely comprehensively adopted without intense government intervention, and even then it is often only a minority of properties or businesses that are insured. Efforts to close this insurance gap include the introduction of parametric (index) insurance products for various catastrophic risks. We compare parametric to indemnity insurance in a simple model where the insurance company has superior information about the probability of the event (reversed asymmetric information). We find that indemnity insurance tends to be welfare superior, because the coverage provided to agents who underestimate the event probability is larger than with parametric cover. Since it could plausibly be argued that a majority of the population is underestimating the risks of many types of extreme events, this difference in social welfare is potentially substantial.

JEL-Codes: D810, D820, G220.

Keywords: business interruption insurance, insurance for pandemics, parametric vs. indemnity insurance, reversed asymmetric information.

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1 Introduction

Insurance for natural hazards (earthquakes, hurricanes, or pandemics) is rarely comprehensively adopted without intrusive government intervention through mandates or heavy subsidies. In many cases – e.g., California, Japan, or Italy for earthquakes, or Louisiana and Texas for flooding – there is still only a minority of homeowners that are insured. This so-called insurance gap became even more visible in 2020, as only a very small fraction of the economic losses caused by the COVID-19 pandemic (directly or indirectly) were covered by explicit insurance contracts (e.g., Hartwig and Gordon (2020); Hilsenrath (2020); Klein and Weston (2020)).

One consequence of this absence of insurance during the pandemic, and after other disasters, has been very large government spending programs, mostly financed by borrowing. These programs aimed not only to mitigate losses for people who suffered them, but also to prevent a long-lasting economic downturn associated with the negative externalities and multiplier effects created by these losses. As such, it is not surprising that attempts to close this well-documented insurance gap are gathering pace as the frequencies and intensities of various types of natural hazard events are increasing in many places around the world, because of global warming, increased exposure due to urbanization and movement to the coasts, or other socio-economic changes that lead to increasing vulnerability.

Efforts to close the insurance gap include the design of public-private partnerships, for example, in earthquake insurance in Japan or California, or solutions where private insurers serve as intermediaries allocating resources provided by the government, as is common in many European countries for floods (Hartwig, Niehaus, and Qiu (2020)), and in the US with its National Flood Insurance Program (Kousky (2018)). Still, many open questions remain with respect to the design, implementation, and maintenance of these insurance schemes. These questions include how to allocate the costs across space and time, what role (if any) the private sector can play vis-à-vis the public sector, and how different kinds of moral hazards associated with different insurance schemes can be ameliorated (Richter and Wilson (2020)).

One proposal to counter some of the perceived reasons for this insurance gap centers around the viability of index/parametric insurance products instead of more conventional indemnity contracts (e.g., Clement et al. (2018); Kousky, Wiley, and Shabman (2021)). The defining feature of these index insurance contracts is that compensation does not depend on a post-event assessment of the actual damages (or operational losses). Instead, claims are based on a pre-defined parameter that is correlated with these actual damages but does not require their assessment. This parameter (the index) can be based on an easily observable variable such as the amount of rainfall in a clearly defined period, a storm's maximum windspeed, a drought index based on measured temperature and humidity, or on a modelled outcome that uses these variables as inputs. Parametric insurance policies are currently intensively being discussed for Business Interruption (BI) Insurance for pandemics, where the parameter triggering compensation could be a continuous pre-determined time period of government mandated lockdown. Crucially for all of these examples, the insurance claim payment is independent of the actual size of the loss. Such insurance contracts can potentially lower the costs of underwriting insurance, can speed up and simplify the claim assessment process, and can also deal with some of the incentive problems associated with insurance cover (such as moral hazard).[1]

In this paper, we develop a simple model to compare parametric and indemnity insurance for catastrophic events. Our model is based on three main assumptions. First, and most importantly, we assume that the (monopolistic) insurer has superior knowledge about the probability of an event (reversed asymmetric information). This seems reasonable whenever probabilities of the event occurring or its expected damage are unrelated to individual attributes. This holds, at least partly, for most catastrophic hazards like floods and pandemics.[2] We consider two types of potential policy holders, a low type who underestimates the risk, and a high type who overestimates it. Both types can either suffer no harm, low harm, or high harm, and are identical except for their different perceptions of the event probability.

Our second assumption is that the benefit of insurance coverage, both to the insured and to society, is not based on their risk-aversion but on its impact on an early recovery of the insured household or business. In other words, the payment of the insurance claim improves the recipient's recovery trajectory. Such effects have empirically been validated e.g. for the 2011 Christchurch (New Zealand) Earthquake (see Poontirakul et al. (2017) and Nguyen and Noy (2020)).

[1]The current BI policies are predominantly based on indemnity insurance which, however, turned out to be very problematic for losses from COVID-19. See French (2020) for a detailed analysis of the problems of loss calculations for indemnity insurance, including some legal obstacles. One exception was PathogenRX, a parametric BI coverage for pandemics that was offered by Marsh since 2018. No company, however, bought it before the outbreak of the COVID-19 pandemic (Banham (2020)). For recent proposals on parametric BI insurance see Lloyds (2020) and OECD (2020). Klein and Weston (2020, Table 3) provide an overview and critical discussion of the pros and cons of some current proposals.

[2]Many papers argue that insurers are better informed than the insured even in cases where individual attributes are vital for the actual risk (see e.g. Spinnewijn (2013), footnote 4). Insurers, for example, might know more about the structural seismic integrity of typical residential housing unit types than do their actual owners (e.g., Filippova et al. (2020)).

The third assumption concerns the institutional restrictions on the design of insurance contracts. For indemnity insurance, we assume that payments to the insured are bounded above by the actual harm incurred, whereas compensation for parametric insurance may well be above the actual level of harm (since the payment is set by the index, irrespective of the actual harm). For both types of insurance, we exclude payments from the policy holder to the insurance company in a damaging event. These assumptions reflect the real-world constraints on the two contract types.

In this framework, we compare indemnity and parametric insurance for three different informational scenarios. In the complete information setting, we assume that the insurer can tailor her contract[3] offer to the insured's biased perception of the event probability. In the second scenario, we assume that the insurer can offer only one contract, which implies that she can choose between either insuring only the agent who overestimates the event probability, or offer a pooling contract that is accepted by both types. This scenario captures, in a stylized form, a situation where the number of different risk perceptions exceeds the number of contract offers. In the third scenario, we assume that the number of contracts offered is identical to the number of types (screening). In this set up, these two contracts are designed to induce the two insured types to choose different contracts (and thus implicitly reveal their type).

Our main results are driven by the fact that the different institutional restrictions on contracts for indemnity and parametric insurance have different impacts on the high- and low-perception types. As the high type has an inflated willingness to pay for coverage due to his perception bias, the insurer benefits from offering compensation that may even exceed the actual harm. This, however, is impossible for indemnity insurance, as the compensation is bounded above by the actual level of harm. Therefore, profits from the high type are lower than with parametric insurance. Conversely, profits from the low type tend to be higher with indemnity insurance, because the compensation can efficiently be tailored to the different levels of harm. This is infeasible for parametric insurance (compensation is by definition independent of the actual harm), and this inefficiency reduces the insurer's profits.

The fact that profits tend to be higher (lower) with parametric insurance for the high type (the low type) is crucial for the scenarios where the insurer has less than complete information about which type she faces. In the second scenario mentioned above where contracts are either pooled or where only the high type is insured, we find that pooling is less likely for parametric

[3] For simplicity, we refer to the insurer as "she" and to the insured as "he".

insurance. This exclusion of the low type yields a higher insurance gap and thereby reduces social welfare compared to indemnity insurance. A similar result arises in our third scenario (screening). For this case, we find that the downwards distortion in the low type's contract is larger with parametric insurance, which again increases the insurance gap and reduces social welfare.

Overall, our results suggest that social welfare tends to be higher with indemnity insurance, because the coverage provided to agents who systematically underestimate the event probability is more widespread than with parametric cover. Since it is often argued that a majority of people underestimate the risks of damaging extreme events (including pandemics), this difference in social welfare between the two types of insurance contracts is potentially substantial. From a public policy perspective, it may justify regulatory interventions that either account for these insurance gaps created by parametric cover through 'nudge-like' actions, or maybe even regulating such cover more vigorously (as in Robinson et al. (2021)).

While we do believe that our results might be useful for the policy decisions of the regulator, we acknowledge that our model neglects important aspects, including risk-aversion, moral hazard and competition. Most notably, moral hazard with regards to loss sizes speaks in favor of parametric insurance since, for any compensation contractually agreed upon, the policy holder is the residual claimant. We discuss all three neglected issues just mentioned and the likely impacts of relaxing these assumptions on our results in section 7.

Our approach is related to other papers assuming that insurers are better informed than the insured. Models with reversed asymmetric information in the context of insurance have been developed by Villeneuve (2000), Jeleva and Villeneuve (2004), Chassagnon and Villeneuve (2005), and Spinnewijn (2013). The models by Villeneuve and his coauthors are in the tradition of Maskin and Tirole (1990), as they assume that agents engage in rational Bayesian updating when observing the insurance company's offer. In Jeleva and Villeneuve (2004), risk-averse agents differ both in their actual risks and in their perceptions. Then, agents who face high risks but have low risk perceptions may be pooled with those who have lower risks but assume a higher loss probability. If their underestimation of the actual risk outweighs the benefit from risk-sharing, then high risk agents may even receive lower coverage.[4]

Most closely related to our framework, Spinnewijn (2013) also considers agents who differ

[4]In a related setting, Chassagnon and Villeneuve (2005) show that the type which values insurance most tends to be over-covered, while the type that values insurance least is undercovered.

only in their risk perceptions and do not update their perception after observing the insurers' offers (they agree to disagree). While we need two levels of harm (and hence three states of the world) to distinguish between indemnity and parametric insurance, they have just two states; one with and one without loss. However, agents can exert effort to reduce the risk and also have different perceptions on the productivity of their effort. Agents hence differ in their perceptions about the likelihood of the risk for the same effort level and about the marginal productivity of this effort. The combination of both perceptions then determines the equilibrium configuration.

The remainder of our paper is organized as follows: The model is introduced in section 2. As a benchmark, section 3 considers the insurer's profit-maximizing contract with complete information and an unrestricted contract space. Section 4 compares indemnity and parametric insurance when (institutionally restricted) contracts can be tailored to the agent's perception on the event probability. Section 5 considers the case of one contract only, and section 6 extends to the screening equilibrium. Section 7 turns to the possible importance of our restrictive assumptions with respect to moral hazard, risk preferences, and competition. We conclude in section 8.

2 The model

For the reasons mentioned in the introduction, we ignore moral hazard issues by assuming that the probability p for the event and the probability distribution over the loss sizes in case of an event are exogenously given. In case of an event, the loss $X \in \{l, h\}$ is either low or high where $l = \lambda - \gamma$ and $h = \lambda + \gamma$, $\gamma > 0$. We assume that both loss sizes are equally likely so that the expected loss is $p \cdot \lambda$. While loss sizes and their probabilities in case of an event are common knowledge, we assume that only the insurer knows the true event probability p.

There are two types of agents who differ solely in their perception of the probability of the event. Agent $\theta \in \{L, H\}$ assumes probability p_θ, where $p_L = p - \delta$ and $p_H = p + \delta$. The percentage of high types is denoted by g. Agents are, on average, unbiased if and only if $g = \frac{1}{2}$.

We consider a monopolistic insurer who offers a contract to agent type θ consisting of an insurance premium ϕ_θ and an amount $D_{\theta X}$ that agent type θ covers by himself in case of loss λ. With slight abuse of terminology, we refer to $D_{\theta X}$ as deductible even when it exceeds the actual loss or when it is negative (i.e. when $D_{\theta X} > X$ or $D_{\theta X} < 0$). Insurance coverage for type θ

and harm X is denoted by $C_{\theta X} = X - D_{\theta X}$. The benefit of insurance coverage is captured by assuming that a deductible $D > 0$ leads to additional costs of αD^2, $\alpha > 0$ for the agent (and thereby also for society). As mentioned in the introduction, the underlying idea is that uninsured losses hamper recovery.

Without insurance, agent type's θ *actual* utility is[5]

$$\begin{aligned} U_\theta^N &= W - p\left(\frac{1}{2}\left((\lambda+\gamma)+\alpha\left(\lambda+\gamma\right)^2\right)+\frac{1}{2}\left((\lambda-\gamma)+\alpha\left(\lambda-\gamma\right)^2\right)\right) \\ &= W - p\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right). \end{aligned} \tag{1}$$

Due to his perception bias, however, type θ wrongly assumes utility

$$\widetilde{U}_\theta^N = W - p_\theta\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right). \tag{2}$$

To avoid tedious and uninteresting case distinctions in the analysis of the two insurance schemes, we assume that the agent's wealth (W) exceeds the high loss as well as the high agent type's perceived loss from an event:

Assumption 1. $W > \max\left(\lambda+\gamma,\ (p+\delta)\left(L+\alpha\left(\lambda^2+\gamma^2\right)\right)\right)$.

