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Abstract

We analyze the effects of optimism and overconfidence when the manager's compensation package includes severance pay and the CEO has bargaining power. We find that optimism does not affect incentive pay but increases severance pay with a negative effect on profit. Overconfidence, on the contrary, reduces incentive pay as shown by the previous literature, while its effect on severance pay depends on the intensity of the bias. High values of overconfidence yield an inefficient level of investment which in turn increases severance pay with a negative impact on firm profit. Thus, the attempt to exploit managerial overconfidence to reduce incentive pay may back.re if the manager is replaced and severance agreements come into effect. Our model explains the large severance payments documented by empirical literature by showing that discretionary pay in excess of contractual severance pay may represent a form of efficient contracting when the manager is overconfident and optimist.

JEL-Codes: J330, D860, D900, L210.

Keywords: overconfidence, optimism, managerial compensation, severance pay, entrenchment.

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Do firms gain from managerial overconfidence? The role of severance pay.

Clara Graziano* and Annalisa Luporini‡

June 2, 2022

1 Introduction

A vast and well-documented empirical evidence suggests that managers often overvalue their probability of success, making overconfidence a common phenomenon among CEOs (Malmendier and Tate, 2015; Santos-Pinto and de la Rosa 2020). Theoretical and empirical literature has shown that managerial biases influence contract design and CEO behavior in many different aspects, such as CEO choice of projects (Malmendier and Tate 2005), CEO hiring (Goel and Thakor 2008) and CEO compensation (de La Rosa 2011, Gervais et al. 2011, Otto 2014). In particular, some authors have suggested that principals can benefit from hiring overconfident managers. In a standard agency model with moral hazard, the optimal contract trades off risk insurance and incentive provision. Managerial overconfidence, and the resulting divergence of beliefs between principal and agent, affect the trade-off between risk and incentives making it easier to satisfy the incentive compatibility constraint (Santos-Pinto 2008, de La Rosa 2011, Gervais et al. 2011, Otto 2014).[1] Firms can take advantage of this effect either by inducing the same level of effort required to an unbiased (rational) manager at a lower cost, or by offering a compensation structure with a particularly heavy incentive pay (the so-called exploitation hypothesis).

Another strand of literature points out the role of severance pay from an optimal contracting point of view. Given the high turnover among executives, severance agreements are an important component

[1] Experimental evidence about the effect of overconfidence on the effort provided by the worker is reported by Chen and Schildberg-Horisch, 2019.

of managerial contracts, even more so when a manager has to make long-lasting investments and the prospect of dismissal may interfere with her decision. If the board cannot commit to retain the manager once the investment is in place, there is room for opportunistic behavior. Severance pay may alleviate the moral hazard problem created by the investment being unobservable and firm-specific (Almazan and Suarez 2003, Inderst and Mueller 2010, Wu and Weng 2018), it may encourage risk taking behavior by a risk-averse manager or by a risk-neutral manager concerned about losing her position (Laux, 2015, Cadman et al. 2016 and Cadman et al. 2021) and it may discourage window dressing or misreporting information to avoid replacement (Vladimirov 2021).

On the other hand, the critics of severance agreements have pointed out that, by insulating the manager from the consequences of poor performance, such payments are simply "rewards for failure" that violate the pay-for-performance principle of agency theory (Bebchuk and Fried 2004) and may discourage investments (Muscarella and Zhao 2015). The fact that high separation payments often come in difficult times for the company and amid layoffs, makes it problematic to justify them in front of the public opinion. For example, despite the dramatic reduction in revenues to football clubs and universities caused by covid-19 pandemic, some football coaches received sizeable severance packages that spurred harsh criticism.[2] Criticism is particularly widespread when severance pay appears as a discretionary payment in excess of contractual provisions. Then, a natural question is whether we can reconcile such practice with efficient behavior on the part of the board instead of following the media interpretation in terms of shareholders expropriation.

The present paper is an attempt to answer this question by analyzing the effect of managerial optimism and overconfidence when a CEO is fired and severance agreements come into effect. We follow the previous literature (see de la Rosa 2011, among others) and we distinguish between *optimism* and *overconfidence*. Optimism occurs when the manager has a subjective belief on the probability of success higher than the “true” probability, while overconfidence distorts the manager’s assessment of the increase in the probability of success due to her effort. In our context, effort takes the form of a firm-specific investment that is observable but unverifiable. We investigate how optimism and overconfidence affect both investment choice and the amount of separation pay necessary to induce the manager to leave when this is profit enhancing. We build upon the analysis of Almazan and Suarez (2003) who suggest that renegotiating severance pay when separation occurs may be optimal because it allows to establish the exact amount of the payment ex-post, once the board knows whether the investment has been made. This is cheaper than just motivating the manager through an incentive pay that has to satisfy an ex-ante incentive compatibility constraint.

Similarly to previous literature on the optimality of severance agreements, in our model severance pay helps inducing the manager to undertake the desired level of investment. However, overconfidence and optimism create a wedge between board’s and manager’s beliefs on expected profit and affect the amount of severance pay asked by the manager. We assume that the incumbent manager has some bargaining power and she/he can credibly threaten to resist being replaced. The idea underlying this assumption is that the incumbent manager can oppose her replacement by making it a costly and contentious process so that valuable opportunities are missed and firm value decreases. To allow a smooth replacement, the board is willing to renegotiate the separation agreement and to consent to a payment high enough to avoid costly opposition. Thus, the board and the manager agree on an amount of money in excess of contractual severance pay. We show that this renegotiated pay depends on optimism and overconfidence,

[2] In November 2020 Will Muschamp received \$15.5 million when leaving the University of South Carolina, shaded just few months later by head coach Gus Malzahn’s reported \$21.5 million buyout from Auburn University. Empirical evidence indicates also that managers often receive large separation payments in excess of contractual severance pay (Goldman and Huang 2015). In February 2020, for instance, MGM Resort CEO Jim Murren, stepped down and got a very generous severance package of \$32 million. In this case the whole amount can be considered discretionary pay since his contract stipulated that he would not receive any severance for leaving voluntarily.

both directly and indirectly through the choice of the investment level.

The main findings of the paper can be summarized as follows. First, managerial optimism and overconfidence, that usually result as beneficial for the firm when only incentive pay is considered, may turn out to be detrimental when turnover and severance pay are taken into account. Second, overconfidence and optimism have different impact on managerial compensation package and firm expected profit. Optimism does not affect incentive pay but raises contractual severance pay and thus reduces firm profit. Overconfidence, on the other hand, may be either advantageous or detrimental, depending on the degree of the bias. Moderate overconfidence reduces both incentive and severance pay without affecting investment choice and consequently results in higher expected profit. Extreme overconfidence, on the contrary, reduces incentive pay but distorts investment choice and thus increases renegotiated severance pay. This effect may offset the reduction in incentive pay with a negative impact on profits. Thus, our model shows that the attempt to exploit executive overconfidence through a heavy use of incentive-pay, documented for example by Humphrey-Jenner et al. (2016), can backfire when the investment choice and the opportunity of replacing the manager are considered.

Finally, our model helps explaining the common practice of granting a separation pay largely exceeding its contractual level as documented by empirical studies (Goldman and Huang 2015) and anecdotal evidence . We rationalize such high payments as a result of the manager's bargaining power coupled with a high level of managerial overconfidence, showing that extreme overconfidence, by inducing the manager to choose an inefficiently high level of investment, leads to a particularly large discretionary pay.

Our paper contributes to the two streams of literature outlined above. First, it contributes to the literature on managerial overconfidence and optimism in a principal-agent relationship (see, for example, Santos-Pinto 2008, de la Rosa 2011, Otto 2014) by showing that models that do not account for the possibility of managerial turnover, and therefore do not consider severance payment, may overstate the positive contribution of overconfidence and optimism. In particular, while the positive effect of moderate overconfidence on expected profit is in line with the results obtained in the previous literature, extreme overconfidence may have a negative effect. Moreover, in our setting, optimism increases renegotiated severance pay and reduces firm expected profits contrary to what happens in the principal-agent literature where it relaxes the incentive/insurance trade-off due to the agent being risk averse, thus making high-powered incentives more profitable for the firm.

Second, our model is related to the literature on the role of severance pay from an optimal contracting point of view. A few papers demonstrate that severance pay, by protecting the manager from the cost of dismissal, can alleviate information revelation problems. For instance, Green and Taylor (2016) show that severance pay may be necessary to induce truthtelling when the manager has an informational advantage over the principal and the latter has to decide whether to terminate a multistage project on the basis of such information. Similarly, Vladimirov (2021) focuses on the interplay between severance pay and contract length to discourage managers from trying to avoid replacement through window dressing or information concealment. In a different setting and closer to our model, Inderst and Mueller (2010) find that offering a combination of severance pay and steep incentive pay may be the cheapest way to induce the manager to disclose information that may lead to her dismissal. A steep incentive scheme makes continuation costly for a "bad" manager and severance pay makes the outside option more attractive. Our paper contributes to this literature by showing that the quasi-rent necessary to induce the manager to leave are likely to be larger when the manager is optimistic and overconfident than in absence of biases. This is particularly important because a steep incentive scheme is more attractive for an optimistic and overconfident manager because of her higher belief of success. Thus it may fail to induce the "bad" manager to leave, leaving severance pay as the only available instrument.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we study the replacement decision and the renegotiation stage. Section 4 analyzes the optimal compensation package. Given the optimal severance and incentive pay, Section 5 investigates the effects of optimism and overconfidence on the firm expected profits. Finally, Section 6 concludes.

2 The model

Consider a board that perfectly represents the shareholders and maximizes firm's value. The board hires a CEO to implement a project. The cash flow generated by the project can take two values, either $r = 0$ or $r = R > 0$. The probability of success of the project, denoted by p_k, depends on a firm-specific investment I_k, $k = L, M, H$, made by the manager after joining the firm. In the absence of investment ($I = I_L = 0$), the probability of success is $p_L > 0$. If the manager makes investment I_M, the probability of success increases to $p_M > p_L$, while it becomes $p_H > p_M$ when the larger investment $I_H > I_M$ is chosen. The cost of the investment $c(I_k) = I_k$ is borne by the CEO. The investment is unverifiable, though it is observable by the board that, consequently, comes to know the manager's probability of success.

