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Abstract

This paper theoretically investigates how labor-market tightness affects market outcomes if firms use informal and self-enforcing agreements to motivate workers. We characterize profit-maximizing equilibria and derive the following results. First, an increase in the supply of homogenous workers can increase wages. Second, even though all workers are identical in terms of skills or productivity, a discrimination equilibrium exists in which a group of majority workers are paid higher wages than a group of minority workers. Third, minimum wages can reduce such discrimination and increase employment. We discuss how these results are consistent with empirical evidence on immigration and a gender pay gap, and provide new testable implications.

JEL-Codes: D210, D860, J210, J380, J610, J710.

Keywords: informal incentives, labor supply, immigration, wage discrimination, minimum wage.

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1 Introduction

Incentivizing workers is an important determinant of a firm's success. As it can be difficult to objectively assess workers' contribution to firm value, informal and self-enforcing agreements are often used to motivate them.[1] Then, firms also need incentives to comply with such an agreement, which is particularly challenging if replacing a worker is easy. Therefore, by determining available alternatives, labor market tightness affects not only workers' but also firms' incentives. Whereas the efficiency wage literature has greatly improved our understanding of workers' incentives to exert effort, the role of labor market tightness for firms' incentives to compensate workers as promised is less well understood.

This paper studies how labor market tightness shapes the design of informal incentive systems and the optimal wage setting of firms. In particular, we take into account that a higher labor supply increases a firm's chances to fill a vacancy, and consequently its temptation to replace a worker instead of compensating him for his effort. Then, firms may find it optimal to endogenously increase the cost of turnover by paying newly-hired workers a rent. The resulting higher commitment allows firms to provide stronger incentives and thus can raise profits. Investigating the interaction between labor market tightness and turnover costs, we derive three key results. First, a higher supply of homogenous workers can actually *increase* wages, a result that differs from both the standard competitive model of a labor market as well as efficiency wage models. Second, discrimination against one group of workers (such as women or immigrants) can be consistent with profit maximization, even if both groups are (otherwise) identical. Third, an appropriate minimum wage can reduce discrimination on labor markets and, at the same time, generate positive employment effects.

While these results are different in nature, they are based on the same channel that firms "voluntarily" pay a rent to workers if a vacancy can readily be filled. We argue that this outcome is preserved by a *social norm* determining which group of market participants — for example, firms or male/native workers — benefits most from informal incentives. Thereby, we follow Greif (1994) and MacLeod and Malcomson (1998), who suggest that social norms can serve as a selection device in settings where multiple equilibria exist. Indeed, Lemieux et al. (2012) and Breza

[1]See Macchiavello (2021) for a survey on the relevance of informal contracts in developing countries; Gibbons and Henderson (2012) present Toyota and Lincoln Electric as two prominent examples in developed countries.

et al. (2021) report that social norms may sustain high wages which are above the market-clearing level.

Setup: Our analysis is based on an infinite-horizon model of an industry with many workers and firms (Sections 2 and 3). We build upon the setup in MacLeod and Malcomson (1998) and extend it by introducing a matching friction on the labor market and allowing for continuous (instead of binary) effort. In every period, each firm can employ exactly one worker. We assume that a firm with a vacancy is randomly matched with an unemployed worker with probability α^F, which increases in the extent of unemployed workers and decreases in the extent of open vacancies. With probability $1 - \alpha^F$, the vacancy remains open until the next period. If a firm is matched with an unemployed worker, the firm makes a take-it-or-leave-it offer which contains an upfront wage and a discretionary bonus potentially paid after a worker's effort choice.

Informal Incentives: Effort increases the firm's revenues, though it is costly for workers. We assume that formal (i.e., court-enforceable) incentive contracts are not feasible, but a worker's effort is observable to his employer.[2] Given this, a firm must use a relational contract to motivate a worker, in which not only the worker has to be incentivized to exert effort, but also the firm to compensate the worker as promised (i.e., a contract must be self-enforcing).

Labor Market Tightness and Turnover Costs: A firm which reneges on a promised payment is punished by the employed worker who subsequently does not exert effort anymore. Still, a firm is able to replace a worker after reneging and start a new employment relationship.[3] In this case, a firm can make a credible promise only if such turnover is sufficiently costly. Because a vacancy causes a production loss, a lower probability of filling a vacancy α^F increases the cost of turnover. Conversely, when α^F is high, the temptation to start a new relationship is large and reduces the willingness to compensate for effort as promised. Then, it can be optimal for a firm to pay a rent to newly employed workers – to *endogenously* increase turnover costs and thereby its own incentives to reward a worker.[4] Inspired by Kandori (1992), Greif (1994), and MacLeod and Malcomson (1998), we suggest that such an equilibrium is selected and sustained by a social norm which lets new workers who are offered a

[2]Note that assuming observable effort is not instrumental for our results.

[3]We exclude multilateral punishments as in Levin (2002) by considering a setting in which deviations cannot be observed by non-involved parties.

[4]We show that it is without loss to distribute this rent evenly over time, so workers receive a rent in each period of employment.

lower wage exert zero effort even if it exceeds their reservation wage.[5] The resulting increased commitment of firms allows for higher equilibrium effort and lets employed workers' payoffs be strictly positive. Therefore, different from approaches with one principal and one agent such as MacLeod and Malcomson (1989) or Levin (2003) where each player's reservation payoff is exogenously given, equilibrium transfers can affect the relationship surplus. Nevertheless, the increased commitment comes with a cost because a firm needs to pay a rent even to a worker whose predecessor has left for exogenous reasons. The optimal level of total turnover costs balances a firm's commitment with such equilibrium costs, and equilibrium effort is below the first best.

Labor Supply and Wages: A higher labor supply increases α^F and makes it easier for firms to fill a vacancy. Because of the self-enforcing nature of contracts, each firm has an incentive to increase compensation to keep the total turnover cost (and consequently equilibrium effort) constant if α^F is high. But a higher labor supply also reduces the chances of unemployed workers to find a job, which in turn lowers a worker's outside option. This "efficiency-wage" mechanism puts *downward* pressure on workers' compensation, so the total effect of an increased labor supply is ambiguous. We show that it is positive if the amount of firms is small. Importantly, though, if the amount of firms is endogenous and determined by a zero-profit condition subject to entry costs, the resulting entry or exit of firms keeps workers' outside options constant. Then, the effect of a higher labor supply on the compensation and utility of employed workers is *unambiguously positive.* Such an outcome is qualitatively different from efficiency-wage models in the spirit of Shapiro and Stiglitz (1984) or MacLeod and Malcomson (1998), where a higher labor supply ceteris paribus reduces workers' incentives to shirk, and (weakly) decreases their compensation.

For lower values of α^F, workers are not paid a rent because the commitment provided by the low chances of filling a vacancy is sufficient. Effort is below the first best for intermediate values of α^F, and at the first best for low values of α^F. In the former case and with a fixed amount of firms, a higher labor supply increases each firm's profits but reduces equilibrium levels of effort and compensation. Allowing for firm entry, however, effort and compensation are pushed up to their original levels. In the latter case, effort and compensation are unaffected by α^F, but higher profits

[5] Breza et al. (2021) provide evidence that workers indeed reject wage offers below the prevailing market wage even in the face of significant unemployment, and argue that this result is driven by social norms.

due to a lower turnover cost yield firm entry.

We argue that these results — an increase in the labor supply can have positive effects on wages (and employment) even if workers are homogeneous — are consistent with observed consequences of immigration and complement other theoretical explanations. As we discuss in Section 5, an abundance of evidence beginning with Card (1990) has found that immigration does not necessarily worsen the labor market conditions for native workers. Although recent studies mostly focus on heterogeneity in worker skills, there is evidence that immigration can benefit native workers even when they work on the same kinds of, mostly low-skilled, tasks. Different from these studies, our mechanism also builds upon firms having wage-setting power, which is in line with recent evidence (Manning, 2003; Dube et al., 2020; Manning, 2021), as well as the discussion by Card (2022) who emphasizes the importance of studying optimal wage setting of firms in labor markets.

Discrimination Equilibrium: In Section 4, we show that labor-market discrimination that is neither based on tastes nor beliefs can be consistent with profit maximization. We derive a discrimination equilibrium in which one (majority) group of workers, "insiders", is treated better than a (minority) group of "outsiders", although both groups of workers only differ in a payoff- and productivity-irrelevant label. The discrimination equilibrium exists because the *expected* rent paid to a new worker determines a firm's commitment, and it is not important how this rent is allocated among different identities. Therefore, it is possible that the rent only benefits insiders (provided the share of outsiders is not too large).[6] The discrimination equilibrium maximizes the profits of individual firms if α^F is sufficiently high. Then, a higher supply of outsiders increases discrimination if their initial share has been low. Eventually, though, discrimination is reduced and disappears once the amount of outsiders is sufficiently large.

The extent of and potential causes for a gender pay gap are one of the most widely analyzed aspects in economics in recent years. Still, an unexplained gap remains that cannot be explained by differences in fundamental worker characteristics (Blau and Kahn, 2017). We argue that such an unexplained gap can be due to a social norm that selects an equilibrium in favor of one dominant group (e.g., male or native workers) over another, smaller, group (e.g., female or immigrant workers).

Minimum Wage: In Section 4.2, we show that a carefully chosen minimum wage can reduce discrimination. If a minimum wage is set between the original

[6] We also show that, as long as the share of insiders is sufficiently large, there is no profit-maximizing equilibrium in which outsiders are treated better than insiders.

wage levels of insiders and outsiders, the latter benefit whereas the former are worse off. Interestingly, such a minimum wage increases employment if we allow for firm entry/exit because it reduces insiders' outside options and thus has a positive effect on firm profits.

In Section 5, we provide a more detailed discussion of evidence on discrimination in labor markets, the role of a minimum wage in reducing a gender pay gap, as well as for negative spillover effects. In Section 6, we discuss the robustness of our main results: general levels of bargaining power between a firm and a worker, other forms of endogenous turnover costs, and different specifications of the probability of filling a vacancy. We also provide additional predictions based on the (un)availability of formal contracts, the severity of labor market frictions, the allocation of bargaining power, and the initial share of outsiders in the discrimination equilibrium — which could help assess the importance of our mechanisms in further empirical research. All proofs are in the Appendix.

Related Theoretical Literature

The standard model of the competitive labor market involves homogeneous-skill workers and no incentive problems between firms and workers; as labor supply goes up, the equilibrium wage goes down (or stays constant after capital has been adjusted). Efficiency-wage models of the labor market acknowledge the need to incentivize workers and assume that this is obtained by a combination of wages above the market-clearing level and a firing threat (Shapiro and Stiglitz, 1984; Yellen, 1984; MacLeod et al., 1994). There, a higher labor supply reduces workers' payoffs once they become unemployed and motivates them to work harder, thus firms decrease wages in response. Consequently, this "efficiency-wage effect" would predict that a higher labor supply *reduces* equilibrium wages. Incorporating the labor-market friction α^F, we demonstrate that a higher labor supply can reduce a firm's credibility when making promises, to which they might optimally respond by *increasing* workers' compensation.

MacLeod and Malcomson (1998) take into account that incentives to workers are often informal and performance pay (such as bonuses) might be used to provide incentives. If firms are on the short side of the market, standard performance pay is not possible because firms would fire and replace workers when supposed to pay a bonus. In this case, firms pay workers a rent to motivate them, which is costly because such a rent has to be paid to new workers as well. Their mechanism involving endogenous turnover costs also appears in our model. However, since the labor market in MacLeod and Malcomson (1998) is frictionless (firms can fill a vacancy with probability one if there is unemployment), a higher labor supply either reduces or has no effect on wages.

Yang (2008) extends the setting of MacLeod and Malcomson (1998) by assuming that turnover is costly. He demonstrates that higher (exogenous) turnover costs reduce total wage payments and unemployment. Fahn (2017) assumes that firms and workers bargain about the terms of the employment relationship. Workers' incentives increase in their bargaining power, thus a minimum wage can increase effort and consequently the efficiency of employment relationships.

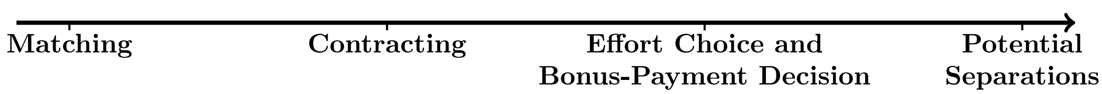
We also contribute to the theoretical literature on labor market discrimination, i.e., pay differences that are not fully accounted for by productivity differences. Notably, different from taste-based or belief-based discrimination, discrimination in our model arises as a profit-maximizing equilibrium. Lindenlaub and Prummer (2021) demonstrate that discrimination might be caused by different amounts and depths of connections in labor market networks, which determine performance and consequently compensation. Prummer and Nava (2021) consider a promotion tournament which not only incentivizes effort, but also extracts information on worker characteristics. Although workers are ex-ante identical, discrimination against one (group) can be optimal because it reduces workers' information rent. As in our paper, discrimination in Prummer and Nava (2021) does not reflect fundamental differences in group characteristics or employer perceptions.

2 Model

Setup There are a mass $F > 0$ of firms and a mass $N > 0$ of workers. We first assume F to be exogenously given and endogenize it below, in Section 3.4. All workers and firms are risk neutral. There are infinitely many periods $t = 1, 2, ...$, and all players have a common discount factor $\delta \in (0, 1)$.

Workers and firms either are part of a match or not, and each firm can employ exactly one worker. At the beginning of every period, unmatched players enter the labor market. An unmatched firm is randomly matched with an unemployed worker with probability $\alpha^F(n, f) \in (0, 1)$, where $f > 0$ is the mass of unmatched firms, $n > 0$ is the mass of unemployed workers, $\alpha^F_f < 0, \alpha^F_n > 0$, $\lim_{n\to 0} \alpha^F(n, f) = 0$, and $\lim_{n\to\infty} \alpha^F(n, f) = 1$. Correspondingly, $\alpha^N(n, f)$ is the probability for an unemployed worker to be matched with a firm, with $\alpha^N_f > 0$, $\alpha^N_n < 0$, $\lim_{f\to 0} \alpha^N(n, f) = 0$, and $\lim_{f\to\infty} \alpha^N(n, f) = 1$.

Once matched, each firm i can make a take-it-or-leave-it (TIOLI) offer to its worker.[7] Formally, the offer made by firm i consists of a wage $w^i_t \in \mathbb{R}$ and the promise to pay a discretionary bonus $b^i_t \in \mathbb{R}$. If a worker rejects the offer, he receives his (exogenous) outside option of zero, the match separates, and firm and worker can re-enter the matching market in the subsequent period. If a worker accepts the offer, he receives w^i_t. Then, the worker exerts effort $e^i_t \in \mathbb{R}_+$ incurring effort costs $c(e^i_t)$, where $c(\cdot) > 0$ is strictly increasing, convex, and $c(0) = c'(0) = 0$. After observing the worker's effort, firm i decides whether to pay a discretionary bonus b^i_t. Then, workers and firms simultaneously decide whether to leave the current match or not, and the match is separated if one of them chooses to leave. All workers and firms who are not part of a match re-enter the labor market. At the end of a period, each worker (whether part of a match or not) leaves the market with exogenous probability $(1 - \gamma)$, after which his utility is set to zero; to keep the size of the labor force constant, we assume that $(1-\gamma)N$ new agents enter the labor market at the beginning of every period. The timing within a period t is summarized in the following graph:



The effort of firm i's worker, e^i_t, generates firm i's revenue $e^i_t\theta$, where $\theta > 0$. Note that if a firm and a matched worker acted as a single entity, they would maximize

$$e^i_t\theta - c(e^i_t).$$

We denote the resulting effort the first best, e^{FB}, characterized by:

$$\theta - c'(e^{FB}) = 0.$$

[7]This incorporates evidence that firms may have considerable wage-setting power even in thick labor markets (Manning, 2021, Card, 2022). We discuss general levels of bargaining power between firms and workers in Section 6.

Contracts, Strategies, and Equilibrium Concept We consider situations in which effort as well as per-worker output can be observed by both, the firm and the worker, but not by anyone outside the respective match. Hence, no verifiable measure of the agent's performance exists, and incentives can only be provided informally, i.e., with relational contracts.

We assume strategies are *contract specific* in the sense of Board and Meyer-Ter-Vehn (2014): actions of firms and workers do not depend on the identity of the worker, calendar time, or history outside the current relationship.[8] Contract-specific strategies imply that firms' and workers' strategies cannot condition on any outcomes of other matches, i.e., no multilateral relational contracts as in Levin (2002) are feasible. We focus on pure strategies.

The equilibrium concept we apply is *social equilibrium.* This concept describes a subgame-perfect equilibrium, which is restricted by the assumptions that strategies are contract-specific.[9] We derive social equilibria that maximize the profits of an individual unmatched firm, taking the behavior of other firms as given. Among these equilibria, we consider those that are (constrained) Pareto optimal.[10] We focus on the *stationary steady state*, which allows us to omit time subscripts. We further discuss this aspect below and show in the proof to Proposition 1 that, for our formulation of the optimization problem, the stationarity assumption is without loss of generality for all periods other than $t = 1$.

Two remarks are in order. First, our setup is based on the model of MacLeod and Malcomson (1998) and extends it by the introduction of labor market frictions and continuous (in contrast to binary) effort. Second, the compensation structure (with an upfront wage and a bonus paid at the end of a period) is assumed for simplicity and does not have to be taken literally. For example, the bonus could also be paid in the form of a salary at the beginning of the next period or correspond to future promotion opportunities, without changing expected payoffs and any of the constraints derived below. It is only important that payment is contingent on the worker exerting equilibrium effort, which effectively means that it is tied to the

[8]In Section 4, we analyze asymmetric equilibria based on a worker's group identity.

