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Alessandra Casarico, Elena Del Rey, Jose I. Silva

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Abstract

We develop a model to study the impact on gender gaps in participation and wages of a liquidity constraint that prevents some households from paying child care. We show that this liquidity constraint generates an inefficiency and amplifies gender gaps in the labour market. In this framework, an extension of paid maternity leave duration has ambiguous effects on gender inequality. In contrast, child care subsidies, which require higher taxes, and loans, which do not, unambiguously reduce gender inequality. We illustrate the mechanisms at play in a numerical example using Spanish data.

JEL-Codes: J160, J180, J130.

Keywords: liquidity constraints, gender wage and participation gaps, statistical discrimination, numerical example.

Alessandra Casarico
Bocconi University
Milan / Italy
alessandra.casarico@unibocconi.it
https://orcid.org/0000-0003-4629-3964

Elena Del Rey
University of Girona / Spain
elena.delrey@udg.edu
https://orcid.org/0000-0001-7644-4490

Jose I. Silva
University of Girona / Spain
jose.silva@udg.edu
https://orcid.org/0000-0002-0425-6199

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1 Introduction

Despite progress, gender gaps in the labour market are still wide. The average gender participation gap in EU countries is around 10 percentage points and the unadjusted gender wage gap is around 15 percent, with large variation across countries.

According to Bertrand (2020), maternity remains a key source of gender inequality in the labour market. There is increasing evidence that, while parenthood is almost a non-event in fathers' labour market outcomes, mothers reduce labour force participation and the number of hours worked; they experience a reduction in hourly wages and in overall earnings. These costs persist throughout women's life rather than being short-term, and are common to many countries irrespective of differences in family policies (Kleven et al. 2019b). Cortes and Pan (2020) discuss factors, both at home and at work, that may contribute to amplify the career-family trade-offs that women face, including increasing returns for inflexible work, and higher parental time demand. Maternity, paternity and parental leaves, monetary and in-kind transfers, tax rebates, child care and early education aim to help parents – and mothers in particular – to combine work and family responsibilities, by reducing the monetary or time cost of raising children, and by offering job-protected time out of the labour market.[1]

Child care costs are certainly critical in parents' labour supply decision, and the source of gender inequality we focus on in this paper. Buying child care in the market allows mothers to work and accumulate experience, granting households a higher lifetime income. However, child care costs are concentrated over a limited time span in a parent career, and they need to be paid before the full benefits of using child care in terms of higher income materialise. This can generate – in the impossibility to borrow from future earnings – a liquidity constraint for some households with children. A binding liquidity constraint at the household level can induce one of the two spouses – in our case mothers – to quit their job after childbirth even when household lifetime income (net of child care costs) is higher when both spouses work. This is clearly inefficient. In addition, when mothers quit, they impose adjustment costs to firms employing them, thus affecting the gender gap in participation and wages during the childbearing period. Since women who quit accumulate less experience, their future wages will be lower, further affecting gender inequality in wages during old age.

In this paper we develop a model to study the impact on gender gaps in participation and wages of liquidity constraints related to the payment of child care costs. In this framework, we study alternative policies and discuss their impact on gender inequality. In particular, we consider an extension in the duration of the paid maternity leave, a proportional subsidy on child care expenses for dual earner households, and a loan. Finally, we propose a numerical example on Spanish data

[1] A vast empirical literature investigates the effects of these policies on maternal labour supply and health, on fertility and time allocation decisions, on children's human capital and health, with a view on the overall impact in terms of reduction in gender gaps in the labour market and in household production. See Olivetti and Petrongolo (2017) and Rossin-Slater (2018) for exhaustive surveys. Österbacka and Räsänen (2022) provide evidence on the effects of child home care and private day care allowances on mothers' return to employment after childbirth in Finland.

to illustrate the mechanisms at work.

To the best of our knowledge, we are the first to isolate the role of liquidity constraints related to the purchase of child care on gender inequality.[2] Affordability of care is often mentioned in surveys as one of the reasons why women do not work or quit their jobs. According to the Eurostat Database, between 0.6 per cent (in Czechia) and 7.5 per cent (in Romania) of mothers do not work because child care is too expensive. In Spain, the share is at 6.4 per cent. Indeed, across European countries, female labour force participation is negatively correlated with the share of mothers not working because child care services are too expensive. In Italy, 46 per cent of mothers who quit their job in 2020 stated that they did so due to the difficulties of combining work and care (Ispettorato Nazionale del Lavoro, 2021), such as lacking care support (no places in child care; costs of care too high; absence of relatives who can support). Not all households face the same market child care costs and, clearly, not all are equally likely to be liquidity constrained, even at similar income levels. In fact, market child care costs can show considerable heterogeneity. Many households rely on friends and relatives, and, therefore, face zero market child care costs. Others only need a few hours of babysitting. Among households needing to place their children in a nursery, some may live near a public facility, while others have to use (more expensive) private institutions. Some have neither. In some households with multiple children, the older one can help take care of the younger, or children may be all taken care of at once. Finally, some households may require special care for one or more of their children. We rely on this heterogeneity to illustrate that some women may be willing but unable to work in otherwise identical households.

We set up a simple unitary model in which households are composed of one man and one woman of given identical productivity. In particular, we build on Bjerk and Han (2007), and develop a model, in which some households will have children. The market cost of child care is randomly distributed across households, to account for the aforementioned heterogeneity in needs for child care in a set-up with exogenous fertility. Differently from Bjerk and Han (2007), we assume that mothers have the right to a paid maternity leave, as it is the case in most developed countries: both the leave and the decision to quit in order to take care of the child generate an adjustment cost for the firm that hired them that is reflected in lower wages for women compared to men. In addition, we account for a second period of work, when all men and women work, wages depend on productivity and accumulated experience, and there is no cost related to child care.

