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Abstract

We analyze a multitasking model with a verifiable routine task and a skill-dependent activity characterized by moral hazard. Contracts negotiated by firm/employee pairs follow from Nash bargaining. High- and low-skilled employees specialize, intermediate productivity employees perform both tasks. Compared to the efficient solution, more employees exert both tasks and effort in the routine task is inefficiently large. As work overload in the routine task is decoupled from a corresponding increase in remuneration, employees perceive a loss of control to allocate effort between the two tasks. Reductions in employees' bargaining power and improvements in monitoring technologies aggravate the issue.

JEL-Codes: D820, D860, J410, M520.

Keywords: multi-tasking, work overload, routine tasks, rent extraction, moral hazard, limited liability, Nash Bargaining.

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Overwhelmed by Routine Tasks: A Multi-Tasking Principal Agent Perspective

Dominique Demougin and Carsten Helm†*

1 Introduction

> "A manager explains that the software developers who report to him are frustrated because "different people are pinging them for information" all day. They are interrupted from writing their code because questions come at them via the chat software the company uses. These IT professionals feel "they go through the whole day, the whole week without doing what they were expected to do" during regular work hours, so they work late nights and weekends ... to try to catch up." (Kelly and Moen, 2020, p. 6)

The recent book "Overload" by Kelly and Moen (2020), describes an evolution in labor contracting where many "good jobs, once characterized by relative autonomy and security, have become bad, with increasing workloads" (Kelly and Moen, 2020, p. 4-5). As the quote above illustrates, dissatisfaction is often due to the underlying job design which uses high-powered incentives that bundle more demanding and less challenging tasks: programming and answering questions via chat software, treating patients and doing paperwork in hospitals, research and administrative duties at universities — to name but a few examples. Employees working under such contracts frequently perceive the more routine activities as an inefficient extra burden that eats away at their working time, preventing them from properly performing in the other task, which is typically perceived as the more central and more rewarding one.

From a positive economics perspective, this raises at least two questions. What are the underlying characteristics of contract negotiations that would lead to a job design where a routine activity ends up distracting an employee from his productive task like software development and patient treatment? What developments of the economic environment could provide a reasonable explanation for the above changes in job design practice?

In order to address these two questions, we introduce a stylized multitasking model where an employment contract may require an employee to perform either one of two tasks or both.[1] The first task is a simple fully verifiable "routine" which does not necessitate any special qualifications. The second task is a "skill-demanding activity" characterized by moral hazard. From the employee's perspective, the two tasks are substitutes, in that greater effort in either activity increases the marginal effort cost of the other task. Employees (the agent) negotiate wage and effort in their labor contract with their respective employers (the principal). Specifically, we assume that the employment contract solves a Nash bargaining game.[2] Due to the moral hazard context, these contracts consist of a fixed payment and an incentive component that is paid when a favorable signal regarding the non-verifiable effort is observed. Employees are risk-neutral but protected by limited liability so that the fixed wage component cannot become negative.

Employees differ in their verifiable skills in the demanding task — also referred to as their productivity. We choose the range of their skills such that low-skilled employees specialize in the verifiable routine task, and very high-skilled ones in the "skill-demanding activity". This obtains independent of the moral hazard context — that is, also in the efficient solution — and even if the employer has all the bargaining power. The reason lies in the cost substitutability between tasks, which makes multi-tasking costly. Despite this cost, employees with "medium productivity" exert both tasks — both in the efficient solution and under the negotiated incentive contract — and our analysis focuses on this group.

The main findings are as follows. For employees who exert effort in the non-verifiable, skill-demanding task, the contract must address the moral hazard problem. As is well-known, in a standard moral hazard environment aligning incentives ensures that the employee will receive an expected wage above the proportion of economic rent that he would obtain based on bargaining power alone. In other words, the employee extracts an additional 'informational' rent, which increases the firm's marginal cost of inducing effort in the skill-demanding activity. Therefore, effort in this task is set below the efficient level, which reflects the well known 'rent versus efficiency' trade-off.

However, the fact that employees of medium productivity also exert the verifiable routine task introduces an additional trade-off. Specifically, for them the contract can also reduce their informational rent by raising effort in this routine task without a corresponding compensation. In addition, more workers are assigned to both tasks than would be efficient. Therefore, these medium productivity employees may feel overloaded by the routine task. They are also likely to feel an acute loss of autonomy. Intuitively, keeping total effort constant, each medium productivity employee would prefer to reduce effort in the routine task and invest more time in the skill-demanding activity, as this would increase their expected bonus. What holds them back is their lack of autonomy in the routine activity because that task is easily verifiable.

By contrast, for "highly productive" employees extracting some of their informational rent by a marginal increase in the routine task would be too costly due to the associated reduction in the skill-demanding activity. Hence these employees specialize and benefit from the usual advantages associated with a moral hazard in an employment relationship; they will be expected to produce less than efficient effort and extract a positive informational rent.

Based on our initial questions, our model suggests that employees who fit our definition of 'medium productivity' will be those who are overworked and lack autonomy. Answering the second question within our model therefore requires identifying variables that would explain why the relevant group of 'medium productivity' employees has become larger and/or why the overload with routine tasks has become greater. We propose two such variables; bargaining power and quality of monitoring. For the former, we argue that the erosion of union power has reduced the bargaining strength of labor,

[1] For an introduction into the economic model of multitasking see the review by Gibbons (1998).

[2] For an introduction into Nash bargaining see Muthoo (1999). For the application of Nash bargaining to contract negotiations under moral hazard see Demougin and Helm (2006).

whereas for the latter, advances in ICT are interpreted as having improved the informational quality of monitoring. Based on these casual empirical interpretations, we find that the comparative statics of our model provide a natural explanation for the aforementioned growing employee dissatisfaction.

Our findings are consistent with a substantial number of studies – much of this in the sociological literature – that reports an increase in work intensity, especially for professionals (Le Fevre, Boxall, and Macky, 2015) that can loosely be associated with the middle class (see Green, Felstead, Gallie, and Henseke (2021) as well as Kelly and Moen (2020) for surveys of this literature). For example, based on survey data for Britain, the Chartered Institute of Personnel and Development (CIPD, 2014) finds that the percentage of "Employee under pressure every single day" is the highest for junior and middle managers. This is also reflected in income characteristics, where the percentage of "Employee under pressure once or twice a week" is the highest for the income group £25-34k, but successively falling for lower as well as for higher income groups.[3] The decline in trade unionism is stated as one reason for the increase in work pressures (Gallie, 2017, p. 234). Another often cited reason in that literature is the increased use of incentive pay systems (see, e.g., Green, 2004a).

In the economic literature our analysis is closely related to articles in contract theory and labor economics which study the implication of moral hazard and multi-tasking environments. In the standard multi-tasking model, all tasks are non-verifiable, but to a different extent (Dewatripont, Jewitt, and Tirole, 2000; Holmstrom and Milgrom, 1991). It seems natural that tasks for which the performance can be measured more precisely should receive higher-powered incentives. However, the agent may then pay too little attention to the less rewarded task. To counteract this, even perfectly measured activities may be left without any incentive (Holmström, 2017). This contrast sharply to our setting where the agents obtain rents from their limited liability, and effort in the perfectly verifiable task is set inefficiently high to extract some of this.

To the best of our knowledge, Laux (2001) was the first to show that assigning multiple projects to a single manager relaxes the limited-liability constraint, which may result in work overload. However, effort choices, success probability and profitability of project are all binary, whereas these are continuous variables in our model. More importantly, Laux (2001) does not consider employees of different productivity, bargaining over contracts, cost substitutability between tasks and monitoring technologies, all aspects that are central for our analysis. Similar to us, Bond and Gomes (2009) extent the model of Laux (2001) in several respects, e.g. allowing tasks to be substitutes (or complements). They focus on insufficient risk taking that results from excessive concentration, an aspect that is not considered in our article. Bénabou and Tirole (2016) also consider performance pay in a multi-tasking model, but one of the tasks cannot be measured so that it requires an intrinsic motivation. Moreover, their focus lies on the over incentivization of high-skilled agents.