Only after the CEO has decided the level of the investment, a new manager materializes. We denote the probability of success of the new manager by $q \in [0, 1]$ with density function $f(q)$. Both the board and the incumbent manager observe the realization of q. Note that the new manager may have a higher probability of success than the incumbent. This can result, for example, from a better match between the new manager's ability and the skills required by the firm (possibly the new manager has made elsewhere an investment in human capital that is valuable also in this firm). In other words, the firm-specific investment of the incumbent may not be sufficient to avoid being less productive than the replacement. In such a case, the board may prefer to fire the incumbent and hire the replacement.

We assume that the CEO can oppose being fired so that replacement can occur only with mutual agreement between board and CEO. Specifically, we assume that the manager has a high enough bargaining power to oppose replacement if contractual severance pay is smaller than what the manager believes she/he would receive by staying with the firm, an amount that can be considered her "outside option" in the bargaining process.[3]

The incumbent manager and the board hold heterogeneous belief regarding the probability of success and are aware of such divergence which affects both the original contract and subsequent renegotiation, if any. Following the previous literature (de la Rosa, 2011), we decompose the managerial bias into two components: optimism, θ, that is independent of the manager's action and is always at work, and overconfidence Δ_i, $i = M, H$, that captures the manager's distorted belief on the productivity of her investment. The effect of such biases on beliefs is made explicit by the following assumption:

Assumption 1: The manager's beliefs about the probability of success are:
i) $p_L + \theta$, if no investment is made, $I = I_L = 0$;
ii) $p_M + \theta + \Delta_M$, if the manager makes investment I_M;
ii) $p_H + \theta + \Delta_H$, if the manager makes investment I_H, where $\Delta_H = \Delta_M(1 + z)$ denotes the high overconfidence resulting from the high investment, with $z > \frac{p_H - p_M}{p_M - p_L} > 0$ and $1 - p_H - \theta \geq \Delta_H > 0$.

Assumption 1 states that optimism has a uniform effect on the manager's belief, while the effect of overconfidence is higher in case of I_H than in case of I_M. Moreover, given $z > 0$, the slope of the

[3] We believe this is a reasonable assumption. Many alternatives, however, can be considered. For instance, the manager could be able to appropriate the whole surplus from the replacement (Almazan and Suarez, 2003, for example, consider this case). Of course, assuming a stronger bargaining power, resulting in a larger fraction of surplus for the incumbent manager, would make firing an overconfident manager even more costly for the firm, thus strengthening our results.

manager's beliefs of success (considered as a function of overconfidence) is everywhere steeper in case of I_H. In what follows, we consider the parameter z as constant and we refer to an increase in Δ_M as an increase in overconfidence. Then, a rise in overconfidence implies a higher increase in the manager's beliefs of success if the latter chooses I_H than if she/he chooses I_M.

The following assumption completes the framework.

Assumption 2: *Investment I_M is efficient: $(p_M - p_L)R > I_M$ while investment I_H is not: $(p_H - p_M)R < I_H - I_M$.*

Note that Assumptions 2 implies: $\frac{I_H - I_M}{p_H - p_M} \geq R \geq \frac{I_M}{p_M - p_L}$ which can be satisfied only if:

$$\frac{I_M}{I_H} \leq \frac{p_M - p_L}{p_H - p_L}.$$

Assumption 2 ensures that I_M is socially profitable, because the additional cost of choosing I_H rather than I_M is larger than the additional expected return from the high investment. The cost of the investment is borne by the manager but, as explained below, it may affect the outcome of the renegotiation in case of dismissal and it may thus become a cost also for the firm.

The contract offered by the board maximizes the expected final cash flow of the project net of managerial compensation. Recalling that the new manager is not necessarily more productive than an incumbent manager who has made a positive investment, we focus on cases where the profit-maximizing board wants to provide the incumbent with the incentive to invest even if replacement may occur with some positive probability. To this end, we consider a simple incentive contract with base salary, incentive pay w contingent on the high return R, and severance pay s. We normalize the reservation level of utility to 0, so that the base salary of the incumbent takes value 0, as well as the compensation of the new manager when replacement occurs.

The manager is protected by limited liability. If the incumbent remains in office, she enjoys benefit of control $B > 0$. Such non-monetary benefit increases the utility of staying with the firm and may thus create a conflict between shareholders and incumbent when replacement is profitable. However, it also reduces the monetary incentive needed to motivate the manager to invest. In any case, we assume that B is too small to induce the manager to choose a positive level of investment I_i, $i = M, H$ in the absence of an additional pay.

The timing of the model can be summarized as follows:

$t = 0$: The board observes whether the manager is overconfident, optimistic or both and offers a compensation contract (w, s) tailored to the manager's type. The manager decides whether to accept the offer.

$t = 1$: If the contract is accepted, the manager decides whether and how much to invest.

$t = 2$: The board observes the investment decision and deduces the probability of success.

$t = 3$: A rival manager appears. Board and incumbent manager observe the rival's ability. The board evaluates whether it is profitable to replace the incumbent. If this is the case and contractual s is too low for the incumbent to accept replacement, renegotiation occurs and a new level of severance pay, s', is agreed upon.

$t = 4$: Cash flow realizes. The manager is paid the compensation/severance pay agreed upon.

The model is solved by working backwardly. We first determine the conditions for replacement and we find the outcome of the renegotiation under an arbitrary initial contract. Then the board's replacement decision is discussed. Given the replacement decision, we determine the investment level chosen by the manager. Finally, the incentive compatible contract (w, s) that maximizes the firm final cash flows is derived.

3 Renegotiation and replacement decision

Let us first establish the condition under which the board is willing to fire the incumbent manager, once the parties have struck a contract designed to induce a positive level of investment I_i, $i = M, H$. The board wants to replace the incumbent whenever the expected profit is higher under the new manager, i.e., when the gain from replacement is higher than the cost:

$$qR - S_i \geq p_i(R - w_{io}), \qquad i = M, H \tag{1}$$

where S_i indicates the severance pay, either contractual (s_i) or renegotiated (s_i'), and w_{io} is the incentive pay offered to induce an overconfident manager o to undertake investment $i = M, H$. The LHS of the condition incorporates the fact that no incentive pay is due to the new CEO when the incumbent is replaced.

The board uses the "right" probability of success. With no constraints on the manager's side, the board would fire her when the probability of success of the replacement is

$$q \geq p_i - \left(\frac{w_{io} - S_i}{R} \right). \tag{2}$$

The firing decision is based on the difference between the probabilities of success of replacement and incumbent, "adjusted" for the difference in the payment to the incumbent manager in case of retention or dismissal. When $S_i > w_{io}$, meaning that the firing cost exceeds the sum that is saved by replacing the manager, the board will opt for replacement for higher values of q than in the opposite case where $S_i < w_{io}$.

Given that the manager can oppose being replaced, severance pay S_i (contractual or renegotiated) must compensate for the loss she will suffer in such a case. If the incumbent has undertaken investment I_i, she believes that her expected compensation is $(p_i + \theta + \Delta_i) w_{io}$. Consequently she consents to replacement only if:

$$S_i \geq (p_i + \theta + \Delta_i) w_{io} + B. \tag{3}$$

where the RHS represents the benefit the CEO expects to receive if she opposes replacement.

If contractual s does not satisfy the above condition, a board willing to fire the incumbent renegotiates the contract by making a take-it-or-leave-it offer, s', to the manager. Whenever s' satisfies the above inequality, the manager accepts the offer. The optimal level of renegotiated severance pay s_{io}' is clearly equal to the RHS of (3). Given that the investment decision has already been made, there is in fact no reason for the board to increase the renegotiated payment above the minimum level necessary to overcome the incumbent's opposition.

Then, for replacement to occur, conditions 1 and 3 must be simultaneously satisfied:

$$(q - p_i)R + p_i w_{io} \geq S_i \geq (p_i + \theta + \Delta_i) w_{io} + B,$$

implying that the increase in the expected return from replacement must be at least as large as what is lost by the incumbent when she leaves the firm:

$$(q - p_i)R \geq (\theta + \Delta_i) w_{io} + B. \tag{4}$$

Provided that this inequality is satisfied, the cutoff value of q, above which the board will replace the incumbent, is determined by (2) taken as an equality. When q is greater than the cutoff, the incumbent should be replaced even if she has made the firm-specific investment, while she should be retained when q is lower than the cutoff.

The lowest feasible value for such cutoff is the one that satisfies also (4) in the form of an equality. Such value obtains when S_i is kept as low as possible and this is what occurs in case of renegotiation. We here anticipate that contractual s will never be higher than the RHS of (4), so that the minimum level of the cutoff will be the relevant one in order to make the replacement decision (see Proposition 2 below). Let us denote such minimum level by:

$$\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i)}{R} w_{io}.$$

In order to establish a benchmark for our analysis, we also consider a rational manager whose subjective beliefs are equal to the "true" probabilities p_i (i.e., a manager with $\theta = \Delta_M = 0$). The minimum cutoff of q for a rational manager is:

$$\widehat{q}_{ir} = p_i + \frac{B}{R}$$

Optimism and overconfidence distort the replacement decision, $\widehat{q}_{ir} < \widehat{q}_{io}$. An entrenchment effect occurs due to the fact that such biases make it more costly to induce the incumbent manager to accept replacement. Note that the cutoff for the overconfident manager $\widehat{q}_{io}$ depends on the incentive pay w_{io}, while $\widehat{q}_{ir}$ the cutoff for the rational one does not.

To fully characterize the contract offered to the CEO, we have to determine w_{io} and S_{io} (which in turn determine $\widehat{q}_{io}$). We have argued that the incumbent manager will consent to replacement only if the severance pay satisfies (3). But should contractual s be so high as to satisfy such condition? Not necessarily, because a high contractual s may discourage the manager from investing while severance pay can be renegotiated if both parties find it profitable to do so. We will show below (in Proposition 2) that it is optimal to offer the manager a low contractual s and to possibly provide the sum necessary to induce him/her to quit at the renegotiation stage. We already know (from (3) above) that such renegotiated severance pay is $s'_{io} = (\theta + \Delta_i)w_{io} + B$.