More specifically, firms meet workers at the beginning of the first period of work, when a contract is signed. Households are formed immediately after and a share of them have children. Mothers are on paid leave for a fraction of the first period. At the end of the leave the household has to decide whether to buy child care in the market or take care of the children at home implying that one of the partners must quit working. As long as lifetime income when both partners work is higher than lifetime income when only one of them does, the household will prefer to buy child care in the market. However, child care costs need to be paid in the first period of work, and

[2] Guner et al. (2020) analyse the effects on household labor supply and welfare of different types of child care policies in a life-cycle model where households cannot borrow, but they do not deal with the effect of these policies on gender inequality.

there may be some households that cannot afford them based on their first period income. In the impossibility to borrow from future income, someone has to quit their job and give up on the accumulation of experience in order to take care of the children, even when not quitting would yield higher lifetime income net of child care costs. The presence of this liquidity constraint is a key ingredient of our analysis.

When offering a work contract, firms form beliefs on the probability that a household will have children, know that mothers will be on maternity leave, and form expectations on whether they will return to work after the leave period expires. Since firms penalise women for their expected periods on leave, they earn less than men for the same productivity and end up being the ones to quit when the household cannot afford to pay for child care costs in the first period of work. We show that the presence of liquidity constraints – besides generating an inefficiency – increases gender wage and participation gaps in equilibrium, compared to a situation in which households interested in buying child care for the woman not to quit can afford to do so. When analysing alternative policies, we find that changes in the duration of the maternity leave have an ambiguous effect on gender inequality and provide conditions for this influence to be positive (or negative).[3] In contrast, proportional child care subsidies and loans targeted to child care expenses unambiguously reduce gender inequality, but the former bear direct burden on public resources.

We calibrate the Spanish economy in 2018 and simulate changes to the duration of the paid maternity leave, as well as the introduction of a subsidy and a loan. We find that, for households in which workers earn average wages, a longer maternity leave increases labour force participation of young women by lowering child care costs. Since more women work when young, the average wage of old women increases, but that of young women decreases because a longer leave is more costly to firms. In contrast, neither subsidies nor loans impose costs on firms and, by increasing female labour force participation, they also raise young and old women's wages.

Our paper is related to contributions studying the role of statistical discrimination in generating gender gaps and how policy can address them (Bjerk and Han, 2007; Dolado et al, 2013; Lommerud and Vagstad, 2015). Liquidity constraints (and loans) do not feature in these set-ups, whereas they are the key issue here. The focus on leave policy is shared with Barigozzi et al. (2018), Bastani et al. (2019), and Del Rey et al. (2017), that all study the role of parental leaves in a theoretical framework. Barigozzi et al. (2018) focus on the endogenous formation of social norms and show that parental leave can reduce social welfare. Bastani et al. (2019) show that a mandatory parental leave can be part of the socially optimal policy when firms are not allowed to offer differentiated contracts due to anti-discrimination legislation. Del Rey et al. (2017) underline the role of the relative bargaining power of firms and workers in determining the effect of leave duration on unemployment and wages.

The rest of the paper is organised as follows. Section 2 presents the model and section 3 the equilibrium. Section 4 presents the effects of the policies. Then, section 5 presents the numerical example based on Spanish data. Section 6 concludes.

[3]Del Rey et al (2021) find an inverted U-shaped relationship between maternity leave duration and female participation using an unbalanced panel of 159 countries for the years 1994, 2004, and 2011.

2 The model

To analyse how household liquidity constraints influence gender gaps in participation and wages, and study the role of policy, we build on Bjerk and Han (2007). Starting from their basic framework, we add a paid maternity leave for women with children and a second period of work, when earnings depend on productivity and accumulated experience. In this setting, we allow for the presence of liquidity constraints for households with children. In the next sections we describe the building blocks of the model, starting from the behaviour of workers and firms, to then determine the equilibrium and its properties.

2.1 Workers

In period t, there is a continuum of young individuals of identical productivity x and gender $g = \{m, f\}$. The total measure of males [resp. females] is normalised to one. Young individuals coexist with an equal mass of children of type x and gender g, that make no economic decision, and an equal mass of old individuals of type x, gender g, and labour market experience ϵ. We neglect time indices because all periods are the same. Population growth rate is zero, as implied from above.

Individuals live for three periods during which they are children, young and old, respectively. From the perspective of individual lifetime, we use first and second period to refer to the periods in which agents are active in the labour market. They are young in the first period and old in the second period. If individuals work during the whole first period, they accumulate high experience h. If they work only during part of the first period, they accumulate intermediate experience i. If they do not work during the first period they accumulate nil experience n. Therefore, experience is $\epsilon = \{h, i, n\}$, with $h > i > n > 0$.

