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The Optimal Design of Assisted Reproductive Technologies Policies

Abstract

This paper studies the optimal design of assisted reproductive technologies (ART) policies in an economy where individuals differ in their reproductive capacity (or fecundity) and in their wage. We find that the optimal ART policy varies with the postulated social welfare criterion. Utilitarianism redistributes only between individuals with unequal fecundity and wages but not between parents and childless individuals. To the opposite, ex post egalitarianism (which gives absolute priority to the worst-off in realized terms) redistributes from individuals with children toward those without children, and from individuals with high fecundity toward those with low fecundity, so as to compensate for both the monetary cost of ART and for the disutility from involuntary childlessness resulting from unsuccessful ART investments. Under asymmetric information and in order to solve for the incentive problem, utilitarianism recommends also to either tax or subsidize ART investments of low-fecundity-low-productivity individuals depending on the degree of complementarity between fecundity and ART in the fertility technology. On the opposite, ex post egalitarianism always recommends marginal taxation.

JEL-Codes: H310, H510, I140, I180, J130.

Keywords: fertility, assisted reproductive technologies, non-linear taxation, utilitarianism, ex-post egalitarianism.

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1 Introduction

More than 100 millions persons were born all around the world in 1978, but among these, one particular name has marked history: Louise Brown, the first live in vitro fertilization (IVF, hereafter) baby (Prag and Mills 2017). Since then, assisted reproductive technologies (ART) have become an increasingly used medical treatment against involuntary childlessness. ART includes all sorts of medical procedures used primarily to address infertility, the most widely used being the IVF.[1] Success in ART varies with many factors, but the most important one is the woman's age. Success rates decline as women age, specifically after the mid-30's.[2]

The direct cost of ART varies across countries, with the U.S. standing out as one of the most expensive places for ART. According to the American Society for Reproductive Medicine (2020), the average cost for an IVF cycle in the U.S. is as high as $12,400. International variations in ART costs generally reflect the costliness of the underlying healthcare system.[3] If unsubsidized, direct ART costs represent a serious economic burden, in particular, for low-income patients. The affordability of fertility treatments then becomes an important driver for utilization, treatment choices, embryo transfer practices and ultimately multiple birth pregnancies. As shown in Bitler and Schmidt (2012), Schmidt (2005, 2007) and Zarezani and Schmidt (2020), more generous (private or public) coverage of ART (as well as a larger access to these treatments) influence the likelihood that couples who use ART will continue with fertility treatments.

The large costs associated to ART imply that, without public intervention, only sufficiently rich households are able to use ART, and, hence, to have children despite their infecundity. Large ART costs lead to the following policy question: should the State encourage investment in ART? If yes, should this intervention be unconditional, or conditional on earnings, so as to better target households that would be unable to invest in ART without the subsidy?

Those questions have not been addressed so far. There is of course a voluminous literature on the economics of fertility, which has built on the work of Becker (1960) and Barro and Becker (1989), but this literature has mostly considered fertility as the outcome of a choice in a riskless environment.[4] True, in the last decade, economics has started to study the production of children by means of fertility functions where different inputs interact in the production of fertility outcomes (Bhattacharya and Chakraborty 2014, Strulik 2017), but the emphasis has been on contraception rather than on reproduction. Exceptions include recent contributions about involuntary childlessness (Gobbi 2013, Baudin et al 2015, 2020, Etner et al 2020). But that literature has remained (mainly) positive, and did not examine the optimal fiscal treatment of ART in the context of heterogeneity in earnings and in fecundity.

The goal of this paper is precisely to study the optimal taxation policy of ART investments in an economy where individuals are unequal in terms of wages and in terms of their reproductive capacity. To explore that issue, we will pay particular attention to two key difficulties raised by

[1] On various ART techniques available, see Trappe (2017).

[2] Part of this decline is due to a lower chance of getting pregnant from ART, and part is due to a higher risk of miscarriage, especially over age 40 (see O'Brien et al. 2017).

[3] For instance, in France, up to four IVF cycles are fully covered by the public health insurance system, while coverage in the US vary across states. In Quebec, between 2011 and 2015, up to 3 IVF cycles were fully covered by the public health insurance system while between 2015 and 2021, this policy was abandoned and individuals had to bear the full cost of these treatments. Since November 2021, only one IVF cycle is now reimbursed. The related medication may be (partially) reimbursed depending on the private insurance plans individuals benefit from.

[4] One important exception is Cigno and Luporini (2011).

that policy design problem.

A first difficulty is that the individual's reproductive capacity is not observable. Only realized fertility is observable, but this depends on both (natural) fecundity and ART investments, and on many other determinants (tastes for children, religious beliefs, etc.). Given this informational constraint, it is hard to observe the relation between wages and the reproductive capacity. One needs to make assumptions on that relation based on the (observable) education/realized fertility relation. A vast number of empirical studies have documented the negative impact of higher education on fertility (Blossfeld 1995, Cigno and Ermisch 1989, Lappegard and Ronsen 2005, Brand and Davis 2011). This suggests that the expansion of higher education is one of the major drivers of fertility decline and of the postponement of births at the aggregate level.[5] In this paper, we follow this strand of literature according to which education leads to postponing the childbearing age, which in turn increases the probability of infertility, implying a negative correlation between the wage and the reproductive capacity.

A second difficulty concerns the selection of a social welfare criterion. In public finance, utilitarianism is often taken as a normative benchmark. However, it is not clear that this ethical criterion is attractive for the design of ART policy. The reason is that utilitarianism allocates resources by giving priority to individuals who have a higher *marginal* utility. But since involuntary childless individuals are unlikely to exhibit a higher marginal utility, they will not have priority under utilitarianism. This is hard to justify, because involuntary childless persons suffer from a major damage: childlessness prevents them from realizing their life-plans and this can have important well-being implications. Moreover, they can hardly be regarded as responsible for this damage. There is thus a strong support for applying the Principle of Compensation (Fleurbaey and Maniquet 2010, Fleurbaey 2008). According to that principle, welfare inequalities due to circumstances should be abolished by governments. Given that pure luck affects fertility outcomes (even under ART), there is a strong case for applying the Principle of Compensation. This leads us to consider the ex post egalitarian criterion, which gives priority to the worst-off in realized terms (once the outcome of the fertility lottery is known).

In order to study the design of the optimal ART policy, this paper develops a model of stochastic fertility, where the probability to give birth depends on individual fecundity (equivalently, his reproductive capacity). When fecundity is low, individuals invest in ART, which acts as a complement to fecundity in the fertility production function. The laissez-faire studies the ART choices of individuals with low fecundity, assuming a negative correlation between wages and fecundity. There is no inefficiency problem so that we can concentrate on redistributive issues between individuals with unequal wages, unequal fecundity and unequal realized fertility. We first describe the optimal ART policy decentralizing the utilitarian optimum under full information (first-best) or asymmetric information (second-best). We then compare these optimal policies with the ones derived under the ex post egalitarian criterion.

Our main results are the following. We show that, under utilitarianism, there is redistribution between individuals with unequal wages and fecundity, as well as from individuals who did not resort to ART to those who did, but no redistribution between individuals with children and

[5]Recently, Ni Bhrolchain and Beaujouan (2012) showed that fertility postponement in France and England is mainly due to educational expansion. Neyer et al. (2017) reach the same conclusion in their comparison between Swedish and Austrian women. Note however that d'Albis et al (2015, 2017) showed that the relationship between the probability to have a first child and the education level exhibits an inverted-U shape. As a consequence, the correlation between education/wage and the reproductive capacity depends on how one partitions the population into high or low education/wage groups.

childless ones. To the contrary, ex post egalitarianism also allows for redistribution from parents toward childless persons. This is quite different from what is most commonly observed in reality, since most countries provide subsidies to families with children.

In addition, under asymmetric information, besides the taxation of labour supply, ART investments are likely to be taxed or subsidized at the margin in order to solve the incentive problem. Under utilitarianism, ART should be taxed (resp. subsidized) under strong (resp. weak) complementarity between fecundity and ART investment in the fertility production function. To the opposite, under ex-post egalitarianism, ART should always be taxed at the margin.

As such, our paper casts original light on a major policy issue of our times: the optimal fiscal treatment of ART. This paper shows that the optimal ART policy varies with the postulated social welfare criterion - utilitarianism or ex post egalitarianism - and, hence, with the postulated ethical treatment of the individuals with lower fecundity as well as of the involuntary childless persons. Another key finding is that the size and the direction of lump sum transfers vary strongly with the social welfare criterion and the presence of informational constraints. Our second-best findings point to a serious dilemma between compensation and incentive constraints.

Our model can be related to the literature on optimal family policy under risky fertility (Cremer et al 2006) and under birth postponement (Pestieau and Ponthiere 2013). With respect to that literature, the contribution of this paper is to focus on the optimal fiscal treatment of ART. This paper is also related to the literature on compensation and fairness (Fleurbaey and Maniquet 2010). The principle of compensation was applied to several policy issues, such as the taxation of savings under unequal lifetime (Fleurbaey et al 2014) or the taxation of inheritance, also under unequal lifetime (Fleurbaey et al 2022). Only two papers focused on the compensation of involuntary infertile individuals: Etner et al (2020) and Leroux et al (2022). Contrary to our model, those papers assume equal wages among individuals, whereas this paper assumes that, on top of differences in fecundity, individuals also differ in wages, and examines the optimal ART policy in that context.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 solves the laissez-faire equilibrium, while Section 4 derives the utilitarian optimal ART policy under symmetric and asymmetric information. Section 5 derives the optimal ART policy under the alternative ex post egalitarian social criterion. Section 6 concludes.

2 The model

The population The population is composed of four types of individuals, who differ in terms of their economic power (i.e., their productivity), and in terms of their fecundity (i.e., their reproductive capacity). Fertility is stochastic: *ex ante*, all individuals face a fertility lottery, whose form depends on productivity and fecundity. *Ex post* (i.e., once the outcome of the fertility lottery is known), some individuals will have children, whereas others will be childless. Table 1 summarizes information about the four types of individuals *ex ante*. Each type i is present in proportion n_i, for $i = 1, 2, 3, 4$.

	productivity w_i	fecundity ε_i
type 1	low ($w_1 = w$)	high ($\varepsilon_1 = \bar{\varepsilon}$)
type 2	low ($w_2 = w$)	low ($\varepsilon_2 = \tilde{\varepsilon} < \bar{\varepsilon}$)
type 3	high ($w_3 = W > w$)	high ($\varepsilon_3 = \bar{\varepsilon}$)
type 4	high ($w_4 = W > w$)	low ($\varepsilon_4 = \breve{\varepsilon} < \tilde{\varepsilon}$)

Table 1: Heterogeneity in productivity and in fecundity.

where $\bar{\varepsilon}$ corresponds to the maximum reproductive capacity. Throughout this paper, we assume that, for individuals with a lower reproductive capacity, there exists a negative correlation between productivity and fecundity: $W > w$ and $\breve{\varepsilon} < \tilde{\varepsilon}$. This assumption can be justified as follows. First, empirical studies show that high-productivity individuals tend to defer childbearing to older ages (for instance, because of longer investment in higher education) and thus have fewer chances to be successful in having children.[6] This mechanism is illustrated in the Appendix, which develops a model in which individuals are initially characterized by a given level of learning ability and a random level of natural fecundity. At the beginning of their life, individuals choose the number of schooling years that will determine their productivity and the (future) age of childbearing. The resulting levels of productivity and fecundity (determined by education and by the natural fecundity factor) are negatively correlated. The model that we present below is a reduced form of that more general model presented in the Appendix.