Note that condition (3) takes it for granted that the incumbent has made investment I_i, $i = M, H$,. In order to determine the optimal contract, however, we also need to consider the optimal level of s' for the hypothetical case where the incumbent has not made any investment so that the probability of success is p_L and (3) takes the form $S_0 \geq (p_L + \theta)w_{io} + B$.[4] This case never occurs in equilibrium but, precisely to provide the appropriate incentives to discourage such behaviour, we need to take into account what payment the incumbent could obtain by not investing and opposing replacement. Thus, the next proposition determines the optimal payments that induce the incumbent to leave both when she has made investment I_i and when she has not invested despite the incentive pay specified in the contract at $t = 0$, was w_{io} in order to induce I_i

Proposition 1: *When contractual s is too low for the biased incumbent to accept replacement at $t = 3$ and q is sufficiently high to induce the board to replace the manager, then*

i) If the incumbent has made investment I_i, $i = M, H$,the optimal renegotiated severance payment to induce her to leave is $s'_{io} = (p_i + \theta + \Delta_i)w_{io} + B$.

ii) If the incumbent has not made any investment ($I = I_L = 0$), the optimal renegotiated payment to overcome her opposition to replacement is $\underline{s}'_{io} = (p_L + \theta)w_{io} + B$.

Proof: The proof immediately follows from the discussion above.□

Corollary 1. *When contractual s is too low for the biased incumbent to accept replacement at $t = 3$, the cutoff value of q is equal to its minimum level $\widehat{q}_{io}$.*

[4] Note that in this case there is no overconfidence, $\Delta_M = 0$.

Given that the incumbent can oppose replacement, she will accept to leave only if the payment is at least as large as what she can gain by staying with the firm. Note that, contrary to contractual severance pay, the renegotiated payments are conditional on the investment because, at the renegotiation stage, the board knows which level of investment (I_M, I_H, or I_L) has been implemented by the manager. The following Corollary establishes the minimum renegotiated payments for a rational manager, anticipating the result of Corollary 5 that a rational manager always chooses I_M.

Corollary 2. *Suppose q is high enough to call for replacement, and the manager is rational ($\theta = \Delta_M = 0$). Then, the optimal renegotiated payment to induce her to leave when she has chosen investment I_M, is $s'_r = p_M w_r + B$ which implies a cutoff value equal to its minimum level $\widehat{q}_r$. In case the manager had not invested ($I = I_L = 0$), the optimal payment to induce her to leave would be $\underline{s}'_r = p_L w_r + B$.*

Even in the case of a rational manager, the renegotiated severance pay corresponds to the amount that the manager expects to obtain by staying with the firm. In summary, any payment lower than s'_{io} and $\underline{s}'_{io}$ (or s'_{ir} and $\underline{s}'_{ir}$) would be rejected by the manager. Such payments are correctly anticipated by both the manager and the board at the time when the contract is struck and thus contribute to the expected returns calculated by both parties.

4 Manager's investment and optimal compensation

Having established the optimal payment that can be renegotiated when the board wants to replace the manager at $t = 3$, we can now determine the levels of contractual severance pay and incentive pay necessary to induce investment I_i $i = M, H$ at $t = 1$. Then we will be able to determine the optimal level of investment.

Let $E_q[W_{io} + B|I_j]$ denote the expectation with respect to q of the compensation of the overconfident manager when she invests I_j under a contract that prescribes investment I_i, $i, j = M, H$, namely a contract that offers incentive and severance pay designed to incentivize I_i. We thus consider both the case where $j = i$ and the manager complies with the contract and the case where $j \neq i$ and the manager chooses a positive investment level different from the one required by the board. The expected compensation if the manager does not make any investment ($I = I_L = 0$) under a contract prescribing I_i is denoted by $E_q[W_{io}|0]$.

Recalling that we have normalized to zero the reservation level of utility, we can write the participation constraints as

$$E_q[W_{io} + B|I_i] - I_i \geq 0 \; i = H, M. \tag{PC}$$

Moreover, in order to induce the manager to choose investment I_i, $i = M, H$, the following two incentive constraints must be satisfied:

$$E_q[W_{io} + B|I_i] - I_i \geq E_q[W_{io}|0] \tag{ICC 1}$$

and

$$E_q[W_{io} + B|I_i] - I_i \geq E_q[W_{io} + B|I_j] - I_j, \qquad i, j = M, H, i \neq j. \tag{ICC2}$$

(ICC 1) guarantees that the manager prefers I_i to not investing, and (ICC 2) that she prefers I_i to I_j when the contract prescribes investment I_i. Note that $E_q[W_{io}|0] \geq 0$ by the limited liability assumption. Consequently, if incentive compatibility constraint (ICC 1) is satisfied, the participation constraint is satisfied as well.[5] We then focus our attention on the ICCs.

[5] A positive level of the reservation utility would not affect this result as long as such level is smaller than the expected compensation in case of no investment, $E_q[W_{io}|0]$.

4.1 *Contractual severance pay*

In order to be more specific about the ICCs, we need to know the value of the payment in case of replacement. In other words, we need to know whether contractual s_{io} is lower than s'_{io} and renegotiation occurs, or whether $s_{io} \geq s'_{io}$ so that s_{io} is paid if the manager is dismissed. This is established in the following proposition.

Proposition 2: *The optimal value of the contractual severance pay for an optimistic and overconfident manager ($\theta > 0$, $\Delta_M > 0$) is $s_{io} = \underline{s}'_{io} = (p_L + \theta)\, w_{io} + B < (p_i + \theta + \Delta_i)w_{io} + B = s'_{io}$ so that the cutoff value for replacement is set at its minimum level $\widehat{q}_{io}$.*

Proof. See Appendix 2.

Corollary 3: *If the manager is rational ($\theta = \Delta_M = 0$), the optimal contractual severance pay is $s_r = \underline{s}'_r = p_L w_r + B < p_M w_r + B = s'_r$ and the cutoff value for replacement is set at its minimum level $\widehat{q}_R$.*

A value of contractual severance pay (s_{io}/s_r) higher than the minimum level of severance pay that would be accepted by a manager who had not invested ($\underline{s}'_{io}/\underline{s}'_r$) is not profitable for the firm because it would make the ICCs more binding (without having any positive effect on the participation constraint because the latter is not binding). This would raise total expected compensation. On the other hand, there is no point in setting $s_{io} < \underline{s}'_{io}$ (or $s_r < \underline{s}'_r$ in case of a rational manager) because this would not relax the ICCs. Consequently the optimal value for the contractual s_{io} and s_r coincide with $\underline{s}'_{io}$ and $\underline{s}'_r$. This, together with Proposition 1 and Corollary 1, implies that the actual cutoff values of the productivity of the new manager that induce replacement are at their minimum levels $\widehat{q}_{io}$ and $\widehat{q}_R$ respectively. Moreover we have

Corollary 4. *When the manager is replaced, renegotiation occurs at $t = 3$ and the severance payment is set at s'_{io} if the manager is biased or at s'_r if the manager is rational.*

4.2 *Incentive pay and investment level*

We now want to determine the incentive pay and the optimal level of investment. We know that I_M is the efficient level, however, this does not ensure that it can always be made incentive compatible. Consider first the incentive-compatibility constraints for investment I_M. Taking into account that severance pay will be renegotiated and set equal to $s'_{Mo} = (p_L + \theta + \Delta_M)w_{Mo} + B$, by Corollary 3, we have

$$\begin{aligned} E_q[W_{Mo} + B|I_M] &= \int_0^{\widehat{q}_{Mo}} [(p_M + \theta + \Delta_M)w_{Mo} + B]f(q)dq + \int_{\widehat{q}_{Mo}}^1 s'_{Mo}f(q)dq \\ &= (p_M + \theta + \Delta_M)w_{Mo} + B \end{aligned}$$

Moreover, we know from Proposition 1 that in case of no investment the manager will receive $\underline{s}'_{Mo}$, independently of whether she is confirmed or replaced, so that it is $E_q[W_{Mo}|0] = (p_L + \theta)\, w_{Mo} + B$. Then, ICC 1 can be written as

$$(p_M + \theta + \Delta_M)w_{Mo} + B - (p_L + \theta)w_{Mo} - B \geq I_M,$$

and the level of the incentive pay that satisfies such constraint is:

$$w_{Mo} \geq \frac{I_M}{(p_M + \Delta_M - p_L)}. \tag{5}$$

Consider then ICC 2. On the RHS we have the utility that the CEO would obtain by choosing I_H when the board wants him to choose I_M. In this case, the level of renegotiated severance pay would be equal to $(p_H + \theta + \Delta_H)w_{Mo} + B$ because this is what the incumbent believes she could obtain by opposing replacement. Then

$$E_q[W_{Mo} + B|I_H] = \int_0^{\widehat{q}_{Mo}} [(p_H + \theta + \Delta_H)w_{Mo} + B]f(q)dq + \int_{\widehat{q}_{Mo}}^1 [(p_H + \theta + \Delta_H)w_{Mo} + B]f(q)dq$$

so that ICC2 can be written as

$$(p_H + \Delta_H - p_M - \Delta_M)w_{Mo} \leq I_H - I_M.$$

Therefore, to guarantee that both ICC1 and ICC2 are satisfied it must be the case that

$$\frac{I_M}{p_M + \Delta_M - p_L} \leq w_{Mo} \leq \frac{I_H - I_M}{p_H + \Delta_H - p_M - \Delta_M}.$$

Recalling that $\Delta_H = \Delta_M(1 + z)$ such inequality can be satisfied if and only if

$$\frac{I_M}{I_H} \leq \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1 + z) - p_L}. \tag{6}$$

From Assumption 1 we know that $z > \frac{p_H - p_M}{p_M - p_L}$. Then, the RHS of (6) is decreasing in Δ_M implying that it is more difficult to satisfy this constraint as overconfidence rises.[6] In other words, for ICC 2 to be satisfied, the manager's beliefs of success when I_H (instead of I_M) is chosen, must not be too large. This implies that, for a high enough level of overconfidence, ICC2 does not hold and the manager, if offered w_{Mo}, will choose I_H. This happens because a high level of overconfidence increases the subjective probability of success to such an extent that, by choosing I_H, the manager expects an increase in her compensation that more than compensates the additional cost of the investment. Consider the following definition.