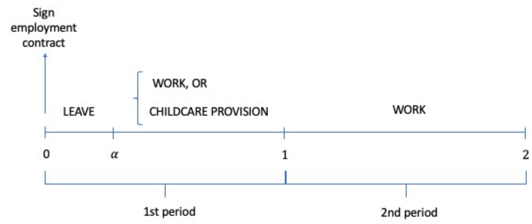
At the beginning of the first period, young individuals sign a work contract involving a wage $w_g(x)$. Immediately after signing the contract, households are formed by a woman and a man, with a proportion ρ of these households having children.[4] Mothers take a paid maternity leave of total length $0 < \alpha \leq 1$, after which they may return to work or remain at home for the rest of the first period. The length of the paid leave is set by the government and cannot be chosen by the households. The government finances the leave with a lump-sum tax τ levied on all workers, young and old. The paid leave, instead, is exempt from taxation by assumption.[5] The interest rate is zero.[6] If mothers stay at home, they will take care of the children. If the mother returns to work, the household has to buy child care in the market at cost $\eta > 0$, where η is a random variable with an increasing and continuous distribution function $F(0, \infty)$, with $F' = f > 0$. As discussed previously, the cost of buying child care η can take different values depending on the availability of relatives, of child care facilities, the number of hours of care, or special needs.

[4]To guarantee constant population, a proportion ρ of households will have $2/\rho$ children.

[5]This is a peculiarity of the Spanish paid maternity leave system, on which we focus in the numerical example and which we consider also in the theoretical part. The assumption does not affect the results.

[6]This assumption is discussed in footnote 8.

Figure 1: Mothers' timeline



Old individuals earn a wage that depends only on the observable productivity and experience $w^{\epsilon}(x)$, with $\epsilon = \{h, i, n\}$. There is no unemployment. Finally, there are no capital markets where households can borrow.

Figure 1 represents mothers' timeline. Men and women without children are assumed to work during the whole first and second periods.

2.2 Firms

There is a continuum of competitive, profit maximising firms, that offer wages $w_g(x)$ to young workers of type x and gender $g = \{m, f\}$, and wages $w^{\epsilon}(x)$ to old workers of type x, and experience $\epsilon = \{h, i, n\}$. The female partner takes a maternity leave, which imposes on the firm a cost $q(x) > 0$ during her absence. This cost can be interpreted in terms of adjustment, reallocation of tasks to cover for the missing worker, or administrative costs. If a worker quits her job during the first period, the firm incurs a cost $p(x) > 0$, which also reflects the presence of adjustment costs related to turnover. To simplify, we assume that $q(x) = p(x)$. When workers are young, they have no experience. When they are old, they no longer need to purchase child care on the market. Wages offered are those which set profits to zero. Profits made when hiring a young male worker of productivity x are

$$\pi_m(x) = x - w_m, \text{then } w_m = x \equiv w_m(x) \tag{1}$$

When hiring young women, firms know that they will have children with probability ρ, take a leave of duration α and return to work with probability λ. Then, *expected* profits when hiring a young woman of productivity x are

$$\pi_f(x) = (1-\rho)(x - w_f) + \rho\,(1-\alpha)\,\lambda(x - w_f) - \rho(1-(1-\alpha)\lambda)q(x) \tag{2}$$

where $q(x)$ is the cost imposed on the firm when a (female) worker of productivity x is absent, either because she is on maternity leave, or because she quits.

To better understand (2), Table 1 summarises the proportion of young female workers in different situations, the associated surplus and costs for the firm. The $(1-\rho)$ women who do not have children produce x and are paid w_f. Since they will work for the entire first period after

Table 1: Proportion of young female workers across different states, surplus and cost to the firm

proportion	surplus	cost
$(1-\rho)$	$x - w_f$	0
$\rho\lambda$	$(1-\alpha)(x - w_f)$	$\alpha q(x)$
$\rho(1-\lambda)$	0	$\alpha q(x) + (1-\alpha)q(x)$

having signed a contract, they impose no cost on the firm. Women who have children (ρ) take a leave of duration α, which costs the firm $\alpha q(x)$. Among those ρ who have children, firms expect a proportion λ to return to work after the leave and generate a surplus $(1-\alpha)(x-w_f)$. Finally, the firm expects $(1-\lambda)$ female workers with children not to return to work; this implies an additional cost $(1-\alpha)q(x)$. The last part of Eq. (2) captures the total expected costs that women impose on firms, given by the sum of $\rho\alpha q(x)$ during the maternity leave, and $\rho(1-\alpha)(1-\lambda)q(x)$, for women who quit.

Using (2), and setting profits equal to zero, we obtain the wage offered to young women of productivity x, given firm's beliefs λ:

$$w_f = x - \frac{\rho(1-(1-\alpha)\lambda)}{(1-\rho)+\rho(1-\alpha)\lambda} q(x) \equiv w_f(x,\alpha) \tag{3}$$

which implies $w_f(x,\alpha) < x$. Note that this wage is the smallest if all mothers are expected to leave and never work, i.e., $\lambda = 0$. Women are willing to sign a work contract before entering the stage of household formation as long as $w_f(x,\alpha) > 0$. We now impose a condition that guarantees that all young women are willing to sign a work contract before forming a household, even when offered the lowest possible wage $w_f(x,\alpha) = x - \frac{\rho}{1-\rho}q(x)$. This ensures that the female participation rate is positive.

Assumption 1

$$(1-\rho)x - \rho q(x) > 0 \tag{4}$$

Since there is no compulsory leave for men, $w_m(x) > w_f(x,\alpha)$, and all men are willing to sign a work contract too.