The fertility technology In our model of stochastic fertility, the probability of being successful in having a child depends on the fecundity of the individual (equivalently, his reproductive capacity) and on her investment in ART. It is denoted by:

$$\pi(\varepsilon_i, a_i) \tag{1}$$

where $\varepsilon_i \in [0, \bar{\varepsilon}]$ accounts for the fecundity of the individual of type i, and a_i is the investment in ART of a person of type i. The probability $\pi(\varepsilon_i, a_i)$ is increasing and quasi-concave in both arguments: $\pi_\varepsilon(\varepsilon_i, a_i) > 0$, $\pi_{\varepsilon,\varepsilon}(\varepsilon_i, a_i) \leq 0 \, \forall a_i$ and $\pi_a(\varepsilon_i, a_i) > 0$, $\pi_{a,a}(\varepsilon_i, a_i) \leq 0 \, \forall \varepsilon_i$. When fecundity is high (types 1 and 3), having a child is a certain event, i.e. $\pi(\bar{\varepsilon}, 0) \to 1$ and we assume that $\pi_a(\bar{\varepsilon}, 0) \to 0$, so that ART is useless. Under low fecundity, we set $\pi_a(\varepsilon_i, 0) \to +\infty$. As we show below, this ensures that individuals with low fecundity (types 2 and 4) invest in ART.[7]

Regarding the sign of the cross derivatives between a_i and ε_i, we assume complementarity between the two inputs in the child production process (i.e. $\pi_{\varepsilon,a} \geq 0$). Complementarity can be justified on the ground that a higher reproductive capacity makes, *ceteris paribus*, the marginal return of ART higher in terms of chances of success in realized fertility.

Preferences In this model of stochastic fertility, individuals must, by the choice of a level of investment in ART, choose a particular lottery, which specifies a probability of having a

[6]See Blossfeld (1995), Cigno and Ermisch, (1989), Lappegard and Ronsen (2005), Brand and Davis (2011).

[7]We could have assumed otherwise that some individuals with very small wages cannot afford ART. This would reinforce our argument in favor of public intervention through a differentiated taxation between individuals with and without children, as well as between individuals with and without fertility problems.

child. Having a child is here assumed to be desirable for all individuals.[8] The pure utility gain associated to having a child is denoted by $\Omega > 0$, whereas the utility of not having a child is normalized to 0. The parameter Ω denotes the (constant) net utility benefit from having a child, which includes both the joy of having a child and the cost (in time and in money) of raising him.

Preferences on lotteries take a Von Neumann Morgenstern form, and are assumed to be additively separable in consumption and in the preference for having children. We further assume that preferences are quasi-linear in consumption, so that the expected utility of individual of type $i = 1, 2, 3, 4$ is:

$$EU_i = u\left(c_i - \frac{\ell_i^2}{2}\right) + \pi(\varepsilon_i, a_i)\Omega \tag{2}$$

where c_i is the consumption of an individual of type i, while ℓ_i is the quantity of labour of an individual of type i. The utility of net consumption, denoted $x_i \equiv c_i - \frac{\ell_i^2}{2}$, is such that $u'(x_i) > 0$ and $u''(x_i) \leq 0$.

3 The laissez-faire

The timing of the model is the following. In a first stage, individuals of type $i = 1, 2, 3, 4$ supply labour ℓ_i and eventually invest in ART for an amount a_i. In a second stage, the lottery of fertility realizes and individuals have a child or not.

Let us first consider individuals with high fecundity (types $i = 1, 3$). Given that they have a child with a probability $\pi(\bar{\varepsilon}, a_i) = 1$ for all levels of a_i, they do not invest in ART ($a_1 = a_3 = 0$) and supply quantities of labour equal to:

$$\ell_1^{LF} = w \text{ and } \ell_3^{LF} = W$$

leading to consumptions $c_i = w_i \ell_i$ equal to:

$$c_1^{LF} = w^2 \text{ and } c_3^{LF} = W^2.$$

Normalizing the price of one unit of ART investment to unity, the problem of individuals with low fecundity (type $i = 2, 4$) is:

$$\max_{\ell_i, a_i} EU_i = u\left(c_i - \frac{\ell_i^2}{2}\right) + \pi(\varepsilon_i, a_i)\Omega$$
$$\text{s. to } a_i + c_i = w_i \ell_i$$

From the first-order condition (FOC) on labour, we obtain that $\ell_i^{LF} = w_i$, so that:[9]

$$\ell_2^{LF} = w < \ell_4^{LF} = W$$

[8] We exclude the cases where agents could have different preferences for children, and thus, the case of childfree persons. This would complicate significantly our model and the shape of compensation at the ex-post egalitarian optimum (see Section 5). On this subject, see Leroux et al. (2022), who derive the ex-post egalitarian optimum when agents have different preferences but equal productivities and reproductive capacities.

[9] We do not model here, an additional time cost of undergoing ART that would reduce labour supply. This cost is implicitly taken into account in the net benefit from having a child Ω. Assuming otherwise would lead us to consider different opportunity costs of ART treatments affecting differently agents with different wages. This would complicate our model without further insights in terms of the taxation of ART.

After replacing for the expression of ℓ_i^{LF} in the FOC for a_i, we obtain that, when a_i is interior,

$$-u'\left(\frac{w_i^2}{2}-a_i\right)+\pi_a(\varepsilon_i,a_i)\Omega=0, \tag{3}$$

where the first term represents the marginal welfare loss of increasing investment in ART and the second one represents its marginal utility gain. On the one hand, the marginal welfare loss of ART is decreasing in the individuals' wage w_i. On the other hand, since $\pi_{\varepsilon,a}(\varepsilon_i,a_i)\geq 0$, the marginal welfare gain $\pi_a(\varepsilon_i,a_i)\Omega$ is increasing in ε_i. Recall that $w_2<w_4$ and $\varepsilon_2>\varepsilon_4$ so that whether $a_2 \gtrless a_4$ is a priori ambiguous. Yet, if we assume that the productivity gap is sufficiently large in comparison to the fecundity gap, the first (marginal cost) effect dominates the second (marginal benefit) effect and $a_4^{LF}>a_2^{LF}$.

Let us now turn to the welfare analysis. Our economy includes 4 types ex ante (i.e., before the outcome of the lottery is known) and 6 types ex post (i.e., after the outcome of the lottery is known). Denoting the realized welfare of individuals of type $i=1,2,3,4$ with realized fertility $j=0,1$ as $U_{i,j}$, we have:[10]

$$\begin{aligned} U_{2,0} &< U_{2,1}<U_{1,1}<U_{3,1}\\ U_{4,0} &< U_{4,1}<U_{3,1}\\ U_{2,j} &< U_{4,j}\forall j \end{aligned}$$

For a given productivity, individuals with high fecundity are better off than individuals who had a child with ART treatments, who are themselves better off than individuals who did not have a child despite ART. Moreover, for a given fertility outcome, high productivity individuals are better off than low productivity ones. Denoting total earnings by $y_i=w_i\ell_i$, Proposition 1 summarizes the results of the laissez-faire.

Proposition 1 *At the laissez-faire equilibrium:*

- *Labour income is higher for high-productivity individuals than for low-productivity ones:* $y_3^{LF}=y_4^{LF}=W^2>y_1=y_2=w^2$.
- *If the productivity gap is sufficiently large with respect to the fecundity gap, high-productivity individuals invest more in ART:* $a_4^{LF}>a_2^{LF}>0=a_1^{LF}=a_3^{LF}$.
- *For a given productivity level, individuals with high fecundity are better off than these with low fecundity but who had a child using ART. Unsuccessful ART users are the worst-off.*
- *For a given realized fertility, individuals with a high productivity are better off than these with a low productivity.*

The above proposition shows that not only differentials in productivity, but also differentials in fecundity and in realized fertility lead to welfare inequalities among individuals. The size of these welfare inequalities depends on the structure of preferences, in particular the shape of the function $u(\cdot)$ and the level of the parameter Ω which captures the net welfare gain from having a child, and, hence, the pure welfare loss of remaining childless.

[10]Since $a_4^{LF}>a_2^{LF}$, $\pi_a(\varepsilon_2,a_2^{LF})>\pi_a(\varepsilon_4,a_4^{LF})$, therefore leading to $x_2^{LF}<x_4^{LF}$ using eq. (3) and the above utility ranking (third line).

4 Utilitarianism

When considering the design of the optimal family policy, a standard ethical benchmark is utilitarianism, based on the Principle of Utility or the Principle of the Greatest Happiness for the Greatest Number (Bentham 1789). In this section, we characterize the social optimum under the utilitarian criterion, and we show how it differs from the laissez-faire equilibrium.

4.1 First-best optimum

The utilitarian problem consists in selecting consumptions, ART investments and labour incomes so as to maximize the sum of individual utilities:

$$\max_{c_i,a_i,y_i} \sum_i n_i \left[u\left(c_i - \frac{(y_i/w_i)^2}{2} \right) + \pi(\varepsilon_i, a_i)\Omega \right] \text{ s. to } \sum_i n_i y_i = \sum_i n_i c_i + n_2 a_2 + n_4 a_4$$

Solving this this problem, we find as usual that net consumptions are smoothed across types: $x_1 = x_2 = x_3 = x_4 = \bar{x}$ and labour supply is equal to the individual's productivity: $\ell_i^* = w_i$. This also implies that $y_3^* = y_4^* > y_1^* = y_2^*$ and that it is independent from fecundity.

