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Extending the Limits of the Abatement Cost

Abstract

The paper examines the relevant cost benefit framework for public authorities investigating the potential of local projects to mitigate climate change. Because these projects are typically limited in time and space, continuation pathways need be introduced to capture the benefits provided by a project over the longer term. This issue is particularly acute in the transition toward carbon neutrality, which aims for the full abatement of emissions by a future end date. The relevant question is not whether or not to decarbonize an activity but when to do so, and how. We propose a new metric that incorporates into the analytical framework the dynamic interactions between a project and its continuation. This metric is defined as the annual overall discounted cost divided by the long term annual abatement. The new metric is a non trivial extension of the standard cost of abatement. It determines when precisely to launch a given project and addresses the question of how to compare competing projects using their on going emissions up to their respective optimal launch dates. Two illustrations make clear the novelty of our approach: the choice of the optimal mix of technologies for the electricity sector and the comparison between competing green technologies for mobility.

JEL-Codes: Q510, Q560, R580.

Keywords: cost benefit analysis, abatement cost, time value of money, learning-by-doing.

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Key policy insights:
- Abatement cost is commonly used by local governments to evaluate climate change mitigation in applied case studies at the sectoral level.
- A non trivial extension of two traditional metrics - the levelized cost of carbon and the levelized cost of carbon abatement - is proposed.
- The proposed metric determines when to launch a given project and addresses the question of how to compare competing projects.

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1 Introduction

While the introduction of a carbon tax would in theory provide the most efficient tool to induce decentralized decisions to mitigate greenhouse gas emissions, in practice, a large array of policies are implemented at the sectoral level. To achieve policy goals, public authorities, whether local or national, often must decide whether to fund and implement decarbonization "projects". These projects aim to decarbonize a given set of polluting activities, and typically involve investing new productive capital over a long time period. The selection of pilot projects plays a major role as these projects are expected to generate future gains through cost reductions and spillovers. Abatement costs are a common tool to evaluate such projects and, involve calculating the cost per ton of carbon dioxide (CO_2) avoided. However, this *a priori* simple definition hides challenges which are addressed in the present paper.

Three recent papers bring attention to these challenges. Gillingham and Stock (2018) explore the impact of using learning-by-doing to assess yearly costs in two case studies (solar panels and electric vehicles). Baker and Khatami (2019) offer a cost benefit foundation for the use of a metric which relates the time value of money to the evolution of the social cost of carbon, where yearly costs dynamics appear as a major question to be addressed. Friedmann et al. (2020) use another metric related to the levelized cost of energy (LCOE) to compare options to decarbonize various industrial activities, they insist on the role of factors related to geography and specific assets.

These empirical challenges are in line with those faced in our own studies, such as those assessing EAS-Hymob (Brunet and Ponssard, 2017), Zero Emission Valley (Teyssier d'Orfeuil, 2020) in France, H-Vision in the Netherlands and HyNet NW in the UK (Athias, 2020), fuel cell electric buses for European metropolitan areas (Meunier et al., 2019). In each of these evaluations a key aspect is left open: what is supposed to happen at the end of the (pilot) project? This is a crucial question since the project is only one element in a much longer trajectory of action that aims to ultimately decarbonize the activity. Because the project is expected to generate future benefits for the overall decarbonization process these longer-term benefits should ideally be integrated during the initial project evaluation. This points to a need to reformulate the relevant cost benefit framework.

To illustrate this need for reformulation consider a project with a finite life time. Two basic metrics are currently available to calculate the abatement cost both of which use the discounted cost as the numerator. For the denominator, the first method (known as the levelized cost of carbon, LCC) uses the sum of the abatements, while the second method (known as the levelized cost of carbon abatement, LCCA) uses the discounted sum of abatements. Suppose, however, that we extend the project to embed its continuation into a permanent trajectory for decarbonizing the activity. Both metrics behave strangely. The LCC will converge to zero suggesting that the project, whatever it is, should be launched immediately. The LCCA, meanwhile, is invariant through any delay in deferring the project launch. We call these outcomes the "paradox of the horizon". The real question is not whether the project should be launched, but rather *when* it should be (since the activity will be decarbonized at some point) and *how* to compare among competing projects.

To address these questions, we derive a new metric by reformulating the cost benefit analysis in the relevant dynamic framework. This new metric bypasses the paradox of the horizon encountered by the LCC and LCCA, providing a precise determination of when to launch a project (and its associated continuation) and also enabling the comparison of competing projects using their on going emissions up to their respective optimal launch dates.

The paper is organized as follows. To appreciate our methodological contribution the reader is invited to briefly revisit some empirical studies and their analysis of carbon abatement issues (see Appendix A for the H-Vision project in Rotterdam and the deployment of fuel cell electric buses in Europe). In section 2, we revisit the traditional metrics and further detail the paradox of the horizon. Then we define the concept of dynamic abatement cost (DAC) for a project given three possible continuation trajectories: simple extension, extension with learning-by-doing and extension with learning-by-doing and spillovers, each of which enhance different dynamic interactions between the project and its continuation. We derive the relevant metrics for addressing the two questions of when to launch an extended project (Proposition 1) and how to select among competing projects (Proposition 2 and 3). This is done assuming that the social cost of carbon grows at the same rate as the social discount rate. In section 3, we revisit this simplifying assumption and show how our results should be modified (Proposition 4). In section 4, we introduce uncertainty through the existence of a backstop technology. In section 5, we apply our approach to two illustrations: the selection of the optimal mix of technologies to decarbonize electricity production in France in 2050, and the choice of batteries versus hydrogen fuel cells for decarbonizing the transport sector. Section 6 presents our conclusions. Appendix B provides the proofs and, Appendix C presents a simple illustratin of the links between the DAC, the LCC and the LCCA.

2 The dynamic cost of abatement

2.1 The standard analysis and the paradox of the horizon

We consider a benevolent social planner who has to decide whether or not to fund a project which enables the decarbonization of a pre-identified polluting activity (denoted as business as usual, BAU). Time is continuous and denoted $t \in \mathbb{R}^+$. BAU emits E tCO_2 per year. The project spreads over T years and reduces emissions by a_t thanks to incremental costs c_t. By construction $a_T = E$: the activity is fully decarbonized at the end of the project. The discount rate is denoted i and the social cost of carbon, denoted P_t at time $t \geq 0$, grows at the rate γ: $P_t = e^{\gamma t} P_0$. For the time being, we assume that $\gamma = i$. In theory, with a carbon budget, γ and i should indeed be equal (Hotelling's rule); we shall come back to this assumption later.