[9]The equilibrium concept is called *social* because — although strategies are contract-specific — a player's strategy will depend on the strategies of all market participants, as the possibility of a re-match determines everyone's *endogenous* outside option. Hence, (as in Ghosh and Ray, 1996; Kranton, 1996; MacLeod and Malcomson, 1998; Fahn, 2017), subgame perfection not only pertains to individual relationships, but the market as a whole has to be in equilibrium. This is because potential deviations also include the opportunity to terminate a current match and go for a new one.

[10]In Section 6, we discuss equilibria which can involve money burning.

worker keeping his job.

3 Symmetric Equilibrium

In this section, we analyze a symmetric profit-maximizing equilibrium. There, any deviation from equilibrium behavior would lead to the static Nash equilibrium with zero effort and zero payments in the respective match; thus, such a match is separated at the end of a period. This is optimal by Abreu (1988) who shows that any observable deviation should trigger the highest feasible punishment for the defector.

Equilibrium Payoffs The discounted utility stream of an employed worker in the stationary steady state equals

$$U = u + \gamma\delta U,$$

where $u = w + b - c(e)$ is an employed worker's per-period utility. Note that discounted continuation utilities are multiplied with γ because workers might leave the market for exogenous reasons with probability $1-\gamma$, then having a utility of zero.

The utility of an unemployed worker is denoted by $\bar{U}$ and equals $\bar{U} = \alpha^N U + \delta\gamma(1-\alpha^N)\bar{U}$. Rearranging it yields

$$\bar{U} = \frac{\alpha^N}{1-\delta\gamma(1-\alpha^N)}U.$$

A matched firm's discounted profit stream is denoted by Π, the expected profits of a firm with an open vacancy are denoted by $\bar{\Pi}$:

$$\Pi = e\theta - b - w + \delta\left[\gamma\Pi + (1-\gamma)\bar{\Pi}\right],$$
$$\bar{\Pi} = \alpha^F\Pi + \delta\left(1-\alpha^F\right)\bar{\Pi}.$$

Rearranging them yields

$$\Pi = \frac{\left(1-\delta\left(1-\alpha^F\right)\right)(e\theta - b - w)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)},$$
$$\bar{\Pi} = \frac{\alpha^F(e\theta - b - w)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)}.$$

3.1 Benchmark: Formal Contracts

We start with a brief analysis of a benchmark case in which formal short-term contracts on an agent's effort are feasible. Then, paying $b = c(e)$ and $w = (1 - \delta\gamma)\,\bar{U}$ maximizes profits. Furthermore, $\bar{U} = 0$ in the symmetric equilibrium. Thus, a firm keeps the full social surplus and maximizes $e\theta - c(e)$, so e^{FB} is implemented. This holds irrespective of the value of α^F, therefore a change in N has no effect on a worker's compensation. Because firms can find a replacement with a larger probability if a worker leaves for exogenous reasons, their profits increase in N.

3.2 Characterization of Equilibrium

To enforce a given effort level, each firm is subject to the following constraints. First, it must be in the worker's interest to exert the agreed-upon effort level. Consider a deviation in which the worker chooses zero effort (which naturally is the optimal deviation). In this case, the worker does not receive the bonus, the respective match splits up, so a worker's continuation utility equals $\delta\gamma\bar{U}$. It follows that equilibrium effort e^* must satisfy the agent's incentive compatibility (IC) constraint:

$$-c(e^*) + b + \delta\gamma U \geq \delta\gamma\bar{U}. \tag{IC}$$

Second, an employed worker's utility must be at least as high as his outside option. This equals $\bar{U}$ for the following reason. At the end of period t, the worker stays only if he expects to receive (at least) $\bar{U}$ in the following period. At the beginning of period $t+1$, the firm could deviate and instead offer a contract with $U = \delta\gamma\bar{U}$ (which constitutes the worker's outside option from the perspective of period $t+1$ because he would have to wait until the next period before potentially finding a new match). However, the worker would respond to such a deviation by collecting the wage and choose $e_{t+1} = 0$, with the match then splitting up at the end of period t. Thus, the following individual rationality (IR) constraint must hold:

$$U \geq \bar{U}. \tag{IR}$$

Note that even though $\bar{U} = \frac{\alpha^N}{1-\delta\gamma(1-\alpha^N)}U$ holds in equilibrium and $\alpha^N < 1$, (IR) cannot be omitted. This is because $\bar{U}$ is constituted by an arrangement the worker (potentially) has with a different firm, hence is regarded as exogenous by individual firms.

Third, a firm must pay a bonus as promised. If the firm reneges and refuses to pay the equilibrium bonus at the end of period t, the match splits up and both parties re-enter the matching market. Therefore, the maximum enforceable bonus payment is given by a dynamic enforcement constraint and equals

$$-b + \delta\gamma\Pi + \delta(1-\gamma)\bar{\Pi} \geq \delta\bar{\Pi}. \tag{DE}$$

There, we also have to take into account that even if a firm pays the bonus, its worker might leave for exogenous reasons (which happens with probability $1-\gamma$). Since

$$\Pi - \bar{\Pi} = \frac{\left(1-\alpha^F\right)(e\theta - b - w)}{\left(1-\delta\gamma\left(1-\alpha^F\right)\right)},$$

(DE) becomes

$$b \leq \delta\gamma\left(1-\alpha^F\right)(e\theta - w). \tag{DE}$$

(DE) describes the maximum bonus the firm can credibly promise in a relational contract. Intuitively, a high bonus may not be self-enforceable because a firm has an incentive to renege and go for a potential new match. Holding other parameters constant, a given bonus is more difficult to sustain as α^F is larger. Also, sticking to its current match has to be optimal for a firm on the equilibrium path, requiring $\delta\gamma\Pi + (1-\gamma)\delta\bar{\Pi} \geq \delta\bar{\Pi}$ and hence $\Pi \geq \bar{\Pi}$. Given $b \geq 0$, this condition is implied by (DE) and hence can be omitted.

Finally, in a stationary steady state, the mass of newly matched firms must be equivalent to the mass of newly matched workers, the mass of unmatched firms at the beginning of a period must be the same as at the end of a period, and the same must hold for workers. Since these conditions do not explicitly appear in Proposition 1 but will be important for comparative statics, we defer a formal characterization of these conditions to Sections 3.3 and 3.4.

Now, we characterize an equilibrium where firms maximize $\bar{\Pi}$, subject to the constraints just derived. Note that our main results would be the same if the objective was to maximize Π, the profits of a matched firm. However, maximizing $\bar{\Pi}$ allows us to (without loss of generality) focus on stationary arrangements. If we maximized Π instead, it would be optimal to treat workers in the first period of their employment differently than in later ones. Thus, maximizing $\bar{\Pi}$ substantially simplifies our exposition.

Our first proposition states how α^F determines equilibrium effort and the utility of workers.

Proposition 1 (Optimal Informal Incentives) *There exists a profit-maximizing equilibrium with the following properties. There are* $\overline{\alpha}^F, \underline{\alpha}^F \in (0,1)$ *such that*

- *For* $\alpha^F \geq \overline{\alpha}^F$*, equilibrium effort is characterized by* $c'(e^*) = \delta\gamma\theta$ *with* $e^* < e^{FB}$*. Each worker's utility is positive (and* $U^* > \bar{U} > 0$*).*
- *For* $\underline{\alpha}^F \leq \alpha^F < \overline{\alpha}^F$*, equilibrium effort is characterized by* $c(e^*)/e^* = \delta\gamma(1-\alpha^F)\theta$*, with* $e^* < e^{FB}$*. Each worker's utility is zero (i.e.,* $U^* = \bar{U} = 0$*).*
- *For* $\alpha^F < \underline{\alpha}^F$*, equilibrium effort is characterized by* $e^* = e^{FB}$*. Each worker's utility is zero (i.e.,* $U^* = \bar{U} = 0$*).*

Because formal contracts are not feasible, a firm's promise to reward a worker for his effort must be credible. As explored above, a worker who does not receive a promised payment responds by not exerting effort anymore. Different from "standard" relational-contracting models with one principal and one agent where only the potential future relationship surplus determines enforceabile effort, a reneging firm can replace a worker and start over. Therefore, a firm in our setting can only make credible promises to reward effort if turnover is costly. This holds in addition to the standard requirement that the future relationship surplus is sufficiently high, which manifests in $\underline{\alpha}^F$ increasing in δ and γ, and effort increasing in both arguments if $\alpha^F \geq \underline{\alpha}^F$.

One form of turnover cost stems from labor market frictions which reduce the chances of finding a replacement. If the market frictions are large (i.e., the probability of finding a new worker α^F is small), such exogenous turnover costs are enough for firms to honor their promises. If frictions are small so that $\alpha^F > \bar{\alpha}^F$, firms make use of another, endogenous, mechanism to make turnover costly by granting new workers a rent as in MacLeod and Malcomson (1998) or Fahn (2017); thus, firms do not utilize their wage-setting power to fully extract the relationship surplus. Therefore, new workers receive an upfront wage which is costly for firms because — different from payments made later on — the wage paid in the first period of a worker's employment cannot be used to provide incentives. This is different from models such as Levin (2003) where outside options are exogenously given and the relationship surplus is orthogonal to transfers.

Such an equilibrium with endogenous turnover costs particularly makes use of the term "social" in social equilibrium. The productivity of a firm's *current* relationship depends on the costs of starting a new relationship in the *future* — although potential new workers are not able to observe anything that happens in the firm's current employment relationships. The social equilibrium specifies that workers regard an offer with a lower rent as a deviation, thus firms have an incentive to compensate their workers as promised. Put differently, this social equilibrium involves a norm that high wages are paid independent of a worker's tenure.[11] Such norms have been identified by Breza et al. (2021), who find that unemployed workers do not accept offers below the prevailing wage.

If α^F is sufficiently small, the presence of the labor-market friction alone is sufficient. Then, firms make use of their wage-setting power and leave no rents to their workers. Furthermore, equilibrium effort is equal to e^{FB}. For intermediate α^F, the bonus is as high as feasible given α^F, effort is below e^{FB} and determined by a binding (DE) constraint.

Generally, the *optimal* level of turnover costs for firms would balance higher incentives that can be provided in a current relationship with the costs of starting new relationships later on. For $\alpha^F < \overline{\alpha}^F$, equilibrium turnover costs are "too large" (firms can only increase but not reduce them). For $\alpha^F \geq \bar{\alpha}^F$, total turnover costs are at the optimal level for an individual firm which takes its worker's outside option as given. Then, equilibrium effort is below e^{FB} because, at e^{FB}, having marginally smaller costs of turnover would only cause a second-order loss in profits due to lower effort. Moreover, because of the response of its competitors, endogenous turnover costs are more expensive for a given firm than the exogenous costs stemming from labor-market frictions – because the former also increase an employed worker's outside option. This aspect is further explored in our next section.

3.3 Comparative Statics with a Fixed Mass of Firms

We now conduct comparative statics with respect to the mass of workers N, holding the mass of firms F constant; we endogenize F and determine its value by a zero-profit condition in Section 3.4. A higher N will increase the mass of unemployed workers n and consequently raise $\alpha^F(n, f)$ but reduce $\alpha^N(n, f)$. To simplify the following analysis, we slightly reduce the generality of the α^F, α^N from now on and assume that $\alpha^F(n, f) = \alpha^F(n - f)$ where $\alpha^F(\cdot)$ is increasing, as well as $\alpha^N(n, f) = \alpha^N(n - f)$ where $\alpha^N(\cdot)$ is decreasing.

As a preliminary step, we formalize the conditions that must hold in the labor market in a stationary steady state, where $f^* > 0$ is the equilibrium mass of

[11] See Kandori (1992), Ghosh and Ray (1996), Kranton (1996), or MacLeod and Malcomson (1998), for more detailed discussions about the role of norms in related settings.

unmatched firms and $n^* > 0$ the equilibrium mass of unemployed workers. First, the mass of newly matched firms must be equivalent to the mass of newly matched workers, hence

$$n^* \alpha^N = f^* \alpha^F. \tag{1}$$

Second, the mass of unmatched firms at the beginning of a period must be the same as at the end of a period, i.e., $f^* = (1 - \alpha^F) f^* + (1 - \gamma)(F - f^*)$, or equivalently

$$f^* = \frac{1 - \gamma}{1 - \gamma + \alpha^F} F. \tag{2}$$

The same holds for unemployed workers, hence $(1 - \alpha^N) n^* + (1 - \gamma)(N - n^*) = n^*$, or equivalently

$$n^* = \frac{1 - \gamma}{1 - \gamma + \alpha^N} N. \tag{3}$$

Plugging (2) and (3) into (1) yields

$$\alpha^N = \frac{(1 - \gamma) F}{(1 - \gamma + \alpha^F) N - F \alpha^F} \alpha^F$$

and, again using (3),

$$n^* - f^* = N - F.$$

Thus, $\alpha^F (n^* - f^*) = \alpha^F (N - F)$, and $\partial \alpha^F / \partial N = \left(\alpha^F\right)'$. Now, we are ready to present the results of comparative statics with respect to N, which depend on the size of α^F in relation to the thresholds $\overline{\alpha}^F$ and $\underline{\alpha}^F$ derived in Proposition 1.

Corollary 1 (Comparative Statics with Constant F) *Assume F is exogenously given.*

- *For $\alpha^F \geq \overline{\alpha}^F$, effort e^* is independent of N, whereas total compensation $w^* + b^*$ and an employed worker's utility U^* may increase or decrease. $w^* + b^*$ and U^* increase in N if F is sufficiently small.*
- *For $\underline{\alpha}^F \leq \alpha^F < \overline{\alpha}^F$, e^* and $w^* + b^*$ decrease in N, whereas U^* is unaffected by N.*
- *For $\alpha^F < \underline{\alpha}^F$, e^*, $w^* + b^*$, and U^* are unaffected by N.*

We first describe the intuition for $\alpha^F \geq \overline{\alpha}^F$. Note that

$$U^* = \frac{\delta\gamma\bar{U}\left(1-\left(1-\alpha^F\right)\delta\gamma\right)-\delta\gamma\left(1-\alpha^F\right)e^*\theta+c(e^*)}{\delta\gamma\alpha^F} \geq 0.$$

We distinguish between (i) a direct effect of a higher N on α^F holding $\bar{U}$ constant and (ii) an indirect effect incorporating changes in $\bar{U}$. For (i), an increase in N directly increases α^F, which raises an employed worker's utility and compensation. This is because total turnover costs are at their optimal level if $\alpha^F \geq \overline{\alpha}^F$, thus an increase in α^F lets firms increase a worker's rent to the same extent.

For (ii), recall that an employed worker's outside option equals $\bar{U} = \alpha^N U / \left[1-\delta\gamma(1-\alpha^N)\right]$. There, workers are paid more in case they are re-employed, but the probability of finding an alternative job, α^N, goes down. The latter resembles the well-known efficiency wage effect in the spirit of Shapiro and Stiglitz (1984).

If the indirect effect on the outside option is positive, workers always benefit from a higher N. Also if it is negative but not too large, the positive direct effect dominates, and wages and utilities increase in N. This holds if F is sufficiently small.

Note that there is no direct impact of N on $\bar{\Pi}$ (total turnover costs and effort remain constant), only an indirect one which is negatively proportional to the effect on $\bar{U}$. If a higher N decreases workers' outside options, firms profits go up, and vice versa; hence, firms can potentially benefit from larger labor market frictions. We further pursue this aspect in Section 3.4.

If $\alpha^F < \overline{\alpha}^F$, the labor market friction is larger than optimal from an individual firm's perspective. Therefore, if N goes up, firms fully "utilize" the decreased friction and request lower effort in response to their reduced commitment. Moreover, there is no indirect effect on the outside option because $\bar{U} = 0$. Since effort goes down, the worker's compensation also goes down. If frictions are so high that e^{FB} is implemented, a change in N has no consequences on effort.

3.4 Comparative Statics with an Endogenous Mass of Firms

Now, we analyze the case in which F is endogenously determined by a zero-profit condition. We assume that there exists a sufficient pool of potential entrant firms, and each of them can enter the industry by paying an entry cost $K > 0$. Then, a zero-profit condition implies $-K + \bar{\Pi} = 0$ (in addition, $\partial\bar{\Pi}/\partial F < 0$ needs to hold in equilibrium).

Because of the zero-profit condition, any change in N must be balanced by a change in F to keep $\bar{\Pi}$ constant, that is,

$$d\bar{\Pi} = \frac{\partial \bar{\Pi}}{\partial N} dN + \frac{\partial \bar{\Pi}}{\partial F} dF = 0,$$

and

$$\frac{dF}{dN} = -\frac{\partial \bar{\Pi}/\partial N}{\partial \bar{\Pi}/\partial F}$$

must hold. This yields the following comparative statics.

Proposition 2 (Comparative Statics with Endogenous F) *Assume that F is endogenously determined to keep $\bar{\Pi}$ constant.*

1. *For $\alpha^F \geq \overline{\alpha}^F$, total compensation $w^* + b^*$ and an employed worker's utility U^* increase in N, whereas effort and $\bar{U}$ are unaffected. F might increase or decrease.*

2. *For $\alpha^F < \overline{\alpha}^F$, F increases in N, whereas compensation, effort, U^* and $\bar{U}$ are unaffected.*

Recall that, with $\alpha^F \geq \bar{\alpha}^F$, there is no direct effect of N on $\bar{\Pi}$, only an indirect effect via $\bar{U}$. The endogenous entry or exit keeps $\bar{U}$ constant, and consequently only the positive direct effect of a higher N on U^* and compensation prevails. Because a higher α^F increases both the hiring probability and the rent paid from a firm to a worker, employment effects of an increased N can be either positive or negative when $\alpha^F \geq \bar{\alpha}^F$. For $\alpha^F < \bar{\alpha}^F$, the direct effects of a higher N on effort and thus compensation are eliminated by firm entry. Thus, a higher N increases employment, leaving effort and utilities unaffected. In sum, with endogenous F the consequences of a higher N on wages are *never negative* in our setting, and are strictly positive if $\alpha^F \geq \bar{\alpha}^F$.[12]

To conclude this section, note that the level of entry costs would determine equilibrium steady-state profits and consequently the values α^F, α^N that are consistent with $\bar{\Pi}$. Proposition 2 implies that profits are increasing in α^F for $\alpha^F < \bar{\alpha}^F$. Moreover, since α^F has no direct effect on profits for $\alpha^F \geq \bar{\alpha}^F$, positive outside options then imply that $\bar{\Pi}$ is maximized at $\bar{\alpha}^F$. Thus, the maximum feasible entry costs for firms to be active are such that $\alpha^F = \bar{\alpha}^F$. For all lower entry costs, α^F can be below or above $\bar{\alpha}^F$, i.e., there is an equilibrium with relatively low and one with relatively

[12] This can change if workers have positive bargaining power, an aspect we discuss in Section 6.

high labor market frictions. We would argue that high levels of α^F might be observed particularly in markets which experience a discrete labor supply increase (for example caused by immigration). This is based on the interpretation in which comparative statics with constant F would describe the short-term, while endogenous F would describe the long-term consequences of a higher labor supply. Moreover, firms' adjustment would probably not be immediate (in particular if reduced profits would call for firm exit), thus a labor supply increase pushing α^F considerably above $\bar{\alpha}^F$ followed by gradual responses of firms would likely yield a new steady state level of $\alpha^F > \bar{\alpha}^F$ (which exists if $\partial\alpha^F/\partial F$ is – in absolute terms – sufficiently small to guarantee $\partial\bar{\Pi}/\partial F < 0$; see the proof to Proposition 2).