The only difference with regard to this standard first-best consumption - labour allocation problem resides in that individuals have uncertain fertility and invest in ART. The FOC with respect to ART investment, a_i writes as follows:

$$\pi_a(\varepsilon_i, a_i)\Omega = \mu \tag{4}$$

where μ is the Lagrange multiplier associated to the resource constraint. This condition shows that for individuals with low fecundity (i.e., types 2 and 4), the optimal level of ART should be set at a level such that the marginal welfare gain of increasing a_i is equal to its marginal welfare cost ($\mu = u'(\bar{x})$ constant for all types i). Hence, whether ART should be higher or lower for type 2 or 4 depends on how the marginal return from ART, i.e. $\pi_a(\varepsilon_i, a_i)\Omega$, varies with a_i and ε_i. Given that $\pi_{a,a}(\varepsilon_i, a_i) \leq 0$, $\pi_{\varepsilon,a}(\varepsilon_i, a_i) \geq 0$ and $\varepsilon_4 < \varepsilon_2$, it follows that $a_4^* < a_2^*$. Thus, among individuals with low fecundity, high-productivity individuals receive, at the utilitarian optimum, less ART than low-productivity ones.

Comparing these results with the laissez-faire (Proposition 1), we show that the decentralization of the utilitarian optimum only requires individualized lump sum taxes and transfers so as to ensure that all individuals consume $\bar{x}$. The direction of redistribution is detailed in the following proposition:

Proposition 2 *The decentralization of the utilitarian first-best optimum requires*

- *lump-sum transfers from individuals with a high wage (types 3 and 4) to individuals with a low wage (types 1 and 2),*
- *lump-sum transfers from individuals with high fecundity (types 1 and 3) to those with low fecundity (types 2 and 4),*
- *no distribution between individuals with and without children.*

The first point in the above proposition is a standard implication of utilitarianism. The second point is directly related to our extension to uncertain fertility, to different reproductive capacities, and to the possibility to invest in ART. The last bullet point is a direct consequence of the government being utilitarian and of the individual's utility being additively separable in the utility of consumption and in the utility from having a child. Under that framework, no distribution should take place between individuals with and without children, something different from what is usually observed in reality.

Regarding inequalities in realized welfare, and using the same notations as in the previous section, we obtain that, at the utilitarian optimum:

$$U_{2,0}^{*} = U_{4,0}^{*} = u(\bar{x}) < U_{1,1}^{*} = U_{2,1}^{*} = U_{3,1}^{*} = U_{4,1}^{*} = u(\bar{x}) + \Omega.$$

Hence, the utilitarian optimum equalizes utilities between all individuals who have a child, independently of whether they had high or low fecundity. It also equalizes utilities across productivity levels, among childless individuals and among parents. Yet, it fails to equalize utilities between parents and childless individuals. Given that $\Omega > 0$, the realized welfare of parents is, at the utilitarian optimum, higher than the one of childless persons. The reason is that the utilitarian planner gives priority to individuals with higher marginal utilities (Sen 1980). Since involuntary childless persons do not exhibit higher marginal utilities, they receive no priority and end up being the worst-off.

4.2 Second-best optimum

In this section, we assume that the social planner can observe neither the agents' characteristics, (w_i, ε_i) nor their labour supply ℓ_i. The planner can only observe total earnings y_i, consumption c_i, the investment in ART a_i, and whether individuals have a child or not. Hence, if mimicking behavior arises, it can only be along the productivity-fecundity (w_i, ε_i) dimensions, since both ART spending and having a child are observable.[11]

Using the traditional approach of optimal income taxation, the planner will offer a menu of second-best contracts $\{y_i, c_i, a_i\}$ to individuals of types $i = 1, 2, 3, 4$, so as to maximize social welfare, granted that the resource constraint and the self-selection constraints are satisfied. At this point, one could object that the planner may easily observe the length of (implicit) education of individuals and, in turn, infer their productivity and fecundity.[12] To avoid this objection, we assume that the optimal policy is offered at the beginning of adulthood, and that the planner commits not to change it over time. This assumption of commitment is standard in dynamic optimal taxation problems.[13]

Among individuals with high fecundity (types 1 and 3), the second-best problem is standard. Only high-productivity individuals (type 3) would have interest in claiming to have a low productivity (type 1).

[11] In other words, individuals cannot declare they had fertility problems when they did not. Neither can they declare they do not have a child when they do (or the reverse).

[12] Recall from the Appendix that productivity and the reproductive capacity result from different length of education.

[13] See, for instance, Battaglini and Coate (2008) who analyse Pareto efficient income taxation with stochastic abilities. They assume that the government can credibly commit to the ex ante optimal tax/transfer system. See also Boadway et al. (1996) and Konrad (2001).

Among individuals with low fecundity (types 2 and 4), the mimicking behavior is less direct. If the first-best allocation were proposed, a type-4 individual would have an interest in pretending to be a type 2 if and only if:

$$u\left(c_2^* - \frac{(y_2^*/W)^2}{2}\right) + \pi(\varepsilon_4, a_2^*)\Omega \geq u\left(c_4^* - \frac{(y_4^*/W)^2}{2}\right) + \pi(\varepsilon_4, a_4^*)\Omega$$

$$\iff \quad \Omega[\pi(\varepsilon_4, a_2^*) - \pi(\varepsilon_4, a_4^*)] \geq u(\bar{x}) - u\left(\bar{x} - \left(\frac{(y_2^*/W)^2}{2} - \frac{(y_2^*/w)^2}{2}\right)\right) \tag{5}$$

where the RHS of this inequality is always negative, while the LHS is positive. Hence a type-4 individual would always have interest in mimicking a type 2. Using a symmetric reasoning, it is also possible to prove that a type-2 individual never has any interest in pretending to be a type-4 one.[14]

The second-best problem consists in solving:

$$\max_{c_i, a_i, y_i} n_i \left[u\left(c_i - \frac{(y_i/w_i)^2}{2}\right) + \pi(\varepsilon_i, a_i)\Omega\right]$$
$$\text{s. to } \sum_i n_i y_i = \sum_i n_i (c_i + a_i)$$
$$\text{s. to } u\left(c_3 - \frac{(y_3/W)^2}{2}\right) \geq u\left(c_1 - \frac{(y_1/W)^2}{2}\right)$$
$$\text{s. to } u\left(c_4 - \frac{(y_4/W)^2}{2}\right) + \pi(\varepsilon_4, a_4)\Omega \geq u\left(c_2 - \frac{(y_2/W)^2}{2}\right) + \pi(\varepsilon_4, a_2)\Omega$$

where the last two constraints are the incentive constraints, which ensure that, respectively, type-3 individuals do not mimic type-1 individuals (third line), and type-4 individuals do not mimic type-2 ones (fourth line).

The FOCs of this problem are provided in the Appendix. Let us first consider the optimal trade-offs for individuals with high fecundity (types 1 and 3). We find the standard Mirrlees (1971) result that the consumption-labour trade-off of the high-income individuals should not be distorted, while that of a low-income type should be distorted downward:

$$y_3^{**} = W^2 \tag{6}$$

$$y_1^{**} = w^2 \left[\frac{n_1 - \lambda \frac{u'(\tilde{x}_1^{**})}{u'(x_1^{**})}}{n_1 - \lambda \frac{u'(\tilde{x}_1^{**})}{u'(x_1^{**})} \frac{w^2}{W^2}}\right] < w^2 \tag{7}$$

where ** accounts for the second-best optimal levels, λ is the Lagrange multiplier associated to the first self-selection constraint, and $\tilde{x}_1 \equiv c_1 - \frac{(y_1/W)^2}{2}$ is the net consumption of a type-3 individual pretending to be a type 1.

Let us now consider the optimal trade-offs for individuals with low fecundity. Rearranging the FOCs of a type-4 individual (see the Appendix), we obtain the usual result of no distortion

[14]Note that low-fecundity individuals (type 2 or 4) never have an interest in claiming to be high fecundity (i.e. type 1 or 3) since in that case, they would receive no ART investment.

at the top for both the ART-consumption trade-off and for the labour-consumption trade-off,

$$\frac{\Omega\pi_a(\varepsilon_4, a_4)}{u'(c_4^{**})} = 1 \tag{8}$$
$$y_4^{**} = W^2. \tag{9}$$

To the opposite, the allocation of a type-2 individual is now distorted so as to avoid mimicking from type 4. Type-2 labour income should be distorted downward:

$$y_2^{**} = w^2 \left[\frac{n_2 - \kappa \frac{u'(\tilde{x}_2^{**})}{u'(x_2^{**})}}{n_2 - \kappa \frac{u'(\tilde{x}_2^{**})}{u'(x_2^{**})} \frac{w^2}{W^2}} \right] < w^2$$

where $\tilde{x}_2 \equiv c_2 - \frac{(y_2/W)^2}{2}$ is the net consumption of a type-4 individual pretending to be of type 2, while κ is the Lagrange multiplier associated to the second incentive-compatibility constraint. Also, the trade-off between consumption and investment in ART is now:

$$\frac{\Omega\pi_a(\varepsilon_2, a_2^{**})}{u'(x_2^{**})} = \frac{n_2 - \kappa \frac{u'(\tilde{x}_2^{**})}{u'(x_2^{**})}}{n_2 - \kappa \frac{\pi_a(\varepsilon_4, a_2^{**})}{\pi_a(\varepsilon_2, a_2^{**})}} \tag{10}$$

with $\tilde{x}_2^{**} > x_2^{**}$ and thus $u'(\tilde{x}_2^{**})/u'(x_2^{**}) < 1$. Given complementarity, we have $\frac{\pi_a(\varepsilon_4, a_2^{**})}{\pi_a(\varepsilon_2, a_2^{**})} < 1$ so that whether the optimal second-best trade-off in equation (10) is distorted downward or upward depends on the comparison between $u'(\tilde{x}_2^{**})/u'(x_2^{**})$ and $\frac{\pi_a(\varepsilon_4, a_2^{**})}{\pi_a(\varepsilon_2, a_2^{**})}$. This comparison implicitly depends on the degree of complementarity between ART and the reproductive capacity. We define the degree of complementarity as the intensity of the relationship between a_i and ε_i in the fertility production function $\pi(\varepsilon_i, a_i)$. Strong (resp. weak) complementarity corresponds to a large (resp. a small) marginal return of a_i on the fertility production function, following a variation in ε_i.