The total net benefit of implementing the project at time zero is:

$$B = \int_0^T e^{-it}[P_t a_t - c_t]dt = P_0 \int_0^T a_t dt - \int_0^T e^{-it} c_t dt \quad (1)$$

A standard lemma follows

Lemma 1 (Baker and Khatami, 2019) *The project should be implemented at date 0 iff:*

$$\frac{\int_0^T e^{-it} c_t dt}{\int_0^T a_t dt} \leq P_0. \quad (2)$$

The left-hand-side of the equation coincides with formula (2) in Baker and Khatami (2019) for the LCC in which $\beta = 0$ since $i = \gamma$. It corresponds to the standard abatement cost (MAC) when $T = 0$, that is $MAC = c_0/E$. Note that if $T > 0$, the integration of the intermediary abatements is done without discounting because of the growth in the carbon price which exactly balances the growth in the social discount rate. The comparison between the LCC and the present carbon price determines whether the project should be implemented. There are two interrelated implicit assumptions in the cost-benefit analysis that leads to the use of the LCC: First, the time horizon is fixed and given by the project. Second, the project can only be launched today or never. This has two troubling consequences : First, even if today the carbon price is lower than the LCC it will eventually be larger which suggests that any project will be worth implementing at some point in time. Second, the choice of the horizon T is somewhat arbitrary, and extending that horizon can make any project worth implementing today; this leads to what we have called the "paradox of the horizon".

Suppose that the life time of the project is long. As T increases, the numerator of the LCC is bounded (because of the discounting) while the denominator will grow toward infinity (because of no discounting) as abatements close to $a_T = E$ are added. Consequently the LCC will go to zero, and any project with a sufficiently long horizon should be implemented regardless of the present carbon price.

Corollary 1 (Paradox of the horizon) *Any project to decarbonize a pre-identified polluting activity will be worth implementing for any present carbon price if its abatements are strictly positive for a long enough duration.*

In some applications, the abatement cost is calculated by using as the denominator the discounted abatements over the life time of the project. The corresponding metric is called the levelized cost of carbon abatement (LCCA) by Friedmann et al. (2020). That is:

$$LCCA = \frac{\int_0^T e^{-it} c_t dt}{\int_0^T e^{-it} a_t dt}$$

.

This indicator would be consistent with a cost benefit analysis in which one assumes that the social cost of carbon is constant (see section 2.4 for more on the economic interpretation of the LCCA). This would give an equivalent result as Lemma 1 for making the decision of whether or not to implement the project based on the LCCA. Note that even if the project is delayed for some time, the LCCA of the delayed project is identical to that of the original project. This is intriguing since their evaluations should differ. The LCCA also faces a paradox of the horizon.

These methodological challenges suggest that the correct question is not "whether" but "when" to implement a given project. However, to formally answer that question one needs to re-examine the choice of the time horizon. Firstly, if we take for granted that the goal is to decarbonize a pre-identified polluting activity, then we should consider a project with finite life only if there is a full decarbonization continuation pathway to the future. Formulating this pathway should be included in the cost benefit analysis. For convenience, we denote the original project and its continuation pathway "the extended project". Secondly, the corollary tells us that the question of interest can be subdivided into two parts: when an extended project should be launched and how to compare several extended projects. We address these two points in sequence.

2.2 Three simple ways to extend the time horizon of a project

For clarity, we now distinguish between the calendar time $t \in \mathbb{R}^+$, and the project time τ which is the time since the beginning of the project. Recall that the project spreads over T years, so $\tau \in [0, T]$, and reduces emissions by a_τ thanks to incremental costs c_τ relative to BAU at time τ after the beginning of the project. Note that this assumes that there are no external factors related to calendar time that affect either the abatements or the project costs. We shall come back to this assumption.

At the end of the project, we consider that the emissions E are perpetually eliminated thanks to a discounted cost $\bar{C}$, which is the discounted sum of yearly expenses needed to operate and maintain the clean capital built during the project. Let us consider three simple continuation paths :

Simple extension: a constant cost $\bar{c}$ per tCO_2 should be spent each year, the continuation discounted cost is then

$$\bar{C} = \int_0^{+\infty} e^{-it} \bar{c} E dt = \frac{\bar{c}}{i} E.$$

Extension with learning-by-doing: the cost per tCO_2 decreases over time at a rate λ so that

$$\bar{C} = \int_0^{+\infty} e^{-it} e^{-\lambda t} \bar{c} E dt = \frac{\bar{c}}{i + \lambda} E.$$

It should be noted that in many applications learning-by-doing may affect both the project and its continuation. This is the case in the situation discussed in (Creti et al., 2018) for analyzing the optimal launch trajectory of fuel cell electric vehicles as a substitute for the current fossil fleet. Our metric generalizes the notion introduced in that paper (see section 5.2.).

Extension with learning-by-doing and spillovers: the project creates side benefits $\bar{B}$ once completed because of learning and spillover externalities for similar polluting activities

$$\bar{C} = \frac{\bar{c}}{i + \lambda} E - \bar{B}.$$

The benefits $\bar{B}$ depend on the existence of similar polluting sites that the project being considered woulod help to decarbonize. At these other sites, similar projects could be implemented, and would also need to be evaluated. Analysis of the coordinated evaluation of a portfolio of closely related projects is beyond the scope of the present article and deserves further research. The two illustrations in Section 5 refine the first two cases to analyze some empirical issues.

2.3 When to launch an extended project and comparison with the LCC

For a project to be launched at time s, the associated extended project involves three parts. It is BAU until $t = s$. It coincides with the project from time s until time $s+T$, with abatements a_τ and incremental costs c_τ. A continuation pathway involving a full decarbonization, $a_t = E$ for all $t \geq T + s$ for a discounted cost $\bar{C}$. The choice of s should minimize the overall discounted social cost noted as $\Gamma(s)$ such that:

$$\Gamma(s) = \int_0^s e^{-it} P_t E dt + \int_s^{s+T} e^{-it}[P_t(E - a_{t-s}) + c_{t-s}]dt + e^{-(s+T)}\bar{C} \tag{3}$$

$$= \int_0^{s+T} e^{-it} P_t E dt + \underbrace{\int_s^{s+T} e^{-it}[c_{t-s} - P_t a_{t-s}]dt}_{\text{the project}} + e^{-(s+T)}\bar{C} \tag{4}$$

For convenience define as I the discounted cost of the project

$$I = \int_0^T e^{-i\tau} c_\tau d\tau \tag{5}$$

We now define a dynamic abatement cost to be denoted as DAC, such that

$$DAC = \frac{i}{E}\left[I + e^{-iT}\bar{C}\right] \tag{6}$$

The optimal time s^* for launching the project is given by the following proposition.