4 Labor Market Discrimination and Minimum Wage

So far, our model has delivered a norm-based explanation for positive wage effects of a higher labor supply, with outcomes for all workers being the same. Now, we demonstrate that also a profit-maximizing "discrimination equilibrium" exists in which one (majority) group receives a persistent wage premium over another (minority) group, although both groups are identical, i.e., are equally productive and have the same (exogenous) outside option. Then, we show how a minimum wage can mitigate such labor market discrimination, however harms the group that is better off.

Assume there are two kinds of workers, "insiders" (the majority group) and "outsiders" (the minority group). These identities can be distinguished by firms but workers are otherwise identical. We assume that firms with a vacancy are randomly matched with workers, hence targeted search is not possible. Moreover, only firms with an open vacancy are (potentially) matched with workers, thus it is not possible for firms with filled positions to look for another type of worker. Alternatively, we could assume a firm's search to be visible to its current employee who then regards it as a violation of the relational contract and exerts no effort.

If a firm has an open vacancy, the probability of being matched with an outsider is α^{FO}, the probability of being matched with an insider is α^{FI}, and $\alpha^F = \alpha^{FI} + \alpha^{FO}$. Furthermore, random matching implies that an unemployed insider has the same chances α^N of finding a job as an unemployed outsider.

Now, a firm's profits when hiring an insider are Π^I, and Π^O when hiring an outsider. Therefore, an unmatched firm's expected profits are

$$\begin{aligned}\bar{\Pi} =& \alpha^{FI}\Pi^{I} + \alpha^{FO}\Pi^{O} + \delta\left(1-\alpha^{F}\right)\bar{\Pi} \\ =& \frac{\alpha^{FI}\Pi^{I} + \alpha^{FO}\Pi^{O}}{1-\delta\left(1-\alpha^{F}\right)}.\end{aligned}$$

4.1 Discrimination Equilibrium

Here, we describe characteristics of a discrimination equilibrium. In particular, we focus on an equilibrium that, among equilibria that maximize individual firms' profits, is best for insiders. As a starting point, take a situation with no outsiders (and in which all insiders have the same arrangement), and with α^{FI} above the threshold $\bar{\alpha}^{F}$ derived in Proposition 1 (i.e., $\alpha^{FI} \geq \bar{\alpha}^{F} = 1 - c(e)/\delta\gamma e\theta$, where e is characterized by $\delta\gamma\theta - c'(e) = 0$). Therefore, insiders are paid a rent, and profit-maximizing effort is below the first best.

Now compare it to a situation in which also outsiders are present. Then, firms' optimal behavior is not uniquely determined, only *expected* turnover costs when starting a new relationship matter. Thus, besides a symmetric equilibrium with identical outcomes for insiders and outsiders, profit-maximizing equilibria in which outsiders' payoffs are lower also exist.[13] There, lower payments to outsiders ceteris paribus increase a firm's profits with outsiders and hence must be accompanied by higher rents for insiders to keep expected profits when starting a new employment relationship constant. The best feasible arrangement for insiders involves firms' profits with outsiders to be as high as possible. As long as α^{FO} is small, such an equilibrium pushes outsiders' payoffs to their outside option of zero and implements an effort level either at e^{FB} or determined by a firm's binding dynamic enforcement constraint. For a higher α^{FO}, firms' profits with outsiders must be reduced to keep them in matches with insiders. Then, outsiders also receive a rent (or their effort is reduced). In any case, though, an outsider's effort e^{O} is strictly larger than an insider's effort e^{I} (which is still characterized by $\delta\gamma\theta - c'(e^{I}) = 0$). Thus, outsiders might *work harder but earn less* than insiders. Only if $\alpha^{FO}$is large, the arrangements of insiders and outsiders coincide.

Proposition 3 (Discrimination Equilibrium) *Assume $\alpha^{FI} > \bar{\alpha}^{F}$ and α^{FO} is sufficiently small. Then, the following, profit-maximizing, discrimination equilib-*

[13]Note that, for $\alpha^{FI} \geq \bar{\alpha}^{F}$, the opposite that insiders are treated worse than outsiders is not feasible.

rium exists and maximizes the payoffs of insiders:

- *insiders' effort is characterized by* $\delta\gamma\theta - c'(e^I) = 0$ *and* $U^I > \bar{U}^I > 0$*;*
- $w^O = 0$*,* $b^O = c(e^O)$*,* $U^O = \bar{U}^O = 0$*, and* $e^O > e^I$*.*

Proposition 3 shows that a higher outside option and seemingly lower productivity of insiders can emerge *endogenously* in a discrimination equilibrium. Note that insiders' effort in the discrimination equilibrium is the same as in the above symmetric equilibrium. Thus, the higher effort of outsiders implies that average productivity is highest in the discrimination equilibrium which maximizes insiders' payoffs.

Importantly, this result is not caused by fundamental differences such as experience or human capital, or personality traits such as risk aversion, preferences for bargaining, or other factors. We argue that social norms, potentially shaped by historical developments, may determine whether a symmetric, non-discriminatory, equilibrium is played, or instead an equilibrium in which one (large) group is treated better than another (small) group.

4.2 Discrimination and Minimum Wage

We now explore the consequences of a minimum wage $\overline{w}$. Such a minimum wage serves as a lower bound on a worker's total compensation, $w + b$. Since b is discretionary, though, the upfront-wage w must exceed $\overline{w}$.

Proposition 4 (Minimum Wage) *Consider the best discrimination equilibrium for insiders in Proposition 3. There exists a* $\hat{\overline{w}} > 0$ *such that the following holds for* $\overline{w} \in \left[0, \hat{\overline{w}}\right)$*:*

- *The minimum wage binds for outsiders but is slack for insiders:* $w^O = \overline{w} < w^I$*.*
- *An increase in* $\overline{w}$
  - *reduces* w^I *as well as* U^I*, but increases* U^O *and* w^O*;*
  - *(weakly) reduces* e^O*;*
  - *increases* $\bar{\Pi}$ *and consequently employment if* F *is endogenous.*

In the best profit-maximizing equilibrium for insiders, $w^I > w^O$. A minimum wage in between these two values reduces the gap between w^I and w^O. This is not only caused by an increase in w^O, but also by a reduction of w^I, thus outsiders are better off and insiders are worse off. Moreover, the minimum wage increases profits because insiders' outside options are reduced (the higher costs of employing outsiders are more than compensated for by lower costs of employing insiders). Therefore, if F is endogenously determined by a zero-profit condition, employment effects of such a minimum wage are positive. This also implies that the discrimination equilibrium is less profitable for firms than the symmetric equilibrium. Still, because different profits are caused by insiders' outside options which are taken as given by individual firms, the discrimination equilibrium is in accordance with (individual-firm) profit maximization.

If the minimum wage is sufficiently high for $w^I = w^O$ to hold, a further increase lets both wages (and utilities) go up. Then, as in Fahn (2017), a higher minimum wage increases equilibrium effort and surplus.

4.3 Comparative Statics

Now, we derive comparative statics with respect to α^{FO}. We show that an inflow of outsiders benefits only insiders if α^{FO} is small; if α^{FO} is sufficiently large, a further increase benefits outsiders and thus reduces the gap. Note that we assume that an inflow of outsiders has no effect on α^{FI} (although one might expect a negative relationship, in particular if many outsiders are present). We argue that this assumption is justified as long as there are not too many outsiders and will further discuss its implications below.

Lemma 1 (Comparative Statics) *Consider the best discrimination equilibrium for insiders and assume $\alpha^{FI} > \bar{\alpha}^F$.*

- *If α^{FO} is sufficiently small, $\partial(w^I+b^I)/\partial\alpha^{FO} > 0$, $\partial U^I/\partial\alpha^{FO} > 0$, $\partial U^O/\partial\alpha^{FO} = 0$, and $\partial e^O/\partial\alpha^{FO} \geq 0$.*

- *For larger α^{FO}, $\partial(w^I + b^I)/\partial\alpha^{FO} = \partial U^I/\partial\alpha^{FO} = 0$, and $\partial U^O/\partial\alpha^{FO} \geq 0$.*

If α^{FO} is small, the best profit-maximizing equilibrium for insiders involves maximizing the profits firms make with outsiders. Then, an increase in α^{FO} requires higher rents for insiders to keep expected profits of an unmatched firm constant; moreover, e^O increases in α^{FO} as long as it is below e^{FB}. If α^{FO} is larger, a further increase of rents for insiders would make it optimal for firms not to hire insiders anymore and instead keep vacancies open in the hope of attracting an outsider.

Therefore, a higher α^{FO} then increases outsiders' rents (until eventually they are the same as insiders').

The total effect of an inflow of outsiders would also have to incorporate the reduction in α^{N} and the (positive or negative) consequences on workers' outside options. As before, though, if we endogenized F the outside option of insiders would stay constant, thus the indirect effect would disappear and only the positive direct effect prevail (as in Section 3.4).

4.4 Further Results and Discussion of Assumptions

In what follows, we show that the positive effect can also extend to the case $\alpha^{FI} \leq \bar{\alpha}^{F}$. The next Lemma states that the threshold of α^{FI} above which insiders are paid a rent is smaller if outsiders are present, and decreases in α^{FO}.

Lemma 2 *There exists a $\bar{\alpha}^{FI} \leq \bar{\alpha}^{F}$ above which insiders are paid a rent. $\bar{\alpha}^{FI}$ is strictly decreasing in α^{FO}.*

Moreover, for $\alpha^{FI} \leq \bar{\alpha}^{FI}$, all profit-maximizing equilibria are symmetric and outsiders are treated exactly as insiders (thus, the outcome is equivalent to the one derived in Section 3).

Lemma 3 *Assume $\alpha^{FI} \leq \bar{\alpha}^{FI}$. Then, $e^{O} = e^{I}$ is uniquely optimal, as well as $w^{O} = w^{I} = 0$ and $b^{I} = b^{O} = c(e)$. Moreover, effort is larger than the level characterized by $\delta\gamma\theta - c'(e) = 0$ and decreasing in α^{FO}. Finally, $\partial\bar{\Pi}/\partial\alpha^{FO} > 0$.*

Note again that we have for simplicity assumed that an inflow of outsiders does not reduce α^{FI}. However, even if we allowed for such an interaction, our results would not change fundamentally as those rely on an increase in α^{F}, the *total* probability of firms being matched.

Finally, we discuss implications of insiders having a strictly positive exogenous outside option. Most of our results then continue to hold as long as α^{FO} is sufficiently small. With high α^{FI}, insiders would still get a rent, and the equilibrium in which insiders are paid more but work less hard could still be sustained. Only with a relatively large mass of outsiders (for example if $\alpha^{FO} \geq \bar{\alpha}^{F}$), excluding insiders could become optimal.

5 Applications

This section presents applications and empirical research related to our results. Regarding the potential positive wage effects of a higher labor supply, we discuss the literature on immigration. Regarding the discrimination equilibrium and potential consequences of a minimum wage, we first discuss research on the gender pay gap and then also on the consequences of immigration.

5.1 Immigration and Positive Wage Effects of a Higher Labor Supply

The immigration literature has extensively analyzed the effects of a higher labor supply on wages. A number of empirical studies lend support to the canonical model of the labor market, finding negative wage effects of immigration (Borjas, 2003; Borjas, 2017). Other studies come to different conclusions. In a seminal paper, Card (1990) studies a large inflow of unskilled Cuban immigrants into Miami in 1980. He finds no significant consequences for employment and wages of low-skilled non-Cubans. Peri and Yasenov (2019) confirm Card's results, with the point estimates of log wages even being positive. Winter-Ebmer and Zweimüller (1996) detect positive effects of immigration on the wages of young Austrians; Friedberg (2001) reports no significant impact of immigration from Soviet Union to Israel, and most point estimates are positive. Peri (2007) states that an increase in average wages of US-born workers is caused by immigration. Furthermore, exploring the consequences of immigration on US workers between 1990 and 2006, Ottaviano and Peri (2012) observe a significantly positive effect on wages of college- and noncollege-educated workers.[14]

To explain these findings, the literature has mostly focused on heterogeneity in worker skills — in particular between immigrants and native workers — and that native workers are able to switch to jobs with different skill demands (see Peri and Sparber, 2009, Ottaviano and Peri, 2012, or Peri, 2016).[15] Then, immigration generally has positive effects on high-skill and negative effects on low-skill native workers. While such an approach explains the effects of immigration on some wages, recent evidence suggests that there can be a non-negative wage effect even among

[14] Also see Dustmann et al. (2008) and Peri (2016) for surveys.

[15] Alternatively, Dustmann et al. (2012) claim that positive wage effects can follow from a perfectly elastic capital supply.

low-skill workers. For example, Foged and Peri (2016) explore how an exogenous inflow of refugees to Denmark affects native workers over the period 1991-2008. Using Danish administrative data on labor market outcomes of individuals, they find that the wages of native workers significantly went up. This wage increase is at least partly driven by native workers who do not change occupations and continue working on the same kinds of tasks as before. Among them, Foged and Peri (2016) do not find negative effects on natives' employment or depressing effects on their wages. Thus, these native workers perform the same kinds of tasks as immigrant workers (who are mostly low-skilled) but still benefit from their entry. Clemens and Hunt (2019) also do not find that immigration has substantial negative effects on low-skilled native workers. Moreover, Tabellini (2019) discovers that immigration across U.S. cities between 1910 and 1930 increased natives' employment, spurred industrial production, and did not generate losses even among those working in occupations highly exposed to immigrants' competition.

Our result highlights that an inflow of immigrants can increase wages in a setting where homogeneous workers need to be incentivized by self-enforcing agreements. Different from previous explanations, we do not assume that workers are paid their marginal productivity. Instead, we follow recent evidence that firms have wage-setting power (Manning, 2003; Dube et al., 2020; Manning, 2021; Card, 2022).

5.2 Discrimination on Labor Markets

5.2.1 Gender Pay Gap

A huge literature has explored the existence and potential causes of a gender pay gap. While a big part of the gap can be explained by fundamental differences such as experience or human capital, personality traits such as risk aversion, or preferences for bargaining, a substantial unexplained gender difference remains (see Blau and Kahn, 2017, for a great survey). Moreover, even though the female labor-force participation increased, the gender wage gap has not disappeared in many countries (see Blau and Kahn, 2007 and Heim, 2007 for evidence on the US). For potential explanations for the unexplained part of the gap, Blau and Kahn (2017) argue that the "formation of norms [...] is a potentially fruitful area of research that has received relatively little attention by economists".

We show that a norm that selects an equilibrium which benefits a majority group of workers facing an inflow of a minority group can lead to discrimination on labor markets. Note that the norm arises as a profit-maximizing equilibrium. Hence, even when a norm might be the consequence of direct discrimination in the past, it can survive after other causes of discrimination have disappeared.

5.2.2 Immigration

There is evidence that immigrant workers face discrimination. For example, Kerr and Kerr (2011) find that immigrants are paid less than natives even when conducting the same kinds of tasks, and this wage gap — although declining over time — persists in the long run. Furthermore, Battisti et al. (2018) analyze the consequences of immigration in 20 OECD countries and observe that, for each country and skill level, native workers are paid higher wages than immigrants. They discuss that natives' wage premia can be driven by either productivity differences or higher outside options. They rationalize their observations with a search-and-matching models of the labor market, where the reservation wages of immigrants are smaller by assumption. Moreover, positive effects of immigration on native workers' wages rely on both, task complementarity between native and immigrant workers and the creation of new jobs. Our paper *endogenously* generates a higher outside option of native workers, and furthermore higher effort of immigrants. The latter is consistent with Dustmann et al. (2012) as well, who argue that the extent of positive wage effects of immigration for some skill levels can be explained by productivity differences only if immigrant workers are more productive than natives and face a wage discount.

5.3 Discrimination and a Minimum Wage

We show that a minimum wage can reduce discrimination on labor markets. Indeed, Blau and Kahn (2000) emphasize the role of the minimum wage as an instrument that helped reduce the gender pay gap. Additional evidence is provided by Blau and Kahn (2003) who explore the role of minimum wages for reducing the gender gap in 22 countries. They identify a negative correlation between a gap and the extent to which a minimum wage binds. Ganguli and Terrell (2009) explore the interaction between a minimum wage and the gender gap in the Ukraine. The find that the gap substantially declined, in particular in the lower half of the wage distribution where a minimum wage is most relevant. Hallward-Driemeier et al. (2017) explore minimum wage increases in Indonesia, which reduced the gender wage gap in particular for educated women at the bottom of a firm's income distribution. Finally, Majchrowska and Strawinski (2018) observe that minimum wage increases in Poland between 2006 and 2010 substantially reduced the gender wage gap, with the reduction being especially pronounced for young workers.