Under strong complementarity, it is more likely that $u'(\tilde{x}_2^{**})/u'(x_2^{**}) > \frac{\pi_a(\varepsilon_4, a_2^{**})}{\pi_a(\varepsilon_2, a_2^{**})}$ and so, that $\frac{\Omega\pi_a(\varepsilon_2, a_2^{**})}{u'(x_2^{**})} < 1$. For incentive compatibility reasons and in order to prevent mimicking from type-4, investment in ART of type-2 individuals should then be distorted downward. The intuition for that result is the following. A type-4 agent claiming to be of type 2 would then obtain a much higher level of ART (indeed, under strong complementarity, a_2 would be very high compared to a_4), so that it is optimal to tax the ART investment of type 2 in order to make it less desirable to a type 4. On the other hand, under weak complementarity, it is more likely that $\frac{\pi_a(\varepsilon_4, a_2^{**})}{\pi_a(\varepsilon_2, a_2^{**})} > u'(\tilde{x}_2^{**})/u'(x_2^{**})$ and thus, that $\frac{\Omega\pi_a(\varepsilon_2, a_2^{**})}{u'(x_2^{**})} > 1$. ART investment of type-2 individuals should then be distorted upward. Under weak complementarity (or even no complementarity), the differences between ART investments of types 2 and 4 would be low (null), so that a type-4 agents would not like to invest more in ART than a type-2. Subsidizing the ART investment of that latter type then makes the allocation less desirable from the point of view of a type-4 agent.

Let us finally explain how the second-best optimum can be decentralized. For individuals with no fertility problems (types 1 and 3), the implementation is standard: only the labour income of type-1 individuals should be taxed at the margin. In the same way, for individuals with low fecundity, only low-productivity (type-2) agents should face a marginal tax on income. This is a way to solve the incentive problem and to make the allocations of low-productivity agents less desirable to high-productivity ones who would like to work more and earn more.

Interestingly, we find that only the ART investment of type-2 agents should be taxed or subsidized at the margin. As demonstrated above (see in particular eq. 10 and explanations below), the ART investment of these individuals should be subject to marginal taxation (resp. subsidization) if ART investment and the reproductive capacity are strong (resp. weak) complements. Finally, lump-sum taxation is also necessary to redistribute between individuals with unequal productivity and fecundity. Like in the first-best, no redistribution is made between individuals with and without children.

Our results are summarized in the following proposition:

Proposition 3 *The decentralisation of the second-best utilitarian optimum requires that:*

- *the labour supply of individuals with low productivity is taxed at the margin,*
- *among individuals with low fecundity, the ART investment of type-2 agents is taxed (resp. subsidized) under strong (resp. weak) complementarity between ART investment and the reproductive capacity,*
- *lump-sum transfers are also necessary in order to redistribute between individuals with different types* (w_i, ε_i).

5 Ex-post egalitarianism

As shown in Section 4, the utilitarian criterion fails to compensate agents for inequalities in fertility outcomes. Utilitarianism implies redistribution across individuals with unequal productivity and fecundity, but does not offer compensation for the welfare losses suffered by persons who fail to have children despite ART investment. Yet, infertility implies a substantial welfare loss for those persons, who cannot be held responsible for their infertility.[15] In our model, fertility is a pure circumstance and thus, a pure matter of luck. However, utilitarianism offers no compensation for bad fertility outcomes: the equalization of marginal utilities - what Sen (1980) called "utilitarian equality" - leads here to inequalities in welfare levels that are hard to justify. This implication of utilitarianism is unattractive, and invites the reliance on another ethical criterion.

Actually, in the context of stochastic fertility, and given the substantial welfare loss caused by involuntary childlessness (measured by Ω), there is a strong support for compensating the persons who remain involuntary childless despite ART investments. This compensation can be justified by referring to Fleurbaey and Maniquet (2010) and Fleurbaey (2008)'s Principle of Compensation: welfare inequalities due to circumstances should be neutralized by the State. In our context, this principle justifies compensation for unfavourable fertility outcomes.

This section focuses on the compensation of inequalities in fertility outcomes. To do so, we rely on the ex-post egalitarian criterion, which consists in maximizing the utility of the worst-off individual in realized terms, that is, her utility once the outcome of the fertility lottery is

[15] Domar et al. (1993) and Fidler and Bernstein (1999) have shown that suffering from infertility can lead to depression problems of the same intensity as those observed for individuals suffering from a cancer.

known (see Fleurbaey et al 2014, Leroux et al 2022). The reason for considering that alternative social welfare criterion is that individuals can hardly be regarded as responsible for their realized fertility, and, as such, they should be compensated for their bad luck in the fertility lottery. We characterize the optimal allocation of resources and the corresponding optimal taxation scheme under that alternative ethical criterion.

5.1 First-best optimum

Let us first remind that ex-post utilities are:

$$
\begin{aligned}
U_{1,1} &= u\left(c_{1,1} - \frac{(y_1/w)^2}{2}\right) + \Omega \quad \text{and} \quad U_{2,1} = u\left(c_{2,1} - \frac{(y_2/w)^2}{2}\right) + \Omega \\
U_{3,1} &= u\left(c_{3,1} - \frac{(y_3/W)^2}{2}\right) + \Omega \quad \text{and} \quad U_{4,1} = u\left(c_{4,1} - \frac{(y_4/W)^2}{2}\right) + \Omega \\
U_{2,0} &= u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) \quad \text{and} \quad U_{4,0} = u\left(c_{4,0} - \frac{(y_4/W)^2}{2}\right)
\end{aligned}
$$

Note that, given the timing of the model, in which we assume that individuals with low fecundity supply labour *before* uncertainty regarding fertility is resolved, same productivity-type individuals supply the same amount of labour (and thus have the same labour income, y_i), independently from being successful or not in having a child.

The ex post egalitarian social planning problem can be written as:

$$
\begin{aligned}
&\max_{c_{i,j},a_i,y_i} \min\{U_{i,j}\} \,\forall i = 1,2,3,4, \forall j = 0,1 \\
&\text{s. to } \sum_i n_i y_i = \sum_i n_i \left[\pi(\varepsilon_i, a_i) c_{i,1} + (1 - \pi(\varepsilon_i, a_i)) c_{i,0}\right] + n_2 a_2 + n_4 a_4
\end{aligned}
$$

Since this objective function is not differentiable, we rewrite that problem as maximizing the utility of the worst-off agent (i.e. a low productivity and low fecundity individual who remains childless *ex post*), subject to the resource constraint and subject to the egalitarian constraints:

$$
\begin{aligned}
&\max_{c_{i,j},a_i,y_i} u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) \\
&\text{s. to } \sum_i n_i y_i = \sum_i n_i \left[\pi(\varepsilon_i, a_i) c_{i,1} + (1 - \pi(\varepsilon_i, a_i)) c_{i,0}\right] + n_2 a_2 + n_4 a_4 \\
\text{s. to} \quad & u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{4,0} - \frac{(y_4/W)^2}{2}\right) \\
& u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{4,1} - \frac{(y_4/W)^2}{2}\right) + \Omega \\
& u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{2,1} - \frac{(y_2/w)^2}{2}\right) + \Omega \\
& u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{1,1} - \frac{(y_1/w)^2}{2}\right) + \Omega \\
& u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{3,1} - \frac{(y_3/W)^2}{2}\right) + \Omega
\end{aligned}
$$

Note that, by definition, the first-best egalitarian optimum ensures that ex post utilities are equalized across types (w_i, ε_i), and across parents and childless individuals. Hence, the ex-post egalitarian optimum not only neutralizes ex-ante welfare inequalities (due to unequal productivity and to unequal fecundity) as under the utilitarian criterion, but also ex-post welfare inequalities (resulting from the luck of having a child or not).[16] The intuition behind this result lies in the assumed structure of preferences. This model assumes some substitutability between consumption and fertility outcomes and it presupposes that a monetary transfer could always compensate for not having a child. Actually, this is equivalent to assuming that w and W are large enough with respect to the preference, Ω, for having a child so that it is always possible to provide full compensation to the involuntary childless by giving them more consumption.

In order to achieve such compensation, the egalitarian optimal allocation will then consist in equalizing net consumptions across ex-ante types, and to provide more consumption to childless individuals:

$$x_{1,1}^E = x_{2,1}^E = x_{3,1}^E = x_{4,1}^E < x_{2,0}^E = x_{4,0}^E \tag{11}$$

where E stands for Egalitarianism.[17] In the Appendix, we also show that the optimal levels of ART investments are given by the following two equations:

$$\pi_a(\varepsilon_2, a_2)(c_{2,0} - c_{2,1}) = 1 \tag{12}$$

$$\pi_a(\varepsilon_4, a_4)(c_{4,0} - c_{4,1}) = 1 \tag{13}$$

Noticing that $c_{2,0} - c_{2,1} = c_{4,0} - c_{4,1}$ because $x_{2,0} = x_{4,0}$ and $x_{2,1} = x_{4,1}$, we obtain that at the egalitarian first-best optimum, $\pi_a(\varepsilon_2, a_2) = \pi_a(\varepsilon_4, a_4)$. Under the assumption of complementarity between fecundity and ART investment and given that $\varepsilon_4 < \varepsilon_2$, we obtain that $a_4^E < a_2^E$, exactly like in the utilitarian optimum. Also, like in the utilitarian first-best optimum, labour supply is not distorted at the egalitarian optimum, for any agent i, and $y_1^E = y_2^E = w^2$ and $y_3^E = y_4^E = W^2$.[18]

Let us now examine how the ex post egalitarian optimum can be decentralized. The timing of the decentralized problem is the following one. In the first stage, the government announces the fiscal policy. It consists in setting an individualized linear tax on ART investment, σ_i and an individualized linear tax on labour income, t_i in the second stage. It will also implement lump-sum taxation /subsidization in the third stage, once uncertainty regarding fertility has realized.[19] In the second stage, before uncertainty realizes, individuals make their choices of labour supply, of consumption and, eventually, of investment in ART, conditional on the taxation scheme. When nature realizes, in the third stage, the government implements (positive or negative) lump-sum taxes, denoted by $T_{i,1}$ and $T_{i,0}$, for all $i = 2, 4$ who resorted to ART and had a child or not, as well as $T_{i,1}$ for individuals with types $i = 1, 3$ with no fecundity problems, so as to

[16]Note that we would obtain a less "radical" solution (i.e. imperfect equalization of ex-post utilities) if instead, we modeled the social welfare function as a concave transformation of ex-post utilities (see Fleurbaey et al 2016). In such a case, we would obtain that some redistribution is still made between individuals with and without children, the extent of which would depend on the concavity of the welfare function. But, we would not obtain full equalization of ex-post utilities anymore.

[17]In the Appendix, we provide also a complete ranking of gross consumptions, $c_{i,j}^E$.

[18]Labour supply is independent from having a child since it is decided before agents effectively have a child.