Proposition 1 *The social cost of carbon at the optimal launch date s^* is equal to the DAC, that is*

$$P_{s^*} = \frac{i}{E}[I + e^{-iT}\bar{C}] \tag{7}$$

Proof. Postponing the launch time by one unit of time reduces the total discounted cost by $i(e^{-is}I + e^{-i(s+T)}\bar{C})$, adds emissions E, and shifts intermediary abatements (a_τ). With a carbon price growing at the interest rate $P_s = e^{is}P_0$ emissions are worth the same independent of their date, and the shift in intermediary abatements has no impact on the total cost. The gain from postponing the cost should be compared with $P_0 E$, that is the cost of adding E emissions. ■

For illustrative purposes, consider a project with $T = 5$, $a_\tau = 1$ and $c_\tau = 100$ for all $\tau \in [1, 5]$. Assume $i = .05$ so that:

$$I = \int_0^T e^{-it} c_t dt = 100(1 - e^{.25})/.05 = 442.40$$

$$LCC = \frac{I}{\int_0^T a_t dt} = 442.40/5 = 88.48$$

The DAC for $\lambda = 0$ is $\bar{c}/E = 100$. It is higher than the LCC of the project. However, if we take $T = 20$, then the LCC drops to 22.12 (it will go to zero as T grows to infinity) whereas the DAC, without learning is unaffected. If we go back to $T = 5$ and suppose that there is learning, we have $DAC = i[I + e^{-iT}\bar{c}/(i+\lambda)]/E$. If $\lambda = .01$, then the DAC=87.0, which is already lower than the LCC. For $\lambda = .03$, the DAC=70.79, a significant drop!

Clearly our new metric does not coincide with the LCC. The LCC gives the false impression that a long enough project should be launched at once whereas the DAC gives the correct answer regarding the optimal launch time of the extended project depending on the formulation of the continuation pathway.

We can reinterpret the DAC as follows. Equation (7) may be restated as

$$P_{s+T} = DACe^{iT} = i[Ie^{iT} + \bar{C}]/E$$

Here, the cost of the project and its continuation are evaluated at the end of the project and not at its beginning. At the optimal launch time, the social cost of carbon should be equal to the cost of decarbonizing a perpetual flow E of emissions, intermediary abatements are irrelevant.

2.4 How to compare several extended projects and comparison with the LCCA

It is enough to consider two extended projects that decarbonize a pre-existing activity. The steps are: Index by $k = 1, 2$ the characteristics of each project: T_k, $a_{\tau,k}$, $c_{\tau,k}$ and $\bar{C}_k$. Denote $\bar{s}_k$ as the optimal launch time for the extended project k. The question is which of the two extended projects to select. The following proposition gives the answer. Let $A_k = \int_0^{T_k} a_{\tau,k} d\tau$ and $\bar{E}_k = (\bar{s}_k + T_k)E - A_k$, $\bar{E}_k$ is the total emissions of the optimal extended project k. Comparing these emissions is enough to make the selection.

Proposition 2 *Between two competing projects* 1 *and* 2 *to decarbonize* E*, project* 1 *should be selected over project* 2 *if and only if it is associated with fewer total emissions computed with the optimal launch dates, that is* $\bar{E}_1 < \bar{E}_2$.

Proof. By definition of the DAC, $P_s = DAC = i[I + e^{-iT} i\bar{C}]/E$. For $\bar{s}_k$ to be optimal, it must be that $P_0 = e^{-i\bar{s}_k} DAC_k$. It follows that comparing the social cost of the optimized extended project $\Gamma(\bar{s}_k)$ becomes

$$\Gamma(\bar{s}_k) = P_0 E(\bar{s}_k + T_s) - P_0 A_k + \underbrace{e^{-i\bar{s}_k} DAC_k}_{=P_0} E$$

Comparing the $\Gamma(\bar{s}_k)$ amounts to comparing the emissions from time 0 to $\bar{s}_k + T_k$, that is $\bar{E}_k$. ■

The total discounted cost $\Gamma(\bar{s}_k)$ has two components: the social cost of emissions and the discounted cash cost. Proposition 2 states that when comparing two projects, only the first component matters since, at the optimal launch time, the discounted cash costs of the two projects taken at time $t = 0$ are identical.

Consider now the special case in which intermediary abatements are null. The following corollary is thus obtained, which nicely extends the standard MAC for ranking different projects.

Corollary 2 *In the case of no intermediary abatements, the selected extended project is the one with the lower DAC and it should be launched at time* $\bar{s}_k$.

Note that in our metric the abatements are not discounted. This seems to contradict some standard applications particularly in the electricity sector, and this point need be clarified. In the calculations of the LCOE, produced quantities are discounted in order to compare the cost per megawatt-hour (MWh) of technologies with different capital intensities and life spans, with the implicit assumption that these technologies could be used to produce similar load curves. Indeed, a well-known issue is the comparison between intermittent and dispatchable technologies (Joskow, 2011).

Actually, when comparing two projects, our metric should be adjusted by a factor related to the discounting of intermediary abatements. In Proposition 2, the characteristics of the project are hidden within the optimal starting date s_k, and Proposition 2 could be reframed as follow:

Proposition 3 *Project 1 should be selected over project 2 if and only if*

$$DAC_1 e^{iT_1} e^{-i\frac{A_1}{E}} < DAC_2 e^{iT_2} e^{-i\frac{A_2}{E}}.$$

See Appendix B.1 for the proof. One way to interpret this result is to see that for project $k \in \{1, 2\}$, A_k/E is the number of years $\tilde{T}$ such that total emissions $\tilde{T}E$ equals the sum of intermediary abatements A_k. A_k saves A_k/E years of emissions, it is as if project k stops decarbonizing the emission flow $T - A_k/E$ years after it starts, instead of T years. Recall that the DAC stands for the annual discounted cost per unit of emissions. The proper assessment consists then in comparing the annualized cost for project discounted at year $T_k - A_k/E$ per unit of emissions.

To better grasp the role of discounting and the difference between the adjusted DAC with the LCCA, it is worth considering that project 1 is to be compared with a flexible (mature) technology that costs $\tilde{c}$ per tCO_2. If that technology is used to mimic the abatement profile (a_τ) followed by E, which would be project 2, then the cost of doing it is

$$\int_0^T e^{-it}\tilde{c}a_t dt + \int_T^{+\infty} e^{-it}\tilde{c}E dt = \tilde{c}\left[\int_0^T e^{-it}a_t dt + \int_T^{+\infty} e^{-it}E dt\right]$$

.

so that

$$LCCA(\text{project 2}) = \tilde{c}$$

And project 1 is less costly than the flexible technology to abate the same profile (i.e. project 2) if and only if

$$LCCA(\text{project 2}) = \tilde{c} > \frac{I + e^{-iT}\bar{C}}{\int_0^T e^{-it}a_t dt + \int_T^{+\infty} e^{-it}E dt} = LCCA(\text{project 1}).$$

This suggests that discounting abatements is the right approach when comparing competing projects. But our broader approach challenges that result, because the flexible technology should actually be used to decarbonize another profile than the profile of project 1. Its flexibility is valuable and should be accounted for in the comparison. The flexible technology should be implemented to decarbonize the whole flow of emissions E when the carbon price is equal to $\tilde{c}$ (the relevant project 2) and the relevant comparison is given by Proposition 3.