Finally, when hiring an old worker of productivity x, and experience ϵ, which are both observable characteristics, firms' profits are:

$$\pi^\epsilon = \epsilon x - w^\epsilon \text{ then } w^\epsilon = \epsilon x \equiv w^\epsilon(x) \tag{5}$$

2.3 Households' lifetime income

Members of households without children work both periods and their net lifetime income is

$$w_m(x) - \tau + w_f(x,\alpha) - \tau + w^h(x) - \tau + w^h(x) - \tau \tag{6}$$

where τ stands for the lump-sum tax paid by each worker in each period to finance the maternity leave.

In households with children, both adult members are active in the labour market at the beginning of the first period. Then, mothers take the paid leave, which is not subject to taxation, for a portion α of the period. If the mother goes back to work when the leave is over, the household has to buy market child care at price η during the period $1-\alpha$. In the second period children are grown up and they no longer impose a cost of care on the parents. Both members of the household continue working since they have more experience and hence higher wages. Fathers work the whole time when young and have experience $\epsilon = h$. Also mothers work in the first period but are on leave a fraction α of it, hence, they accumulate less experience ($\epsilon = i$). Thus, net lifetime income of households where young mothers work is

$$w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - (1-\alpha)\eta + w^h(x) - \tau + w^i(x) - \tau \tag{7}$$

If the mother does not work when the leave is over, the household does not buy child care in the market and the woman accumulates no experience ($\epsilon = n$). Assumption 2 states that all wages are larger than taxes and implies, in particular, that old women always work irrespective of experience.[7]

Assumption 2 $w_g(x) - \tau > 0$ for $g = \{m, f\}$, $w^\epsilon(x) - \tau > 0$ for $\epsilon = \{h, i, n\}$

Net lifetime income of households with children where the mother does not work when young is:

$$w_m(x) - \tau + \alpha w_f(x,\alpha) + w^h(x) - \tau + w^n(x) - \tau \tag{8}$$

Note that children affect women's wages in two distinct ways. First, the compulsory leave α and the fact that some women quit their jobs to take care of children increase firms' expected costs and, hence, reduce wages for all young women, whether they are mothers or not, because of statistical discrimination. Second, there is a penalty children impose only on mothers, through a reduction in second period wages due to lower experience ($w^n < w^i < w^h$).

3 Equilibrium

The choice to return to work after the leave by mothers and the simultaneous setting of wages by firms, together with a balanced government budget constraint determine the equilibrium.

3.1 Decision to return to work after maternity leave

Households with children decide on whether the mother goes back to work after the leave by comparing household lifetime income when she does (and the household buys child care) and when she does not (and the mother stays at home to provide care). The return to work after the leave period affects mothers' experience and their wage when old.

Comparing household lifetime income if young mothers go back to work after the leave period and if they do not, i.e., (7) and (8), it will be optimal for the household that the mother goes back

[7] Accounting also for the case where inexperienced old women do not work complicates the exposition without adding insight.

to work if:

$$(1-\alpha)\left(w_f(x,\alpha)-\tau-\eta\right)+w^i(x)-w^n(x)>0 \tag{9}$$

This condition allows to identify a threshold level of child care costs η^*, below which households are better off if women returning to work after the leave period, for given wages:

$$\eta<w_f(x,\alpha)-\tau+\frac{w^i(x)-w^n(x)}{(1-\alpha)}=\eta^*\left(x,\alpha\right). \tag{10}$$

If (10) is satisfied, households want to buy child care on the market. Otherwise, it is better if the mother stays at home. By Assumption 2, $\eta^*\left(x,\alpha\right)$ defined by (10) is positive. At equilibrium, the number of mothers who wish to return to work at the end of the paid leave for given wages is $F(\eta^*\left(x,\alpha\right))$.

Note that η^* is also the threshold of child care costs below which it is efficient for women to return to work after the leave, given taxes and paid leave duration. If child care costs are larger than η^*, the additional income earned by mothers by going back to work and accumulating more experience is smaller than the costs incurred.

So far, we have not considered whether households have enough income when young to pay for child care costs. All that mattered was lifetime income as if there were perfect credit markets where households could borrow. When households cannot use their future earnings as collateral for a loan, for the mother to go back to work at the end of the paid leave it must hold that two earner households can afford to buy child care in the first period.

To consider the question of affordability, let c denote unavoidable household consumption (food, housing, etc). The following assumption guarantees that households can always pay for minimum consumption in the first period, even if mothers do not work.

Assumption 3 $w_m-\tau+\alpha w_f(x,\alpha)>c$.

This assumption also guarantees that households can pay for minimum consumption in the first period when the mother goes back to work at the end of the leave.[8] However, for them to be able to pay for child care costs we further need that:

$$w_m(x)-\tau+w_f(x,\alpha)-(1-\alpha)\tau-c\geqslant(1-\alpha)\eta \tag{11}$$

We can now identify a threshold η^c, above which households *cannot afford* the purchase of child care, i.e., (11) is *not* satisfied:

$$\eta>\frac{w_m(x)-\tau+w_f(x,\alpha)-(1-\alpha)\tau-c}{1-\alpha}=\eta^c\left(x,\alpha\right) \tag{12}$$