The agent's actual utility with premium ϕ_θ and deductible $D_{\theta X}$ is

$$U_\theta = W - \phi_\theta - p\left(\frac{1}{2}\left(D_{\theta h}+I_{\theta h}\alpha D_{\theta h}^2\right)+\frac{1}{2}\left(D_{\theta l}+I_{\theta l}\alpha D_{\theta l}^2\right)\right), \tag{3}$$

where $I_{\theta X} \in \{0,1\}$ is an index variable that takes the value "1" if and only if $D_{\theta X} > 0$. Otherwise, the term $\alpha D_{\theta X}^2$ disappears (i.e. $I_{\theta X} = 0$), as no part of the loss remains uninsured. For the wrongly expected utility, p needs again to be substituted by p_θ, i.e.

$$\widetilde{U}_\theta = W - \phi_\theta - p_\theta\left(\frac{1}{2}\left(D_{\theta h}+I_{\theta h}\alpha D_{\theta h}^2\right)+\frac{1}{2}\left(D_{\theta l}+I_{\theta l}\alpha D_{\theta l}^2\right)\right). \tag{4}$$

The insurer's expected profit R_θ from type θ is

$$R_\theta = \phi_\theta - p\left(\lambda - \frac{1}{2}\left(D_{\theta h}+D_{\theta l}\right)\right). \tag{5}$$

[5] Superscript "N" denotes the case of "No insurance".

As for social welfare, note that everything except $\alpha D_{\theta X}^2$ is purely distributive. Social welfare is thus maximized iff all losses are at least fully insured. Adding up over the two types, expected welfare losses WL compared to the first best are thus

$$WL = p\left[g\alpha\left(\frac{1}{2}I_{Hh}\cdot D_{Hh}^2 + \frac{1}{2}I_{Hl}D_{Hl}^2\right) + (1-g)\,\alpha\left(\frac{1}{2}I_{Lh}\cdot D_{Lh}^2 + \frac{1}{2}I_{Ll}D_{Ll}^2\right)\right]. \qquad (6)$$

Without insurance, welfare losses boil down to $WL^N = p\alpha\left(\lambda^2+\gamma^2\right)$.

3 Benchmark: Unrestricted contract space

Before we turn to the profit-maximizing contracts offered by the insurer, we first consider an unrestricted contract space as benchmark. Agent type θ accepts an insurance contract if his wrongly expected utility with insurance weakly exceeds the one without, $\widetilde{U}_\theta \geq \widetilde{U}_\theta^N$. Thereby, we need to distinguish between the cases with and without (at least) full coverage.[6] Making use of the agent's perceived utilities with and without insurance as given by Equations (2) and (4), respectively, we get:

$$\widetilde{U}_\theta^{D_{\theta X}>0} \geq \widetilde{U}_\theta^N \Leftrightarrow \phi \leq \phi_\theta^{D_{\theta X}>0} = p_\theta\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - p_\theta\left(\frac{1}{2}D_{\theta l}+\frac{1}{2}\alpha D_{\theta l}^2+\frac{1}{2}D_{\theta h}+\frac{1}{2}\alpha D_{\theta h}^2\right) \text{ for } D_{\theta X}>0,\ X \qquad (7)$$

$$\widetilde{U}_\theta^{D_{\theta X}\leq 0} \geq \widetilde{U}_\theta^N \Leftrightarrow \phi \leq \phi_\theta^{D_{\theta X}\leq 0} = p_\theta\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - p_\theta\left(\frac{1}{2}D_{\theta l}+\frac{1}{2}D_{\theta h}\right) \text{ for } D_{\theta X}\leq 0,\ X=h,l. \qquad (8)$$

For each type and for both cases, the insurer maximizes profits as given in Equation (5), thereby observing the agent's participation constraint $\widetilde{U}_\theta \geq \widetilde{U}_\theta^N$. As the insurer has no reason to leave money on the table, the agent's participation constraint is binding. The maximum insurance premium the insurer can charge is thus $\phi_\theta^{D_{\theta X}>0}$ or $\phi_\theta^{D_{\theta X}\leq 0}$, respectively. Substituting

[6] There are actually $2\cdot 2=4$ cases, as there are two deductibles that can either be positive or not. However, as we will show below that $D_{\theta h}=D_{\theta l}$ in the profti-maximizing contract, we directly restrict attention to the two cases where both deductibles are either positive or not.

the respective premiums into the insurer's profit function for the two cases yields

$$R_\theta^{D_{\theta X}>0} = \left[p_\theta\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - p_\theta\left(\frac{1}{2}D_{\theta l}+\frac{1}{2}\alpha D_{\theta l}^2+\frac{1}{2}D_{\theta h}+\frac{1}{2}\alpha D_{\theta h}^2\right)\right] - \left(\frac{1}{2}p\left(\lambda-D_{\theta h}\right)+\frac{1}{2}p\left(\lambda-D_{\theta l}\right)\right). \tag{9}$$

$$R_\theta^{D_{\theta X}\leq 0} = \left[p_\theta\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - p_\theta\left(\frac{1}{2}D_{\theta l}+\frac{1}{2}D_{\theta h}\right)\right] - \left(\frac{1}{2}p\left(\lambda-D_{\theta h}\right)+\frac{1}{2}p\left(\lambda-D_{\theta l}\right)\right). \tag{10}$$

The derivatives with respect to the deductibles are

$$\begin{aligned}\frac{\partial R_\theta^{D_{\theta X}>0}}{\partial D_{\theta X}} &= \frac{1}{2}\left(p-p_\theta\right)-\alpha D_{\theta\lambda}p_\theta,\ X=h,l, \text{ and}\\ \frac{\partial R_\theta^{D_{\theta X}\leq 0}}{\partial D_{\theta X}} &= \frac{1}{2}\left(p-p_\theta\right),\ X=h,l.\end{aligned} \tag{11}$$

The first order conditions show that the profit-maximizing deductibles are identical for the two loss sizes, because the additional costs of uninsured losses are independent of the actual loss size.

For the high type, $p_H = p+\delta$ yields $\frac{\partial R_H^{D_{HX}>0}}{\partial D_{HX}} = -\frac{1}{2}\delta - \alpha D_{H\lambda}\left(p+\delta\right) < 0$ and $\frac{\partial R_H^{D_{H\lambda}\leq 0}}{\partial D_{H\lambda}} = -\frac{1}{2}\delta < 0$, i.e. profits are always strictly decreasing in the deductible. If the contract space is unrestricted, the insurer hence charges the agent's full wealth as premium ($\phi_H^* = W$) and overcompensates in case of an event ($D_{HX} < 0$).[7] The reason is that overcompensating the agent enables the insurer to take full advantage of the his perception bias. Social welfare is still maximized because all that matters is that there are no uninsured losses.

For the low type, we get $\frac{\partial R_L^{D_{LX}\leq 0}}{\partial D_{LX}} = \frac{1}{2}\delta > 0$, i.e. profits are strictly increasing in deductibles when those are negative. The reason is that the insurer has no reason to compensate more than losses if the agent underestimates the event probability. For positive deductibles, we get $\frac{\partial R_L^{D_{LX}>0}}{\partial D_{LX}} = \frac{1}{2}\delta - \alpha D_{LX}\left(p-\delta\right)$ and $\frac{\partial\left(R_L^{D_{LX}>0}\right)^2}{\partial^2 D_{LX}} = -\alpha\left(p-\delta\right) < 0$, and hence the interior solution $D_{LX} = \frac{\delta}{2\alpha(p-\delta)}\ \forall X$. However, as the deductible is bounded above by the the agent's wealth, the profit-maximizing deductible is effectively given by $D_{LX}^* = \min\left\{\frac{\delta}{2\alpha(p-\delta)}, W\right\}$.

[7] The insurer's incentive to overinsure the high type would be lower if we accounted in addition for risk aversion. In this case, the agent wants to smoothen his consumption utility and would hence not want to pay the full wealth up-front. The effect we are interested in would then be lower but not eliminated, and the analysis would be very convoluted without additional insights.

Note that the deductible for the low type may well be above actual harm, as this enables the insurer to take full advantage of his underestimation of the event probability. In this case, the insurance premium is negative.[8] The deductible is determined by the following trade-off: On the one hand, optimal risk-sharing requires full coverage. And as the insurer can make a take-it-or-leave it offer, any welfare loss is ultimately borne by her. But on the other hand, the agent's willingness to pay for insurance coverage is reduced by his underestimation of the event probability. Note also that the deductible converges to zero if the perception bias becomes arbitrarily small, $D^*_{LX}(\delta \to 0) \to 0$.

Interestingly, the insurer's profit is non-monotone in the low type's perception bias: If the perception bias δ is so small that the profit-maximizing deductible is below harm, then any increase in δ leads first to lower profits, as the bias reduces the agent's willingness to pay for insurance coverage. But if δ is so large that the deductible exceeds the harm, which implies a negative insurance premium, then any further increase in δ reduces the required up-front payment from the insurer to the agent, and hence yields higher profits.[9] Note for later reference that profits from the low type can only increase in δ if the insurer can charge a deductible above actual harm.

We summarize these results in Proposition 1.

Proposition 1 (Unrestricted contract space).

(i) For both agent types, deductibles are independent of the actual loss sizes, $D_h = D_l$.

(ii) High perception type: The insurer charges the agent's full wealth as premium, $\phi^*_H = W$. The deductible is negative, $D^*_H = \frac{(p+\delta)\left(\lambda+\alpha\left(\gamma^2+L^2\right)\right)-W}{p+\delta} < 0$. Social welfare is maximized. The insurer's profit increases in the perception bias δ.

(iii) Low perception type: The deductible is weakly positive and given by $D^*_L = \min\left\{\frac{\delta}{2\alpha(p-\delta)}, W\right\}$. There exists $\widehat{\delta}$ such that profits are decreasing (increasing) in δ if $\delta \leq \widehat{\delta}$ $(\delta > \widehat{\delta})$. Social welfare is not maximized and expected welfare losses in case of $\frac{\delta}{2\alpha(p-\delta)} \leq W$ are $WL_L = p\,(1-g)\,\frac{\delta^2}{4\alpha(p-\delta)^2}$.

Proof. All proofs are in the Appendix.

[8] We are aware that such a contract can hardly be interpreted as an "insurance contract". Still, it is a contract that maximizes the sum of the *perceived* utilities of the two parties involved, and is hence interesting as a benchmark for the more realistic common contracts subsequently considered.

[9] The case distinctions are actually somewhat more subtle as there are two levels of harm, but the basic intuition still holds.

4 Institutional restrictions on contracts

We now consider our two insurance schemes, $S \in \{I, P\}$ for indemnity and parametric insurance, respectively. We introduce the following institutional restrictions:

Assumption 2. With indemnity insurance, $D^I \in (0, X)$ $\forall\theta$, $\forall X$.

The lower bound says that insurers cannot pay more than actual losses. The upper bound excludes payments from agents to insurers in case of an event. Both restrictions resemble standard insurance practice.

By definition of parametric insurance, coverage (C) is independent of actual loss sizes, i.e. $C_l^P = C_h^P \equiv C^P$.

Furthermore, we introduce the following institutional restriction:

Assumption 3. With parametric insurance, $C^P \in (0, X + \gamma)$.

The lower bound again excludes payments from the agent to the insurer in case of an event. The upper bound limits the insurer's compensation to the highest possible loss, as almost all insurance contracts include a cap on the insurer's liability. We analyze how these restrictions influence the relative decline in the injurer's profit for the two insurance schemes; depending on the agent's perception bias.

4.1 Indemnity insurance

From Proposition 1, we know that the profit-maximizing deductibles are identical if both loss sizes are insured at all. For the high type, we know that the insurer wants to overcompensate the agent in case of an event. As this is excluded by the lower bound $D = 0$, the insurer can no longer take full advantage of the agent's bias. This reduces her profit. Social welfare is still maximized since the agent is fully insured. As social welfare is the same while profits are lower, the agent's utility is higher than without restrictions. The reason is that the lower bound $D = 0$ partially protects him from his own perception bias.

For the low type, recall from Proposition 1 that the lower bound $D = 0$ is non-binding. We also know from Proposition 1 that the unrestricted profit-maximizing contract entails a deductible $D > X$ iff δ is sufficiently large. The restriction $D_X^I \leq X$ hence leads to a change in the insurer's contracts iff the unrestricted deductible exceeds at least the low level of harm.

Analyzing the different cases leads to the regions summarized in Proposition 2.[10] The formal analysis is relegated to the Appendix.

Proposition 2 (Indemnity insurance).

(i) High perception type. The deductible is zero for both loss sizes, $D_H^I = 0$. Social welfare is maximized. Profits are lower than without institutional restrictions.

(ii) Low perception type. There are three regions:

(iia) Region I1. For $\delta < \widetilde{\delta}_L^I = \frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}$, the upper bound on the deductible is non-binding. The deductible is $D_L^I\left(\delta < \widetilde{\delta}_L^I\right) = \frac{\delta}{2\alpha(p-\delta)}$.

(iib) Region I2. For $\delta \in \left(\widetilde{\delta}_L^I, \widehat{\delta}_L^I = \frac{2p\alpha(\lambda+\gamma)}{1+2\alpha(\lambda+\gamma)}\right)$, only the high loss is insured and $D_{Lh}^I\left(\widetilde{\delta}_L^I, \widehat{\delta}_L^I\right) = \frac{\delta}{2\alpha(p-\delta)}$.

(iic) Region I3. For $\delta > \widehat{\delta}_L^I$, the low type is not insured.

(iid) In cases *(iib)* and *(iic),* profits are lower than without restrictions.

(iie) Welfare losses are increasing from Region (iia) to (iic), $WL_L^{I,\delta<\widetilde{\delta}_L^I} < WL^{I,\delta\in\left(\widetilde{\delta}_L^I,\widehat{\delta}_L^I\right)} < WL^{I,\delta>\widehat{\delta}_L^I}$.