Our article also addresses the question whether to bundle or to disentangle tasks. Related contributions include Schmitz (2005) who focuses on sequential actions, as well as Kragl and Schöttner (2014) who analyze the interactive effects of imperfect performance measures and wage floors. Finally, we should mention that other economic explanations for work overload have been suggested in the literature. A prominent example is Akerlof (1976), who argues that adverse selection forces managers into a 'rat race' that leads to overwork. In Aghion and Tirole (1997), the principal overloads himself with work as a commitment not to interfere too much with the tasks of the agent. Skott and Guy (2007) explain increases in work intensity with improved technologies to monitor especially low-skilled employees, an aspect that we also address in our comparative static analysis. Besides the different modelling approach, our article differs from these contributions by focusing on work overload of medium-skilled employees.

The remainder of the article is structured as follows. Section 2 introduces the basic model and Section 3 the efficient solution that serves as a benchmark. The main part is Section 4, where we derive the optimal contract and discuss its properties. In Section 5 we then identify and discuss some comparative statics, before Section 6 concludes. An Appendix contains all proofs.

[3] With the exception of the highest income group £60k+ for which pressure increases again.

2 The model

We consider an environment populated by a continuum of risk neutral employees and risk neutral firms of equal measure. Firms are identical. Employees differ in their skills, $\gamma > 0$, which determines the productivity of the firm/employee match.

Employees carry out two different tasks with respective efforts $a_1, a_2 \geq 0$. These efforts generate an output, $f(a_1, a_2) = a_1 + \gamma a_2$, to the firm and effort costs, $c(a_1, a_2) = \frac{1}{2}a_1^2 + \beta a_1 a_2 + \frac{1}{2}a_2^2$, to the employee, where $0 \leq \beta < 1$. The cross-term implies that increasing the effort level in one task raises the marginal cost of effort in the other task. Applied to the introductory example, it reflects the natural intuition that it becomes more difficult for developers to spend time writing code (effort a_2) when they are forced to give more time to answer questions on a chat line (effort a_1).

Effort in task 1 is verifiable, whereas effort in task 2 and output are not. We assume, however, the existence of a monitoring technology which generates a verifiable binary proxy variable, σ, that is correlated with a_2. Specifically, we have $\sigma \in \{0, 1\}$, where $\sigma = 1$ denotes the favorable signal (see Milgrom, 1981). We follow the model by Demougin and Fluet (2001) and assume

$$p(a_2) = \left(\frac{a_2}{\kappa}\right)^{\frac{1}{\mu}}, \tag{1}$$

where $p(a_2) := \Pr[\sigma = 1 | a_2, \mu]$ is the probability of observing the favorable outcome as a function of effort, a_2, for a given μ. As $p(a_2)$ is a probability, we assume that κ is large enough so that $\frac{a_2}{\kappa} < 1$. The ratio $1/\mu$ is interpreted as the fraction of time which the monitor spends in observing the agent. Accordingly, we require $\mu > 1$ and take the monitoring technology characterized by μ as exogenously determined. Observe that $\mu > 1$ ensures concavity of the function $p(a_2)$. Moreover, in this specification $1/\mu$ can be interpreted as the constant effort elasticity of the probability to observe the favorable signal, i.e.

$$a_2 \frac{p'(a_2)}{p(a_2)} = \frac{1}{\mu}. \tag{2}$$

Accordingly, a reduction in μ means that the probability of observing a favorable signal becomes more sensitive to changes in effort a_2, which we interpret in the discussion of the results as an improvement in the monitoring technology.

Employees and firms are matched in pairs and negotiate an incentive contract $\mathcal{C} := \{F, B, a_1, a_2\}$, where F is a fixed payment, B a bonus and a_1, a_2 the effort levels which the employee is told to implement. With respect to the verifiable effort in task 1, we assume that the contract works as follows. If the employee does not satisfy the prescribed effort, i.e. if he undertakes effort $\hat{a}_1$ with $\hat{a}_1 < a_1$, he is laid off and receives no payment (similar to Lazear, 2000). Otherwise, if $\hat{a}_1 \geq a_1$, the employee obtains F. In addition, the employee receives the bonus B if the favorable signal, $\sigma = 1$, is realized. Moreover, feasible contracts are restricted by a financial constraint on the part of the agent which requires non-negative payments, i.e. $F \geq 0$ and $F + B \geq 0$.

Following standard terminology, we call a contract $\mathcal{C}$ incentive feasible if it is incentive compatible, individually rational and satisfies the payment constraints. Given an incentive feasible contract, the employee's and the firm's payoffs are

$$u := F + p(a_2) B - c(a_1, a_2), \tag{3}$$
$$\pi := a_1 + \gamma a_2 - F - p(a_2) B. \tag{4}$$

We depart from the standard Principal-Agent paradigm and assume that both parties have bargaining power (as in Demougin and Helm, 2011). In particular, we use the Nash bargaining solution (NBS) and denote by $\varphi \in (0, 1)$ the employees' bargaining power coefficient. Accordingly, negotiations lead the parties to select a contract which maximizes $u^{\varphi}\pi^{1-\varphi}$.

For the sake of simplicity, the parties' outside options are normalized to zero, and we assume that the parties choose their outside option in case of indifference. Hence the negotiated contract $\mathcal{C}$ is said to be "mutually beneficial" if and only if $u > 0$ and $\pi > 0$. The simple example $a_1 = 1, a_2 = 0, F = 3/4$, and $B = 0$, which generates $u = \pi = 1/4$, shows that such contracts always exist so that the participation constraints can be ignored henceforth. For later reference, we state this result formally.

Lemma 1. *For all γ over the support, there exists an incentive feasible contract that is mutually beneficial.*

3 The benchmark solution

In this section, we determine the Pareto efficient solution for an arbitrary γ and briefly discuss the implication for a negotiated contract under symmetric information. The Pareto efficient solution is obtained by maximizing the aggregate payoff, $u + \pi$, subject to the feasibility of effort, $a_1, a_2 \geq 0$:

$$R^* := \max_{a_1,a_2 \geq 0} \; a_1 + \gamma a_2 - \left(\frac{1}{2}a_1^2 + \beta a_1 a_2 + \frac{1}{2}a_2^2 \right), \tag{5}$$

where the value R^* represents the maximum economic rent. Accounting for the non-negativity requirements, the Lagrangian is:

$$\mathcal{L} = a_1 + \gamma a_2 - \left(\frac{1}{2}a_1^2 + \beta a_1 a_2 + \frac{1}{2}a_2^2 \right) + \xi a_1 + \zeta a_2. \tag{6}$$

The optimal solution follows from the first-order conditions,

$$1 - a_1 - \beta a_2 + \xi = 0 \tag{7}$$
$$\gamma - \beta a_1 - a_2 + \zeta = 0, \tag{8}$$

together with the complementary slackness conditions, $\xi a_1 = \zeta a_2 = 0$, and the requirement $\xi, \zeta \geq 0$. In Appendix A, we very the following result.

Proposition 1. *The Pareto efficient solution is*

$$a_1^* = \begin{cases} 1 & \text{if } \gamma < \beta \\ \frac{1-\beta\gamma}{1-\beta^2} & \text{if } \beta \leq \gamma \leq \frac{1}{\beta} \\ 0 & \gamma > \frac{1}{\beta} \end{cases} \quad \text{and} \quad a_2^* = \begin{cases} 0 & \text{if } \gamma < \beta \\ \frac{\gamma-\beta}{1-\beta^2} & \text{if } \beta \leq \gamma \leq \frac{1}{\beta} \\ \gamma & \gamma > \frac{1}{\beta}. \end{cases} \tag{9}$$

For $\beta = 0$, it follows immediately from the objective function that equalizing marginal benefits and marginal costs to obtain the efficient solution requires $a_1^* = 1$ and $a_2^* = \gamma$. For $\beta > 0$, the cross term $\beta a_1 a_2$ raises the respective marginal costs of effort if both tasks are exerted. Intuitively, this fosters specialization. Employees that are sufficiently more productive in task 2 (high γ) fully specialize in that task ($a_1 = 0$), thereby eliminating the cross term. In contrast, low-γ employees have a comparative advantage in task 1 and set $a_2 = 0$. In the intermediary case, employees should undertake both tasks.