[19]We consider here individualized linear tax functions for labour income and ART investments together with lump-sum taxation, which is equivalent to non-linear tax schedules.

redistribute resources between individuals with unequal productivity and fecundity as well as to compensate them for (ex-post) inequalities in realized fertility.[20]

In the Appendix, we derive the individual's problem facing such a tax-and-transfer scheme. We are able to show that in order to implement the egalitarian first-best optimum, we only need lump sum transfers as follows. The ranking of net consumptions $x_{i,j}^E$ in eq. (11) leads to the following ranking of lump-sum taxes:

$$\begin{aligned} T_{3,1} &> T_{1,1}, \quad T_{4,0} > T_{2,0} \quad \text{and} \quad T_{4,1} > T_{2,1} \\ T_{3,1} &> T_{4,1} > T_{4,0} \quad \text{and} \quad T_{1,1} > T_{2,1} > T_{2,0}. \end{aligned}$$

The first line above shows that high-productivity individuals should be more taxed than low-productivity individuals for a given realization of nature (whether they have a child or not, after using ART or not). The second line ensures that no welfare inequality remains between parents with no fecundity problems, those who invested in ART, and those who remained childless, for a given productivity level. Such transfers aim at compensating agents for both the monetary cost of investing in ART and the non-monetary cost of ending up childless.

Proposition 4 *The decentralization of the egalitarian first-best optimum requires to redistribute resources from high-productivity toward low-productivity individuals, from high-fecundity toward low-fecundity individuals and from individuals with children toward childless individuals. No taxation or subsidisation of ART investment is needed.*

This proposition shows that, under the ex-post egalitarian criterion, lump-sum redistribution goes from individuals with children toward those without children, and from individuals with high fecundity toward those with low fecundity. This tax-and-transfer policy is quite different from what is usually observed in our societies, where families with children usually benefit from specific allowances. Also, in some countries (like in Canada and the U.S), redistribution toward couples with fertility problems is limited. This is also quite different from the results obtained under the utilitarian criterion, where there was no redistribution from families with children toward those without.

5.2 Second-best optimum

At the egalitarian first best, redistribution involves three dimensions: productivity, fecundity, and being successful or not in having a child. As mentioned in Section 4.2, investing in ART and having a child are both observable so that the mimicking behavior can only arise on the productivity-fecundity (w_i, ε_i) dimension.

Looking at the first-best egalitarian allocation, it is straightforward to see that among individuals with high fecundity, only high-productivity individuals have an interest in declaring to be low productivity. Among individuals with low fecundity, whether type-2 or type-4 have an interest in mimicking the other type depends on the precise levels of consumptions, labour

[20] One could instead model two different types of lump-sum taxes: one compensating for differences in productivity and in investment in ART, implemented in the second stage and, one compensating agents for not having a child, implemented ex post, in the third stage. Since agents can anticipate these taxes, it is equivalent to assume a single tax that would achieve redistribution on these three dimensions.

income and ART investment. Equivalently, if the social planner were to propose the first-best allocation, type-4 individuals would have an interest in claiming to be a type-2, if and only if:

$$\pi(\varepsilon_4, a_2^E)[u(\tilde{x}_{2,1}^E) + \Omega] + (1-\pi(\varepsilon_4, a_2^E))u(\tilde{x}_{2,0}^E) \geq \pi(\varepsilon_4, a_4^E)[u(x_{4,1}^E) + \Omega] + (1-\pi(\varepsilon_4, a_4^E))u(x_{4,0}^E) \tag{14}$$

where $\tilde{x}_{2,1}^E \equiv c_{2,1}^E - \frac{(y_2^E/W)^2}{2}$ and $\tilde{x}_{2,0}^E \equiv c_{2,0}^E - \frac{(y_2^E/W)^2}{2}$ are the net consumptions obtained by a type-4 individual claiming to be a type 2. Note that since, at the first-best optimum, the egalitarian constraints require that $x_{i,1}^E$ and $x_{i,0}^E$ are equalized across types i, we obtain:

$$x_{2,1}^E = c_{2,1}^E - \frac{(y_2^E/w)^2}{2} = x_{4,1}^E = c_{4,1}^E - \frac{(y_4^E/W)^2}{2} < \tilde{x}_{2,1}^E = c_{2,1}^E - \frac{(y_2^E/W)^2}{2}$$
$$x_{2,0}^E = c_{2,0}^E - \frac{(y_2^E/w)^2}{2} = x_{4,0}^E = c_{4,0}^E - \frac{(y_4^E/W)^2}{2} < \tilde{x}_{2,0}^E = c_{2,0}^E - \frac{(y_2^E/W)^2}{2}$$

A type-4 individual claiming to be a type-2 would then obtain more net consumption $\tilde{x}_{2,j}^E$ than truthfully revealing his type and obtaining net consumption $x_{4,j}^E$. This is true independently from whether the individual is successful with ART or not. In addition, in the first best, $a_2^E > a_4^E$ so that a type-4 individual would always have an interest in claiming to be of type 2, if the first-best allocation was proposed (i.e. condition 14 would always hold).

Hence, the second-best ex-post egalitarian problem now includes incentives constraints and is written as:

$$\max_{c_{i,j},a_i,y_i} u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) \tag{15}$$
$$\text{s. to } \sum_i n_i y_i = \sum_i n_i \left[\pi(\varepsilon_i, a_i)c_{i,1} + (1-\pi(\varepsilon_i, a_i))c_{i,0}\right] + n_2 a_2 + n_4 a_4$$
$$\text{s. to } u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{4,0} - \frac{(y_4/W)^2}{2}\right) \tag{16}$$
$$\text{s. to } u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{4,1} - \frac{(y_4/W)^2}{2}\right) + \Omega \tag{17}$$
$$\text{s. to } u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{2,1} - \frac{(y_2/w)^2}{2}\right) + \Omega \tag{18}$$
$$\text{s. to } u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{1,1} - \frac{(y_1/w)^2}{2}\right) + \Omega \tag{19}$$
$$\text{s. to } u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) = u\left(c_{3,1} - \frac{(y_3/W)^2}{2}\right) + \Omega \tag{20}$$
$$\text{s. to } u\left(c_{3,1} - \frac{(y_3/W)^2}{2}\right) \geq u\left(c_{1,1} - \frac{(y_1/W)^2}{2}\right) \tag{21}$$
$$\text{s. to } \pi(\varepsilon_4, a_4)\left[u\left(c_{4,1} - \frac{(y_4/W)^2}{2}\right) + \Omega\right] + (1-\pi(\varepsilon_4, a_4))u\left(c_{4,0} - \frac{(y_4/W)^2}{2}\right)$$
$$\geq \pi(\varepsilon_4, a_2)\left[u\left(c_{2,1} - \frac{(y_2/W)^2}{2}\right) + \Omega\right] + (1-\pi(\varepsilon_4, a_2))u\left(c_{2,0} - \frac{(y_2/W)^2}{2}\right) \tag{22}$$

where the last two constraints are the incentive constraints preventing high productivity individuals from mimicking low productivity individuals, when, respectively, individuals have either high or low fecundity. That problem is solved in the Appendix.

Before studying the optimal trade-offs, let us first show how the introduction of the incentive constraints modifies the ranking of ex-post utilities. For individuals with high fecundity, the incentive constraint (eq. 21) cannot be satisfied at the same time as the egalitarian constraint stating that $U_{1,1} = U_{3,1}$. Indeed, when the incentive constraint is binding, one necessarily has:

$$u\left(c_{3,1} - \frac{(y_3/W)^2}{2}\right) = u\left(c_{1,1} - \frac{(y_1/W)^2}{2}\right) > u\left(c_{1,1} - \frac{(y_1/w)^2}{2}\right).$$

Hence, at the ex post optimum, there will remain welfare inequalities between agents with high fecundity but unequal productivity. Leaving a rent to the high-productivity individual is a necessary condition in order to prevent him from mimicking a low-productivity type, but as a result, it does not allow for the equalization of ex-post utilities across different productivity types.

In the same way, for individuals with low fecundity, the incentive constraint (eq. 22) cannot hold at the same time as the equality of ex post utilities across productivity types, i.e. $U_{4,1} = U_{2,1}$ and $U_{4,0} = U_{2,0}$, do. We prove this by contradiction by assuming instead that it could be the case; we could then rewrite eq. (22) as

$$\begin{aligned} & \pi(\varepsilon_4, a_4)\left[u\left(c_{2,1} - \frac{(y_2/w)^2}{2}\right) + \Omega\right] + (1 - \pi(\varepsilon_4, a_4))u\left(c_{2,0} - \frac{(y_2/w)^2}{2}\right) \\ \geq\ & \pi(\varepsilon_4, a_2)\left[u\left(c_{2,1} - \frac{(y_2/W)^2}{2}\right) + \Omega\right] + (1 - \pi(\varepsilon_4, a_2))u\left(c_{2,0} - \frac{(y_2/W)^2}{2}\right) \end{aligned}$$

which obviously cannot be satisfied under $W > w$ and $a_4 < a_2$.

Therefore, for the optimal allocation to be incentive-compatible under asymmetric information, not all egalitarian constraints can be binding, and we will obtain that $U_{3,1} > U_{1,1}$, $U_{4,1} > U_{2,1}$ and $U_{4,0} > U_{2,0}$. Nonetheless, it is still possible to equalize the utility of agents with the same low fecundity but different realized fertility: $U_{4,1} = U_{4,0}$ and $U_{2,1} = U_{2,0}$. Ex-post utilities between individuals with unequal fecundity but the same productivity level can also still be equalized. We therefore obtain the following ranking of ex-post utilities,

$$U_{3,1} = U_{4,1} = U_{4,0} > U_{1,1} = U_{2,1} = U_{2,0}.$$

To sum up, while the first-best egalitarian optimum could equalize *all* ex-post utilities (across productivity types, fecundity and realized fertility), it is not anymore the case at the second-best optimum, across productivity types. The reason is that the government needs to leave an informational rent to the high-productivity type so as to make sure he truthfully reveals his type but at the same time, it prevents the equalization of all ex-post utilities. This ranking of utilities therefore implies that net consumptions are such that:

$$x_{4,0}^{E^{SB}} > x_{3,1}^{E^{SB}} = x_{4,1}^{E^{SB}} \gtrless x_{2,0}^{E^{SB}} > x_{1,1}^{E^{SB}} = x_{2,1}^{E^{SB}} \quad (23)$$

where E^{SB} stands for optimal Egalitarian Second-Best level.

Let us show how the optimal trade-offs for labour income and ART investments are modified at the second-best optimum. For individuals with high fecundity (types $1, 3$), we obtain (see the Appendix):[21]

$$y_3^{E^{SB}} = W^2 \tag{24}$$

$$y_1^{E^{SB}} = w^2 \left[\frac{\gamma_4 - \lambda \frac{u'(\tilde{x}_{1,1})}{u'(x_{1,1})}}{\gamma_4 - \lambda \frac{w^2}{W^2} \frac{u'(\tilde{x}_{1,1})}{u'(x_{1,1})}} \right] < w^2 \tag{25}$$

where λ and γ_4 are the Lagrange multipliers associated with the incentive constraint (21) and the egalitarian constraint, eq. (19), and $\tilde{x}_{1,1} = c_{1,1} - (y_1/W)^2/2$ is the net consumption of a type-3 agent pretending to be a type 1. These trade-offs are similar to those obtained at the second-best utilitarian optimum (see eq. 6 and 7). Among individuals with high fecundity (types 1 and 3), the labour income of high-productivity individuals should not be distorted, while that of low-productivity individuals should be distorted downward so as to avoid mimicking from high-productivity ones. This also implies that $y_3^{E^{SB}} > y_1^{E^{SB}}$.