The intuition for the high type has already been discussed before the Proposition. For the low type, the insurer can still charge the unrestricted profit-maximizing deductible if the bias is sufficiently low (*Region* I1). We show in the Appendix that the critical threshold for that is given by $\widetilde{\delta}_L^I$. If this deductible is infeasible because it would exceed the low loss, then the low loss is not insured at all. In *Region I2*, the high loss will still be insured. If the perception bias becomes too large, $\delta > \widehat{\delta}_L^I$, then there is no profitable contract where the agent gets a positive compensation. The full benefit from risk-sharing is then lost due to the agent's perception bias. Profits from the low type decrease whenever the upper bound on the deductible is binding for at least one level of harm (part *(iid)* of the Proposition). Straightforwardly, welfare losses are highest without insurance, followed by the case where only the high loss is insured.

4.2 Parametric Insurance

As we know from Proposition 1 that the insurer's profit from the high type is strictly increasing in the coverage, she now offers the feasible upper bound $C_H^P = \lambda + \gamma$.[11] By definition of parametric

[10] Superscript I indicates indemnity insurance.
[11] Superscript P indicates parametric insurance.

insurance, coverage is the same for both loss sizes. Similar to indemnity insurance, the contract for the low type requires some case distinctions. Leaving the formal analysis to the Appendix, we summarize the results in Proposition 3.

Proposition 3 (Parametric insurance).

(i) High perception type: The insurer offers the maximum feasible coverage, $C_H^P = \lambda + \gamma$. Social welfare is maximized. Profits are lower than without restrictions.

(ii) Low perception type: There are three regions:
(iia) Region P1. For $\delta < \widetilde{\delta}_L^P = \frac{2\gamma p\alpha}{1+2\gamma\alpha}$, the compensation is
$C_L^P = (\lambda + \gamma) - \frac{\delta}{\alpha(p-\delta)} \geq \lambda - \gamma$.
(iib) Region P2. For $\delta \in \left(\widetilde{\delta}_L^P, \widehat{\delta}_L^P = \frac{2Lp\alpha}{1+2L\alpha}\right)$, the compensation is
$C_L^P = \left(\lambda - \frac{\delta}{2\alpha(p-\delta)}\right) < \lambda - \gamma$.
(*iic) Region* $P3$. For $\delta > \widehat{\delta}_L^P$, the low type is not insured.
(*iid)* Profits are always lower than without institutional restrictions.
(*iie)* Welfare losses are increasing from Region (iia) to (iic), $WL_L^{P,\delta<\widetilde{\delta}_L^P} < WL_L^{P,\delta\in\left(\widetilde{\delta}_L^P,\widehat{\delta}_L^P\right)} < WL_L^{P,\delta>\widehat{\delta}_L^P}$.

The results for the high type follow directly from the restriction $C^P \leq \lambda + \gamma$. For the low type, note first that the insurer has no incentive to offer a compensation above the high loss, which implies that the upper bound is non-binding. Furthermore, the compensation decreases strictly in the perception bias δ: If the bias is rather low, $\delta < \widetilde{\delta}_L^P$, then the insurer offers a compensation between the two levels of harm (*Region P1*). The parameter space for this case increases in the difference between the two loss sizes, i.e. $\frac{\partial \widetilde{\delta}_L^P}{\partial \gamma} > 0$. The reason is that the agent's perceived expected costs from uninsured high losses, $(p - \delta)\,\alpha\left(\lambda + \gamma - C_L^P\right)$, and hence also his willingness to pay for coverage, increases with γ. If δ further increases but is still below the upper threshold $\widehat{\delta}_L^P$, then the insurer offers a compensation that is even below the low loss (*Region P2*). Finally, if the bias becomes too large, $\delta > \widehat{\delta}_L^P$, then the agent's willingness to pay is so low that there is no insurance coverage at all (*Region P3*). Profits and social welfare both decrease in the bias for reasons similar to those discussed for indemnity insurance. Finally, in contrast to indemnity insurance, profits are always lower than without restrictions, because the insurer cannot implement the same deductible for both loss sizes.

4.3 Comparison

We now compare the two insurance schemes with respect to the insurer's profit and social welfare. The comparison is straightforward for the high type. The welfare comparison for the low type requires several case distinctions and can go in either direction.

Proposition 4 (Comparison of indemnity and parametric insurance)

High type

(i) Profits are higher with parametric than with indemnity insurance, $R_H^P > R_H^I$.

(ii) Social welfare is maximized for both insurance schemes.

Low type

(iii) Profits are weakly higher with indemnity than with parametric insurance, $R_L^I \geq R_L^P$.

(iv) For the comparison of social welfare, there are the following cases:

Case 1. For $\delta > \widehat{\delta}_L^I = \frac{2p\alpha(\lambda+\gamma)}{1+2\alpha(\lambda+\gamma)}$, we are in Region 3 for both insurance schemes. As the low type is not insured in either insurance scheme, welfare losses are maximal and identical.

Case 2. For $\delta \in \left(\widehat{\delta}_L^P, \widehat{\delta}_L^I\right)$, we are in Region P3 for parametric insurance where the low type is not insured. With indemnity insurance, we are in Region I2 where only the low loss of the low type is not insured. Welfare losses are lower for indemnity insurance.

Case 3. For $\delta < \widehat{\delta}_L^P$, we are neither in Region I3 nor P3. Then, two cases need to be distinguished:

Case 3A. If $\gamma \geq \frac{\lambda}{2}$ or, equivalently, $\widetilde{\delta}_L^I \leq \widetilde{\delta}_L^P$, then welfare losses are lower for indemnity insurance.

Case 3B. If $\gamma < \frac{\lambda}{2}$ or, equivalently, $\widetilde{\delta}_L^I > \widetilde{\delta}_L^P$, then welfare losses can be lower for either of the two insurance schemes.

The intuition for the high type is as follows: We know from Proposition 1 that the insurer's profit is strictly increasing in the compensation in case of an event. With indemnity insurance, coverage is bounded above by the actual harm, $D_X^I \geq 0$. With parametric insurance, the insurer can still overcompensate for the low level of harm as $C_{\max}^P = \lambda + \gamma$. This yields higher profits compared t indemnity insurance (*part (i)*). Welfare is maximized for both insurance schemes as the high type is (at least) fully covered (*part (ii)*).

For the low type, recall that the insurer never compensates more than the actual harm. The potentially binding constraints are hence the lower bound for the compensation with parametric insurance ($C^P \geq 0$) and the upper bound for the deductible ($D_X^I \leq X$) with indemnity insurance. The advantage of indemnity insurance is that the insurer has one more degree of freedom as she can differentiate the compensation between the two levels of harm. To see why this yields higher profits, simply assume that the indemnity insurer copies the compensation with parametric insurance for one level of harm, and then optimizes the compensation for the other level of harm. The inequality in *part (iii)* of the Proposition is only weak, as not insuring the low type at all is profit-maximizing in both insurance schemes if δ is sufficiently large.[12]

The welfare comparison is more subtle. *Case 1* in *part (iv)* of the Proposition captures the case where the perception bias δ is so large that, in both insurance schemes, the insurer prefers to not insure the low type at all. Welfare losses are then the same. In *Case 2*, δ is still so high that the low type is not insured with parametric insurance. With indemnity insurance, the high loss is insured, which hence reduces the welfare losses compared to no insurance. The reason for this advantage of indemnity insurance is the possibility to insure just one loss. This is impossible for parametric insurance, as the compensation is independent of the actual loss size.

If the perception bias δ further decreases, then the low type is at least partially insured in both insurance schemes. Then the welfare comparison depends on the other parameters of the model. We show in the Appendix that a sufficient condition for the superiority of indemnity insurance is then that the difference in the two loss sizes is sufficiently high ($\gamma \geq \frac{\lambda}{2}$; see *Case 3A*). The intuition is that a high difference in the two loss sizes makes it more important from a social welfare perspective to differentiate the compensation to the agent. If the difference in the two loss sizes becomes small, however, then differentiating between the two levels of compensation is less important, and then either of the two insurance schemes may lead to lower social losses (*Case 3B*).

Summing up, indemnity insurance tends to lead to lower social losses with the low type than parametric insurance: A first sufficient condition for this is that at least some insurance is provided with indemnity insurance, but no insurance at all with parametric insurance. The

[12] One might object that, with indemnity insurance, the insurer cannot copy the compensation with parametric insurance in case (iia) of Proposition 3 where $C_L^P \geq \lambda - \gamma$. While true, this does not matter because the only reason to compensate more than actual losses for the low type is that the same compensation needs to be paid to both types (see the proof of Proposition 4).

opposite case cannot happen. A second sufficient condition for this is that $\gamma > \frac{1}{2}$. And even in the opposite case, indemnity insurance may well be superior if γ is sufficiently close to $\frac{1}{2}$ (also depending on all other parameters of the model).

To summarize: For the high type, social welfare is maximized for either insurance scheme. The insurer's profit is higher with parametric insurance. For the low type, the insurer prefers indemnity insurance, and this also tends to lead to lower social losses than parametric insurance.

5 One contract only

So far, we have assumed that the insurer can tailor its contract offer to the agent's type. If so, the insurer will offer as much coverage as possible to the high type, and coverage below the maximum loss to the low type. We now assume that p_θ is unobservable, and that the insurer offers just one contract. From an applied perspective, this can be interpreted as a short-cut for a situation where the number of different perceptions on the event probability exceeds the number of different contract offers, so that at least some pooling occurs in equilibrium. The insurer then chooses between two options:

Option 1. She offers a contract that is only accepted by the high type. This yields a profit of gR_H^S, $S \in \{I, P\}$.

Option 2. She offers a contract that is accepted by both types. This yields expected profit of R_L^S, $S \in \{I, P\}$.

In option 2, only the low type's participation constraint will be binding. For both insurance schemes, it is therefore profit-maximizing to offer a contract identical to the one offered to the low type in the complete information case (see Propositions 2 and 3, respectively). The reason is that the two types differ only in their willingness to pay for coverage, so that there is no reason to deviate from the contract for the low type. The profit in option 2 is thus R_L^S, as it makes no difference if the insurer faces the low or the high type.

To avoid tedious case distinctions that do not add much from an economic point of view, we restrict attention to the case where we are in Region 1 of Propositions 2 and 3, i.e. we assume that $\delta \leq \min\left(\widetilde{\delta}_L^I, \widetilde{\delta}_L^P\right)$. This yields

Proposition 5 (One contract only)

(i) Suppose the insurer offers a contract that is accepted by both types. Then, the contract is the same as the one for the low type with complete information.

(ii) There exists $\widetilde{g}_R^S$, $S \in \{I, P\}$ such that insuring only the high type yields higher profits iff $g > \widetilde{g}_R^S$. The threshold is higher for indemnity insurance, $\widetilde{g}_R^I > \widetilde{g}_R^P$.

(iii) There exists $\widehat{g}_R^S$, $S \in \{I, P\}$ such that insuring only the high type yields higher social welfare if and only if $g > \widehat{g}_R^S$. The threshold is higher for indemnity insurance, $\widehat{g}_R^I > \widehat{g}_R^P$.

(iv) (a) With indemnity insurance, the insurer's incentive to exclude the low type is too strong from a social welfare perspective. *(b)* With parametric insurance, the insurer's incentive to exclude the low type may be either too strong or too weak.

The intuition for *part (i)* has already been provided above. The first part of *part (ii)* is straightforward: Profits with only the high type being insured increase in the percentage of high types, while this percentage is meaningless when both types are insured with the same contract. The second part of *part (ii)* says that the incentive to exclude the low type is larger with parametric insurance. This is a Corollary to Propositions 2 and 3 which say that profits from the high type (low type) are larger with parametric (indemnity) insurance.

A similar reasoning holds for *part (iii)* of the Proposition. Note first that excluding the low type is *not* always welfare-inferior, as we need to compare the welfare with the contracts *actually chosen* by the insurer in case she insures only the high type or both types. The trade-off then arises from the fact that the coverage for the high type is lower in case both types are insured, as the high type then gets the same coverage as the low type. Again, insuring both types is welfare superior iff g is sufficiently low. This threshold is higher for indemnity insurance. The intuition is that insuring both types is better under indemnity insurance than under parametric insurance because the compensation can be differentiated between the two loss sizes, thereby implementing the same deductible for both losses. Hence, while the low type is more often excluded with parametric insurance, this may also be better from a welfare perspective.

The last claim is investigated in *part (iv)*. The first part says that, given the contracts the insurer offers with indemnity insurance, it would *always* be better to insure both types. Hence, the low type is not only too often excluded from a first best perspective, but also from a second best perspective, i.e. when the insurer offers her profit-maximizing contracts. Mandatory insurance, which yields a pooling contract, would in our model hence be superior to excluding the low type from accessible insurance they would choose to purchase.

This is not the case for parametric insurance. If the distortion in the agents' perception captured by δ is small, then there are cases where welfare is lower if the insurer covers both types: With δ being small, the insurer may have an inefficiently high incentive to cover both types, as she can extract high profits even from the contract that she offers to the low type. This implies that mandatory insurance, which leads to a pooling equilibrium, may reduce social welfare for parametric insurance.

6 Screening

We now consider the case where the number of contract offers equals the number of types (screening). Instead of characterizing the insurer's full maximization problem under either policy type, we streamline the analysis with the following observations: Suppose the insurer offers the same contracts as with complete information. Then, the low type has no incentive to imitate the high type as, due to his underestimation of the event probability, he is not willing to pay the premium charged from the high type.[13] For the high type, things are less clear: On the one hand, he benefits from the low insurance premium required for the low type's participation constraint. But on the other hand, he gets less coverage than he actually wants. If it is optimal for the insurer to cover only the low loss, then the high type might have no imitation incentive at all. Then, the fact that the insurer's does not know the agent's type has no impact at all.