Corollary 3 *Project 1 is preferable to a flexible technology with cost $\tilde{c}$ per tCO_2 if and only if*

$$DACe^{iT}e^{-i\frac{A}{E}} < \tilde{c}.$$

Proof. For the flexible technology we have $DAC = \tilde{c}$; $T = 0$ and $A = 0$. ■

To conclude this discussion observe that in the special case of stationary abatements, the LCCA of the project is equal to the DAC of the corresponding extended project through a simple extension. This provides a simple economic interpretation for using the LCCA as well as making explicit its limitations.

3 When the social cost of carbon does not grow with the social discount rate

Consider that the price of carbon grows at a rate γ: $P_t = e^{\gamma t}P_0$, and denote $\beta = i - \gamma$. This is the situation considered by Baker and Khatami (2019) who define the LCC as a function of β:

$$LCC(\beta) = \frac{\int_0^T e^{-it}c_\tau d\tau}{\int_0^T e^{-\beta t}a_\tau d\tau}$$

This LCC and its comparison with the current carbon price determines whether the benefits from abatements over the life time of the project are superior to its discounted cost. The same limitations apply, and we propose choosing the optimal launch date of the extended project by minimizing the overall social discounted cost $\Gamma(s)$ given by equation (4).

Proposition 4 *The optimal launching date (if positive) is such that*

$$P_s = \frac{i[I + e^{-iT}\bar{C}]}{\beta\int_0^T e^{-\beta t}a_t dt + Ee^{-\beta T}} \tag{8}$$

See Appendix B.2 for the proof. As before, the choice of the optimal launch date induces a trade-off between the benefit of postponing the cost and the cost of postponing abatements. The right-hand side generalizes the DAC obtained for $\beta = 0$. With a carbon price not growing at the interest rate postponing intermediary abatements has an impact on the total discounted cost, namely.

Corollary 4 *If the carbon price grows more quickly than the interest rate ($\beta < 0$) the optimal launch time occurs earlier and vice versa.*

Proof. The denominator of the right-hand side is positive and decreasing with respect to β if the abatements a_τ are increasing,[1] an increase in the growth rate of the carbon price advances the optimal launch time since it increases the value of subsequent abatements.[2] ■

[1] The derivative of the denominator with respect to β is $\int_0^T e^{-\beta t}(1-\beta t)a_t dt - Te^{-\beta T}$ which is, after an integration by part of the first term: $-\int_0^T te^{-\beta t}\dot{a}_t dt$.

[2] Indeed, in theory the project costs and abatements should be optimized according to the growth rate of the carbon price.

Another way to obtain this result is to consider the carbon price at the end of the project:

$$P_{s+T} = \frac{i[e^{iT}I + \bar{C}]}{\beta \int_0^T e^{\beta(T-t)} a_t dt + E}.$$

The numerator of the right-hand side is the discounted cost at T of the project and its continuation, and the denominator is increasing with respect to β (if a_τ is increasing), so that an increase in the growth rate of the carbon price decreases the carbon price at the end of the project.

Finally, it is interesting to note that the denominator in equation (8) is equal to β times the discounted sum of emissions $(a_\tau)_{\tau \in [0,T]}$ followed by E after T, discounted at rate β, so that

$$P_s = \frac{i}{\beta} \frac{I + e^{-iT}\bar{C}}{\int_0^T e^{-\beta t} a_t dt + \int_T^{+\infty} e^{-\beta t} E dt}$$

and the second fraction of the right-hand side could be interpreted as the LCC of the extended project, with the practical issue that the denominator is only well defined for $\beta > 0$ whereas $\beta < 0$ is well possible.

In terms of applications, it is ordinarily assumed that the rate of growth of the carbon price eventually converges to the interest rate. In France Quinet et al. (2019), the official reference price of CO_2, the so-called "valeur tutélaire du carbone", is growing at a non constant, decreasing, rate that is larger than the discount rate (thus $\beta < 0$ but is increasing to zero over time). The underlying value was not determined as a welfare maximization under a carbon budget constraint, which would have resulted in a Hotelling's rule, but rather as a carbon price that would ensure carbon neutrality in 2050, together with a carbon budget consistent with the Paris Agreement and political constraints. As a consequence, according to corollary 4, cost benefit analysis based on this social cost of carbon has advanced green investments. However one may also argue that, for political acceptance, the Quinet social cost of carbon in 2020 is lower than what it should be according to economic reasoning. This delays green investments...

4 Uncertainty and backstop technology

If a backstop technology may appear in the future to decarbonize the pre-existing polluting activity at no cost, the extended project should then be interrupted and the backstop implemented. If the backstop appears according to a Poisson process with rate μ, then at date t the backstop did not materialize with probability $e^{-\mu t}$. Taking into account the possibility of the backstop into the computation of the social cost given by equation (4) amounts to discounting by $i + \mu$ instead of i. Our analysis can then be reproduced replacing i with $i + \mu$, so the cost I should be computed with $i + \mu$ and the continuation cost $\bar{C}$ also. For a continuation pathway such as replication with learning-by-doing it becomes:

$$\bar{C}(\mu) = \int_0^{+\infty} e^{-(i+\mu)t} e^{-\lambda t} \bar{c} dt E = \frac{\bar{c}}{i + \lambda + \mu} E.$$

And Proposition 4 becomes

Corollary 5 *With a backstop technology characterized by a Poisson process with rate μ the optimal launching date s is such that*

$$P_s = \frac{(i+\mu)\left[\int_0^T e^{-(i+\mu)t} c_t dt + e^{-(i+\mu)T} \bar{C}(\mu)\right]}{(\beta+\mu)\int_0^T e^{-(\beta+\mu)t} a_t dt + E e^{-(\beta+\mu)T}}. \tag{9}$$

An increase of the probability μ decreases the denominator and has an ambiguous impact on the annualized cost in the numerator. If costs are decreasing (e.g. the project consists mainly in costly capital expenditures before cheaper operating expeditures) the numerator is increasing with respect to μ and an increase of the probability of a backstop should delay the implementation of the project: A familiar result in the theory of option. Note also that intermediary abatements do intervene even if $\beta = 0$, because the costs of the project are affected by the calendar time t.

5 Illustrations

5.1 The optimal electricity mix of technologies in France at the 2050 horizon

Many empirical studies discuss the abatement cost based on LCOE calculations. This is especially the case in the electricity sector. Typically a reference year is selected, and several portfolios of technologies are compared through their respective LCOE and potential emissions. One portfolio is taken as the reference (business as usual, BAU), and it is then a simple matter to derive the abatement cost of the portfolios relative to BAU.