Definition 1: *The manager is moderately overconfident when* $\Delta_M \leq \Delta_M^* = \frac{I_M(p_H - p_L) - I_H(p_M - p_L)}{I_H - I_M(1+z)}$ *so that (6) is satisfied. Conversely, the manager is extremely overconfident when* $\Delta_M > \Delta_M^*$ *so that (6) does not hold.*

When the manager is moderately overconfident, I_M is incentive compatible and a board willing to induce such level of investment will offer the lowest possible level of w_{Mo} satisfying 5, that is

$$w_{Mo} = \frac{I_M}{(p_M + \Delta_M - p_L)}. \tag{7}$$

Conversely, when the manager is extremely overconfident I_M cannot be implemented. Let us then consider the incentive compatibility constraints for the high level of investment, I_H. ICC1 and ICC2 respectively imply

$$w_{Ho} \geq \frac{I_H}{(p_H + \Delta_H - p_L)} \tag{8}$$

[6] Note that $\frac{\partial}{\partial \Delta_M}\left(\frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1+z) - p_L}\right) = \frac{(p_H - p_L) - (1+z)(p_M - p_L)}{(p_H + \Delta_M(1+z) - p_L)^2} < 0.$

and

$$w_{Ho} \geq \frac{I_H - I_M}{(p_H + \Delta_H - p_M - \Delta_M)} = \frac{I_H - I_M}{p_H - p_M + z\Delta_M}. \tag{9}$$

The incentive pay offered by the board depends on which constraint is binding. It is immediate to verify that extreme overconfidence corresponds to the case where (8) is the binding constraint. In fact this happens iff $\frac{I_H}{(p_H+\Delta_H-p_L)} > \frac{I_H-I_M}{(p_H+\Delta_H-p_M-\Delta_M)}$ or

$$\frac{I_M}{I_H} > \frac{p_M + \Delta_M - p_L}{p_H + \Delta_M(1+z) - p_L}, \tag{10}$$

corresponding to $\Delta_M > \Delta_M^*$. In such a case, the only feasible level of investment is I_H and the optimal incentive pay is given by the lowest value of w satisfying (8)[7]

$$w_{Ho} = \frac{I_H}{p_H + \Delta_H - p_L}.$$

Investment I_H can be incentive compatible even under moderate overconfidence, though it can be proved that it is generally unprofitable. The following proposition allows us to restrict our attention to two mutually exclusive cases: extreme overconfidence with investment level I_H, and moderate overconfidence with investment level I_M.

Proposition 3: *When the manager is moderately overconfident, the board generally offers* $w_{Mo} = \frac{I_M}{(p_M+\Delta_M-p_L)}$ *and the manager chooses* I_M*. When the manager is extremely overconfident, the board offers* $w_{Ho} = \frac{I_H}{(p_H+\Delta_H-p_L)}$ *and the manager chooses* I_H*.*

Proof: See Appendix 3.

Corollary 5. *When the manager is rational* ($\theta = \Delta_M = 0$) *only* I_M *is incentive compatible. The optimal bonus is*

$$w_r = \frac{I_M}{(p_M - p_L)}.$$

Proof. In order to check that I_H is not incentive compatible consider that in this case ICC 2 implies

$$w_{Ho} \geq \frac{I_H - I_M}{(p_H - p_M)},$$

but by Assumption 2 we know that $\frac{I_H-I_M}{(p_H-p_M)} > R$ so that the board will never offer such incentive pay. The rest of the corollary follows from the discussion above, setting $\theta = \Delta_M = 0$.□

When the manager is rational and holds correct beliefs about the probability of success, it is not possible to implement I_H because the manager is aware that the increase in the cost is not compensated by the increase in the expected compensation. In fact, the rise in the cost is higher than the gain in expected return (see Assumption 2). However, this may not be enough to prevent an overconfident manager from choosing the inefficient investment because of the biased assessment of the probability of success. Thus, our model accounts for the possibility, documented by a large literature (see, among others, Malmendier and Tate 2005 and 2015), that an optimistic and overconfident manager may choose an investment level higher than the optimal one.

Let us compare the outcome of a rational manager to that of a biased manager choosing I_M. By comparing w_{Mo} to w_r it is immediately evident that overconfidence decreases the incentive pay necessary to induce the manager to choose investment I_M, making it easier to satisfy the ICCs. This is the incentive

[7] Note that under 10, it is $w_{Ho} = \frac{I_H}{p_H+\Delta_H-p_L} < \frac{I_M}{p_M+\Delta_M-p_L} = w_{Mo}$.

effect of overconfidence which also persists when overconfidence increases to the extreme values that result in the choice of investment I_H. We can then prove the following

Corollary 6. *w_{io} is continuously decreasing in Δ_M, for $i = M, H$.*

Proof. The corollary immediately follows from $w_{Mo} = \frac{I_M}{(p_M + \Delta_M - p_L)}$ and $w_{Ho} = \frac{I_H}{(p_H + \Delta_M(1+z) - p_L)}$, considering that at Δ_M^* it is $\frac{I_M}{(p_M + \Delta_M - p_L)} = \frac{I_H}{(p_H + \Delta_M(1+z) - p_L)}$.□

The result that incentive pay is smaller when the manager is overconfident is in line with previous theoretical literature (De La Rosa, 2011) and with empirical evidence (Otto 2014, Humphrey-Jenner et al. 2016). It simply derives from the fact that overconfidence induces the manager to overestimate the effect of her investment on the probability of success so that a lower bonus is needed to incentivize the same level of investment. Contrary to overconfidence, in our framework, optimism (θ) has no impact on the bonus because it induces no distortion in the marginal probability of success as this is uniformly shifted upwards. Note that optimism has no impact on the marginal probability of success in the previous principal/agent literature either (De La Rosa, 2011). There, however, optimism tends to make the incentive pay steeper because it relaxes the incentive/insurance trade-off due to the manager being risk-averse. Whether the incentive pay will be lower or higher than in the case of a rational manager will then depend on the degree of the overall bias ("slight" or "significant overconfidence overall").[8]

4.3 *The cutoff value* $\widehat{q}_{io}$

Once we know the optimal value of the bonus, we can analyze the impact of optimism and overconfidence on the cutoff value of the new manager productivity that triggers replacement. The following proposition shows that the entrenchment effect identified above, namely the fact that $\widehat{q}_r < \widehat{q}_{io}$ implying that a biased manager will be replaced for higher values of q than a rational one, is increasing in managerial biases.

Proposition 4. *The cutoff value, $\widehat{q}_{io}$, is increasing both in optimism and in overconfidence.*

Proof. see Appendix 4.

Optimism and overconfidence introduce a distortion in the replacement decision by increasing the cutoff value above which the board wants to replace the manager. Given that $\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i) w_{io}}{R}$ this is immediate as far as optimism is concerned. However, it also occurs in the case of overconfidence even if the incentive bonus is decreasing in Δ_M (see Corollary 6). In fact, the decrease in w_{io} is more than compensated by the increase in the managerial belief of success (reflected in the multiplicative term $(\theta + \Delta_i)$). Finally, the cutoff is increasing in the biases both for a given value of investment and when the increase in overconfidence induces the shift from I_M to I_H.

4.4 Severance pay

We now discuss the properties of the contractual and the renegotiated severance pays determined above. Recall that, in case of replacement, the payment will always be renegotiated. First of all, the relationship between renegotiated severance pay and the two managerial biases, is established both for a given level of investment I_i and at Δ_M^* where the shift from I_M to I_H occurs.

[8] Note our different use of the terms moderate and extreme overconfidence that only refer to the bias in the belief of the manager's investment productivity (overconfidence in a strict sense) with respect to De la Rosa who considers high or low overconfidence overall referring to the sum of the two biases.

Proposition 5. *For a given investment I_i, the severance payment renegotiated in case of replacement, $s'_{io} = (p_i + \theta + \Delta_i)w_i + B$, is increasing in optimism and decreasing in overconfidence. At Δ^*_M where the shift from I_M to I_H occurs, it is increasing in both optimism and overconfidence.*

Proof. See Appendix 5.

For a given level of investment I_i, an increase in optimism (θ) always raises the renegotiated severance pay $s'_{io} = (p_i + \theta + \Delta_i)w_{io} + B$ because w_{io} is not affected by optimism. On the contrary, an increase in the level of overconfidence reduces s'_{io} because the reduction in the bonus w_{io} (see Corollary 6) counterbalances the increase in the managerial belief $(p_i + \theta + \Delta_i)$. Only at Δ^*_M, where the shift from I_M to I_H induces a spike in the belief of success, there is an increase in s'_{io}. For $\Delta_M > \Delta^*_M$ however s'_{Ho} is again decreasing in Δ_M.

Let us define as discretionary pay the difference between renegotiated and contractual severance pay, $s'_{io} - s_{io}$. Using Propositions 1 and 3 and the expressions for w_{io}, $i = M, H$, as well as Corollary 3 and 5 for a rational manager, it is immediate to verify that the discretionary amount paid in addition to the contractual one is given by the cost of the investment.

Result 1. *The discretionary severance pay is equal to the investment: $s'_{io} - s_{io} = I_i$, $i = M, H$, for a biased manager, and $s'_R - s_R = I_M$ for a rational manager.*

The manager anticipates that, in case of replacement, she will be able to recover the investment made, by renegotiating the contractual severance pay. This clearly provides the incentive to invest, despite the risk of being replaced. In other words, renegotiation allows the board to provide the necessary ex-ante incentive by reimbursing the manager for the investment only if this has been undertaken. Note that, both in case of a rational and of a moderately overconfident manager, the discretionary pay is equal to I_M while the high level I_H obtains in case of extreme overconfidence.