Importantly, in our setting the minimum wage not only increases outsiders' wages, but also reduces insiders' wages. Although positive spillover effects have been observed and for example explained by firms wanting to preserve their wage distribution, there also is evidence that a minimum wage can cause negative spillover effects.[16] Analyzing the British minimum wage, Stewart (2012) finds that the minimum wage reduced wage growth for levels slightly above. Neumark et al. (2004) observe mixed results, in that immediate spillover effects are positive, lagged effects negative. Hirsch et al. (2015) argue that wages above the minimum wage increase less than they would without such a lower bound. To the best of our knowledge, however, there is no study that explores whether a reduced gender pay gap due to a minimum wage indeed is caused by higher female and lower male wages (or wage growth), as our model would indicate.

Finally, our model can provide a novel explanation for positive employment effects of a minimum wage. Excellent surveys such as Belman and Wolfson (2014) or Schmitt (2015), as well as recent papers (Cengiz et al., 2019) state that there is no evidence for a systematic negative employment effect of a (moderate) binding minimum wage. Several forces that could counteract the resulting higher wage costs of an increased minimum wage have been identified, among them higher productivity of workers (Fahn, 2017; Coviello et al., 2022). We show that positive employment effects can be a side effect of reduced discrimination, in that it increases the costs of employing one group and decreases the costs of employing another group of workers, with the latter dominating.

6 Discussion and Conclusion

We have demonstrated how norms on labor markets that determine which group of market participants benefits most if multiple equilibria exist might affect wages and cause discrimination, and how a minimum wage can reduce such discrimination and at the same time increase employment. To conclude, we discuss the robustness of our results once we relax some assumptions and suggest additional predictions which can be used to assess the validity of our model.

[16] An alternative explanation for negative spillover effects of a minimum wage is provided by Fahn and Seibel (2021).

Robustness First, we explore the consequences of workers having positive bargaining power in wage negotiations. Then, outcomes would rely on the exact specification of the bargaining process, whether disagreement payoffs are determined by separation or only by non-production (as in Hall and Milgrom, 2008), and to what extent renegotiation would happen. Here, we discuss one particular setting which is motivated by dynamic bargaining approaches such as Ramey and Watson (1997) or Fahn (2017).[17] Assume that, at the beginning of a period, firm and worker bargain about how the relationship surplus is shared. The relationship surplus contains the expected discounted sum of payoffs generated in this relationship (i.e., $(e\theta - c(e)) / (1 - \delta\gamma)$) minus disagreement payoffs. Disagreement would cause a termination of the match and let both players enter the matching market in the subsequent period. The bargaining outcome would determine a worker's *minimum payoff*, which however could unilaterally be increased by a firm; thus in equilibrium utility levels of workers would be higher than their bargaining outcomes if this also increased firms' profits. Finally, any deviation from equilibrium behavior would lead to a termination of the employment relationship.

Given the above bargaining setting, we now discuss the role of endogenous turnover costs. Endogenous turnover costs increase a firm's commitment and induce workers to exert higher effort. A positive bargaining power also provides incentives for workers to exert higher effort because they want to remain employed to secure the associated rent in the future. If this rent is sufficiently high, a "voluntary" increase by firms is not profitable. However, if the bargaining outcome is not sufficient to implement firms' desired effort, it remains optimal to increase the costs of turnover by paying workers an additional rent. The latter case is more likely if workers' bargaining power is low or if α^F is high, since a high α^F increases a firm's disagreement payoff and thus reduces the relationship surplus. Then, a higher labor supply will continue to increase an employed worker's compensation, making his bargaining power effectively irrelevant in determining his payoff. To the contrary, if bargaining outcomes would determine equilibrium payoffs (i.e., if worker bargaining power was large or α^F small), but also if formal contracts on effort were possible, an increase in N would reduce a worker's compensation via the negative effect on his disagreement payoffs.

Second, we discuss the specific form of endogenous turnover costs. Firms would

[17] Those are hybrid models where individual choices are made non-cooperatively but bargaining follows the cooperative Nash-bargaining regime (see Miller and Watson, 2013, for an axiomatic foundation); moreover, deviations cause a termination of the relationship .

be indifferent between increasing a worker's compensation (as in our setting) or using different measures, for example letting workers temporarily reduce their effort or conduct inefficient trainings, or doing anything else that destroys surplus by "money burning."[18] To assess a firm's credibility, though, it is necessary for workers to observe the realization of turnover costs. Thus, we would argue that the safest way for firms to ensure this is using options such as wages or effort reductions. Those directly affect workers and are obviously costly for firms without supplying direct benefits to them. It remains to discuss why, to make turnover more costly, firms do not use effort reductions in early periods instead of higher wages. Effort reductions would actually increase industry-wide profits because workers' outside options would be zero throughout. However, if we extended the model slightly, by introducing a product market where prices decrease in total output and allowing a firm to employ more than one worker, paying higher wages would dominate effort reductions for individual firms. The reason is that a rent paid in firm A increases workers' outside options in firm B. Thus, production becomes more expensive for firm B which would consequently reduce its employment and output, allowing firm A to boost its sales (naturally, firm B would do the same, causing adverse effects on firm A).

Third, we assume that a firm's chances to fill a vacancy, α^F, are exogenous to a firm's efforts. One might argue that firms should be able to increase α^F, for example by conducting costly search. Even then, our results continue to hold if firms are able to hide their *previous* search effort from a newly hired worker. To illustrate this argument, assume that α^F is exactly at $\bar{\alpha}^F$. Now, holding search effort fixed, an increase in N and the resulting higher α^F would make it optimal to increase a worker's compensation to keep effort (and consequently the firm's profits) constant. But then, the firm would be better off reducing costly search and keeping α^F at $\bar{\alpha}^F$. However, if workers believe that the firm has reduced its search effort but are not able to observe whether this has actually occurred, firms would have an incentive to secretly increase search effort once a vacancy opens up (without having to pay a higher wage), reducing their incentives to pay a promised bonus in their current employment relationship.

[18]See Carmichael and MacLeod (1997) or McAdams (2011) who demonstrate that inefficiencies in the early periods of repeated interactions with anonymous re-matching might be needed to sustain cooperation later on.

Additional Predictions In the following, we describe possibilities to further determine the relevance of our setup. The empirical assessments of the following predictions would not only help to evaluate the usefulness of our model, but also generate new insights on the consequences of a higher labor supply.

First, our results rely on the unavailability of formal, court-enforceable contracts to adequately motivate workers: if effort was verifiable, a higher labor supply would either have zero or negative effects on wages. Thus, we would predict that our mechanism is particularly relevant in settings where informal incentives and subjective performance measures are more important to motivate workers. We would argue that this holds in the service industry, where aspects such as friendliness or customer orientation are important but difficult to measure objectively. But also high-skill tasks or those that are R&D-intensive are often difficult to be incentivized with the use of formal contracts alone. Furthermore, firms have to rely on informal incentives if the legal system they operate in is weak. As an example for the latter, Fallah et al. (2019) investigate the impact of the Syrian refugee influx on labor market outcomes in Jordan. They find that employment and unemployment were unaffected, whereas hourly wages went up.

Second, the severity of labor market frictions plays an important role in our setting. Recall that, with endogenous F, the effect of a higher labor supply on wages is strictly positive for $\alpha^F \geq \bar{\alpha}^F$ and zero for $\alpha^F < \bar{\alpha}^F$; employment effects are ambiguous for $\alpha^F \geq \bar{\alpha}^F$ and strictly positive for $\alpha^F < \bar{\alpha}^F$. Thus, we would predict larger positive wage and smaller (positive or negative) employment effects in markets in which firms can fill a vacancy relatively easily, whereas tighter labor markets would be associated with a smaller (or even negative) wage effect but a larger positive impact on employment.

Third, our results are stronger whenever firms have more pronounced wage-setting or higher bargaining power. Put differently, we would expect negative wage effects of a higher labor supply in markets where workers are paid their marginal productivities, and non-negative or even positive effects if firms have the power to set the terms of employment.

Finally, an inflow of outsiders in the discrimination equilibrium increases the wage gap if α^{FO} is small, and reduces it once α^{FO} is sufficiently large. Therefore, our predictions regarding the consequences of an increased labor supply in majority-dominated industries depend on the initial share of the minority group.

Appendix

Proof of Proposition 1:

Here, we first show that our stationarity assumption is without loss of generality. Standard arguments can be applied to confirm that stationary arrangements are optimal from the second period of an employment relationship. In the first such period, though, wages might be different (if first-period effort or bonus were different than later values, the problem could be transformed into one that is payoff equivalent but in which only wages differ). Denote w_1 as the wage paid in the first, w the wage paid in all later periods of an employment relationship. Then, the optimization problem is to maximize

$$\max \bar{\Pi} = \frac{\alpha^F \left[e\theta - b - w_1 + \delta\gamma\left(w_1 - w\right)\right]}{\left(1-\delta\right)\left(1 - \delta\gamma\left(1-\alpha^F\right)\right)},$$

subject to

$$-c(e) + b + \delta\gamma w - \delta\gamma\left(1-\delta\gamma\right)\bar{U} \geq 0 \qquad \text{(IC)}$$

$$-b + \delta\gamma\left[\left(1-\alpha^F\right)e\theta + \alpha^F w_1 - w\right] \geq 0 \qquad \text{(DE)}$$

$$w_1\left(1-\delta\gamma\right) + w\delta\gamma + b - c(e) - \left(1-\delta\gamma\right)\bar{U} \geq 0 \qquad \text{(IR1)}$$

$$w + b - c(e) - \left(1-\delta\gamma\right)\bar{U} \geq 0 \qquad \text{(IR)}$$

To show that it is weakly optimal to set $w_1 = w$, let us to the contrary assume that there is a profit-maximizing social equilibrium with $w_1 > w$. Then, we can reduce b by $\delta\gamma\varepsilon$ and increase w by ε. This operation leaves Π_1, (IC), (DE), and (IR1) unaffected, but relaxes (IR). The opposite operation can be applied if $w_1 < w$, thus it is weakly optimal to set $w_1 = w$. Note that this holds for all periods besides the very first of the game.

Therefore, (IR) is implied by (IR1) and can be omitted. The Lagrange function becomes

$$\begin{aligned}\mathcal{L} = & \frac{\alpha^F\left[e\theta - b - w\right]}{\left(1-\delta\right)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} \\ & + \lambda_{IC}\left[-c(e) + b + \delta\gamma w - \delta\gamma\left(1-\delta\gamma\right)\bar{U}\right] \\ & + \lambda_{DE}\left[-b + \delta\gamma\left(1-\alpha^F\right)\left(e\theta - w\right)\right] \\ & + \lambda_{IR}\left[w + b - c(e) - \left(1-\delta\gamma\right)\bar{U}\right],\end{aligned}$$

with first-order conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial e} &= \frac{\alpha^F \theta}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} - \lambda_{IC} c'(e) + \lambda_{DE} \delta\gamma \left(1-\alpha^F\right)\theta - \lambda_{IR} c'(e) = 0 \\
\frac{\partial \mathcal{L}}{\partial b} &= -\frac{\alpha^F}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} + \lambda_{IC} - \lambda_{DE} + \lambda_{IR} = 0 \\
&\Rightarrow \lambda_{IR1} = \frac{\alpha^F}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} - \lambda_{IC} + \lambda_{DE} \\
\frac{\partial \mathcal{L}}{\partial w} &= -\frac{\alpha^F}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} + \lambda_{IC}\delta\gamma - \lambda_{DE}\delta\gamma\left(1-\alpha^F\right) + \lambda_{IR} = 0 \\
&\Rightarrow \lambda_{DE} = \lambda_{IC} \frac{(1-\delta\gamma)}{\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} \\
&\Rightarrow \lambda_{IR} = \frac{\alpha^F - (1-\delta)\,\delta\gamma\alpha^F\lambda_{IC}}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)}
\end{aligned}
$$

Note also that, if $\lambda_{IC} = \lambda_{DE} = 0$, then $\lambda_{IR} > 0$. Therefore, we have the following three cases: **1)** $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} = 0$, **2)** $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} > 0$, **3)** $\lambda_{IC} = \lambda_{DE} = 0$ and $\lambda_{IR} > 0$. In the following, we will derive the outcomes for all three cases, as well as the conditions for each of them to hold.

Case 1: $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} = 0$.

Now,

$$
\begin{aligned}
\lambda_{IR} =& \frac{\alpha^F - (1-\delta)\,\delta\gamma\alpha^F\lambda_{IC}}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)} = 0 \\
\Rightarrow& \lambda_{IC} = \frac{1}{(1-\delta)\,\delta\gamma} \\
\Rightarrow& \lambda_{DE} = \frac{(1-\delta\gamma)}{\left(1-\delta\gamma\left(1-\alpha^F\right)\right)(1-\delta)\,\delta\gamma},
\end{aligned}
$$

and effort is characterized by

$$
\theta - \frac{c'(e)}{\delta\gamma} = 0.
$$

Moreover, binding (IC) and (DE) constraints yield

$$
\begin{aligned}
w =& \frac{\delta\gamma\left(1-\delta\gamma\right)\bar{U}-\delta\gamma\left(1-\alpha^{F}\right)e\theta+c(e)}{\delta\gamma\alpha^{F}}\\
b =& \left(1-\alpha^{F}\right)\frac{\delta\gamma e\theta-c(e)-\delta\gamma\left(1-\delta\gamma\right)\bar{U}}{\alpha^{F}}\\
\Rightarrow w+b =& \frac{\delta\gamma\left(1-\delta\gamma\right)\bar{U}\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)}{\delta\gamma\alpha^{F}}\\
&+\frac{\left(1-\delta\gamma\left(1-\alpha^{F}\right)\right)c(e)-\delta\gamma\left(1-\alpha^{F}\right)e\theta\left(1-\delta\gamma\right)}{\delta\gamma\alpha^{F}},
\end{aligned}
$$

as well as

$$
\begin{aligned}
\bar{\Pi} &= \left[e\theta\delta\gamma-\delta\gamma\left(1-\delta\gamma\right)\bar{U}-c(e)\right]/\left[\delta\gamma\left(1-\delta\right)\right]\\
U &= \left[\delta\gamma\bar{U}\left(1-\left(1-\alpha^{F}\right)\delta\gamma\right)-\delta\gamma\left(1-\alpha^{F}\right)e\theta+c(e)\right]/\delta\gamma\alpha^{F}.
\end{aligned}
$$

Using $\overline{U}=\alpha^{N}\left[c(e)-\delta\gamma\left(1-\alpha^{F}\right)e\theta\right]/\left[\delta\gamma\left(1-\delta\gamma\right)\left(\alpha^{F}-\alpha^{N}\right)\right]$ yields

$$
\begin{aligned}
U =& \frac{\left[1-\delta\gamma\left(1-\alpha^{N}\right)\right]\left(c(e)-\delta\gamma\left(1-\alpha^{F}\right)e\theta\right)}{\delta\gamma\left(1-\delta\gamma\right)\left(\alpha^{F}-\alpha^{N}\right)}\\
w+b =& \frac{\left[1-\delta\gamma\left(1-\alpha^{N}\right)\right]\left(c(e)-\delta\gamma\left(1-\alpha^{F}\right)e\theta\right)}{\delta\gamma\left(\alpha^{F}-\alpha^{N}\right)}+c(e)\\
\Pi =& \frac{\left(1-\delta\left(1-\alpha^{F}\right)\right)\frac{\delta\gamma e\theta\left(1-\alpha^{N}\right)-c(e)}{\delta\gamma\left(\alpha^{F}-\alpha^{N}\right)}}{\left(1-\delta\right)}\\
\bar{\Pi} =& \frac{\alpha^{F}}{1-\delta\left(1-\alpha^{F}\right)}\Pi=\alpha^{F}\frac{\delta\gamma e\theta\left(1-\alpha^{N}\right)-c(e)}{\delta\gamma\left(1-\delta\right)\left(\alpha^{F}-\alpha^{N}\right)}
\end{aligned}
$$

Moreover, $w=\left[c(e)-\delta\gamma\left(1-\alpha^{F}\right)e\theta\right]/\left[\delta\gamma\left(\alpha^{F}-\alpha^{N}\right)\right]$ and $b=\left(1-\alpha^{F}\right)\left[\delta\gamma e\theta\left(1-\alpha^{N}\right)-c(e)\right]/\left(\alpha^{F}-\alpha^{N}\right)$.
The consistency requirement is

$$
\begin{aligned}
&U\geq\bar{U}\\
\Leftrightarrow&\alpha^{F}\geq\frac{\delta\gamma\left(e\theta-\bar{U}\left(1-\delta\gamma\right)\right)-c(e)}{\delta\gamma\left(e\theta-\bar{U}\left(1-\delta\gamma\right)\right)}
\end{aligned}
$$

Due to symmetry, $\bar{U}=0$ at the threshold. Hence, this case holds if

$$
\alpha^{F}\geq1-\frac{c(e^{*})}{\delta\gamma e^{*}\theta},
$$

where e^* is characterized by

$$\theta - \frac{c'(e^*)}{\delta\gamma} = 0.$$

Case 2: $\lambda_{IC}, \lambda_{DE} > 0$ and $\lambda_{IR} > 0$.