Subsequently, we restrict attention to the more interesting case where the high type would pick the low type's contract if the insurer offered the same contracts as with complete information. A sufficient condition for this is that we are in Region 1 of Propositions 2 and 3, i.e. that the perception bias is not too large. Just as with one contract only, we therefore restrict attention to the case where $\delta \leq \min\left(\widetilde{\delta}_L^I, \widetilde{\delta}_L^P\right)$, which implies that the bounds for the low type's contract are non-binding for both insurance schemes. We then get

Proposition 6 (Screening)

(i) High type: For both insurance schemes, coverage is the same as with complete information. The insurance premium is lower than with complete information.

(ii) Low type: For both insurance schemes, coverage is lower than with complete information.

(iii) Distortion of contract: Compared to complete information, the downwards distortion in

[13] It can easily be shown that the low type's perceived (and actual) utility is negative when he picks the high type's contract.

the low type's coverage is larger for parametric than for indemnity insurance.

The first two parts of Proposition 6 resemble the textbook case of monopolistic screening: As the high type has the imitation incentive, coverage is the same as before (no distortion at the top). However, the high type earns a perceived rent, as the premium is lowered in order to reduce his imitation incentive, and thereby the necessary distortion in the low type's contract required for a separating equilibrium. For the low type, coverage is further reduced, and the premium is determined by his binding participation constraint (no rent at the bottom). The novel result is hence *part (iii)* which expresses that the downwards distortion in the low type's contract is higher for parametric insurance. There are two reasons for that, First, the insurer's profit from the low type is lower with parametric insurance, so that she cares less about this type. Second, the high type's imitation incentive is ceteris paribus higher with parametric insurance as, in Region 1, coverage for the low type is still above the low level of harm. Importantly, *part (iii)* implies that social welfare with screening is lower with parametric insurance due to the lower coverage for the low type who underestimates the event probability.

7 Possible Extensions and Discussion

In this section, we discuss how our results would be affected if we relaxed some of our key assumptions.

8 Possible Extensions and Discussion

In this section, we discuss how our results would be affected if we relaxed some of our key assumptions.

8.1 Risk-aversion

We modelled the benefit from insurance by assuming that the agent and society suffer additional costs when losses are uninsured. This assumption is motivated by the empirical literature on catastrophic disasters and pandemics that demonstrates that individuals and firms recover more quickly when they have access to insurance payments (e.g. von Dahlen and Saxena (2012), Nguyen and Noy (2020)). This implies that insurance coverage is beneficial even when agents are risk-neutral. While our assumption leads, operationally, to similar conclusions as risk aversion, it does not acknowledge that risk aversion provides, in addition, an incentive to smoothen the agent's net wealth between the three different states (no event, low harm, and high harm). However, with risk aversion, we cannot derive closed form solutions for insurance fees and deductibles, and thus cannot meaningfully compare the two insurance schemes.

Arguably, our main results should be qualitatively robust. Consider first the contract for the high type. With indemnity insurance, recall that the insurer fully covers both levels of harm even with risk neutrality, i.e., the deductible is zero. As the high type's net wealth is hence identical anyway in all states, the optimal contract is the same with and without risk aversion. Social welfare is still maximized, but the agent's already inflated willingness to pay for coverage increases further. For parametric insurance, the compensation without risk aversion is $\lambda + \gamma$. There are then two possible cases: If the compensation $C = \lambda + \gamma$ the unrestricted profit-maximizing compensation, then the insurer will reduce the compensation to smoothen the agent's net wealth between the different states. If the insurer's unrestricted compensation is above $\lambda+\gamma$ because the agent's perception bias is large, then she might still offer $C = \lambda+\gamma$. Most importantly from our paper's perspective, profits should still be higher for parametric insurance, as the benefit of covering more than actual harm in case of low harm is reduced by risk aversion, but does not disappear altogether.

Consider next the low type, and start again with indemnity insurance. As the profit-maximizing deductible is positive, the agent's net wealth is lower in case of an event. Risk aversion thus leads to a lower profit-maximizing deductible in order to smoothen the agent's net wealth. The lower the perception bias δ, the higher is this effect. Furthermore, the threshold $\widetilde{\delta}$ such that a loss is only insured if $\delta \leq \widetilde{\delta}$ increases, because risk aversion leads to an upwards shift in the low type's willingness to pay for insurance. This also increases social welfare, as it (partially) countervails the low type's reduced willingness to pay for coverage.

For parametric insurance, we again need to distinguish between two cases. If the profit-maximizing compensation with risk neutrality is below the low level of harm. $C^P < \lambda - \gamma$, then the argument is similar to indemnity insurance. The net wealth is then lowest in case of high harm, followed by low harm, and no event. The insurer has hence an incentive to increase the compensation to benefit from the agent's higher willingness to pay. Again, this increases social welfare. If the compensation is between the two levels of harm $C^P \in (\lambda - \gamma, \lambda + \gamma)$, then the agent's net wealth is lowest in case of high harm, followed by no event, and low harm. In this case, the insurer might either increase or decrease the compensation, depending on whether the lower wealth in case of high harm or the higher wealth in case of low harm dominates. Social welfare increases (decreases) if the compensation increases (decreases).

8.2 Moral hazard

For the case of disasters caused by natural hazards or pandemics, it is reasonable to assume that the agent cannot influence the event probability. However, he might still influence the loss size; for example, by investing beforehand in mitigation actions (like cloud backup systems for businesses). One could hence model moral hazard by assuming that the probability for the high loss decreases at a decreasing rate in the agent's unobservable effort.

Consider first indemnity insurance. With exogenous probabilities for the two levels of harm, the insurer fully insures the high type for both loss sizes in order to take maximum advantage of his perception bias. In case of moral hazard, the insurer needs to set incentives for effort. As this incentive depends only on the difference in the deductibles between the two levels of harm, the insurer still provides full coverage for the low loss. For the high loss, the deductible would be positive. This deductible is decreasing in the high type's perception bias, as a high type with a high perception bias puts high weight on the non-compensated loss anyway.

For the low type, assume that we are in Region 1 where both losses are insured. As the effort incentive again depends only on the difference in the deductibles for the two loss sizes, the insurer has now an incentive to decrease the deductible for the low loss and, at the same time, increase the deductible for the high loss. The profit-maximizing difference between the two loss sizes is larger than for the high type, as the low type's effort incentive is reduced by his underestimation of the event probability.

For parametric insurance, the insurer cannot differentiate the compensation between the two loss sizes (as the compensation is only conditional on the parametrization of the index). As the difference in the agent's payment with high and low loss is simply the difference in the two losses,[14] the agent does not face any moral hazard problem at all if he can only influence the size of the loss but not its probability of occurrence. This, however, seems reasonable in the case of a pandemic (and most other types of natural hazards). As the agent's effort is therefore socially efficient under parametric insurance, the insurer has no incentive to change her contractual offer.

[14] $\Delta = (\lambda + \gamma - C(P) - (\lambda - \gamma - C(P)) = 2\gamma$.

Most importantly in our context, moral hazard with respect to loss sizes straightforwardly works in favor of parametric insurance. As a caveat, we hence acknowledge that our conclusion that indemnity insurance tends to be superior may no longer hold when we take moral hazard with respect to the probability distribution over loss sizes into account.

8.3 Competition

In our basic model with complete information, and with neither risk aversion nor moral hazard, the only impact of perfect competition would be distributive: In the unique equilibrium, companies would still fully insure the high type (with a compensation above the low level of harm for parametric insurance), and would charge a premium such that they just break even. Other equilibria do not exist, as companies would underbid each other until they earn zero profits. Then, not only the high type's perceived consumer surplus, but also his actual surplus would be positive and equal to the social surplus from insurance. A similar result holds for the low type, who is still insured if and only if the monopoly profit would be positive. This profit would then be fully re-distributed to the low type, but neither the terms of the contract nor social welfare will be influenced by competition.

For the screening model, note that the different perceptions of the event probability have consequences similar to adverse selection. We did not analyze this case, but one would assume that the results developed after the seminal paper by Rothschild and Stiglitz (1976) should carry over to our setting, which implies that an equilibrium in pure strategies exists only if the least cost separating allocation is interim efficient.

9 Conclusion

We compared indemnity and parametric insurance in a model where the insurance company is better informed than the insured agents about the probability of an event. This assumption seems most relevant for catastrophic risks such as earthquakes, floods, pandemics, or even acts of terrorism where probabilities are to a large degree. Our motivation here is to understand the consequences of underwriting indemnity vs. parametric insurance contracts under this assumption of asymmetric information – where the asymmetry is assumed to be the opposite of the more traditional analysis of insurance for micro-scale events, where the prevailing view is that the insured agent possesses more information about the risk than the insurer.

We restricted attention to two main differences between indemnity and parametric insurance. First, with indemnity insurance, insurers can offer different compensations for different losses, while the compensation is the same for parametric insurance (since compensation is only conditional on the parametric trigger). This implies that risk-sharing is never optimal with parametric insurance. Second, compensation is limited by the actual loss size for indemnity insurance, while insurers can overcompensate low losses with parametric cover. We find that the latter difference yields higher profits with parametric insurance for the insured who overestimate the event probability, while the former difference tends to yield higher profits, higher coverage, and higher social welfare for those insureds who underestimate the event probability.

Our main result is that, in situations where the insurer cannot observe the different types, social welfare tends to be higher with indemnity insurance. We considered two cases. In the first case, we assume that the number of contracts that the number of different contracts that the insurer can reasonably falls short of the many different (mis)perceptions of the among potential policy holders. This implies that some agents will choose the same contract (pooling), or that some agents remain uninsured. We find that the institutional restrictions set higher incentives to exclude customers with parametric insurance. This yields a higher insurance gap and thereby reduces social welfare compared to indemnity insurance. A similar result arises in our second case where we assume that the number of contracts equals the number of different perceptions on the event probability (screening). For this case, we find that the downwards distortion in the low type's contract is larger with parametric insurance, which again increases the insurance gap and reduces social welfare. An additional result of our analysis is that, at least in our framework, mandatory insurance would yield higher social welfare with indemnity insurance. Conversely, it might reduce social welfare with parametric insurance because it might lead to too low a coverage even for types who overestimate the probability of an event. Notwithstanding that we neglected important aspects of insurance markets, including risk-aversion, moral hazard and competition, our results might be useful for the policy decisions of the regulator.

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Appendix

Proof of Proposition 1.

Part (i). Has been proven in the text.

Part (ii). That the insurer will charge the agent's full wealth as premium, $\phi_H^* = W$, follows from $\frac{\partial R_H^{D_{HX}>0}}{\partial D_{HX}} = -\frac{1}{2}\delta - \alpha D_{HX}(p+\delta) < 0$ and $\frac{\partial R_H^{D_{HX}\leq 0}}{\partial D_{HX}} = -\frac{1}{2}\delta < 0$ as derived in the text. The agent's participation constraint is thus

$$\phi_H^{D_H \leq 0} = W \leq (p+\delta)\left(\lambda + \alpha\left(\lambda^2 + \gamma^2\right)\right) - (p+\delta) D_{H\lambda}, \tag{12}$$

which yields a deductible of

$$D_H^{D_H \leq 0} = \frac{(p+\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right) - W}{p+\delta} < 0. \tag{13}$$

The insurer's profit from the high type with an unrestricted contract is

$$R_H^{D_H \leq 0} = \phi_H - p(\lambda - D_H) = \frac{\delta W}{p+\delta} + \alpha p\left(\lambda^2+\gamma^2\right), \tag{14}$$

and thus increasing in the perception bias as $\frac{\partial R_H^{D_H \leq 0}}{\partial \delta} = \frac{pW}{(p+\delta)^2} > 0$.

Part (iii). The maximum insurance premium the low type is willing to pay with $D > 0$ is

$$\phi_L^{D_L>0} = (p-\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right) - \left(D + \alpha D^2\right)\right), \tag{15}$$

which yields profits of

$$R_L^{D_L>0} = (p-\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right) - \left(D+\alpha D^2\right)\right) - \left(p\left(\lambda - D\right)\right). \tag{16}$$

Solving $\frac{\partial R_L^{D>0}}{\partial D} = 0$ for D yields the profit-maximizing deductible $D_L^{D_L>0} = \left(\frac{\delta}{2\alpha(p-\delta)}\right)$ as derived in the text. Inserting $D_L^{D_L>0} = \left(\frac{\delta}{2\alpha(p-\delta)}\right)$ into the profit function yields after simplifying

$$R_L^{D_L>0} = (p-\delta)\,\alpha\left(\lambda^2+\gamma^2\right) - \delta\lambda + \frac{\delta^2}{4\alpha(p-\delta)} \tag{17}$$

with

$$\frac{\partial R_L^{D_L>0}}{\partial \delta} = \frac{\delta(2p-\delta)}{4\alpha(p-\delta)^2} - \left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right), \frac{\partial^2 R_L^{D>0}}{\partial \delta^2} = \frac{1}{2}\frac{p^2}{\alpha(p-\delta)^3} > 0. \tag{18}$$

The first derivative is positive if and only if $\delta \geq \widehat{\delta}$ where $\widehat{\delta}$ is implicitly given by $\frac{\delta(2p-\delta)}{4\alpha(p-\delta)^2} - \left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right) = 0$. For $\delta \to 0$, we get $\frac{\partial R_L^{D_L>0}}{\partial \delta}(\delta \to 0) = -\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right) < 0$. For $\delta \to p$, we get $\frac{\partial R_L^{D_L>0}}{\partial \delta}(\delta \to p) > 0$, as $4\alpha(p-\delta)^2 \to 0$. Furthermore, $\frac{\partial^2 R_L^{D_L>0}}{\partial \delta^2} = \frac{1}{2}\frac{p^2}{\alpha(p-\delta)^3} > 0$ so that $\widehat{\delta}$ exists by the intermediate value theorem. Finally, as the deductible is the same for both losses, we get an expected welfare loss of

$$WL_L = p(1-g)\alpha\left(\frac{1}{2}\left(\frac{\delta}{2\alpha(p-\delta)}\right)^2 + \frac{1}{2}\left(\frac{\delta}{2\alpha(p-\delta)}\right)^2\right) = \frac{p(1-g)\delta^2}{4\alpha(p-\delta)^2}. \blacksquare \tag{19}$$

Proof of Proposition 2.