Corberand et al. (2021) provide such an analysis using the LCCA to compare various portfolios for the French electricity mix for a given level of consumption for the year 2050. The main assumptions are the following: The demand level is given and fluctuates by hour and day within the year. A portfolio is characterized by a mix of technologies. An optimization model is used to define the required capacities to meet the fluctuating demand. The investment and operating costs of the corresponding capacities are defined, given their respective life time and assuming that they are greenfield; all costs are based on what is expected to be the state of the art of the technologies in the year 2050.

For the sake of illustration consider the case in which BAU is 30% nuclear, 50% renewables, 10% hydropower, and 10% natural gas whereas alternative portfolios would show reductions in natural gas (and related emissions). We focus on the portfolio Proxy AMS which consists of 34% nuclear, 50% renewables, 10% hydropower, complemented by methanation and methanisation. The capacities in the two scenarios are different because the fluctuations in demand are matched with a different mix of technologies. Electrolysers, batteries and gas turbines are used in Proxy AMS to cope with fluctuations. For a consumption level at 532 TWh, the yearly equivalent cost for BAU is estimated to be € 45 815 million, while the excess yearly equivalent cost for Proxy AMS is estimated to be € 6 900 million, which gives an LCOE at 86 € /MWh versus 99€ /MWh (Table 18, page 66). The calculation of the avoided emissions amounts to 18.7 Mt CO_2, that is .035 tCO_2/MWh. This gives a LCCA of 370 € /tCO_2 (Table 21, page 75).

A simple way to illustrate our approach would be to proceed as follows. Suppose that a smooth transition from BAU to Proxy AMS starts at some predefined year to be completed over ten years, $T = 10$. Assume that the excess yearly cost would linearly converge from some multiple of € 6 900 million to € 6 900 million . Say that the multiple is 2. The actual excess cost at year τ is computed as this excess cost times the percentage of substitution at that year; for instance at year $\tau = 1$, the percentage of implementation is 10% and $c_1 = 6900*(1+.9)*.1 = 1311$, at year $\tau = 2$ the percentage is 20% and $c_2 = 6900*(1+.8)*.2 = 2884$, and so on until year $\tau = 10$, when the percentage of implementation is 100% and $c_{10} = 6900*(1+0)*1 = 6900$. For years after T, we assume that they remain constant (no learning-by-doing, no backstop technology).

As noted in Corollary 1 if one were to calculate the LCC for the extended Proxy AMS portfolio it would be zero since the discounted excess cost is finite while the total avoided emissions go to infinity (assuming a social cost of carbon that grows at the discount rate). The relevant question is at what time should one launch the Proxy AMS portfolio? The answer suggested in this paper is to use the DAC. With our set of parameters using a discount rate of 3%, recalling that intermediary abatements are irrelevant, we get DAC=322 € $/tCO_2$. If the social cost of carbon is taken as 500 € tCO_2 in line with Quinet et al. (2019), the optimal launch time for Proxy AMS would be year 2035. Similar calculations could be done for the other porfolios so as to select the best one.

More realistic assumptions should be introduced such as having the set of costs for a portfolio depend on the calendar date accounting for exogenous technological change and a specific learning-by-doing for the future portfolio. This exercise suggests some directions in which the LCCA analysis could be extended to compare portfolios.

5.2 The choice of batteries versus hydrogen fuel cells for mobility

The transport sector needs to be decarbonized and two green technologies stand out for achieving this goal: batteries and hydrogen fuel cell vehicles. The choice between these technologies depends on a number of factors such as range, and refuelling time among others. The rate of learning-by-doing of each technology will play a significant role. To assess the evolution of the total cost of ownership (TCO) of each technology one needs to have an idea of the respective volumes and learning-by-doing. For instance consider the case of a diesel bus (DB) operator who wants to go green using either battery electric buses (BEB) or fuel cell electric buses (FCEB). In the short term FCEB are more expensive than BEB, but what about the long term? The long term TCOs of FCEB and BEB depend on the respective market shares, that will be achieved which depend on the cost evolution, which depend on the learning-by-doing rates, etc. Our proposed metric provides an interesting starting point to formalize this issue as a closed loop optimization problem.

Start with a simple formalization of the situation. Time is continuous and denoted t. The discount rate is constant and denoted i. The social cost of carbon $P_0(t)$ grows at the rate i. There are N DB to green, and the life time of a bus is one unit, whatever the technology. The cost of a DB is constant and normalized to zero. Its rate of emissions is also constant and denoted E_D. The cost of a green bus exhibits learning-by-doing, the form of which is assumed to be:

$$C(X_t, x_t) = [\underline{c} + (\bar{c} - \underline{c})e^{-\lambda X_t}]x_t$$

in which, the variables X_t and x_t respectively stand for the cumulative production and the production at time t; the parameters $\underline{c}$ and $\bar{c}$ respectively stands for the long and short term marginal costs; and λ represents learning-by-doing. These variables and parameters will be indexed by B or H to distinguish between BEB and FCEB.

The cost function is a special case of the one considered in Creti et al. (2018) in two respects. Firstly there is no convexity so that Lemma 2 in that paper applies: the transition to either green technology considered independently would take place instantaneously at a date s to be determined for BEB and FCEB respectively. Secondly the precise form of learning-by-doing, which may be seen as an extension of the commonly used form in which $\underline{c} = 0$, facilitates the calculation of the DAC. Indeed, once the transition takes place the future discounted cost denoted $\bar{C}$ becomes:

$$\bar{C} = \int_0^{+\infty} e^{-it}[\underline{c} + (\bar{c} - \underline{c})e^{-\lambda N t}]N dt = \frac{1}{i}\underline{c}N + (\bar{c} - \underline{c})\frac{N}{i + \lambda N}$$

so that the total social discounted cost $\Gamma(s)$, given by equation (4), simply becomes:

$$\Gamma(s) = P_0 s(NE_D) + e^{-is}\bar{C},$$

which is mimimized when s solves

$$P_0 NE_D = ie^{-is}\bar{C}$$

that is,

$$P_0 e^{is} = P_s = \frac{i\bar{C}}{NE_D} = \frac{1}{E_D}\left[\bar{c}\frac{i}{i + \lambda N} - \underline{c}\frac{\lambda N}{i + \lambda N}\right]$$

The right hand-sides corresponds to the DAC. It may be seen as a weighted average between two static MACS: one with the short term marginal cost $\bar{c}$, and the other with the long term marginal cost $\underline{c}$, with the weights depending on the rate of learning-by-doing λ and the size of the market N. The higher λ and N, the lower the DAC which eventually converge to the standard MAC with the long term marginal cost. On the contrary, if λ is null, the DAC coincides with the standard MAC with the short term marginal cost whatever the size of the market. These results are intuitive and nicely quantified with our metric. Coming back to the choice between BEB and FCEB, according to Corollary 2, the technology to be chosen is the one with the lowest DAC.