Binding (IC) and (DE) constraints yield

$$\begin{aligned}
w =& \frac{\delta\gamma\,(1-\delta\gamma)\,\bar{U} + c(e) - \delta\gamma\left(1-\alpha^F\right)e\theta}{\delta\gamma\alpha^F} \\
b =& \left(1-\alpha^F\right)\frac{\delta\gamma e\theta - c(e) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}}{\alpha^F} \\
\Rightarrow w + b =& \frac{\delta\gamma\,(1-\delta\gamma)\,\bar{U}\left(1-\left(1-\alpha^F\right)\delta\gamma\right)}{\delta\gamma\alpha^F} \\
&+ \frac{\left(1-\delta\gamma\left(1-\alpha^F\right)\right)c(e) - \delta\gamma\left(1-\alpha^F\right)e\theta\,(1-\delta\gamma)}{\delta\gamma\alpha^F},
\end{aligned}$$

The binding (IR) constraint delivers $U = \bar{U} = 0$ and equilibrium effort which is characterized by

$$\delta\gamma\left(1-\alpha^F\right)e^*\theta - c(e^*) = 0. \tag{4}$$

This case holds if the condition from case 1 is not satisfied, and if e^* here is below e^{FB}. Finally, incorporating (4) yields

$$\begin{aligned}
w =& \frac{c(e^*) - \delta\gamma\left(1-\alpha^F\right)e^*\theta}{\delta\gamma\alpha^F} = 0 \\
b =& c(e^*) \\
\bar{\Pi} =& \frac{\alpha^F\left(e^*\theta - c(e^*)\right)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)}
\end{aligned}$$

Case 3: $\lambda_{IC} = \lambda_{DE} = 0$, $\lambda_{IR} > 0$

Now, $e^* = e^{FB}$, hence $\theta - c'(e) = 0$. This case holds if

$$\alpha^F < 1 - \frac{c(e^{FB})}{\delta\gamma e^{FB}\theta},$$

and

$$\begin{aligned}
w =& 0 \\
b =& c(e^{FB}) \\
\bar{\Pi} =& \frac{\alpha^F \left(e^{FB}\theta - c(e^{FB})\right)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)}
\end{aligned}$$

■

Proof of Corollary 1:

1. $\alpha^F \geq \bar{\alpha}^F$

$c'(e^*) = \delta\gamma\theta$ if $\alpha^F \geq \bar{\alpha}^F$, and hence $de^*/d\alpha^F = 0$.

For the following, note that $1-\alpha^N = \frac{(1-\gamma+\alpha^F)N-(2-\gamma)F\alpha^F}{(1-\gamma+\alpha^F)N-F\alpha^F}$, $\alpha^F-\alpha^N = \frac{1-\gamma+\alpha^F}{(1-\gamma+\alpha^F)N-F\alpha^F}(N-F)\alpha^F$, $1-\gamma+\alpha^N = \frac{(1-\gamma)(1-\gamma+\alpha^F)N}{(1-\gamma+\alpha^F)N-F\alpha^F}$, and $\frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N} = \frac{(1-\delta\gamma)(1-\gamma+\alpha^F)N-(1-2\delta\gamma+\delta\gamma^2)F\alpha^F}{(1-\gamma)F\alpha^F}$.

Now,

$$\begin{aligned}
\overline{U} =& \frac{[c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta](1-\gamma)F}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)(N-F)}, \\
U =& \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N}\overline{U} \\
=& \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N}\frac{[c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta](1-\gamma)F}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)(N-F)} \\
=& \frac{[c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta][(1-\delta\gamma)(1-\gamma+\alpha^F)N-(1-2\delta\gamma+\delta\gamma^2)F\alpha^F]}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)(N-F)\alpha^F},
\end{aligned}$$

$$\begin{aligned}
w+b =& (1-\delta\gamma)\,U + c(e^*) \\
=& \frac{[c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta][(1-\delta\gamma)(1-\gamma+\alpha^F)N-(1-2\delta\gamma+\delta\gamma^2)F\alpha^F]}{\delta\gamma(1-\gamma+\alpha^F)(N-F)\alpha^F} + c(e^*), \\
\bar{\Pi} =& \frac{(1-\gamma+\alpha^F)N(\delta\gamma e^*\theta - c(e^*)) - F\alpha^F[(2-\gamma)\delta\gamma e^*\theta - c(e^*)]}{\delta\gamma\left(1-\delta\right)\left(1-\gamma+\alpha^F\right)(N-F)}.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\frac{\partial\overline{U}}{\partial\alpha^F} =& \frac{(1-\gamma)F\left[\delta\gamma e^*\theta\left(2-\gamma\right)-c(e^*)\right]}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)^2(N-F)} > 0, \\
\frac{\partial\overline{U}}{\partial N} =& -\frac{(1-\gamma)F\left(c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta\right)}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)(N-F)^2} > 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{dU}{dN} =& \frac{d\left(\frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N}\right)}{dN}\overline{U} + \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N}\frac{d\overline{U}}{dN} \\
=&(1-\delta\gamma)\left(\frac{(1-\gamma+\alpha^F)}{(1-\gamma)F\alpha^F} - \frac{N\left(\alpha^F\right)'}{F\left(\alpha^F\right)^2}\right)\overline{U} + \frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N}\frac{d\overline{U}}{dN} \\
=&\frac{\left(c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta\right)}{\delta\gamma(N-F)}\left[\frac{1}{\alpha^F} - \frac{\left[1-\gamma\delta(1-\alpha^N)\right](1-\gamma)F}{\alpha^N(1-\delta\gamma)(1-\gamma+\alpha^F)(N-F)}\right] \\
&-N\left(\alpha^F\right)'\frac{(1-\gamma)\left(c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta\right)}{\delta\gamma(1-\gamma+\alpha^F)(N-F)\left(\alpha^F\right)^2} \\
&+\frac{1-\gamma\delta(1-\alpha^N)}{\alpha^N}\frac{(1-\gamma)F\left[\delta\gamma e^*\theta(2-\gamma)-c(e^*)\right]}{\delta\gamma(1-\delta\gamma)\left(1-\gamma+\alpha^F\right)^2(N-F)}\left(\alpha^F\right)' \\
=&N\left(\alpha^F\right)'\frac{(\delta\gamma e^*\theta - c(e^*))}{\left(\alpha^F\right)^2\delta\gamma(N-F)} - F\left(\alpha^F\right)'\frac{(1-2\delta\gamma+\delta\gamma^2)\left[\delta\gamma e^*\theta(2-\gamma)-c(e^*)\right]}{\delta\gamma(1-\delta\gamma)\left(1-\gamma+\alpha^F\right)^2(N-F)} \\
&-F(1-\gamma)\frac{c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta}{\delta\gamma(N-F)^2\alpha^F(1-\delta\gamma)}\left(\frac{1-\delta\gamma+\delta\gamma\alpha^F}{1-\gamma+\alpha^F}\right)
\end{aligned}$$

For any strictly positive $\left(\alpha^F\right)'$ and N, this is strictly positive if F is sufficiently small. Furthermore,

$$\begin{aligned}
\frac{d\overline{U}}{dN} =& -(1-\gamma)F\frac{c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta}{\delta\gamma(1-\delta\gamma)(1-\gamma+\alpha^F)(N-F)^2} \\
&+(1-\gamma)F\frac{(2-\gamma)\delta\gamma e^*\theta - c(e^*)}{\delta\gamma(1-\delta\gamma)\left(1-\gamma+\alpha^F\right)^2(N-F)}\left(\alpha^F\right)'
\end{aligned}$$

which is smaller than dU/dN but can still be positive.

If $dU/dN > 0$, the same holds for $d(w+b)/dN$, and vice versa.

Finally,

$$\begin{aligned}
\frac{d\bar{\Pi}}{dN} =&(1-\gamma)F\frac{c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta}{\delta\gamma(1-\delta)(1-\gamma+\alpha^F)(N-F)^2} \\
&-(1-\gamma)F\frac{(2-\gamma)\delta\gamma e^*\theta - c(e^*)}{\delta\gamma(1-\delta)(1-\gamma+\alpha^F)^2(N-F)}\left(\alpha^F\right)',
\end{aligned}$$

which reveals

$$\frac{d\bar{\Pi}}{dN} = -\frac{d\overline{U}}{dN}.$$

2. $\alpha^F \leq \bar{\alpha}^F$

The above optimization problem yields

$$w + b = \frac{\left(1 - \delta\gamma\left(1 - \alpha^F\right)\right) c(e) - \delta\gamma\left(1 - \alpha^F\right) e\theta\left(1 - \delta\gamma\right)}{\delta\gamma\alpha^F} = c(e)$$
$$\bar{\Pi} = \frac{\alpha^F\left(e^*\theta - c(e^*)\right)}{\left(1 - \delta\right)\left(1 - \delta\gamma\left(1 - \alpha^F\right)\right)}$$

Therefore,

$$\frac{\partial e^*}{\partial N} = \frac{\delta\gamma e^*\theta}{\delta\gamma\left(1 - \alpha^F\right)\theta - c'(e^*)}\left(\alpha^F\right)' < 0$$

$$\frac{\partial\left(w + b\right)}{\partial N} = c'(e^*)\frac{\partial e^*}{\partial N} < 0$$

If $\alpha^F \leq \underline{\alpha}^F$ (and $e^* = e^{FB}$) comparative statics are equivalent, only $\partial e^*/\partial N = 0$.

■

Proof of Proposition 2:

Again we distinguish between $\alpha^F \geq \bar{\alpha}^F$ and $\alpha^F < \bar{\alpha}^F$ and conduct comparative statics for each case separately.

1. $\alpha^F \geq \bar{\alpha}^F$

In the proof to Corollary 1 we have derived

$$\bar{\Pi} = \frac{(1 - \gamma + \alpha^F)N(\delta\gamma e^*\theta - c(e^*)) - F\alpha^F[(2 - \gamma)\delta\gamma e^*\theta - c(e^*)]}{\delta\gamma\left(1 - \delta\right)\left(1 - \gamma + \alpha^F\right)(N - F)}.$$

Thus,

$$
\begin{aligned}
\frac{\partial\bar{\Pi}}{\partial N} =& F(1-\gamma)\frac{c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta}{\delta\gamma\left(1-\delta\right)\left(1-\gamma+\alpha^F\right)(N-F)^2} \\
&- F(1-\gamma)\frac{\left[(2-\gamma)\delta\gamma e^*\theta-c(e^*)\right]}{\delta\gamma\left(1-\delta\right)\left(1-\gamma+\alpha^F\right)^2(N-F)}\left(\alpha^F\right)' \\
\frac{\partial\bar{\Pi}}{\partial F} =& -N\left(1-\gamma\right)\frac{c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta}{\delta\gamma\left(1-\delta\right)\left(1-\gamma+\alpha^F\right)(N-F)^2} \\
&+ F(1-\gamma)\frac{\left[(2-\gamma)\delta\gamma e^*\theta-c(e^*)\right]}{\delta\gamma\left(1-\delta\right)\left(1-\gamma+\alpha^F\right)^2(N-F)}\left(\alpha^F\right)',
\end{aligned}
$$

where $\partial\bar{\Pi}/\partial F < 0$ in equilibrium (otherwise, more firms would directly increase profits, causing additional entry).

Moreover,

$$
\begin{aligned}
\frac{dU}{dN} =& \frac{\partial U}{\partial N}+\frac{\partial U}{\partial F}\frac{dF}{dN}+\frac{\partial U}{\partial\alpha^F}\left(\left(\alpha^F\right)'-\left(\alpha^F\right)'\frac{dF}{dN}\right) \\
=& \frac{\left[c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta\right]\left(1-\gamma\right)\left(1-\delta\gamma+\delta\gamma\alpha^F\right)}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)\alpha^F(N-F)^2}\left(N\frac{dF}{dN}-F\right) \\
&+\left[N\frac{\left(\delta\gamma e\theta-c(e)\right)}{\left(\alpha^F\right)^2}-F\frac{\left(1-2\delta\gamma+\delta\gamma^2\right)\left[\delta\gamma e\theta\left(2-\gamma\right)-c(e)\right]}{\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)^2}\right]\frac{\left(1-\frac{dF}{dN}\right)}{\delta\gamma(N-F)}\left(\alpha^F\right)',
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{dF}{dN} =& \frac{F\frac{c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta}{(N-F)}-F\frac{\left[(2-\gamma)\delta\gamma e^*\theta-c(e^*)\right]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'}{N\frac{c(e^*)-(1-\alpha^F)\delta\gamma e^*\theta}{(N-F)}-F\frac{\left[(2-\gamma)\delta\gamma e^*\theta-c(e^*)\right]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'} \\
1-\frac{dF}{dN} =& \frac{c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta}{N\frac{c(e^*)-(1-\alpha^F)\delta\gamma e^*\theta}{(N-F)}-F\frac{\left[(2-\gamma)\delta\gamma e^*\theta-c(e^*)\right]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'} > 0 \\
N\frac{dF}{dN}-F =& -\frac{F\left(N-F\right)\frac{\left[(2-\gamma)\delta\gamma e^*\theta-c(e^*)\right]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'}{N\frac{c(e^*)-(1-\alpha^F)\delta\gamma e^*\theta}{(N-F)}-F\frac{\left[(2-\gamma)\delta\gamma e^*\theta-c(e^*)\right]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'} < 0.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{dU}{dN} =& \frac{\frac{\left(c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta\right)}{\alpha^F\delta\gamma(N-F)}\left(\frac{N(\delta\gamma e\theta-c(e))}{\alpha^F}-F\frac{[(2-\gamma)\delta\gamma e^*\theta-c(e^*)]}{(1-\gamma+\alpha^F)}\right)}{N\frac{c(e^*)-(1-\alpha^F)\delta\gamma e^*\theta}{(N-F)}-F\frac{[(2-\gamma)\delta\gamma e^*\theta-c(e^*)]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'}\left(\alpha^F\right)' \\
=& \frac{(1-\delta)\left[c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta\right]\bar{\Pi}}{\left(\alpha^F\right)^2\left[N\frac{c(e^*)-(1-\alpha^F)\delta\gamma e^*\theta}{(N-F)}-F\frac{[(2-\gamma)\delta\gamma e^*\theta-c(e^*)]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'\right]} > 0,
\end{aligned}
$$

since $\partial\bar{\Pi}/\partial F < 0$ in equilibrium yields a positive denominator.

Moreover,

$$
\begin{aligned}
\frac{d\overline{U}}{dN} =& \frac{\partial\overline{U}}{\partial N}+\frac{\partial\overline{U}}{\partial F}\frac{dF}{dN}+\frac{\partial\overline{U}}{\partial\alpha^F}\left(1-\frac{dF}{dN}\right)\left(\alpha^F\right)' \\
=& \left(N\frac{dF}{dN}-F\right)\frac{(1-\gamma)\left(c(e^*)-\delta\gamma\left(1-\alpha^F\right)e^*\theta\right)}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)(N-F)^2} \\
&+\frac{(1-\gamma)F\left(\delta\gamma e^*\theta\left(2-\gamma\right)-c(e^*)\right)}{\delta\gamma\left(1-\delta\gamma\right)\left(1-\gamma+\alpha^F\right)^2(N-F)}\left(1-\frac{dF}{dN}\right)\left(\alpha^F\right)' \\
=& \frac{\left(c(e^*)-\left(1-\alpha^F\right)\delta\gamma e^*\theta\right)\left(\delta\gamma e^*\theta\left(2-\gamma\right)-c^*(e)\right)\frac{(1-\gamma)(F-F)}{(1-\gamma+\alpha^F)^2(N-F)}}{\delta\gamma\left(1-\delta\gamma\right)\left[N\frac{c(e^*)-(1-\alpha^F)\delta\gamma e^*\theta}{(N-F)}-F\frac{[(2-\gamma)\delta\gamma e^*\theta-c(e^*)]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'\right]}\left(\alpha^F\right)' \\
=& 0.
\end{aligned}
$$

$dU/dN > 0$ implies that $d\left(w+b\right)/dN > 0$ as well. Now, we explore how the total effect on compensation is driven by changes in wage and bonus. Recall that

$$
\begin{aligned}
w =& \frac{\delta\gamma\left(1-\delta\gamma\right)\bar{U}-\delta\gamma\left(1-\alpha^F\right)e\theta+c(e)}{\delta\gamma\alpha^F} \\
b =& \left(1-\alpha^F\right)\frac{\delta\gamma e\theta-c(e)-\delta\gamma\left(1-\delta\gamma\right)\bar{U}}{\alpha^F}.
\end{aligned}
$$

Because $d\overline{U}/dN = 0$, we can treat $\overline{U}$ as a constant, hence

$$
\begin{aligned}
\frac{dw}{dN} =& \frac{\delta\gamma e\theta-c(e)-\delta\gamma\left(1-\delta\gamma\right)\bar{U}}{\delta\gamma\left(\alpha^F\right)^2}\left(1-\frac{dF}{dN}\right)\left(\alpha^F\right)' > 0 \\
\frac{db}{dN} =& -\frac{\delta\gamma e\theta-c(e)-\delta\gamma\left(1-\delta\gamma\right)\bar{U}}{\left(\alpha^F\right)^2}\left(1-\frac{dF}{dN}\right)\left(\alpha^F\right)' < 0.
\end{aligned}
$$

Finally, note that dF/dN is positive if $\partial\bar{\Pi}/\partial N > 0$, with the sign of $\partial\bar{\Pi}/\partial N$ being identical to the sign of

$$\frac{c(e^*) - \left(1-\alpha^F\right)\delta\gamma e^*\theta}{(N-F)} - \frac{[(2-\gamma)\delta\gamma e^*\theta - c(e^*)]}{(1-\gamma+\alpha^F)}\left(\alpha^F\right)'.$$