A natural question is whether the severance pay obtained by a biased manager is larger or smaller than the one received by a rational one. Proposition 5 shows that there are opposing effects at work when the manager is both optimistic and overconfident. Hence, the overall effect on the renegotiated severance pay (contractual plus discretionary pay) depends on the mix of the two biases. A rational manager receives $s'_r = p_M \frac{I_M}{(p_M - p_L)} + B$ and a biased one $s'_{io} = (p_i + \theta + \Delta_i)\frac{I_i}{p_i + \Delta_i - p_L} + B$ where s'_{io} is increasing in optimism and decreasing in overconfidence. Then, by comparing the two payments we obtain the following result.

Result 2. *The renegotiated severance pay of a biased manager is higher than that of a rational one only for a sufficiently high level of the ratio of optimism to overconfidence:*
i) when the manager is moderately overconfident a necessary and sufficient condition is $\frac{\theta}{\Delta_M} \geq \frac{p_L}{p_M - p_L}$;
ii) when the manager is extremely overconfidence, a necessary condition is $\frac{\theta}{\Delta_M} + \frac{\Delta_M(1+z)+(p_H - p_M)}{\Delta_M} > \frac{p_M}{p_M - p_L}$.

This result emphasizes that optimism is necessary to have a severance pay higher for a biased manager than for a rational one, when the manager is moderately overconfident. When the manager is extremely overconfident, the severance payment may be higher even with zero optimism if p_H and z are sufficiently large. Indeed, in this case, the manager believes she would get the bonus with a very high probability, and therefore requires a high payment to accept replacement.

5 The effect of optimism and overconfidence on expected profits

So far, we have discussed the characteristics of the optimal payments in each possible state (manager retention or dismissal). We now analyze the overall impact of managerial biases on expected profits V_{io}, considering first the effect of an increase in optimism θ, for a given level of overconfidence Δ_M, and then the effect of an increase in overconfidence Δ_M, taking the level of optimism θ as given. As the manager will receive s'_{io} when replaced, we can write

$$V_{io} = \int_0^{\widehat{q}_{io}} \left[p_i(R - w_{io})\right] f(q)dq + \int_{\widehat{q}_{io}}^{1} (qR - s'_{io}) f(q)dq \tag{11}$$

The first term on the RHS is the expected profit of the firm when the incumbent is confirmed, while the second term measures the expected profit when the manager is replaced. Consider the effect of an increase in optimism, recalling that optimism does not affect the investment choice so that we can take the level of investment as given.

Proposition 6: *For a given value of overconfidence $\Delta_M \geq 0$, the expected profit of the firm is decreasing in optimism θ.*

Proof: The proposition immediately follows from:

$$\frac{\partial V_{io}}{\partial \theta} = -\int_{\widehat{q}_{io}}^{1} w_{io} f(q)dq - \frac{\partial \widehat{q}_{io}}{\partial \theta} \underbrace{\left[(\widehat{q}_{io} - p_i)R - ((\theta + \Delta_i)w_{io} + B)\right]}_{=0} f(\widehat{q}_{io}) < 0, \quad i = M, H.$$

Corollary 7: *If the firm hires an optimistic but not overconfident manager ($\theta > 0$, $\Delta_M = 0$) the expected profit is lower than under a rational manager.*

Proposition 6 proves that optimism has always a negative effect on profits. Indeed, optimism has no impact on the incentive bonus w_{io} while it raises the severance payment s'_{io}.[9] In the absence of overconfidence, this drives expected profits below the level obtained by a rational manager. This is at odds with previous results in the literature where optimism generally raises expected profits by relaxing the incentive/insurance trade-off due to the agent being risk-averse and by making high-powered incentives more attractive (de la Rosa, 2011).

Let us now evaluate the effect of an increase in overconfidence for a given level of optimism. We know that, following an increase in overconfidence for a given level of investment I_i, the incentive bonus paid in case of retention decreases (incentive effect), as well as the severance payment paid in case of dismissal (severance effect).[10] Consequently, the expected cost to induce investment I_i, $i = M, H$, is reduced. However, we cannot ensure that expected profits are continuously increasing in Δ_M because of the discontinuity at Δ_M^* where the increase in overconfidence induces the shift from investment I_M to investment I_H and severance pay has a sudden increase (see Proposition 5). This point is clarified in the following proposition.

[9] We also know that optimism contributes to the entrenchment effect making the cutoff value $\widehat{q}_{io}$ rise but this has no impact on profits because the cutoff value is determined by balancing what is gained from replacement and the payment necessary to have the incumbent leave so that the net effect is equal to zero. In fact the terms concerning the entrenchment effect cancel out in the derivative $\frac{\partial V_{io}}{\partial \theta}$ (maximum theorem).

[10] Again the rise in $\widehat{q}_{io}$ has no effect because the payment necessary to have the incumbent leave cancels out with the sum gained from replacement (see previous footnote).

Proposition 7: *For any given level of optimism* $\theta \geq 0$*, expected profit increases in* Δ_M*, as long as* $0 < \Delta_M < \Delta_M^*$ *and* I_M *is chosen. At* Δ_M^**, the manager is indifferent between* I_M *and* I_H *but* I_M *generally yields higher expected profit and is thus chosen. A further increase to* $\Delta_M > \Delta_M^*$*, implies a shift to* I_H*, and leads to a discontinuity: the expected profit has an initial drop and then resumes an increasing trend.*

Proof: See Appendix.

Corollary 8: *Expected profit is higher with a moderately overconfident but not optimistic manager (*$\theta = 0$*,* $0 < \Delta_M \leq \Delta_M^*$*) than with a rational manager.*

Profit is increasing in overconfidence as long as overconfidence is moderate and does not lead to an inefficiency in the investment level. When overconfidence is so high as to induce the shift from I_M to I_H, there is a discontinuity with a drop in profit because of the sudden increase both in the severance payment and in $\widehat{q}_{io}$. Once the new level of the investment I_H is chosen, profit is again increasing in overconfidence. Note, however, that there is no guarantee that it will reach again the level corresponding to Δ_M^*, because the increase in Δ_M is bounded by the constraint that the belief of success cannot exceed one. Hence, a moderate level of overconfidence is beneficial for the firm but this may not hold true for extreme levels of overconfidence. In particular, a moderately overconfident but not optimistic manager yields higher expected profits than a rational one, but the reverse may hold in the case of an extremely overconfident manager.

This issue is analyzed by the following proposition. To evaluate the drop in the expected profit occurring at Δ_M^*, we need to specify the probability that a better manager shows up. Since we have no reason to consider any particular value of q more/less likely to occur than the other values, we assume that q is uniformly distributed over the interval $[0, 1]$.

Proposition 8. *When* $\theta = 0$*, the drop in profits occurring at* Δ_M^* *can be high enough to result in* $V_{Ho} < V_R$*.*

Proof. See Appendix.

Proposition 8 highlights the negative impact on profit resulting from the shift in the investment from I_M to I_H and provides an indication as to the size of the drop that takes place at Δ_M^*. In particular, it shows that expected profit with an overconfident manager can be reduced below the level attained by a rational manager even when we eliminate optimism. This leads to the conclusion that, in the absence of optimism, the firm benefits from hiring a moderately overconfident manager while it may be damaged by an extremely overconfident one.

When the manager is both optimistic and overconfident, the question arises whether the negative effect of optimism can counterbalance the positive effect of moderate overconfidence. Is an optimistic and moderately overconfident manager always better than a rational one? The following proposition deals with such question maintaining the assumption that q is uniformly distributed over the interval $[0, 1]$

Proposition 9. *There always exist a value* $\widetilde{\theta}$*:* $1 - p_H > \widetilde{\theta} \geq 0$ *such that for* $\theta > \widetilde{\theta}$*,* $V_{Mo} < V_R$*.*

Proof. See Appendix .

Proposition 9 confirms the detrimental effect of optimism and shows that, for sufficiently high values of θ, the negative effect of optimism prevails on the positive effect of moderate overconfidence, similarly to what we found in Result 2. As a consequence, a biased manager with a high level of optimism generates an expected cash flow smaller than the one obtained by a rational manager, even when the manager is only moderately overconfident. For levels of θ below $\widetilde{\theta}$, the outcome is indeterminate in the sense that it depends on the level of overconfidence. If overconfidence is high enough, it may balance the negative impact of optimism. However, when low levels of overconfidence are coupled with optimism, a rational manager leads to higher expected profits.

Summarizing, if the manager is both optimistic and overconfident, the overall effect of managerial bias is likely to be detrimental for the firm, once severance costs are taken into account.

6 Conclusion

The paper examines the effects of managerial optimism and overconfidence on severance pay and firm profit in a setting where the firm aims at motivating the CEO to undertake a firm-specific and unverifiable investment. The manager has to choose one among three possible alternatives: zero investment, a positive efficient investment level and a higher inefficient investment level. Severance pay helps motivating the manager to invest despite the anticipated possibility of being replaced. It turns out that the cheapest way to motivate the manager is to offer a low contractual severance pay and to renegotiate the payment ex post in case replacement becomes profitable. Renegotiating ex post allows to determine the payment once the board has observed which investment has been undertaken. This lowers the cost of incentivizing the investment, no matter the fact that the incumbent manager can use her bargaining power. While optimism does not affect investment choice, the degree of overconfidence is crucial for such decision: under moderate overconfidence the efficient level is implemented but under extreme overconfidence the inefficient level is chosen.

Optimism and overconfidence have different effects also on the components of the compensation package. Optimism does not affect incentive pay but raises severance pay (contractual and renegotiated) and this leads to lower expected profit than would be obtained by a rational (i.e. unbiased) manager. Overconfidence, on the contrary, decreases incentive pay with a positive effect on profit while its effect on severance pay depends on the degree of the bias through the choice of the investment. A moderate level of overconfidence reduces severance pay with no distortion in the investment level, but a sufficiently high level of overconfidence induces the manager to choose the inefficient investment, which in turn results in a very high renegotiated severance pay, lowering expected profit. Hence, there is a discontinuity with a drop in expected profits at the level of overconfidence that induces the switch from the efficient to the inefficient investment. In summary, the firm always benefits from moderate overconfidence while extreme overconfidence may be detrimental.