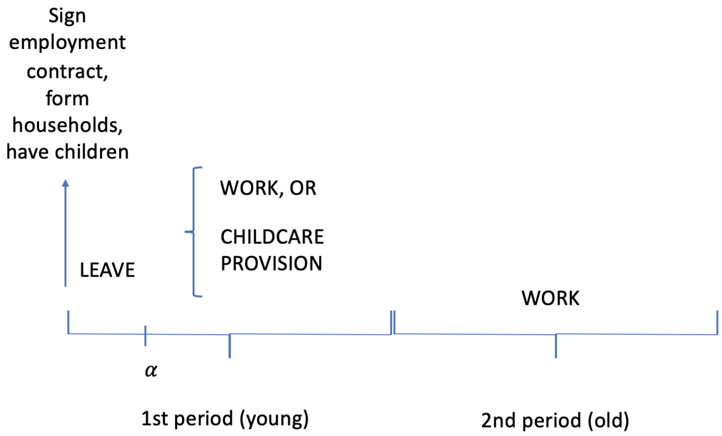
where $\eta^c>0$. If $\eta^c\left(x,\alpha\right)\geq\eta^*\left(x,\alpha\right)$, all those households willing to buy child care are able to. If $\eta^c\left(x,\alpha\right)<\eta^*\left(x,\alpha\right)$, or, by (10) and (12):

$$w_m(x)-\tau+\alpha w_f(x,\alpha)-c<w^i(x)-w^n(x) \tag{13}$$

some households will be liquidity constrained, i.e., unable to buy child care and let the mother return to work, in spite of this choice generating more net lifetime income. That is, in spite of

[8]In households where mothers work, earnings in the first period is $w_m-\tau+w_f(x,\alpha)-(1-\alpha)\tau$. Since all wages exceed taxes by Assumption 2, households where mothers work can also afford minimum consumption c.

Figure 2: Thresholds and types of households with children



this being the efficient choice. In this case, the equilibrium number of mothers that return to work is $F(\eta^c(x,\alpha))$. In households with $\eta \in (\eta^c(x,\alpha), \eta^*(x,\alpha))$ women would like to go back to work after the leave but cannot afford to do so. Hence, the number of liquidity constrained households is $F(\eta^*(x,\alpha)) - F(\eta^c(x,\alpha))$. Note that the number of liquidity constrained households depends both on how many households find it optimal to buy child care in the market and how many of them are able to pay for it. Eliminating this liquidity constraint is efficient because it increases aggregate income. As we will see below, it also reduces gender inequality. Figure 2 represents the relevant thresholds and preferences over/affordability of market child care.

3.2 Participation and wages

With perfect competition (firm's zero profits at equilibrium), young and old male and old female workers' wages coincide with their respective marginal productivities, because the firm observes gender and experience (see equations (1) and (5)). The wage paid to a young woman is given by (3). In equilibrium, firm's beliefs on how many mothers will return to work at the end of the paid leave coincide exactly with how many do, i.e.,

$$\lambda = F(\hat{\eta}) \tag{14}$$

where

$$F(\hat{\eta}) = \min\{F(\eta^*(x,\alpha)), F(\eta^c(x,\alpha))\}. \tag{15}$$

Then, the equilibrium wage of a young woman is:

$$w_f(x,\alpha) = x - \rho\frac{1-(1-\alpha)F(\hat{\eta})}{1-\rho+\rho(1-\alpha)F(\hat{\eta})}q(x) \tag{16}$$

For the existence of the equilibrium, it is *necessary and sufficient* that,

$$\frac{\rho(1-\alpha)f(\eta^*(x,\alpha))q(x)}{(1-\rho+\rho(1-\alpha)F(\eta^*(x,\alpha)))^2} < 1 \tag{17}$$

if all households willing to buy child care can afford to do so, i.e., $\hat{\eta} = \eta^*(x, \alpha)$, and

$$\frac{\rho f\left(\eta^c(x,\alpha)\right) q(x)}{\left(1-\rho+\rho\left(1-\alpha\right) F\left(\eta^c(x,\alpha)\right)\right)^2} < 1 \tag{18}$$

if some households are constrained. Appendix 1 provides the formal proof.

We conclude this section with a proposition characterising gender inequality at equilibrium. To this end, first, we compute labour force participation of young men and women. All men participate for the entire first period, that is, $MLF = 1$. Women participate the whole first period if they have no children or, if they have children and return to work at the end of the leave, since women on maternity leave are also part of the labour force. Labour force participation of young women at equilibrium is, hence:

$$FLF = (1-\rho) + \rho\alpha + \rho(1-\alpha)F(\hat{\eta}) \tag{19}$$

Second, we calculate the gender gap in wages of old workers who, by assumption, always work. Note that wages of old workers depend on accumulated experience and this is different on average for men and women. In particular, we denote the average wage of old workers of productivity x and gender $g = \{m, f\}$, $\bar{\omega}_g(x)$. It holds that $\bar{\omega}_m(x) = w^h(x)$, since all men have high experience. The average wage of old women writes:

$$\bar{\omega}_f(x) = (1-\rho)w^h(x) + \rho F(\hat{\eta})w^i(x) + \rho(1-F(\hat{\eta}))w^n(x) \tag{20}$$

The first term on the right hand side captures wages of old women without children, who all have high experience. The term $\rho F(\hat{\eta})$ refers to women who have children, return to work after the leave, and earn $w^i(x)$ in the second period of work; $\rho(1-F(\hat{\eta}))$ are women who have children and go back to work only when the children are grown-ups and earn $w^n(x)$ in the second period of work.