This is positive if $\left(\alpha^F\right)'$ is small. To the contrary, at $\alpha^F = \left(\delta\gamma e^*\theta - c(e^*)\right)/\delta\gamma e^*\theta$,this term equals

$$-\delta\gamma e^*\theta\left(\alpha^F\right)' < 0.$$

2. $\alpha^F < \bar{\alpha}^F$

Now, equilibrium effort is given by

$$\delta\gamma\left(1-\alpha^F\right)e^*\theta - c(e^*) = 0 \tag{5}$$

or $e^* = e^{FB}$, whichever is smaller. If $e^* = e^{FB}$, $de^*/dN = 0$. If e^* is determined by (5),

$$\frac{\partial e^*}{\partial\alpha^F} = \frac{\delta\gamma e^*\theta}{\delta\gamma\left(1-\alpha^F\right)\theta - c'(e^*)} < 0.$$

and

$$\bar{\Pi} = \frac{\alpha^F\left(e\theta - c(e^*)\right)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)},$$

with

$$\begin{aligned}
\frac{\partial\bar{\Pi}}{\partial F} &= \left(\frac{\left(e\theta - c(e^*)\right)(1-\delta\gamma)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)^2} + \frac{\alpha^F\left(\theta - c'(e^*)\right)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)}\frac{\partial e^*}{\partial\alpha^F}\right)\left(\alpha^F\right)' \\
&= \frac{e^*\theta}{(1-\delta)}\frac{\delta\gamma\theta - c'(e^*)}{\left[\delta\gamma\left(1-\alpha^F\right)\theta - c'(e^*)\right]}\left(\alpha^F\right)' < 0 \\
\frac{\partial\bar{\Pi}}{\partial N} &= -\left(\frac{\left(e\theta - c(e^*)\right)(1-\delta\gamma)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)^2} + \frac{\alpha^F\left(\theta - c'(e^*)\right)}{(1-\delta)\left(1-\delta\gamma\left(1-\alpha^F\right)\right)}\frac{\partial e^*}{\partial\alpha^F}\right)\left(\alpha^F\right)' \\
&= -\frac{e^*\theta}{(1-\delta)}\frac{\delta\gamma\theta - c'(e^*)}{\left[\delta\gamma\left(1-\alpha^F\right)\theta - c'(e^*)\right]}\left(\alpha^F\right)' > 0.
\end{aligned}$$

Therefore,

$$\frac{dF}{dN} = 1 > 0,$$

i.e., employment effects are positive, and

$$\frac{de^*}{dN} = \frac{\partial e^*}{\partial \alpha^F}\left(\left(\alpha^F\right)' - \frac{dF}{dN}\left(\alpha^F\right)'\right) = 0.$$

It follows that $d(w+b)/dN = 0$. ■

Proof of Proposition 3 and Lemma 1:
The set of constraints is

$$-b^I + \delta\gamma\left(\Pi^I - \bar{\Pi}\right) \geq 0 \tag{DEI}$$
$$-b^O + \delta\gamma\left(\Pi^O - \bar{\Pi}\right) \geq 0 \tag{DEO}$$
$$U^I - \bar{U}^I \geq 0 \tag{IRI}$$
$$U^O - \bar{U}^O \geq 0 \tag{IRO}$$
$$-c(e^I) + b^I + \delta\gamma\left[U^I - \bar{U}^I\right] \geq 0 \tag{ICI}$$
$$-c(e^O) + b^O + \delta\gamma\left[U^O - \bar{U}^O\right] \geq 0 \tag{ICO}$$

Now, $\Pi^I = \pi^I + \delta\left[\gamma\Pi^I + (1-\gamma)\bar{\Pi}\right]$ and
$\Pi^O = \pi^O + \delta\left[\gamma\Pi^O + (1-\gamma)\bar{\Pi}\right]$, where $\pi^I = e^I\theta - w^I - b^I$ and $\pi^O = e^O\theta - w^O - b^O$.
Thus,

$$\begin{aligned}
\Pi^I =& \frac{\pi^I\left(1-\delta+\delta\alpha^{FI}\right)(1-\delta\gamma) + \delta\alpha^{FO}\left[(1-\delta)\gamma\pi^I + (1-\gamma)\pi^O\right]}{(1-\delta)(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
\Pi^O =& \frac{\pi^O\left(1-\delta+\delta\alpha^{FO}\right)(1-\delta\gamma) + \delta\alpha^{FI}\left[(1-\delta)\gamma\pi^O + (1-\gamma)\pi^I\right]}{(1-\delta)(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
\bar{\Pi} =& \frac{\alpha^{FI}\pi^I + \alpha^{FO}\pi^O}{(1-\delta)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
\Pi^I - \bar{\Pi} =& \frac{\left(1-\alpha^{FI}-\delta\gamma\left(1-\alpha^F\right)\right)\pi^I - \alpha^{FO}\pi^O}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
\Pi^O - \bar{\Pi} =& \frac{\left(1-\alpha^{FO}-\delta\gamma\left(1-\alpha^F\right)\right)\pi^O - \alpha^{FI}\pi^I}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}
\end{aligned}$$

This allows us to rewrite the optimization problem, which becomes to maximize $\alpha^{FI}\pi^I + \alpha^{FO}\pi^O$, subject to

$$-b^I + \delta\gamma \frac{\left(1-\alpha^{FI} - \delta\gamma\left(1-\alpha^F\right)\right)\pi^I - \alpha^{FO}\pi^O}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \geq 0 \tag{DEI}$$

$$\Leftrightarrow \frac{-b^I\left(1-\delta\gamma+\delta\gamma\alpha^{FO}\right) + \delta\gamma\left(e^I\theta - w^I\right)\left[1-\alpha^{FI}-\delta\gamma\left(1-\alpha^F\right)\right] - \delta\gamma\alpha^{FO}\pi^O}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \geq 0 \tag{DEI}$$

$$-b^O + \delta\gamma \frac{\left(1-\alpha^{FO} - \delta\gamma\left(1-\alpha^F\right)\right)\pi^O - \alpha^{FI}\pi^I}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \geq 0 \tag{DEO}$$

$$\Leftrightarrow \frac{-b^O\left(1-\delta\gamma+\delta\gamma\alpha^{FI}\right) + \delta\gamma\left(1-\alpha^{FO}-\delta\gamma\left(1-\alpha^F\right)\right)\left(e^O\theta - w^O\right) - \delta\gamma\alpha^{FI}\pi^I}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \geq 0 \tag{DEO}$$

$$-c(e^I) + b^I + \delta\gamma\left(w^I - (1-\delta\gamma)\,\bar{U}^I\right) \geq 0 \tag{ICI}$$

$$w^I + b^I - c(e^I) - (1-\delta\gamma)\,\bar{U}^I \geq 0 \tag{IRI}$$

$$-c(e^O) + b^O + \delta\gamma\left(w^O - (1-\delta\gamma)\,\bar{U}^O\right) \geq 0 \tag{ICO}$$

$$w^O + b^O - c(e^O) - (1-\delta\gamma)\,\bar{U}^O \geq 0 \tag{IRO}$$

There, $\bar{U}^I = \alpha^N U^I + (1-\alpha^N)\delta\gamma\bar{U}^I$ and $\bar{U}^O = \alpha^N U^O + (1-\alpha^N)\delta\gamma\bar{U}^O$ are taken as given by firms. Also note that b^I and b^O cannot be negative. We omit these conditions for now and will later check whether they are satisfied.

Setting up the Lagrange function and obtaining first-order conditions yields

$$
\begin{aligned}
\frac{\partial L}{\partial e^I} =& \alpha^{FI}\theta + \lambda_{DEI}\delta\gamma\frac{\theta\left(1-\alpha^{FI}\right)(1-\delta\gamma)+\alpha^{FO}\delta\gamma\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
& - \lambda_{DEO}\delta\gamma\frac{\alpha^{FI}\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} - c'(e^I)\left(\lambda_{ICI}+\lambda_{IRI}\right) = 0 \\
\frac{\partial L}{\partial e^O} =& \alpha^{FO}\theta - \lambda_{DEI}\delta\gamma\frac{\alpha^{FO}\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
& + \lambda_{DEO}\delta\gamma\frac{\theta\left(1-\alpha^{FO}\right)(1-\delta\gamma)+\alpha^{FI}\delta\gamma\theta}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} - c'(e^O)\left(\lambda_{ICO}+\lambda_{IRO}\right) = 0 \\
\frac{\partial L}{\partial b^I} =& -\alpha^{FI} - \lambda_{DEI}\frac{1-\delta\gamma+\alpha^{FO}\delta\gamma}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
& + \lambda_{DEO}\frac{\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} + \lambda_{ICI} + \lambda_{IRI} = 0 \\
& \Rightarrow \lambda_{ICI} = \alpha^{FI} + \frac{\lambda_{DEI}\left(1-\delta\gamma+\alpha^{FO}\delta\gamma\right) - \lambda_{DEO}\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} - \lambda_{IRI} \\
\frac{\partial L}{\partial w^I} =& -\alpha^{FI} - \lambda_{DEI}\delta\gamma\frac{\left(1-\alpha^{FI}\right)(1-\delta\gamma)+\alpha^{FO}\delta\gamma}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} \\
& + \lambda_{DEO}\frac{\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)} + \lambda_{ICI}\delta\gamma + \lambda_{IRI} = 0
\end{aligned}
$$

$$
\begin{aligned}
& \Rightarrow \lambda_{IRI} = \alpha^{FI}\left(1 - \frac{\delta\gamma\left(\lambda_{DEI}+\lambda_{DEO}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^{F}\delta\gamma\right)}\right) \\
& \Rightarrow \lambda_{ICI} = \frac{\lambda_{DEI}}{(1-\delta\gamma)}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial L}{\partial b^O} =& -\alpha^{FO} + \lambda_{DEI}\frac{\delta\gamma\alpha^{FO}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
& -\lambda_{DEO}\frac{1-\delta\gamma+\delta\gamma\alpha^{FI}}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} + \lambda_{ICO} + \lambda_{IRO} = 0 \\
\Rightarrow & \lambda_{ICO} = \alpha^{FO} - \frac{\lambda_{DEI}\delta\gamma\alpha^{FO} - \lambda_{DEO}\left(1-\delta\gamma+\delta\gamma\alpha^{FI}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} - \lambda_{IRO}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial w^O} =& -\alpha^{FO} + \delta\gamma\frac{\lambda_{DEI}\alpha^{FO} - \lambda_{DEO}\left[\left(1-\alpha^{FO}\right)(1-\delta\gamma)+\alpha^{FI}\delta\gamma\right]}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)} \\
& + \lambda_{ICO}\delta\gamma + \lambda_{IRO} = 0 \\
\Rightarrow & \lambda_{IRO} = \alpha^{FO}\left(1 - \frac{\delta\gamma\left(\lambda_{DEI}+\lambda_{DEO}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}\right) \\
\Rightarrow & \lambda_{ICO} = \frac{\lambda_{DEO}}{(1-\delta\gamma)}
\end{aligned}
$$

Thus, effort levels are characterized by

$$
\alpha^{FI}\left(\theta - c'(e^I)\right)\left(1 - \frac{\delta\gamma\left(\lambda_{DEI}+\lambda_{DEO}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}\right) + \frac{\delta\gamma\theta - c'(e^I)}{(1-\delta\gamma)}\lambda_{DEI} = 0
$$

$$
\alpha^{FO}\left(\theta - c'(e^O)\right)\left(1 - \frac{\delta\gamma\left(\lambda_{DEO}+\lambda_{DEI}\right)}{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}\right) + \frac{\delta\gamma\theta - c'(e^O)}{(1-\delta\gamma)}\lambda_{DEO} = 0
$$

Results The analysis yields the following outcomes: Either, $\lambda_{IRI} = \lambda_{IRO} = 0$ or $\lambda_{IRI}, \lambda_{IRO} > 0$. Moreover, either $\lambda_{DEI}, \lambda_{ICI} > 0$ or $\lambda_{DEI} = \lambda_{ICI} = 0$, and equivalently for λ_{DEO} and λ_{ICO}.

For the proposition, we focus on the case $\lambda_{IRI} = \lambda_{IRO} = 0$ and differentiating with respect to λ_{DEI} and λ_{DEO}. The case $\lambda_{IRI}, \lambda_{IRO} > 0$ is explored in Lemma 3.

With $\lambda_{IRI} = \lambda_{IRO} = 0$, $\lambda_{DEI} + \lambda_{DEO} = \frac{(1-\delta\gamma)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}{\delta\gamma}$, thus effort levels are characterized by

$$
\begin{aligned}
\left(\delta\gamma\theta - c'(e^I)\right)\frac{\lambda_{DEI}}{(1-\delta\gamma)} &= 0 \\
\left(\delta\gamma\theta - c'(e^O)\right)\frac{\lambda_{DEO}}{(1-\delta\gamma)} &= 0.
\end{aligned}
$$

First, we show that we can ignore the cases ($\lambda_{ICO}, \lambda_{DEO} > 0$; $\lambda_{ICI} = \lambda_{DEI} = 0$) and ($\lambda_{ICO}, \lambda_{DEO} > 0$; $\lambda_{ICI}, \lambda_{DEI} > 0$).

To the contrary, assume

I) $\lambda_{ICO}, \lambda_{DEO} > 0$**,** $\lambda_{ICI} = \lambda_{DEI} = 0$

Then, e^O is characterized by $\delta\gamma\theta - c'(e^O) = 0$, e^I is not uniquely identified. Binding (ICO) and (DEO) constraints yield

$$\begin{aligned} w^O =& \frac{\delta\gamma\alpha^{FI}\pi^I + \delta\gamma e^O\theta\alpha^{FO}(1-\delta\gamma) - \left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1-\delta\gamma)\bar{U}^O\right)\left(1-\delta\gamma+\alpha^{FI}\delta\gamma\right)}{\delta\gamma(1-\delta\gamma)\alpha^{FO}} \\ b^O =& \delta\gamma\frac{-\delta\gamma\alpha^{FI}\pi^I + \left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1-\delta\gamma)\bar{U}^O\right)\left(1-\delta\gamma+\alpha^F\delta\gamma-\alpha^{FO}\right)}{\delta\gamma(1-\delta\gamma)\alpha^{FO}} \\ w^O + b^O =& \frac{\delta\gamma\alpha^{FI}\pi^I + \delta\gamma e^O\theta\alpha^{FO} - \left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1-\delta\gamma)\bar{U}^O\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}{\delta\gamma\alpha^{FO}}, \end{aligned}$$

and

$$\bar{\Pi} = \frac{\left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1-\delta\gamma)\bar{U}^O\right)}{\delta\gamma(1-\delta)}$$

Consistency requires that these values satisfy (DEI) and (ICI), which become

$$\begin{aligned} b^I \leq& \delta\gamma\left(e^I\theta - w^I\right) - \left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1-\delta\gamma)\bar{U}^O\right) && \text{(DEI)} \\ b^I \geq& c(e^I) - \delta\gamma\left(w^I - (1-\delta\gamma)\bar{U}^I\right), && \text{(ICI)} \end{aligned}$$

thus the following condition is necessary:

$$\delta\gamma e^O\theta - c(e^O) - \delta\gamma(1-\delta\gamma)\bar{U}^O \leq \delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)\bar{U}^I$$

This condition requires $\bar{U}^I \leq \bar{U}^O$ because e^O is characterized by $\delta\gamma\theta - c'(e^O) = 0$ and thus maximizes $\delta\gamma e\theta - c(e)$. If $\bar{U}^I = \bar{U}^O$, both types are treated the same, a situation we will analyze in case III) below. If $\bar{U}^I < \bar{U}^O$, it is optimal for firms to deviate and only employ insiders. Since $\alpha^{FI} \geq 1 - \frac{c(e)}{\delta\gamma e\theta}$ and the right hand side decreases in a worker's outside option, a firm's expected profits then would amount to

$$\bar{\Pi} = \frac{\left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)\bar{U}^I\right)}{\delta\gamma(1-\delta)},$$

with e^I also characterized by $\delta\gamma\theta - c'(e^I) = 0$. Therefore, $\bar{U}^I < \bar{U}^O$ is not possible because only insiders would be hired in this case, driving down $\bar{U}^O$.

II) $\lambda_{ICO}, \lambda_{DEO} > 0$**,** $\lambda_{ICI}, \lambda_{DEI} > 0$

Then, e^O is characterized by $\delta\gamma\theta - c'(e^O) = 0$, e^I is characterized by $\delta\gamma\theta - c'(e^I) = 0$, hence

$$e^I = e^O.$$

Binding (IC) and (DE) constraints yield

$$\begin{aligned}
w^I =& \frac{\delta\gamma\alpha^{FO}\pi^O + \delta\gamma e^I\theta\alpha^{FI}\left(1-\delta\gamma\right) - \left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right)\left(1-\delta\gamma+\alpha^{FO}\delta\gamma\right)}{\delta\gamma\left(1-\delta\gamma\right)\alpha^{FI}} \\
b^I =& \delta\gamma\frac{-\delta\gamma\alpha^{FO}\pi^O + \left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right)\left(1-\delta\gamma+\alpha^F\delta\gamma-\alpha^{FI}\right)}{\delta\gamma\left(1-\delta\gamma\right)\alpha^{FI}} \\
w^I + b^I =& \frac{\delta\gamma\alpha^{FO}\pi^O + \delta\gamma e^I\theta\alpha^{FI} - \left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right)\left[1-\delta\gamma+\alpha^F\delta\gamma\right]}{\delta\gamma\alpha^{FI}} \\
w^O + b^O =& \frac{\delta\gamma\alpha^{FI}\pi^I + \delta\gamma e^O\theta\alpha^{FO} - \left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^O\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}{\delta\gamma\alpha^{FO}}
\end{aligned}$$

Plugging this into the profit functions yields

$$\begin{aligned}
\pi^O = e^O\theta - w^O - b^O =& \frac{\left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^O\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right) - \delta\gamma\alpha^{FI}\pi^I}{\delta\gamma\alpha^{FO}} \\
\pi^I = e^I\theta - w^I - b^I =& \frac{\left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right)\left[1-\delta\gamma+\alpha^F\delta\gamma\right] - \delta\gamma\alpha^{FO}\pi^O}{\delta\gamma\alpha^{FI}},
\end{aligned}$$

and

$$\begin{aligned}
\left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^O\right) &= \left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right) \\
\left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right) &= \left(\delta\gamma e^O\theta - c(e^O) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^O\right).
\end{aligned}$$

Given $e^I = e^O$, this case can consequently only hold if $\bar{U}^I = \bar{U}^O$ and outsiders and insiders are treated identically, a case we will analyze below.

