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Alistair Macaulay

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Abstract

Using a novel decomposition, I show that systematic relationships between information and subjective models across agents distort the aggregate transmission of shocks in a general class of macroeconomic models. I document evidence of such a systematic correlation between household information and subjective models around inflation using unique features of the Bank of England Inflation Attitudes Survey: on average, households with more negative beliefs about the impacts of inflation obtain more information about inflation. A model in which acquiring information about inflation is costly, and observed information affects the perceived relationship of inflation and real incomes, can explain the empirical variation in information and subjective models in the cross-section and over time. The model generates time-varying shock transmission, and a selection effect that weakens the role of information frictions in aggregate dynamics. Through a novel channel, transitory spikes in inflation may become 'baked in' to inflation expectations, but only among those with the most positive subjective models of the effects of inflation.

JEL-Codes: D830, D840, E310, E710.

Keywords: information frictions, subjective models, heterogeneous agents, expectations, shock transmission.

Alistair Macaulay
Department of Economics & St. Anne's College
University of Oxford / United Kingdom
alistair.macaulay@economics.ox.ac.uk

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1 Introduction

The Full Information Rational Expectations (FIRE) assumption underlying many macroeconomic models is composed of two parts: the first (FI) concerns what agents observe, while the second (RE) concerns the models agents use to turn their information into expectations of the future. Indeed, whether it is a sophisticated forecaster running inflation data through a structural VAR, or a naive household using a rule-of-thumb to map their current experiences to future expectations, forming an expectation relies on the combination of information and a model.

However, when studying plausible departures from the extreme sophistication of FIRE, recent literature tends to consider these two assumptions separately. Models acknowledging that agents may not fully observe all variables in their environment in real time typically assume the agents know the true model governing the evolution of those variables.[1] In contrast, the literatures on learning, imperfect common knowledge, and others explore misperceptions about equilibrium laws of motion, but assume that agents have full information about current realisations.[2] Both strands of literature have amassed empirical evidence in support of relaxing their component of the FIRE assumptions.[3]

In this paper I consider information and subjective models jointly. Specifically, I ask: how do agent information and subjective models interact? And how does that interaction affect macroeconomic dynamics? In answering, I make three main contributions:

1. I introduce a novel decomposition of the aggregate response to shocks in a general log-linear model with arbitrary information and subjective models. I show that shock transmission depends on the covariance of the amount of information observed and subjective models across agents.
2. I document key facts about information and subjective models around inflation in UK household survey data, highlighting their relationship and behaviour over time.
3. I build a model in which the interaction of rational inattention and endogenous subjective models accounts for the empirical findings. The interaction has implications for the role of information frictions in aggregate dynamics, and the evolution of expectations and consumption following inflationary shocks.

The novel covariance highlighted in the first part of the paper arises because agents receiving potentially noisy information about a variable use it for two purposes: they update their expectations about that variable directly, but they also update their expectations of other variables, depending on how the variables are related in their subjective

[1] e.g. Sims (2003), Reis (2006), Angeletos and La'O (2010), Maćkowiak and Wiederholt (2009, 2015), Angeletos and Sastry (2021), among many others.

[2] e.g. Bullard and Mitra (2002), Eusepi and Preston (2011), Ilut and Schneider (2014), Angeletos and Lian (2018), Farhi and Werning (2019), among many others.

[3] For information, see e.g. Link et al. (2021). For subjective models, see e.g. Andre et al. (2022).

models. Information about a given shock therefore causes some agents to increase their expectations of other variables, and others to decrease them, depending on their subjective models. If that information is concentrated among agents with particular subjective models, their expectations are the ones most influenced by the shock, and their model therefore has a disproportionate impact on aggregate expectations. Formally, the aggregate expectations of any imperfectly observed variable y depend on the cross-sectional covariance of direct information on the shock x ($\partial \mathbb{E}x/\partial x$) and cross-learning ($d\mathbb{E}y/d\mathbb{E}x$), which summarises agent subjective models.[4]

The covariance of information and subjective models therefore affects aggregate responses to shocks, as long as expectations of the variables updated through the subjective model matter for agent choices. Agents with different subjective models react differently to the same piece of information, and so the aggregate response depends on the distribution of information among those with heterogeneous subjective models. Since a simple definition of a narrative is that it is composed of a state of the world (information) and a series of perceived consequences (subjective model) (Gibbons and Prusak, 2020), I refer to this effect as the narrative heterogeneity channel of shock transmission.[5] Importantly, if an agent's information affects their subjective model, or their subjective model affects the information they receive, that will lead to systematic relationships between the two components of expectations, and so to the narrative heterogeneity channel.

In the second part of the paper I explore the joint distribution of information and subjective models empirically. Since expectations combine information and subjective models, data on expectations is insufficient. I therefore use unique features of the Bank of England's Inflation Attitudes Survey to separate information and subjective models about inflation. Specifically, respondents are asked about the information sources they used to arrive at their expectations, and how a hypothetical rise in inflation would affect the strength of the UK economy. The first of these questions concerns information without involving the conclusions drawn from it. The second concerns the respondent's subjective model of how inflation relates to the rest of the economy, without asking what they believe inflation is or will be.

Using this data, I document three facts about household subjective models and information about inflation. First, households who believe inflation makes no difference to the strength of the economy are less likely to use information about inflation when forming their expectations, while households who believe it is positively or negatively

[4]Andre et al. (2022) similarly use cross-learning to summarise an agent's subjective model.

[5]Eliaz and Spiegler (2020) define a narrative as a causal chain of variables represented by a DAG. This can be seen as a restriction on the kinds of subjective models that qualify as part of narratives. Unlike Shiller (2017), this part of the paper does not consider how narratives spread, but rather studies the impact of a given distribution of narratives on shock transmission.

associated with the strength of the economy use similar information sources. Second, the proportion of households who believe inflation is associated with a weaker economy rises strongly with recent realised inflation. Third, households who believe inflation weakens the economy expect substantially higher inflation, and perceive that inflation has been higher in the last year, than other households. Altogether, these facts suggest that in the case of inflation, information and subjective models are not independent of each other, and their joint distribution varies over time. The decomposition in the first part of the paper indicates that this will have implications for the aggregate transmission of shocks.

To explore those implications, in the final part of the paper I develop a model that is consistent with the empirical facts. The key ingredients required to match the data are that households face costs of acquiring information about inflation, and that they adjust their subjective models of how inflation affects real incomes when they observe the realisations of their chosen information.

With costly information as in the rational inattention literature (Sims, 2003), subjective models influence optimal information choices, as the expected benefit of information depends on the extent to which it affects household choices. If a household has a subjective model that implies inflation is irrelevant for their choices, there is no benefit to information, so they will not pay for it. This explains the cross-sectional relationship of information and subjective models.

If households then update their subjective models as a result of the realisations of their noisy signals, the model can also match the remaining empirical results. I propose a reduced-form process in which perceiving that inflation is high causes households to update their subjective model towards the belief that inflation erodes real incomes.[6] When realised inflation rises, average inflation perceptions rise, and so the proportion of households with negative models of the effects of inflation rises. Within the cross-section, high perceived inflation is associated with such negative subjective models.

This interaction between information and subjective models has several implications for aggregate behaviour. First, the effect of heterogeneous subjective models on information choices implies a selection effect: the households who process the most information about inflation are the ones who react the most to that information. This selection reduces the importance of information frictions at the aggregate level, relative to the level of frictions implied by micro-evidence on inattention.[7] This may explain why, despite the evidence of widespread inattention to macroeconomic variables, representative-agent DSGE models only require a small amount of inattention to match empirical aggregate impulse responses (Maćkowiak and Wiederholt, 2015).

[6] While the implications of the model are derived using this reduced-form link from information to subjective models, I offer a microfoundation in Appendix D.4.

[7] This is akin to the selection effect in menu cost models of price setting (Golosov and Lucas, 2007).

Second, the interaction of information and subjective models implies that aggregate shock transmission varies substantially over time. Larger spikes in inflation cause more households to switch to believing inflation harms the economy, which means that the elasticity of aggregate consumption to inflation becomes more negative. As inflation perceptions are persistent, so is this effect of inflation shocks on the distribution of information and subjective models. In particular, these effects are powerful among households who are attentive to inflation, as their expectations are more responsive to the initial shock. The covariance of information and subjective models therefore shifts after inflationary shocks. In a quantification to the UK over 2001-2019, the narrative heterogeneity channel accounts for 39% of the time-variation in shock transmission.

Finally, if observed information can also affect a household's long-run inflation expectations, the interaction of information and subjective models can further lead to high expected inflation becoming 'baked in' among certain households, as has recently concerned policymakers and analysts.[8] Specifically, households who start out with subjective models in which inflation strengthens the real economy react to higher perceived inflation by adjusting towards a more neutral view, in which inflation does not matter for their choices. If they also increase their long-run expectations, they carry this more neutral view into the following periods, which means they will not respond as strongly to future inflation information. That information therefore has less value to them, so they reduce the amount of information they acquire, and so never adjust their long-run expectations back down, even if inflation subsequently remains low. This persistently changes the joint distribution of information and subjective models, and so alters the future dynamics of the economy through the narrative heterogeneity channel. It also implies that policymakers should focus their attention on the expectations of those households with positive subjective models of the effects of inflation, as they are the households who would be hardest to bring back if their inflation expectations became unanchored.

Related literature. This paper principally contributes to the broad literatures on information frictions, subjective models, and heterogeneity in macroeconomics. A large number of papers have studied the role of limited information in macroeconomic outcomes (see reviews in Hubert and Ricco, 2018; Coibion et al., 2018). These models typically assume that agents know the true equilibrium model of the economy: if they could observe the realisations of the exogenous shocks, they could map perfectly from those into all endogenous variables. Similarly, literatures on learning (Eusepi and Preston, 2018), model uncertainty (Hansen and Sargent, 2008; Ilut and Schneider, 2014), imperfect common knowledge (Angeletos and Lian, 2018), level-k thinking (Farhi and Werning, 2019; Iovino

[8]See for example Michael Schumacher (Wells Fargo), quoted in Domm (2021): "[Jerome] Powell has sounded concerned about [inflation] expectations getting baked in."

and Sergeyev, 2022), and others study the macroeconomic implications of misperceptions of the true structural relationships in the economy.[9] These generally assume that agents observe all variable realisations up to the current period. To my knowledge, this paper is the first to systematically study the combination of heterogeneous information frictions and subjective models, and how their interaction shapes aggregate dynamics.

In much of this theoretical literature, the departure from FIRE naturally implies heterogeneity in expectations, as has been well-documented empirically (e.g. Carroll, 2003). Noisy information implies agents receive idiosyncratic signals (e.g. Angeletos and Pavan, 2007; Maćkowiak and Wiederholt, 2009), and recent work has also explored differences in the incentives to acquire information across agents (Broer et al., 2020; Macaulay, 2021). When agents learn from experience about the process determining certain variables, different cohorts will form different subjective models (Malmendier and Nagel, 2016). Other theories of subjective model formation also frequently feature such heterogeneity (see Hommes, 2021, for a review). However, in relaxing only one aspect of FIRE at a time, these models necessarily miss the narrative heterogeneity channel explored in this paper, as this relies on simultaneous heterogeneity in both information and subjective models.

Where existing papers do simultaneously depart from both full information and rational expectations, they typically consider the effects of these departures in representative-agent settings (Ryngaert, 2018; Angeletos et al., 2020; Pfäuti, 2022). Models with diagnostic expectations (Bordalo et al., 2018, 2020; Bianchi et al., 2021) similarly feature neither full information nor rational expectations in how information is used to update expectations, but to date this literature has focused on models with a representative agent. I extend these literatures by considering heterogeneity in the components of expectation formation, which gives rise to the narrative heterogeneity channel.

I also contribute to the literature on the role of narratives in economic decisions. While theoretical models of narratives have been developed for questions in microeconomics and political economy (Bénabou et al., 2018; Eliaz and Spiegler, 2020), and policymakers have used them to explain aggregate behaviour (e.g. Haldane, 2020), most work in macroeconomics has been concerned with empirically tracking particular narratives and their impacts (Shiller, 2017; Larsen et al., 2021). The framework developed in this paper is a first step towards incorporating economic narratives into macroeconomic models.

The empirical part of the paper also contributes to our understanding of the narratives households use to understand inflation. This therefore relates to early work on the reasons many households dislike inflation (Shiller, 1997), and more recent work studying how this relates to expectations of other variables and actions (Kamdar, 2019; Candia et al., 2020). More generally, several papers have found evidence for heterogeneous information (Link

[9] Molavi (2019) shows how these can all be cast as forms of misspecification in subjective models.

et al., 2021, 2022) and heterogeneous subjective models (Laudenbach et al., 2021; Andre et al., 2022) in a variety of contexts. Beutel and Weber (2021) and Macaulay and Moberly (2022) find evidence for simultaneous heterogeneity along both dimensions.[10] I extend this literature by separating information from subjective models in a survey with a long time series and rich data on household characteristics and other expectations. This allows me to observe information and subjective models at the individual level, over two decades in which macroeconomic conditions changed a great deal.

Michelacci and Paciello (2020) use different questions in the IAS to elicit preferences over inflation and interest rates, and explain the heterogeneity in these preferences by wealth using a model with Knightian uncertainty. However, while subjective models and preferences over aggregate variables are closely linked, this paper differs from them in studying the relationship of subjective models with information, rather than wealth.[11]

Finally, the model with rational inattention and endogenous subjective models also contributes to the literatures on selection effects in information (Yang, 2019; Afrouzi and Yang, 2021) and the time-varying transmission of aggregate shocks.[12] In particular, the model suggests a novel channel through which inflation expectations may remain high after a transitory inflation shock, but only for those with positive subjective models of inflation, which affects the aggregate reaction to future inflationary shocks.

The rest of the paper is structured as follows. Section 2 derives the novel decomposition of aggregate responses to shocks in a general log-linear model with arbitrary information and subjective models. Section 3 explores information and subjective models about inflation in the IAS data. Section 4 develops the model of rational inattention and endogenous subjective models that matches the empirical findings, and Sections 5 and 6 explore the implications of that model. Section 7 concludes.

2 General decomposition

I begin by presenting a decomposition of the aggregate household response to an arbitrary shock, in a general log-linear model. The decomposition highlights the roles played by information and subjective models, and their distribution across households, in determining the strength of aggregate shock transmission. Specifically, I show that the aggregate consumption response to a shock comes through three channels: the representative agent channel, the preference heterogeneity channel, and the narrative heterogeneity channel.

[10] Link et al. (2022) also find no association between information acquisition and disagreement, suggesting substantial heterogeneity in subjective models alongside the heterogeneity in information.

[11] See Dräger et al. (2020) for more evidence on preferences over macroeconomic variables.

[12] See e.g. Primiceri (2005), Galí and Gambetti (2009, 2015), Paul (2020), among many others.

2.1 The agent

Household $h \in H$ chooses a $N_x \times 1$ vector of choice variables $\boldsymbol{X}_t^h$ in period t. Letting lower case letters be log-deviations of variables from some arbitrary point, a log-linear approximation of their policy function can be written:[13]

$$\boldsymbol{x}_t^h = \boldsymbol{\mu}_t^h \mathbb{E}_t^h \boldsymbol{z}_t^h \tag{1}$$

where $\boldsymbol{z}_t^h$ is a $N_z \times 1$ vector of variables exogenous to the household,[14] and $\boldsymbol{\mu}_t^h$ is a $N_x \times N_z$ matrix of coefficients. For simplicity, I will refer to these coefficients as representing the household's preferences, though note they could also come from any constraints the household faces.

The vector $\boldsymbol{z}_t^h$ may include both aggregate and idiosyncratic variables. Some elements of $\boldsymbol{z}_t^h$ may be known or observed by household h, but for the unknown elements, the household-specific expectations operator $\mathbb{E}_t^h$ may or may not coincide with rational expectations. The elements of $\boldsymbol{z}_t^h$ may also be realised in any period: the indexation at time t simply reflects that they are the variables that matter for period t choices. This setup therefore encompasses a wide range of models. I show a particular example with a standard consumption function in Section 2.2.

I now consider a shock to one of the variables in the policy function z_{nt}^h. The reaction of household choices is determined by the changes in the expectation of each element of the policy function:

$$\frac{d\boldsymbol{x}_t^h}{dz_{nt}^h} = \boldsymbol{\mu}_t^h \frac{d\mathbb{E}_t^h \boldsymbol{z}_t^h}{dz_{nt}^h} \tag{2}$$

Applying the chain rule to the derivative of each element of $\mathbb{E}_t^h \boldsymbol{z}_t^h$ leads to a simple expression for the household's response to the shock.

Proposition 1 *For any household with policy function described by equation 1, the response to a shock to z_{nt}^h is given by:*

$$\frac{d\boldsymbol{x}_t^h}{dz_{nt}^h} = \boldsymbol{\mu}_t^h (\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^h)^{-1} \boldsymbol{\delta}_{nt}^h \tag{3}$$

[13] This linearisation does not need to be taken about a steady state, or about the same point for each household. If two households have different idiosyncratic state variables, they can therefore have different responses to aggregate variables and expectations, just as they would in a fully non-linear model. This is why the coefficients $\boldsymbol{\mu}_t^h$ are indexed by household and by period, as the linearisation could be taken about different points each period.

[14] This is without loss of generality, as any endogenous choice variable can also be expressed as a linear function of other elements of $\boldsymbol{z}_t^h$. Substituting out using that function, and repeating for any remaining endogenous variables, gives a policy function only in terms of variables exogenous to the household.

where:

$$\boldsymbol{\mathcal{M}}_t^h = \begin{pmatrix} 0 & \mathcal{M}_{12,t} & \dots & \mathcal{M}_{1N_z,t} \\ \mathcal{M}_{21,t} & 0 & \dots & \mathcal{M}_{2N_z,t} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{M}_{N_z1,t} & \mathcal{M}_{N_z2,t} & \dots & 0 \end{pmatrix}, \qquad \mathcal{M}_{ij,t}^h \equiv \frac{\partial \mathbb{E}_t^h z_{it}^h}{\partial \mathbb{E}_t^h z_{jt}^h} \tag{4}$$

$$\boldsymbol{\delta}_{nt}^h = \left(\frac{d\mathbb{E}_t^h z_{1t}^h}{dz_{nt}^h}\bigg|_{\mathbb{E}_t^h z_{j\neq 1,t}}, \frac{d\mathbb{E}_t^h z_{2t}^h}{dz_{nt}^h}\bigg|_{\mathbb{E}_t^h z_{j\neq 2,t}}, \dots, \frac{d\mathbb{E}_t^h z_{N_zt}^h}{dz_{nt}^h}\bigg|_{\mathbb{E}_t^h z_{j\neq N_z,t}} \right)'$$

Proof. The derivative of the expectation of each element z_{it}^h of $\boldsymbol{z}_t^h$ can be decomposed using the chain rule:

$$\frac{d\mathbb{E}_t^h z_{it}^h}{dz_{nt}^h} = \frac{d\mathbb{E}_t^h z_{it}^h}{dz_{nt}^h}\bigg|_{\mathbb{E}_t^h z_{j\neq i,t}} + \sum_{j\neq i}^{N_z} \frac{\partial \mathbb{E}_t^h z_{it}^h}{\partial \mathbb{E}_t^h z_{jt}^h} \frac{d\mathbb{E}_t^h z_{jt}^h}{dz_{nt}^h} \tag{5}$$

Stacking this expression over all elements of $\boldsymbol{z}_t^h$ and rearranging gives:

$$\frac{d\mathbb{E}_t^h \boldsymbol{z}_t^h}{dz_{nt}^h} = (\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^h)^{-1} \boldsymbol{\delta}_{nt}^h \tag{6}$$

which substituted into equation 2 gives the result. ■

Equation 3 is useful because it distinctly highlights the separate roles played by the household's information, subjective model, and preferences. When the shock occurs, household h receives some direct information about how each of the variables in $\boldsymbol{z}_t^h$ have changed, and update expectations accordingly according to $\boldsymbol{\delta}_{nt}^h$. This update to the expectation of each variable further causes the household to engage in a second round of updating, where they use their newly updated expectations of each z_{it} to inform their expectations of all other variables linked to z_{it} through their subjective model. This secondary updating is reflected by $(\boldsymbol{I}-\boldsymbol{\mathcal{M}}_t^h)^{-1}$. Once all expectations have been updated, the preferences $\boldsymbol{\mu}_t^h$ determine the choice response.

Importantly, while $\boldsymbol{\mathcal{M}}_t^h$ only captures the direct effect of expectations of one variable on another, variables may also be linked indirectly. That is, an update to $\mathbb{E}_t^h z_{it}^h$ may affect $\mathbb{E}_t^h z_{jt}^h$ directly, but also indirectly through its effect on the expectation of some other variable $\mathbb{E}_t^h z_{kt}^h$, which is linked in the household's subjective model to both z_{it}^h and z_{jt}^h. The matrix $(\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^h)^{-1}$ captures all such direct and indirect links between variables. From here, it will be convenient to work directly with this, which I refer to as the cross-learning matrix:[15]

$$\boldsymbol{\chi}_t^h \equiv (\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^h)^{-1} \tag{7}$$

where the $(i,j)^{th}$ element of $\boldsymbol{\chi}_t^h$ will be denoted $\chi_{ij,t}^h$. It is these values that are measured in the empirical literature on cross-learning (e.g. Roth and Wohlfart, 2020).

Finally, having updated all of their expectations using their information, and filtering it through their subjective model, household choices are determined by their reaction to each of those expectations, which is contained in the preference matrix $\boldsymbol{\mu}_t^h$. The information, subjective model, and response components of the household's economic narrative are therefore represented by $\boldsymbol{\delta}_{nt}^h$, $\boldsymbol{\chi}_t^h$, and $\boldsymbol{\mu}_t^h$ respectively.

Notice that full information rational expectations is nested in this framework, as the special case in which all current variables are observed, and the subjective model coincides with the true model in equilibrium. This therefore differs from models in which narratives are represented by Directed Acyclic Graphs (DAGs) (Spiegler, 2020), as most general equilibrium models do not have a recursive causal ordering of variables, so the true equilibrium laws of motion cannot be expressed as a DAG.

2.2 An example

Consider the common textbook setup where infinitely lived households have CRRA utility over consumption, and can trade one-period risk-free bonds for intertemporal consumption smoothing. The consumption function log-linearised around the steady state is:

$$c_t^h = (1-\beta)\sum_{s=0}^{\infty}\beta^s \mathbb{E}_t^h y_{t+s}^h - \sigma\beta\sum_{s=0}^{\infty}\beta^s(\mathbb{E}_t^h i_{t+s} - \mathbb{E}_t^h \pi_{t+s+1}) \tag{8}$$

where y_t^h is real income in period t, i_t is the nominal interest rate, and π_t is gross inflation. The parameters β and σ are the discount factor and coefficient of relative risk aversion respectively. See Appendix A.1 for the derivation.

This is the familiar result that consumption depends on the present value of future income and all future real interest rates. Within the framework of equation 1, $\boldsymbol{z}_t^h$ contains all current and future realisations of y_t^h, i_t, and π_{t+1}. The coefficients $\boldsymbol{\mu}_t^h$ contain the relevant combinations of the preference parameters β and σ.

To see the interpretation of Proposition 1 in more detail, assume that there is no heterogeneity in real incomes among households, so $y_t^h = y_t$. The households believe

[15] This has a parallel in the literature on production networks (Carvalho and Tahbaz-Salehi, 2019). The direct links in $\boldsymbol{\mathcal{M}}_t^h$ are analogous to the elements of the input-output matrix, and $\boldsymbol{\chi}_t^h$ is the corresponding Leontief inverse. As with production networks, this Leontief inverse regulates the transmission of shocks.

inflation and income are linked according to the simple subjective model:

$$\begin{aligned} y_t &= \kappa^h \pi_t + u_{yt}, \quad u_{yt} \sim N(0, \sigma_y^2) \\ \pi_t &= u_{\pi t}, \quad u_{\pi t} \sim N(0, \sigma_\pi^2) \end{aligned} \tag{9}$$

That is, inflation may have causal effects on real incomes, but there is believed to be no feedback from real incomes to inflation. For this example, assume that the household does not believe i_t is related to either y_t or π_t, so we can leave that out of the analysis.