Let us now study the optimal trade-offs for individuals with low fecundity (types 2 and 4). We have:

$$\begin{aligned} y_2^{E^{SB}} &= w^2 \left[\frac{\Lambda u'(x_{2,0}) + u'(x_{2,1})\mu_3 - \kappa[u'(\tilde{x}_{2,1})\pi(\varepsilon_4, a_2) + (1-\pi(\varepsilon_4, a_2))u'(\tilde{x}_{2,0})]}{\Lambda u'(x_{2,0}) + u'(x_{2,1})\gamma_3 - \kappa \frac{w^2}{W^2}[u'(\tilde{x}_{2,1})\pi(\varepsilon_4, a_2) + (1-\pi(\varepsilon_4, a_2))u'(\tilde{x}_{2,0})]} \right] \\ &< w^2 \end{aligned} \tag{26}$$

The expression inside brackets being smaller than 1, it is optimal to distort downward the labour income of type-2 individuals, in order to avoid the mimicking from type-4 ones. We also obtain that the labour income of a type-4 individuals should be left undistorted with respect to the first-best, and such that $y_4^{E^{SB}} = W^2$.

Regarding the optimal trade-offs for the optimal investment in ART, we show (see the Appendix) that the investment in ART of a type-4 individual is not distorted with respect to the first-best egalitarian optimum (i.e. the trade-off is identical to eq. 13). To the opposite, the optimal level of ART of a type 2 individual should now be distorted downward in comparison to the egalitarian first-best optimum (see eq. 12) because of the presence of the incentive constraint:

$$\pi_a(\varepsilon_2, a_2)(c_{2,0} - c_{2,1}) = 1 - \frac{\lambda}{\mu n_2} \pi_a(\varepsilon_4, a_2)(u(\tilde{x}_{2,1}) - u(\tilde{x}_{2,0}) + \Omega) \tag{27}$$

with $u(\tilde{x}_{2,1}^{E^{SB}}) - u(\tilde{x}_{2,0}^{E^{SB}}) + \Omega > 0$. This result is quite different from what we had obtained at the utilitarian second-best optimum where the direction of the distortion was unclear and depended on the degree of complementarity between ART and fecundity (see explanations below eq. 10 in Section 4.2). Here, the incentive constraint (eq. 22) and the necessity to leave a rent to type-4 makes the solution unambiguous.

Let us finally study the tax-and transfer scheme implementing the second-best egalitarian optimum. We find the usual result of no distortion at the top for type-3 and type-4 individuals,

[21] For ease of notation, we drop the exponent E^{SB} on the RHS of this equality, but ART investment and consumption levels are measured at their second-best egalitarian level.

so that for them, marginal taxation of labour income is unnecessary. To the opposite, for incentive reasons, we find that the labour income of low-productivity individuals (type 1 and 2) should be distorted at the margin (see eq. 25 and 26) and hence, taxed at the margin. In the Appendix, we provide the precise formula for the labour income tax rates.

In addition, as mentioned above, for incentive compatibility reasons, only the ART investment of a type-2 individual should be distorted downward and hence, taxed at the margin. The expression for the tax on ART should then be set to:[22]

$$\sigma_2^{E^{SB}} = \frac{\lambda}{\mu n_2} \pi_a(\varepsilon_4, a_2^{E^{SB}})(u(\tilde{x}_{2,1}^{E^{SB}}) - u(\tilde{x}_{2,0}^{E^{SB}}) + \Omega) > 0, \tag{28}$$

Finally, in order to achieve redistribution and to obtain the second-best ranking of net consumptions (condition 23), the following lump-sum taxes and transfers need to be implemented:

$$\begin{array}{rcl} & & T_{3,1}^{E^{SB}} > T_{4,1}^{E^{SB}} > T_{4,0}^{E^{SB}} \\ & & T_{1,1}^{E^{SB}} \gtrless T_{2,1}^{E^{SB}} > T_{2,0}^{E^{SB}} \\ T_{3,1}^{E^{SB}} & > & T_{1,1}^{E^{SB}}, T_{4,1}^{E^{SB}} > T_{2,1}^{E^{SB}}, T_{4,0}^{E^{SB}} > T_{2,0}^{E^{SB}} \end{array}$$

As in the first-best, we find that, among high productivity individuals (first line), parents who did not use ART should be more taxed that those who used ART. Moreover, among individuals who used ART, those who have a child should be more taxed than those who were unlucky in having one. For low productivity individuals (second line), among those who resorted to ART, those who succeeded in having a child should also be more taxed. Nonetheless, it is not clear now whether they should be more or less taxed than individuals with high fecundity (as a result of the incentive problem and of the distortions on labour income so that $y_2^{E^{SB}} \gtrless y_1^{E^{SB}}$). Like in the previous section, we find that it is optimal to transfer resources from parents toward childless persons and to some extent from high fecundity toward low fecundity individuals, which is quite different from what is usually observed in reality. The last line ensures that some redistribution is still made from high-productivity toward low-productivity agents.

Our results are summarized in this last proposition.

Proposition 5 *The decentralization of the second-best egalitarian optimum requires that:*

- *the labour supply of individuals with low productivity is taxed at the margin,*
- *among individuals with low fecundity, the ART investment of type-2 agents is taxed,*
- *lump sum redistribution goes from high-productivity toward low-productivity individuals, from high-fecundity toward low-fecundity individuals and from individuals with children toward childless individuals.*

6 Conclusion

This paper studied the optimal design of ART investment policies when individuals differ in wage and in fecundity, under both the utilitarian and the ex post egalitarian criteria. We assume away any efficiency problem and concentrate exclusively on redistributive issues.

[22] Computations are detailed in the Appendix.

We first find that under both social welfare criteria, only lump-sum taxation is necessary under full information. Yet, while the utilitarian optimum redistributes resources only between individuals with unequal productivity and unequal fecundity, the ex-post egalitarian optimum redistributes resources also between parents (whether they used ART or not) and childless individuals. At the first-best egalitarian optimum, all ex-post utilities are equalized, that is between high and low productivity individuals, as well as between childless persons and parents who, for some of them, also had to bear ART costs. To the opposite, at the second-best egalitarian optimum and in order to satisfy the incentive constraints, ex-post utilities need to remain higher for high-earning individuals, but they can still be equalized across parents (whether they invested in ART or not) and childless individuals of a given income type. All in all, contrary to what is usually observed in reality, public policies decentralizing the ex-post egalitarian optimum foster transfers both toward individuals who resorted to ART and toward childless persons.

Second, we find that, in addition to the taxation of the labour income of low-productivity agents, the ART of the low-productivity-low-fecundity individuals should also be distorted under asymmetric information as a way to relax incentive constraints. Under utilitarianism, whether investment in ART should be taxed or subsidized at the margin depends on whether the reproductive capacity and the investment in ART are weak or strong complements. On the contrary, the decentralization of the ex-post egalitarian optimum unambiguously requires to tax the ART investment of the low-wage-low-fecundity individual.

Let us also finally mention some extensions to be investigated in the future. First, we plan to extend our model to include differences in preferences for children and, to include "childfree" persons, i.e., persons who do not want to have a child. The difficulty with this extension is that we have to deal with interpersonal welfare comparisons between persons with distinct preferences, which is far from trivial. In Leroux et al. (2022), this problem is studied, but assuming that all individuals have the same wage and fecundity, unlike in this paper. Second, we do not model here the time cost of ART, which is likely to reduce labour supply. Assuming otherwise, the opportunity cost of ART is higher for high-productivity individuals, potentially decreasing their investment in ART and thus, their probability of success. This would certainly affect the optimal non-linear taxation of ART and labour as well as optimal lump-sum redistribution. These extensions are on our research agenda.

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Appendix

6.1 On the relation between productivity and fecundity

In this appendix, we present a model that explains the relationship between education and the reproductive capacity, ε_i. The model used in the core of this paper is a reduced form of this more extensive model.[23]

For simplicity, we abstract here from individuals with high fecundity (types $1, 3$). We consider an economy populated with two types of individuals $i = \{2, 4\}$, who differ both in their learning ability, A_i and in natural fecundity η_i such that $A_4 > A_2$ and $\eta_4 \gtreqless \eta_2$. They live one period of unitary length.

As in Boadway et al. (1996), individuals' productivity depends on the number e of years they study as well as on their learning ability. We denote it by $w(e, A_i)$ with $w_e(e, A_i) > 0$, $w_{A_i}(e, A_i) > 0$ and $w(0, A_i) = w \, \forall A_i$. We also assume complementarity between the learning ability and the number of school years, such that $w_e(e, A_4) > w_e(e, A_2) \, \forall e$.

Individuals consider having children at the end of their schooling, that is at the start of their working career. Nonetheless, having a child is uncertain and the probability to have one, denoted by $p\,(e, a, \eta_i)$, depends on the investment in ART, a, the number of school years, e, and natural fecundity, η_i, with $p_e \leq 0, p_a \geq 0, p_{\eta_i} \geq 0$.

Individual preferences are additive and separable in consumption, leisure and in the expected net utility benefit from having a child. The individual's utility has the following form:

$$U_i(e, \ell, a) = v((w(e, A_i)\ell - h(\ell))\,(1 - e) - a) + p\,(e, a, \eta_i)\,\Omega$$

[23] The model developed in this Appendix is static, but allows to give a first idea of the relation between education, childbearing age and fertility. For a study of this relation using a dynamic overlapping generations model, see d'Albis et al (2018).

where ℓ is the labor supply, $h(\ell)$ the disutility of work (with $h'(\ell) \geq 0$, $h''(\ell) \geq 0$) and Ω is the net utility benefit obtained from having a child. The utility of consumption $v(c)$ is increasing and concave and, we assume quasi linearity in labour. In this specification, the individual devotes e years to his education and works for the remaining time, $(1-e)$ years.

Maximization of the above utility function with respect to e yields the following FOCs:

$$\frac{\partial U_i}{\partial e} = v'(c)\left[w_e(e, A_i)\ell - (w(e, A_i)\ell - h(\ell))\right] + p_e\left(e, a, \eta_i\right)\Omega \leq 0.$$

Without loss of generality, we assume that type-2 agents, with a low learning ability A_2, always choose not to invest in education so that $e_2 = 0$ and end up with productivity $w_2 = w(0, A_2) = w$.[24] To the opposite, the solution is interior for agents with learning ability A_4 and we denote by $e_4 = \bar{e}$, the level of education chosen. In turn, they have productivity $w_4 = w(\bar{e}, A_4) = W > w$.