Three simple assumptions should be revisited to seriously address the choice between the two technologies. Firstly the non convexity of the cost function implies that the transition occurs simultaneously for the whole fleet of DB, whereas a progressive transition would be more realistic. Secondly BEB and FCEB are assumed to be perfectly substitutable, a more elaborate market model should be introduced in which the competitive advantages of each technology be made explicit. Finally the link between the learning-by-doing and the market shoud be clarified: for instance can we expect some cost sharing between buses and trucks for some components? Despite these limitations, we believe that our preliminary analysis provides an interesting starting point.

6 Conclusion

The paper is intended to provide the relevant cost benefit framework for public authorities investigating the potential of emission mitigation projects. Such projects typically explore different ways to decarbonize a pre-existing activity through the deployment of new technologies. As such they qualify as pilot projects, making it critical to incorporate into the analysis future cost reductions and externalities. We propose a new metric that incorporates the effects of dynamics into the analytical framework. It is based on the notion of the extended project that is,, the project itself and a proposed continuation pathway in which decarbonization persists thanks to some incremental costs. We suggest different ways to define these incremental costs to capture the future benefits provided by the project.

The new metric is a non trivial extension of either the levelized cost of carbon (LCC) or the levelized cost of carbon abatement (LCCA) as operationalized in (Baker and Khatami, 2019) and (Friedmann et al., 2020) respectively. Both methods typically lead to what we call the paradox of the horizon: For the LCC, any long enough project which decarbonizes a pre-existing activity would be good to launch immediately; For the LCCA, it is not affected in any delay introduced in launching the project. We reformulate the question of whether or not to launch a project as when to launch it, and derive the associated metric. It is calculated as the annual discounted cost divided by the long term abatement. We also show how to use this new metric to compare several competing projects.

The emphasis here is on methodology. In any specific field study, however, a number of idiosyncratic features would appear. In some circumstances a local project would need to be enlarged through a life cycle analysis to meaningfully address the decarbonization issue. Sometimes a project cannot be discussed independently of similar parallel projects, the coordination of which has important consequences in terms of synergies and externalities. We provide some clues on how to proceed through two illustrations: the choice of the optimal mix of technologies for the electricity sector (a typical case in which the LCCA is used), and the comparison between competing green technologies for mobility (an important new topic for the energy transition in which the LCC is used).

As a concluding word of caution, this paper does not address the question of abatement curves inferred from large multi-sector scenarios made at the national or international level for the whole economy (note that such models have their own weaknesses; see Kesicki and Ekins, 2012). Projects are analyzed in this paper through a partial equilibrium framework whereas the analysis of global multi-sector scenarios requires a general equilibrium framework. While intrinsically simpler, a number of issues remain unaddressed. Enlarging a local project study to encompass externalities, as suggested in sectoral applied studies, may be a more operational route than embedding a project into a multi-sector scenario. We think that such projects are worth of study, and enriching the methodology to analyze them is a relevant exercise.

References

Athias, C. (2020). *Déploiement de l'hydrogène dans les zones industrialo-portuaires*, Master's thesis, ENS Lyon and chair Energy and Prosperity.

Baker, E. D. and Khatami, S. N. (2019). The levelized cost of carbon: a practical, if imperfect, method to compare co2 abatement projects, *Climate Policy* **19**(9): 1132–1143.

Brunet, J. and Ponssard, J.-P. (2017). Policies and deployment for fuel cell electric vehicles an assessment of the normandy project, *International Journal of Hydrogen Energy* **42**(7): 4276–4284.

Corberand, J., Domergue, S. D., de Guibert, O., Memmi, E. and Pommeret, A. (2021). Coûts d'abattement dans le secteur de l'énergie, *Les coûts d'abattement, rapport de la Commission présidée par Patrick Criqui*, France Stratégie.

Creti, A., Kotelnikova, A., Meunier, G. and Ponssard, J.-P. (2018). Defining the abatement cost in presence of learning-by-doing: Application to the fuel cell electric vehicle, *Environmental and resource economics* **71**(3): 777–800.

Friedmann, J., Fan, Z., Byrum, Z., Ochu, E., Bhardwaj, A. and Sheerazi, H. (2020). Levelized cost of carbon abatement: An improved cost-assessment methodology for a net-zero emissions world, *Columbia University SIPA Center on Global Energy Policy: New York, NY, USA* .

Gillingham, K. and Stock, J. H. (2018). The cost of reducing greenhouse gas emissions, *Journal of Economic Perspectives* **32**(4): 53–72.

Joskow, P. L. (2011). Comparing the costs of intermittent and dispatchable electricity generating technologies, *American Economic Review* **101**(3): 238–41.

Kesicki, F. and Ekins, P. (2012). Marginal abatement cost curves: a call for caution, *Climate Policy* **12**(2): 219–236.

Lak, S. (2019). Blue hydrogen as accelerator and pioneer for energy transition in the industry, *H-vision, Feasibility study report* .

Meunier, G., Moulin, L. and Ponssard, J.-P. (2019). Why local initiatives for the energy transition should coordinate. the case of cities for fuel cell buses in europe, *Working Paper chair Energy and Prosperity* .

Meunier, G., Moulin, L. and Ponssard, J.-P. (2022). Why local initiatives for the energy transition should coordinate. the case of cities for fuel cell buses in europe, *Revue d'Economie Industrielle* .

Quinet, A., Bueb, J., Le Hir, B., Mesqui, B., Pommeret, A. and Combaud, M. (2019). La valeur de l'action pour le climat, *France stratégie* pp. 3624–48.

Teyssier d'Orfeuil, F. (2020). *Regional clusters for the energy transition, a cost-benefit analysis*, Master's thesis, Ecole Polytechnique and chair Energy and Prosperity.