Under symmetric information, the negotiated contract involves a fixed payment F and feasible effort levels $a_1, a_2 \geq 0$ which maximize the Nash bargaining product (NBP hereafter):

$$\max_{F,a_1 a_2} \; u^{\varphi}\pi^{1-\varphi} = \left(F - \left(\frac{1}{2}a_1^2 + \beta a_1 a_2 + \frac{1}{2}a_2^2 \right) \right)^{\varphi} (a_1 + \gamma a_2 - F)^{1-\varphi}. \tag{10}$$

From standard results under Nash bargaining theory (e.g., Muthoo, 1999), we know that the solution to (10) implements the Pareto efficient effort level given by (9). Moreover, we know that the fixed payment distributes the economic rent R^* between the firm and the employee according to their respective bargaining power, i.e. such that $\varphi\pi^* = (1-\varphi)\, u^*$.[4]

[4] Formally, the surplus distribution problem can be written as $\max_u u^{\varphi}\pi^{1-\varphi}$ such that $R^* = u + \pi$. Substitution for π and taking the first-order condition yields $u^* = \varphi R^*$. Finally, substituting for R^* and rearranging terms verifies $\varphi\pi^* = (1-\varphi)\, u^*$.

4 Optimal incentive contracts

In this section, we return to the problem of contract negotiations between a firm and a employee characterized by the productivity factor γ. As outlined in Section 2, the negotiations lead the parties to select the incentive feasible contract $\mathcal{C}$ which maximizes the Nash bargaining product, subject to $a_1, a_2 \geq 0$. The constraints lead to a second-best contract that we denote by $\mathcal{C}^c = \{F^c, B^c, a_1^c, a_2^c\}$.

Let $i(\cdot)$ be an indicator function that takes the value 0 if $\hat{a}_1 < a_1^c$ and 1 otherwise. This notation is useful to formally represent the assumption that an employee is laid off without payments if effort in task one falls below the contractually agreed level. Taking a logarithmic transformation of $u^{\varphi}\pi^{1-\varphi}$, and using the payoffs as given in (3) and (4), the maximization program can be written as

$$\max_{\mathcal{C}} \ \varphi \ln \left[F + p\left(a_2\right) B - c(a_1, a_2)\right] + (1-\varphi)\ln[a_1 + \gamma a_2 - F - p\left(a_2\right) B] \ \text{ s.t.} \tag{11}$$

$$(a_1, a_2) = \arg \max_{\hat{a}_1, \hat{a}_2 \geq 0} \left[F + p\left(\hat{a}_2\right) B\right] \cdot i(\hat{a}_1 - a_1) - \left(\frac{1}{2}\hat{a}_1^2 + \beta \hat{a}_1 \hat{a}_2 + \frac{1}{2}\hat{a}_2^2\right) \qquad \text{(IC)}$$

$$a_1, a_2 \geq 0 \qquad \text{(EF)}$$

$$F, F+B \geq 0. \qquad \text{(PC)}$$

Accordingly, (IC) is the agent's Incentive Compatibility requirement, (EF) are the Effort Feasibility conditions and (PC) the Payment Constraints.

Observe that by Lemma 1, the solution will satisfy $u > 0$ and $\pi > 0$. Hence, we have ignored the participation constraints. In addition, we know that for any contract that is mutually beneficial if abided by, the employee will set $\hat{a}_1 \geq a_1$ as otherwise he would be fired and receive a non-positive payoff. Moreover, the agent would never set $\hat{a}_1 > a_1$ as doing so would not affect his payments but raise his cost. Altogether, we conclude $i(\cdot) = 1$ so that (IC) simplifies to

$$a_2 = \arg \max_{\hat{a}_2 \geq 0} \ F + p\left(\hat{a}_2\right) B - \left(\frac{1}{2}a_1^2 + \beta a_1 \hat{a}_2 + \frac{1}{2}\hat{a}_2^2\right). \tag{12}$$

This is a strictly concave optimization problem so that the necessary and sufficient first-order condition is

$$Bp'(a_2) - (\beta a_1 + a_2) + \psi = 0, \tag{13}$$

where $\psi \geq 0$ is the Lagrange multiplier associated with the constraint $\hat{a}_2 \geq 0$. For $a_2 > 0$, the complementary slackness yields $\psi = 0$ so that using (2) we have

$$p\left(a_2\right) B = \mu\left(a_2^2 + \beta a_1 a_2\right). \tag{14}$$

Moreover, from the definition of $p\left(a_2\right)$ in (1), the expression for the expected bonus also holds for $a_2 = 0$.[5] Accordingly, we can further simplify the principal's optimization problem by eliminating (12) and replacing $p\left(a_2\right) B$ with the right-hand side of (14). Altogether, (11) can be rewritten as:

$$\begin{aligned} \max_{F, a_1, a_2} \quad & \varphi \ln\left[F + \mu\left(a_2^2 + \beta a_1 a_2\right) - \left(\tfrac{1}{2}a_1^2 + \beta a_1 a_2 + \tfrac{1}{2}a_2^2\right)\right] \\ & +(1-\varphi)\ln[a_1 + \gamma a_2 - F - \mu\left(a_2^2 + \beta a_1 a_2\right)] \qquad \text{s.t.} \end{aligned} \tag{15}$$

$$a_1, a_2 \geq 0 \qquad \text{(EF)}$$

$$F \geq 0, \qquad \text{(PC')}$$

[5] For this case, $\psi = \beta a_1$ satisfies (13).

where the optimal bonus follows from (14) if $a_2^c > 0$ and $B^c = 0$ otherwise. The Lagrange function associated with problem (15) is

$$\begin{aligned}\mathcal{L} &= \varphi \ln\left[F + \mu\left(a_2^2 + \beta a_1 a_2\right) - \left(\frac{1}{2}a_1^2 + \beta a_1 a_2 + \frac{1}{2}a_2^2\right)\right] \\ &\quad + (1-\varphi)\ln[a_1 + \gamma a_2 - F - \mu\left(a_2^2 + \beta a_1 a_2\right)] + \lambda F + \phi a_1 + \eta a_2,\end{aligned} \tag{16}$$

where λ, ϕ and η are the respective multipliers. The Kuhn-Tucker conditions yield a 6×6 equation system

$$\varphi \frac{(\mu - 1)\beta a_2 - a_1}{u} + (1-\varphi)\frac{1 - \mu a_2 \beta}{\pi} + \phi = 0 \tag{17}$$

$$\varphi \frac{\mu(2a_2 + \beta a_1) - \beta a_1 - a_2}{u} - (1-\varphi)\frac{\mu(2a_2 + \beta a_1) - \gamma}{\pi} + \eta = 0 \tag{18}$$

$$\frac{\varphi}{u} - \frac{1-\varphi}{\pi} + \lambda = 0 \tag{19}$$

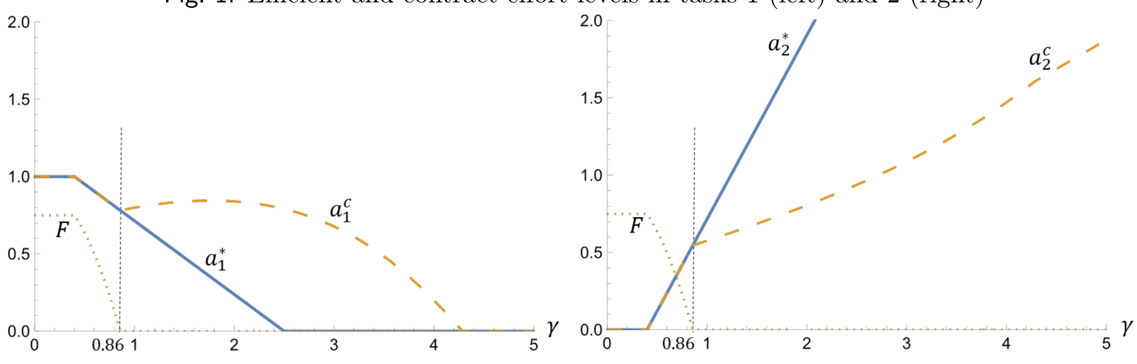
$$\lambda F = \phi a_1 = \eta a_2 = 0 \tag{20}$$

together with the inequalities $\lambda, \eta, \phi \geq 0$ and $a_1, a_2, F \geq 0$.