Part (i) (High type). From $\frac{\partial R_H^{D_H>0}}{\partial D_H} = -\frac{1}{2}\delta - \alpha D(p+\delta) < 0$ and $\frac{\partial R_H^{D_H\leq 0}}{\partial D_H} = -\frac{1}{2}\delta < 0$ as derived in the text before Proposition 1, we know that the insurer will choose the lowest feasible deductible. Hence, $D_H = 0$ for noth loss sizes. With $D_H = 0$, the maximum premium the high type is willing to pay is $\phi_H^{D=0} = (p+\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right)$, which yields profits of

$$R_H^{I,D=0} = \phi_H^{I,D=0} - p\lambda = (p+\delta)\alpha\left(\lambda^2+\gamma^2\right) + \delta\lambda. \tag{20}$$

Profits are lower than without restrictions as

$$R_H - R_H^{I,D=0} = \frac{\delta}{p+\delta}\left(W - (p+\delta)\left(\lambda + \alpha\left(\gamma^2+\lambda^2\right)\right)\right) > 0, \tag{21}$$

where the sign follows from Assumption 1.

Part (ii) (Low type). We know from Proposition 1 that $D_L = \frac{\delta}{2\alpha(p-\delta)}$, which is still feasible with indemnity insurance iff $\lambda - \gamma \geq \frac{\delta}{2\alpha(p-\delta)}$. Hence, nothing changes in this case. Solving $\frac{\delta}{2\alpha(p-\delta)} = \lambda - \gamma$ for δ gives $\widetilde{\delta}_L^I = \frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}$ as stated in part *(iia)* of the Proposition. Solving $\frac{\delta}{2\alpha(p-\delta)} = \lambda + \gamma$ for δ gives the upper bound $\widehat{\delta}_L^I$ in part *(iib)*, which also proves part *(iic)*.

It remains to prove that the insurer prefers not to insure loss size X if $\frac{\delta}{2\alpha(p-\delta)} > X$: From Proposition 1, we know that $\frac{\partial R_L^{D_L>0}}{\partial D_L} = \frac{1}{2}\delta - \alpha D_L(p-\delta)$. Solving for δ shows that this is strictly positive if $D_L < \frac{\delta}{2\alpha(p-\delta)}$. The insurer will hence choose the maximum feasible deductible, which is $D = X$. This, however, means that loss size X will not be insured at all, as $C_{LX}^I = X - D_{LX}^I = 0$.

Note also that, for all $D_{LX}^{I} < X$, profits are negative if $\frac{\delta}{2\alpha(p-\delta)} > X$.

(iid) In Region 3, profits they are zero because no contract is offered. In Region 2, profits are lower than without restrictions as it is no longer profitable to insure the low loss.

(iie) For further reference, we calculate welfare losses: $WL_{L}^{I,\delta<\widetilde{\delta}_{L}^{I}} = p(1-g)\frac{\delta^2}{4\alpha(p-\delta)^2}$; $WL^{I,\delta\in\left(\widetilde{\delta}_{L}^{I},\widehat{\delta}_{L}^{I}\right)} = p(1-g)\frac{\alpha}{2}\left(\left(\frac{\delta}{2\alpha(p-\delta)}\right)^2 + (\lambda-\gamma)^2\right)$, and $WL_{L}^{I,\delta>\widehat{\delta}_{L}^{I}} = p(1-g)\,\alpha\left(\lambda^2+\gamma^2\right)$. These welfare losses follow immediately from inserting the deductibles into the welfare loss function given by Eqn. (6). The welfare ranking follows from $\frac{\partial WL}{\partial D} > 0\ \forall D > 0$. ■

Proof of Proposition 3.

In the agent's utility function, and hence also in the insurer's profit maximization problem, we need to distinguish between two cases. In the first case, the insurer offers a compensation weakly below the low level of harm. The agent's perceived expected utility is then

$$\widetilde{U}_{\theta}^{P,C\leq\lambda-\gamma} = W - \phi_{\theta}^{P,C\leq\lambda-\gamma} - p_{\theta}\left(\frac{1}{2}\left(\alpha(\lambda+\gamma-C)^2 + (\lambda+\gamma-C)\right) + \frac{1}{2}\left(\alpha(\lambda-\gamma-C)^2 + (\lambda-\gamma-C)\right)\right) \tag{22}$$

$$= W - \phi_{\theta}^{P,C\leq\lambda-\gamma} - p_{\theta}\left(\alpha\left(\gamma^2 + C^2 + \lambda^2\right) - C(1+2\lambda\alpha) + \lambda\right).$$

The maximum premium follows from $\widetilde{U}_{L}^{P,C\leq\lambda-\gamma} \leq \widetilde{U}_{L}^{N}$ and is

$$\phi_{\theta}^{P,C,\leq\lambda-\gamma} = p_{\theta}\left(C_{\theta} - \alpha C_{\theta}^2 + 2\lambda\alpha C_{\theta}\right). \tag{23}$$

Expected profits are

$$R_{\theta}^{P,C\leq\lambda-\gamma} = \phi_{\theta}^{P,C<\lambda-\gamma} - pC_{\theta} = p_{\theta}\left(C_{\theta} - \alpha C_{\theta}^2 + 2\lambda\alpha C_{\theta}\right) - pC_{\theta} \tag{24}$$

with $\frac{\partial R_{\theta}^{P,C\leq\lambda-\gamma}}{\partial C_{\theta}} = p_{\theta} - p + 2\alpha p_{\theta}(\lambda - C_{\theta})$ and $\frac{\partial^2 R_{\theta}^{P,C\leq\lambda-\gamma}}{\partial C_{\theta}^2} = -2\alpha p_{\theta} < 0$.

In the second case, the insurer offers a compensation weakly above the low level of harm. The agent's wrongly expected utility is then

$$\widetilde{U}_{\theta}^{P,C\geq\lambda-\gamma} = W - \phi_{\theta}^{P,C\geq\lambda-\gamma} - p_{\theta}\left(\frac{1}{2}\left(\alpha\left(\lambda+\gamma-C_{\theta}\right)^{2}+\left(\lambda+\gamma-C_{\theta}\right)\right)-\frac{1}{2}\left(C_{\theta}-\left(\lambda-\gamma\right)\right)\right) \tag{25}$$

$$= W - \phi_{\theta}^{P,C\geq\lambda-\gamma} - p_{\theta}\left(\frac{1}{2}\alpha\left(\lambda+\gamma-C_{\theta}\right)^{2}-\left(C_{\theta}-\lambda\right)\right),$$

because there is no uncompensated loss in case of low harm. The maximum premium $\phi_{\theta}^{P,C\geq\lambda-\gamma}$ follows now from $\widetilde{U}_{\theta}^{P,C\geq\lambda-\gamma} \leq \widetilde{U}_{\theta}^{N}$ and is

$$\phi_{\theta}^{P,C\geq\lambda-\gamma} = p_{\theta}\left(\alpha\left(\lambda^{2}+\gamma^{2}\right)+C_{\theta}-\frac{1}{2}\alpha\left(\lambda+\gamma-C_{\theta}\right)^{2}\right). \tag{26}$$

Substituting for $\phi^{P,C\geq\lambda-\gamma}$ in the insurer's profit function yields

$$R_{\theta}^{P,C\geq\lambda-\gamma} = \phi_{\theta}^{P,C\geq\lambda-\gamma} - pC_{\theta} = p_{\theta}\left(\alpha\left(\lambda^{2}+\gamma^{2}\right)+C_{\theta}-\frac{1}{2}\alpha\left(\lambda+\gamma-C_{\theta}\right)^{2}\right) - pC_{\theta} \tag{27}$$

with $\frac{\partial R_{\theta}^{P,C\geq\lambda-\gamma}}{\partial C_{\theta}} = \left(p_{\theta}-p\right)+\alpha p_{\theta}\left(\lambda+\gamma-C_{\theta}\right)$ and $\frac{\partial^{2} R_{\theta}^{P,C\in(\lambda-\gamma,\lambda+\gamma)}}{\partial C_{\theta}^{2}} = -\alpha p_{\theta} < 0$.

We can now move on to the contracts for the two agent types.

Part (i) (High type).

For $C_H \geq \lambda-\gamma$, we get $\frac{\partial R_{H}^{P,C\geq\lambda-\gamma}}{\partial C_{H}} = \left(\delta+\alpha\left(p+\delta\right)\left(\lambda+\gamma-C_{H}\right)\right) > 0$, where the sign follows from $C_H \leq \lambda+\gamma$. The insurer hence offers the maximum feasible coverage, $C_H^P = \lambda+\gamma$. Social welfare is maximized as deductibles are non-positive. Profits are

$$R_{H}^{P,C\geq\lambda-\gamma} = \phi_{H}^{C\geq L-\gamma} - pC_{H} = p_{H}\left(\alpha\left(\lambda^{2}+\gamma^{2}\right)+C_{H}-\frac{1}{2}\alpha\left(\lambda+\gamma-C_{H}\right)^{2}\right) - pC_{H}. \tag{28}$$

Substituting for $p_H = (p+\delta)$ and $C_H = (\lambda+\gamma)$ yields

$$R_{H}^{P} = \delta\left(\lambda+\gamma\right)+\alpha\left(p+\delta\right)\left(\gamma^{2}+\lambda^{2}\right). \tag{29}$$

Profits are lower than without restrictions as

$$R_{H} - R_{H}^{P} = \delta\left(\frac{W}{p+\delta}-\alpha\left(\gamma^{2}+^{2}\right)-\left(\lambda+\gamma\right)\right) > 0. \tag{30}$$

Part (ii) (Low type).

Parts *(iia)* to *(iic)*. Assume first that $C_L^P \leq \lambda - \gamma$, i.e. that even the low harm is not fully compensated. Then, $\frac{\partial R_L^{P,C\leq\lambda-\gamma}}{\partial C_L} = -\delta + 2\alpha\left(p-\delta\right)\left(\lambda - C_L\right)$. $\frac{\partial R_L^{P,C\leq\lambda-\gamma}}{\partial C_L} \leq 0$ for all $C_L \leq \lambda - \gamma$ if $\delta \geq \widehat{\delta}_L^P \equiv \frac{2\lambda p\alpha}{2\lambda\alpha+1}$. Thus, if the perception bias is too large, the low type is not insured at all. For $\delta < \widehat{\delta}_L^P$, the profit-maximizing coverage follows from solving $\frac{\partial R_L^{P,C\leq\lambda-\gamma}}{\partial C_L} = 0$ for C_L and is given by $C_L^{P,C\leq\lambda-\gamma} = \left(\lambda - \frac{\delta}{2\alpha(p-\delta)}\right)$. We know from above that

$$\phi_L^{P,C\leq\lambda-\gamma} = p_L\left(C_L - \alpha C_L^2 + 2\lambda\alpha C_L\right). \tag{31}$$

Expected profits are

$$R_L^{P,C\leq\lambda-\gamma} = \phi_L^{P,C<\lambda-\gamma} - pC_L = p_L\left(C_L - \alpha C_L^2 + 2\lambda\alpha C_L\right) - pC_L. \tag{32}$$

Substituting $p_L = p - \delta$ and $C_L^{P,C\leq\lambda-\gamma} = \left(\lambda - \frac{\delta}{2\alpha(p-\delta)}\right)$ yields the premium

$$\phi_L^{P,C\leq\lambda-\gamma} = \left(p - 2\delta\right)\lambda - \frac{\delta}{2\alpha} - \alpha\left(p-\delta\right)\left(\lambda - \frac{\delta}{2\alpha\left(p-\delta\right)}\right)^2 + 2\lambda\alpha\left(p-\delta\right)\lambda. \tag{33}$$

As profits, we get

$$\begin{aligned} R_L^{P,C\leq\lambda-\gamma} &= \phi_L^{P,C\leq\lambda-\gamma} - pC_L^{P,C\leq\lambda-\gamma} && (34)\\ &= \alpha\left(p-\delta\right)\left(\lambda - \frac{\delta}{2\alpha\left(p-\delta\right)}\right)\left(\lambda + \frac{\delta}{2\alpha\left(p-\delta\right)}\right) - \delta\left(\lambda - \frac{\delta}{2\alpha\left(p-\delta\right)}\right). && (35) \end{aligned}$$