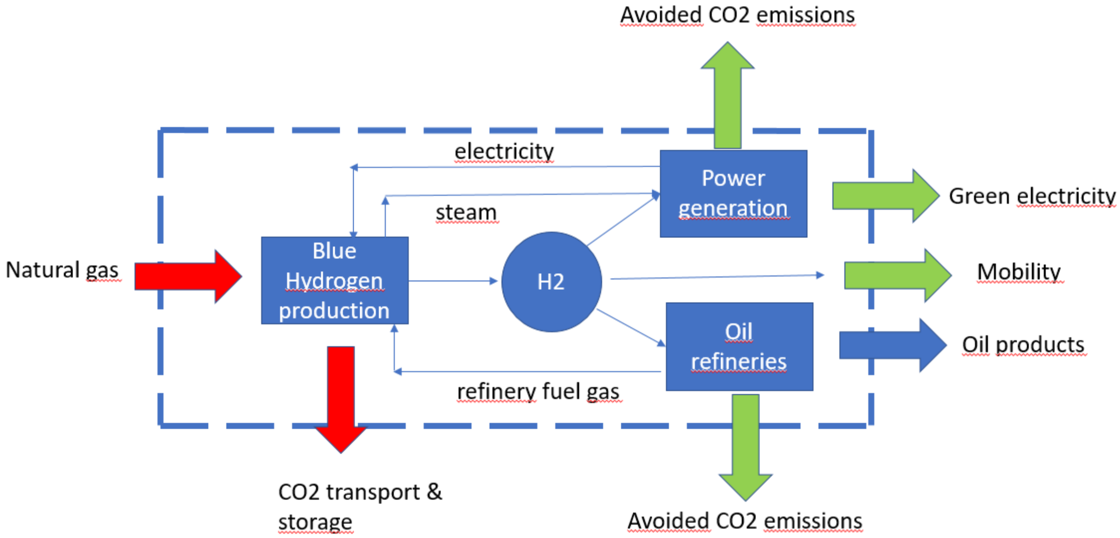

Figure 1: Overview of the energy and cash flows considered in the H-vision project (from Lak, 2019, Figure 6B)

A Two empirical studies

A.1 The H-Vision project in Rotterdam

Port areas are often at the heart of industrial activities that have large carbon intensities (e.g., gas turbine electricity generation, oil refineries, steel, aluminum, chemicals). Ports are also often located close to major metropolitan areas (e.g., Rotterdam, Hamburg, Liverpool, Dunkirk, Marseille Fos), where the population is deeply affected by air pollution. As such decarbonizing these areas is a national objective and many related projects have been launched or or been explored launched.[3] In the case of Rotterdam, the H-Vision project, launched in 2019 [4] will enable massive decarbonization in the area. The scope of analysis is illustrated in Figure 1. The project is expected to be operational in 2026, decarbonizing an estimated 1 to 2 million tons of CO_2 per year, then ramping up to 4 million tons per year in 2030, and achieving a steady state thereafter. The project is based on the development of blue hydrogen, produced by reforming natural gas, as a substitute to natural gas or coal in two local industries (power generation and oil refineries), and a limited amount in mobility uses. Hydrogen is produced through the ATR (auto-thermal reforming) process, the CO_2 emissions being captured and stored in the North Sea. The industrial units benefit from avoiding the cost of permits under the Europena Trading Scheme; the power generation unit also benefits from extended sales of green electricity. Several scenarios are constructed: a business as usual (BAU) aligned with the Current Policy scenario of the International Energy Agency (in this scenario coal is progressively substituted by biomass which induces a loss of production for power generation); an economic world scenario and a more aggressive

[3] For a review of some port projects including the Rotterdam project see Athias, 2020 ; note that professional conferences are regularly devoted to this subject https://meet4hydrogen.com/en/program.php#hyports-conferences

[4] Steven Lak, 2019, Blue Hydrogen as accelerator and pioner for energy transition in the industry, Pdf

decarbonization scenario called the sustainable world scenario. We only report on the financial analysis of the economic world scenario versus BAU. Several components are detailed: capital expenditures, operating expenses, and revenues over the life time of the project for the period 2023-2045. The net cash flows for each component, assuming a 3 % social discount rate, are summarized in Table 1.

Component	Value in billion €
H_2 **production capex**	-2.1
H_2 **production opex**	-5.8
H_2 **transport and storage**	-3.2
Refineries retrofitting	-0.3
Power plants retrofitting	-0.5
Power plants revenues	7.7
Refineries avoided ETS certificates	3.4
Rounding errors	0.1
Net present value	-0.7

Table 1: Components of the economic model (Lak, 2019, Project economics, section 9)

The LCCA of the economic world scenario versus BAU is obtained by dividing the net cash-flow (-0.7 billion €) by the discounted CO_2 avoided from 2026 to 2045 (47.9 Mt CO_2); this gives an LCCA of 146 € /tCO_2. Note that this includes the revenue received from avoiding ETS permits of refineries and the sale of green electricity (including avoiding ETS permits) for power generation. If the abatements were not discounted the corresponding LCC would be 89 € /tCO_2. The economic analysis does not introduce what will happen after 2045, nor the fact that this project is part of a larger national plan in which it is expected to be interconnected with other projects developed in industrial areas in the Netherlands, as well as in Belgium and Germany. Furthermore, the overall national plan will progressively rely on the production of green hydrogen, taking advantage of the large deployment of offshore wind farms. Neither learning-by-doing nor spillovers issues are considered.

A.2 Fuel cell electric buses in Europe

Many studies have focused on decarbonization of the transport sector. According to EU statistics this sector was responsible for around 25 % of greenhouse gas emissions in 2019 (https://appsso.eurostat.ec.europa.eu/). The sector involves different segments (small vehicles, heavy duty vehicles, trains, maritime transport, airplanes), and the cost-benefit analysis of alternative low-carbon technologies (batteries, biofuels, fuel cells, etc.) depends on the segment under consideration. Over the years, the EU has introduced new legislation to tackle transport challenges and to meet its climate and energy targets. National legislation as well as city rules and private initiatives have also played a strong role in triggering decarbonization of specific segments.

One example is the EU program known as JIVE (Joint Initiative for Hydrogen Vehicles across Europe, https://www.fuelcellbuses.eu/projects/jive), designed to encourage the deployment of fuel cell electric buses in major EU cities. The program provides generous subsidies to public transport operators in municipalities that commit to substituting diesel buses with FCEB. For each of these deployments, the operator elaborates a business plan in the corresponding city. The business plan is used to secure further subsidies from national and regional agencies as well as from private financial institutions. Some agencies require that a cost benefit study be undertaken to evaluate the sustainability of the project evaluating its cost and benefits in terms of avoided greenhouse house gas emissions, local pollutants and employment.

Meunier et al. (2022) have undertaken an economic analysis of the JIVE program. As part of the analysis the authors derive the total cost of ownership (TCO) for three alternative technologies: diesel, fuel cells and batteries, using published sources and interviews. Abatement costs can then be obtained using the average emissions of each technology, taking into account local pollutants and traffic congestion. The underlying metric used in the calculation is the LCCA (annual extra cost of the substitution / avoided emissions). It appears that the current and future abatement costs of FCEB remain much higher than the corresponding costs of BEB, which suggests a deployment of FCEB limited to niche markets.