Intuitively, in the absence of a constraint on the fixed payment F, the parties would negotiate a contract which (i) sets effort in the two tasks at the efficient level (a_1^*, a_2^*) so as to maximize the overall economic surplus, and (ii) use F to distribute that surplus so as to maximize the NBP, $u^{\varphi}\pi^{1-\varphi}$. As we saw in the foregoing section, this requires $\varphi\pi = (1-\varphi)u$, which from (19) is only true if the multiplier for the fixed payment constraint, λ, is equal to zero. However, for some γ-type employees the fixed payment that would be required to allocate the surplus according to the distribution of bargaining power violates the non-negativity constraint on F, thereby rendering this solution infeasible. In the following, we analyze how this affects effort levels in the negotiated contract.

To gain some initial insights into the optimal contract, we used Mathematica to solve the above system for parameters $\beta = 0.4, \varphi = 0.5$, $\mu = 2$ and depict the solution for the different employee types $\gamma > 0$ in Figure 1. The efficient effort levels in tasks 1 (left graph) and task 2 (right graph) are depicted as solid lines, effort levels in the contract solution as dashed lines, and the fixed payment F as a dotted line. In line with the above intuition, efficient and contracted effort levels coincide as long as $F > 0$, which is the case for $\gamma \leq 0.86$.

Fig. 1: Efficient and contract effort levels in tasks 1 (left) and 2 (right)



Following Proposition 1, consider first the employees characterized by $\gamma < \beta$ for which the efficient solution requires $a_1^* = 1$ and $a_2^* = 0$. Implementing this solution as a contract requires no bonus $(B = 0)$. Therefore, $F > 0$ can be used for compensating the employee in order to implement the optimal surplus distribution, $\varphi\pi = (1-\varphi)u$. Accordingly, we have $a_1^c = a_1^* = 1$ and $a_2^c = a_2^* = 0$, as shown in Figure 1. Substitution into $\varphi\pi = (1-\varphi)u$ then yields the constant fixed payment $F^c = 0.5(\varphi + 1)$.

Second, consider employees with a productivity parameter in the range $\gamma \in [\beta, \frac{1}{\beta}]$ for whom the efficient solution satisfies $a_1^* = \frac{1-\beta\gamma}{1-\beta^2}, a_2^* = \frac{\gamma-\beta}{1-\beta^2}$. Substituting these effort levels into the condition for the optimal surplus distribution yields the fixed payment that would be required to satisfy $(1-\varphi)\,u = \varphi\pi$:

$$\widehat{F} = \frac{1+\varphi+\gamma^2\,(1+\varphi-2\mu)+2\beta\gamma\,(\mu-(1+\varphi))}{2\,(1-\beta^2)}. \tag{21}$$

Due to the constraint $F \geq 0$, this solution can only be implemented if the numerator in (21) is positive. Setting the numerator to zero yields a quadratic equation in γ with two roots. Given that $\gamma > 0$, only the positive root matters, which we denote by $\hat{\gamma}_1$:

$$\hat{\gamma}_1 = \frac{\beta\,(\mu-1-\varphi)+\sqrt{\beta^2\,(\mu-1-\varphi)^2+(2\mu-1-\varphi)\,(1+\varphi)}}{2\mu-1-\varphi}. \tag{22}$$

Noting that the expression for $\widehat{F}$ in (21) is strictly positive for $\gamma = 0$, we conclude that $\widehat{F} \geq 0$ if and only if $\gamma \leq \hat{\gamma}_1$. Moreover, in the proof of Proposition 2, we show that $\hat{\gamma}_1 \in (\beta, \frac{1}{\beta})$ so that it falls indeed within the range that we currently consider. For the parameter values of the example, (22) yields $\hat{\gamma}_1 \simeq 0.86$ (see Figure 1).

Altogether, over the region $\gamma \in (\beta, \hat{\gamma}_1]$, we have $a_1^c = a_1^*$, $a_2^c = a_2^*$ and $F^c = \widehat{F}$ from (21). Employees $\gamma \in (\hat{\gamma}_1, \frac{1}{\beta}]$ also exert effort in each task both in the efficient as well as in the negotiated solution. However, because $\widehat{F}$ is now negative, the payment constraint becomes binding, $F^c = 0$, and the two solutions no longer coincide.

Third, consider employees with a productivity parameter in the remaining range, $\gamma > \frac{1}{\beta}$. For them it is also true that the efficient solution with its specialization on task 2, $a_1^* = 0$ and $a_2^* = \gamma$, cannot coincide with the contract negotiated by the parties. Suppose to the contrary, substituting the efficient solution into $(1-\varphi)\,u = \varphi\pi$ and solving for the fixed payment yields

$$\widehat{F} = 2\gamma^2\,(1+\varphi-2\mu) < 0, \tag{23}$$

where the sign follows from $\varphi < 1$ and $\mu > 1$.

We now consider the optimal adjustment of the contract for employees $\gamma > \hat{\gamma}_1$ for which the payment constraint binds. As illustrated by the example in the right-hand graphic of Figure 1, a_2^* increases in the productivity parameter γ. However, as task 2 is non-verifiable, implementing $a_2 = a_2^*$ would require a large bonus payment to the employee, which — due to the latter's financial constraint $F \geq 0$ — renders the optimal surplus distribution infeasible. Hence the contract sets $F^c = 0$, $a_2^c < a_2^*$ and implements $\varphi\pi^c < (1-\varphi)\,u^c$. In other words, the employee receives a greater share of the economic rent than his bargaining power alone would warrant.[6] This reflects the trade-off between efficiency in effort and the optimal surplus distribution, which works through two channels. The first channel resembles the standard "rent versus efficiency" argument in the one task moral hazard model: to diminish the agent's informational rent, the bonus B is reduced, but as a result effort in task 2 is below its efficient level.

There is, however a second adjustment channel, which works through a modification of the expectation in the (routine) task 1. *Ceteris paribus*, by slightly increasing the effort in task 1, the parties are able to come closer to the optimal surplus distribution because the employee does not receive any additional compensation for it. However, the cross term, $\beta a_1 a_2$, in the cost function introduces a countervailing effect because raising a_1 increases the marginal costs of task 2 and, therefore, the bonus necessary to induce this task.

In Proposition 2, we show that there exists a range of medium productivity employee, $\gamma \in (\hat{\gamma}_1, \hat{\gamma}_2]$, for which the intended distributional effect of raising a_1 dominates the unintended countervailing

[6] Note, however, that this does not imply that the employee's total benefit is larger because the size of the economic rent is smaller than in the efficient solution.

effect from multi-tasking. This has two consequences; first, for these employees the optimal effort in task 1 is set inefficiently high, and, second, there are more employees than in the efficient solution who are required to exercise both tasks (technically, $\hat{\gamma}_2 > \frac{1}{\beta}$). In our numerical example a_1^c is even increasing in γ for types immediately to the right of $\hat{\gamma}_1$, although the marginal productivity of task 1 is independent of γ.

Altogether, from the perspective of employees with medium-productivity the private information on task 2 comes with a trade-off. On the one hand, it enables them to obtain an informational rent so that their payoff exceeds that of the firm after accounting for the bargaining weights, i.e. $(1-\varphi)\,u > \varphi\pi$. On the other hand, they are overloaded with work in the verifiable task 1, although they are no more qualified in this task than low-γ employees.

For high-productivity employees ($\gamma > \hat{\gamma}_2$) the countervailing effect from multi-tasking always dominates and a_1^c falls to the efficient level of 0. Intuitively, for these employees a_2 is large, making a positive effort level in task 1 increasingly costly due the cross term in the cost function. To conclude, we can distinguish between three groups of employees. Low-γ types ($\gamma < \hat{\gamma}_1$) exert efficient effort in both activities and their share of the economic rent is solely determined by their bargaining power φ. Medium-γ types ($\hat{\gamma}_1 \leq \gamma \leq \hat{\gamma}_2$) undertake inefficiently low effort in task 2, receive an information rent from effort in that task and are overloaded with work in task 1. Finally, high-γ types ($\gamma > \hat{\gamma}_2$) receive a large informational rent without facing the trade-off of having to work in task 1. The following proposition and corollary summarize the above elaborations.