We can further define an indicator of the individual's reproductive capacity ε_i that depends on e and η_i, such that

$$\varepsilon_i = \eta_i f(e)$$

with $f_e \leq 0$. If $\eta_2 \geq \eta_4$, we unambiguously obtain that $\varepsilon_2 > \varepsilon_4$. To the opposite, if $\eta_4 > \eta_2$, we obtain that $\varepsilon_2 > \varepsilon_4$ if at the time of the childbearing decision, the effect of schooling years dominates the effect of natural fecundity. Hence, we are back to our original model (see Section 2) where we assumed two types of individuals, $i = \{2, 4\}$, characterized by different wages and reproductive capacity such that $w_4 = W > w = w_2$ and $\varepsilon_2 > \varepsilon_4$.

We can also rewrite the probability of success of having a child as depending exclusively on the investment in ART and on the reproductive capacity:

$$\pi(\varepsilon_i, a_i) = \pi(\eta_i f(e), a_i) \equiv p(e, a_i, \eta_i)$$

with $\pi_a \geq 0$, $\pi_\varepsilon \geq 0$, $\pi_{a,a} \leq 0$, $\pi_{\varepsilon,\varepsilon} \leq 0$. It is also possible to find specific forms of $u(.)$ $v(.)$ and $h(.)$ so that the utility function presented above exactly corresponds to the one derived in Section 2 (see equation 2) and such that:

$$\begin{aligned}
U_2(0, \ell, a) &= v(w(0, A_2)\ell - h(\ell) - a) - p(0, a, \eta_2)\Omega \\
&= u(w\ell - \frac{\ell^2}{2} - a) - \pi(\varepsilon_2, a)\Omega \\
U_4(e_4, \ell, a) &= v((w(e, A_4)\ell - h(\ell))(1 - e_4) - a) + p\left(e_4, a, \eta_i\right)\Omega \\
&= u(W\ell - \frac{\ell^2}{2} - a) - \pi(\varepsilon_4, a)\Omega.
\end{aligned}$$

[24] The above FOC with respect to e for the agent with learning ability A_2 is always negative for any positive level of education. This would effectively be the case if $w_e(0, A_2)$ or $p_e(0, a, \eta_2)$ are small.

6.2 The utilitarian second-best problem

The second-best utilitarian problem consists in solving

$$\max_{c_i,a_i,y_i} n_i \left[u\left(c_i - \frac{(y_i/w_i)^2}{2} \right) + \pi(\varepsilon_i, a_i)\Omega \right]$$

$$\text{s. to } \sum_i n_i y_i = \sum_i n_i c_i + n_2 a_2 + n_4 a_4$$

$$\text{s. to } u\left(c_3 - \frac{(y_3/W)^2}{2} \right) \geq u\left(c_1 - \frac{(y_1/W)^2}{2} \right)$$

$$\text{s. to } u\left(c_4 - \frac{(y_4/W)^2}{2} \right) + \pi(\varepsilon_4, a_4)\Omega \geq u\left(c_2 - \frac{(y_2/W)^2}{2} \right) + \pi(\varepsilon_4, a_2)\Omega$$

Denoting the Lagrange multipliers associated with the self-selection and the resource constraints by λ, κ, and μ respectively, the FOCs can be rearranged as follows:

$$u'(x_1)\left[n_1 - \lambda \frac{u'(\tilde{x}_1)}{u'(x_1)} \right] - \mu n_1 = 0 \quad (29)$$

$$u'(x_2)\left[n_2 - \kappa \frac{u'(\tilde{x}_2)}{u'(x_2)} \right] - \mu n_2 = 0 \quad (30)$$

$$u'(x_3)\left[n_3 + \lambda \right] - \mu n_3 = 0 \quad (31)$$

$$u'(x_4)\left[n_4 + \kappa \right] - \mu n_4 = 0 \quad (32)$$

$$\Omega\pi_a(\varepsilon_2, a_2)\left[n_2 - \kappa \frac{\Omega\pi_a(\varepsilon_4, a_2)}{\Omega\pi_a(\varepsilon_2, a_2)} \right] - \mu n_2 = 0 \quad (33)$$

$$\Omega\pi_a(\varepsilon_4, a_4)\left[n_4 + \kappa \right] - \mu n_4 = 0 \quad (34)$$

$$-\frac{y_1}{w^2} u'(x_1)\left[n_1 - \lambda \frac{u'(\tilde{x}_1)}{u'(x_1)} \frac{w^2}{W^2} \right] + \mu n_1 = 0 \quad (35)$$

$$-\frac{y_2}{w^2} u'(x_2)\left[n_2 - \kappa \frac{u'(\tilde{x}_2)}{u'(x_2)} \frac{w^2}{W^2} \right] + \mu n_2 = 0 \quad (36)$$

$$-\frac{y_3}{W^2} u'(x_3)(n_3 + \lambda) + \mu n_3 = 0 \quad (37)$$

$$-\frac{y_4}{W^2} u'(x_4)[n_4 + \lambda] + \mu n_4 = 0 \quad (38)$$

with $\tilde{x}_1 = c_1 - \frac{(y_1/W)^2}{2}$ and $\tilde{x}_2 = c_2 - \frac{(y_2/W^2)}{2}$ are the net consumptions of a type-3 individual pretending to be a type-1 individual, and of a type-4 individual pretending to be of type 2.

Rearranging these equations, we obtain the second-best trade-offs, eq. 6 to 10.

6.3 The ex post egalitarian first-best problem

Denoting respectively $\mu, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$, the Lagrange multipliers associated to the resource and egalitarian constraints, the FOCs are:

$$\frac{\partial \mathcal{L}}{\partial c_{2,0}} = u'(x_{2,0})\Gamma - \mu n_2(1-\pi(\varepsilon_2, a_2)) = 0 \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial c_{2,1}} = u'(x_{2,1})\gamma_3 - \mu n_2 \pi(\varepsilon_2, a_2) = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial y_2} = -[\Gamma u'(x_{2,0}) + u'(x_{2,1})\gamma_3]\frac{y_2}{w_2^2} + \mu n_2 = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial a_2} = \mu n_2[\pi_a(\varepsilon_2, a_2)(c_{2,0} - c_{2,1}) - 1] = 0 \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial c_{4,0}} = u'(x_{4,0})\gamma_1 - \mu n_4(1-\pi(\varepsilon_4, a_4)) = 0 \tag{43}$$

$$\frac{\partial \mathcal{L}}{\partial c_{4,1}} = u'(x_{4,1})\gamma_2 - \mu n_4 \pi(\varepsilon_4, a_4) = 0 \tag{44}$$

$$\frac{\partial \mathcal{L}}{\partial y_4} = -[\gamma_1 u'(x_{4,0}) + \gamma_2 u'(x_{4,1})]\frac{y_2}{w_2^2} + \mu n_4 = 0 \tag{45}$$

$$\frac{\partial \mathcal{L}}{\partial a_4} = \mu n_4[\pi_a(\varepsilon_4, a_4)(c_{4,0} - c_{4,1}) - 1] = 0 \tag{46}$$

$$\frac{\partial \mathcal{L}}{\partial c_{1,1}} = \gamma_4 u'(x_{1,1}) - \mu n_1 = 0 \tag{47}$$

$$\frac{\partial \mathcal{L}}{\partial c_{3,1}} = \gamma_5 u'(x_{3,1}) - \mu n_3 = 0 \tag{48}$$

$$\frac{\partial \mathcal{L}}{\partial y_1} = -\gamma_4 u'(x_{1,1})\frac{y_1}{w_1^2} + \mu n_1 = 0 \tag{49}$$

$$\frac{\partial \mathcal{L}}{\partial y_3} = -\gamma_5 u'(x_{3,1})\frac{y_3}{w_3^2} + \mu n_3 = 0 \tag{50}$$

where $\Gamma = 1 - \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5$.

First, rearranging eq. (39)-(41), eq. (43)-(45) as well as eq. (47)-(50), we obtain that labour is not distorted for any agent i: $y_1^E = y_2^E = w^2$ and $y_3^E = y_4^E = W^2$.

We derive here the complete ranking of the individuals' gross consumptions. Condition (11) implies that $c_{i,0}^E > c_{i,1}^E \, \forall i$: the ex post egalitarian optimum involves extra consumption for childless individuals and thus, it ensures that, for a given type, no ex-post inequality remains from being lucky or not in having a child.

In order to equalize the ex-post utilities, i.e. $U_{2,1} = U_{4,1}$ and $U_{2,0} = U_{4,0}$, one needs to set $c_{2,1}^E < c_{4,1}^E$ and $c_{2,0}^E < c_{4,0}^E$. This is due to the fact that $y_2^E < y_4^E$, so that an individual with high productivity, therefore, has higher overall disutility from labour and thus, needs to be compensated for that disutility. In addition, we can equalize the ex-post utility of a type-4 individual who was unsuccessful in having children with that of a successful type 2, i.e. $U_{4,0} = U_{2,1}$. This can be done by setting $c_{4,0}^E > c_{2,1}^E$, which leads us to:

$$c_{1,1}^E = c_{2,1}^E < c_{3,1}^E = c_{4,1}^E \gtreqless c_{2,0}^E < c_{4,0}^E.$$

The indeterminacy between $c^E_{4,1}$ and $c^E_{2,0}$, results from the fact that, at the ex-post egalitarian optimum, one needs to ensure that

$$u\left(c^E_{4,1}-\frac{W^2}{2}\right)+\Omega=u\left(c^E_{2,0}-\frac{w^2}{2}\right).$$

On the one hand, a type-4 successful individual supplies more labour than a type-2; but, on the other hand, he was lucky enough to have a child, which was not the case for the unsuccessful type-2 individual. Whether $c^E_{4,1}\gtreqless c^E_{2,0}$ depends on the relative sizes of these two effects and thus, on the magnitude of Ω, on the form of $u(\cdot)$ as well as on the gap between w and W.

6.4 Decentralisation of the first-best egalitarian optimum

For individuals with high fecundity (types $i=1,3$), the decentralization of the egalitarian optimum involves no income taxation, since for them, at the egalitarian first-best optimum, $y_i=w_i^2\,\forall i=1,3$, like in the laissez-faire. Only lump sum taxation is required.