The model also shows that optimism increases *contractual* severance pay, while overconfidence may increase the *discretionary* amount bargained at the replacement stage. Hence, we suggest that the high payments observed in several turnover events may be explained by managerial overconfidence coupled with some bargaining power originated by the possibility to oppose replacement. Overall, our model indicates that it is important to consider severance agreements when studying the effect of managerial optimism and overconfidence because of their impact on contractual and discretionary separation pay.

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8 Appendix

Proof of Proposition 2

For any given level of investment, the board will maximize profits by keeping both incentive and severance pay as low as possible, considering the incentive compatibility constraints (recall that the participation constraint is never binding). We then want to prove that raising contractual s above $\underline{s}'_{io}$ makes the ICCs more binding, thus raising both w and the severance pay (either contractual or renegotiated) that is paid in case of replacement. On the other hand, setting s below $\underline{s}'_{io}$ does not help relaxing the ICCs and results in the same renegotiated severance pay.

Define $\widehat{\widehat{q}}_{io} \equiv p_i + \frac{S_i - w_{io}}{R} \geq \widehat{q}_{io} \equiv p_i + \frac{(\theta+\Delta_i)w_{io}}{R}$ and $\widehat{\widehat{q}}_{jio} \equiv p_j + \frac{S_i - w_{io}}{R}$, $i = M, H$ $J = L, M, H$ where $\widehat{\widehat{q}}_{jio}$ is the cutoff level of q if the incumbent chooses I_j under a contract designed to incentivize I_i. Then, $E_q[W_{io} + B|I_i] = \int_0^{\widehat{\widehat{q}}_{io}} [(p_i + \theta + \Delta_i)w_{io} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{io}}^1 sf(q)dq$ and $E_q[W_{io} + B|I = 0] = \int_0^{\widehat{\widehat{q}}_{Lio}} [(p_L + \theta)w_{io} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{Lio}}^1 sf(q)dq$ $i = H, M$ because when no investment is made under a contract prescribing I_i and w_{io} the manager is replaced when q is greater than $\widehat{\widehat{q}}_{Lio} \equiv p_L + \frac{S_i - w_{io}}{R}$. Consequently, ICC1 can be written as

$$\int_0^{\widehat{\widehat{q}}_{io}} [(p_i + \theta + \Delta_i)w_{io} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{io}}^1 sf(q)dq - I_i \geq \int_0^{\widehat{\widehat{q}}_{Lio}} [(p_L + \theta)w_{io} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{Lio}}^1 sf(q)dq....i = H, M$$

or

$$\int_{\widehat{\widehat{q}}_{Lio}}^{\widehat{\widehat{q}}_{io}} [(p_i + \theta + \Delta_i)w_{io} + B]f(q)dq + \int_0^{\widehat{\widehat{q}}_{Lio}} [(p_i - p_L + \Delta_i)w_{io}]f(q)dq - I_i \geq \int_{\widehat{\widehat{q}}_{Lio}}^{\widehat{\widehat{q}}_{io}} sf(q)dq$$

Clearly the board wants to keep both w_{io} and s as low as possible in order to maximize its profit. Note in fact that, if $s \leq s'_{io}$, the cutoff value is kept at its minimum level $\widehat{q}_{io}$, from Proposition 1, thus also minimizing the distortion in the replacement decision. We can easily verify that the minimization of both w_{io} and s can be reached, by setting s as low as possible, because the RHS of the above inequality is increasing in s

$$\frac{d \int_{\widehat{\widehat{q}}_{Lio}}^{\widehat{\widehat{q}}_{io}} sf(q)dq}{ds} = \frac{s}{R}\left[f(\widehat{\widehat{q}}_{io}) - f(\widehat{\widehat{q}}_{Lio})\right] + \int_{\widehat{\widehat{q}}_{Lio}}^{\widehat{\widehat{q}}_{io}} f(q)dq > 0$$

as $\frac{s}{R} < 1$ so that in the first term we subtract a number which is lower than the first term of the sum contained in the integral.

There is however no point in setting $s < \underline{s}'_{io}$ because in that case, severance pay would be renegotiated even in the case of no investment, making this case exactly equal to the one in which $s = \underline{s}'_{io}$. Then it should be $s = \underline{s}'_{io} = (p_L + \theta)w_i + B$ and $\widehat{\widehat{q}}_{io} = \widehat{q}_{io}$ so that ICC1 becomes

$$\int_0^{\widehat{q}_{io}} [(p_i + \theta + \Delta_i)w_{io} + B]f(q)dq + \int_{\widehat{q}_{io}}^1 s'_{io} f(q)dq - I_i =$$

$$\int_0^1 [(p_i + \theta + \Delta_i)w_{io} + B]f(q)dq - I_i \geq$$

$$\int_0^1 [(p_L + \theta)w_{io} + B]f(q)dq = \int_0^1 \underline{s}'_{io} f(q)dq.$$

Consider now ICC2. Note that if s were lower than s'_{io} it would be renegotiated at $t = 3$, and ICC2 would not be affected by contractual severance pay. We can then restrict our attention to values of $s \geq s'_{io}$. In case of investment I_M, ICC2 takes the form

$$\int_0^{\widehat{\widehat{q}}_{Mo}} [(p_M + \theta + \Delta_M)w_{Mo} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{Mo}}^1 sf(q)dq - I_M \equiv$$

$$\int_0^{\widehat{\widehat{q}}_{Mo}} [(p_M + \theta + \Delta_M)w_{Mo} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{Mo}}^1 [(p_M + \theta + \Delta_M)w_{Mo} + B + x_M]f(q)dq - I_M >$$

$$\int_0^{\widehat{\widehat{q}}_{HMo}} [(p_H + \theta + \Delta_H)w_{Mo} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{HMo}}^1 [(p_H + \theta + \Delta_H)w_{Mo} + B + x_H]f(q)dq - I_H \equiv$$

$$\int_0^{\widehat{\widehat{q}}_{HMo}} [(p_H + \theta + \Delta_H)w_{Mo} + B]f(q)dq + \int_{\widehat{\widehat{q}}_{HMo}}^1 sf(q)dq - I_H$$

where x_H, $x_M \geq 0$.

This can be written as

$$[(p_M + \theta + \Delta_M)w_{Mo} + x'_M] - I_M \geq [(p_H + \theta + \Delta_H)w_{Mo} + x'_H] - I_H$$

or

$$w_{Mo} \leq \frac{I_H - I_M + x'_H - x'_M}{(p_H + \theta + \Delta_H) - (p_M + \theta + \Delta_M)}$$

where $x'_H - x'_M < 0$. Thus IIC2 is not relaxed by having x_H, $x_M > 0$. In fact, as we want to keep w_{Mo} as low as possible, in this case the binding constraint is ICC1, so that it is preferable to set $s = \underline{s}'_{io}$, as argued above. Notice that the argument still holds in case it is $(p_M + \theta + \Delta_M)w_{Mo} + B < s < (p_H + \theta + \Delta_H)w_{Mo} + B$.

Consider now investment I_H, ICC2 has the form

$$\int_0^{\widehat{\widehat{q}}_{Ho}} [(p_H+\theta+\Delta_H)w_{Ho}+B]f(q)dq + \int_{\widehat{\widehat{q}}_{Ho}}^1 sf(q)dq - I_H \equiv$$

$$\int_0^{\widehat{\widehat{q}}_{Ho}} [(p_H+\theta+\Delta_H)w_{Ho}+B]f(q)dq + \int_{\widehat{\widehat{q}}_{Ho}}^1 [(p_H+\theta+\Delta_H)w_{Ho}+B+x_H]f(q)dq - I_H >$$

$$\int_0^{\widehat{\widehat{q}}_{HMo}} [(p_M+\theta+\Delta_M)w_{Ho}+B]f(q)dq + \int_{\widehat{\widehat{q}}_{HMo}}^1 [(p_H+\theta+\Delta_H)w_{Ho}+B+x_M]f(q)dq - I_M \equiv$$

$$\int_0^{\widehat{\widehat{q}}_{HMo}} [(p_H+\theta+\Delta_H)w_{Ho}+B]f(q)dq + \int_{\widehat{\widehat{q}}_{HMo}}^1 sf(q)dq - I_M$$

where x_H, $x_M \geq 0$.

This can be written as

$$[(p_H+\theta+\Delta_H)w_{Mo}+x'_H] - I_H \geq [(p_M+\theta+\Delta_M)w_{Mo}+x'_M] - I_M$$

which is made more binding by setting $x_M, x_H > 0$ as $x_M > x_H$. Again, as we want to keep w_{Ho} as low as possible, the binding constraint is ICC1, so that it is preferable to set $s = \underline{s}'_{io}$.□

Proof of Proposition 3

The part of the proposition concerning extreme overconfidence is proved in the text. In order to prove the part on moderate overconfidence we must that in this case both I_M and I_H can be made incentive compatible. Incentive compatibility of I_M has been discussed in the text where we have shown that it implies offering $w_{Mo} = \frac{I_M}{(p_M+\Delta_M-p_L)}$. Consider now the incentive compatibility constraints for I_H and note that the condition for moderate overconfidence 6 can be written as $\frac{I_H}{(p_H+\Delta_H-p_L)} \leq \frac{I_H-I_M}{(p_H+\Delta_H-p_M-\Delta_M)}$ implying that 9 is binding, and the lowest value of incentive pay in this case is

$$w_{Ho} = \frac{I_H - I_M}{(p_H+\Delta_H-p_M-\Delta_M)}.$$

To determine the contract offered by the board, observe that, given 6, $w_{Ho} = \frac{I_H-I_M}{(p_H+\Delta_H-p_M-\Delta_M)}$ is greater than $w_{Mo} = \frac{I_M}{(p_M+\Delta_M-p_L)}$. Furthermore, the higher bonus raises $\widehat{q}_{Ho}$ above $\widehat{q}_{Mo}$. We must prove that these two effects make I_H generally unprofitable for the firm which will consequently offer the manager w_{Mo} to induce I_M. In other words we must prove that the expected profit of the firm is higher under a contract based on $w_{Mo} = \frac{I_M}{(p_M+\Delta_M-p_L)}$ (to induce the choice of I_M) than under a contract based on $w_{Ho} = \frac{I_H-I_M}{p_H+\Delta_H-(p_M+\Delta_M)}$ (to induce the choice of I_H). Taking into account that renegotiation occurs when the manager is replaced (Corollary 3) and that $s'_{io} = (p_i+\theta+\Delta_i)w_{io}+B$, the expected profit of the firm V_{io} can be written as

$$V_{io} = \int_0^{\widehat{q}_{io}} [p_i(R-w_{io})]\, f(q)dq + \int_{\widehat{q}_{io}}^1 qR - (p_i+\theta+\Delta_i)\, w_{io} - B)f(q)dq$$