Proposition 2. *Consider a continuum of employees that differ only in their productivity $\gamma > 0$. Let $\hat{\gamma}_1$ be as given in (22) and*

$$\hat{\gamma}_2 := \frac{4\mu - 2}{(2\mu - 1 + \varphi)\,\beta} > \frac{1}{\beta}. \tag{24}$$

The optimal contract that follows from problem (15) is as follows:

1. *For $\gamma \leq \hat{\gamma}_1$: $a_1^c = a_1^*$ and $a_2^c = a_2^*$ — i.e., for low-productivity employees effort is efficient in both tasks — and $F^c > 0$. Specifically, $F^c = 0.5\,(\varphi + 1)$ for $\gamma \leq \beta$ and $F^c = \frac{1+\varphi+\gamma^2(1+\varphi-2\mu)+2\beta\gamma(\mu-(1+\varphi))}{2(1-\beta^2)}$ for $\gamma \in (\beta, \hat{\gamma}_1]$.*

2. *For $\gamma \in (\hat{\gamma}_1, \hat{\gamma}_2]$: $a_1^c > a_1^*$ and $0 < a_2^c < a_2^*$ — i.e., for medium-productivity employees effort is inefficiently high in task 1, but inefficiently low in task 2 — and $F^c = 0$.*

3. *For $\gamma > \hat{\gamma}_2$: $a_1^c = a_1^* = 0$ and $0 < a_2^c = \frac{1+\varphi}{2\mu}\gamma < a_2^* = \gamma$ — i.e., for high-productivity employees effort is efficient in task 1, but inefficiently low in task 2 — and $F^c = 0$.*

Corollary 1. *Medium-productivity employees $\gamma \in (1/\beta, \hat{\gamma}_2]$ specialize on task 2 in the efficient solution, but they are assigned both tasks in the contract solution so as to reduce their informational rent from moral hazard.*[7]

5 Comparative statics and interpretations

5.1 Effects of bargaining power and monitoring quality on work overload

In the introduction, we motivated our analysis by referring to a mainly sociological literature which reports increasing complaints by employees that they are distracted from their main work by being inundated with routine tasks. In our analysis, these employees correspond to medium-productivity employees who are required to undertake an inefficiently high effort in task 1 (see Proposition 2). In this section, we consider two candidates for reasonable variations in the underlying parameters of the model that might offer plausible explanations for this trend: a decrease in φ and μ, which we rationalize by the decline in unionism and rapid developments in ICT that have enhanced the capacity of firms to monitor tasks that are prone to moral hazard. The next result summarizes the impact of φ and μ on the number of employees affected by excessive effort in the routine task.

[7] The corollary follows straightforwardly from the Propositions 1 and 2(2).

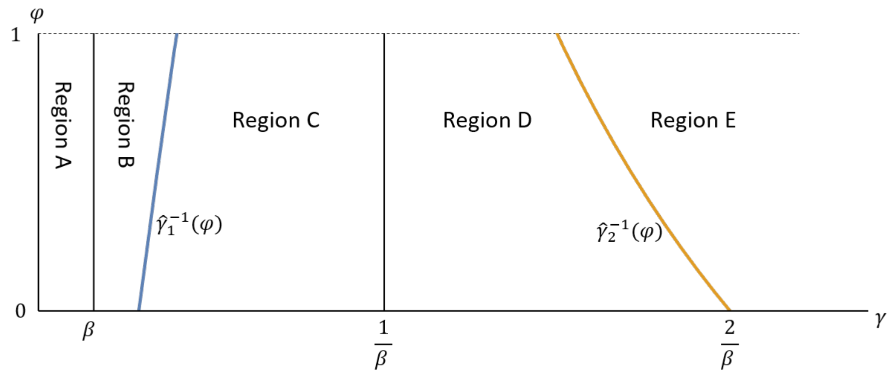
Proposition 3. *The mass of employees that exert inefficiently high effort in task 1 is*

1. *strictly increasing in the bargaining power of firms (lower φ), and*
2. *strictly decreasing in improvements of the monitoring technology (lower μ).*

We start by discussing the first point. Figure 2 and the table below show how the elements of an optimal contract depend on the bargaining power parameter φ.[8] Regions A and B represent low-productivity employees for whom the payment constraint at the optimal contract is not binding, so both efforts are set at the efficient level. High productivity employees who fully specialize in task 2 are found in region E. Finally, regions C and D constituted the group of medium productivity employees. These employees exert an inefficiently high effort in the "routine" task 1. In particular, in region D, it would even be efficient to have no work at all in the routine task. In line with Proposition 3, these two regions increase in the bargaining power of firms (lower φ).

Intuitively, as employees' bargaining power falls, extracting their rent by raising effort in the routine task 1 becomes more prevalent. More formally, note that Region C is bounded to the left by $\hat{\gamma}_1^{-1}(\varphi)$, i.e. by the critical employee for which the contract implements the efficient effort levels (a_1^*, a_2^*), the optimal surplus allocation $\frac{\pi}{u} = \frac{1-\varphi}{\varphi}$, and $F^c = 0$. Now imagine a reduction in the employees' bargaining power to $\hat{\varphi} = \varphi - \epsilon$, where ϵ is a small positive term. Maintaining the optimal surplus allocation would require that the ratio $\frac{\pi}{u}$ increases. As the efficient effort levels do not depend on φ, this would require a lower F. But this would violate the employee's financial constraint. Hence, $\hat{\gamma}_1(\varphi)$ becomes smaller for lower φ and Region C increases.

Fig. 2: Different solution regions and bargaining power



	A	B	C	D	E
a_1	$a_1^c = a_1^* = 1$	$a_1^c = a_1^* = \frac{1-\beta\gamma}{1-\beta^2}$	$a_1^c > a_1^* = \frac{1-\beta\gamma}{1-\beta^2}$	$a_1^c > a_1^* = 0$	$a_1^c = a_1^* = 0$
a_2	$a_2^c = a_2^* = 0$	$a_2^c = a_2^* = \frac{\gamma-\beta}{1-\beta^2}$	$a_2^c < a_2^* = \frac{\gamma-\beta}{1-\beta^2}$	$a_2^c < a_2^* = \gamma$	$a_2^c = \frac{1+\varphi}{2\mu}\gamma < a_2^* = \gamma$
F^c	$0.5(\varphi+1)$	$\frac{(1+\varphi)(1+\gamma^2-2\beta\gamma)+2\gamma\mu(\beta-\gamma)}{2(1-\beta^2)}$	0	0	0
		$\hat{\gamma}_1^{-1}(\varphi) = \frac{2\mu(\gamma-\beta)\gamma}{1+(\gamma-2\beta)\gamma} - 1$			$\hat{\gamma}_2^{-1}(\varphi) = \frac{(2-\beta\gamma)(2\mu-1)}{\beta\gamma}$

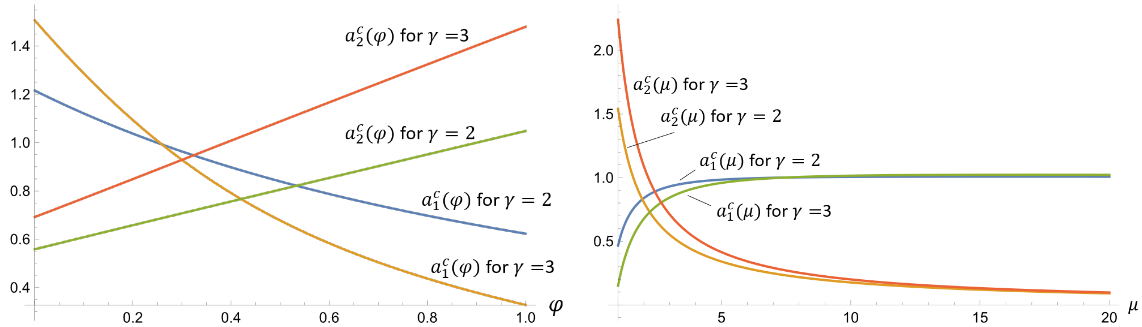
Next consider Region D, which is limited to the right by the $\hat{\gamma}_2^{-1}(\varphi)$-curve. In that region, the contract trades-off the impact of an unequal surplus allocation of the Nash Bargaining product, which can be measured by the distance $d := \frac{\pi}{u} - \frac{1-\varphi}{\varphi}$, against the efficiency loss due to the misallocation of effort levels. For a given bargaining power, $\hat{\gamma}_2(\varphi)$ is then defined by the critical γ for which this trade-off just implements a zero effort in task 1. Now consider again a small reduction in the employees'

[8] The graphic in Figure 2 has been derived using the same numerical example as for Figure 1 ($\beta = 0.4$ $\mu = 2$).

bargaining power to $\hat{\varphi}$ so that *ceteris paribus* the distance d decreases. To counteract this, the optimal contract marginally raises effort in task 1, thereby raising the payoff ratio π/u and, thus, the distance d. Consequently, a smaller φ means that Region D becomes larger.