It is easily checked that this profit is just zero at $\delta_L^P \equiv \frac{2\lambda p\alpha}{2\lambda\alpha+1}$ and positive for $\delta < \delta_L^P$. Finally, recall that we so far considered the case where $C_L^P \leq \lambda - \gamma$. This is compatible with $C_L^{P,C\leq\lambda-\gamma} = \left(\lambda - \frac{\delta}{2\alpha(p-\delta)}\right)$ iff $\delta \geq \widetilde{\delta}_L^P \equiv \frac{2p\alpha\gamma}{2\alpha\gamma+1} < \widehat{\delta}_L^P \equiv \frac{2\lambda p\alpha}{2\lambda\alpha+1}$. If $\delta \geq \widetilde{\delta}_L^P$, then the restriction $C_L^P \leq \lambda - \gamma$ is binding and the insurer sets $C_L^P = \lambda - \gamma$. For further reference, note that

$$R_L^{P,C\leq L-\gamma}\left(\widetilde{\delta}_L^P \equiv \frac{2p\alpha\gamma}{2\alpha\gamma+1}\right) = \frac{p\alpha\left(\lambda-\gamma\right)^2}{2\alpha\gamma+1}. \tag{36}$$

We now turn to the case $C_L^P \geq \lambda - \gamma$. As profits are decreasing in C_L^P for $C_L^P \leq \lambda - \gamma$, they are a forteriori decreasing in C_L^P for $C_L^P > \lambda - \gamma$. If there is an interior solution, then

$$C_L^{P,C\geq\lambda-\gamma} = (\lambda+\gamma) - \frac{\delta}{\alpha\,(p-\delta)}. \tag{37}$$

After calculating the insurance premium, substituting the premium into the profit function, and simplifying, profits can be written as

$$R_L^{P,C\geq\lambda-\gamma} = (p-\delta)\,\alpha\left(\lambda^2+\gamma^2\right) - \delta\,(\lambda+\gamma) + \frac{\delta^2}{2\alpha\,(p-\delta)}. \tag{38}$$

Again, we need to take into account that we considered the case where $C \geq \lambda-\gamma$. Substituting $C_L^{P,C\geq\lambda-\gamma} = \lambda-\gamma$ into Eqn. (29) yields the familiar threshold $\widetilde{\delta}_L^P \equiv \frac{2p\alpha\gamma}{2\alpha\gamma+1}$. Thus, the profit maximizing coverage is only feasible if $\delta \leq \delta^{\max} = \frac{2p\alpha\gamma}{1+2\alpha\gamma}$, i.e. if the perception bias is not too large. For any $\delta \geq \widetilde{\delta}_L^P \equiv \frac{2p\alpha\gamma}{2\alpha\gamma+1}$, we get the same profit as before, i.e.

$$R_L^{P,C\geq\lambda-\gamma}\left(\widetilde{\delta}_L^P \equiv \frac{2p\alpha\gamma}{2\alpha\gamma+1}\right) = R_L^{P,C\leq\lambda-\gamma}\left(\widetilde{\delta}_L^P \equiv \frac{2p\alpha\gamma}{2\alpha\gamma+1}\right) = \frac{p\alpha\,(\lambda-\gamma)^2}{2\alpha\gamma+1}. \tag{39}$$

As the two feasibility conditions are mutually exclusive, we get the three regions expressed in parts *(iia)* to *(iic)* in Proposition 3.

Part *(iid)*. Profit-maximizing for the low type requires that deductibles are the same for both loss sizes, which cannot be achieved by definition of parametric insurance. This is sufficient to prove *part (iid)*.

Part *(iie)*. For further reference, we calculate welfare losses compared to the first best: $WL_L^{P,\delta<\widetilde{\delta}_L^P} = p\,(1-g)\,\frac{\alpha}{2}\left(\left(\frac{\delta}{\alpha(p-\delta)}\right)^2 + \left(\frac{\delta}{\alpha(p-\delta)} - 2\gamma\right)^2\right)$,
$WL_L^{P,\delta\in\left(\widetilde{\delta}_L^P,\widehat{\delta}_L^P\right)} = p\,(1-g)\,\frac{\alpha}{2}\left(\left(\frac{\delta}{2\alpha(p-\delta)} + \gamma\right)^2 + \left(\frac{\delta}{2\alpha(p-\delta)} - \gamma\right)^2\right) > WL_L^{P,\delta<\widetilde{\delta}_L^P}$,
and $WL_L^{P,\delta>\widehat{\delta}_L^P} = p\,(1-g)\,\alpha\left(\lambda^2+\gamma^2\right)$.

These welfare losses follow from inserting the deductibles into the welfare loss function given in Eqn. (6). The welfare ranking follows from the fact that welfare strictly increases in the compensation iff $C < X$. ■

Proof of Proposition 4.

Part (i). Recall from the proofs of Proposition 2 and 3 that profits with indemnity and parametric insurance from the high type are $R_H^{I;} = \phi_H^{D=0} - p\lambda = (p+\delta)\,\alpha\left(\lambda^2+\gamma^2\right) + \delta\lambda$ and

$R_H^P = \delta(\lambda+\gamma) + \alpha(p+\delta)\left(\lambda^2+\gamma^2\right)$, respectively. It follows that $R_H^P - R_H^{I,D=0} = \gamma\delta > 0$.

Part (ii). As both losses are at least fully insured with both insurance schemes, social welfare is maximized in either case.

Part (iii). To prove that profits from the low type are weakly higher with indemnity insurance, we do not need to compare the profits for all combinations of the different cases stated in Propositions 2 and 3. Instead, confine attention to the three cases for parametric insurance in Proposition 3: In *Region P3*, the low type is not insured with parametric insurance. Profits from the low type with indemnity insurance are then weakly higher, as the insurer will insure the low type iff this yields positive profits.

In *Region P2*, suppose the indemnity insurer suboptimally sets $D_{Ll}^I = \lambda - \gamma - C_L^P$. Profits from the low loss are then identical with both insurance schemes. But while compensation for the high level of harm is the same with parametric insurance (and hence not profit-maximizing, as this requires a compensation that increases with harm), the indemnity insurer can optimally choose D_{Lh}^I. Profits are hence strictly higher by definition of optimality, and hence a forteriori higher if the indemnity insurer also optimizes for the low level of harm, instead of just mirroring C_L^P.

In *Region P1*, the insurer cannot mirror C_L^P, as this would require $D > \lambda - \gamma$, thereby violating Assumption 2. However, as we know from Proposition 1 that the profit-maximizing deductible from the low type is always positive, this restriction is non-binding in the profit-maximizing indemnity contract. By contrast, with parametric insurance, the insurer sets a compensation above $\lambda - \gamma$ for the low loss solely because the same compensation applies to $\lambda + \gamma$. This reinforces the argument. Taking the three cases together, it follows that profits from the low type are strictly higher with indemnity insurance if the low type is insured at all.

Part (iv). Note first that $\widehat{\delta}_L^I - \widehat{\delta}_L^P = \frac{2p\alpha\gamma}{(2\lambda\alpha+1)(2\lambda\alpha+2\alpha\gamma+1)} > 0$, while $\widetilde{\delta}_L^I \geq \widetilde{\delta}_L^P \Leftrightarrow \gamma \leq \frac{\lambda}{2}$. Recall from Propositions 2 and 3 that, within each insurance scheme, welfare losses are highest in Region 3, followed by Region 2, and Region 1.

Case 1. Follows directly from Propositions 2 and 3.

Case 2. That we are in Region P3 for parametric insurance and in Region I2 for indemnity insurance follows from $\delta \in \left(\widehat{\delta}_L^P = \frac{2\lambda p\alpha}{2\lambda\alpha+1}, \widehat{\delta}_L^I\right)$ as deduced in Propositions 2 and 3. We get

$$WL_L^{P,\delta>\widehat{\delta}_L^P} - WL^{I,\delta\in\left(\widetilde{\delta}_L^I,\widehat{\delta}_L^I\right)} = p\left(1-g\right)\frac{\alpha}{2}\left(\left(\lambda+\gamma\right)^2 - \left(\frac{\delta}{2\alpha\left(p-\delta\right)}\right)^2\right) > 0, \tag{40}$$

where the sign follows from the fact that $(\lambda+\gamma)^2$ is the welfare loss from the high loss without insurance, and $\left(\frac{\delta}{2\alpha(p-\delta)}\right)^2$ the welfare loss with partial insurance.

For all other cases we need to distinguish between $\gamma \geq \frac{\lambda}{2}$ (*Case 3A*) and $\gamma < \frac{\lambda}{2}$ (*Case 3B*)

Case 3A: $\gamma \geq \frac{\lambda}{2}$ and thus $\widetilde{\delta}_L^I \leq \widetilde{\delta}_L^P$.

Case 3A1. If $\delta \in \left(\widetilde{\delta}_L^P = \frac{2\gamma p\alpha}{2\gamma\alpha+1}, \widehat{\delta}_L^P\right)$, we are in Region 2 for both insurance schemes. After simplifying, we get

$$\begin{aligned}\Delta WL^{A1} &\equiv WL^{I,\delta\in\left(\widetilde{\delta}_L^I,\widehat{\delta}_L^I\right)} - WL_L^{P,\delta\in\left(\widetilde{\delta}_L^P\equiv\frac{2\gamma p\alpha}{2\gamma\alpha+1},\widehat{\delta}_L^P\right)} \\ &= p\left(1-g\right)\frac{\alpha}{2}\left(\lambda^2-\gamma^2-2\lambda\gamma-\frac{\delta^2}{4\alpha^2\left(p-\delta\right)^2}\right).\end{aligned} \tag{41}$$

We claim that $\Delta WL^{A1} < 0$ for all $\gamma \geq \frac{\lambda}{2}$. As $\frac{\partial\left(\Delta WL^{A1}\right)}{\partial\gamma} = -2p\left(1-g\right)\frac{\alpha}{2}\left(\lambda+\gamma\right) < 0$, we need to consider the minimum $\gamma = \frac{\lambda}{2}$. We then get $\Delta WL^{A1}_{\gamma=\frac{L}{2}} = -p\left(1-g\right)\frac{\alpha}{2}\left(\frac{\lambda^2}{4}+\frac{\delta^2}{4\alpha^2(p-\delta)^2}\right) < 0$.

Case 3A2. If $\delta \in \left(\widetilde{\delta}_L^I, \widetilde{\delta}_L^P\right)$, we are in Region I2 and in Region P1. After simplifying, we get

$$\begin{aligned}\Delta WL^{A2} &\equiv WL^{I,\delta\in\left(\widetilde{\delta}_L^I,\widehat{\delta}_L^I\right)} - WL_L^{P,\delta<\widetilde{\delta}_L^P} \\ &= p\left(1-g\right)\left(\left(\lambda-\gamma\right)^2-4\gamma^2-\frac{3\delta^2}{4\alpha^2\left(p-\delta\right)^2}-\frac{\delta^2}{\left(p\alpha-\alpha\delta\right)^2}+\frac{4\gamma\delta}{p\alpha-\alpha\delta}\right).\end{aligned} \tag{42}$$

We claim that $\Delta WL^{A2} < 0$ for all $\gamma \geq \frac{\lambda}{2}$. For ease of notation, define $-\frac{3\delta^2}{4\alpha^2(p-\delta)^2} - \frac{\delta^2}{(p\alpha-\alpha\delta)^2} + \frac{4\gamma\delta}{p\alpha-\alpha\delta} \equiv A$. We get

$$\frac{\partial A}{\partial\delta} = -\frac{p\left(7\delta-8p\alpha\gamma+8\alpha\gamma\delta\right)}{2\alpha^2\left(p-\delta\right)^3} > 0 \Leftrightarrow \delta > \frac{8p\alpha\gamma}{8\alpha\gamma+7}. \tag{43}$$

In case of $\frac{\partial A}{\partial\delta} > 0$, we need to consider the maximum $\widetilde{\delta}_L^P = \left(\frac{2\gamma p\alpha}{2\gamma\alpha+1}\right)$, as we want to prove that $\Delta WL^{A2} < 0$. We then get $A\left(\widetilde{\delta}_L^P\right) = \gamma^2$. Thus, if $\frac{\partial A}{\partial\delta} > 0$, ΔWL^{A2} is bounded above by $\Delta WL^{A2}(1) = (\lambda-\gamma)^2 - 4\gamma^2 + \gamma^2$. As $\frac{\partial \Delta WL^{A2}(1)}{\partial \gamma} < 0$, we consider the minimum $\gamma = \frac{\lambda}{2}$, which yields $\Delta WL^{A2}(1) = -\frac{1}{2}\lambda^2 < 0$. Turn next to the case where $\frac{\partial A}{\partial \delta} < 0$. Then, we need to consider the minimum $\widetilde{\delta}_L^I = \frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}$, as we want to prove that $\Delta WL^{A2} < 0$. We get $A\left(\widetilde{\delta}_L^P\right) = (-7\lambda + 15\gamma)(\lambda-\gamma)$. Thus, if $\frac{\partial A}{\partial \delta} < 0$, ΔWL^{A2} is bounded above by $\Delta WL^{A2}(2) = \left((\lambda-\gamma)^2 - 4\gamma^2 + (-7\lambda+15\gamma)(\lambda-\gamma)\right)$, where $\frac{\partial \Delta WL^{A2}(1)}{\partial \gamma} < 0$ as $\gamma \geq \frac{\lambda}{2}$. We hence consider again $\gamma = \frac{\lambda}{2}$, which yields $\Delta WL^{A2}(2) = -\frac{1}{2}\lambda^2 < 0$.