The household observes a noisy signal about each variable of interest in period t:

$$\begin{aligned} s^h_{yt} &= y_t + \varepsilon^h_{yt}, \quad \varepsilon^h_{yt} \sim N(0, \sigma^2_{\varepsilon y}) \\ s^h_{\pi t} &= \pi_t + \varepsilon^h_{\pi t}, \quad \varepsilon^h_{\pi t} \sim N(0, \sigma^2_{\varepsilon \pi}) \end{aligned} \tag{10}$$

If the household follows Bayes' rule to incorporate these signals into their expectations of y_t and π_t, their posterior expectations of each will be a linear combination of s^h_{yt} and $s^h_{\pi t}$, with the coefficients depending on the relative signal-to-noise ratios of each signal. Importantly, those ratios will depend on κ^h, as that determines how strongly the variables are believed to be linked, and therefore how informative s^h_{yt} is about π_t, and how informative $s^h_{\pi t}$ is about y_t. Rearranging these expectation functions gives:

$$\begin{aligned} \mathbb{E}^h_t y_t &= \frac{\sigma_y^2}{\sigma_y^2 + \sigma^2_{\varepsilon y}} s^h_y + \kappa^h \frac{\sigma^2_{\varepsilon y}}{\sigma_y^2 + \sigma^2_{\varepsilon y}} \mathbb{E}^h_t \pi_t \\ \mathbb{E}^h_t \pi_t &= \frac{\sigma_\pi^2}{\sigma_\pi^2 + \sigma^2_{\varepsilon\pi}\left(1 + \kappa^{h2}\frac{\sigma_y^2}{\sigma_\pi^2}\right)} s^h_\pi + \kappa^h \frac{\sigma^2_{\varepsilon\pi}\frac{\sigma_y^2}{\sigma_\pi^2}}{\sigma_\pi^2 + \sigma^2_{\varepsilon\pi}\left(1 + \kappa^{h2}\frac{\sigma_y^2}{\sigma_\pi^2}\right)} \mathbb{E}^h_t y_t \end{aligned} \tag{11}$$

After a shock to an arbitrary variable z_{nt}, these expectations change according to:

$$\begin{aligned} \frac{d\mathbb{E}^h_t y_t}{dz_{nt}} &= \frac{\sigma_y^2}{\sigma_y^2 + \sigma^2_{\varepsilon y}} \frac{dy_t}{dz_{nt}} + \kappa^h \frac{\sigma^2_{\varepsilon y}}{\sigma_y^2 + \sigma^2_{\varepsilon y}} \frac{d\mathbb{E}^h_t \pi_t}{dz_{nt}} \\ \frac{d\mathbb{E}^h_t \pi_t}{dz_{nt}} &= \frac{\sigma_\pi^2}{\sigma_\pi^2 + \sigma^2_{\varepsilon\pi}\left(1 + \kappa^{h2}\frac{\sigma_y^2}{\sigma_\pi^2}\right)} \frac{d\pi_t}{dz_{nt}} + \kappa^h \frac{\sigma^2_{\varepsilon\pi}\frac{\sigma_y^2}{\sigma_\pi^2}}{\sigma_\pi^2 + \sigma^2_{\varepsilon\pi}\left(1 + \kappa^{h2}\frac{\sigma_y^2}{\sigma_\pi^2}\right)} \frac{d\mathbb{E}^h_t y_t}{dz_{nt}} \end{aligned} \tag{12}$$

These are of the same form as equation 5. The first terms of each equation contain the elements of $\boldsymbol{\delta}^h_{nt}$, and the coefficients in the second terms contain the elements of $\boldsymbol{\mathcal{M}}^h_t$.

Consider first the change in $\mathbb{E}^h_t y_t$. The first term has two components: the signal-to-noise ratio of the income signal s^h_{yt}, and the underlying response of y_t to the shock. That is, if they precisely observe y_t, then $\mathbb{E}^h_t y_t$ responds to the shock in exactly the same way as the realised variable, regardless of changes in $\mathbb{E}^h_t \pi_t$. The noisier the household's direct information about y_t, the smaller that direct response. At the extreme with no direct information observed about y_t ($\sigma^2_{\varepsilon y} \to \infty$), the direct effect of the shock on expectations approaches 0 and the only way the household can update $\mathbb{E}^h_t y_t$ is through $\mathbb{E}^h_t \pi_t$.[16]

The coefficient in the second term also has two components. First, a change in expected inflation only affects expected income if the household believes that the two are linked in their subjective model ($\kappa^h \neq 0$). The slope of the perceived relationship between them, κ^h, therefore regulates the updating from $\mathbb{E}^h_t \pi_t$ to $\mathbb{E}^h_t y_t$. Second, this slope from the subjective model is scaled by one minus the signal-to-noise ratio. Intuitively, this scaling reflects how strongly the household weights the information in $\mathbb{E}^h_t \pi_t$ relative to the other information they have about y_t.

Now turn to the change in $\mathbb{E}^h_t \pi_t$. All of the effects described above are present, but there is a further nuance. The weights on $d\pi_t/dz_{nt}$ and $d\mathbb{E}^h_t y_t/dz_{nt}$ are no longer determined by the simple signal-to-noise ratio in the relevant direct signal. This is because the first term of the $\mathbb{E}^h_t \pi_t$ updating equation reflects the extent of updating due to $s^h_{\pi t}$, holding $\mathbb{E}^h_t y_t$ constant. Since in the household's subjective model π_t is a direct cause of y_t, this conditioning involves assuming the structural shock u_{yt} offsets the perceived rise in π_t, effectively reducing the informativeness of $s^h_{\pi t}$ when it is used in this way. This distortion is smaller if income shocks are believed to be more volatile relative to inflation shocks, as then y_t is less strongly correlated with π_t in the subjective model.

The core insights, however, remain the same: the direct response varies between 0 (if $\sigma^2_{\varepsilon\pi} \to \infty$) and the realised change in inflation (if $\sigma^2_{\varepsilon\pi} = 0$), and the coefficient on $d\mathbb{E}^h_t y_t/dz_{nt}$ is determined by the association between π_t and y_t in the subjective model (κ^h), and how the household weights that information relative to the direct information.

2.3 Aggregate behaviour

I now return to the general case. Consider a unit mass of the households modeled in Section 2.1. The vector of aggregate choices is given by:

$$\bar{\boldsymbol{x}}_t = (\bar{x}_{kt}), \quad \bar{x}_{kt} = \int_0^1 \omega^h_{kt} x^h_{kt} dh \tag{13}$$

where ω^h_{kt} denotes a weighting applied to household h's choice x^h_{kt}, such that:

$$\bar{\boldsymbol{x}}_t = \mathbb{E}_H \boldsymbol{x}^h_t \tag{14}$$

where $\mathbb{E}_H$ denotes the expected value across households.

[16] If the households are not Bayesian, as for example in models with diagnostic expectations (Bordalo et al., 2020), the information terms will not reflect the true signal-to-noise ratios, but rather the non-optimal responses to signals.

Again, consider a shock to z_{nt}. The only change to the shock considered in Proposition 1 is that now the shock is assumed to be to an aggregate variable, so the h superscript is dropped. Proposition 1 and the properties of covariances lead us to the decomposition of the aggregate choice response:

Proposition 2 *The response of aggregate choice $\bar{x}_{kt}$ to a shock to z_{nt} is given by:*

$$\frac{d\bar{x}_{kt}}{dz_{nt}} = \sum_{i=1}^{N_z}\sum_{j=1}^{N_z}\left[\bar{\mu}_{ki,t}\bar{\chi}_{ij,t}\bar{\delta}_{jn,t} + Cov_H(\mu^h_{ki,t}, \chi^h_{ij,t}\delta^h_{jn,t}) + \bar{\mu}_{ki,t}Cov_H(\chi^h_{ij,t}, \delta^h_{jn,t})\right] \tag{15}$$

where $\delta^h_{jn,t}$ and $\mu^h_{ki,t}$ denote the j^{th} element of $\boldsymbol{\delta}^h_{nt}$ and the $(k,i)^{th}$ element of $\boldsymbol{\mu}^h_t$ respectively, $\bar{\delta}_{jn,t}$ and $\bar{\mu}_{ki,t}$ are their aggregate counterparts, and $\bar{\chi}_{ij,t}$ is the aggregate value of $\chi^h_{ij,t}$.

Proof. Appendix A.2 ■

This decomposition shows the three channels determining the aggregate response to shocks. The first term is the *representative agent channel*: changes in the average preferences, subjective model, and information about each variable will affect the aggregate response to shocks. This is the term that most empirical work on narratives in macroeconomics have focused on (e.g. Shiller, 2017; Larsen et al., 2021). The second term is the *preference heterogeneity channel*, and the third is the *narrative heterogeneity channel.* These show that along with representative agent effects, aggregate responses to shocks may be strongly influenced by the distribution of preferences, subjective models, and information across households.

To see the intuition for these channels, consider again the textbook consumption function in equation 8, and a positive shock to future inflation π_{t+1}. If all households believe that higher inflation is associated with lower real incomes, then the average $\chi^h_{y\pi,t}$ is negative, and the aggregate consumption response to the future inflation will be negative. This is the effect that would be seen in a representative agent model.

If, however, this pessimistic subjective model of the effects of inflation only takes hold among hand-to-mouth households, then aggregate consumption will respond much more positively to the shock than the average would suggest, because the households who update their expected future real incomes down are also those who do not react to their expectations. This is the preference heterogeneity channel. These effects are the focus of a large literature in heterogenous-agent macroeconomics: for example, the earnings heterogeneity channel of monetary transmission in Auclert (2019) reflects the correlation between the marginal propensity to consume out of income (an element of $\boldsymbol{\mu}^h_t$), and the exposure of household income to monetary policy. Since $\boldsymbol{\chi}^h_t\boldsymbol{\delta}^h_{nt}$ is equal to $d\mathbb{E}^h_t\boldsymbol{z}^h_t/dz^h_{nt}$ (equation 6), for households with full information and rational expectations this vector simply reflects such heterogeneous shock exposure.

Finally, if all households are unconstrained, but the pessimistic model of inflation is concentrated among households who do not obtain any information about future inflation, then this again raises $d\bar{c}_t/d\pi_{t+1}$. Those households who would update $\mathbb{E}_t^h y_{t+1}^h$ down and reduce consumption if they learned that inflation was about to rise are also the households who do not observe the shock, and so do not update $\mathbb{E}_t^h \pi_{t+1}^h$. This is the effect of the narrative heterogeneity channel.

It is important to be clear that this is a decomposition, not a solution for aggregate actions. $\boldsymbol{\delta}_{nt}^h$ captures direct information received by the household, but the information received depends on the true reaction of $\boldsymbol{z}_t^h$ to the shock, which I have taken as given here. In a general equilibrium setting that underlying reaction will contain equilibrium outcomes, involving general equilibrium effects, which may in turn depend on aggregate actions. This is nonetheless a useful exercise, as for a given movement in aggregate variables it highlights the channels through which shocks transmit to aggregate household behaviour. In this way it is similar to the decomposition in Auclert (2019), which takes movements in several aggregate variables as given to find the transmission to aggregate consumption.

In the following section I go on to provide survey evidence on these effects when considering a shock to inflation, which suggests that there is important heterogeneity in information and subjective models beyond that implied by heterogeneity in shock exposure, as studied in Auclert (2019) and Bilbiie (2019). I then build a model with rational inattention and endogenous subjective models that matches this data. The empirical evidence and the model will suggest particular signs and time-series behaviours for the three terms in the context of inflation shocks. The results in this section, however, are more general. If we take seriously the notion that agents have heterogeneous information and heterogeneous subjective models of the economy, understanding aggregate behaviour requires understanding these three channels.

3 Survey evidence on information and subjective models of inflation

In this section I document three stylised facts about household information and subjective models. Specifically, the facts refer to the information households obtain about inflation, and their subjective models of how inflation is related to aggregate economic performance. These facts will inform the model in Section 4.

First, I find that households with subjective models in which inflation has no real effects obtain less direct information about inflation. Second, more households hold subjective models in which inflation damages the real economy when realised inflation has recently been high. Third, households with subjective models in which inflation damages the economy have persistently higher inflation perceptions and expectations than those with other models.

3.1 Data

To study the joint behaviour of information and subjective models, we need data that is informative about each separately. This is a challenge, as most empirical papers on information frictions or subjective models use data on realised expectations, which combine both information and subjective models (as shown in Section 2), and so cannot be used to identify the narrative heterogeneity channel. To disentangle information and subjective models I use data from the Bank of England Inflation Attitudes Survey (IAS), which contains several unique questions which enable me to measure these components of expectation formation separately.

The IAS has been fielded quarterly since 2001 to a repeated cross-section of UK households. In the first quarter of each year approximately 4000 households are surveyed, while in other quarters approximately 2000 are surveyed. I use the individual-level response data from 2001-2019, omitting the quarters conducted after the onset of the Covid-19 pandemic, as the implementation of the survey had to be changed substantially at this time (see Bank of England, 2020).

Alongside questions on expected inflation, interest rates, and other macroeconomic and personal variables, respondents are asked several questions which do not commonly appear in other household surveys. These questions are helpful in disentangling information and subjective models about inflation.

The first of these is Question 3, which asks households about their subjective model of the relationship between inflation and the 'strength of the economy'.

Question 3 *If prices started to rise faster than they are now, do you think Britain's economy would end up stronger, or weaker, or would it make little difference?*

This differs from standard questions on expected future economic outcomes because it does not invoke the use of information about the state of the world. Similarly to the hypothetical vignettes used in Andre et al. (2022), the answers to this question are informative about cross-learning, which is denoted $\chi^h_{ij,t}$ in Section 2 and summarises the household's subjective model.[17] In the analysis below, I will refer to a respondent

[17] In Section 2 I noted that $\chi^h_{ij,t}$ comprised subjective models and any weighting the household put on expectations of z^h_{jt}. Since these weights do not change the sign of $\chi^h_{ij,t}$, the qualitative responses to Question 3 still reflect the sign of the cross-learning from expected inflation to expectations of the state of the real economy, as long as no household perfectly observes the 'overall state of the economy'.

answering that inflation would make the economy stronger/no difference/weaker as having a positive/neutral/negative subjective model of inflation respectively.

There are two possible interpretations of this question. Households may view it as asking about the causal effects of inflation on the economy (as in the model of Spiegler, 2021). Alternatively, they could see it as asking about the most likely source of a rise in inflation, if they believe supply- and demand-driven inflation is associated with different real outcomes (Kamdar, 2019). For the purposes of this section, this distinction does not matter, as $\chi^h_{ij,t}$ in the decomposition of aggregate actions (Proposition 2) is simply the degree to which households update their expectations of one variable when their expectation of another changes. In this case, it is the updating of expectations about the strength of the economy when expected inflation rises. This is captured by the question, whether the updating occurs because of a perceived direct causal link from inflation to the real economy, or a belief about the type of shocks hitting the economy.

The second set of novel questions concern the information households use to arrive at their inflation expectations, without asking what those expectations are. This allows us to learn about household information ($\delta^h_{jn,t}$) without contamination from cross-learning ($\chi^h_{ij,t}$).

Question 2f *What were the most important factors in getting to your expectation for how prices in the shops would change over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

We can divide the possible answers into four categories. First, options 1 and 2 concern past experienced price rises. Options 3 and 4 are direct information about inflation. Options 5-8 concern other macroeconomic variables, either current or expected, and options 9 and 10 are extras. A rational household may well use the information sources in options 1,2 and 5-8 to forecast inflation, but in the decomposition in Proposition 2 this would represent cross-learning from information about other variables. The only answers that represent the use of direct information about inflation are options 3 and 4.[18]

Question 2f was only asked in 2016Q1, but very similar questions were asked at other times. In each, the respondent is asked about the information sources they used to arrive at their expectation of inflation (either over the next 12 months or a longer horizon), or the information sources that led them to change their expectation over the last 12 months, again for expectations of 12-month ahead or longer-term inflation. For each of these questions I construct a dummy variable equal to 1 if the respondent reports using direct information about inflation, and equal to 0 if they do not. The full details of each question, and the options representing direct information on inflation, are in Appendix B. Combining these dummy variables gives one overall indicator for if the respondent used direct information on inflation in forming their expectations, that is whether $\delta^h_{\pi n,t} > 0$. This indicator is observed for 8 separate quarters between 2009Q1-2019Q1.

In Appendix C.1 I confirm that these measures of information and subjective models correlate with planned household consumption, and that the signs of these correlations are consistent with the measures picking up the desired elements of household beliefs. Details of the auxiliary questions used for that analysis can be found in Appendix C.1.

Another possible test of the information indicator would ask if households who obtain direct information about inflation make more accurate forecasts. However, if information about the level of inflation influences subjective models, that may in turn affect information choices, so it is not clear ex ante what correlations to expect between information and forecast accuracy. I therefore leave discussion of this test for Section 4, after the model has been developed. The results are consistent with the model, adding further evidence that the information indicator reliably measures the object of interest.

The other questions used in this section are much more standard, asking households to give point estimates for "how prices have changed over the last twelve months" and "how much would you expect prices in the shops generally to change over the next twelve months". For each question, respondents choose from a list of ranges, and follow-up questions may then asked with more precise ranges, until the respondent has selected an inflation rate bin from the set:

$$\mathbb{E}^h_t \pi_s \in \{\leq -5\%, (-5\%, -4\%], (-4\%, -3\%], (-3\%, -2\%], (-2\%, -1\%], (-1\%, 0\%), 0\%, \\ (0\%, 1\%), [1\%, 2\%), [2\%, 3\%), [3\%, 4\%), [4\%, 5\%), [5\%, 6\%), [6\%, 7\%), [7\%, 8\%), [8\%, 9\%), \\ [9\%, 10\%), \geq 10\%\}$$

[18]Past experienced price rises are indirect information because to use them for forecasting, the household needs a model of the persistence of inflation. Macaulay and Moberly (2022) find that this perceived persistence is very heterogeneous across households.

For the exercises in Section 3.4, I code perceptions and expectations at the midpoint of the selected bin, with the lowest and highest bins coded as -5.5% and 10.5% respectively. I refer to these answers as perceived and expected inflation respectively.

3.2 Information and subjective models in the cross-section

The first stylised fact concerns the cross-sectional distribution of information and subjective models, the key relationship in the narrative heterogeneity channel identified in Section 2. Table 1 shows the results of a probit regression of the information indicator defined in Section 3.1 on the respondent's subjective model of inflation, represented by their answer to Question 3. The first column shows this with quarter fixed effects only, while the second also includes a range of household controls.[19]

Table 1: Information correlates with subjective models

	(1)	(2)
end up stronger	-0.0102	-0.00827
	(0.0191)	(0.0192)
make little difference	-0.0356***	-0.0315**
	(0.0128)	(0.0129)
dont know	-0.0627***	-0.0605***
	(0.0172)	(0.0172)
Controls	None	All
Time FE	Yes	Yes
Observations	8270	8270

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the average marginal effects from estimating a probit regression of the information indicator on the responses to Question 3. The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

Those answering that inflation makes no difference to the aggregate economy, or who don't know the effect of inflation, are significantly less likely to use information about inflation than someone who believes inflation makes the economy weaker. There is no significant difference in the probability of using direct inflation information between those holding this view and those with positive subjective models of inflation. The coefficients displayed are average marginal effects, so the probability of using direct inflation information is 3-3.5 percentage points lower for those with a neutral model of the effects of

[19]These are gender, age, class, employment status, income, education, region, and home-ownership status. Age, class, income and education are all reported in bands, and included as categorical variables.

inflation than those who believe inflation weakens the economy. Over the whole population 23% of respondents use direct inflation information, so this difference is non-trivial.

Fact 1 *Households who believe inflation makes no difference to the economy acquire less information about inflation than households who believe inflation does affect the economy (in either direction).*

The information indicator is composed of answers to several slightly different questions. In particular, some questions concern information used to arrive at the respondent's point forecast for inflation, while others concern the information they used in changing their inflation expectations over the last year. Some questions concern expected inflation over the next 12 months, while others ask about a longer horizon. In Appendix C.3 I repeat the regressions of Table 1 on subsets of the questions, and find that the results are qualitatively robust to these alternatives. The results also remain significant for all such splits that maintain a substantial sample size.

This is not consistent with models with exogenous information, as there would be no reason for information to be systematically correlated with household subjective models. It is, however, consistent with models of endogenous information acquisition, as the value of inflation information is lower for households who believe inflation makes little difference to other variables that matter for their decisions. The (broadly defined) strength of the aggregate economy is such a variable as long as households believe there is some relationship between the aggregate economy and their personal decisions, which is supported by evidence in Roth and Wohlfart (2020), among others. The implications of this link from subjective models to information acquisition are discussed further in Section 4.

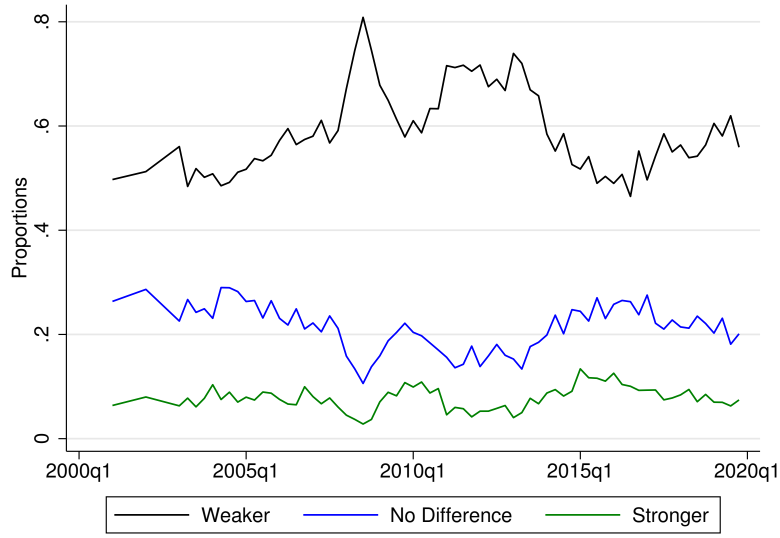
3.3 Time-series properties of subjective beliefs about inflation

I next turn to the time-series behaviour of subjective models of the effects of inflation. Figure 1 shows the proportions answering Question 3 with each subjective model of inflation over time ('don't know' omitted for figure clarity).

The majority of households answer that inflation would make the economy weaker in all quarters, in keeping with the findings in Shiller (1997), and more recently the experimental evidence of Andre et al. (2022). Combined with empirical Fact 1, this suggests that the covariance of information on inflation and cross-learning from inflation to the strength of the economy is negative. Assuming households consume more when they believe the economy is strong, the narrative heterogeneity channel will act to reduce the consumption response to inflationary shocks.

The relatively long time series of the IAS also allows us to see that the distribution of answers varies substantially over time, and that much of this variation can be explained

Figure 1: Proportions giving each answer to Question 3: "If prices started to rise faster than they are now, do you think Britain's economy would end up stronger, or weaker, or would it make little difference?"



by recent inflation experiences. The proportion of respondents saying that inflation would make the economy weaker rose sharply in 2008 and 2011, as inflation was rising in the UK, and that proportion subsequently fell alongside inflation after 2013. The correlation between annual CPI inflation and the proportion of respondents with negative models of inflation is extremely high, at 0.799. Tests in Appendix C.2 show that this correlation is robust to the addition of various macroeconomic controls, which themselves explain far less of the variation in the distribution of responses than realised inflation.