Regarding the second statement in the proposition, we know that improving the quality of monitoring leads to a reduction in the marginal incentive costs associated with the moral hazard problem. The parties will therefore agree to marginally increase the effort in task 2. Due to the cross term $\beta a_1 a_2$, this increases the marginal costs of task 1. As a result, less employees are required to exert inefficiently high effort in task 1.

Fig. 3: Effort levels and variations in φ (left graph) and μ (left graph)



Proposition 3 and the discussion above have focused on how the borders of Regions C and D shift outwards as φ falls, implying that more employee types face an inefficiently high effort burden in task 1 — and conversely for lower μ. However, to explain the feeling of being overloaded by routine tasks, also the effects of changes in φ and μ on employees that already belonged to regions C and D matter. Unfortunately, the model is too complex to analytically determine the relevant comparative statics. Nevertheless, in line with the preceding discussion, our numerical simulation for one γ-type each from regions C and D ($\gamma = 2$ and $\gamma = 3$) show that the effort level in task 1 rises as the employees' bargaining power falls (lower φ), but decreases for improved monitoring (lower μ) (see Figure 3).[9]

5.2 Effects of bargaining power and monitoring quality on task discretion

Above we have found that a reduction in employees' bargaining power and better monitoring technologies have countervailing effects on effort levels in the routine task. Irrespective of this, whether high work effort is perceived as a burdening overload will also depend on how it is remunerated and on employees' ability to freely choose their effort level. Therefore, in the remainder we propose a different approach to interpret our results that rests on the idea of a decrease in tasks' discretion. It is motivated by empirical evidence based on UK data presented in Green (2004b), where the decline in employee well-being at work is shown to be "largely associated with a combination of rising effort and task discretion" (p. 616).

In our model, both levels of effort are incentivized; task 1 by the threat of dismissal and task 2 with the promise of a bonus. However, these distinct mechanisms imply a fundamental difference in the employee discretion associated with these activities. Intuitively, employees have full discretion over task 2 and no discretion over task 1. To see this more formally, consider the optimization problem of an employee after he accepted the contract $\mathcal{C}^c = \{F^c, B^c, a_1^c, a_2^c\}$. As the contract is incentive-compatible, we already know that the employee does not want to be dismissed, which requires $a_1 \geq a_1^c$.[10] Adding

[9] As for the other figures, we use the parameters $\beta = 0.4$ as well as $\mu = 2$ (left hand graphic) and $\varphi = 0.5$ (right hand graphic).
[10] Note that we used the same logic to eliminate the constraint $a_1 \geq a_1^c$ in (12).

this as a constraint, his maximization problem is

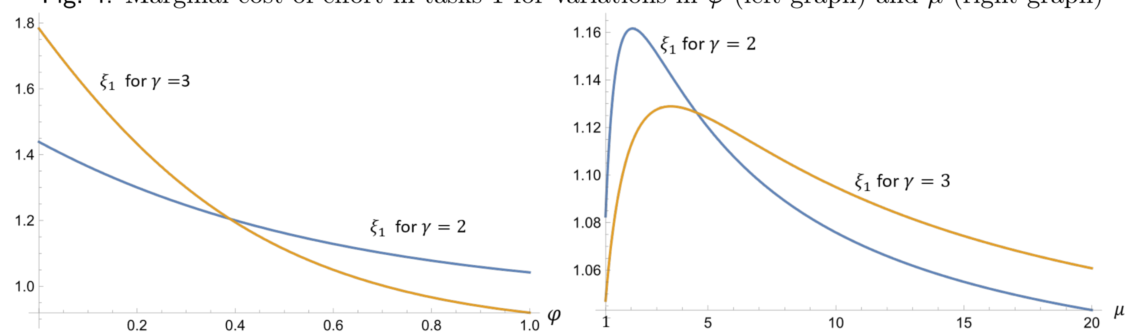
$$v(\mathcal{C}^c) := \max_{a_1,a_2} F^c + B^c p(a_2) - c(a_1,a_2) \text{ s.t. } a_1 \geq a_1^c, \tag{25}$$

where $v(\cdot)$ denotes the maximum value function. At this stage of the game, a_1^c, a_2^c are parameters. Hence, writing the Lagrangian of (25), and applying the envelope theorem yields $\frac{\partial v(\mathcal{C}^c)}{\partial a_1^c} = -\xi_1$ and $\frac{\partial v(\mathcal{C}^c)}{\partial a_2^c} = 0$, where ξ_1 is the multiplier associated with the constraint $a_1 \geq a_1^c$. The variable ξ_1 is the shadow (or virtual) price of a_1^c. It measures how much the employee would be willing to pay for marginally loosening the constraint. From the first-order condition of (25), we know that $\xi_1 = \frac{\partial}{\partial a_1} c(a_1^c, a_2^c) > 0$. Therefore, we obtain the following result.

Proposition 4. *Employees that exert both tasks under the negotiated contract would always be willing to pay a positive price for a marginal reduction in a_1^c, but nothing for a marginal change in a_2^c.*

In Green's (2004) terminology, this reflects that the employee has discretion over effort 2, but no discretion over effort 1.

Fig. 4: Marginal cost of effort in tasks 1 for variations in φ (left graph) and μ (right graph)



In the figure 4, we have plotted ξ_1 as functions of φ and μ for the values of $\gamma = 2$ and $\gamma = 3$. The resulting curves are decreasing except for the segment of the graph on the right where μ becomes close to 1. Intuitively, in this segment the quality of monitoring is so large that any further improvement sufficiently reduces the moral hazard associated with the second task to induce a significant shift away from the routine task to the skill demanding activity. Therefore, the moral hazard problem and the use of task 1 to extract the associated rent disappear. But of course, these are precisely the cases which are uninteresting for our analysis, hence we focus on the downward sloping section of these curves.

With this caveat, Figure 4 suggests that reducing union power (lower φ) and better monitoring capabilities through ICT improvements (lower μ) have lead to an increase in the shadow price associated with the required effort in the routine task. Intuitively, it makes the lack of discretion in the routine task costlier to the employee, providing a natural explanation for the aforementioned growing employee dissatisfaction.

6 Concluding remarks

There is a widespread perception that a substantial part of the working population is currently struggling with increasing work overload. One aspect of this evolution involves routine activities such as phone calls, text messages, emails, and paperwork that end up distracting employees from what they perceive to be their core business. To analyze reasons for this, we designed a parsimonious multitasking employment model in which firm/employee pairs negotiate a contract that may involve either a verifiable routine task, or a complex task characterized by moral hazard, or both. The model is further characterized by three key variables; the employee's productivity (or skill) in the complex task, the distribution of bargaining power in the Nash bargaining game, and a measure of quality associated with the monitoring signal used in the contract to decide on the payment of a bonus.

Low- and high-skilled employees specialize in the routine or demanding task, respectively, whereas medium-productivity employees are active in both tasks. For them, work overload in the routine task is used to reduce the informational rent they obtain due to the moral hazard problem associated with the complex activity. We also argued that these employees perceive this contractual structure as a loss of control. Intuitively, given their contract and keeping their cost of effort constant, they would always prefer, at the margin, to shift part of their effort from the routine task against an appropriate increase towards the complex activity.