Proposition 1 *The equilibrium exhibits gender gaps in labour force participation and wages. In particular,*

1. *The ratio of male to female labour force participation for young workers is:*

$$\frac{1}{(1-\rho)+\rho\alpha+\rho(1-\alpha)F\left(\hat{\eta}\right)} > 1 \tag{21}$$

2. *The ratio of male to female wages for young workers is:*

$$\frac{w_m(x)}{w_f(x,\alpha)} = \frac{x}{x-\rho\frac{1-(1-\alpha)F(\hat{\eta})}{1-\rho+\rho(1-\alpha)F(\hat{\eta})}q(x)} > 1 \tag{22}$$

3. *There is no gender participation gaps among old workers by assumption.*

4. *The ratio of male to female average wages for old workers is:*

$$\frac{\bar{\omega}_m(x)}{\bar{\omega}_f(x)} = \frac{w^h(x)}{(1-\rho)w^h(x)+\rho F\left(\hat{\eta}\right)w^i(x)+\rho(1-F\left(\hat{\eta}\right))w^n(x)} > 1 \tag{23}$$

Proof. By construction, all men, women without children, and older women work. Participation of young mothers is given by (19). Wages are given by (1), (5) and (16). ■

We now investigate the effects of enabling women in constrained households to return to work. This amounts to raising the equilibrium threshold from $\eta^c(x)$ to $\eta^*(x)$. In the equilibrium in which some households cannot afford to pay child care costs, i.e., $\hat{\eta} = \eta^c(x)$, gender gaps in participation and wages result from a combination of statistical discrimination and liquidity constraints in young age, and lower accumulated experience by mothers, with negative repercussions on female wages in old age. Lifting the liquidity constraint, the labour force participation of young mothers increases and the ratio in (21) goes down. In addition, the wage of young women increases. Indeed, differentiating (16) with respect to η^c we get:

$$\frac{\partial w_f(x,\alpha)}{\partial \eta^c} = \frac{\rho\,(1-\alpha)\,f\,(\eta^c)\,q(x)}{\left(1-\rho+\rho\,(1-\alpha)\,F\,(\eta^c)\right)^2} > 0 \tag{24}$$

Then, the gender wage gap in (22) goes down. Finally, more women accumulate labour market experience and the average wage of old women increases. Hence, the gender wage gap in (23) is reduced. This allows us to write the following

Corollary 1 *Enabling women in constrained households to return to work when young increases efficiency and reduces gender gaps in participation and wages:*

$$\frac{1}{(1-\rho)+\rho\alpha+\rho(1-\alpha)F\,(\eta^*)} < \frac{1}{(1-\rho)+\rho\alpha+\rho(1-\alpha)F\,(\eta^c)},$$

$$\left.\frac{w_m(x)}{w_f(x,\alpha)}\right|_{\eta=\eta^*} < \left.\frac{w_m(x)}{w_f(x,\alpha)}\right|_{\eta=\eta^c}$$

and

$$\left.\frac{\bar{\omega}_m(x)}{\bar{\omega}_f(x)}\right|_{\eta=\eta^*} < \left.\frac{\bar{\omega}_m(x)}{\bar{\omega}_f(x)}\right|_{\eta=\eta^c}.$$

In Appendix 2, we study how gaps in participation and wages change with individual productivity x through a comparative statics exercise.

3.3 Balanced government budget constraint

The government funds the benefits accruing to mothers on leave by levying a lump-sum tax τ on all workers. Letting $F(\hat{\eta})$ denote the number of households with child care costs smaller than $\hat{\eta}$, where $\hat{\eta}$ is the child care cost born by the last household where the mother goes back to work at equilibrium, the government budget constraint reads:

$$\rho\alpha w_f(x,\alpha) = \left(3+(1-\rho)+(1-\alpha)\rho F(\hat{\eta})\right)\tau \tag{25}$$

4 Policy

In this section we explore the effect of alternative family policies on gender inequality when some households are constrained at equilibrium. We first discuss the effects of increasing the duration of the maternity leave. Then, we explore the role of child care subsidies to dual earner households. Finally, we consider a government loan. To study this instrument, we assume that, unlike households, the government can borrow in international markets to obtain the funds required to cover part of the child care expenses. We also assume that the government has the power to seize incomes directly, in case households do not repay the loan.

4.1 Extending the duration of paid maternity leave

We first consider the impact of changes in the duration of the paid maternity leave on gender gaps when some households are liquidity constrained. In principle, longer periods of paid maternity leave reduce the market cost of child care, because households in which mothers return to work will have to pay it for a shorter period of time. However, they also affect wages directly, because longer leave periods are more costly to firms. This has repercussions on participation, which may feed back into wages. We state the following proposition.

Proposition 2 *If some households are liquidity constrained, increasing the duration of the paid maternity leave α has the following effects on labour market outcomes:*

a) *More young women of productivity x return to work after maternity leave and their wages increase iff*

$$\frac{w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c}{(1-\alpha)} > \frac{F(\eta^c(x,\alpha))}{f\left(\eta^c(x,\alpha)\right)} \tag{26}$$

Then, the participation of young women in the labour market increases. As a result, gender gaps in participation and wages for young workers decrease, and so does the gender wage gap of old workers.

b) *More young women of productivity x return to work after maternity leave and their wages decrease iff*

$$\frac{F(\eta^c(x,\alpha))}{f\left(\eta^c(x,\alpha)\right)} > \frac{w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c}{(1-\alpha)} > \frac{\rho F(\eta^c(x,\alpha))q(x)}{\left(1-\rho+\rho\left(1-\alpha\right)F(\eta^c(x,\alpha))\right)^2} \tag{27}$$

Then, the participation of young women in the labour market increases, and the gender gap in participation for young workers and in wages for old workers decrease, whereas that in wages of young workers increases.

c) *Fewer young women of productivity x return to work after maternity leave and their wages decrease iff*

$$\frac{F(\eta^c(x,\alpha))}{f\left(\eta^c(x,\alpha)\right)} > \frac{\rho F(\eta^c(x,\alpha))q(x)}{\left(1-\rho+\rho\left(1-\alpha\right)F(\eta^c(x,\alpha))\right)^2} > \frac{w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c}{(1-\alpha)} \tag{28}$$

Then, the effect on the participation of young women in the labour market is ambiguous and so is the effect on the gender gap in participation for young workers. The gender wage gap increases both for young and old workers.