For individuals with low fecundity (types $i=2,4$), the individual's problem, including the tax-and-transfer scheme is written as follows:

$$\begin{array}{cl}\max\limits_{c_{i,j},a_i,y_i} & \pi(\varepsilon_i,a_i)\left[u\left(c_{i,1}-\frac{(y_i/w_i)^2}{2}\right)+\Omega\right]+(1-\pi(\varepsilon_i,a_i))u\left(c_{i,0}-\frac{(y_i/w_i)^2}{2}\right)\\ \text{s. to} & y_i(1-t_i)=\pi(\varepsilon_i,a_i)c_{i,1}+(1-\pi(k,\varepsilon_i))c_{i,0}+a_i(1+\sigma_i)\end{array}$$

where $c_{i,1}=c_i-T_{i,1}$ and $c_{i,0}=c_i-T_{i,0}$.[25] The FOCs with respect to c_i, a_i and y_i are written as follows,

$$\begin{aligned}\pi(\varepsilon_i,a_i)u'(x_{i,1})+(1-\pi(\varepsilon_i,a_i))u'(x_{i,0}) &= \mu\\ \pi_a(\varepsilon_i,a_i)[u(x_{i,1})+\Omega-u(x_{i,0})]-\mu[(1+\sigma_i)+\pi_a(\varepsilon_i,a_i)(c_{i,1}-c_{i,0})] &= 0 \quad (51)\\ -(\pi(\varepsilon_i,a_i)u'(x_{i,1})+(1-\pi(\varepsilon_i,a_i))u'(x_{i,0}))\frac{y_i}{w_i^2}+\mu(1-t_i) &= 0 \quad (52)\end{aligned}$$

where μ is the Lagrange multiplier associated to the budget constraint.

Since at the egalitarian optimum, no ex-post inequality remains, i.e. $u(x^E_{i,1})+\Omega=u(x^E_{i,0})\,\forall i$, condition (51) simplifies to

$$\pi_a(\varepsilon_i,a_i)(c_i^{s,E}-c_i^{f,E})=1+\sigma_i$$

Comparing this equation with the FOCs with respect to a_i of the first-best egalitarian problem, i.e. equations (42) and (46), we then obtain that no tax on ART investment is required, $\sigma_i^E=0\,\forall i$.

In the same way, equation (52) simplifies to $y_i=w_i^2(1-t_i)$. Hence, no labour taxation is required here to decentralize the egalitarian first-best optimum, since under that optimum, $y_i^E=w_i^2$ for all agents.

[25]Note that since all individual decisions are taken ex-ante, i.e. before uncertainty is realized, the agent i decides about a *unique* consumption level, c_i.

6.5 The second-best ex post egalitarian problem

The first-order conditions of the second-best egalitarian problem stated in Section 5.2 are:

$$\frac{\partial \mathcal{L}}{\partial c_{2,0}} = u'(x_{2,0})\Lambda - \kappa(1-\pi(\varepsilon_4,a_2))u'(\tilde{x}_{2,0}) - \mu n_2(1-\pi(\varepsilon_2,a_2)) = 0 \quad (53)$$

$$\frac{\partial \mathcal{L}}{\partial c_{2,1}} = u'(x_{2,1})\gamma_3 - \kappa u'(\tilde{x}_{2,1})\pi(\varepsilon_4,a_2) - \mu n_2\pi(\varepsilon_2,a_2) = 0 \quad (54)$$

$$\frac{\partial \mathcal{L}}{\partial c_{1,1}} = u'(x_{1,1})\gamma_4 - \lambda u'(\tilde{x}_{1,1}) - \mu n_1 = 0 \quad (55)$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial y_2} &= -\frac{y_2}{w^2}[\Lambda u'(x_{2,0}) + u'(x_{2,1})\gamma_3] + \mu n_2 \\ &+ \kappa\frac{y_1}{W^2}[u'(\tilde{x}_{2,1})\pi(\varepsilon_4,a_2) + (1-\pi(\varepsilon_4,a_2))u'(\tilde{x}_{2,0})] = 0 \quad (56)\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial y_1} = -\frac{y_1}{w^2}\gamma_4 u'(x_{1,1}) + \mu n_1 + \lambda\frac{y_1}{W^2}u'(\tilde{x}_{1,1}) = 0 \quad (57)$$

$$\frac{\partial \mathcal{L}}{\partial a_2} = \mu n_2[\pi_a(\varepsilon_2,a_2)(c_{2,0}-c_{2,1}) - 1] - \lambda\pi_a(\varepsilon_4,a_2)(u(\tilde{x}_{2,1}) - u(\tilde{x}_{2,0}) + \Omega) = 0 \quad (58)$$

$$\frac{\partial \mathcal{L}}{\partial c_{4,0}} = u'(x_{4,0})[\gamma_1 + \kappa(1-\pi(\varepsilon_4,a_4))] - \mu n_4(1-\pi(\varepsilon_4,a_4)) = 0 \quad (59)$$

$$\frac{\partial \mathcal{L}}{\partial c_{4,1}} = u'(x_{4,1})[\gamma_2 + \kappa\pi(\varepsilon_4,a_4)] - \mu n_4\pi(\varepsilon_4,a_4) = 0 \quad (60)$$

$$\frac{\partial \mathcal{L}}{\partial c_{3,1}} = u'(x_{3,1})[\gamma_5 + \lambda] - \mu n_3 = 0 \quad (61)$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial y_4} &= -\frac{y_4}{W^2}[(\gamma_1 + \kappa(1-\pi(\varepsilon_4,a_4)))u'(x_{4,0}) + (\gamma_2 + \kappa\pi(\varepsilon_4,a_4))u'(x_{4,1})] \\ &+ \mu n_4 = 0 \quad (62)\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial y_3} = -\frac{y_3}{W^2}u'(x_{3,1})(\lambda + \gamma_5) + n_3\mu = 0 \quad (63)$$

$$\frac{\partial \mathcal{L}}{\partial a_4} = \mu n_4[\pi_a(\varepsilon_4,a_4)(c_{4,0}-c_{4,1}) - 1] + \kappa\pi_a(\varepsilon_4,a_4)[u(x_{4,1}) - u(x_{4,0}) + \Omega] = 0 \quad (64)$$

where μ, γ_1, γ_2, γ_3, γ_4, γ_5, λ and κ are the Lagrange multipliers associated to the resource constraint, the egalitarian constraints, and the incentive constraints respectively, and $\Lambda = 1 - \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5$.

Using eq. (55) and (57), we obtain the expression for $y_1^{E^{SB}}$. Using (61) and (63), we obtain the expression for $y_3^{E^{SB}}$. Using (53) and (54) and (56), we obtain the expression for $y_2^{E^{SB}}$. Using (59) and (60) and (62), we obtain the expression for $y_4^{E^{SB}}$.

Looking at eq. (64) for the optimal level of ART of a type-4 individual, one can see that the last term is null, since ex post, $U_{4,1} = U_{4,0}$. The FOC for $a_4^{E^{SB}}$ is then identical to the first-best one (eq. 46).

In the same way, looking at equation (58), we derive the optimal second-best trade-off for $a_2^{E^{SB}}$, eq. (27).

Let us then study the second-best ranking of gross consumptions. Using condition (23), we can show that $c_{3,1}^{E^{SB}} = c_{4,1}^{E^{SB}} < c_{4,0}^{E^{SB}}$ since $y_4^{E^{SB}} = y_3^{E^{SB}} = W^2$. Also, since $y_3^{E^{SB}} > y_1^{E^{SB}}$ and together with the egalitarian and the incentive constraints, we obtain that $c_3^{E^{SB}} > c_1^{E^{SB}}$.

For the low-productivity individuals, $c_{2,1}^{E^{SB}} < c_{2,0}^{E^{SB}}$ since they both earn $y_2^{E^{SB}}$. Like in the egalitarian first-best, we find that, for a given productivity type, among individuals with low fecundity, those who could not have a child should obtain higher consumption than those who managed to have one. Yet, contrary to the egalitarian first-best, since, for a low productivity-type w, the size of the labour income distortions may be different between those with low or high fecundity, we have $y_2^{E^{SB}} \gtreqless y_1^{E^{SB}}$ and thus $c_{1,1}^{E^{SB}} \gtreqless c_{2,1}^{E^{SB}}$.[26] Also, since $y_2^{E^{SB}} < y_4^{E^{SB}}$, condition (23) implies that, for individuals with low fecundity, at the egalitarian second-best, $c_{2,1}^{E^{SB}} < c_{4,1}^{E^{SB}}$ and $c_{2,0}^{E^{SB}} < c_{4,0}^{E^{SB}}$, but $c_{2,0}^{E^{SB}} \gtreqless c_{4,1}^{E^{SB}}$.[27] All in all, except for $c_{1,1}^{E^{SB}} \gtreqless c_{2,1}^{E^{SB}}$, these rankings are close to the first-best ones (see eq. 6.3).

Decentralisation of the second-best egalitarian problem. Comparing equations (25) and (26) to equation (52) of the decentralization problem, we obtain the values of the labour incomes taxes decentralizing the second-best egalitarian problem:

$$t_1^{E^{SB}} = 1 - \left[\frac{\gamma_4 - \lambda \frac{u'(\tilde{x}_{1,1})}{u'(x_{1,1})}}{\gamma_4 - \lambda \frac{w^2}{W^2} \frac{u'(\tilde{x}_{1,1})}{u'(x_{1,1})}} \right] > 0$$

$$t_2^{E^{SB}} = 1 - \left[\frac{\Lambda u'(x_{2,0}) + u'(x_{2,1})\mu_3 - \kappa [u'(\tilde{x}_{2,1})\pi(\varepsilon_4, a_2) + (1 - \pi(\varepsilon_4, a_2))u'(\tilde{x}_{2,0})]}{\Lambda u'(x_{2,0}) + u'(x_{2,1})\gamma_3 - \kappa \frac{w^2}{W^2} [u'(\tilde{x}_{2,1})\pi(\varepsilon_4, a_2) + (1 - \pi(\varepsilon_4, a_2))u'(\tilde{x}_{2,0})]} \right] > 0$$

In order to obtain the precise formula for the taxation of ART investment of a type-2 individuals, we compare equation (58) for the optimal level of $a_2^{E^{SB}}$, with that from the decentralized problem, eq. (51). In order for these two equations to coincide, we set the tax on ART investment equal to (28). No tax is needed for $a_4^{E^{SB}}$ as shown above.

[26] Indeed, it is not possible to show analytically whether the expression inside bracket is higher or smaller in equation (25) than in equation (26).

[27] Like in the egalitarian first-best optimum, it is not possible to rank $c_{2,0}^{E^{SB}}$ and $c_{4,1}^{E^{SB}}$, since on the one hand, a type-4 agent should receive more consumption than a type 2 (to compensate for higher labour supply) but on the other hand, the type-4 here was lucky enough to have child.