Fact 2 *A greater proportion of households believe inflation weakens the economy when realised inflation is high.*

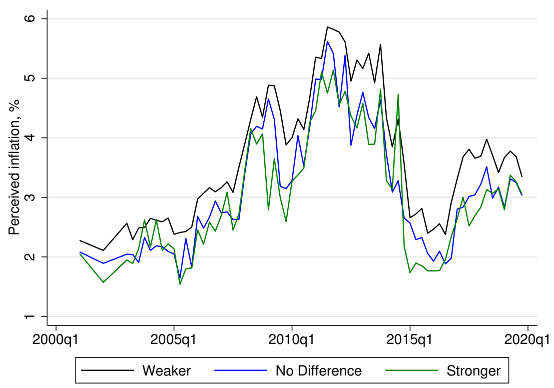
This is not what we would expect from a rational expectations model. The question is about the effect of an aggregate variable (inflation) on the aggregate performance of the economy. Even if households are differentially exposed to the shock, if they all had model-consistent beliefs they would all give the same answer to this question. The fact that there is heterogeneity at all is evidence that at least some household subjective models are inconsistent with rational expectations.

These patterns also suggest that the majority of households are not using New Keynesian-style models. In a textbook New Keynesian model, a rise in inflation causes the central bank to raise the nominal interest rate. If the Taylor Principle is satisfied, the real interest rate rises, so output falls. If it is not, the real rate falls, and output rises. If most households used this model, they should respond that inflation would make the economy weaker in the periods before interest rates reached the Zero Lower Bound, and they should switch to the view that inflation would make the economy stronger once we reach the ZLB in 2009. There is little evidence for this in Figure 1, and indeed statistical tests in Appendix C.2 find no evidence of such a shift.[20]

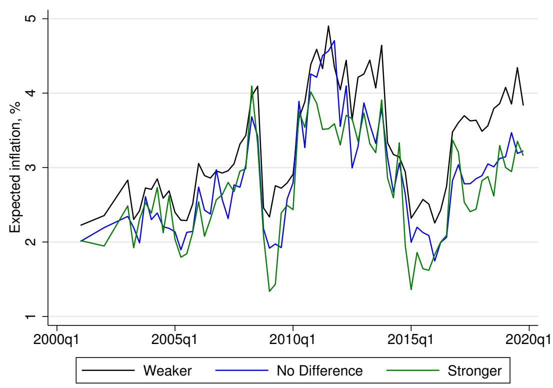
3.4 Inflation expectations and perceptions vary with subjective models

Finally, I compare perceived and expected inflation across households with different subjective model beliefs. Figures 2a and 2b show the time series of perceived and expected inflation by qualitative subjective model of inflation.

Figure 2: Inflation perceptions and expectations by subjective model.



(a) Perception over the past 12 months $\mathbb{E}_t\pi_{t,t-12}$



(b) Expectation for next 12 months $\mathbb{E}_t\pi_{t+12,t}$

There is a persistent wedge between the perceptions and expectations of the different groups. Respondents who believe inflation weakens the economy systematically perceive that inflation has been higher, and expect it to be higher over the next year, than those who believe inflation makes no difference to the economy. They, in turn, perceive and expect higher inflation than those with a positive subjective model of inflation.[21]

The differences are large: Table 2 shows that even after controlling for the full set of available household characteristics, those with a negative model of inflation perceive that inflation has been 54 basis points higher than those with a neutral view, and 70 basis points higher than those with a positive view. The gaps are similarly large and strongly significant for expected inflation.

[20]In principle, after 2009 a New Keynesian model would predict that a sufficiently large rise in inflation would lift the economy away from the ZLB, implying higher real interest rates and lower output. However, in 2013 the Bank of England issued forward guidance that interest rates would not rise until unemployment fell below 7% (Bank of England, 2013), so it is unlikely that households were expecting them to contract in response to small rises in inflation at this time.

[21]Dräger et al. (2020) similarly find for German households that inflation expectations are higher among those reporting that they would prefer inflation to be lower.

Fact 3 *Households who believe inflation weakens the economy perceive higher inflation, and expect higher future inflation, than those with less negative subjective models.*

This is not driven by the households using different kinds of information to arrive at their perceptions and expectations: Table 1 shows that the households with positive subjective models use similar information sources to those with negative models. It is, however, consistent with information about high inflation causing households to update their subjective models towards more negative views, as in the model in Section 4. This simultaneously accounts for Facts 2 and 3, as within a period those who receive signals that inflation is high shift to more negative subjective models, and when realised inflation rises more households receive such signals.

Table 2: Perceived and expected inflation are higher for those with more negative subjective models.

	(1) Perceived inflation	(2) Expected inflation
end up stronger	-0.696***	-0.565***
	(0.0371)	(0.0353)
make little difference	-0.543***	-0.466***
	(0.0226)	(0.0207)
dont know	-0.462***	-0.413***
	(0.0315)	(0.0294)
Controls	Yes	Yes
Time FE	Yes	Yes
R-squared	0.179	0.113
Observations	85803	85201

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of regressing perceived and expected inflation on the responses to Question 3. The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

4 Narrative heterogeneity in a consumption-savings model

In this section I present a heterogeneous agent model in partial equilibrium that can rationalise the empirical findings documented in Section 3. The key elements needed to match the data are that households face costs of processing information about inflation, and that the perceived effect of inflation on real incomes changes with perceptions of recent inflation. These features imply a two-way relationship between information and subjective models, which explains the empirical facts from Section 3.

4.1 Model setup

Time is discrete, and the period is denoted by t. In each period, household h chooses consumption C_t^h to maximise expected discounted utility:

$$\tilde{\mathbb{E}}_0^h U_0^h = \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \frac{(C_t^h)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{16}$$

subject to:

$$C_t^h + B_t^h = \frac{R_{t-1}}{\Pi_t} B_{t-1}^h + W_t \tag{17}$$

where W_t is the real wage faced by all households in period t, R_t is the gross nominal interest rate on one-period bonds B_t^h bought in period t, and Π_t is gross inflation between periods $t-1$ and t. All wages and prices are observed before the consumption choice in period t, but future wages and prices are unknown. The operator $\tilde{\mathbb{E}}_t^h$ reflects the expectations of household h in period t, which may not coincide with the rational expectations operator $\mathbb{E}_t$. However, I will assume that given the household's subjective model for the evolution of r, π, w, the household uses their information optimally. Any non-rationality in expectations therefore comes only from misperceptions in these laws of motion.

While households observe the current price level when choosing consumption, I assume that they may not perfectly observe the current rate of inflation. This assumption is common in models where agents use a Kalman filter to update their inflation expectations (e.g. Coibion and Gorodnichenko, 2015), and is consistent with the evidence in Macaulay and Moberly (2022), who find substantial uncertainty about inflation over the past year.[22]

The first order condition is a standard consumption Euler equation:

$$(C_t^h)^{-\frac{1}{\sigma}} = \beta \tilde{\mathbb{E}}_t^h \frac{R_t}{\Pi_{t+1}} (C_{t+1}^h)^{-\frac{1}{\sigma}} \tag{18}$$

To proceed, I take a log-quadratic approximation to utility, as is common in the rational inattention literature (e.g. Maćkowiak and Wiederholt, 2009). Specifically, I substitute the budget constraint into expected utility $\tilde{\mathbb{E}}_t^h U_t^h$, then take a log-quadratic approximation about a steady state with $\Pi = 1, R = \beta^{-1}$. After this approximation, the expected discounted utility loss relative to a household fully informed about current

[22] One way to microfound this is to assume that households consist of a forecaster, who forms expectations without observing current inflation, and a shopper who uses those forecasts (along with observed current prices) to make consumption decisions. A similar assumption is made in Pfäuti (2022).

inflation each period is given by Proposition 3:

Proposition 3 *Let $\tilde{\mathbb{E}}_0^{h*} U_0^{h*}$ denote the expected utility of an otherwise identical household who observes Π_t precisely before choosing C_t^h. Furthermore, let $\hat{U}_0^{h*}$ and $\hat{U}_0^h$ denote the log-quadratic approximation to the discounted utility of the fully-informed and uninformed households respectively. The expected utility loss from imperfect information about π_t each period is:*

$$\tilde{\mathbb{E}}_0^h(\hat{U}_0^{h*} - \hat{U}_0^h) = -\frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma} \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t (c_t^h - c_t^{h*})^2 \tag{19}$$

where lower-case letters are log-deviations of the corresponding variables from steady state, and c_t^{h*} denotes the period-t consumption of the fully-informed household.

Proof. Appendix D.1 ■

Notice that the fully-informed household here uses the same potentially non-rational expectations operator as the uninformed household. That is, they have the same subjective model, but different information. This will be helpful in solving for optimal information choices.

To focus on the feedback between subjective models and information choices, I take steady state assets $\bar{B}^h \to 0$. This implies that wealth plays no role in information choices, so abstracts away from wealth as an alternative source of information heterogeneity (as in e.g. Broer et al., 2020).[23] With this assumption, the problem of a fully-informed household is identical to that in Appendix A.1, and so the consumption function is:

$$c_t^{h*} = (1-\beta) \sum_{s=0}^{\infty} \beta^s \tilde{\mathbb{E}}_t^{h*} w_{t+s}^h - \sigma\beta \sum_{s=0}^{\infty} \beta^s (\tilde{\mathbb{E}}_t^{h*} r_{t+s} - \tilde{\mathbb{E}}_t^{h*} \pi_{t+s+1}) \tag{20}$$

Since utility losses from deviating from this are quadratic, a household with imperfect information sets $c_t^h = \tilde{\mathbb{E}}_t^h c_t^{h*}$.

The expectations of future real wages and inflation are therefore critical in determining consumption choices. The remainder of this section studies how these expectations are formed when information processing is costly, and realisations of that information can affect the household's subjective model.

The timing of expectation formation is as follows. The households start period 1 with some prior subjective model, which they use in making their information decisions (see Section 4.3). Once the household observes the realisation of their chosen signals, they will use that information to update the parameters in their subjective model (Section

[23] Michelacci and Paciello (2020) show that with ambiguity aversion, wealth heterogeneity implies heterogeneity in subjective models. Combining this with the effects on information choice, wealth could therefore form an additional reason for a systematic relationship between information and subjective models. This is beyond the scope of this paper.

4.4). The realised signals and updated subjective model will then be used to form the expectations the household will use to choose consumption.

4.2 Subjective models

Households form expectations of future variables by taking information on current real wages, interest rates, and inflation, and forecasting forward using their subjective models. I assume that subjective models of all households take the simple form:

$$\pi_t = \rho_\pi^h \pi_{t-1} + u_{\pi t} \tag{21}$$

$$r_t = \phi^h \pi_t + u_{it} \tag{22}$$

$$w_t = \alpha^h \pi_t + \lambda^h r_t + \rho_w^h w_{t-1} + u_{wt} \tag{23}$$

where $u_{xt} \sim N(0, \sigma_x^2)$. Note that the parameters of these subjective models may differ across households, so even if equations 21 - 23 nest the rational expectations solution to some general equilibrium model, it will not be the case that all households have rational expectations.

Unlike in Section 3, this specification of the subjective model does restrict the interpretation of Question 3 in the IAS. The only shock driving inflation is $u_{\pi t}$, so there is no way for demand-like shocks to affect inflation through real incomes. This model therefore rules out that heterogeneous cross-learning from inflation to the real economy is driven by heterogeneous beliefs about the type of shocks driving inflation. Rather, such heterogeneity can only come here from heterogeneous beliefs about the causal effects of inflation. This assumption aids tractability, but also reflects the fact that the distribution of survey answers is very consistent over time, in levels and in how it correlates with realised inflation. If the answers reflected beliefs about the type of shocks driving inflation, we would expect this distribution to change across time periods characterised by different types of shocks. Since the distribution of subjective models evolved in the same way with the largely demand-driven run-up in inflation before the Great Recession as the supply-driven spike after the Brexit referendum, it does not appear that the source of inflation shocks plays a key role in the majority of survey answers.

With this subjective model, the expectations of a fully-informed household are (derivation in Appendix D.2):

$$\tilde{\mathbb{E}}_t^{h*} \pi_{t+s} = (\rho_\pi^h)^s \pi_t \tag{24}$$

$$\tilde{\mathbb{E}}_t^{h*} i_{t+s} = \phi^h (\rho_\pi^h)^s \pi_t \tag{25}$$

$$\tilde{\mathbb{E}}_t^{h*} w_{t+s} = \frac{(\alpha^h + \lambda^h \phi^h)\rho_\pi^h}{\rho_\pi^h - \rho_w^h} \big((\rho_\pi^h)^s - (\rho_w^h)^s\big)\pi_t + (\rho_w^h)^s w_t \tag{26}$$

Substituting these into the consumption function (20), the consumption function of the fully informed household is:

$$c_t^* = \frac{1-\beta}{1-\beta\rho_w^h} w_t - \sigma\beta r_t + \frac{\beta\rho_\pi^h[(1-\beta)(\alpha^h + \lambda^h\phi^h) - \sigma(\phi^h\beta - 1)(1-\beta\rho_w^h)]}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\pi_t \tag{27}$$

The corresponding consumption function for an uninformed household is therefore:

$$c_t = \frac{1-\beta}{1-\beta\rho_w^h} w_t - \sigma\beta r_t + \frac{\beta\rho_\pi^h[(1-\beta)(\alpha^h + \lambda^h\phi^h) - \sigma(\phi^h\beta - 1)(1-\beta\rho_w^h)]}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\tilde{\mathbb{E}}_t^h\pi_t \tag{28}$$

where current inflation appears in expectation because the household may be imperfectly informed about current inflation. They still observe w_t and r_t precisely. I refer to $\tilde{\mathbb{E}}_t^h\pi_t$ as perceived inflation below, in keeping with the evidence in Section 3.

For given information, households with different subjective models expect different future real wages and inflation, as in the general model in Section 2.

4.3 Optimal information processing

The household chooses the structure and precision of the signals they will receive to maximise expected utility. Substituting the consumption functions of informed and uninformed households (equations 27 and 28) into the expected utility loss from imperfect information in equation 19 gives:

$$\tilde{\mathbb{E}}_0^h(\hat{U}_0^{h*} - \hat{U}_0^h) = \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^h}{\partial\tilde{\mathbb{E}}_t^h\pi_t}\right)^2 \tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t(\pi_t - \tilde{\mathbb{E}}_t^h\pi_t)^2 \tag{29}$$

where:

$$\frac{\partial c_t^h}{\partial\tilde{\mathbb{E}}_t^h\pi_t} = \frac{\beta\rho_\pi^h[(1-\beta)(\alpha^h + \lambda^h\phi^h) - \sigma(\beta\phi^h - 1)(1-\beta\rho_w^h)]}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)} \tag{30}$$

is the elasticity of the household's consumption to perceived inflation.

That is, utility losses are proportional to the mean squared error in inflation perceptions, which will depend on the precision of the household's signals. Importantly, the parameters of the household's subjective model determine the expected utility loss from errors in perceived inflation, because they determine how those errors translate into errors in consumption, through the squared elasticity of consumption to perceived inflation. This is how subjective models affect the household's information choices.

Acquiring more precise information reduces the expected squared error in the household's inflation perception, but following the rational inattention literature I assume that increasing information precision is costly to the household. Specifically, the cost of a signal s_t^h is given by:

$$\mathcal{C}(\{s_t^h\}^t) = \psi \sum_{t=0}^{\infty} \beta^t I(\pi^t; s_t^h | \mathcal{I}_{t-1}^h) \tag{31}$$

where $\psi > 0$ is a positive constant and $I(\pi^t; s_t^h|\mathcal{I}_{t-1}^h)$ is the Shannon mutual information between priors and posteriors in period t. That is, the cost is proportional to the extra information provided by the signal s_t^h about the history of inflation to that point which was not contained in the previous period's information set. This cost function is common in the rational inattention literature (Maćkowiak et al., 2020).

To solve for optimal information processing, I make the simplifying assumption that the household chooses information as if they are certain about the parameters of their subjective model. Similarly, they ignore that they will update those parameters after receiving information. This is akin to the anticipated utility assumption in many models with least-squares learning, where agents do not consider that their perceived law of motion will change as they observe new periods of data in the future (see Bullard and Suda (2016) for a discussion of this in the learning literature).

I also assume that the household does not infer anything about π_t from the w_t and r_t that they observe each period. In principle, these are also noisy signals about π_t, but for simplicity I will not account for them in the information decision.[24]

The household information choice problem then has the same form as the firm's rational inattention problem in Maćkowiak and Wiederholt (2009). As in that paper, I proceed by making three further assumptions.

Assumption 1: (π_t, s_t) has a stationary Gaussian distribution.

Assumption 2: When the household decides on their information strategy in period 0, they receive a long sequence of signals of their chosen form. This implies that $\tilde{\mathbb{E}}_t^h(\pi_t^2|\mathcal{I}_t^h)$ is constant over time.

Assumption 3: In period t, households can only process information about variables realised up to period t. They cannot process any information about realisations of inflation in future periods.[25]

With these assumptions, Maćkowiak and Wiederholt (2009) show that the optimal signal is of the form:

$$s_t^h = \pi_t + \varepsilon_t^h, \quad \varepsilon_t^h \sim N(0, \sigma_{\varepsilon h}^2) \tag{32}$$

The household therefore uses the standard linear-quadratic-Gaussian Kalman filter to

[24] Strictly, the rational inattention setup assumes that the agent chooses among all possible signals. So w_t and r_t are available signals, but the household chooses not to pay to process them when forming their inflation perception.

[25] This ensures that as the cost of information approaches zero the household information set in period t contains realised values of all period t variables, but not realisations of variables in future periods, as in standard full-information DSGE models. See Jurado (2021) for a detailed discussion of this assumption.

predict inflation:

$$\tilde{\mathbb{E}}_t^h \pi_t = K^h(\pi_t + \varepsilon_t^h) + (1 - K^h)\rho_\pi^h \tilde{\mathbb{E}}_{t-1}^h \pi_{t-1} \tag{33}$$

The household's information choice problem therefore reduces to choosing the variance of noise $\sigma_{\varepsilon h}^2$ in the signal s_t^h, which in turn implies a Kalman gain K^h. Assumption 2 ensures that the household always uses the steady state K^h to form inflation perceptions.

The optimal information choice is given in Proposition 4:

Proposition 4 *The utility-maximising signal structure is as in equation 32, with $\sigma_{\varepsilon h}^2$ such that K^h satisfies:*

$$\begin{cases} K^h = 0 & \text{if } \Gamma^h < \psi(1 - (\rho_\pi^h)^2)^2 \\ \dfrac{1 - K^h}{(1 - (\rho_\pi^h)^2(1 - K^h))^2} = \dfrac{\psi}{\Gamma^h} & \text{if } \Gamma^h \geq \psi(1 - (\rho_\pi^h)^2)^2 \end{cases} \tag{34}$$

where:

$$\Gamma^h = \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right)^2 \tag{35}$$

Proof. Appendix D.3. ■

This implies that if the elasticity of consumption to perceived inflation is close to 0, the household pays no attention to inflation. Once that elasticity is sufficiently positive or negative, perceived inflation affects decisions enough to warrant paying for some information, and $K^h > 0$. As the consumption elasticity to perceived inflation grows further, attention rises and K^h approaches 1, at which point perceived inflation is equal to the true realised π_t. These properties can be seen graphically in Figure 3.

To further explore the properties of optimal information choices, note that household h processes no information if:

$$\left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right)^2 \leq \frac{2\psi\sigma(1 - (\rho_\pi^h)^2)^2}{(\bar{C}^h)^{1-\frac{1}{\sigma}}\sigma_\pi^2 \log(2)} \tag{36}$$

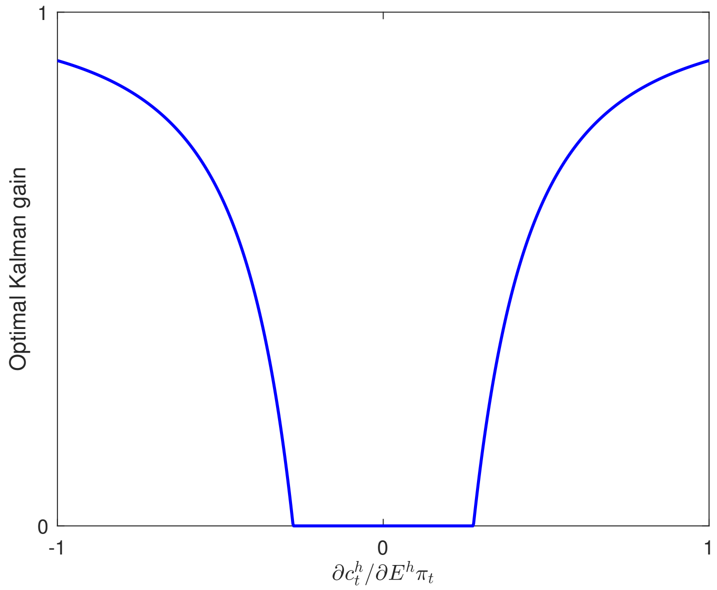
As long as $\psi > 0$, a household with $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t = 0$ will never pay attention to inflation. This elasticity of consumption to perceived inflation is in turn determined by the household's subjective model. From equation 30, we have that:

$$\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} = 0 \quad \text{if} \quad (1 - \beta)(\alpha^h + \lambda^h \phi^h) = \sigma(\beta\phi^h - 1)(1 - \beta\rho_w^h) \tag{37}$$

That is, consumption is unresponsive to perceived inflation if the perceived income and substitution effects of a change in inflation exactly cancel out.

The no-attention region is wider if the perceived volatility of the inflation process

Figure 3: Optimal K^h against the elasticity of consumption to perceived inflation, for parameters listed in Appendix E.



$\sigma_\pi^2/(1-(\rho_\pi^h)^2)$ is lower. This is consistent with evidence in Cavallo et al. (2017) and Pfäuti (2022) that households pay less attention to inflation when it is less volatile. A higher information cost also implies a wider no-attention region. Similarly, outside of the no-attention region, attention is increasing in $\sigma_\pi^2/(1-(\rho_\pi^h)^2)$ and decreasing in ψ.

Information choices are therefore determined by the household's subjective model, and this naturally implies the model matches empirical Fact 1. A simple proxy for 'the strength of the economy' might be aggregate consumption. If households believe others hold beliefs similar to their own, then the households who report in the survey that inflation makes no difference to the economy are those with subjective models such that $\partial c_t^h/\partial \tilde{\mathbb{E}}_t^h \pi_t$ is close to zero. They therefore process less information about inflation than those with stronger positive or negative $\partial c_t^h/\partial \tilde{\mathbb{E}}_t^h \pi_t$.

4.4 Subjective model updating

After processing their information, and forming a perception of current inflation, the households update their subjective model. Specifically, I assume that the only update is to the parameter α^h, the effect of inflation on real wages. Denoting the parameter value used in making information choices at the start of the period as α_0^h, the updated parameter $\hat{\alpha}_t^h$ is given by:

$$\hat{\alpha}_t^h = \alpha_0^h + \alpha_1^h \tilde{\mathbb{E}}_t^h \pi_t \tag{38}$$

That is, each household takes the parameter from their subjective model at the start of the period, and distorts it up or down depending on the realisation of perceived inflation. Specifically, to match empirical Facts 2 and 3, I assume that $\alpha_1^h < 0$, so when households perceive higher inflation they update their subjective model towards the view that inflation erodes real wages.