Proof. See Appendix 2.

■

The intuition of this proposition is as follows. Increasing the duration of the maternity leave has two different effects on the number of women returning to work. First, longer duration reduces child care costs and incentivises women to go back to work. Second, wages can increase or decrease, with a further impact on the number of women who return to work.

In fact, with a longer leave, mothers are less likely to quit –which reduces costs for firms– but are also absent from work for a longer period, which increases costs for firms. Depending on which of the two effects dominates, wages can increase or decrease. If wages increase, the incentive to go back to work is stronger. This is case a in the Proposition. If wages decrease, this weakens the incentives to return to work. In case b in the Proposition, more women return to work in spite of the decrease in wages. In case c, the negative effect on wages dominates the reduction in child care costs and fewer women go back to work after the leave. This reinforces the negative effect on wages further.

With respect to the female labour force participation, given by (19), a longer duration of the leave keeps women attached to the labour force for longer, and can increase or decrease the number of mothers going back to work after the leave. If more women return to work after the leave, participation increases (cases a and b). Otherwise, the effect of a longer leave on female labour force participation is ambiguous. Finally, the effect on the average wage of old women hinges on the proportion of women returning to work after child birth. Hence, the average wage of old women increases in case a and b in the Proposition and decreases in case c when the duration of the leave is extended.

To conclude this analysis, note that funding a longer maternity leave will require adjusting the government budget constraint. We can show that increasing taxes has a negative effect on gender inequality (see Appendix 2). In particular, it holds that

$$sign\frac{dw_f}{d\tau} = sign\left(-\frac{\rho\,(2-\alpha)\,f\,(\eta^c)\,q(x)}{\left(1-\rho+\rho\,(1-\alpha)\,F(\eta^c)\right)^2}\right) < 0 \tag{29}$$

$$sign\frac{d\eta^c}{d\tau} = sign\left(-\frac{1+(1-\alpha)}{1-\alpha}\right) < 0 \tag{30}$$

Hence, increasing taxes limits the positive effects of extending paid leave duration in case a, and exacerbates the negative effects in cases b and c.

4.2 Child care subsidies to dual earner households

The government could subsidise households with child care costs $\eta \in (\eta^c, \eta^*)$, that is, households for which it is optimal that the mother returns to work, but cannot afford it. However, since η

is not observed, the government does not know the child care needs of one particular family and, thus, cannot subsidise constrained households only. Under these circumstances, we assume that the government subsidises a proportion s of all child care bought in the market. The first period income of a constrained household would then become

$$w_m(x) - \tau + w_f(x, \alpha) - (1-\alpha)\tau - c - (1-s)(1-\alpha)\eta$$

Hence, the households that can now afford child care are those with

$$\eta < \frac{w_m(x) - \tau + w_f(x, \alpha) - (1-\alpha)\tau - c}{(1-s)(1-\alpha)} = \eta^s(x, \alpha, s) \tag{31}$$

Clearly, $\eta^s(x, s) > \eta^c(x, \alpha)$: more households can afford for the woman to work after child birth, given α. From *Corollary 1*, this reduces gender gaps in participation and wages. Subsidising child care, however, requires higher taxes. The government budget constraint becomes:

$$\rho\alpha w_f(x, \alpha) + s(1-\alpha)\int_0^{\eta^s} \eta dF(\eta) = (3 + (1-\rho) + (1-\alpha)\rho F(\eta^s))\,\tau \tag{32}$$

As before, taxes limit the positive effects of the subsidy since, with subsidies,

$$sign\frac{dw_f}{d\tau} = sign\left(-\frac{\rho\,(2-\alpha)\,f\,(\eta^s)\,q(x)}{(1-s)\,(1-\rho+\rho\,(1-\alpha)\,F(\eta^s))^2}\right) < 0 \tag{33}$$

$$sign\frac{d\eta^s}{d\tau} = sign\left(-\frac{1+(1-\alpha)}{(1-s)(1-\alpha)}\right) < 0 \tag{34}$$

The details of these calculations are available in Appendix 2.

Summing up, subsidising child care costs in two earner households can alleviate liquidity constraints and reduce gender inequality in the labour market, but the required taxes will hinder their effectiveness in doing so.

We now assume that the government can borrow in the international capital market to lend constrained households what they need to buy child care. Since the government will not aim to make a profit on this loan, we assume that it lends at the same rate at which it borrows. This justifies our assumption that interest rates are zero for simplicity.[9]

4.3 Loans

In this section we characterise a simple loan program run by the government to alleviate liquidity constraints and, by *Corollary 1*, reduce gender inequality. We show that only constrained households have incentives to apply for a loan that can only be used to pay for child care services. Our assumption is that the government can borrow in international markets, and that it can directly seize household income so that non-repayment is not an option.