III) $\lambda_{ICO}, \lambda_{DEO} = 0$**,** $\lambda_{ICI} = \lambda_{DEI} > 0$

Then, e^I is characterized by $\delta\gamma\theta - c'(e^I) = 0$, e^O is not uniquely identified. Binding

(ICI) and (DEI) constraints yield

$$\begin{aligned}
b^I =& c(e^I) - \delta\gamma \left(w^I - (1-\delta\gamma)\,\bar{U}^I\right) \\
w^I =& \frac{\delta\gamma e^I \theta\,(1-\delta\gamma)\,\alpha^{FI} + \delta\gamma\alpha^{FO}\pi^O - \left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right]\left(1-\delta\gamma+\delta\gamma\alpha^{FO}\right)}{\delta\gamma\,(1-\delta\gamma)\,\alpha^{FI}} \\
w^I + b^I =& \frac{\delta\gamma\alpha^{FO}\pi^O + \alpha^{FI}\delta\gamma e^I\theta - \left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right]\left(1-\delta\gamma+\delta\gamma\alpha^F\right)}{\delta\gamma\alpha^{FI}}
\end{aligned}$$

Also note that expected profits are

$$\bar{\Pi} = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right]}{\delta\gamma\,(1-\delta\gamma)},$$

hence π^O has no direct effect on an individual firm's expected profits.

Consistency requires that these values satisfy (DEO), (IRO) and (ICO), which become

$$\begin{aligned}
b^O \leq& \delta\gamma\left(e^O\theta - w^O\right) - \left(\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right) && \text{(DEO)} \\
b^O \geq& c(e^O) - \delta\gamma\left(w^O - (1-\delta\gamma)\,\bar{U}^O\right) && \text{(ICO)} \\
b^O \geq& c(e^O) - \left(w^O - (1-\delta\gamma)\,\bar{U}^O\right). && \text{(IRO)}
\end{aligned}$$

(DEO) and (ICO) yield the necessary condition

$$\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I \leq \delta\gamma e^O\theta - c(e^O) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^O. \tag{6}$$

Because e^I maximizes $\delta\gamma e\theta - c(e)$, this is only possible if $\bar{U}^I \geq \bar{U}^O$. In the following we separately analyze the cases $\bar{U}^I > \bar{U}^O$ and $\bar{U}^I = \bar{U}^O$.

A) $\bar{U}^I = \bar{U}^O$ Now, condition (6) can only hold if $e^O = e^I$. Moreover, since matching probabilities are the same for insiders and outsiders, consistency (i.e., $\bar{U}^I = \bar{U}^O$) requires $w^O + b^O = w^I + b^I$, which is achieved by setting $w^O = w^I$ and $b^O = b^I$. This also implies that

$$\pi^I = \pi^O = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right]\left(1-\delta\gamma+\delta\gamma\alpha^F\right)}{\delta\gamma\alpha^F}.$$

Then, (IR) constraints of insiders and outsiders are identical, and satisfied for

$$\alpha^F \geq 1 - \frac{c(e^I)}{\delta\gamma\left(e^I\theta - (1-\delta\gamma)\,\bar{U}^I\right)}.$$

In a symmetric social equilibrium in which all firms' actions are identical, this condition becomes

$$\alpha^F \geq 1 - \frac{c(e^I)}{\delta\gamma e^I\theta}.$$

B) $\bar{U}^I > \bar{U}^O$

Since $\bar{\Pi} = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)\bar{U}^I\right]}{\delta\gamma(1-\delta\gamma)}$ and $\bar{U}^I$ is taken as given by firms, several profit-maximizing arrangements with outsiders exist. For example, setting $w^O = (1-\delta\gamma)\,\bar{U}^O$ and $b^O = c(e^O)$ satisfies (ICO) and (IRO), then (DEO) becomes $\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I \leq \delta\gamma e^O\theta - c(e^O) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^O$, and values of e^O exist that satisfy this condition.

Moreover, the following consistency requirements must hold. The first is (IRI), i.e.,

$$\begin{aligned} &\delta\gamma\alpha^{FO}\pi^O + \left(\alpha^F - \alpha^{FO}\right)(1-\delta\gamma)\,c(e^I) \\ &- \left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right]\left((1-\delta\gamma)\left(1-\alpha^F\right) + \alpha^{FO}\right) \geq 0. \end{aligned} \tag{7}$$

Since $\alpha^{FI} \geq 1 - \frac{c(e)}{\delta\gamma e\theta}$, condition (7) would hold for any $\pi^O \geq \pi^I$. Rewriting this condition yields

$$\alpha^{FI} \geq \bar{\alpha}^{FI} = \frac{\left(\delta\gamma e^I\theta - c(e^I)\right)\left(1-\delta\gamma\left(1-\alpha^{FO}\right)\right) - \alpha^{FO}\delta\gamma\pi^O}{\delta\gamma e^I\theta\,(1-\delta\gamma)},$$

where we also take into account that (IRI) binds at $\bar{\alpha}^{FI}$, in which case $\bar{U}^I = 0$.

Additionally, $b^I \geq 0$ (and consequently $\Pi^I \geq \bar{\Pi}$) must be satisfied, which yields

$$\pi^O \leq \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right]\left(\left(1-\alpha^F\right)(1-\delta\gamma) + \alpha^{FO}\right)}{\delta\gamma\alpha^{FO}}, \tag{8}$$

and is equivalent to $\pi^O \leq \pi^I\frac{\left(\left(1-\alpha^F\right)(1-\delta\gamma)+\alpha^{FO}\right)}{\alpha^{FO}}$, thus $\pi^O > \pi^I$ is indeed feasible.[19]

[19] Here, we take into account that (DEI) and (ICI) continue to bind if $b^I = 0$. This can easily

Also note that, if condition (8) binds, the consistency requirement (7) becomes

$$\alpha^{FI}\left(1-\delta\gamma\right)c(e^{I})\geq 0,$$

thus holds for all levels α^{FI}.

It follows that any π^{O} that satisifies conditions (7) and (8) can be supported by a profit maximizing social equilibrium, and levels of e^{O}, w^{O} and b^{O} exist that generate such a π^{O}, with (DEO), (ICO) and (IRO) holding as well.

Then,

$$w^{I}+b^{I}=\frac{\delta\gamma\alpha^{FO}\pi^{O}+\alpha^{FI}\delta\gamma e^{I}\theta-\left[\delta\gamma e^{I}\theta-c(e^{I})-\delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right]\left(1-\delta\gamma+\delta\gamma\alpha^{F}\right)}{\delta\gamma\alpha^{FI}}$$

is increasing in π^{O}. Since $U^{I}=\frac{w^{I}+b^{I}-c(e^{I})}{1-\delta\gamma}$ and e^{I} independent of π^{O}, (employed and unemployed) insiders benefit from a higher π^{O}.

In the following, we compute an equilibrium in which π^{O} and consequently the payoffs of insiders are maximized. An arrangement maximizing π^{O} subject to (ICO), (IRO) and (DEO) would involve setting setting $w^{O}=\left(1-\delta\gamma\right)\bar{U}^{O}$ and $b^{O}=c(e^{O})$, yielding $w^{O}=\bar{U}^{O}=0$. Thus, $\pi^{O}=e^{O}\theta-c(e^{O})$, and e^{O} is constrained by

$$\delta\gamma e^{O}\theta-c(e^{O})\geq\delta\gamma e^{I}\theta-c(e^{I})-\delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}. \tag{9}$$

If e^{FB} satisfies (9), then $e^{O}=e^{FB}$. Otherwise, e^{O} is characterized by the binding (9), and

$c(e^{O})=\delta\gamma e^{O}\theta-\left(\delta\gamma e^{I}\theta-c(e^{I})-\delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right).$

Note that condition (9) implies that $e^{O}=e^{I}$ at $\bar{\alpha}^{FI}$ (where $\bar{U}^{I}=0$). For higher values of $\bar{\alpha}^{FI}$, $\bar{U}^{I}>0$, then $e^{O}>e^{I}$ is indeed possible.

The maximized π^{O} must satisfy condition (8), i.e.,

$$\alpha^{FI}\leq\tilde{\alpha}^{FI}=\frac{\left[\delta\gamma e^{I}\theta-c(e^{I})-\delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right]\left(1-\delta\gamma\left(1-\alpha^{FO}\right)\right)-\pi^{O}\delta\gamma\alpha^{FO}}{\left(1-\delta\gamma\right)\left[\delta\gamma e^{I}\theta-c(e^{I})-\delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right]},$$

be confirmed: with $b^{I}=0$, the problem is to maximize $\alpha^{FI}\left(e^{I}\theta-w^{I}\right)+\alpha^{FO}\pi^{O}$ subject to (DEI) and (ICI). Then, if either (DEI) or (ICI) did not bind, either e^{I} or π^{O} could be increased without violating a constraint, thereby increasing expected profits.

or, as a constraint on α^{FO},

$$\alpha^{FO} \leq \tilde{\alpha}^{FO} = \frac{\left(1-\alpha^{FI}\right)(1-\delta\gamma)\left[\delta\gamma e^{I}\theta - c(e^{I}) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right]}{\delta\gamma\left[\pi^{O} - \left(\delta\gamma e^{I}\theta - c(e^{I}) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right)\right]}.$$

Otherwise, the binding condition (8) determines π^{O}. Then, either e^{O} is reduced or outsiders are also paid a rent. In any case, $\pi^{O} > \pi^{I}$, and the consistency requirement (7) is satisfied given $\alpha^{FI} \geq 1 - \frac{c(e)}{\delta\gamma e\theta}$. Also note that $\tilde{\alpha}^{FI} = 1$ at $\alpha^{FO} = 0$ and $\tilde{\alpha}^{FI} < 1$ for $\alpha^{FO} > 0$. Finally, if $\alpha^{FI} > \tilde{\alpha}^{FI}$ (or, equivalently, $\alpha^{FO} > \tilde{\alpha}^{FO}$), $b^{I} = 0$, and

$$\pi^{O} = \frac{\left[\delta\gamma e^{I}\theta - c(e^{I}) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right]\left[\left(1-\alpha^{F}\right)(1-\delta\gamma) + \alpha^{FO}\right]}{\delta\gamma\alpha^{FO}},$$

or

$$\pi^{O} = \pi^{I}\frac{\left[\left(1-\alpha^{F}\right)(1-\delta\gamma) + \alpha^{FO}\right]}{\alpha^{FO}}.$$

Outcomes and Comparative Statics In the following we conduct partial comparative statics with respect to α^{F} (holding α^{N} constant) in a market equilibrium that maximizes insiders' payoffs. Now, we also have to take effects on outside options into account. Recall that

$$\bar{U}^{I} = \frac{\alpha^{N}U^{I}}{1-\delta\gamma(1-\alpha^{N})} = \frac{\alpha^{N}}{1-\delta\gamma(1-\alpha^{N})}\frac{w^{I}+b^{I}-c(e^{I})}{1-\delta\gamma}.$$

Moreover,

$$w^{I}+b^{I} = \frac{\delta\gamma\alpha^{FO}\pi^{O} + \alpha^{FI}\delta\gamma e^{I}\theta - \left[\delta\gamma e^{I}\theta - c(e^{I}) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^{I}\right]\left(1-\delta\gamma+\delta\gamma\alpha^{F}\right)}{\delta\gamma\alpha^{FI}}.$$

We analyze all three possible cases separately:

1. $\alpha^{FI} < \tilde{\alpha}^{FI}$ and $e^{O} = e^{FB}$

Then, $\pi^{O} = e^{FB}\theta - c(e^{FB})$ and

$$w^I + b^I$$

$$= \left(1-\delta\gamma(1-\alpha^N)\right) \frac{\delta\gamma\alpha^{FO}\pi^O - \left(1-\delta\gamma+\delta\gamma\alpha^{FO}\right)\delta\gamma e^I\theta + \frac{c(e^I)(1-\delta\gamma)\left(1-\delta\gamma+\delta\gamma\alpha^F\right)}{[1-\delta\gamma(1-\alpha^N)]}}{\delta\gamma\left[\left(\alpha^{FI}-\alpha^N\right)(1-\delta\gamma)-\alpha^N\delta\gamma\alpha^{FO}\right]},$$

hence

$$\frac{\partial\left(w^I+b^I\right)}{\partial\alpha^{FO}}$$

$$= \left(1-\delta\gamma(1-\alpha^N)\right)(1-\delta\gamma)\frac{\pi^O\left(\alpha^{FI}-\alpha^N\right)-\alpha^{FI}\left(\delta\gamma e^I\theta-c(e^I)\right)}{\left[\left(\alpha^{FI}-\alpha^N\right)(1-\delta\gamma)-\alpha^N\delta\gamma\alpha^{FO}\right]^2}$$

This term is positive because the (DEO) constraint,

$$\delta\gamma e^{FB}\theta - c(e^{FB}) \geq \delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)\bar{U}^I,$$

becomes

$$\pi^O\left(\alpha^{FI}-\alpha^N\right) \geq \alpha^{FI}\left(\delta\gamma e^I\theta - c(e^I)\right) + e^{FB}\theta\left[\left(\alpha^{FI}-\alpha^N\right)(1-\delta\gamma)-\alpha^N\delta\gamma\alpha^{FO}\right],$$

where $\left[\left(\alpha^{FI}-\alpha^N\right)(1-\delta\gamma)-\alpha^N\delta\gamma\alpha^{FO}\right] > 0$ because $w^I+b^I>0$.

$\partial\left(w^I+b^I\right)/\partial\alpha^{FO} > 0$ also implies $\partial U^I/\partial\alpha^{FO} > 0$ and $\partial\bar{U}^I/\partial\alpha^{FO} > 0$, since $U^I = \left(w^I+b^I-c(e^I)\right)/(1-\delta\gamma)$ and e^I is independent of α^{FO}.

Finally,

$$\bar{\Pi} = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)\bar{U}^I\right]}{(1-\delta)\delta\gamma},$$

thus

$$\frac{\partial\bar{\Pi}}{\partial\alpha^{FO}} = -\frac{(1-\delta\gamma)\frac{\partial\bar{U}^I}{\partial\alpha^{FO}}}{(1-\delta)} < 0.$$

2. $\alpha^{FI} < \tilde{\alpha}^{FI}$ **and** $e^O < e^{FB}$**, characterized by binding (DEO)**

Now, $\delta\gamma e^O\theta - c(e^O) = \delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I$, hence

$\pi^O = e^O\theta - c(e^O) = (1-\delta\gamma)\,e^O\theta + \delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I$ and

$$w^I + b^I = \frac{\left[\delta\gamma\alpha^{FO}\,(1-\delta\gamma)\,e^O\theta - \left(1-\alpha^{FI}\right)\delta\gamma e^I\theta\right]\left(1-\delta\gamma(1-\alpha^N)\right) + c(e^I)\,(1-\delta\gamma)}{\delta\gamma\,(1-\delta\gamma)\,(\alpha^{FI}-\alpha^N)}.$$

Therefore, the binding (DEO) constraint becomes

$$\begin{aligned}&\delta\gamma e^O\theta\left(\alpha^{FI} - \alpha^N + \alpha^N\alpha^{FO}\right) - c(e^O)\left(\alpha^{FI}-\alpha^N\right)\\ =&\delta\gamma e^I\theta\left(\frac{\left(1-\alpha^N\right)\alpha^{FI} - \delta\gamma\left(\alpha^{FI}-\alpha^N\right)}{(1-\delta\gamma)}\right) - \frac{c(e^I)\left((1-\delta\gamma)\,\alpha^{FI} + \delta\gamma\alpha^N\right)}{(1-\delta\gamma(1-\alpha^N))},\end{aligned}$$

with

$$\frac{\partial e^O}{\partial\alpha^{FO}} = -\frac{\delta\gamma e^O\theta\alpha^N}{\delta\gamma\theta\left(\alpha^{FI}-\alpha^N+\alpha^N\alpha^{FO}\right) - c'(e^O)\left(\alpha^{FI}-\alpha^N\right)} > 0.$$

Thus,

$$\frac{\partial\left(w^I + b^I\right)}{\partial\alpha^{FO}} = \frac{\left(e^O + \alpha^{FO}\frac{\partial e^O}{\partial\alpha^{FO}}\right)\theta\left(1-\delta\gamma(1-\alpha^N)\right)}{(\alpha^{FI}-\alpha^N)} > 0.$$

Again, $\partial U^I/\partial\alpha^{FO}$ and $\partial\bar{U}^I/\partial\alpha^{FO} > 0$, and

$$\frac{\partial\bar{\Pi}}{\partial\alpha^{FO}} = -\frac{(1-\delta\gamma)\,\frac{\partial\bar{U}^I}{\partial\alpha^{FO}}}{(1-\delta)} < 0.$$

3. $\alpha^{FI} \geq \tilde{\alpha}^{FI}$

Now, the binding condition (8) yields

$$\pi^O = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I\right]\left(\left(1-\alpha^F\right)(1-\delta\gamma) + \alpha^{FO}\right)}{\delta\gamma\alpha^{FO}},$$

thus

$$w^I + b^I = \frac{c(e^I) + \delta\gamma\,(1-\delta\gamma)\,\bar{U}^I}{\delta\gamma}$$

and

$$w^I + b^I = \frac{c(e^I)}{\delta\gamma\left(1-\alpha^N\right)}.$$

This gives $\partial U^I/\partial\alpha^{FO} = \partial\bar{U}^I/\partial\alpha^{FO} = \partial\bar{\Pi}/\partial\alpha^{FO} = 0$, whereas

$$\begin{aligned}
\pi^O =& \frac{\left[\delta\gamma e^I\theta - \frac{c(e^I)}{(1-\alpha^N)}\right]\left(\left(1-\alpha^F\right)(1-\delta\gamma) + \alpha^{FO}\right)}{\delta\gamma\alpha^{FO}} \\
\frac{\partial\pi^O}{\partial\alpha^{FO}} =& -\frac{\left[\delta\gamma e^I\theta - \frac{c(e^I)}{(1-\alpha^N)}\right](1-\delta\gamma)\left(1-\alpha^{FI}\right)}{\delta\gamma\left(\alpha^{FO}\right)^2} < 0.
\end{aligned}$$

■

Proof of Proposition 4:

We assess the two cases $\alpha^{FI} < \tilde{\alpha}^{FI}$ and $\alpha^{FI} \geq \tilde{\alpha}^{FI}$, where $\tilde{\alpha}^{FI}$ has been defined in the proof to Proposition 3. Also note that $\alpha^{FI} \geq \bar{\alpha}^F$ implies that e^I is constant and characterized by $\delta\gamma\theta - c'(e^I) = 0$ (this holds as long as $\overline{w}$ is not too large).