The applications below are concerned with variation in perceived inflation, so the reduced-form specification here is sufficient. There are however many possible microfoundations for equation 38. For example, if households believe there is an optimal level of inflation, such that real wages are increasing in inflation below that bliss point, but are decreasing beyond it, their α^h parameter would behave in this way. Appendix D.4 provides an alternative formal microfoundation, in which households are ambiguity averse, and face Knightian uncertainty about α^h. In that environment households distort their subjective model towards the worst case, which varies with perceived inflation.[26]

This assumption on subjective models matches empirical Fact 3 in a straightforward way. Substituting the expression for $\hat{\alpha}_t^h$ (equation 38) into the consumption function (equation 28):

$$\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h} = \left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\alpha_0^h} + \frac{\beta(1-\beta)\rho_\pi^h \alpha_1^h}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)} \tilde{\mathbb{E}}_t^h \pi_t \tag{39}$$

All households are assumed to have $\alpha_1^h < 0$, so this implies that higher perceived inflation is associated with more negative consumption responses to perceived inflation. That consumption response reflects the household's beliefs about future aggregate variables, so this matches the empirical finding that households who believe that higher inflation would weaken the economy on average perceive higher recent inflation.

The updating process also implies that the model matches empirical Fact 2, that more households hold negative models of the effects of inflation when realised inflation rises. Equation 39 implies:

$$\begin{aligned} &\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h} < X \\ \iff &\tilde{\mathbb{E}}_t^h \pi_t > \frac{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}{\beta(1-\beta)\rho_\pi^h \alpha_1^h}\left(X - \left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\alpha_0^h}\right) \end{aligned} \tag{40}$$

where X is an arbitrary threshold.

That is, household h's consumption response to inflation, after updating their subjective model, is below any threshold X if their perceived inflation is sufficiently high. Using the Kalman filter equation for inflation perceptions (equation 33), and holding priors and

[26] This approach relates to that of Michelacci and Paciello (2020), who note that ambiguity aversion naturally generates the negative correlation between preferences and expectations I observe for inflation.

realised inflation fixed, the household's perceived inflation is distributed according to:

$$\tilde{\mathbb{E}}_t^h \pi_t \sim N(K^h \pi_t + (1 - K^h)\rho_\pi^h \tilde{\mathbb{E}}_{t-1}^h \pi_{t-1}, (K^h)^2 \sigma_{\varepsilon h}^2) \tag{41}$$

Since $K^h \in [0, 1]$, we therefore have:

$$\frac{\partial}{\partial \pi_t} \left[\Pr \left(\left. \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} \right|_{\hat{\alpha}_t^h} < X \right) \right] \geq 0 \tag{42}$$

where the inequality is strict if $K^h > 0$, and is an equality otherwise. That is, households who acquire some information ($K^h > 0$) become more likely to hold negative subjective models of inflation when inflation is currently high. As inflation rises, there will therefore be fewer households who believe the appropriate response to inflation is to increase consumption, and more households with subjective models implying they should reduce consumption. More households therefore believe that higher inflation would weaken the economy when inflation is high.

In principle, as subjective models change, the value of information changes, so some households may wish to go back and acquire more information about inflation. For simplicity, in the main analysis of the paper I abstract away from this. However, in Appendix D.5 I allow for such multiple rounds of information processing, and derive a testable implication: among those with negative subjective models of inflation, higher perceived inflation encourages more information processing, so there should be a positive correlation between $\tilde{\mathbb{E}}_t^h \pi_t$ and information.[27] Among those with positive models, that correlation should be reversed. I find evidence of these correlations in the IAS data, further supporting the mechanisms in the model.[28]

5 Implications of narrative behaviour

In this section I show that the feedback between information and subjective models has important implications for macroeconomic behaviour, because the resulting systematic relationships between the components of expectation formation lead to a large and time-varying narrative heterogeneity channel of shock transmission. Calibrating the model to the UK over the period of the survey data, the narrative heterogeneity channel accounts for 36% of the elasticity of aggregate consumption to inflation in steady state, and 39%

[27] As in other surveys, households overestimate inflation on average (Carroll, 2003; Kumar et al., 2015), so this implies households with more information make larger forecast errors.

[28] This result is consistent with the finding in Link et al. (2022) that greater information acquisition about inflation is associated with higher expected inflation, since most households believe inflation weakens the economy. A similar mechanism arises in the model in Section 6, where perceived long-run inflation can affect information choices.

of its volatility over the period.

These effects, of course, only relate to the partial equilibrium response of aggregate consumption to inflation, and omit general equilibrium effects. Nonetheless, it is useful to observe these changes in this partial equilibrium laboratory where we can cleanly analyse the mechanisms, as Auclert (2019) does for the effects of MPC heterogeneity. Moreover, Wolf (2021) finds evidence that the 'missing intercept' of these general equilibrium effects is close to zero, implying that partial equilibrium responses to shocks to consumption are a good approximation to the full shock response in general equilibrium.

5.1 Selection in attention

First, consider the effect of subjective models on information choice, as discussed in Section 4.3. To isolate this, assume for now that $\alpha_1^h = 0$, so the only heterogeneity in subjective models is that present at the start of each period, when households choose their information.

Consider a shock that increases inflation in period t. The partial equilibrium effect of this on the consumption of household h on impact is given by:

$$\frac{\partial c_t^h}{\partial \pi_t} = \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} \frac{\partial \tilde{\mathbb{E}}_t^h \pi_t}{\partial \pi_t} = \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} K^h \tag{43}$$

where the second equality follows from equation 33.

The partial equilibrium response of aggregate consumption is therefore:

$$\frac{\partial \bar{c}_t}{\partial \pi_t} = \int_0^1 \omega^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} K^h dh = \int_0^{q_0} \omega^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} K^h dh \tag{44}$$

where ω^h is a weight on household h as in equation 13. The fraction of households who pay no attention ($K^h = 0$) is $1 - q_0$, and they are assumed to be indexed by $h \in [q_0, 1]$.

To see how the relationship between information and subjective models affects aggregate outcomes, compare this to a model in which all households have the same Kalman gain $\bar{K}$, equal to the average K^h from the baseline model:

$$\bar{K} = \mathbb{E}_H(K^h) = \mathbb{E}_H(K^h | K^h > 0) \cdot q_0 \tag{45}$$

This, for example, could reflect an economist calibrating a model with homogeneous information frictions to micro-level evidence on household information. In such a homogeneous-K^h model the aggregate partial equilibrium response of consumption to the inflation shock can be decomposed into two integrals:

$$\begin{aligned}\left.\frac{\partial \bar{c}_t}{\partial \pi_t}\right|_{K^h=\bar{K}} &= \int_0^1 \omega^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} \bar{K} dh \\ &= \int_0^{q_0} \omega^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} K^h \frac{\bar{K}}{K^h} dh + \int_{q_0}^1 \omega^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} \bar{K} dh\end{aligned} \tag{46}$$

The first term is identical to the expression for $\partial\bar{c}_t/\partial\pi_t$ in the baseline model with endogenous attention, except that each household's response is weighted by $\bar{K}/K^h$. Relative to the baseline model, the consumption responses of more attentive households receive a lower weight, while less attentive households are over-weighted. The definition of $\bar{K}$ (equation 45) implies that the re-weighting for the mean household within $h \in [0, q_0]$ is $q_0 \leq 1$. The down-weighting of attentive households is therefore stronger if a greater proportion of the population is inattentive (q_0 is lower), as this brings down $\bar{K}$.

The second integral concerns the consumption responses of inattentive households. In the baseline model, their response to perceived inflation is irrelevant, because their inattention means their inflation perceptions are unaffected by the shock. Here, however, their perceptions react to the shock with elasticity $\bar{K}$. Relative to the baseline model with endogenous attention, this alternative with homogeneous attention therefore under-weights the most attentive households, and over-weights the least attentive.

This leads to systematic differences in aggregate consumption responses, because the most attentive households have high K^h in the baseline model precisely because they have the largest consumption responses to perceived inflation. Formally, the difference between the aggregate consumption responses in the endogenous-K^h baseline and the homogeneous-K^h model is:

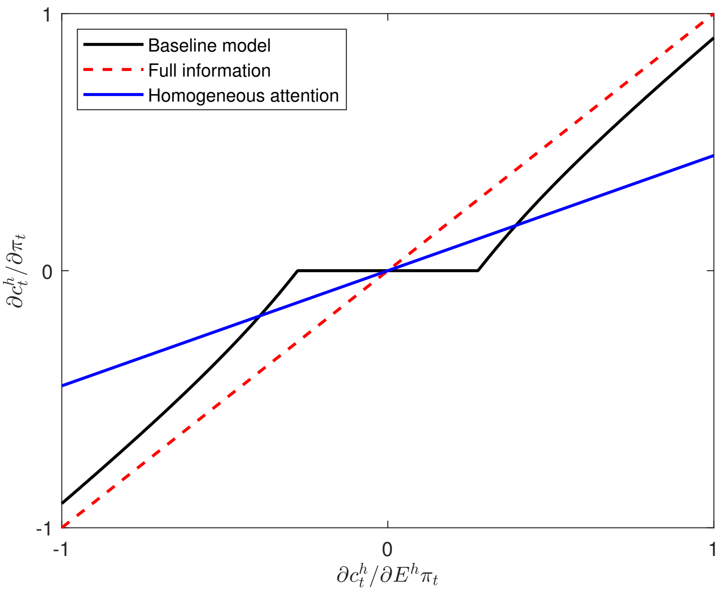
$$\frac{\partial \bar{c}_t}{\partial \pi_t} - \left.\frac{\partial \bar{c}_t}{\partial \pi_t}\right|_{K^h=\bar{K}} = \mathbb{E}_H\left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}(K^h - \bar{K})\right) = Cov_H\left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}, K^h\right) \tag{47}$$

The difference therefore depends on the covariance of information and subjective models: by making attention exogenous, the homogeneous-K^h model omits the narrative heterogeneity channel of shock transmission discussed in Section 2. This covariance depends on the distribution of subjective models, as K^h is increasing in the absolute value of $\partial c_t^h/\partial\tilde{\mathbb{E}}_t^h\pi_t$. Among households with $\partial c_t^h/\partial\tilde{\mathbb{E}}_t^h\pi_t > 0$, the covariance of consumption responses and K^h is positive, but among those with $\partial c_t^h/\partial\tilde{\mathbb{E}}_t^h\pi_t < 0$ it is negative.

This implies that the narrative heterogeneity channel typically amplifies the partial equilibrium aggregate consumption response to the shock relative to the homogeneous-K^h model. If most households increase consumption when perceived inflation rises, then the baseline aggregate consumption response to a π_t increase is positive. At the same time, the narrative heterogeneity channel in expression 47 is positive. Conversely, if most households have strong negative subjective models of inflation, the baseline aggregate response is negative, as is the narrative heterogeneity channel in expression 47.

Figure 4 shows this effect graphically. It plots the consumption response of an individual household to a shock to π_t against the same household's response to an increase in perceived inflation $\tilde{\mathbb{E}}_t^h \pi_t$. If households observed inflation precisely, this would simply be the 45° line (red dashed line).

Figure 4: Consumption response to a change in actual inflation against response to perceived inflation. Parameters listed in Appendix E.



The black solid line shows this relationship in the baseline model with endogenous K^h. Households with $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t$ close to zero pay no attention to current inflation, and so their perceptions of inflation do not change when the shock hits. They therefore do not react. Households with greater $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t$ pay more attention, and so their perceptions are more sensitive to the inflation change, and their elasticity of consumption to π_t is therefore closer to the 45° line.

If the endogenous K^h is replaced by a fixed $\bar{K}$ for all households, the elasticity of c_t^h to π_t is instead given by the blue solid line. Relative to the baseline model, consumption responses are drawn closer to the full-information line for all households with $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t$ such that $K^h < \bar{K}$ in the baseline model. Conversely, consumption responses are reduced towards zero for all those who are more attentive than average in the baseline model. Since the less attentive households are the ones who would react the least under full information, removing the narrative heterogeneity channel in this way weakens the effect of the shock.

This effect is analogous to the selection effect in menu cost models of price setting

(Caplin and Spulber, 1987; Golosov and Lucas, 2007). In those models, the aggregate price level is less sticky than the average of firm-level stickiness, because firms change prices when their current price is far from the optimal price. Price adjustments are therefore disproportionately drawn from firms desiring large price changes.

In the model presented here, households obtaining information about inflation are disproportionately drawn from those who would react strongly to that information. Just as the price level in a menu cost is more flexible than the average flexibility at the firm level, this implies that aggregate consumption is more responsive to inflation than is implied by micro-level estimates of household attention. The narrative heterogeneity channel can therefore explain why representative agent models typically require only small information frictions to match aggregate data (Maćkowiak and Wiederholt, 2015), while micro-level studies find very large degrees of inattention (Link et al., 2021).[29]

A further implication of this selection effect is that information treatment experiments aimed at estimating the causal effects of expectations (see Candia et al., 2020, for a review) will disproportionately measure the responses of a non-representative subset of agents. The standard approach in these studies is to regress the outcome variable on the expectation of interest, instrumented using an indicator for whether the respondent was in the treatment or control group.[30] The estimate is therefore consistent for the local average treatment effect on those who update their expectations as a result of the information provision, and is most influenced by those who update the furthest. The selection effect studied here suggests that those agent will disproportionately be those with the smallest responses to information, as they start out with the most uncertain beliefs due to their lack of attention. They therefore update expectations the most when shown publicly available information. However, when a shock hits the economy it is the attentive households who get the most information about it, and react most strongly.[31]

5.2 State-dependent shock transmission

I now return to the two-way feedback between information and subjective models. The interaction implies that the transmission of inflation shocks to aggregate consumption depends on the size and recent history of realised inflation deviations from steady state,

[29] Relatedly, Afrouzi and Yang (2021) find that firms acquire information only in periods when they are changing prices, which is when their expectations matter for the dynamics of the price level.

[30] It is also common to use a second instrument, the interaction of the treatment indicator with the agent's prior expectation (e.g. Coibion et al., 2019). This does not substantially change the intuition discussed here.

[31] In some settings the response of inattentive households is precisely the object of interest. For example, when studying central bank communication with the general public the researcher is typically interested in how previously inattentive households respond to the provision of more accessible information (Coibion et al., 2022; Haldane et al., 2021).

as this alters the distribution of subjective models, and their correlation with information choices, in each period.

To explore these effects, I begin by showing how the aggregate consumption response to an inflation shock depends on the distribution of inflation perceptions, before showing how that distribution varies with the size of inflation shocks and recent inflation history.

The distribution of $\tilde{\mathbb{E}}_t^h \pi_t$.

The elasticity of aggregate consumption to an inflation shock is given by:

$$\frac{\partial \bar{c}_t}{\partial \pi_t} = \int_0^1 \omega^h K^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t} dh \tag{48}$$

where the Kalman gain K^h gives the response of household h's perceived inflation to a rise in realised π_t.

Using equation 39, we can decompose this as follows:

$$\begin{aligned} \frac{\partial \bar{c}_t}{\partial \pi_t} &= \int_0^1 \omega^h K^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\bigg|_{\alpha_0^h} dh + \int_0^1 \omega^h K^h \frac{\beta(1-\beta)\rho_\pi^h \alpha_1^h}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)} \tilde{\mathbb{E}}_t^h \pi_t dh \\ &= \mathbb{E}_H\left(K^h \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\bigg|_{\alpha_0^h}\right) - \mathbb{E}_H(K^h)\mathbb{E}_H(\Omega^h \tilde{\mathbb{E}}_t^h \pi_t) - Cov_H(K^h, \Omega^h \tilde{\mathbb{E}}_t^h \pi_t) \end{aligned} \tag{49}$$

where the second equality uses the definition of a covariance, and Ω^h is a strictly positive combination of preferences and subjective model parameters:

$$\Omega^h = -\frac{\beta(1-\beta)\rho_\pi^h \alpha_1^h}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)} \tag{50}$$

The first term of the aggregate elasticity to inflation is a function of underlying parameters only. Since the initial subjective models held by households at the start of each period are assumed to be fixed here, this is unaffected by realised shocks.

The second term, however, shows that the average subjective model will adjust towards lower values of $\hat{\alpha}_t^h$ as perceived inflation rises. This more negative average subjective model will reduce the aggregate consumption elasticity to inflation. The third term shows that such a rise in perceived inflation will have more of an effect if it occurs in households who process a lot of information about inflation. These are the time-varying components of the representative agent and narrative heterogeneity channels identified in Section 2.

Size dependence.

I now show how this aggregate consumption elasticity to inflation varies with the size of the inflation shock. Differentiating equation 49 with respect to current inflation, and using the Kalman filtering equation (33) to extract the response of perceived inflation, we obtain:

$$\frac{d}{d\pi_t}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -\mathbb{E}_H(K^h)\mathbb{E}_H(\Omega^h K^h) - Cov_H(K^h, \Omega^h K^h) \tag{51}$$

The effects on each of the terms is especially clear if we further assume that all households share the same α_1^h, ρ_π^h, and ρ_w^h, and so the same Ω^h. In that case equation 51 becomes:

$$\frac{d}{d\pi_t}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -\Omega\big(\mathbb{E}_H(K^h)\big)^2 - \Omega Var_H(K^h) \tag{52}$$

The elasticity of aggregate consumption to inflation therefore falls for two reasons as the inflationary shock gets larger. First, the average inflation perception rises, so the average subjective model becomes more negative about inflation. This matches up with the survey data: the large 0.9% point rise in annual CPI inflation from August to November 2021 in the UK coincided with a 9% point increase in the share of households responding that inflation weakens the economy in the IAS.

Second, the narrative heterogeneity channel also contributes to a fall in $\partial\bar{c}_t/\partial\pi_t$. As the shock size increases, the difference between the inflation perceptions of attentive (high K^h) and less attentive (low K^h) households grows. The most attentive households therefore adjust their subjective models more towards lower $\hat{\alpha}_t^h$ relative to inattentive households, which makes the covariance of K^h and $\partial c_t^h/\partial\tilde{\mathbb{E}}_t^h\pi_t$ more negative. Intuitively, as the most attentive households adjust their perceptions by the most, larger shocks lead to a greater concentration of very negative subjective models among the most attentive households. This effect is particularly strong if information choices are very heterogeneous across households, as suggested by the empirical evidence in Link et al. (2021).

History dependence.

If households believe inflation is persistent, recent inflation history will also affect the distribution of inflation perceptions. Differentiating equation 49 with respect to realised inflation in period $t-1$ gives:

$$\frac{d}{d\pi_{t-1}}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -\mathbb{E}_H(K^h)\mathbb{E}_H(\Omega^h K^h(1-K^h)\rho_\pi^h) - Cov_H(K^h, \Omega^h K^h(1-K^h)\rho_\pi^h) \tag{53}$$

The first effect is as with the size dependence: high inflation in the previous period implies high average inflation perceptions in period t, which lowers $\partial\bar{c}_t/\partial\pi_t$. This is because high π_{t-1} implies high prior beliefs about π_t. Since households receive noisy signals about inflation in period t, those elevated priors lead to higher perceived π_t.

The narrative heterogeneity effect is more subtle. Again assuming that households all share the same α_1^h, ρ_π^h, and ρ_w^h, equation 53 becomes:

$$\frac{d}{d\pi_{t-1}}\left(\frac{\partial \bar{c}_t}{\partial \pi_t}\right) = -\Omega\rho_\pi\mathbb{E}_H(K^h)\mathbb{E}_H(K^h(1-K^h)) - \Omega\rho_\pi Cov_H(K^h, K^h(1-K^h)) \tag{54}$$

The second term may be positive or negative, because there are two opposing effects: on the one hand, as for the size dependence, any inflation shock has the greatest effects within the period on the perceptions of the most attentive households. This acts to reduce the covariance of information and subjective models. However, on the other hand, the most attentive households are the least reliant on their prior beliefs when forming perceptions of π_t, and so are least affected by their past inflation perceptions. If average K^h is sufficiently large, this second effect dominates and high past inflation increases the covariance of information and $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t$, because the most attentive are least influenced by past inflation and so hold less negative subjective models of the effects of inflation.

5.3 Quantifying the narrative heterogeneity channel

To understand the relative sizes of the effects derived above, I now calibrate the model to the UK over 2001-2019, the sample period of the IAS data in Section 3. The narrative heterogeneity channel accounts for substantial fractions of the steady state aggregate consumption elasticity to inflation, and its variation over time.

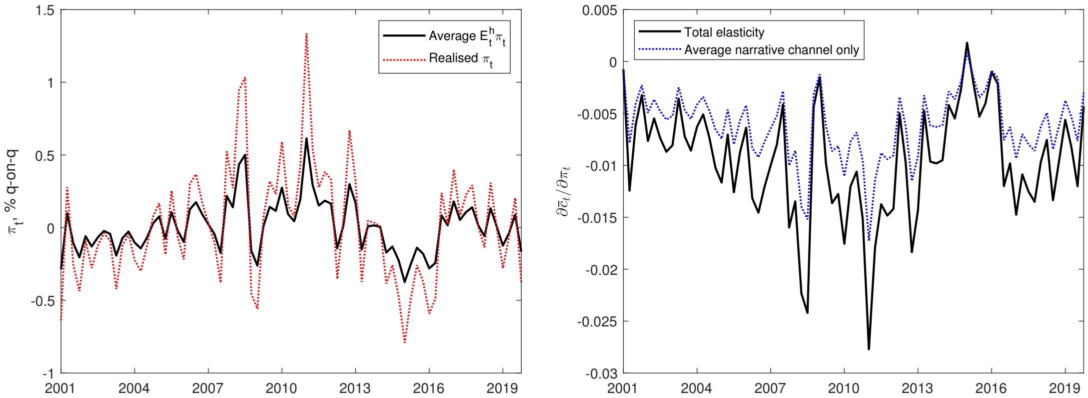
To calibrate the model, I first set some preference parameters to standard values. I then run a naive estimation of equations 21 - 23 to obtain all parameters of household subjective models except for α_0^h and α_1^h. I set these parameters to be the same for all households, and use the estimated α from those regressions as the mean α_0^h across households. Finally, I assume that α_0^h has a normal distribution, and all households share the same α_1^h. I calibrate the variance of the α_0^h distribution, along with α_1^h and ψ, to match three targets from the IAS data: the average proportion of households who believe inflation makes the economy weaker, the average elasticity of this proportion to increases in inflation, and an estimate of the average Kalman gain in inflation perceptions. Full details of the calibration are in Appendix E.

Note that the regressions of equations 21 - 23 used in this calibration, while naive from the point of view of modern empirical macroeconomics, are not naive from the households' point of view. If their subjective model has the correct structure, then these regressions will uncover the intended underlying parameters. In this it is important that w_t does not appear in the law of motion for inflation, as in that case inflation would be endogenous in equation 23.

I obtain a stationary distribution of inflation perceptions by assuming that $\pi_t = 0$ for many periods, so the only variation in $\tilde{\mathbb{E}}_t^h \pi_t$ comes from idiosyncratic noise in household signals. In the steady state with $\tilde{\mathbb{E}}_t^h \pi_t$ drawn from this distribution, $\partial \bar{c}_t / \partial \pi_t$ is negative. This is because the majority of households believe inflation weakens the economy in the survey, so most have negative $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t$. As identified in the IAS data, there is a negative correlation between information K^h and $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t$, so the narrative heterogeneity channel is also negative, accounting for 36% of the steady state $\partial \bar{c}_t / \partial \pi_t$.

I next simulate the model for 1000000 households, using the path of de-meaned quarterly CPI inflation observed in the UK over the sample period for realisations of π_t. Figures 5a and 5b show the paths of average perceived inflation and $\partial \bar{c}_t / \partial \pi_t$. Compared to realised inflation, perceived inflation is relatively smooth, despite the selection effect discussed in Section 5.1. However, this still implies substantial volatility in $\partial \bar{c}_t / \partial \pi_t$. When inflation spiked in 2016Q3 after the Brexit referendum, the elasticity of aggregate consumption to inflation was 5.5x larger than in the previous quarter.

Figure 5: Simulated inflation perceptions and aggregate consumption elasticity to inflation. Calibration and simulation details are in Appendix E.