These results are of particular relevance in the context of current discussions about the 'squeezed middle class' (OECD, 2019). Our comparative static analysis also indicate potential policy measures that might be suitable for addressing work overload. An example would be policies aimed at strengthening the bargaining power of employees, which has notably suffered from a significant decline in trade unionism in many countries. Our analysis suggests that such policies would not only improve wages, but also alleviate the growing work overload in routine activities. With regard to advances in ICT, it is often claimed that they facilitate the monitoring of employees and reduce slack time (Kelly and Moen, 2020). However, Gallie (2017) notes that the trends in perceived intensity do not line up nicely with the introduction of key technologies. Such a mixed picture also obtains from our analysis, where better monitoring technologies for complex activities reduce the work overload in routine tasks, but employees nevertheless perceive an increasing loss of control, making them feel dissatisfied.

Obviously, the possibility to reduce informational rents from one activity by work intensification in another depends on multitasking. We have examined the natural case where multitasking is costly in that greater effort in one activity raises the costs of the other. If these costs fall, which is captured by a lower β in our model, it is straightforward to see that the mechanism to extract rent by work overload in routine tasks will be used more intensively.[11] One might argue that advances in ICT such as the digitalization of administrative processes or speech-to-text software point in this direction.

Future research could extend the analysis in numerous directions. One avenue would be to remove some of the technical restrictions which were imposed for the sake of parsimony. An example of this would be to increase the range of β into the negative numbers in order to capture the idea of task complementarity. A different avenue would be to switch from a purely positive perspective to a more normative approach which would include welfare considerations and examine possible policy responses. For instance, one could embed the firm/employee negotiations in a general dynamic equilibrium environment which includes saving and investments. In such a setup one could easily introduce policy variables and evaluate their impact on contracting and more broadly on the income distribution as well as the investment by firms. All these items provide interesting avenues which are left for future research.

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[11] A formal analysis of this is available upon request.

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Appendix

A Proof of Proposition 1

Observe that the objective function in (5) is strictly concave and the constraints are linear. Hence, the first-order conditions together with the complementary slackness requirements are sufficient. From these requirements, there are, *a priori*, four possible cases which we examine in turn.

1. $\xi^* = \zeta^* = 0 \Longrightarrow a_1^*, a_2^* \geq 0$. Setting the value of the multipliers in (7) and (8) yields $a_1^* = \frac{1-\beta\gamma}{1-\beta^2}$ and $a_2^* = \frac{\gamma-\beta}{1-\beta^2}$. As the denominator is strictly positive, the condition $a_1^*, a_2^* \geq 0$ necessitates $\gamma - \beta \geq 0$ and $1 - \beta\gamma \geq 0$, which implies $\beta \leq \gamma \leq \frac{1}{\beta}$.

2. $\xi^* = 0, \zeta^* > 0 \Longrightarrow a_1^* \geq 0, a_2^* = 0$. Substituting $\xi^* = a_2^* = 0$ into (7) and (8) yields $a_1^* = 1$ and $\zeta^* = \beta - \gamma$. Accordingly, $\zeta^* > 0$ implies $\gamma < \beta$.

3. $\xi^* > 0, \zeta^* = 0 \Longrightarrow a_1^* = 0, a_2^* \geq 0$. Now substituting $\zeta^* = a_1^* = 0$ into (7) and (8) yields $a_2^* = \gamma$ and $\xi^* = \gamma\beta - 1$. Hence $\xi^* > 0$ implies $\gamma > \frac{1}{\beta}$.

4. $\xi^*, \zeta^* > 0 \Longrightarrow a_1^* = a_2^* = 0$. From (7), this would imply $\xi = -1$, leading to a contradiction so that this case cannot occur.

B Proof of Proposition 2

The proposition is organized along the employee types and the threshold values $\hat{\gamma}_1$ and $\hat{\gamma}_2$. We start by showing that $\hat{\gamma}_1 \in (\beta, \frac{1}{\beta})$ and $\hat{\gamma}_2 \in (\frac{1}{\beta}, \frac{2}{\beta})$, which yields the stated ranges for the three cases in the proposition ($\gamma \leq \hat{\gamma}_1, \gamma \in (\hat{\gamma}_1, \hat{\gamma}_2]$ and $\gamma > \hat{\gamma}_2$). In particular, $\beta < \hat{\gamma}_1$ is equivalent to

$$\beta < \frac{\beta\left(\mu - 1 - \varphi\right) + \sqrt{\beta^2\left(\mu - 1 - \varphi\right)^2 + \left(2\mu - 1 - \varphi\right)\left(1 + \varphi\right)}}{2\mu - 1 - \varphi}. \tag{26}$$

Multiplying both sides with the denominator of the right-hand fraction, canceling common terms, isolating the square root and squaring both side yields:

$$0 < \left(\varphi + 1\right)\left(1 - \beta^2\right)\left(2\mu - 1 - \varphi\right). \tag{27}$$

This inequality is clearly satisfied because $\beta, \varphi < 1$ and $\mu > 1$.

Next, consider $\hat{\gamma}_1 < \frac{1}{\beta}$. Performing similar operations, the inequality holds if and only if

$$0 < \left(1 - \beta^2\right)\left(2\mu - 1 - \varphi\right), \tag{28}$$

which is again clearly satisfied. Using similar steps, it straightforward to show that $\frac{1}{\beta} < \hat{\gamma}_2 < \frac{2}{\beta}$.

Similarly to the approach of proving Proposition 1, we structure the proof of claims 1 to 3 along the different cases that can obtain for the multipliers in the Kuhn-Tucker conditions (17 to (20). That the maximization program (15) has a solution that satisfies these conditions for any $\gamma > 0$ follows from Lemma 1.

Suppose $\lambda = 0$ so that $\frac{\varphi}{u} = \frac{1-\varphi}{\pi}$ from (19). Substitution into the optimality conditions (17) and (18), these simplify to

$$\begin{aligned} 1 - a_1 - \beta a_2 + \phi\frac{u}{\varphi} &= 0 \\ \gamma - \beta a_1 - a_2 + \eta\frac{u}{\varphi} &= 0, \end{aligned} \tag{29}$$

where $\dfrac{u}{\varphi} > 0$. Comparing the equation system (29) with (7) and (8) implies that the parties will negotiate efficient effort a_1^* and a_2^*. From the discussion of Eqs. (22) and (21), this solution is feasible if and only if $\gamma \leq \hat{\gamma}_1$. Noting that the values for F^c have already been derived in the main text, this concludes the proof of statement 1.

From the above, we know that for all other types of employees, $\gamma > \hat{\gamma}_1$, we have $\lambda > 0$ and, thus, $F^c = 0$. Accordingly, the employee's payoff simplifies to

$$u = \mu\left(a_2^2 + \beta a_1 a_2\right) - \left(\frac{1}{2}a_1^2 + \beta a_1 a_2 + \frac{1}{2}a_2^2\right). \tag{30}$$

This requires $a_2^c > 0$ because otherwise $u < 0$, which cannot be optimal by Lemma 1. Hence, by complementary slackness we obtain $\eta = 0$ for all $\gamma > \hat{\gamma}_1$ and, thus, for all $\lambda > 0$. Altogether, with

$\gamma > \hat{\gamma}_1$ two situations can occur that are now examined in turn: $\lambda > 0, \phi = 0$ and $\lambda > 0, \phi > 0$ that will be shown to coincide with cases 2 and 3 in the Proposition. We start with the latter.

With $\lambda > 0$ and $\phi > 0$, complementary slackness implies $F^c = a_1^c = 0$. Substituting this together with $\eta = 0$ into (18), and solving for a_2 yields:

$$a_2^c = \frac{1+\varphi}{2\mu}\gamma. \tag{31}$$

Substituting a_2^c together with $F^c = a_1^c = 0$ into (17) and rearranging terms yields

$$\frac{2}{\gamma} - \frac{2\mu - 1 + \varphi}{2\mu - 1}\beta + \frac{1+\varphi}{2\mu}\gamma\phi = 0. \tag{32}$$

The condition $\phi \geq 0$ requires that the first two terms add up to a non-positive value. Observing that these terms are decreasing in γ, we obtain the critical productivity level $\hat{\gamma}_2$ defined by (24). In particular, $\phi > 0$ obtains if and only if $\gamma > \hat{\gamma}_2$. Moreover, observe that $\hat{\gamma}_2 > 1/\beta$ so that $a_1^* = 0$ and $a_2^* = \gamma$ from Proposition 1. Therefore, $a_1^c = a_1^* = 0$ and from (31) we have $a_2^c < a_2^*$ because $\varphi < 1$ and $\mu > 1$. This concludes the proof of statement 3.