1. $\alpha^{FI} < \tilde{\alpha}^{FI}$

We first collect some results for $\overline{w} = 0$. Then, $w^O = \bar{U}^O = 0$, $b^O = c(e^O)$, where either $e^O = e^{FB}$ or characterized by the binding (DEO), $\delta\gamma e^O\theta - c(e^O) = \delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)\bar{U}^I$, whichever is smaller. Thus, any $\overline{w} > 0$ binds for outsiders. Moreover, $w^I > 0$ and increases in $\pi^O = e^O\theta - c(e^O)$. Let us now compute results for an arbitrary $w^O > 0$ (taking into account that $\alpha^{FI} < \tilde{\alpha}^{FI}$ holds, where we will see below that $\tilde{\alpha}^{FI}$ increases in $\overline{w}$). Recall that the best equilibrium for insiders maximizes $w^I + b^I$ (e^I is constant) and consequently $\pi^O = e^O\theta - w^O - b^O$. In the following, we how having $w^O > 0$ affects π^O, e^O, w^I and U^I. To compute π^O, we plug

$$\begin{aligned}
U^O =& \frac{w^O + b^O - c(e^O)}{1-\delta\gamma} \text{ and} \\
\bar{U}^O =& \frac{\alpha^N}{1-(1-\alpha^N)\delta\gamma} U^O
\end{aligned}$$

into (ICO), which yields

$$-c(e^O) + b^O + \delta\gamma\left(1-\alpha^O\right)w^O \geq 0.$$

(ICO) binds in an equilibrium that maximizes π^O. Therefore, $b^O = c(e^O) - \delta\gamma\left(1-\alpha^O\right)w^O$ and

$$\pi^O = e^O\theta - c(e^O) - \left(1-\delta\gamma\left(1-\alpha^O\right)\right)w^O.$$

Moreover, $U^O = \frac{1-\delta\gamma\left(1-\alpha^O\right)}{1-\delta\gamma}w^O$, and (DEO) becomes

$$\begin{aligned}
&\delta\gamma e^O\theta - c(e^O) - \delta\gamma\frac{\alpha^N\left(1-\delta\gamma\left(1-\alpha^O\right)\right)}{1-(1-\alpha^N)\delta\gamma}w^O \\
\geq&\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I,
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow&\frac{\left(\alpha^F-\alpha^N-\alpha^{FO}\right)\left(\delta\gamma e^O\theta - c(e^O)\right)+\alpha^{FO}\alpha^N\delta\gamma e^O\theta}{\left[\left(\alpha^{FI}-\alpha^N\right)\left(1-\delta\gamma\right)-\alpha^N\delta\gamma\alpha^{FO}\right]} \\
&-w^O\frac{\delta\gamma\alpha^N\left(1-\delta\gamma\left(1-\alpha^{FO}\right)\right)\left(\alpha^F-\alpha^N\right)}{\left(1-(1-\alpha^N)\delta\gamma\right)\left[\left(\alpha^{FI}-\alpha^N\right)\left(1-\delta\gamma\right)-\delta\gamma\alpha^N\alpha^{FO}\right]} \\
&\geq\frac{\alpha^{FI}\left(\delta\gamma e^I\theta - c(e^I)\right)}{\left(\alpha^{FI}-\alpha^N\right)\left(1-\delta\gamma\right)-\alpha^N\delta\gamma\alpha^{FO}}
\end{aligned}$$

The latter takes into account that

$$\begin{aligned}
w^I =&\frac{c(e^I)-\delta\gamma e^I\theta\left[\left(1-\delta\gamma\right)\left(1-\alpha^F\right)+\alpha^{FO}\right]+\delta\gamma\alpha^{FO}\pi^O}{\delta\gamma\left[\left(1-\delta\gamma\right)\left(\alpha^F-\alpha^N\right)-\alpha^{FO}\left(1-\delta\gamma(1-\alpha^N)\right)\right]}\left(1-\delta\gamma+\delta\gamma\alpha^{FO}\right) \\
&\text{and} \\
w^I+b^I =&\frac{\alpha^{FO}\pi^O\left(1-\delta\gamma(1-\alpha^N)\right)}{\left[\left(1-\delta\gamma\right)\left(\alpha^F-\alpha^N\right)-\alpha^{FO}\left(1-\delta\gamma(1-\alpha^N)\right)\right]} \\
&-\frac{e^I\theta\left(1-\delta\gamma(1-\alpha^N)\right)\left[\left(1-\alpha^F\right)\left(1-\delta\gamma\right)+\alpha^{FO}\right]}{\left[\left(1-\delta\gamma\right)\left(\alpha^F-\alpha^N\right)-\alpha^{FO}\left(1-\delta\gamma(1-\alpha^N)\right)\right]} \\
&+\frac{c(e^I)\left(1-\delta\gamma+\delta\gamma\alpha^F\right)\left(1-\delta\gamma\right)}{\delta\gamma\left[\left(1-\delta\gamma\right)\left(\alpha^F-\alpha^N\right)-\alpha^{FO}\left(1-\delta\gamma(1-\alpha^N)\right)\right]},
\end{aligned}$$

as well as

$$\bar{U}^I =\frac{\alpha^N}{1-\delta\gamma(1-\alpha^N)}\frac{w^I+b^I-c(e^I)}{1-\delta\gamma}.$$

Thus, (DEO) is tightened by a higher w^O, which implies that e^O decreases in w^O if (DEO) binds. Therefore, a higher w^O reduces $\pi^O = e^O\theta - c(e^O) - \left(1-\delta\gamma\left(1-\alpha^O\right)\right)w^O$ directly, but potentially also indirectly via a smaller e^O. All this holds as long as $w^I > w^O$, and $\hat{\overline{w}}$ is defined as the w^O for which $w^O = w^I$.

Finally, in the proof to proposition (3) we have shown that, as long as $\bar{U}^I > \bar{U}^O$, $\bar{\Pi} = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma(1-\delta\gamma)\bar{U}^I\right]}{\delta\gamma(1-\delta\gamma)}$. Thus, a smaller $\bar{U}^I$ caused by a higher $\overline{w}$ increases $\bar{\Pi}$ and – if F is endogenous – raises employment (see the proof to Proposition **2**).

2. $\alpha^{FI} \geq \tilde{\alpha}^{FI}$

Recall that

$$\tilde{\alpha}^{FI} = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right]\left(1-\delta\gamma\left(1-\alpha^{FO}\right)\right) - \pi^O\delta\gamma\alpha^{FO}}{\left(1-\delta\gamma\right)\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right]},$$

and that

$$\pi^O = \frac{\left[\delta\gamma e^I\theta - c(e^I) - \delta\gamma\left(1-\delta\gamma\right)\bar{U}^I\right]\left(\left(1-\alpha^F\right)\left(1-\delta\gamma\right) + \alpha^{FO}\right)}{\delta\gamma\alpha^{FO}} \tag{10}$$

if $\alpha^{FI} \geq \tilde{\alpha}^{FI}$. In this case $\pi^O < e^O\theta - c(e^O)$, with e^O either at e^{FB} or characterized by the binding (DEO). Thus, there exist strictly positive values $\overline{w}$ such that π^O is still characterized by (10). For the following, assume that e^O is chosen as high as feasible given π^O (recall that e^O has not been uniquely defined for $\alpha^{FI} \geq \tilde{\alpha}^{FI}$, and many combinations of e^O and w^O could generate π^O characterized by (10)). This also implies that w^O is as high as feasible given π^O, with $\partial\pi^O/\partial w^O < 0$.

Now, replacing $\bar{U}^I$ yields

$$\begin{aligned}\tilde{\alpha}^{FI} =& \frac{\left(1-\delta\gamma\left(1-\alpha^{FO}\right)\right)}{\left(1-\delta\gamma\right)} \\ &- \frac{\pi^O\delta\gamma\alpha^{FO}\left[\left(1-\delta\gamma\right)\left(\alpha^F-\alpha^N\right) - \alpha^{FO}\left(1-\delta\gamma(1-\alpha^N)\right)\right]}{\left(1-\delta\gamma\right)\left\{\alpha^{FI}\left(1-\delta\gamma\right)\left[\left(1-\alpha^N\right)\delta\gamma e^I\theta - c(e^I)\right] - \delta\gamma\alpha^N\alpha^{FO}\pi^O\right\}},\end{aligned}$$

with

$$\begin{aligned}\frac{\partial\tilde{\alpha}^{FI}}{\partial\pi^O} =& -\frac{\delta\gamma\alpha^{FO}\left[\left(1-\delta\gamma\right)\left(\alpha^F-\alpha^N\right) - \alpha^{FO}\left(1-\delta\gamma(1-\alpha^N)\right)\right]\alpha^{FI}\left[\left(1-\alpha^N\right)\delta\gamma e^I\theta - c(e^I)\right]}{\left\{\alpha^{FI}\left(1-\delta\gamma\right)\left[\left(1-\alpha^N\right)\delta\gamma e^I\theta - c(e^I)\right] - \delta\gamma\alpha^N\alpha^{FO}\pi^O\right\}^2} \\ &< 0.\end{aligned}$$

Then a binding $\overline{w}$ requires a reduction of π^O, which lets $\tilde{\alpha}^{FI}$ go up and consequently induces $\alpha^{FI} < \tilde{\alpha}^{FI}$, for which the results derived above hold.

■

Proof of Lemma 2:

Recall from the proof to Proposition 3 that the (IRI) constraint, condition (7), equals

$$\alpha^{FI} \geq \bar{\alpha}^{FI} = \frac{\left(\delta\gamma e^I\theta - c(e^I)\right)\left(1-\delta\gamma\left(1-\alpha^{FO}\right)\right) - \alpha^{FO}\delta\gamma\pi^O}{\delta\gamma e^I\theta\left(1-\delta\gamma\right)}. \tag{11}$$

It follows that $\bar{\alpha}^{FI}$ is larger with $\alpha^{FO} = 0$ than with $\bar{\alpha}^{FO} > 0$.

At $\bar{\alpha}^{FI}$, (DEO) reveals that $e^O = e^I$ and both, (IRI) and (IRO), just bind. Thus, the optimal π^O is uniquely determined at $\bar{\alpha}^{FI}$, and

$$\frac{\partial\bar{\alpha}^{FI}}{\partial\alpha^{FO}} < 0.$$

■

Proof of Lemma 3:

If $\alpha^{FI} < \bar{\alpha}^{FI}$, (IRI) binds which implies that (IRO) binds as well (see the proof to Proposition 3). Thus, $w^I = c(e^I) - b^I$ and $w^O = c(e^O) - b^O$. Plugging these values into the respective (DE) functions, taking into account (IC) constraints, and that our objective is to maximize

$$\bar{\Pi} = \frac{\alpha^{FI}\left(e^I\theta - w^I - b^I\right) + \alpha^{FO}\left(e^O\theta - w^O - b^O\right)}{\left(1-\delta\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)},$$

reveals that it is weakly optimal to set $b^I = c(e^I)$ and $b^O = c(e^O)$. Thus, for $\alpha^{FI} < \bar{\alpha}^{FI}$ the problem becomes to maximize

$$\frac{\alpha^{FI}\left(e^I\theta - c(e^I)\right) + \alpha^{FO}\left(e^O\theta - c(e^O)\right)}{\left(1-\delta\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)},$$

subject to

$$-c(e^I) + \delta\gamma\frac{\left[\left(1-\alpha^{FI}\right)\left(1-\delta\gamma\right)+\delta\gamma\alpha^{FO}\right]e^I\theta - \alpha^{FO}\left(e^O\theta - c(e^O)\right)}{\left(1-\delta\gamma+\delta\gamma\alpha^{FO}\right)} \geq 0 \tag{DEI}$$

$$-c(e^O) + \delta\gamma\frac{\left[\left(1-\alpha^{FO}\right)\left(1-\delta\gamma\right)+\delta\gamma\alpha^{FI}\right]e^O\theta - \alpha^{FI}\left(e^I\theta - c(e^I)\right)}{\left(1-\delta\gamma+\delta\gamma\alpha^{FI}\right)} \geq 0. \tag{DEO}$$

It is immediate that $e^I = e^O = e^{FB}$ if these values satisfy (DEI) and (DEO). We now show that $e^I = e^O$ must always hold. To do so, we first rewrite constraints to

$$\begin{aligned}
&-c(e^I)\left(1-\delta\gamma\right)-\delta\gamma\alpha^{FO}\left(c(e^I)-c(e^O)\right)\\
&+\delta\gamma\left[\left(1-\delta\gamma+\delta\gamma\alpha^F\right)e^I\theta-\alpha^{FI}e^I\theta-\alpha^{FO}e^O\theta\right]\geq 0 \qquad \text{(DEI)}\\
&-c(e^O)\left(1-\delta\gamma\right)-\delta\gamma\alpha^{FI}\left(c(e^O)-c(e^I)\right)\\
&+\delta\gamma\left[\left(1-\delta\gamma+\delta\gamma\alpha^F\right)e^O\theta-\alpha^{FI}e^I\theta-\alpha^{FO}e^O\theta\right]\geq 0, \qquad \text{(DEO)}
\end{aligned}$$

and define Δ as the difference between the left-hand side of (DEI) and the left-hand side of (DEO):

$$\Delta=\left[\left(\delta\gamma e^I\theta-c(e^I)\right)-\left(\delta\gamma e^O\theta-c(e^O)\right)\right]\left(1-\delta\gamma+\delta\gamma\alpha^F\right)$$

By the definition of Δ, (DEI) is slack if $\Delta > 0$, whereas (DEO) is slack if $\Delta < 0$. For the following, we also define $\bar{e}$ is the effort level characterized by $\delta\gamma\theta - c'(\bar{e}) = 0$.

Assume $e^I > e^O$, hence at least one of the (DE) constraints binds and restricts profits. We show that we can increase e^O and e^I in a way that does not directly affect profits but relaxes the binding constraint, thus allows firms to eventually increase their profits.

Applying the total differential, a marginal change in e^O, by de^O, does not change profits if

$$de^I=-\frac{\alpha^{FO}\left(\theta-c'(e^O)\right)}{\alpha^{FI}\left(\theta-c'(e^I)\right)}de^O.$$

This operation changes the left-hand side of (DEI) by

$$\frac{\alpha^{FO}\left(\theta-c'(e^O)\right)}{\alpha^{FI}\left(\theta-c'(e^I)\right)}de^O\left(\frac{-\left(\delta\gamma\theta-c'(e^I)\right)\left(1-\delta\gamma+\delta\gamma\alpha^F\right)}{\left(1-\delta\gamma+\delta\gamma\alpha^{FO}\right)}\right),$$

and the right-hand side of (DEO) by

$$\frac{\left(\delta\gamma\theta-c'(e^O)\right)\left(1-\delta\gamma+\delta\gamma\alpha^F\right)}{\left(1-\delta\gamma+\delta\gamma\alpha^{FI}\right)}de^O$$

First, assume $e^O < e^I < \bar{e}$, hence $\Delta > 0$ and (DEI) is slack whereas (DEO) binds. Then, this operation with $de^O > 0$ tightens (DEI) and relaxes (DEO), which allows firms to increase profits.

Second, assume $e^O < \bar{e} \leq e^I$. Then, this operation with $de^O > 0$ relaxes both constraints, which allows firms to increase profits.

Third, assume $e^I > e^O \geq \bar{e}$, hence $\Delta < 0$ and (DEO) is slack whereas (DEI) binds. Then, this operation with $de^O > 0$ tightens (DEO) and relaxes (DEI), which allows firms to increase profits.

Summing up, $e^I > e^O$ is not optimal. Equivalently, we can show that $e^I < e^O$ cannot be optimal as well, allowing us conclude that $e^I = e^O = e$ in a profit-maximizing social equilibrium with $\alpha^{FI} < \bar{\alpha}^{FI}$.

Therefore, (DEI) and (DEO) coincide, and the optimization problem becomes to maximize

$$\frac{\alpha^F \left(e\theta - c(e)\right)}{\left(1-\delta\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)},$$

subject to

$$-c(e) + \delta\gamma\left(1-\alpha^F\right)e\theta \geq 0. \tag{DE}$$

Naturally, $e = e^{FB}$ if it satisfies the (DE) constraint. Otherwise, the binding (DE) constraint determines equilibrium effort, with

$$\frac{de}{d\alpha^{FO}} = \frac{\delta\gamma e\theta}{-c'(e) + \delta\gamma\left(1-\alpha^F\right)\theta} < 0.$$

Since $e = \bar{e}$ at $\alpha^{FI} = \bar{\alpha}^{FI}$, this implies that $e > \bar{e}$ for $\alpha^{FI} < \bar{\alpha}^{FI}$.

Moreover,

$$\begin{aligned}\frac{\partial\bar{\Pi}}{\partial\alpha^{FO}} &= \frac{\left(e\theta - c(e)\right)\left(1-\delta\gamma\right)}{\left(1-\delta\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)^2} + \frac{\alpha^F\left(\theta - c'(e)\right)}{\left(1-\delta\right)\left(1-\delta\gamma+\alpha^F\delta\gamma\right)}\frac{de}{d\alpha^{FO}} \\ &= \frac{e\theta}{\left(1-\delta\right)}\frac{\delta\gamma\theta - c'(e)}{\left[-c'(e) + \delta\gamma\left(1-\alpha^F\right)\theta\right]} > 0,\end{aligned}$$

where the denominator – the partial derivative of the left-hand side of (DE) with respect to e – must be negative if (DE) binds. ■

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