(a) Average $\tilde{\mathbb{E}}^h_t \pi_t$ and realised π_t (b) $\partial \bar{c}_t / \partial \pi_t$

The transmission of inflation shocks therefore varies a great deal over time due to the interaction of information and subjective models. Using the decomposition from Section 2, we can further split that variation into the representative agent and narrative heterogeneity channels. The blue line in Figure 5b shows $\partial \bar{c}_t / \partial \pi_t$ without the narrative heterogeneity channel. It is substantially less volatile: fluctuations in the covariance of information and subjective models account for 39% of the standard deviation of $\partial \bar{c}_t / \partial \pi_t$. As discussed in Section 5.2, when inflation rises the narrative heterogeneity channel becomes more negative, widening the gap between the total $\partial \bar{c}_t / \partial \pi_t$ and that implied by representative agent effects alone.

6 Endogenous long-run expectations

So far in this analysis, information about inflation has mostly affected expectations about aggregate variables in the near future, as all variables are perceived to be stationary. Policymakers, however, frequently also consider how recent events affect longer-term expectations (e.g. Powell, 2021). In this section I extend the model to allow households to use current information to update their expectations of long-run inflation. Inflationary shocks may become 'baked in' to expectations after an inflationary shock, but only among households who held positive subjective models of the effects of inflation before the shock. This in turn has persistent effects on the transmission of inflationary shocks, with the majority of the long-run effect driven by the narrative heterogeneity channel.

Suppose that the household's subjective model for inflation includes a long-run mean of inflation $\bar{\pi}_t$ which is not necessarily equal to 0:

$$\pi_t = \rho_\pi^h \pi_{t-1} + (1 - \rho_\pi^h)\bar{\pi}_t + u_{\pi t} \tag{55}$$

To begin with, assume that households treat the long-run mean of inflation as a parameter of their subjective model (rather than a time-varying variable). Following the anticipated utility assumption used above, they therefore make information choices expecting $\bar{\pi}_t$ to remain constant at their current estimate for certain. This assumption greatly simplifies the analysis and allows for analytic results, but is not critical for the mechanisms. In Appendix F I relax this and assume that $\bar{\pi}_t$ is treated as a time-varying variable in the information choice problem. The qualitative results below continue to hold.

Re-deriving the consumption function with this new subjective model for inflation gives (derivation in Appendix D.6):

$$c_t^h = \frac{1-\beta}{1-\beta\rho_w^h} w_t - \sigma\beta r_t + \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\left(\tilde{\mathbb{E}}_t^h \pi_t + \frac{1-\rho_\pi^h}{\rho_\pi^h(1-\beta)} \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t \right) \tag{56}$$

where $\tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t$ is household h's estimate of $\bar{\pi}_t$ before information processing in period t. This consumption function is as in equation 28, except for the additional term in $\bar{\pi}_t$.

In the previous sections, household information choices were determined by the constant subjective model parameter α_0^h. However, as the household may now expect inflation to deviate from 0 in the long term, I allow the perceived long-run mean of inflation to affect that initial model:

$$\alpha_t^{h,prior} = \alpha_0^h + \alpha_1^h \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t \tag{57}$$

In this way the model allows us to understand the consequences of a rise in long-term inflation expectations for both information and subjective models.

These assumptions imply that the expected utility loss from imperfect information is given by:

$$\tilde{\mathbb{E}}_0^h(\hat{U}_0^{h*} - \hat{U}_0^h) = \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\alpha_t^{h,prior}} \right)^2 \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \left((\pi_t - \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t) - (\tilde{\mathbb{E}}_t^h \pi_t - \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t) \right)^2 \tag{58}$$

Rewriting equation 55 with the assumption that $\bar{\pi}_t$ will remain at $\tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t$ for all t gives:

$$(\pi_t - \tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t) = \rho^h_\pi(\pi_{t-1} - \tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t) + u_{\pi t} \tag{59}$$

The information choice problem is therefore isomorphic to that in Section 4.3, with π_t replaced with $\pi_t - \tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t$ and the constant in the objective function adjusted for $\alpha^{h,prior}_t$. The optimal signal is of the form:

$$s^h_t = \pi_t - \tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t + \varepsilon^h_t, \quad \varepsilon^h_t \sim N(0, \sigma^2_{\varepsilon ht}) \tag{60}$$

and the optimal choice of $\sigma^2_{\varepsilon ht}$ is as implied by Proposition 4, with the relevant coefficient Γ^h computed using $\alpha^{h,prior}_t$.

The household then uses this signal to update their beliefs about current inflation, and also their beliefs about the long-run mean $\bar{\pi}_t$. For that updating they therefore acknowledge that $\bar{\pi}_t$ may in fact change over time. Specifically, they assume that $\bar{\pi}_t$ follows a random walk (as in e.g. Cogley and Sbordone, 2008; Fisher et al., 2021):

$$\bar{\pi}_t = \bar{\pi}_{t-1} + v_t, \quad v_t \sim N(0, \sigma^2_v) \tag{61}$$

With this assumption, we can write the household's forecasting problem in state-space form:

$$\xi_t = F^h\xi_{t-1} + e^h_t \tag{62}$$

$$(s^h_t + \tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t) = C'\xi_t + \varepsilon^h_t \tag{63}$$

where:

$$\xi_t = \begin{pmatrix} \pi_t \\ \bar{\pi}_t \end{pmatrix}, \quad F^h = \begin{pmatrix} \rho^h_\pi & 1-\rho^h_\pi \\ 0 & 1 \end{pmatrix}, \quad e^h_t = \begin{pmatrix} u_{\pi t} + (1-\rho^h_\pi)v_t \\ v_t \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{64}$$

It therefore remains optimal for households to incorporate signals into their perceptions of π_t and $\bar{\pi}_t$ using the Kalman filter:

$$\tilde{\mathbb{E}}^h_t\xi_t = (I - K^h_tC')F^h\tilde{\mathbb{E}}^h_{t-1}\xi_{t-1} + K^h_ts^h_t \tag{65}$$

where K^h_t is a 2×1 vector of gain parameters.

This does mean that households use their signals in a different way to what they expected when they made their information decisions, as they did not anticipate the update to beliefs about $\bar{\pi}_t$. This is a direct consequence of the anticipated utility assumption, relaxed in Appendix F.

Perceived long-run inflation affects optimal attention and expectation updating according to Proposition 5.

Proposition 5 *Let $\sigma^{2*}_{\varepsilon ht}$ denote the optimally chosen noise variance in s^h_t. Then, for $\sigma^{2*}_{\varepsilon ht} < \infty$:*

$$\frac{\partial \sigma^{2*}_{\varepsilon ht}}{\partial \tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t} < 0 \quad \text{if and only if} \quad \left.\frac{\partial c^h_t}{\partial \tilde{\mathbb{E}}^h_t \pi_t}\right|_{\alpha^{h,prior}_t} < 0 \tag{66}$$

$$\frac{\partial K^h_t}{\partial \tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t} > 0 \quad \text{if and only if} \quad \left.\frac{\partial c^h_t}{\partial \tilde{\mathbb{E}}^h_t \pi_t}\right|_{\alpha^{h,prior}_t} < 0 \tag{67}$$

Proof. Appendix D.7. ■

That is, if a household starts the period with a negative subjective model, such that they reduce consumption when perceived inflation rises, then higher long-run inflation expectations cause them to pay more attention to inflation, acquiring more precise signals. Higher long-run expected inflation causes them to update their subjective model further towards inflation eroding real wages (equation 57), which increases the magnitude of their consumption response to inflation. Information gets more valuable, so they pay to acquire more of it. Their perceptions of π_t and $\bar{\pi}_t$ become more responsive to realised π_t as a result.

The reverse is true for a household with $\partial c^h_t / \partial \tilde{\mathbb{E}}^h_t \pi_t > 0$ under $\alpha^{h,prior}_t$. Higher long-run expected inflation similarly reduces their $\alpha^{h,prior}_t$, but that shift pulls the consumption response to inflation towards zero, reducing the value of inflation information. Perceived current and long-run inflation get less responsive to realised π_t.

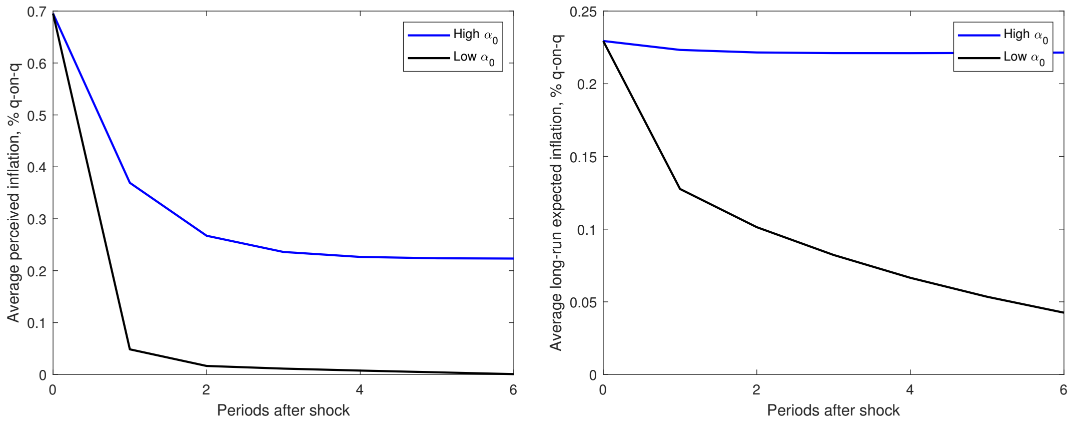
Information about π_t therefore not only affects the subjective model used that period, but also the subjective model used to make information choices in the next period, through perceptions of $\bar{\pi}_t$. These interdependencies imply that the expectations of different households may follow very different paths after a shock. To show this, Figure 6 plots the average perceived π_t and $\bar{\pi}_t$ for two groups of households after a 1 percentage point i.i.d. inflation shock. Within a group, all households share the same subjective model parameters, but obtain idiosyncratic signals.

The figure is drawn assuming all households have $\tilde{\mathbb{E}}^h_{t-1}\bar{\pi}_t = 0$ when the shock hits, and prior beliefs in the period of the shock are drawn from the stationary distribution obtained in the absence of aggregate shocks.

The first group of households, shown in black, begin the shock period with low α^h_0, so they have $\partial c^h_t / \partial \tilde{\mathbb{E}}^h_t \pi_t < 0$ and $K^h_t > 0$. Since they process some information, both their perceived current and long-run inflation rise when the shock hits. However, as this leads them to increase their information processing, they observe that inflation has fallen in the periods after the shock, and their perceptions quickly return to 0.

The second group of households, shown in blue, are identical to the first except that they have a higher α^h_0, such that $\partial c^h_t / \partial \tilde{\mathbb{E}}^h_t \pi_t > 0$. Their α^h_0 has been chosen such that

Figure 6: Simulated average $\tilde{\mathbb{E}}_t^h \pi_t$ and $\tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t$ for two household groups after an i.i.d. inflation shock. Calibration and simulation details are in Appendix E.



(a) Average $\tilde{\mathbb{E}}_t^h \pi_t$

(b) Average $\tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t$

both groups have the same K_t^h in the period of the shock, so average inflation perceptions initially rise by the same amount. However, the rise in perceived $\bar{\pi}_t$ causes this second group to pay *less* attention to inflation, as their subjective models shift towards inflation making less difference for their consumption. This slows down the return of long-run expectations, and perceived current inflation, to steady state among this group, as they do not precisely observe the fall in inflation after the shock. In turn, this means their attention remains low. Indeed, many high-α_0 households cease processing information completely, so they never reduce their long-run expectations from their elevated post-shock level.

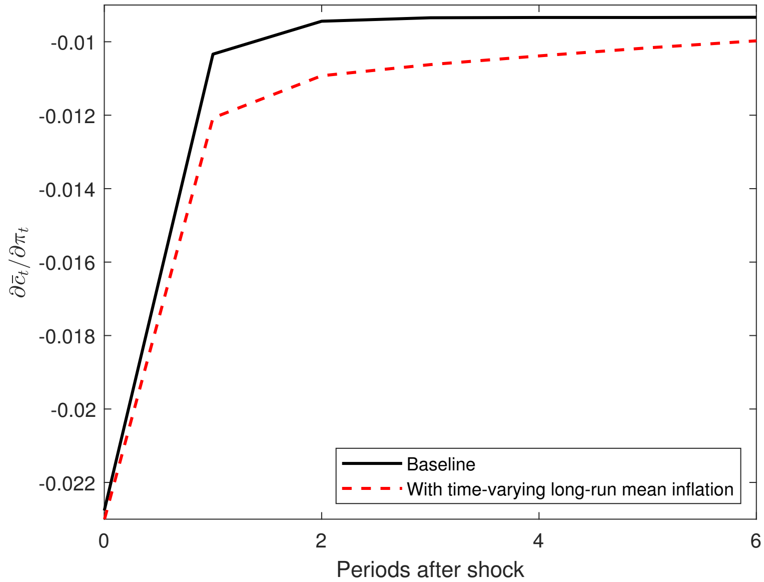
High inflation can therefore become 'baked in' to expectations, but only among households who start out believing inflation strengthens the economy, and who subsequently reduce their attention after an inflationary shock. This is a novel effect from the interaction of the two components of expectations: if households had limited information but knew the true equilibrium law of motion for inflation, they would know that the shock is transitory, and would not update their long-run expectations. If households didn't know the true model but had full information, they would all observe inflation returning to 0 after the shock.

Importantly, the fact that this 'baking in' is correlated with subjective models implies that it has a persistent effect on the aggregate transmission of inflationary shocks. Figure 7 shows $\partial \bar{c}_t / \partial \pi_t$ with and without time-varying long-run perceptions after the same one-off inflationary shock used to draw Figure 6.

The aggregate consumption elasticity to inflation returns quickly to its pre-shock level when the long-run inflation mean is known to be constant at 0, because the perceived persistence of inflation in the calibration is low. However, with time-varying perceived long-run inflation, $\partial \bar{c}_t / \partial \pi_t$ remains depressed persistently after the shock, because of the households whose expectations have become 'baked in' at a high level.

Decomposing the changes in $\partial\bar{c}_t/\partial\pi_t$ reveals that the narrative heterogeneity channel explains the majority of the extra persistence, accounting for 62% of the difference between $\partial\bar{c}_t/\partial\pi_t$ and its pre-shock value after 6 quarters. This is because the households who started with more positive subjective models persistently process less information, which persistently lowers the covariance of information and subjective models.

Figure 7: Simulated aggregate consumption elasticity to inflation after an i.i.d. inflation shock. Calibration and simulation details are in Appendix E.



7 Conclusion

This paper explores the interaction of the two components of expectation formation, information and subjective models, which previous literature has tended to treat separately.

I show that in a general log-linear model, shocks pass through to aggregate responses along three channels. The first is the transmission that would be seen in a representative agent model. The second comes from heterogeneity in the parameters of policy functions, as is well-known from previous literature on heterogeneous-agent macroeconomics. The third channel, though, is novel. The narrative heterogeneity channel operates when information and subjective models covary systematically across agents. Heterogeneous subjective models imply heterogeneous responses to information, so systematic patterns in how information is distributed among agents with different subjective models distort the transmission of shocks to aggregate actions.

I use unique features of the Bank of England Inflation Attitudes Survey to document that subjective models and information about inflation do indeed covary systematically with each other, and with inflation perceptions and expectations. The distribution of subjective models also varies systematically with realised inflation. These results suggest that the transmission of shocks to aggregate consumption is affected by the interaction of information and subjective models, and changes over time as a result.

Finally, I propose a model with rational inattention and time-varying subjective models as a way to explain the empirical findings. The model generates a selection effect on information, size- and history-dependent shock transmission, and the possibility that expectations of high inflation may become 'baked in' after large transitory inflation spikes, but only among certain households.

This implies that when watching for the possibility that high inflation is becoming 'baked in' to expectations, not all households are of equal concern. It is the households who believed before the shock that more inflation would make the economy stronger who pose the most risk, because they reduce their attention to inflation as perceived inflation rises. If their expectations become unanchored, reducing realised inflation will not be sufficient to bring their expectations back down. From August to November 2021, the perceived inflation of households reporting that inflation makes the economy stronger in the IAS did indeed rise, but only by 26 basis points. This is substantially smaller than the average rise in perceived inflation across all households in the survey (78 b.p.), suggesting that the cat was not yet out of the bag in UK inflation expectations at the end of 2021.

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A Log-linear model proofs and derivations

A.1 Consumption function in a standard household problem

This derivation closely follows that in Bilbiie (2019) appendix A, and is also similar to consumption functions derived in Farhi and Werning (2019) and others.

Suppose that a household maximises:

$$\mathbb{E}_t^h \sum_{s=0}^{\infty} \beta^s \frac{(C_{t+s}^h)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \text{ s.t. } C_{t+s}^h + B_{t+s}^h = R_{t+s-1} B_{t+s-1}^h + Y_{t+s}^h \tag{68}$$

Where C_t^h is consumption, σ is the intertemporal elasticity of substitution, B_t^h are real one-period bonds bought in period t, R_t is the gross interest rate on such a bond, and Y_t^h is real income.

The first order condition is the standard Euler equation, which when log-linearised about steady state becomes:

$$c_t^h = \mathbb{E}_t^h c_{t+1}^h - \sigma r_t \tag{69}$$

Substituting forward we obtain:

$$c_t^h = \mathbb{E}_t^h c_{t+s}^h - \sigma \sum_{k=0}^{s-1} r_{t+k} \tag{70}$$

Assuming that $b_t^h = 0$ (as it is in equilibrium in a standard representative-agent or two-agent New Keynesian model), the present value budget constraint is:

$$\mathbb{E}_t^h \sum_{s=0}^{\infty} C_{t+s}^h \prod_{k=0}^{s-1} R_{t+k}^{-1} = \mathbb{E}_0^h \sum_{t=0}^{\infty} Y_{t+s}^h \prod_{k=0}^{s-1} R_{t+k}^{-1} \tag{71}$$

Log-linearising (recalling that in steady state $R = \beta^{-1}$):

$$\sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^h (c_{t+s}^h - \sum_{k=0}^{s-1} r_{t+k}) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^h (y_{t+s}^h - \sum_{k=0}^{s-1} r_{t+k}) \tag{72}$$

Use the Euler equation to substitute out for $\mathbb{E}_t^h c_{t+s}^h$ to obtain:

$$\sum_{s=0}^{\infty} \beta^s (c_t^h - (1-\sigma)\mathbb{E}_t^h \sum_{k=0}^{s-1} r_{t+k}) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^h (y_{t+s}^h - \sum_{k=0}^{s-1} r_{t+k}) \tag{73}$$

Rearranging:

$$\begin{aligned} \frac{1}{1-\beta} c_t^h &= \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^h y_{t+s}^h - \sigma \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^h (\sum_{k=0}^{s-1} r_{t+k}) \\ &= \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^h y_{t+s}^h - \frac{\sigma\beta}{1-\beta} \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^h r_{t+s} \end{aligned} \tag{74}$$

Multiplying through by $1-\beta$, and applying the Fisher equation $r_t = i_t - \pi_{t+1}$, we obtain equation 8.

A.2 Proof of Proposition 2

From the definition of $\bar{x}_{kt}$ (equation 14), we have:

$$\frac{d\bar{x}_{kt}}{dz_{nt}} = \mathbb{E}_H \frac{dx^h_{kt}}{dz_{nt}} \tag{75}$$

The k^{th} row of equation 3 can be written as:

$$\frac{dx^h_{kt}}{dz_{nt}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \mu^h_{ki,t} \chi^h_{ij,t} \delta^h_{jn,t} \tag{76}$$

Substituting this into equation 75 gives:

$$\frac{d\bar{x}_{kt}}{dz_{nt}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbb{E}_H \mu^h_{ki,t} \chi^h_{ij,t} \delta^h_{jn,t} \tag{77}$$

From the definition of covariance, $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y) + Cov(X,Y)$ for any X, Y. Applying this to equation 77 implies:

$$\frac{d\bar{x}_{kt}}{dz_{nt}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\bar{\mu}_{ki,t} \mathbb{E}_H(\chi^h_{ij,t} \delta^h_{jn,t}) + Cov_H(\mu^h_{ki,t}, \chi^h_{ij,t} \delta^h_{jn,t}) \right] \tag{78}$$

Applying the covariance formula again to the first term inside the sum in equation 78 implies equation 15.

B Defining the direct information indicator in the IAS

The full set of questions used to construct the information dummy is set out below, along with the dates at which each was asked and how the answers are mapped into the information indicator used above. All of the questions were only asked in the first quarter of the year(s) indicated. In the main exercises I exclude questions 2e and 2g from the total information variable, to ensure that there are no periods in which two questions are asked. I remove these rather than the short run questions in those periods to keep the majority of questions as short run expectations. The cross-sectional results are robust to including these extra questions, but the time series results are not, because including 2e and 2g means a much higher proportion of people are recorded as being informed in those periods, which affects the time series estimates. Only using short run questions (i.e. dropping 2018 and 2019 when Question 2civ is used) does not affect the cross-sectional results.

Question 2aiv *What were the most important factors that led you [to change (insert their response to how expectation has changed)] your expectation of prices in the shops over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *The level of the exchange rate (the value of sterling)*
10. *Other factors*
11. *None*

Asked: 2017
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2civ *What were the most important factors that led you to change/not change your expectation of prices in the shops in the longer term?*

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *The level of the exchange rate (the value of sterling)*
10. *Other factors*
11. *None*

Asked: 2018, 2019
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2d *When you said prices would go up in the next 12 months, how important were the following things in getting to that answer?*

For each option, possible answers are:

- *Very important*
- *Fairly important*
- *Not very important*
- *Not at all important*
- *Don't know*
- *Refused*

Options:

1. *How prices have changed in the shops in your most recent visits (i.e. the last 1 to 6 months).*
2. *How prices have changed in the shops over the longer term (i.e. the last 12 months or more)*
3. *The current level of interest rates.*
4. *The current strength of the British Economy.*
5. *The inflation target set by the government.*
6. *Reports on inflation outlook in the media.*
7. *Reports of VAT changes in the media.*
8. *Other factor(s).*

Asked: 2009, 2010, 2011, 2013
Information indicator: =1 if 'very important' selected for option 6, =0 otherwise.

Question 2e *And which, if any, of the same factors were important in getting to your expectation of how prices will change over the longer term (say in 5 years time)?*

1. *How prices have changed in the shops in your most recent visits (i.e. the last 1 to 6 months).*
2. *How prices have changed in the shops over the longer term (i.e. the last 12 months or more)*
3. *The current level of interest rates.*
4. *The current strength of the British Economy.*
5. *The inflation target set by the government.*
6. *Reports on inflation outlook in the media.*
7. *Reports of VAT changes in the media.*
8. *Other factor(s).*

Asked: 2011, immediately after Question 2d
Information indicator: =1 if item 6 selected, =0 otherwise.

Question 2f *What were the most important factors in getting to your expectation for how prices in the shops would change over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

Asked: 2016
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2g *And what were the most important factors in getting to your expectation for how prices in the shops would change over the longer term (say in 5 years' time)?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

Asked: 2016
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

C Further empirical results

C.1 The relationship of planned consumption with measured information and subjective models

To confirm that the survey measures of information and subjective models uncover meaningful aspects of household beliefs, I consider how they correlate with planned consumption behaviour. To this end, I use the following survey question:

Question 17.2 *Which, if any, of the following actions are you taking, or planning to take, in the light of your expectations of price changes over the next twelve months?*

- *Cut back spending and save more.*

Crucially, this asks about consumption choices which are explicitly driven by expected inflation.[32] A household answering 'yes' to this question, and who reports elsewhere in the survey that they expect prices to rise in the next year, is therefore indicating that $dc_t^h/d\mathbb{E}_t^h p_{t+1} < 0$.[33] A question that only asked about consumption or consumption changes, without reference to the cause of the behaviour, would conflate this with reactions to expectations of other variables, which might also be influenced by the same shocks as expected inflation, either directly or through cross-learning. Question 17.2 is therefore informative about the sign of $\frac{dc_t^h}{d\mathbb{E}_t^h p_{t+1}}$. If current prices are assumed to be fixed by the household, then this is the same as the sign of $\frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}}$.