Recall that $w_{Ho} = \frac{I_H-I_M}{p_H+\Delta_H-(p_M+\Delta_M)} > \frac{I_M}{(p_M+\Delta_M-p_L)} = w_{Mo}$ when $\frac{I_M}{I_H} < \frac{p_M+\Delta_M-p_L}{p_H+\Delta_H-p_L}$. This implies that $\widehat{q}_{Mo} = p_M + \frac{B}{R} + \frac{\theta+\Delta_M}{R} w_{Mo} < p_H + \frac{B}{R} + \frac{\theta+\Delta_H}{R} w_{Ho} = \widehat{q}_{Ho}$. The difference between the expected profit that can be obtained by offering w_{Ho} and offering w_{Mo} is

$$\begin{aligned} V_{Ho} - V_{Mo} = & \int_0^{\widehat{q}_{Mo}} (p_{H-}p_M)\,(R - w_{Mo}) f(q)dq - \int_0^{\widehat{q}_{Mo}} p_H(w_{Ho} - w_{Mo}) f(q)dq \\ & + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R - w_{Ho}) f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (p_H + \Delta_H - p_M - \Delta_M) w_{Ho} f(q)dq \\ & - \int_{\widehat{q}_{Ho}}^{1} (p_M + \Delta_M) w_{Mo} f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta)\, w_{Mo} - B]\, f(q)dq \end{aligned}$$

This expression can be written as

$$\begin{aligned} & \int_0^{\widehat{q}_{Mo}} (p_{H-}p_M)\,(R - w_{Mo}) f(q)dq + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{p_H\,(R - w_{Mo}) - [qR - (p_M + \Delta_M + \theta)\, w_{Mo} - B]\}\, f(q)dq \\ & - \int_0^{\widehat{q}_{Mo}} p_H(w_{Ho} - w_{Mo}) f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(w_{Ho} - w_{Mo}) f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (p_H + \Delta_H - p_M - \Delta_M) w_{Ho} f(q)dq + \\ & - \int_{\widehat{q}_{Ho}}^{1} (p_M + \Delta_M) w_{Mo} f(q)dq \end{aligned}$$

where $\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta)\, w_{Mo} - B]\, f(q) > \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_H + \Delta_H + \theta)\, w_{Mo} - B]\, f(q)dq >$ $\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R - w_{Mo}) f(q)dq$. Considering that $I_H - I_M \geq (p_{H-}p_M)\, R$, we then have

$$\begin{aligned} V_{Ho} - V_{Mo} \quad \leq \quad & \int_0^{\widehat{q}_{Mo}} [I_H - I_M - (p_{H-}p_M)\, w_{Mo}]\, f(q)dq - \int_0^{\widehat{q}_{Mo}} (p_{H-}p_M)\,(w_{Ho} - w_{Mo}) f(q)dq \\ & - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(w_{Ho} - w_{Mo}) f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[qR - (p_M + \Delta_M + \theta)\, w_{Mo} - B] - [p_H\,(R - w_{Mo})]\}\, f(q)dq \\ & - \int_{\widehat{q}_{Ho}}^{1} (p_H + \Delta_H - p_M - \Delta_M) w_{Ho} f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (p_M + \Delta_M) w_{Mo} f(q)dq \\ = \quad & \int_0^{\widehat{q}_{Mo}} [I_H - I_M - (p_{H-}p_M)\, w_{Ho}]\, f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(w_{Ho} - w_{Mo}) f(q)dq \\ & - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[q - (p_M + \Delta_M + \theta)\, w_{Mo} - B] - [p_H(R - w_{Mo})]\}\, f(q)dq \\ & - \int_{\widehat{q}_{Ho}}^{1} (p_H + \Delta_H - p_M - \Delta_M) w_{Ho} f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (p_M + \Delta_M) w_{Mo} f(q)dq \end{aligned}$$

Substituting $I_H - I_M = [p_H + \Delta_H - (p_M + \Delta_M)]\, w_{Ho}$, the above inequality becomes

$$V_{Ho} - V_{Mo} \leq \int_0^{\widehat{q}_{Mo}} [p_H + \Delta_H - (p_M + \Delta_M) - (p_{H} - p_M)]\, w_{Ho} f(q)dq - \int_{\widehat{q}_{Ho}}^1 (p_H + \Delta_H - p_M - \Delta_M) w_{Ho} f(q)dq$$

$$- \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(w_{Ho} - w_{Mo}) f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[qR - (p_M + \Delta_M + \theta)\, w_{Mo} - B] - [p_H(R - w_{Mo})]\} f(q)dq$$

$$- \int_{\widehat{q}_{Ho}}^1 (p_M + \Delta_M) w_{Mo} f(q)dq$$

$$= \int_0^{\widehat{q}_{Mo}} z\Delta_M w_{Ho} f(q)dq - \int_{\widehat{q}_{Ho}}^1 (p_H - p_M + z\Delta_M) w_{Ho} f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(w_{Ho} - w_{Mo}) f(q)dq$$

$$- \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[qR - (p_M + \Delta_M + \theta)\, w_{Mo} - B] - [p_H(R - w_{Mo})]\} f(q)dq$$

$$- \int_{\widehat{q}_{Ho}}^1 (p_M + \Delta_M) w_{Mo} f(q)dq$$

Considering that $z\Delta_M$ is very small and all the other terms are negative, $V_{Ho} - V_{Mo}$ will generally be negative. Recall in fact that $z\Delta_M \leq \frac{I_H - I_M}{w_{Mo}} - (p_H - p_M)$ for $\Delta_M < \Delta_M^*$,where Δ_M^* is the value of Δ_M where there is the shift from moderate to extreme overconfidence.□

Proof of Proposition 4

To evaluate the effect of the biases, substitute $w_{io} = \frac{I_i}{(p_i + \Delta_i - p_L)}$ in the expression for the cutoff value $\widehat{q}_{io}$, obtaining $\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta + \Delta_i) I_i}{R(p_i + \Delta_i - p_L)}$. Clearly, for a given level of investment, $\widehat{q}_{io}$ is increasing in the optimism component θ and it is is also increasing in the overconfidence parameter Δ_M as

$$\frac{\partial \widehat{q}_{io}}{\partial \Delta_M} = \frac{I_i R\,(p_i + \Delta_i - p_L) - I_i R(\theta + \Delta_i)}{[R\,(p_i + \Delta_i - p_L)]^2} = \frac{I_i (p_i - p_L - \theta)}{R\,(p_i + \Delta_i - p_L)^2} > 0, \;\; i = M, H$$

because $p_i - p_L > \theta$ by Assumption 1.

To evaluate what happens at Δ_M^* where the shift from I_M to I_H occurs, note that at Δ_M^* it is $w_{Mo} = w_{Ho} \equiv w$ implying that $\widehat{q}_{Mo} = p_M + \frac{B}{R} + \frac{\theta + \Delta_M^*}{R} w < p_H + \frac{B}{R} + \frac{\theta + \Delta_M^*(1+z)}{R} w = \widehat{q}_{Ho}$. Then the cutoff value is increasing also in this point (even if there is a discontinuity).□

Proof of Proposition 5

That s'_{io} is increasing in optimism and decreasing in overconfidence for a given level of investment I_i immediately follows from

$$\frac{\partial s'_{io}}{\partial \theta} = (p_i + \Delta_i)\, w_{io} > 0$$

and from

$$\frac{\partial s'_{Mo}}{\partial \Delta_M} = -\left(\frac{I_M}{(p_M + \Delta_M - p_L)}\right)\left(\frac{(p_L + \theta)}{(p_M + \Delta_M - p_L)}\right) < 0.$$

$$\frac{\partial s'_{Ho}}{\partial \Delta_M} = -\left(\frac{(1+z)\, I_H}{(p_H + \Delta_M\,(1+z) - p_L)}\right)\left(\frac{(p_L + \theta)}{(p_H + \Delta_M\,(1+z) - p_L)}\right) < 0.$$

To verify that s'_{io} is increasing in both optimism and overconfidence at Δ^*_M where the shift from I_M to I_H occurs, note that at at Δ^*_M it is $w_{Mo} = w_{Ho} \equiv w$ implying $s'_{Mo} = (p_M + \theta + \Delta^*_M)w < (p_H + \theta + \Delta^*_M(1+z))w = s'_{Ho}$.□

Proof of Proposition 7

In order to prove the proposition we must show that a) when I_i is chosen, profits are increasing in Δ_i and b) when 6 holds as an equality, profits are generally higher if I_M is chosen. Note that, substituting the value of s'_{io}, V_{io} can be written as

$$V_{io} = p_i(R - w_{io}) + \int_{\widehat{q}_{io}}^{1} \left[(q - p_i)R - (\theta + \Delta_i)\, w_{io} - B\right] f(q)dq$$

a) For $i = M, H$ it is

$$\frac{\partial V_{io}}{\partial \Delta_i} = -p_i \frac{\partial w_{io}}{\partial \Delta_i} - \int_{\widehat{q}_{io}}^{1} [w_{io} + (\theta + \Delta_i)\frac{\partial w_{io}}{\partial \Delta_i}] f(q)dq - \frac{\partial \widehat{q}_{io}}{\partial \Delta_M} \underbrace{[(\widehat{q}_{io} - p_i)R - ((\theta + \Delta_i)w_{io} + B)]}_{=0} f(\widehat{q}_{io}).$$