TCO € /km 2020	FCEV	BEB	Diesel
1 Fixed capital	1.71	1.23	0.55
Purchase price	650 000	470 000	210 000
2 Maintenance	0.40	0.80	0.30
Personnel costs	2.63	2.63	2.63
3 Fuel	.80	0.31	0.48
Unit price € (kg H2, kWh, l)	10	0.24	1.60
Consumption per 100 km	8.00	1.30	30
Total 1+2+3	5.53	4.97	3.96

Table 2: Estimates of the TCO for FCEB, BEB and diesel buses (Meunier et al., 2022)

Such an economic analysis is limited: the TCO's are computed at various point in time without making explicit two important factors: 1) how to introduce interregional dependencies since the future cost in one city will depend on the policies adopted in other cities; and 2) how will a European scenario contribute to the decline of future TCO's by providing a large enough volume to decrease vehicles costs.

B Proofs

B.1 Proof of Proposition 3

With a given project $k = 1, 2$ the optimal launch date $\bar{s}_k$ is such that $P_0 e^{i\bar{s}_k} = DAC_k$ so

$$\bar{s}_k = \frac{1}{i}[\ln(DAC_k) - \ln(P_0)]$$

Recall that $A_k = \int_0^{T_k} a_{\tau,k} d\tau$ stands for the total intermediary abatements of the project for $k = 1, 2$. Project 1 has fewer emissions$\bar{E}_1$ than project 2 if and only if

$$\begin{aligned}
\bar{E}_1 < \bar{E}_2 &\Leftrightarrow (\bar{s}_1 + T_1)E - A_1 < (\bar{s}_2 + T_2)E - A_2 \\
&\Leftrightarrow (\frac{1}{i}\ln(DAC_1) + T_1)E - A_1 < (\frac{1}{i}\ln(DAC_2) + T_2)E - A_2 && \text{replacing } \bar{s}_k \\
&\Leftrightarrow \ln(DAC_1) + iT_1 - i\frac{A_1}{E} < \ln(DAC_2) + iT_2 - i\frac{A_2}{E} && \text{multiplying by } i/E \\
&\Leftrightarrow DAC_1 e^{iT_1} e^{-i\frac{A_1}{E}} < DAC_2 e^{iT_2} e^{-i\frac{A_2}{E}}
\end{aligned}$$

B.2 Proof of Proposition 4

With $P_t = P_0 e^{\gamma t}$ the project abatements are worth (from today point of view):

$$\int_s^{s+T} e^{-it} P_t a_{t-s} dt = \int_s^{s+T} e^{-\beta t} P_0 a_{t-s} dt = P_0 e^{\beta s} \int_0^T e^{-\beta t} a_t dt.$$

The total discounted cost (4) is then (using I):

$$\Gamma(s) = \int_0^{s+T} e^{-it} P_t E dt + e^{-is}[I + e^{-iT}\bar{C}] - P_0 e^{-\beta s} \int_0^T e^{-\beta t} a_t dt$$

Then the derivative with respect to the starting date s:

$$\begin{aligned}
\Gamma'(s) &= e^{-i(s+T)} P_{s+T} E - i e^{-is}[I + e^{-iT}\bar{C}] + \beta P_0 e^{-\beta s} A(\beta) \\
&= e^{-i(s+T)} P_s e^{\gamma T} E - i e^{-is}[I + e^{-iT}\bar{C}] + \beta P_s e^{-is} A(\beta)
\end{aligned}$$

Which is then cancelled for s that solves equation (8).

C Comparison of the metrics: a simple illustration

Consider a situation in which one wants to substitute an electric car for a fossil fuel car. The extra purchase price is K, the lifetime of either car is N years and the electric car would abate a tCO_2 per year. For simplicity we neglect fuel costs. The interest rate is i and defined as $\delta = \frac{1}{1+i}$. The discounted investment cost is K, which occurs at year 0. The annualized cost over the lifetime is

$$k = K\frac{1-\delta}{1-\delta^N}$$

$(K = \sum_{t=0}^{N-1} \delta^t k)$

C.1 Metrics with no intermediary abatements

Take the case of a single renewal:

$$LCC(1) = \frac{K}{Na}$$

$$LCCA(1) = \frac{discounted\, cost}{discounted\, abat} = \frac{K}{[\sum_{t=0}^{N-1} \delta^t]a} = \frac{k}{a} = \frac{annual\ cost}{annual\ abat}$$

If the car is renewed every N years X times, the time horizon is $T = XN$. The discounted cost becomes:

$$C(X) = K + \delta^N K ... = K\frac{1-\delta^{XN}}{1-\delta^N} = K\frac{1-\delta^T}{1-\delta^N}$$

and

$$LCC(X) = \frac{C(X)}{XNa}$$

which converges toward zero as X increases toward infinity (Corollary 1). However

$$LCCA(X) = \frac{C(X)}{a\frac{1-\delta^T}{1-\delta}} = K\frac{1-\delta^T}{1-\delta^N}\frac{1-\delta}{1-\delta^T}\frac{1}{a} = K\frac{1-\delta}{1-\delta^N}\frac{1}{a} = \frac{k}{a} = LCCA(1)$$

LCCA(X) is independent of X. To compute the DAC, let X go to infinity, this corresponds an extension of the situation with replication. Note that $C(+\infty) = \frac{K}{1-\delta^N} = \frac{k}{1-\delta}$ so that k is also the annualized cost of C). Consequently:

$$DAC = \frac{k}{a}$$

In that special case DAC is equal to LCCA.

C.2 Metrics with intermediary abatements

Suppose there are two cars, and we introduce a simple deployment scenario with ramping for exogenous constraints: only one car is renewed the first year, the second car the second year, then each car at the end of its life time and this continues over an infinite horizon.

- Discounted costs: For the first car: K in year zero and in years N, $2N$...total discounted cost $C = K/(1-\delta^N)$.

For the second car: K in year one and in years $N+1$...total discounted cost δC

Total discounted cost $C + \delta C$ and annualized cost $(1-\delta)(1+\delta)C$

- Abatements: a then $2a$ forever.

Discounted abatements

$$a + \delta 2a + \delta^2 2a + ... = \frac{1+\delta}{1-\delta}a$$

The metrics for this case are respectively:

$$LCC = 0$$

$$LCCA = \frac{(1+\delta)C}{\frac{1+\delta}{1-\delta}a} = \frac{(1-\delta)C}{a} = \frac{k}{a}$$

$$DAC = \frac{(1-\delta)(1+\delta)C}{2a} = \frac{1+\delta}{2}\frac{k}{a} < \frac{k}{a} = LCCA$$

The discounted cost and abatement do not depend on the metric but each metric suggests a different “launch date”: zero for the LCC, and an earlier date for the DAC than for the LCCA. The DAC does integrate the inertia due to ramping constraints (only one car the first year), which induces a launch earlier than without inertia.