The vast majority of respondents (98%) expect positive inflation over the next 12 months.[34] For these households, yes and no responses to Question 17.2 respectively indicate that:

$$\frac{dc_t^h}{d\mathbb{E}_t^h p_{t+1}} \begin{cases} < 0 & \text{if answer yes} \\ \geq 0 & \text{if answer no} \end{cases} \tag{79}$$

For the minority who expect deflation, these inequalities are reversed: responding with 'yes' indicates consumption is being cut because of an expected fall in prices. I therefore

[32] Question 17.1 in the survey is also about consumption, asking if the respondent will "bring forward major purchases such as furniture or electrical goods". I do not use this in my main measure of consumption responses for two reasons. First, as Nunes and Park (2020) note, the question refers specifically to durable goods, which may not respond to prices in the same way as aggregate consumption, which is the object of interest here. Second, it is very rarely chosen: just 6% of respondents said they would bring forward major purchases. In contrast, 40% report that they will cut back spending and save more. Any estimation on this variable will therefore be heavily influenced by a small subset of agents.

[33] This interpretation is discussed in more detail at the end of this appendix.

[34] The analysis in this section excludes any households who report expecting zero inflation over the next 12 months, or who do not answer the inflation expectation question, as Question 17.2 is difficult to interpret for these households. I discuss the appropriate counterfactual implicit in the question below. Including these people, 79% of respondents to Question 17.2 expect positive inflation, 7% expect zero inflation, 2% expect deflation, and 12% do not answer.

define the following indicator:

$$\frac{\widetilde{dc_t^h}}{d\mathbb{E}_t^h p_{t+1}} = \begin{cases} 1 & \text{if Q17.2='no' and } \mathbb{E}_t^h \pi_{t+1} > 0 \\ 0 & \text{if Q17.2='yes' and } \mathbb{E}_t^h \pi_{t+1} > 0 \\ 1 & \text{if Q17.2='yes' and } \mathbb{E}_t^h \pi_{t+1} < 0 \\ 0 & \text{if Q17.2='no' and } \mathbb{E}_t^h \pi_{t+1} < 0 \end{cases} \tag{80}$$

For the large majority who expect inflation, this is equal to 1 if $\frac{dc_t^h}{d\mathbb{E}_t^h p_{t+1}} \geq 0$, and equal to 0 if the reaction to expected price rises is strictly negative. The same is true of the minority who expect deflation, except that any household with $\frac{dc_t^h}{d\mathbb{E}_t^h p_{t+1}} = 0$ would respond 'no' to Question 17.2, and so is counted as if their response to expected price rises is strictly negative. The mislabeling is not a large issue, as less than 1% of respondents to Question 17.2 both expect deflation and answer 'no'. The results below are robust to removing the few households who expect deflation (see Table 3 column 2).

Table 3 shows how this is related to the information indicator and the subjective models (responses to Question 3). Column 1 shows the results from estimating a probit regression of $\frac{\widetilde{dc_t^h}}{d\mathbb{E}^{h}_{+1}}$ on the information indicator interacted with subjective models (Question 3), plus the standard household controls and time fixed effects used above. The coefficient on information is significantly negative for those with negative subjective models of inflation, despite the fact that we would expect $\frac{dc_t^h}{d\mathbb{E}_t^h p_{t+1}} \geq 0$ in a standard New Keynesian model. Being informed is therefore associated with a *lower* probability of responding positively to expected inflation for these households.

However, for those who believe inflation makes the economy stronger, being informed is associated with a significantly higher $\Pr(\frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}} \geq 0)$. For those who believe inflation makes no difference, the average value of $\Pr(\frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}} \geq 0)$ with and without information, which is also consistent with the interpretation of these variables as $\frac{\widetilde{dc_t^h}}{d\mathbb{E}^{h}_{+1}} = 1$ includes the case where $\frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}} = 0$.

This is consistent with individuals filtering information through their subjective models of the economy. If a household who believes inflation weakens the economy gets more information about future positive inflation, their subjective model implies that they should cut consumption, because bad times lie ahead. If instead a household believes inflation strengthens the economy, then they will react in the opposite way to the same inflation. The overall correlation of information and consumption response is negative because the majority of households believe inflation makes the economy weaker. This therefore supports the claim that the information indicator and answers to Question 3 reflect the information and subjective models used by households in making their

Table 3: Consumption response to inflation correlates with information, by subjective model

	(1) c response to Eπ	(2) c response to Eπ
information indicator=1	-0.213***	-0.224***
	(0.0611)	(0.0613)
end up stronger	0.0108	0.0392
	(0.0891)	(0.0906)
information indicator=1 × end up stronger	0.348*	0.313*
	(0.185)	(0.186)
make little difference	0.130**	0.157***
	(0.0594)	(0.0600)
information indicator=1 × make little difference	0.0240	-0.0149
	(0.126)	(0.128)
dont know	0.0958	0.0978
	(0.0833)	(0.0846)
information indicator=1 × dont know	-0.0158	-0.0342
	(0.186)	(0.187)
Expected Inflation	All	Exclude Deflation
Controls	All	All
Time FE	Yes	Yes
Observations	4940	4871

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of probit regressions of the $\widetilde{\frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}}}$ indicator on the information indicator, interacted with responses to Question 3. The omitted category is a household with information indicator=0 who holds the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

consumption decisions.

The analysis here assumes that when asked whether they will cut back consumption and save more, households are comparing their actions to a counterfactual in which there are no price rises over the next 12 months. An alternative possibility is that they are comparing with a consumption plan made in the past, in which case the relevant counterfactual is where expected inflation is unchanged from the level expected when the plan was made. I consider this in two ways, and find that the qualitative patterns in reported consumption responses to inflation are the same for households expecting inflation to increase or decrease relative to the previous year. It does not therefore appear that past inflation is the relevant counterfactual for most respondents.

First, column 2 of Table 3 re-runs the regression in column 1, excluding any respondent who reports expecting prices to fall over the next year. All results are qualitatively the same as over the full sample, showing that the few respondents expecting deflation are not driving the results.

Second, I split the sample by the sign of the respondent's expected change in inflation, computed as the sign of the difference between 12-month ahead inflation forecast and their perception of inflation over the previous 12 months. The results are in Table 4.[35] The sample sizes in each group are substantially smaller than over the full sample, so some significance is lost, but importantly the signs of the key coefficients remain the same. In each group, households who believe inflation makes the economy weaker are less likely to have $\frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}} \geq 0$ when they get inflation information. For households who believe inflation makes the economy stronger, this effect is reversed. The similarity of these patterns suggests that most respondents use 'no price change' as the counterfactual when answering Question 17.2, not 'no inflation change'. If the latter was used, we would expect to see changes of sign across the columns in Table 4, as a household expecting a fall in inflation would be reporting $-1 \times \frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}}$, while one expecting a rise in inflation would report $\frac{dc_t^h}{d\mathbb{E}_t^h \pi_{t+1}}$.

[35]For brevity I only include the results of the regression including interaction effects, but repeating column 1 of Table 3 with this split also yields no changes in key coefficient signs.

Table 4: Consumption response to inflation correlates with information, by subjective model and sign of perceived Eπ change.

	(1)	(2)	(3)
	E$\Delta\pi < 0$	E$\Delta\pi = 0$	E$\Delta\pi > 0$
c response to Eπ			
information	-0.140	-0.305***	-0.257**
indicator=1	(0.116)	(0.101)	(0.107)
end up stronger	0.0668	-0.178	0.195
	(0.164)	(0.151)	(0.165)
information	0.586	0.349	0.397
indicator=1 × end up stronger	(0.441)	(0.293)	(0.307)
make little	0.165	0.136	0.181
difference	(0.111)	(0.0957)	(0.112)
information	0.129	-0.300	0.113
indicator=1 × make little difference	(0.241)	(0.211)	(0.216)
dont know	0.156	0.0293	0.0264
	(0.176)	(0.128)	(0.167)
information	-0.141	0.469	0.117
indicator=1 × dont know	(0.354)	(0.359)	(0.325)
Controls	All	All	All
Time FE	Yes	Yes	Yes
Observations	1384	1876	1463

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of probit regressions of the $\frac{\widetilde{dc_t^h}}{d\mathrm{E}_t^h \pi_{t+1}}$ indicator on the information indicator, interacted with responses to Question 3, split by the sign of the respondent's inflation expectations. The omitted category in all cases is a household with information indicator=0 who holds the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

C.2 Time series patterns in subjective models of inflation

Bhandari et al. (2019) also study the time series of responses to Question 3, and conclude that households are more pessimistic about inflation when output growth is low. To explore this, I regress the proportion of households responding 'end up weaker' on realised annual CPI inflation and quarterly GDP growth. The results are in column 2 of Table 5. Consistent with Bhandari et al. (2019), the coefficient on GDP growth is significantly negative. However, the R^2 is only slightly higher than that of a regression on inflation only (column 1), so GDP growth does not account for much of the variation in survey answers. Indeed, GDP growth does not have any significant relationship with the proportion of households with a negative view of inflation outside of the four worst months of the Great Recession (column 3).

Table 5: Regressions of the proportion of households answering weaker to question 3 of the Inflation Attitudes Survey on aggregate variables.

	(1)	(2)	(3)
	Proportion weaker	Proportion weaker	Proportion weaker
Inflation	0.0568***	0.0517***	0.0501***
	(0.00489)	(0.00479)	(0.00469)
GDP growth		-0.0261***	-0.0110
		(0.00869)	(0.0180)
Constant	0.466***	0.487***	0.482***
	(0.0109)	(0.0123)	(0.0152)
Omitted quarters	None	None	2008Q2-2009Q1
R-squared	0.615	0.647	0.554
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of regressing the proportion of households answering Question 3 that inflation makes the economy weaker on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights.

Similar patterns in reverse are observed for the other answers. Tables 6-8 repeat the regressions of Table 5, replacing the dependent variable with the proportion of respondents choosing each of the other possible answers to Question 3. In all cases, inflation accounts for a large share of the variation in survey answers, and higher inflation is associated with significantly lower proportions giving each answer. Higher GDP growth is associated with higher proportions on these other answers, but that relationship is not significantly different from zero for any answer when excluding the worst of the Great Recession.

Table 9 repeats the regressions of Table 5, replacing each variable with its first difference. The key result of Table 5 is maintained: the proportion believing inflation weakens the economy rises as inflation rises.

To test if the distribution of beliefs about inflation shifts when the economy reaches the Zero Lower Bound, I estimate an ordered probit regression of subjective models of inflation in the zero lower bound period, and a variety of controls.[36] A response that inflation makes the economy stronger is coded as the highest value, and inflation makes the economy weaker is the lowest value (I exclude the 'don't know' answers). A

[36]The first column of Table 10 has no controls, the second includes the set of household-level covariates used throughout the paper, and the third adds inflation and GDP growth.

Table 6: Regressions of the proportion of households answering no difference to question 3 of the Inflation Attitudes Survey on aggregate variables.

	(1) Proportion no diff.	(2) Proportion no diff.	(3) Proportion no diff.
Inflation	-0.0292***	-0.0262***	-0.0257***
	(0.00303)	(0.00313)	(0.00314)
GDP growth		0.0150***	0.0106
		(0.00473)	(0.0107)
Constant	0.277***	0.264***	0.266***
	(0.00772)	(0.00883)	(0.0103)
Omitted quarters	None	None	2008Q2-2009Q1
R-squared	0.534	0.569	0.470
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of regressing the proportion of households answering Question 3 that inflation makes no difference to the economy on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights.

positive coefficient on the zero lower bound period would therefore imply a shift towards believing inflation makes the economy stronger, as we would expect if households follow a standard New Keynesian model. This is not what the results in Table 10 show: there is no significant shift towards a positive view of inflation in the ZLB period.

C.3 Cross-sectional patterns in information on inflation

The first three columns of Table 11 show the results of probit regressions of the information indicator on subjective models, controls, and period fixed-effects, for three subsamples. The first only uses questions about the information used to arrive at the respondent's *change* in expected inflation, and the second uses only questions about information used to form point forecasts. The third column excludes questions relating to forecast horizons longer than 12 months. The signs of the marginal effects are the same as in the main exercise in Table 1, though they are not significant in the case of the revisions questions, as the sample size is small.

The fourth and fifth columns of Table 11 repeat the regression for broader definitions of the information dummy than that used in Table 1. In the fourth column, the information indicator includes Questions 2e and 2g. Again, the results are robust, but I leave these questions out in the main analysis to ensure consistency in the number of questions per quarter, which is useful when considering the time series of information. In the

Table 7: Regressions of the proportion of households answering stronger to question 3 of the Inflation Attitudes Survey on aggregate variables.

	(1) Proportion stronger	(2) Proportion stronger	(3) Proportion stronger
Inflation	-0.0123***	-0.0116***	-0.0108***
	(0.00193)	(0.00215)	(0.00221)
GDP growth		0.00346	-0.00392
		(0.00363)	(0.00646)
Constant	0.104***	0.102***	0.104***
	(0.00431)	(0.00550)	(0.00638)
Omitted quarters	None	None	2008Q2-2009Q1
R-squared	0.388	0.395	0.311
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of regressing the proportion of households answering Question 3 that inflation makes the economy stronger on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights.

final column, I extend the criteria for setting the information indicator equal to 1 in Question 2d to account for the fact that some people may be unwilling to select the highest importance box for any information source. I therefore set the information indicator to 1 if in answer to Question 2d, the respondent selects 'very important' for direct inflation information (as before), or if they do not select 'very important' for any option, but do respond that four or fewer options were 'fairly important', and direct inflation information is among them. Again, the results are robust.

Table 8: Regressions of the proportion of households answering no idea to question 3 of the Inflation Attitudes Survey on aggregate variables.

	(1)	(2)	(3)
	Proportion no idea	Proportion no idea	Proportion no idea
Inflation	-0.0154***	-0.0139***	-0.0135***
	(0.00249)	(0.00262)	(0.00267)
GDP growth		0.00762*	0.00428
		(0.00423)	(0.00987)
Constant	0.153***	0.147***	0.148***
	(0.00687)	(0.00757)	(0.00884)
Omitted quarters	None	None	2008Q2-2009Q1
R-squared	0.355	0.376	0.282
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of regressing the proportion of households answering Question 3 that they have no idea how inflation affects the economy on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights.

Table 9: Regressions of the proportion of households answering weaker to question 3 of the Inflation Attitudes Survey on aggregate variables, first differences.

	(1)	(2)	(3)
	D.Proportion weaker	D.Proportion weaker	D.Proportion weaker
D.Inflation	0.0342***	0.0368***	0.0254**
	(0.00947)	(0.0101)	(0.0116)
D.GDP growth		0.0108	0.0153*
		(0.00777)	(0.00786)
Constant	0.0000325	0.000153	-0.000207
	(0.00445)	(0.00445)	(0.00454)
Omitted quarters	None	None	2008Q2-2009Q1
R-squared	0.179	0.196	0.109
Observations	67	67	63

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of regressing the quarterly change in the proportion of households answering Question 3 that inflation makes the economy weaker on quarterly changes in annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights.

Table 10: Ordered probit regressions of subjective models of inflation on whether the economy is at the zero lower bound on nominal interest rates.

	(1) Subjective model	(2) Subjective model	(3) Subjective model
Subjective model			
ZLB	-0.00801	-0.00785	-0.00513
	(0.00937)	(0.00962)	(0.00972)
Controls	None	Household	Household + macro
Observations	83526	83526	83526

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of an ordered probit regression of answers to Question 3 on an indicator for whether the UK economy was at the zero lower bound, defined as the period from 2009Q2 to the end of 2019 (end of the sample). The ordering is: "stronger", "no difference", "weaker". Those answering "no idea" are omitted. All regressions are weighted using the survey weights provided in the IAS.

Table 11: Information correlates with subjective models, split by information question type

	(1) Revision	(2) Point forecast	(3) Short horizon	(4) Extra Qs	(5) Q2d wider
end up stronger	0.0575	-0.0335	-0.0123	0.00114	-0.00126
	(0.0380)	(0.0218)	(0.0206)	(0.0196)	(0.0196)
make little difference	-0.0191	-0.0331**	-0.0392***	-0.0310**	-0.0312**
	(0.0233)	(0.0155)	(0.0141)	(0.0132)	(0.0131)
dont know	-0.0408	-0.0715***	-0.0622***	-0.0663***	-0.0472***
	(0.0297)	(0.0206)	(0.0192)	(0.0174)	(0.0180)
Controls	All	All	All	All	All
Time FE	Yes	Yes	Yes	Yes	Yes
Observations	2364	5906	6848	8306	8270

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the average marginal effects from estimating probit regressions of the information indicators constructed from subsets of the questions listed in Appendix B on the responses to Question 3. The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

D Dynamic model: derivations and proofs

D.1 Proof of Proposition 3

The proof is an adaptation of the derivation of expression (34) in Maćkowiak and Wiederholt (2015).

First, substitute the budget constraint (17) into the utility function (16) to obtain:

$$\tilde{\mathbb{E}}_0^h U_0^h = \tilde{\mathbb{E}}_0^h \beta^t \frac{1}{1-\frac{1}{\sigma}} \left(\frac{R_{t-1}}{\Pi_t} B_{t-1}^h + W_t - B_t^h \right)^{1-\frac{1}{\sigma}} \tag{81}$$

Write this in log-deviations from steady state:

$$\tilde{\mathbb{E}}_0^h U_0^h = \tilde{\mathbb{E}}_0^h \beta^t \frac{1}{1-\frac{1}{\sigma}} \left(\bar{R}\bar{B}^h \exp(r_{t-1} - \pi_t + b_{t-1}^h) + \bar{W}\exp(w_t) - \bar{B}^h \exp(b_t^h) \right)^{1-\frac{1}{\sigma}} \tag{82}$$

where $\bar{X}$ denotes the steady state value of the corresponding variable X_t, and $x_t \equiv \log(X_t/\bar{X})$ is the corresponding log-deviation.

We then take a quadratic approximation of this with respect to each variable in log-deviation, about the steady state. For this, define $z_t = (r_{t-1}, \pi_t, w_t)'$ as the vector of exogenous variables taken as given by the household in period t. The past asset choice b_{t-1}^h is also taken as given in period t, and b_t^h is the only choice variable. After the quadratic approximation, expected discounted utility is given by:

$$\begin{aligned} \tilde{\mathbb{E}}_0^h U_0^h \approx \tilde{\mathbb{E}}_0^h \hat{U}_0^h = \bar{U}^h + \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \Bigg[& h_b b_t^h + h_z z_t + \frac{1}{2} H_{bb,-1} b_t^h b_{t-1}^h + \frac{1}{2} H_{bb,0} (b_t^h)^2 + \frac{1}{2} H_{bb,1} b_{t+1}^h b_t^h \\ & + \frac{1}{2} b_t^h H_{bz,0} z_t + \frac{1}{2} b_t^h H_{bz,1} z_{t+1} + \frac{1}{2} z_t' H_{zz,0} z_t + \frac{1}{2} z_t' H_{zb,-1} b_{t-1}^h + \frac{1}{2} z_t' H_{zb,0} b_t^h \Bigg] \\ & + \beta^{-1} \left(h_{-1} b_{-1}^h + \frac{1}{2} H_{-1} (b_{-1}^h)^2 + \frac{1}{2} H_{bb,1} b_{-1}^h b_0^h + \frac{1}{2} b_{-1}^h H_{bz,1} z_0 \right) \end{aligned} \tag{83}$$

where $\beta^t h_b$ denotes the first derivative of U_0^h with respect to b_t^h, evaluated at the steady state. Similarly, h_z denotes the vector of first derivatives of U_0^h with respect to z_t, evaluated at steady state. The matrices $\beta^t H_{xy,\tau}$ denote the second derivatives of U_0^h with respect to x_t and $y_{t+\tau}$, for $x_t, y_t \in \{b_t^h, z_t\}$, evaluated at steady state. Finally, $\beta^{-1} h_{-1}$ and $\beta^{-1} H_{-1}$ are the first and second derivatives of U_0^h with respect to initial wealth b_{-1}^h, evaluated at steady state.

As in Maćkowiak and Wiederholt (2015), note that there are no cross-products of b_t and z_{t-1}, because from equation 82 the first derivative of U_0^h with respect to b_t^h does not depend on any elements of z_{t-1}. Similarly, there are no terms in the interaction of z_t and z_{t-1} or z_{t+1}.

We now simplify this, using several properties of the coefficient vectors and matrices. First, we have that $z_t' H_{zb,0} b_t^h = b_t^h H_{bz,0} z_t$. Second:

$$\begin{aligned}\tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \frac{1}{2} H_{bb,-1} b_t^h b_{t-1}^h &= \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} H_{bb,1} b_t^h b_{t-1}^h \\ &= \frac{1}{2} \beta^{-1} H_{bb,1} b_{-1}^h b_0^h + \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} H_{bb,1} b_t^h b_{t+1}^h\end{aligned} \tag{84}$$

Similarly:

$$\begin{aligned}\tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \frac{1}{2} z_t' H_{zb,-1} b_{t-1}^h &= \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} b_{t-1}^h H_{bz,1} z_t \\ &= \frac{1}{2} \beta^{-1} b_{-1}^h H_{zb,1} z_0 + \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \frac{1}{2} b_t^h H_{zb,1} z_{t+1}\end{aligned} \tag{85}$$

Using these, and the fact that $h_b = 0$, the log-quadratic approximation to utility becomes:

$$\begin{aligned}\tilde{\mathbb{E}}_0^h \hat{U}_0^h = \bar{U}^h + \tilde{\mathbb{E}}_0^h \sum_{t=0}^{\infty} \beta^t \Bigg[& h_z z_t + \frac{1}{2} H_{bb,0} (b_t^h)^2 + H_{bb,1} b_{t+1}^h b_t^h \\ & + b_t^h H_{bz,0} z_t + b_t^h H_{bz,1} z_{t+1} + \frac{1}{2} z_t' H_{zz,0} z_t \Bigg] \\ & + \beta^{-1} \left(h_{-1} b_{-1}^h + \frac{1}{2} H_{-1} (b_{-1}^h)^2 + H_{bb,1} b_{-1}^h b_0^h + b_{-1}^h H_{bz,1} z_0 \right)\end{aligned} \tag{86}$$

Next, we find b_t^{h*}, the optimal asset holdings chosen each period by a fully-informed household. This satisfies the first order condition:

$$\tilde{\mathbb{E}}_0^{h*} \left[H_{bb,0} b_t^{h*} + H_{bb,1} b_{t+1}^{h*} + \beta^{-1} H_{bb,1} b_{t-1}^{h*} \right] = -\tilde{\mathbb{E}}_0^{h*} \left[H_{bz,0} z_t + H_{bz,1} z_{t+1} \right] \tag{87}$$

Define the expected utility of a fully-informed household, $\tilde{\mathbb{E}}_0^{h*} \hat{U}_0^{h*}$, as the expected discounted utility if the household chooses this optimal saving behaviour. The expected utility loss from deviating from this rule is:

$$\begin{aligned}\tilde{\mathbb{E}}_0^h(\hat{U}_0^{h*}-\hat{U}_0^h)=\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}H_{bb,0}(b_t^{h*})^2+H_{bb,1}b_{t+1}^{h*}b_t^{h*}-\frac{1}{2}H_{bb,0}(b_t^h)^2-H_{bb,1}b_{t+1}^hb_t^h\right]\\+\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t(b_t^{h*}-b_t^h)(H_{bz,0}z_t+H_{bz,1}z_{t+1})+\tilde{\mathbb{E}}_0^h\beta^{-1}\left(H_{bb,1}b_{-1}^hb_0^{h*}-H_{bb,1}b_{-1}^hb_0^h\right)\end{aligned}\tag{88}$$

where I have used that $b_{-1}^{h*}=b_{-1}^h$.