By substituting $\widehat{q}_{io} = p_i + \frac{B}{R} + \frac{(\theta+\Delta_i)w_{io}}{R}$, we can immediately verify that the square bracket in the last term of the RHS is equal to zero. Substituting $\frac{\partial w_{io}}{\partial \Delta_i} = -\frac{I_i}{(p_i + \Delta_\theta - p_L)^2} = -\frac{w_{io}}{(p_i + \Delta_i - p_L)} < 0$, we then obtain:

$$\frac{\partial V_{io}}{\partial \Delta_i} = p_i \frac{w_{io}}{(p_i + \Delta_i - p_L)} - \int_{\widehat{q}_{io}}^{1} [w_{io} - \frac{(\theta + \Delta_i)w_{io}}{(p_i + \Delta_i - p_L)}] f(q)dq$$

$$\frac{w_{io}}{p_i + \Delta_i - p_L} \left[p_i - \int_{\widehat{q}_{io}}^{1} (p_i - \theta - p_L) f(q)dq \right] > 0, \; i = M, H.$$

b) When 6 holds as an equality, the manager is indifferent between I_M and I_H but profits are generally higher in the former case. Note that in this case $w_{Mo} = w_{Ho} \equiv w$ while $\widehat{q}_{Mo} = p_M + \frac{B}{R} + \frac{\theta + \Delta_M}{R} w < p_H + \frac{B}{R} + \frac{\theta + \Delta_H}{R} w = \widehat{q}_{Ho}$. Consider the difference

$$\begin{aligned} V_{Ho} - V_{Mo} \;\; = \;\; & \int_0^{\widehat{q}_{Ho}} p_H(R - w) f(q)dq + \int_{\widehat{q}_{Ho}}^{1} [qR - (p_H + \Delta_H + \theta)w - B] f(q)dq + \\ & - \int_0^{\widehat{q}_{Mo}} p_M(R - w) f(q)dq - \int_{\widehat{q}_{Mo}}^{1} [qR - (p_M + \Delta_M + \theta)w - B] f(q)dq \\ = \;\; & \int_0^{\widehat{q}_{Mo}} (p_H - p_M)(R - w) f(q)dq + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R - w) f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (p_H + \Delta_H - p_M - \Delta_M) w f(q)dq \\ & - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta)\, w - B]\, f(q)dq \end{aligned}$$

Considering that in this case it is

$$w_{Mo} = \frac{I_M}{p_M + \Delta_M - p_L} = \frac{I_H}{p_H + \Delta_H - p_L} = \frac{I_H - I_M}{p_H + \Delta_H - p_M - \Delta_M} = w_{Ho} = w$$

the above expression can be written as

$$\int_0^{\widehat{q}_{Mo}} (p_{H-}p_M)\,(R-w)f(q)dq + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R-w)f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [qR - (p_M + \Delta_M + \theta)\,w - B]\,f(q)$$

$$= \int_0^{\widehat{q}_{Mo}} (p_{H-}p_M)\,(R-w)f(q)dq + \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{p_H\,(R-w) - [q - (p_M + \Delta_M + \theta)\,w - B]\}\,f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq$$

where $\int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [q - (p_M + \Delta_M + \theta)\,w - B]\,f(q) > \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} [q - (p_H + \Delta_H + \theta)\,w - B]\,f(q)dq > \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} p_H(R{-}w)f(q)dq.$

Note that $I_H - I_M = (p_{H-}p_M)\,w + z\Delta_M w$, and considering that $I_H - I_M \geq (p_{H-}p_M)\,R$, we have that

$$V_{Ho} - V_{Mo} \leq \int_0^{\widehat{q}_{Mo}} [I_H - I_M - (p_{H-}p_M)\,w]\,f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq +$$

$$- \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[q - (p_M + \Delta_M + \theta)\,w - B] - [p_H(R-w)]\}\,f(q)dq$$

$$= \int_0^{\widehat{q}_{Mo}} z\Delta_M w f(q)dq - \int_{\widehat{q}_{Ho}}^{1} (I_H - I_M)f(q)dq - \int_{\widehat{q}_{Mo}}^{\widehat{q}_{Ho}} \{[q - (p_M + \Delta_M + \theta)\,w - B] - [p_H(R-w)]\}\,f(q)dq$$

Considering that $z\Delta_M$ is very small and both the second and the third term are negative, $V_{Ho} - V_{Mo}$ will generally be negative. Recall that $z\Delta_M \leq \frac{I_H - I_M}{w_{Mo}} - (p_H - p_M)$ for $\Delta_M < \Delta_M^*$.□

Proof of Proposition 8

In case of a rational manager, the expected profit is

$$V_r = p_M(R - w_R) + \int_{\widehat{q}_R}^{1} [(q - p_M)R - B)]\,f(q)dq.$$

Then, considering 11, $V_{Ho} - V_r$ is equal to

$$V_{Ho} - V_r = p_H(R - w_{Ho}) - p_M(R - w_r) +$$
$$+ \int_{\widehat{q}_{Ho}}^{1} [q - (p_H + \Delta_H + \theta)w_{Ho} - B]f(q)dq - \int_{\widehat{q}_r}^{1} [q - p_M w_r - B]f(q)dq.$$

Under the assumption that q is uniformly distributed over the interval $[\underline{q}, 1]$, such expression becomes

$$V_{Ho} - V_r = \frac{(p_H - p_M)}{2(1-\underline{q})}(\widehat{q}_{Ho} + \widehat{q}_r - 2\underline{q})R - \frac{(2 - \widehat{q}_{Ho} - \widehat{q}_r)}{2(1-\underline{q})}\Delta_H w_{Ho} - p_H w_{Ho} + p_M w_r$$

Given that $(p_H - p_M)\,R \leq I_H - I_M$ by assumption 2, a sufficient condition for $V_{Ho} - V_r < 0$ then is

$$(I_H - I_M)\,\frac{(\widehat{q}_{Ho} + \widehat{q}_r - 2\underline{q})}{2(1-\underline{q})} - \frac{(2 - \widehat{q}_{Ho} - \widehat{q}_r)}{2(1-\underline{q})}\Delta_H w_{Ho} - p_H w_{Ho} + p_M w_r < 0. \tag{12}$$

In order to show that the condition 12 can be satisfied at $\Delta_M = \Delta_M^*$ take into account that for $\Delta_M = \Delta_M^*$ it is:

- $w_{Ho} = \frac{I_H}{p_H+\Delta^*_H-p_L} = \frac{I_M}{p_M+\Delta^*_M-p_L} = w_{Mo}$, where $\Delta^*_H = (1+z)\Delta^*_M$, implying

$$I_H - I_M = \frac{p_H + \Delta^*_H - p_L}{p_M + \Delta^*_M - p_L} I_M$$

- $p_M w_r - p_H w_{Ho} = \frac{p_M I_M}{p_M+\Delta^*_M-p_L} - \frac{p_H I_H}{p_H+\Delta^*_H-p_L} = \left(\frac{p_M \Delta^*_M}{(p_M-p_L)\left(p_M+\Delta^*_M-p_L\right)} - \frac{(p_H-p_M)}{\left(p_M+\Delta^*_M-p_L\right)} \right) I_M$

Substituting these expressions in condition 12 we obtain

$$-(p_H - p_M)(2 - \widehat{q}_{Ho} - \widehat{q}_r) - z\Delta^*_M 2(1 - \widehat{q}_{Ho} - \widehat{q}_r + \underline{q}) + \frac{\Delta^*_M}{(p_M - p_L)}\left[2(1-\underline{q}) - (2 - \widehat{q}_{Ho} - \widehat{q}_r)\right] < 0$$

Considering that, from condition 6, the value of Δ^*_M depends on $\frac{I_M}{I_H}$, and that $1 < \frac{I_M}{I_H} < \frac{p_H-p_L}{p_M-p_L}$ there clearly exist sufficiently low values of the ratio $\frac{I_M}{I_H}$ that can satisfy the condition.□

Proof of Proposition 9

From Proposition 7 we have that for any given level of θ, V_{Mo} is increasing in Δ_M. Consider, however, that for any given level of θ, the maximum value that the parameter Δ_M can take is $\Delta^M_M(\theta) = \frac{1-p_H-\theta}{1+z}$. For any level of θ we can then calculate the maximum level of V_{io}, that is the level of expected profit corresponding to $\Delta^M_M(\theta)$. Let us call it $V_{io}^{\Delta^M_M(\theta)}$.

From Assumption 1, the maximum level that can θ can take is to $\theta^M(\Delta_M = 0) = 1 - p_H$. We know from Corollary 3 that at such level of θ, it is $V_r > V_{Mo}^{\Delta^M_M(\theta)}$ (note that I_M is chosen when $\Delta_M = 0$). Let θ diminish, which makes V_{Mo} increase. Correspondingly $\Delta^M_M(\theta)$ increases, further increasing $V_{Mo}^{\Delta^M(\theta)}$. Three cases are then possible: i) it may be $V_{Mo}^{\Delta^M_M(\theta)} = V_r$ for some $\Delta^M_M(\theta) < \Delta^*_M$ so that for further decreases in θ, it is $V_{Mo}^{\Delta^M(\theta)} > V_r$; ii) for low enough values of Δ^*_M, it may still be $V_r > V_{Mo}^{\Delta^M(\theta)}$ at $\Delta^M_M(\theta) = \Delta^*_M$; iii) it may also happen that $\theta \to 0$ for $\Delta^M_M(\theta) < \Delta^*_M$, implying that I_H is never chosen because $p_M + \Delta^*_M > 1$. In the first two cases, there clearly exists $\widetilde{\theta} > 0$ such that $V_r > V_{Mo}^{\Delta^M(\theta)}$ for $\theta \geq \widetilde{\theta}$. But even in the third case $V_r > V_{Mo}^{\Delta^M(\theta)}$ will be reached for $\theta > 0$ because we know from Corollary 7 that $V_r > V_{Mo}^{\Delta^M(\theta)}$ for $\theta \to 0$.