Finally, suppose $\lambda > 0$ and $\phi = 0$. From the above, this case applies to all $\gamma \in (\hat{\gamma}_1, \hat{\gamma}_2]$, for which we have already shown that $\eta = 0$ and $a_1^c > 0$. Solving (18) for φ/u and substitution into (19) yields

$$\frac{1-\varphi}{\pi}\frac{\gamma - \beta a_1 - a_2}{\mu\left(2a_2 + \beta a_1\right) - \left(a_2 + \beta a_1\right)} = \lambda > 0. \tag{33}$$

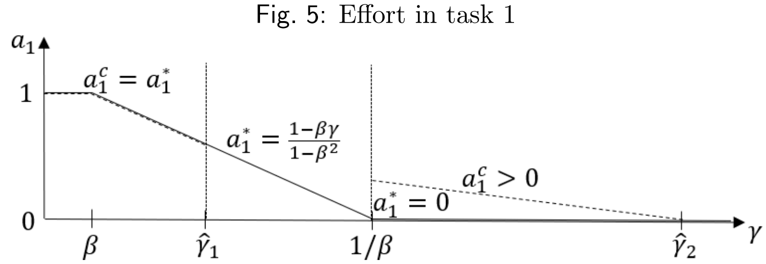
Note that the denominator is strictly positive because $\mu > 1$. Thus, for $\lambda > 0$ the contract solution requires $\gamma - \beta a_1^c - a_2^c > 0$.

First, consider the types $\gamma \in (1/\beta, \hat{\gamma}_2]$ where the efficient effort levels are $a_1^* = 0 < a_1^c$ and $a_2^* = \gamma$. Accordingly, the solution satisfies $\gamma - \beta a_1^c - a_2^c > 0 = \gamma - a_2^*$. Canceling common terms and rearranging yields $a_2^* > \beta a_1^c + a_2^c$ which verifies $a_2^c < a_2^*$. Second, consider the types $\gamma \in (\hat{\gamma}_1, 1/\beta]$. From the first-order conditions (17)–(18) by using $\eta, \phi = 0$ and canceling common terms (π, u and φ) we obtain

$$\left(1 - \beta a_2 - a_1\right)\left(\mu\left(2a_2 + \beta a_1\right) - \gamma\right) + \left(1 - \mu a_2 \beta\right)\left(\gamma - \beta a_1 - a_2\right) = 0. \tag{34}$$

Remember that $a_1^c = a_1^*$ for $\gamma < \hat{\gamma}_1$, $a_1^* = \frac{1-\beta\gamma}{1-\beta^2}$ for $\gamma \in (\hat{\gamma}_1, 1/\beta]$ and $a_1^c > a_1^* = 0$ for $\gamma > 1/\beta$. Figure 5 depicts this situation.

Fig. 5: Effort in task 1



By contradiction to the claim in the Proposition, suppose that there exists a $\gamma \in (\hat{\gamma}_1, 1/\beta]$ with $a_1^c \leq a_1^*$. By continuity of a_1^c in γ, there must then be a $\gamma \in (\hat{\gamma}_1, 1/\beta]$ for which $a_1^c = a_1^* = \frac{1-\beta\gamma}{1-\beta^2}$. Substitution into (34) yields an equation in a_2 with two possible roots

$$a_{2,1} = \frac{\gamma - \beta}{1 - \beta^2} \text{ and } a_{2,2} = \left(\beta\gamma - 1\right)\frac{\beta^2\left(\mu - 1\right) + 1}{\beta\mu\left(1 - \beta^2\right)}. \tag{35}$$

The first root is the efficient effort level, a_2^*. As a_1^c is also equal to a_1^* and $\gamma > \hat{\gamma}_1$ it yields a contradiction to our finding that for all $\gamma > \hat{\gamma}_1$ the constraint $F^c \geq 0$ binds so that the solution is inefficient. The second root is also not feasible because with $\gamma \leq 1/\beta$ it would imply $a_{2,2} \leq 0$. However, we have already shown that $a_2^c > 0$ for all $\gamma \in (\hat{\gamma}_1, \hat{\gamma}_2]$. Altogether, we conclude that $a_1^c > a_1^*$ for all $\gamma \in (\hat{\gamma}_1, 1/\beta]$. From condition (8) for the efficient solution, this yields $\gamma - \beta a_1^c - a_2^* < 0$ (note that $\zeta = 0$ for the relevant range $\gamma \in (\hat{\gamma}_1, 1/\beta]$). Moreover, we have already shown that $\gamma - \beta a_1^c - a_2^c > 0$. Taken together this implies $a_2^c < a_2^*$, which concludes the proof of statement 2.

C Proof of Proposition 3

We start with statement 1. From Proposition 2 we know that $a_1^c > a_1^*$ if and only if $\gamma \in (\hat{\gamma}_1, \hat{\gamma}_2]$. As illustrated in Figure 2, the claim therefore follows immediately if $\hat{\gamma}_1$ is (weakly) increasing and $\hat{\gamma}_2$ is strictly decreasing in φ. The latter is obvious from (24). Moreover, differentiation and rearranging terms yields

$$\frac{d\hat{\gamma}_1}{d\varphi} = \frac{\mu}{(\varphi - 2\mu + 1)^2} \left(\frac{\left(1-\beta^2\right)(\mu - \varphi) + \beta^2 + \mu - 1}{\sqrt{\beta^2\varphi^2 + \mu - \varphi^2 + \left((2\varphi+1)\left(1-\beta^2\right) + \beta^2\mu\right)(\mu - 1)}} - \beta \right),$$

where the two fractions are strictly positive. Therefore, $\frac{d\hat{\gamma}_1}{d\varphi} > 0$ if and only if

$$\begin{aligned} &\left[\left(1-\beta^2\right)(\mu-\varphi) + \beta^2 + \mu - 1\right]^2 - \beta^2\left[\beta^2\varphi^2 + \mu - \varphi^2 + \left((2\varphi+1)\left(1-\beta^2\right) + \beta^2\mu\right)(\mu-1)\right] \\ &= \left(1-\beta^2\right)(\varphi - 2\mu + 1)^2 > 0, \end{aligned}$$

which is clearly satisfied.

The proof of statement 2 is done in the same way. In particular, the claim follows immediately if $\hat{\gamma}_1$ is (weakly) decreasing and $\hat{\gamma}_2$ is strictly increasing in μ. Differentiation of $\hat{\gamma}_2$ w.r.t. μ yields $d\hat{\gamma}_2/d\mu = \frac{4\varphi}{\beta(2\mu+\varphi-1)^2} > 0$. Similarly, differentiation of $\hat{\gamma}_1$ w.r.t. μ yields

$$\frac{d\hat{\gamma}_1}{d\mu} = \frac{(1+\varphi)\left(1 - 2\mu + \varphi + \beta\left(\beta(\mu - \varphi - 1) + \sqrt{\beta^2(1-\mu+\varphi)^2 + (1+\varphi)(2\mu - 1 - \varphi)}\right)\right)}{(\varphi + 1 - 2\mu)^2\sqrt{\beta^2(-\mu+\varphi+1)^2 + (\varphi+1)(2\mu - \varphi - 1)}}. \quad (36)$$

The denominator is clearly positive. Hence we need to show that the numerator is negative, i.e.,

$$\beta\sqrt{\beta^2(1-\mu+\varphi)^2 + (1+\varphi)(2\mu-1-\varphi)} < 2\mu - 1 - \varphi - \beta^2(\mu - \varphi - 1) = (1-\beta^2)(2\mu - 1 - \varphi) + \beta^2\mu. \quad (37)$$

Observe that the terms on both sides of the inequality are strictly positive. Squaring them yields after some rearrangements and cancellation of common terms

$$0 < (1-\beta^2)(2\mu - 1 - \varphi). \quad (38)$$

Given the restrictions on the parameters, this is clearly satisfied, which concludes the proof.