Finally, assume that there is an interior solution that maximizes $\frac{\partial A}{\partial \delta}$ given by $\delta = \frac{8p\alpha\gamma}{8\alpha\gamma+7}$. Then, we get $A\left(\frac{\partial A}{\partial \delta} = 0\right) = \frac{16}{7}\gamma^2$ and $\Delta WL^{A2}(3) = \left((\lambda-\gamma)^2 - 4\gamma^2 + \frac{16}{7}\gamma^2\right)$, where $\frac{\partial \Delta WL^{A2}(1)}{\partial \gamma} < 0$. We hence consider again $\gamma = \frac{\lambda}{2}$, which yields $\Delta WL^{A2}(3) = -\frac{5}{28}\lambda^2 < 0$.

Case 3A3. If $\delta < \widetilde{\delta}_L^I$, we are in Region 1 for both insurance scheme. Then, welfare losses are lower with indemnity insurance. After simplifying, we get

$$WL_L^{I,\delta \leq \widetilde{\delta}_L^I} - WL_L^{P,\delta < \widetilde{\delta}_L^P} = -p(1-g)\frac{3\delta^2 + 8\alpha\gamma(p-\delta)(\alpha\gamma(p-\delta)-\delta)}{4\alpha(p-\delta)^2}. \tag{44}$$

The sign depends on the numerator $N \equiv 3\delta^2 + 8\alpha\gamma(p-\delta)(\alpha\gamma(p-\delta)-\delta)$. For the maximum $\delta \to p$ we get $N = 3p^2$. For the minimum $\delta \to 0$ we get $N = 8p^2\alpha^2\gamma^2$. As $\frac{\partial N^2}{\partial^2 \delta} = 16\alpha^2\gamma^2 + 16\alpha\gamma + 6 > 0$, any interior solution is a minimum. From $\frac{\partial N}{\partial \delta} = 0$ we get $\delta = \frac{8p\alpha^2\gamma^2 + 4p\alpha\gamma}{8\alpha^2\gamma^2 + 8\alpha\gamma + 3}$. Substituting gives $N = \frac{8p^2\alpha^2\gamma^2}{8\alpha^2\gamma^2+8\alpha\gamma+3}$, which proves that $N > 0$. Hence, $WL_L^{I,\delta \leq \widetilde{\delta}_L^I} - WL_L^{P,\delta < \widetilde{\delta}_L^P} < 0$.

Case 3B: $\gamma < \frac{\lambda}{2}$ and thus $\widetilde{\delta}_L^I > \widetilde{\delta}_L^P$.

Case 3B1. If $\delta \in \left(\widetilde{\delta}_L^I = \frac{2p\alpha(L-\gamma)}{1+2\alpha(L-\gamma)}, \widehat{\delta}_L^P\right)$ we are in Region 2 for both insurance schemes. We know that

$$\Delta WL^{B1} = WL^{I,\delta \in \left(\widetilde{\delta}_L^I, \widehat{\delta}_L^I\right)} - WL_L^{P,>\delta \in \left(\widetilde{\delta}_L^P \equiv \frac{2\gamma p\alpha}{2\gamma\alpha+1}, \widehat{\delta}_L^P\right)} = p(1-g)\frac{\alpha}{2}\left(\lambda^2 - \gamma^2 - 2\lambda\gamma - \frac{\delta^2}{4\alpha^2(p-\delta)^2}\right). \tag{45}$$

We claim that there exists $\widetilde{\gamma}^{B1}$ such that $\Delta WL^{B1} > 0 \Leftrightarrow \gamma < \widetilde{\gamma}^{B1}$. First, $\frac{\partial\left(\Delta WL^{B1}\right)}{\partial \gamma} = -2p(1-g)(\lambda+\gamma) < 0$. Second, we know from the proof of Case (A1) that $\Delta WL^{B1} < 0$ for $\gamma = \frac{\lambda}{2}$. Third, for $\gamma \to 0$, we get $\Delta WL^{B1}_{\gamma \to 0} = p(1-g)\frac{\alpha}{2}\left(\lambda^2 - \frac{\delta^2}{4\alpha^2(p-\delta)^2}\right)$, which decreases in δ. For the minimum $\delta = \frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}$ of the case under consideration, we get $\Delta WL^{B1}_{\gamma \to 0} = p(1-g)\frac{\alpha}{2}\gamma(2\lambda-\gamma) > 0$. All three features together prove that $\widetilde{\gamma}^{B1}$ exists by the intermediate value theorem. Hence, in Case 3B1, welfare may be higher with either of the two insurance schemes.

Case 3B2. If $\delta \in \left(\widetilde{\delta}_L^P, \widetilde{\delta}_L^I\right)$ we are in Regions P2 and I1. After simplifying, we get as difference in social losses

$$\Delta WL^{B2} \equiv WL_L^{I,\delta<\widetilde{\delta}_L^I} - WL^{P,\delta\in\left(\widetilde{\delta}_L^P,\widehat{\delta}_L^P\right)} = \frac{p(1-g)\left(4\lambda^2\alpha^2(p-\delta)^2-\delta^2\right)}{4\alpha(p-\delta)^2}, \tag{46}$$

where $\frac{\partial\left(4\lambda^2\alpha^2(p-\delta)^2-\delta^2\right)}{\partial\delta} = -\left(8\lambda^2\alpha^2(p-\delta)+2\delta\right) < 0$. As we want to prove that $\Delta WL^{B2} > 0$, we need to consider the maximum $\widetilde{\delta}_L^I = \frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}$ to get $\Delta WL^{B2}_{\min} = p\alpha\gamma(2\lambda-\gamma)(1-g) > 0$.

Case 3B3. The proof is identical to the proof of case A3. ■

Proof of Proposition 5.

Part (i). We restrict attention to indemnity insurance; the proof for parametric insurance proceeds analogously. We know from Propositions 1 and 2 that the deductible is the same for both loss sizes, and that the maximum insurance premium is then given by

$$\phi_\theta = p_\theta\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)-\left(D+\alpha D^2\right)\right). \tag{47}$$

Due to $p_H = p+\delta > p_L = p-\delta$, the critical constraint for ϕ_θ is the one for the low type. Profits are thus

$$\begin{aligned} R^I(H,L) &= \phi_L - p(\lambda-D) \\ &= (p-\delta)\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)-\left(D+\alpha D^2\right)\right)-p(L-D). \end{aligned} \tag{48}$$

We get $\frac{\partial R^I(H,L)}{\partial D} = 0 \Leftrightarrow D^I(H,L) = \frac{\delta}{2\alpha(p-\delta)}$ as in the contract for the low type.

Part (ii). For existence of $\widetilde{g}_R^S$, we can also restrict attention to indemnity insurance. With $\delta \le \widetilde{\delta}_L^I$, we know from Proposition 2 that

$$R^I(H,L) = (p-\delta)\left(\alpha\left(\lambda^2+\gamma^2\right)\right)-\delta\lambda+\frac{\delta^2}{4\alpha(p-\delta)} \tag{49}$$

if the insurer insures both types, and that

$$R^I(H) = g\left((p+\delta)\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right)-p\lambda\right) \tag{50}$$

if the insurer insures only the high type. Define $\Delta R^I \equiv R^I(H) - R^I(H,L)$. Then, *(i)* $\frac{\partial(\Delta R)}{\partial g} = \frac{\partial R^I(H)}{g} > 0$. *(ii)* For $g \to 1$, we get after simplifying

$$\Delta R(g \to 1) = 2\delta\lambda + 2\alpha\delta\left(\lambda^2 + \gamma^2\right) - \frac{\delta^2}{4\alpha(p-\delta)} > 0.\text{[15]} \tag{51}$$

(iii) For $g \to 0$, we get $\Delta R(g \to 0) = -R^I(H,L) < 0$. From *(i)* - *(iii)*, it follows from the intermediate value theorem that there exists some $\widetilde{g}^I$ such that $gR_H^I > R_L^I$ if and only if $g > \widetilde{g}^I$.

To see that $\widetilde{g}_R^I > \widetilde{g}_R^P$, note that $\widetilde{g}_R^S$ is implicitly given by $\Delta R^S = R^S(H) - R^S(H,L) = 0$. From $\frac{\partial R^S(H)}{\partial g} > 0$ and $\frac{\partial R^S(H,L)}{\partial g} = 0$, it follows that $\frac{\partial \widetilde{g}^S}{\partial R^S(H)} < 0$ and $\frac{\partial \widetilde{g}^S}{\partial R^S(H,\lambda)} > 0$. The claim then follows from $R^P(H) > R^I(H)$ and $R^P(H,L) < R^I(H,L)$ (see Proposition 4).

Part (iii). The proof that $\widehat{g}_R^S$ exists follows the same logic as the proof for $\widetilde{g}_R^S$. For the comparison of the two thresholds, note that social losses from the contract to the high type are zero for both insurance schemes if only the high type is insured, while social losses from not-insuring the low type are $SL_L^N = p(1-g)\alpha\left(\lambda^2 + \gamma^2\right)$ in either case. This given, $\widetilde{g}_R^I > \widetilde{g}_R^P$ is implied by the proof of Proposition 4, Case 3A3.

Part (iv)

a) For indemnity insurance, recall the expressions for the profits $gR^I(H)$ and $R^I(H,L)$ from the proof of *part (ii)*. The difference is

$$\begin{aligned}\Delta R^I &= R^I(H,L) - gR(H) \\ &= (p-\delta)\alpha\left(\lambda^2+\gamma^2\right) - \delta\lambda + \frac{\delta^2}{4\alpha(p-\delta)} - g\left((p+\delta)\alpha\left(\lambda^2+\gamma^2\right) + \delta\lambda\right).\end{aligned} \tag{52}$$

If $\Delta R^I > 0$, then both types will be insured. Losses in social welfare are $(1-g)WL^I(H) = (1-g)p\alpha\left(\lambda^2+\gamma^2\right)$ if only the high type is insured, and $WL^I(H,L) = \frac{p\delta^2}{4\alpha(p-\delta)^2}$ if both types are insured. Insuring both types is thus welfare superior if

$$\Delta WL^I \equiv (1-g)WL^I(H) - WL^I(H,L) = (1-g)p\alpha\left(\lambda^2+\gamma^2\right) - \frac{p\delta^2}{4\alpha(p-\delta)^2} > 0. \tag{53}$$

That both types are insured too often (too seldom) can only happen if $Y^I > 0$ ($Y^I < 0$), where $Y^I \equiv \Delta R^I - \Delta W^I$. After simplifying, we get

$$Y^I = \frac{\delta^2(2p-\delta) - 4\alpha(p-\delta)^2(1+g)\delta\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right)}{4\alpha(p-\delta)^2}. \tag{54}$$

The sign depends on the numerator N. Recall that the insurer's incentive to exclude the low type is inefficiently high if $Y^I < 0$. As we claim in part *(iv,a)* that $Y^I < 0$, and because $\frac{\partial N}{\partial \lambda} < 0$ and $\frac{\partial N}{\partial g} < 0$, we need to consider the minimum $g \to 0$ and the minimum λ, which follows from $\delta \leq \widetilde{\delta}^I = \left(\frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}\right)$. We get $\widetilde{\lambda}^I_{\text{min}} = \frac{\delta+2\alpha\gamma(p-\delta)}{2\alpha(p-\delta)}$. Substituting for $\widetilde{\lambda}^I_{\text{min}}$ and $g = 0$ gives

$$N = -4\alpha\gamma\delta\left(p-\delta\right)\left(p+2\alpha\gamma\left(p-\delta\right)\right) < 0, \tag{55}$$

and hence also $Y^I < 0$.

b) For parametric insurance, define $Y^P \equiv \Delta R^P - \Delta WL^P$, where

$$\begin{aligned}\Delta WL^P &= WL^P\left(H,L\right) - WL^P\left(L\right) \\ &= \left(1-g\right)p\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - p\frac{\alpha}{2}\left(\left(\frac{\delta}{\alpha\left(p-\delta\right)}\right)^2 + \left(\frac{\delta}{\alpha\left(p-\delta\right)} - 2\gamma\right)^2\right).\end{aligned} \tag{56}$$

After simplifying, we get

$$\begin{aligned}Y^P &= \alpha\left(\gamma^2+\lambda^2\right)\left(2gp-\left(1-g\right)\delta\right) - \left(1-g\right)\lambda p - \delta\left(1+g\right)\left(\lambda+\gamma\right) \\ &\quad + \frac{2\delta^2 - \alpha\delta^3 + 8\alpha\gamma\left(p-\delta\right)\left(\alpha\gamma\left(p-\delta\right)-\delta\right)}{2\alpha^2\left(p-\delta\right)^2}\end{aligned} \tag{57}$$

with

$$\frac{\partial Y^P}{\partial g} = \lambda\left(p-\delta\right) + \alpha\gamma^2\delta + 2\lambda^2 p\alpha + \lambda^2\alpha\delta + 2p\alpha\gamma^2 - \gamma\delta > 0.\text{[16]} \tag{58}$$

As we want to check if, similar to indemnity insurance, $Y^P < 0$ always holds, and because of $\frac{\partial Y^P}{\partial g} > 0$, we need to consider $g \mapsto 0$ to get

$$Y^P\left(g \mapsto 0\right) = -\delta\left(\lambda+\gamma+\alpha\left(\gamma^2+\lambda^2\right)\right) + \frac{2\delta^2 - \alpha\delta^3 + 8\alpha\gamma\left(p-\delta\right)\left(\alpha\gamma\left(p-\delta\right)-\delta\right)}{2\alpha^2\left(p-\delta\right)^2}, \tag{59}$$

which decreases in λ. Of course, $Y^P\left(g \mapsto 0, \lambda^{\max}\right) < 0$ as λ is unbounded above. To check if Y^P is always negative, we need to consider the minimum λ, which follows from $\delta \leq \widetilde{\delta}^I = \left(\frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}\right)$. Hence $\widetilde{\lambda}^P_{\text{min}} = \left(\frac{\delta+2\alpha\gamma(p-\delta)}{2\alpha(p-\delta)}\right)$. Substituting for $\widetilde{\lambda}^P_{\text{min}}$ in $Y^P\left(g \mapsto 0\right)$ yields

$$Y^P\left(g \mapsto 0, \widetilde{\lambda}^P_{\text{min}}\right) = \begin{array}{c} -\delta\left(\frac{\delta+2\alpha\gamma(p-\delta)}{2\alpha(p-\delta)}\right) - \delta\gamma - \delta\alpha\gamma^2 - \delta\alpha\left(\frac{\delta+2\alpha\gamma(p-\delta)}{2\alpha(p-\delta)}\right)^2 \\ +\frac{2\delta^2-\alpha\delta^3-8\alpha\gamma(p-\delta)\delta}{2\alpha^2(p-\delta)^2} + 4\gamma^2. \end{array} \tag{60}$$

Suppose next that $\delta \mapsto 0$. Then, $Y^P\left(g \mapsto 0, \widetilde{\lambda}^P_{\min}, \delta \mapsto 0\right) = 4\gamma^2 > 0$. Hence, both cases are possible. ■

Proof of Proposition 6.