Substituting in equation 87 we have:

$$\begin{aligned}\tilde{\mathbb{E}}_0^h(\hat{U}_0^{h*}-\hat{U}_0^h)=\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}H_{bb,0}(b_t^{h*})^2+H_{bb,1}b_{t+1}^{h*}b_t^{h*}-\frac{1}{2}H_{bb,0}(b_t^h)^2-H_{bb,1}b_{t+1}^hb_t^h\right]\\-\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t(b_t^{h*}-b_t^h)(H_{bb,0}b_t^{h*}+H_{bb,1}b_{t+1}^{h*}+\beta^{-1}H_{bb,1}b_{t-1}^{h*})+\tilde{\mathbb{E}}_0^h\beta^{-1}\left(H_{bb,1}b_{-1}^hb_0^{h*}-H_{bb,1}b_{-1}^hb_0^h\right)\end{aligned}\tag{89}$$

Collecting terms and rearranging:

$$\begin{aligned}\tilde{\mathbb{E}}_0^h(\hat{U}_0^{h*}-\hat{U}_0^h)=\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t\Bigg[-\frac{1}{2}H_{bb,0}(b_t^{h*})^2-\frac{1}{2}H_{bb,0}(b_t^h)^2+H_{bb,0}b_t^hb_t^{h*}+H_{bb,1}b_{t+1}^{h*}b_t^h\\-H_{bb,1}b_{t+1}^hb_t^h\Bigg]+\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t\beta^{-1}H_{bb,1}b_{t-1}^{h*}(b_t^h-b_t^{h*})+\tilde{\mathbb{E}}_0^h\beta^{-1}H_{bb,1}b_{-1}^h(b_0^{h*}-b_0^h)\end{aligned}\tag{90}$$

The second summation can be written as:

$$\begin{aligned}\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t\beta^{-1}H_{bb,1}b_{t-1}^{h*}(b_t^h-b_t^{h*})=\beta^{-1}H_{bb,1}b_{-1}^hb_0^h-\tilde{\mathbb{E}}_0^h\beta^{-1}H_{bb,1}b_{-1}^hb_0^{h*}\\+\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^tH_{bb,1}b_t^{h*}(b_{t+1}^h-b_{t+1}^{h*})\end{aligned}\tag{91}$$

where I have again used $b_{-1}^{h*}=b_{-1}^h$.

Substituting this into the expected utility loss and collecting terms:

$$\tilde{\mathbb{E}}_0^h(\hat{U}_0^{h*}-\hat{U}_0^h)=\tilde{\mathbb{E}}_0^h\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2}H_{bb,0}(b_t^h-b_t^{h*})^2-H_{bb,1}(b_t^h-b_t^{h*})(b_{t+1}^h-b_{t+1}^{h*})\right]\tag{92}$$

Differentiating the instantaneous utility function $U_{p,t}$ twice gives:

$$H_{bb,0} = \frac{\partial^2 U^h_{p,t}}{\partial (b^h_t)^2}\bigg|_{ss} = -\frac{1}{\sigma}(\bar{C}^h)^{-\frac{1}{\sigma}-1}(\bar{B}^h)^2(1+\beta^{-1}), \quad H_{bb,1} = \frac{\partial^2 U^h_{p,t}}{\partial b^h_t \partial b^h_{t+1}}\bigg|_{ss} = \frac{1}{\sigma}(\bar{C}^h)^{-\frac{1}{\sigma}-1}(\bar{B}^h)^2 \tag{93}$$

Therefore:

$$\tilde{\mathbb{E}}^h_0(\hat{U}^{h*}_0 - \hat{U}^h_0) = -\frac{1}{\sigma}(\bar{C}^h)^{-\frac{1}{\sigma}-1}(\bar{B}^h)^2 \tilde{\mathbb{E}}^h_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1+\beta^{-1}}{2}(b^h_t - b^{h*}_t)^2 + (b^h_t - b^{h*}_t)(b^h_{t+1} - b^{h*}_{t+1}) \right] \tag{94}$$

Next, we transform this into an equation involving consumption choices, rather than asset choices. Log-linearising the budget constraint (17) gives:

$$\bar{C}^h c^h_t = \beta^{-1}\bar{B}^h(r_{t-1} - \pi_t + b^h_{t-1}) - \bar{B}^h b^h_t + \bar{W} w_t \tag{95}$$

Subtracting the equivalent for the fully-informed household:

$$\bar{C}^h(c^h_t - c^{h*}_t) = \beta^{-1}\bar{B}^h(b^h_{t-1} - b^{h*}_{t-1}) - \bar{B}^h(b^h_t - b^{h*}_t) \tag{96}$$

We substitute this into equation 94 and rearrange. To see how the rearrangement works, define $\Delta^h_t = \bar{B}^h/\bar{C}^h \cdot (b^h_t - b^{h*}_t)$, so that equation 96 becomes:

$$\Delta^h_t = \beta^{-1}\Delta^h_{t-1} - (c^h_t - c^{h*}_t) \tag{97}$$

Substituting out for $(b^h_t - b^{h*}_t)$ and $(b^h_t - b^{h*}_t)$ in equation 94 using the definition of Δ^h_t gives:

$$\tilde{\mathbb{E}}^h_0(\hat{U}^{h*}_0 - \hat{U}^h_0) = -\frac{1}{\sigma}(\bar{C}^h)^{1-\frac{1}{\sigma}} \tilde{\mathbb{E}}^h_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1+\beta^{-1}}{2}(\Delta^h_t)^2 + \Delta^h_t \Delta^h_{t+1} \right] \tag{98}$$

The terms inside the square brackets can be rearranged to:

$$\begin{aligned}
-\frac{1}{2}(\Delta^h_t)^2 - \frac{1}{2\beta}(\Delta^h_t)^2 + \Delta^h_t\Delta^h_{t+1} &= -\frac{1}{2}\frac{1}{\beta^2}(\Delta^h_{t-1})^2 + \frac{1}{\beta}\Delta^h_{t-1}(c^h_t - c^{h*}_t) - \frac{1}{2}(c^h_t - c^{h*}_t)^2 \\
&\quad - \frac{1}{2\beta}(\Delta^h_t)^2 + \Delta^h_t\Delta^h_{t+1} \\
= -\frac{1}{2}\frac{1}{\beta^2}(\Delta^h_{t-1})^2 + \frac{1}{\beta}\Delta^h_{t-1}(c^h_t - c^{h*}_t) - \frac{1}{2}(c^h_t - c^{h*}_t)^2 &- \frac{1}{2\beta}(\Delta^h_t)^2 + \Delta^h_t(\beta^{-1}\Delta^h_t - (c^h_{t+1} - c^{h*}_{t+1})) \\
= -\frac{1}{2}(c^h_t - c^{h*}_t)^2 + \frac{1}{2\beta}\big((\Delta^h_t)^2 - \frac{1}{\beta}(\Delta^h_{t-1})^2\big) &- \big(\Delta^h_t(c^h_{t+1} - c^{h*}_{t+1}) - \frac{1}{\beta}\Delta^h_{t-1}(c^h_t - c^{h*}_t)\big)
\end{aligned} \tag{99}$$

where the first and second equalities involve substituting out using equation 97.

Substituting this into equation 98, canceling terms when they appear from multiple periods, and noting that $\Delta^h_{-1} = 0$, we obtain:

$$\tilde{\mathbb{E}}^h_0(\hat{U}^{h*}_0 - \hat{U}^h_0) = -\frac{1}{2\sigma}(\bar{C}^h)^{1-\frac{1}{\sigma}}\tilde{\mathbb{E}}^h_0 \sum_{t=0}^{\infty} \beta^t (c^h_t - c^{h*}_t)^2 \tag{100}$$

D.2 Forecasts using the subjective model

The subjective model represented by equations 21 - 23 can be written in VAR form as:

$$Y_t = A^h Y_{t-1} + B^h U_t \tag{101}$$

where:

$$\begin{aligned}
Y_t &= (\pi_t, w_t, i_t)' \\
A &= \begin{bmatrix} \rho^h_\pi & 0 & 0 \\ (\alpha^h + \lambda^h \phi^h)\rho^h_\pi & \rho^h_w & 0 \\ \phi\rho_\pi & 0 & 0 \end{bmatrix} \\
U_t &= (u_{\pi t}, u_{wt}, u_{it})' \\
B &= \begin{bmatrix} 1 & 0 & 0 \\ \alpha^h + \lambda^h \phi^h & 1 & \lambda \\ \phi^h & 0 & 1 \end{bmatrix}
\end{aligned} \tag{102}$$

To form a forecast of future variables, the fully-informed agent uses:

$$\tilde{\mathbb{E}}^{h*}_t Y_{t+s} = (A^h)^s Y_t \tag{103}$$

That is, their forecasts are optimal given their subjective model.

To find $(A^h)^s$, first find diagonal matrix D^h and matrix P^h such that:

$$A^h = P^h D^h (P^h)^{-1} \tag{104}$$

This is satisfied with:

$$
\begin{aligned}
P &= \begin{bmatrix} 0 & (\phi^h)^{-1} & 0 \\ 0 & \dfrac{(\alpha^h + \lambda^h \phi^h)\rho_\pi^h}{\phi^h(\rho_\pi^h - \rho_w^h)} & 1 \\ 1 & 1 & 0 \end{bmatrix} \\
D^h &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_\pi^h & 0 \\ 0 & 0 & \rho_w^h \end{bmatrix}
\end{aligned}
\tag{105}
$$

We then have that:

$$
\begin{aligned}
(A^h)^s = P^h (D^h)^s (P^h)^{-1} &= P^h \cdot \begin{bmatrix} 0^s & 0 & 0 \\ 0 & (\rho_\pi^h)^s & 0 \\ 0 & 0 & (\rho_w^h)^s \end{bmatrix} \cdot (P^h)^{-1} \\
&= \begin{bmatrix} (\rho_\pi^h)^s & 0 & 0 \\ \dfrac{(\alpha^h + \lambda^h \phi^h)\rho_\pi^h}{\rho_\pi^h - \rho_w^h}((\rho_\pi^h)^s - (\rho_w^h)^s) & (\rho_w^h)^s & 0 \\ \phi^h (\rho_\pi^h)^s & 0 & 0 \end{bmatrix}
\end{aligned}
\tag{106}
$$

This implies equations 24 - 26.

D.3 Proposition 4

Using the signal structure (equation 32), the utility cost of period-t signal s_t^h is given by:

$$
\begin{aligned}
I(\pi^t; s_t^h | \mathcal{I}_{t-1}^h) \equiv H(\pi_t | s^{t-1,h}) - H(\pi_t | s^{t,h}) &= \frac{1}{2}\log_2\left(\frac{Var(\pi_t|\mathcal{I}_{t-1}^h)}{Var(\pi_t|\mathcal{I}_t^h)}\right) \\
&= \frac{1}{2}\log_2\left(\frac{1}{1-K^h}\right)
\end{aligned}
\tag{107}
$$

where the final equality uses standard properties of the Kalman filter.

Assumption 2 further implies that $\tilde{\mathbb{E}}_0^h(\pi_t - \mathbb{E}_t\pi_t)^2$ is constant over time. From the properties of the Kalman filter:

$$
\tilde{\mathbb{E}}_0^h(\pi_t - \mathbb{E}_t\pi_t)^2 = \frac{(1-K^h)\sigma_\pi^2}{1-(\rho_\pi^h)^2(1-K^h)} \tag{108}
$$

Using these results, and evaluating the resulting geometric series in the utility losses and costs of information, the household information choice problem reduces to:

$$\min_{K} \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right)^2 \frac{(1-K^h)\sigma_\pi^2}{1-(\rho_\pi^h)^2(1-K^h)} + \frac{\psi}{2}\log_2\left(\frac{1}{1-K^h}\right) \tag{109}$$

subject to $K^h \in [0,1]$.

The first order condition for an interior solution is:

$$\frac{1-K^h}{(1-(\rho_\pi^h)^2(1-K^h))^2} = \frac{\psi}{\Gamma^h} \tag{110}$$

where Γ^h is as defined in Proposition 4:

$$\Gamma^h = \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2)\cdot\left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right)^2 \tag{111}$$

Since $\psi/\Gamma^h > 0$, the K^h implied by this first order condition is always strictly less than 1. However, it remains to find the region where the constraint $K^h \geq 0$ binds. First, note that the interior solution implies $K^h = 0$ when:

$$\Gamma^h = \psi(1-(\rho_\pi^h)^2)^2 \tag{112}$$

Differentiating the left hand side of equation 110 with respect to K^h gives:

$$\frac{\partial}{\partial K^h}\left(\frac{1-K^h}{(1-(\rho_\pi^h)^2(1-K^h))^2}\right) = -\frac{1+(\rho_\pi^h)^2(1-K^h)}{(1-(\rho_\pi^h)^2(1-K^h))^3} \tag{113}$$

The left hand side of equation 110 is therefore strictly decreasing in K^h. The constraint therefore binds, and optimal $K^h = 0$, whenever the right hand side is sufficiently large, that is when:

$$\Gamma^h < \psi(1-(\rho_\pi^h)^2)^2 \tag{114}$$

D.4 Microfounding subjective model updating

This section provides one way to microfound the process described in Section 4.4, in which households update their subjective models towards the view that inflation erodes real wages when $\tilde{\mathbb{E}}_t^h \pi_t$ is high. The household faces Knightian uncertainty about the α^h parameter in their subjective model. After observing the realisation of s_t^h, the household updates their subjective model to reflect this: following the literature on ambiguity aversion they make decisions using worst-case beliefs (Hansen and Sargent, 2008).

This leads to distorted subjective models in which α^h falls as perceived inflation rises. Intuitively, when perceived inflation is high, the worst case is that high inflation is associated with low real incomes. However, when perceived inflation is lower, the reverse is true. The worst case is then that inflation supports real wages, and so the ambiguity averse household distorts their subjective model in that direction, with a positive $\hat{\alpha}_t^h$.

Formally, the household selects beliefs and actions as if they are playing a game with an 'evil agent', who distorts α^h to minimise expected utility, while the household simultaneously chooses c_t^h to maximise expected utility. The maximisation problem is solved by the consumption function in equation 28 with the updated α^h. To solve the evil agent problem, we then need to find the indirect expected utility when households follow this consumption function.

To find this indirect utility, begin with the expected utility of a household who is fully-informed about inflation each period. To simplify the problem, here I assume that $\sigma \to 1$, so the instantaneous utility from consumption C_t^h is $\log(C_t^h)$. As this is already log-linear, a log-quadratic approximation to this instantaneous utility simply yields $\log(\bar{C}^h)+c_t^h$. The log-quadratic approximation of expected discounted utility, substituting in the consumption function of the informed household (equation 27), is therefore:

$$\tilde{\mathbb{E}}_0^{h*}\hat{U}_0^{h*} = \tilde{\mathbb{E}}_0^{h*}\sum_{t=0}^{\infty}\beta^t\bigg(\frac{1-\beta}{1-\beta\rho_w^h}w_t - \sigma\beta r_t + \frac{\beta\rho_\pi^h[(1-\beta)(\alpha^h+\lambda^h\phi^h) - \sigma(\phi^h\beta-1)(1-\beta\rho_w^h)]}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\pi_t\bigg) \tag{115}$$

Substituting out for expected future inflation, interest rates, and real wages using the subjective model (equations 24 - 26) gives indirect utility as a function of current observables and subjective model parameters:

$$\tilde{\mathbb{E}}_0^{h*}\hat{U}_0^{h*} = \frac{1-\beta}{(1-\beta\rho_w^h)^2}w_0 - \sigma\beta r_0 + \frac{1}{1-\beta\rho_\pi^h}\left(\frac{\beta\rho_\pi^h(\alpha^h+\lambda^h\phi^h)}{1-\beta\rho_w^h} - \sigma\beta^2\phi^h\rho_\pi^h + \frac{\partial c_t^h}{\partial\tilde{\mathbb{E}}_t^h\pi_t}\right)\pi_0 \tag{116}$$

Finally, use the expression for the expected utility loss from limited information (equation 29) to find the expected indirect utility of the potentially uninformed household:

$$\tilde{\mathbb{E}}_0^{h}\hat{U}_0^{h} = \frac{1-\beta}{(1-\beta\rho_w^h)^2}w_0 - \sigma\beta r_0 + \frac{1}{1-\beta\rho_\pi^h}\left(\frac{\beta\rho_\pi^h(\alpha^h+\lambda^h\phi^h)}{1-\beta\rho_w^h} - \sigma\beta^2\phi^h\rho_\pi^h + \frac{\partial c_t^h}{\partial\tilde{\mathbb{E}}_t^h\pi_t}\right)\tilde{\mathbb{E}}_0^h\pi_0 - \frac{\log(\bar{C}^h)}{2(1-\beta)}\left(\frac{\partial c_t^h}{\partial\tilde{\mathbb{E}}_t^h\pi_t}\right)^2\frac{(1-K^h)\sigma_\pi^2}{1-(\rho_\pi^h)^2(1-K^h)} \tag{117}$$

where I have used that the expected variance of inflation perception gaps is constant at:

$$\tilde{\mathbb{E}}_0^h(\pi_t - \mathbb{E}_t\pi_t)^2 = \frac{(1-K^h)\sigma_\pi^2}{1-(\rho_\pi^h)^2(1-K^h)} \tag{118}$$

Differentiating the expected indirect utility with respect to α^h gives:

$$\frac{\partial \tilde{\mathbb{E}}_0^h \hat{U}_0^h}{\partial \alpha^h} = \frac{\beta(2-\beta)\rho_\pi^h}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\tilde{\mathbb{E}}_0^h\pi_0 - \frac{\beta\rho_\pi^h \log(\bar{C}^h)(1-K^h)\sigma_\pi^2}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)(1-(\rho_\pi^h)^2(1-K^h))} \cdot \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h\pi_t} \tag{119}$$

Expected indirect utility is therefore increasing in α^h if:

$$\tilde{\mathbb{E}}_0^h\pi_0 > \frac{\log(\bar{C}^h)(1-K^h)\sigma_\pi^2}{(2-\beta)(1-(\rho_\pi^h)^2(1-K^h))} \cdot \frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h\pi_t} \tag{120}$$

That is, expected indirect utility is increasing in α^h if and only if perceived inflation is sufficiently high, and otherwise it is decreasing.[37] Whether the worst-case distortions to α^h are restricted to a set $\hat{\alpha}_t^h \in [\alpha_0^h - \alpha^*, \alpha_0^h + \alpha^*]$, or the evil agent can pay a convex cost to distorting $\hat{\alpha}_t^h$ away from α_0^h, the solution will therefore have the distorted $\hat{\alpha}_t^h$ falling as $\tilde{\mathbb{E}}_t^h\pi_t$ rises, as in equation 38.

D.5 Allowing repeat information decisions within the period

Suppose that after forming $\tilde{\mathbb{E}}_t^h\pi_t$ and updating their subjective model to $\hat{\alpha}_t^h$ using equation 38, households are able to return to the information processing step. They can process more information about inflation in period t if they choose to. I will refer to the original information processing described in Section 4.3 as 'stage 1' information, and any subsequent information as the 'stage 2' information choice.

In keeping with the anticipated utility assumption made throughout this model, assume that the household makes their stage 2 information decision assuming that their current $\hat{\alpha}_t^h$ will be maintained in all future periods.

First, we find the desired Kalman gain using the same steps as in Section 4.3. Denoting the desired Kalman gain as K_t^{h*}, the solution is:[38]

[37] Note that since K^h is decided before any distortion to α^h, it is also not a function of expected inflation. Everything on the right hand side of condition 120 is a function of underlying parameters and the parameters of the subjective model only.

[38] Note that this is an approximation, as the solution to this problem assumes the household uses the steady state Kalman filter. If they have already processed more information in period t than their desired amount, they would not strictly be using the steady state Kalman filter in the next periods.

$$\begin{cases} K_t^{h*} = 0 & \text{if } \Gamma_t^{h*} < \psi(1-(\rho_\pi^h)^2)^2 \\ \dfrac{1-K_t^{h*}}{(1-(\rho_\pi^h)^2(1-K_t^{h*}))^2} = \dfrac{\psi}{\Gamma_t^{h*}} & \text{if } \Gamma_t^{h*} \geq \psi(1-(\rho_\pi^h)^2)^2 \end{cases} \tag{121}$$

where:

$$\Gamma_t^{h*} = \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h} \right)^2 \tag{122}$$

If this desired Kalman gain is below K^h, the gain after the stage 1 information choice, then the Kalman gain used by the household in period t remains at K^h. That is, I assume that they do not forget information they have just processed. However, if their desired Kalman gain implies more precise signals than they have currently processed, they will now process more information in stage 2 to reach the desired K_t^{h*}. The final information choices after stage 2 information processing are therefore given by:

$$\begin{cases} \hat{K}_t^h = 0 & \text{if } \Gamma^h < \psi(1-(\rho_\pi^h)^2)^2 \ \&\ \Gamma_t^{h*} < \psi(1-(\rho_\pi^h)^2)^2 \\ \dfrac{1-\hat{K}_t^h}{(1-(\rho_\pi^h)^2(1-\hat{K}_t^h))^2} = \dfrac{\psi}{\Gamma^h} & \text{if } \Gamma^h \geq \psi(1-(\rho_\pi^h)^2)^2 \ \&\ \Gamma_t^{h*} \leq \Gamma^h \\ \dfrac{1-\hat{K}_t^h}{(1-(\rho_\pi^h)^2(1-\hat{K}_t^h))^2} = \dfrac{\psi}{\Gamma_t^{h*}} & \text{if } \Gamma_t^{h*} \geq \psi(1-(\rho_\pi^h)^2)^2 \ \&\ \Gamma_t^{h*} > \Gamma^h \end{cases} \tag{123}$$

Within the first two groups, perceived inflation is unrelated to information choices, as the relevant information choices are those that were made before perceived inflation was first realised. However, in the final group, perceived inflation and information choices are related, because the signals received in stage 1 information processing inform the perceived inflation, which then affects the stage 2 information choices.

Differentiating the information first order condition for that group with respect to perceived inflation gives:

$$\frac{d\hat{K}_t^h}{d\tilde{\mathbb{E}}_t^h \pi_t} = \frac{2\psi(1-(\rho_\pi^h)^2(1-\hat{K}_t^h))^3}{\Gamma_t^{h*}(1+(\rho_\pi^h)^2(1-\hat{K}_t^h))} \cdot \left(\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h} \right)^{-1} \cdot \frac{d}{d\tilde{\mathbb{E}}_t^h \pi_t}\left(\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h} \right) \tag{124}$$

To find how the consumption elasticity to perceived inflation changes with perceived inflation, recall from Equations 39 and 50 that:

$$\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h} = \left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\alpha_0^h} - \Omega^h \tilde{\mathbb{E}}_t^h(\pi_t|\text{stage 1}) \tag{125}$$

Therefore:

$$\frac{d}{d\tilde{\mathbb{E}}_t^h \pi_t}\left(\left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h}\right) = -\Omega^h \frac{d\tilde{\mathbb{E}}_t^h(\pi_t|\text{stage 1})}{d\tilde{\mathbb{E}}_t^h \pi_t} \tag{126}$$

where $d\tilde{\mathbb{E}}_t^h(\pi_t|\text{stage 1})/d\tilde{\mathbb{E}}_t^h \pi_t > 0$ because both are positively related to the same underlying inflation rate, and the noise in the initial signal. The consumption elasticity to perceived inflation therefore falls with perceived inflation. Plugging this into Equation 124, among this group who process more information after the update to their subjective models, we therefore have that:

$$\frac{d\hat{K}_t^h}{d\tilde{\mathbb{E}}_t^h \pi_t} > 0 \iff \left.\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right|_{\hat{\alpha}_t^h} < 0 \tag{127}$$

Final information processing is increasing in perceived inflation, but only among those with negative subjective models of the effects of inflation. Intuitively, for households with negative models, higher perceived inflation causes them to update their models to even lower values of $\hat{\alpha}_t^h$, so they react even more negatively to inflation, and so information about inflation becomes more valuable to them. For households starting with positive models of inflation, higher perceived inflation implies less positive responses to inflation, so information becomes less valuable, and they do not return to process any more information. Those households only obtain more information in this second stage if their initial signal implies a low perceived inflation, and within this group lower perceived inflation is associated with more information.