Parts (i) and (ii) for indemnity insurance.

We know from the proof of Proposition 2 that $D^I_{H\lambda} = 0$, that $\phi^*_H = (p+\delta)\left(\lambda + \alpha\left(\lambda^2 + \gamma^2\right)\right)$, and that the high type's perceived utility with complete information is the same as without insurance:

$$\widetilde{U}^I_H = W - \phi^I_H = W - (p+\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right). \tag{61}$$

For the low type, we know that $D^I_L = \frac{\delta}{2\alpha(p-\delta)}$ in Region I1, and thus that

$$\begin{aligned} \phi^I_L &= (p-\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right) - \left(D^I_L + \alpha\left(D^I_L\right)^2\right)\right) \\ &= (p-\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right) - \frac{\delta\left(2p-\delta\right)}{4\alpha\left(p-\delta\right)^2}\right). \end{aligned} \tag{62}$$

We first confirm that, with these two contract offers, the high type has an incentive to imitate. If so, his perceived utility is

$$\begin{aligned} \widetilde{U}^I_H\left(D^I_L\right) &= W - \phi^I_L - (p+\delta)\left(D^I_L + \alpha\left(D^I_L\right)^2\right) \\ &= W - (p-\delta)\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - \frac{\delta^2\left(2p-\delta\right)}{2\alpha\left(p-\delta\right)^2}. \end{aligned} \tag{63}$$

After simplifying, we get

$$\widetilde{U}^I_H\left(D^I_L\right) - \widetilde{U}^*_H = 2\delta\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - \frac{\delta^2\left(2p-\delta\right)}{2\alpha\left(p-\delta\right)^2}. \tag{64}$$

The derivatives are $\frac{\partial\left(\widetilde{U}^I_H\left(D^I_L\right)-\widetilde{U}^*_H\right)}{\partial\delta} = 2\left(\lambda+\alpha\left(\lambda^2+\gamma^2\right)\right) - \frac{\delta\left(4p^2-3p\delta+\delta^2\right)}{2\alpha(p-\delta)^3}$ and $\frac{\partial^2\left(\widetilde{U}_H\left(D^I_L\right)-\widetilde{U}^*_H\right)}{\partial\delta^2} = -\frac{p^2(2p+\delta)}{\alpha(p-\delta)^4} < 0$. Hence, $\widetilde{U}^I_H\left(D^I_L\right) - \widetilde{U}^*_H$ is either continuously decreasing in δ or inversely U-shaped. Next, $\widetilde{U}_H\left(D^I_L\right) - \widetilde{U}^*_H < 0$ if $\delta \mapsto p$ as $\frac{\delta^2(2p-\delta)}{2\alpha(p-\delta)^2}$ becomes arbitrarily large. Hence, if the perception bias converges to zero, then the high type has no incentive to choose a contract with less than full coverage. But for the maximum $\delta = \widetilde{\delta}^I_L = \left(\frac{2p\alpha(\lambda-\gamma)}{1+2\alpha(\lambda-\gamma)}\right)$ in Region 1, we get $\widetilde{U}^I_H\left(D^I_{L\lambda}\right) - U^*_H = \frac{(4p\alpha\gamma(\lambda-\gamma))(1+2\alpha\lambda)}{1+2\alpha(\lambda-\gamma)} > 0$, in which case the high type does have an imitation incentive.

As the high type has the incentive to imitate, we know that the high type's participation constraint and the low type's incentive compatibility constraint are slack. The insurer hence maximizes[17]

$$\begin{aligned} R^{I,SC} &= gR_H^{I,SC} + (1-g)\, R_L^{I,SC} \\ &= g\left(\phi_H^{I,SC} - p\lambda\right) + (1-g)\left(\phi_L^{I,SC} - p\left(\lambda - D_L^{I,SC}\right)\right), \end{aligned} \tag{65}$$

subject to the low type's participation constraint (PCL) and the high type's incentive compatibility constraint (ICH):

$$\begin{aligned} (\mathrm{PCL^I})\ \phi_L^{I,SC} &\leq (p-\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right) - \left(D_L^{I,SC} + \alpha\left(D_L^{I,SC}\right)^2\right)\right) \\ (\mathrm{ICH^I})\ \phi_H^{I,SC} &\leq \phi_L^{I,SC} + (p+\delta)\left(D_L^{I,SC} + \alpha\left(D_L^{I,SC}\right)^2\right). \end{aligned} \tag{66}$$

In equilibrium, both constraints are binding. Substituting for $\phi_L^{I,SC}$ in the binding (ICH) and solving for $\phi_H^{I,SC}$ gives

$$\phi_H^{I,SC} = (p-\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right)\right) + 2\delta\left(D + \alpha D^2\right). \tag{67}$$

Substituting for $\phi_L^{I,SC}$ and $\phi_H^{I,SC}$ in the profit function and simplifying gives

$$R^{I,SC} = (p-\delta)\,\alpha\left(\gamma^2+\lambda^2\right) - \lambda\delta + D_L^{I,SC}\left(\delta\,(1+g) - \alpha D_L^{I,SC}\left(p\,(1-g) - \delta\,(1+g)\right)\right) \tag{68}$$

with

$$\begin{aligned} \frac{\partial R^{I,SC}}{\partial D_L^{I,SC}} &= (1+g)\,\delta - 2\alpha D_L^{I,SC}\left(p\,(1-g) - \delta\,(1+g)\right) \\ \frac{\partial^2 R^{I,SC}}{\partial\left(D_L^{I,SC}\right)^2} &= 2\alpha\left(g\,(p+\delta) - (p-\delta)\right). \end{aligned} \tag{69}$$

An interior solution requires that $\frac{\partial^2 R^{I,SC}}{\partial\left(D_L^{I,SC}\right)^2} < 0$, and hence that the frequency of high types

[17] Superscript "SC" expresses "Screening".

is sufficiently small (otherwise, the insurer will only insure the high type). In case there is an interior solution, $\frac{\partial R^{I,SC}}{\partial D_L^{I,SC}} = 0$ yields $D_L^{I,SC} = \frac{\delta(1+g)}{2\alpha((p-\delta)-g(p+\delta))} < D_L^I = \frac{\delta}{2\alpha(p-\delta)}$. For $g \mapsto 0$, the insurer offers the same contract to the low type as with complete information. Finally, the distortion increases in the frequency of high types, i.e. $\frac{\partial D_L^{I,SC}}{\partial g} = \frac{p\delta}{\alpha(\delta-p+g\delta+gp)^2} > 0$.

Parts (i) and (ii) for parametric insurance.

We omit the (straightforward) proof that the high type again has an incentive to imitate the low type in case the insurer offers the same two contracts as with complete information (proof available on request). The insurer now maximizes

$$\begin{aligned} R^{P,SC} &= gR_H^{P,SC} + (1-g)\, R_L^{P,SC} \\ &= g\left(\phi_H^{P,SC} - p\,(\lambda+\gamma)\right) + (1-g)\left(\phi_L^{I,SC} - pC_L^{P,SC}\right), \end{aligned} \tag{70}$$

again by observing the low type's participation constraint and the high type's incentive compatibility constraint. The high type's perceived utility with the high type-contract is

$$\begin{aligned} \widetilde{U}_H^{P,SC} &= W - \phi_H^{P,SC} - (p+\delta)\left(\frac{1}{2}\,(\lambda+\gamma-(\lambda+\gamma)) + \frac{1}{2}\,(\lambda-\gamma-(\lambda+\gamma))\right) \\ &= W - \phi_H^{P,SC} + (p+\delta)\,\gamma. \end{aligned} \tag{71}$$

This utility must be weakly above his utility from imitation, which is

$$\begin{aligned} \widetilde{U}_H^{P,SC}\left(C_L^{P,SC}\right) &= W - \phi_L^{P,SC} \\ &\quad - (p+\delta)\left(\frac{1}{2}\left(\lambda-\gamma-C_L^{P,SC}\right) + \frac{1}{2}\left(\left(\lambda+\gamma-C_L^{P,SC}\right) + \alpha\left(\lambda+\gamma-C_L^{P,SC}\right)^2\right)\right) \\ &= W - \phi_L^{P,SC} - (p+\delta)\left(\lambda - C_L^{P,SC} + \frac{1}{2}\alpha\left(\lambda+\gamma-C_L^{P,SC}\right)^2\right). \end{aligned} \tag{72}$$

As the low type's participation constraint is the same as with complete information, the insurer maximizes $R^{P,SC}$ by observing the two constraints

$$\begin{aligned} &(\mathrm{PCL^P})\ \phi_L^{P,SC} \leq (p-\delta)\left(\lambda + \alpha\left(\lambda^2+\gamma^2\right) - \frac{1}{2}\left(\lambda-\gamma-C_L^{P,SC}\right) - \frac{1}{2}\left(\left(\lambda+\gamma-C_L^{P,SC}\right) + \alpha\left(\lambda+\gamma-C_L^{P,SC}\right)^2 \right.\right. \\ &(\mathrm{ICH^P})\ \phi_H^{P,SC} \leq \phi_L^{P,SC} + (p+\delta)\left(\lambda+\gamma-C_L^{P,SC} + \frac{1}{2}\alpha\left(\lambda+\gamma-C_L^{P,SC}\right)^2\right). \end{aligned} \tag{73}$$

Again, both constraints are binding. Substituting for $\phi_L^{P,SC}$ from (PCL^P) in (ICH^P) gives

$$\phi_H^{P,SC} = (\lambda + \gamma)(p + \delta) + \alpha\delta\left(C_L^{P,SC}\right)^2 + p\alpha\left(\lambda^2 + \gamma^2\right) + 2\lambda\alpha\gamma\delta - 2\delta C_L^{P,SC}\left(1 + \alpha(\lambda + \gamma)\right). \quad (74)$$

Substituting for $\phi_L^{P,SC}$ and $\phi_H^{P,SC}$ in the profit function, and taking the derivatives with respect to the compensation for the low type gives

$$\frac{\partial R^{P,SC}}{\partial C_L^{P,SC}} = \alpha\left(\lambda + \gamma - C_L^{P,SC}\right)\left((p - \delta) - g(p + \delta)\right) - \delta(1 + g),$$
$$\frac{\partial^2 R^{P,SC}}{\partial\left(C_L^{P,SC}\right)^2} = -\alpha(p - \delta) + g\alpha(p + \delta). \quad (75)$$

An interior solution requires that $\frac{\partial^2 R^{P,SC}}{\partial\left(C_L^{P,SC}\right)^2} < 0$, and hence again that the frequency of high types is sufficiently small (otherwise, the insurer will, similar to the case without screening, only insure the high type). In case there is an interior solution, $\frac{\partial R^{P,SC}}{\partial C_L^{P,SC}} = 0$ yields $C_L^{P,SC} = \lambda + \gamma - \frac{(1+g)\delta}{\alpha((p-\delta)-g(p+\delta))} < C_L^{P,SC} = \lambda + \gamma - \frac{\delta}{\alpha(p-\delta)}$. For $g \mapsto 0$, the insurer offers the same contract to the low type as with complete information. Again, the downwards distortion increases in the frequency of high types, i.e. $\frac{\partial C_L^{P,SC}}{\partial g} = -\frac{2p\delta}{\alpha(\delta-p+g\delta+gp)^2} < 0$.

Part (iii). From $\frac{\partial D_L^{I,SC}}{\partial g} = \frac{p\delta}{\alpha(\delta-p+g\delta+gp)^2}$ and $\frac{\partial C_L^{P,SC}}{\partial g} = -\frac{2p\delta}{\alpha(\delta-p+g\delta+gp)^2}$, it follows that the reduction in compensation is larger for parametric insurance: $\frac{\partial C_L^{P,SC}}{\partial g} - \left(\frac{\partial D_L^{I,SC}}{\partial g}\right) = -\frac{p\delta}{\alpha(\delta-p+g\delta+gp)^2} < 0$. ■