To test this, I regress perceived and expected inflation on the information indicator described in Section 3.1. For each dependent variable, I first run the regression for the households who report negative subjective models of inflation in response to Question 3, corresponding to those with $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t < 0$. I then repeat the regression for those reporting non-negative subjective models.[39]

The results are in Table 12. Within the group with negative subjective models of inflation, both perceived and expected inflation are significantly higher among those obtaining direct information about inflation. This relationship turns negative among those with other subjective models, though this is not significant. These results are therefore in line with the model presented here.

[39] As the information indicator is not observed every quarter there are too few observations to draw conclusions from regressions on each non-negative subjective model option individually.

Table 12: Information is associated with higher perceived and expected information among those with negative subjective models.

	(1) Perceived	(2) Perceived	(3) Expected	(4) Expected
Information	0.226**	-0.122	0.311***	-0.0109
	(0.102)	(0.138)	(0.0990)	(0.119)
Subjective Model	Negative	Non-negative	Negative	Non-negative
Controls	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
R-squared	0.111	0.127	0.111	0.115
Observations	5114	2787	5298	2923

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The table reports the results of regressing perceived and expected inflation on the information indicator, split by responses to Question 3. The first and third columns are the results using those who answer that inflation would make the economy weaker, and the second and fourth columns use all other respondents. All regressions are weighted using the survey weights provided in the IAS.

D.6 Consumption function with time-varying long-run inflation

Changing the subjective model of inflation does not change anything about the model before the initial consumption function of a fully informed household (equation 20).[40]

However, the change in subjective model to include long-run inflation $\bar{\pi}_t$ does affect how we evaluate the expectation terms. Specifically, the subjective model in VAR(1) form is now:

$$Y_t = A^h Y_{t-1} + B^h U_t \tag{128}$$

where:

$$\begin{aligned}
Y_t &= (\pi_t, w_t, i_t, \bar{\pi}_t)' \\
A &= \begin{bmatrix} \rho_\pi^h & 0 & 0 & 1-\rho_\pi^h \\ (\alpha^h + \lambda^h\phi^h)\rho_\pi^h & \rho_w^h & 0 & (\alpha^h + \lambda^h\phi^h)(1-\rho_\pi^h) \\ \phi\rho_\pi & 0 & 0 & \phi^h(1-\rho_\pi^h) \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
U_t &= (u_{\pi t}, u_{wt}, u_{it}, v_t)' \\
B &= \begin{bmatrix} 1 & 0 & 0 & 1-\rho_\pi^h \\ \alpha^h + \lambda^h\phi^h & 1 & \lambda & (\alpha^h + \lambda^h\phi^h)(1-\rho_\pi^h) \\ \phi^h & 0 & 1 & \phi^h(1-\rho_\pi^h) \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{129}$$

[40]Note we assume this fully-informed household observes $\bar{\pi}_t$ as well as π_t.

This is the same for the case where $\bar{\pi}_t$ is a random walk and where it is assumed to be constant. In the latter case, simply set $\sigma_v^2 = 0$.

To form a forecast of future variables, the fully-informed agent uses:

$$\tilde{\mathbb{E}}_t^{h*} Y_{t+s} = (A^h)^s Y_t \tag{130}$$

To find $(A^h)^s$, first find diagonal matrix D^h and matrix P^h such that:

$$A^h = P^h D^h (P^h)^{-1} \tag{131}$$

This is satisfied with:

$$\begin{aligned} P &= \begin{bmatrix} 1 & 0 & (\phi^h)^{-1} & 0 \\ \dfrac{\alpha^h + \lambda^h \phi^h}{1-\rho_w^h} & 0 & \dfrac{(\alpha^h + \lambda^h \phi^h)\rho_\pi^h}{\phi^h(\rho_\pi^h - \rho_w^h)} & 1 \\ \phi^h & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ D^h &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\pi^h & 0 \\ 0 & 0 & 0 & \rho_w^h \end{bmatrix} \end{aligned} \tag{132}$$

We then have that:

$$(A^h)^s = \begin{bmatrix} (\rho_\pi^h)^s & 0 & 0 & 1-(\rho_\pi^h)^s \\ \dfrac{(\alpha^h + \lambda^h\phi^h)\rho_\pi^h}{\rho_\pi^h - \rho_w^h}((\rho_\pi^h)^s - (\rho_w^h)^s) & (\rho_w^h)^s & 0 & \Lambda^h(s) \\ \phi^h(\rho_\pi^h)^s & 0 & 0 & \phi^h(1-(\rho_\pi^h)^s) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{133}$$

where

$$\Lambda^h(s) = \frac{(\alpha^h + \lambda^h\phi^h)\big(\rho_\pi^h - \rho_w^h - (\rho_\pi^h)^{s+1}(1-\rho_w^h) + (\rho_w^h)^{s+1}(1-\rho_\pi^h)\big)}{(\rho_\pi^h - \rho_w^h)(1-\rho_w^h)} \tag{134}$$

Using this to evaluate the infinite sums in the consumption function (20) gives:

$$\begin{aligned} c_t^{h*} = \frac{1-\beta}{1-\beta\rho_w^h} w_t - \sigma\beta r_t &+ \frac{\beta\rho_\pi^h[(1-\beta)(\alpha^h + \lambda^h\phi^h) - \sigma(\phi^h\beta - 1)(1-\beta\rho_w^h)]}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\pi_t \\ &+ \frac{\beta(1-\rho_\pi^h)[(1-\beta)(\alpha^h + \lambda^h\phi^h) - \sigma(\phi^h\beta - 1)(1-\beta\rho_w^h)]}{(1-\beta)(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\bar{\pi}_t \end{aligned} \tag{135}$$

The consumption function of an uninformed household, who believes $\bar{\pi}_t = \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t$ for certain, is therefore:

$$c_t^h = \frac{1-\beta}{1-\beta\rho_w^h} w_t - \sigma\beta r_t + \frac{\beta\rho_\pi^h[(1-\beta)(\alpha^h+\lambda^h\phi^h) - \sigma(\phi^h\beta - 1)(1-\beta\rho_w^h)]}{(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\tilde{\mathbb{E}}_t^h\pi_t$$
$$+ \frac{\beta(1-\rho_\pi^h)[(1-\beta)(\alpha^h+\lambda^h\phi^h) - \sigma(\phi^h\beta-1)(1-\beta\rho_w^h)]}{(1-\beta)(1-\beta\rho_\pi^h)(1-\beta\rho_w^h)}\tilde{\mathbb{E}}_{t-1}^h\bar{\pi}_t \quad (136)$$

Simplifying the final two terms, we obtain equation 56.

D.7 Proof of Proposition 5

First, define $\tilde{K}_t^h$ as the Kalman gain the household expects to use when they make their information decision (that is, assuming no updating of $\bar{\pi}_t$ beliefs). From Proposition 4 we have:

$$\begin{cases} \tilde{K}_t^h = 0 & \text{if } \Gamma_t^h < \psi(1-(\rho_\pi^h)^2)^2 \\ \dfrac{1-\tilde{K}_t^h}{(1-(\rho_\pi^h)^2(1-\tilde{K}_t^h))^2} = \dfrac{\psi}{\Gamma_t^h} & \text{if } \Gamma_t^h \geq \psi(1-(\rho_\pi^h)^2)^2 \end{cases} \quad (137)$$

where:

$$\begin{aligned} \Gamma_t^h &= \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\bigg|_{\alpha_t^{h,prior}} \right)^2 \\ &= \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\bigg|_{\alpha_0^h} - \Omega^h \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t \right)^2 \end{aligned} \quad (138)$$

Among those with $\Gamma_t^h \geq \psi(1-(\rho_\pi^h)^2)^2$, we have that:

$$\frac{\partial \tilde{K}_t^h}{\partial \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t} = \frac{\psi(1-(\rho_\pi^h)^2(1-\tilde{K}_t^h))^3}{(\Gamma_t^h)^2(1+(\rho_\pi^h)^2(1-\tilde{K}_t^h))} \frac{\partial \Gamma_t^h}{\partial \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t} \quad (139)$$

where:

$$\begin{aligned} \frac{\partial \Gamma_t^h}{\partial \tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t} &= -\frac{\Omega^h(\bar{C}^h)^{1-\frac{1}{\sigma}}}{\sigma}\sigma_\pi^2 \ln(2) \cdot \left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\bigg|_{\alpha_t^{h,prior}} \right) \\ &> 0 \text{ if and only if } \left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\bigg|_{\alpha_t^{h,prior}} \right) < 0 \end{aligned} \quad (140)$$

Since ρ_π^h and $\tilde{K}_t^h$ are both $\in [0,1]$, the coefficient in front of $\partial\Gamma_t^h/\partial\tilde{\mathbb{E}}_{t-1}^h\bar{\pi}_t$ in equation 139 is always positive. This proves that, for households with $\tilde{K}_t^h > 0$, and so $\sigma_{\varepsilon ht}^{2*} < \infty$, $\tilde{K}_t^h$ strictly increases in $\tilde{\mathbb{E}}_{t-1}^h\bar{\pi}_t$ if and only if $\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\big|_{\alpha_t^{h,prior}} < 0$. The statement in equation 66 then follows from the inverse relationship between $\sigma_{\varepsilon ht}^{2*}$ and $\tilde{K}_t^h$, from the standard properties of the steady state Kalman filter. Those with $\Gamma_t^h \geq \psi(1-(\rho_\pi^h)^2)^2 < 0$ do not change attention with marginal changes in $\tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t$.

Second, we turn to the actual Kalman gains employed by the household. Define Σ as the steady state variance-covariance matrix of ξ_t conditional on the information set in period $t-1$. From the standard properties of the steady state Kalman filter:

$$\Sigma = F(\Sigma - \Sigma C(C'\Sigma C + \sigma_{\varepsilon ht}^2)^{-1} C'\Sigma)F' + Q \tag{141}$$

The Kalman gain vector is then given by:

$$K_t^h = \Sigma C(C'\Sigma C + \sigma_{\varepsilon ht}^2)^{-1} \tag{142}$$

The statement in equation 67 then follows from equation 66 and the fact that the elements of the Kalman gain vector grow as signal precision improves ($\partial K_t^h / \partial \sigma_{\varepsilon ht}^2 < 0$).

E Parameters for figures in Sections 5 and 6

Figures 3 and 4:

All parameters as in Table 13, except α_0^h distributed such that $\partial c_t^h / \partial \tilde{\mathbb{E}}_t^h \pi_t$ is in the range $[-1, 1]$, and ψ set at 0.2×10^{-3}. This scales all attention so that the change in attention with subjective models is clear in the figures.

Figures 5a and 5b:

The calibrated parameters are set out in Table 13. β and σ are set to standard values, and ϕ is set such that the Taylor principle is just satisfied. For the remaining parameters of subjective models, including the mean of the α_0^h distribution, I estimate equations 21-23 using OLS on UK data from 1993-2019. The longer sample than the survey data is to allow for more precise estimation of model parameters. It is not extended further back because of the structural break in many UK macroeconomic time series at the end of 1992 identified by Benati (2006).

For the inflation data, I take the log-first difference of quarterly CPI (ONS series MM23). I de-mean and remove obvious seasonal variation by regressing the series on quarter-of-the-year dummies, and taking the residual as my quarter-on-quarter inflation series. As well as being used in the calibration, this series is fed into the model to generate the simulated paths for perceived inflation and the aggregate consumption elasticity to inflation.

The interest rate data is 3-month money market rates, taken from the OECD Main Economic Indicators. To be consistent with the model equations, I transform this annualised rate into a gross quarterly interest rate, then take logs and de-mean. Following

Harrison and Oomen (2010), I allow the mean of interest rates to vary when there are changes in the broad regime of UK monetary policy, which I take to occur in 2009Q1 as interest rates hit the ZLB.

For real wages, I begin by summing ONS series ROYH, ROYK, and ROYJ to obtain a measure of total nominal wages. I then divide this by total hours (ONS series YBUS) and working age population (ONS series MGSL) to obtain nominal wages per worker per hour. Finally, I divide by the level of CPI (including the seasonal adjustment carried out in the computation of inflation) to obtain real wages. I then take logs and hp-filter the series to obtain the cyclical component.

For $\sigma(\alpha_0^h)$, α_1^h, and ψ I target three moments from the IAS data. The first is the average ratio of 'weaker' to 'stronger' answers in response to Question 3. The raw proportions are inappropriate since we do not know how far either side of a true $dc_t^h/d\tilde{\mathbb{E}}_t^h\pi_t = 0$ is considered 'little difference' by the respondents, but the ratio still gives a sense of the balance between negative and positive models of the economy. That ratio is on average 7.533.

The second target is the estimated elasticity of the proportion with negative models to inflation, that is the coefficient from regressing Pr('weaker') on current inflation and a constant. That elasticity is 0.090.

Finally, the third target is an estimate of the average Kalman gain across the population, which helps to identify the information cost parameter ψ. For this, take Equation 33 and average across households to give:

$$\mathbb{E}_H\big(\tilde{\mathbb{E}}_t^h\pi_t\big) = \mathbb{E}_H\big(K^h\big)\pi_t + \big(1 - \mathbb{E}_H\big(K^h\big)\big)\rho_\pi\mathbb{E}_H\big(\tilde{\mathbb{E}}_{t-1}^h\pi_{t-1}\big) \tag{143}$$

where I have used the fact that all households are calibrated to have the same ρ_π, and in the model information, and so K^h, is decided before the households update their subjective models, and so is independent of perceived inflation. Denoting $\bar{\mathbb{E}}_t\pi_t$ as the average perceived inflation in time t, I therefore estimate:

$$\bar{\mathbb{E}}_t\pi_t = \gamma_1\pi_t + \gamma_2\bar{\mathbb{E}}_{t-1}\pi_{t-1} \tag{144}$$

by OLS, restricting $\gamma_2 = \rho_\pi(1 - \gamma_1)$, where ρ_π is as in Table 13. The estimated γ_1 therefore gives an estimate of the average Kalman gain across the population. This target is 0.448.

Table 13: Calibration

Parameter	Value	Source	Parameter	Value	Source
β	0.99	standard	σ_π	0.003	estimated model
σ	1	standard	σ_i	0.004	estimated model
ϕ	β^{-1}	Lee et al. (2013)	σ_w	0.008	estimated model
$\mathbb{E}_H\alpha_0^h$	-0.732	estimated model	$\sigma(\alpha_0^h)$	0.613	targets
λ	-0.037	estimated model	α_1^h	-234	targets
ρ_π	0.329	estimated model	ψ	0.787×10^{-9}	targets
ρ_w	0.731	estimated model			

Figures 6 and 7:

All shared parameters are as in Table 13, except for ψ, which is set to 0.453×10^{-9} to ensure that average K_{1t}^h remains equal to the target level from the survey (0.448) in the period before the shock. For Figure 6, the high-α group have $\alpha_0^h = 0.997$, while the low-α group have $\alpha_0^h = -0.923$. These are chosen such that both households have the same initial $K_{1t}^h = 0.7$. The variance of v_t in equation 61 is set at $\sigma_v^2 = \sigma_\pi^2/10$.

To simulate these figures, optimal attention is derived using equation 137. The variance of noise in the signals is then given by:

$$\sigma_{\varepsilon ht}^2 = \frac{\sigma_\pi^2(1-\tilde{K}_t^h)}{\tilde{K}_t^h(1-(\rho_\pi^h)^2(1-\tilde{K}_t^h))} \tag{145}$$

Plugging this into equations 141 and 142 for each household in the simulation each period gives the Kalman gain vector, to be used to simulate the path of each household's expectations.

F Relaxing anticipated utility in Section 6

In this section I relax the assumption that households make information choices assuming $\bar{\pi}_t$ will remain constant at $\partial\tilde{\mathbb{E}}_{t-1}^h\bar{\pi}_t$ for certain. Instead, they know that $\bar{\pi}_t$ follows the persistent process:

$$\bar{\pi}_t = \bar{\rho}\bar{\pi}_{t-1} + v_t, \quad v_t \sim N(0, \sigma_v^2) \tag{146}$$

where $\bar{\rho}$ is close to but strictly less than 1. That is, I now assume that $\bar{\pi}_t$ is very persistent but stationary. This ensures that it is possible for households to pay no attention to inflation, without their utility losses from inattention becoming infinite.

Repeating the steps in Appendix D.6, the consumption function becomes:

$$c_t^h = \frac{1-\beta}{1-\beta\rho_w^h}w_t - \sigma\beta r_t + \frac{\partial c_t^h}{\partial\tilde{\mathbb{E}}_t^h\pi_t}\left(\tilde{\mathbb{E}}_t^h\pi_t + \frac{(1-\rho_\pi^h)\bar{\rho}}{\rho_\pi^h(1-\beta\bar{\rho})}\tilde{\mathbb{E}}_t^h\bar{\pi}_t\right) \tag{147}$$

For simplicity, I restrict households to obtaining signals of the same form as in the model without $\bar{\pi}_t$:

$$s_t^h = \pi_t + \varepsilon_t^h \quad \varepsilon_t^h \sim N(0, \sigma^2_{\varepsilon ht}) \tag{148}$$

In Sections 4 and 5, this was the optimal signal structure chosen endogenously by households. This is no longer the case here, for two reasons. First, without this restriction households would also like to acquire information about $\bar{\pi}_t$ directly. Forcing households to estimate long-run inflation from realised inflation is in line with the approach taken by the literature on inflation forecasting (Stock and Watson, 2007). Intuitively, $\bar{\pi}_t$ is a latent variable that cannot be observed directly in the data, so it is plausible that households cannot obtain direct signals about it. Just like empirical researchers, they must infer it from observing other variables.

Second, π_t no longer follows an AR(1) process, so unrestricted households would not choose the simple Gaussian signal over current π_t only.[41] Restricting households to the simple signal form in equation 148 is a common way to simplify rational inattention problems (e.g. Lei, 2019).

In state-space form, the subjective model is:

$$\xi_t = F^h \xi_{t-1} + e_t^h \tag{149}$$

$$s_t^h = C' \xi_t + \varepsilon_t^h \tag{150}$$

where:

$$\xi_t = \begin{pmatrix} \pi_t \\ \bar{\pi}_t \end{pmatrix}, \quad F^h = \begin{pmatrix} \rho_\pi^h & (1-\rho_\pi^h)\bar{\rho} \\ 0 & \bar{\rho} \end{pmatrix}, \quad e_t^h = \begin{pmatrix} u_{\pi t} + (1-\rho_\pi^h) v_t \\ v_t \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{151}$$

It therefore remains optimal for households to incorporate signals into their perceptions of π_t and $\bar{\pi}_t$ using the Kalman filter:

$$\tilde{\mathbb{E}}_t^h \xi_t = (I - K_t^h C') F^h \tilde{\mathbb{E}}_{t-1}^h \xi_{t-1} + K_t^h s_t^h \tag{152}$$

where K_t^h is a 2×1 vector of gain parameters.

The household's attention problem is to choose the noise in their signals $\sigma^2_{\varepsilon ht}$ to minimise expected utility losses from limited information plus information costs, as in Section 4.3. Formally, define Σ_0 and Σ_1 as the steady state variance-covariance matrices of ξ_t conditional on the information sets in period t and $t-1$ respectively.

[41] It can be shown that π_t follows an ARMA(2,1) process. Even without the incentives to forecast $\bar{\pi}_t$ accurately, the optimal signal in period t would therefore also contain information on π_{t-1} and the current shock realisation, as these help to forecast π_{t+1} (Maćkowiak et al., 2018).

The per-period expected utility loss from limited information in steady state is given by:

$$\frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right)^2 (\zeta' \Sigma_0 \zeta) \tag{153}$$

where:

$$\zeta = \left(1, \frac{(1-\rho_\pi^h)\bar{\rho}}{\rho_\pi^h(1-\beta\bar{\rho})}\right)' \tag{154}$$

Following Maćkowiak et al. (2018), the attention problem can therefore be written:

$$\min_{\sigma^2_{\varepsilon ht}} \frac{(\bar{C}^h)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^h}{\partial \tilde{\mathbb{E}}_t^h \pi_t}\right)^2 \zeta' \Sigma_0 \zeta + \frac{\psi}{2}\log_2\left(\frac{C'\Sigma_1 C}{\sigma^2_{\varepsilon ht}} + 1\right) \tag{155}$$

Where in the steady state Kalman filter, Σ_1 and Σ_0 are defined by:

$$\Sigma_1 = F(\Sigma_1 - \Sigma_1 C(C'\Sigma_1 C + \sigma^2_{\varepsilon ht})^{-1} C'\Sigma_1)F' + Q \tag{156}$$

$$\Sigma_0 = \Sigma_1 - \Sigma_1 C(C'\Sigma_1 C + \sigma^2_{\varepsilon ht})^{-1} C'\Sigma_1 \tag{157}$$

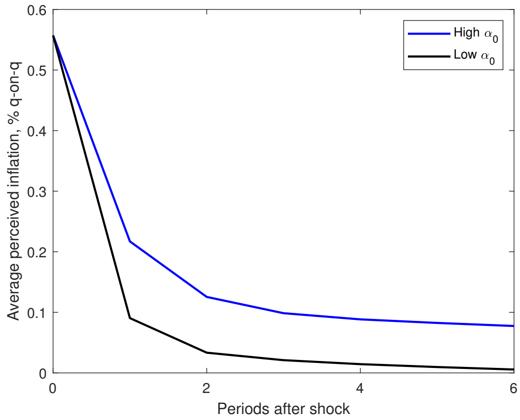
And the Kalman gain vector is:

$$K_t^h = \Sigma_1 C(C'\Sigma_1 C + \sigma^2_{\varepsilon ht})^{-1} \tag{158}$$

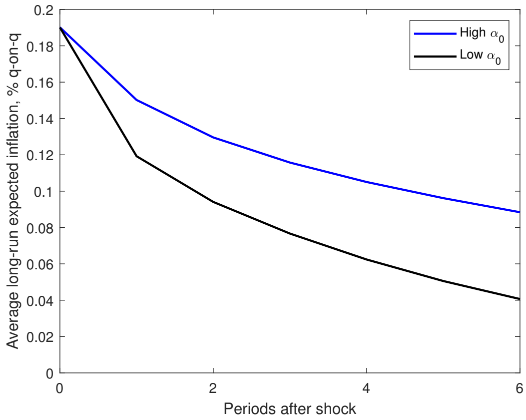
Note that I am maintaining the assumption here that households immediately use the steady state Kalman filter each period, even though their attention is potentially changing. This is an approximation to maintain tractability, and is related to a remaining aspect of the anticipated utility assumption. Households do not expect their subjective model to change in the future, so do not expect their information processing decisions to change, even though they account for changing $\bar{\pi}_t$ in their decisions.

In Figure 8 I repeat the exercise of Figure 6 above, using $\bar{\rho} = 0.99$, and adjusting ψ to 0.485×10^{-5} to ensure average $K_{1t}^h = 0.448$ before the shock hits. All other parameters are the same. The core mechanism from Section 6 remains: after the shock, low-α_0 households increase attention, and so quickly learn that inflation has fallen. High-α_0 households reduce attention, and so their perceived current and long-run inflation fall much more slowly.

Figure 8: Simulated average $\tilde{\mathbb{E}}_t^h \pi_t$ and $\tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t$ for two household groups after an i.i.d. inflation shock, with time-varying $\bar{\pi}_t$ taken into account in information decision.



(a) Average $\tilde{\mathbb{E}}_t^h \pi_t$



(b) Average $\tilde{\mathbb{E}}_{t-1}^h \bar{\pi}_t$