[9]In particular, let r denote the cost of borrowing for the government. This is also both the interest households would obtain from lending (opportunity cost of waiting) and the interest households would pay for a government loan (since the government will not intend to make a profit). Then, with $R = 1 + r$, the present value of lifetime income of a household where the mother goes back to work after borrowing B and repays it in the second period is: $w_m(x) - \tau + w_f(x, \alpha) - (1-\alpha)\tau - (1-\alpha)\eta + B + \frac{w^h(x) - \tau + w^i(x) - \tau}{R} - \frac{RB}{R}$. Assuming $r = 0$ in this context is innocuous.

Proposition 3 *Let* $0 < \eta^c(x, \alpha) < \eta^*(x, \alpha)$*. If the government provides loans in the form of child care vouchers:*

a) *Households with* $\eta < \eta^c(x, \alpha)$ *do not borrow.*

b) *Households with* $\eta \in (\eta^c(x, \alpha), \eta^*(x, \alpha))$ *borrow, and repay, the amount*

$$B(x, \eta) = (1 - \alpha)(\eta - \eta^c(x, \alpha)) \tag{35}$$

c) *Households with* $\eta > \eta^*(x, \alpha)$ *do not borrow.*

Proof. First, note that over-borrowing and default are not interesting options for the households. On the one hand, no household has an interest in borrowing more than they need, since borrowing can only be used to pay for child care services and has to be paid back. This prevents over-borrowing. On the other hand, the government can seize an amount of income that could even be larger than the amount owed in case of non-repayment. This eliminates incentives for default. Let us now look at each type of households in turn:

a) For households with $\eta < \eta^c(x, \alpha)$, first period income is larger than child care costs, hence they do not need to borrow and borrowing would not lead to an increase in lifetime income.

b) For households with $\eta \in (\eta^c(x, \alpha), \eta^*(x, \alpha))$, first period income $w_m(x) - \tau + w_f(x, \alpha) - (1 - \alpha)\tau - c$ is lower than child care costs $(1 - \alpha)\eta$. They need to borrow that difference, which we can write $(1 - \alpha)(\eta - \eta^c(x, \alpha))$. If they borrow, women in these households will go back to work and the additional income earned will be larger than their loan repayment since $\eta < \eta^*(x, \alpha)$.

c) In households with $\eta > \eta^*(x, \alpha)$, since (9) is not satisfied, it holds that

$$w_f(x, \alpha) - (1 - \alpha)\tau - (1 - \alpha)\eta + w^i(x) < \alpha w_f(x, \alpha) + w^n(x)$$

i.e., households attain higher lifetime income if mothers do not go back to work after the leave. Hence, they are better off staying at home and providing care themselves, instead of buying child care to return to work.

■

Clearly, more complex environments (e.g. the inclusion of uncertainty, different attitudes towards risk, or asymmetric information) provide additional challenges to the design of a loan program. Chapman and Higgins (2009) were the first to propose household loans to help women with children to return to work. A very similar tool, that of student loans, has, however, been discussed for a long time. Like higher education investments, child care can be seen as an investment that improves women's future earning prospects. Hence, all the insights gained about the implementation of student loans can be applied to child care loans. Income contingent loans, in particular, have gained prominence as a way to deal with asymmetric information and uncertain future outcomes.[10]

We now propose a numerical example to compare the effects on gender inequality of the three policies anlysed in this section.

5 A numerical example

The theoretical model presented before shows that some households' inability to afford child care, besides generating an inefficiency, amplifies gender gaps in participation and wages. It also demonstrates how different policies affect the extent of gender inequality, by altering households' constraints. In particular, the model illustrates that a longer paid maternity leave has unclear effects on female labour force participation and wages, and that the effects of subsidies and loans differ due to the role played by taxes. In this section, we calibrate and simulate the model using Spanish data. Since there are many aspects of the real world that are currently not captured by our model, our goal is not to reach quantitative conclusions. Instead, we wish to provide an example of how the different policies affect gender inequality at equilibrium when some households are liquidity constrained.

5.1 Calibration

We calibrate the model in yearly terms for households with average earnings in the Spanish economy in 2018. Table 2 presents the calibrated parameters and variables. Next, we describe the calibration procedure.

Households and benchmark leave duration Young individuals are between 30 and 49 years old. Old individuals are 50 and above. We set the proportion of households with children at $\rho = 0.704$, which reflects the percentage of women aged 30 to 49 who are mothers in 2018 according to the Spanish National Institute of Statistics. In the benchmark calibration we consider a scenario of young households with two adults (one man, one woman) and $2/\rho$ children. Mothers receive 4 months of fully paid maternity leave per child. Thus, we set $\alpha = (4months \times 2children)/(19years \times 12months \times \rho) = 0.0496$, implying that a woman aged between 30 and 49 years spends 5 per cent of her available time on leave. Older households consist of two members (one man, one woman).

Wages We use the 2018 Spanish Wages Structure Survey to calibrate the wage distribution of full time workers. Our model has young and old women and men. Old men have high job experience, while old women may have high, intermediate or no job experience. We consider that male and female workers with high job experience are those with more than 11 years of job seniority. In contrast, female workers with no job experience have less than one year of job seniority. For female

[10]See Barr et al. (2019), Britton et al. (2019) and Van Long (2019), for some lessons from income contingent loan design around the world. Quiggin (2014) shows the advantages of income contingent loans under asymmetric information.

workers with intermediate experience we want to focus on those who only stopped working during maternity leave. For this reason, for medium experience, we consider women with between 10 and 11 years of job seniority.[11]

Using a total sample of 28,500 establishments with around 220,000 employees, we compute the average annual wage of old experienced men, which is equal to 42,953 euros, and normalise it to $w^h(x) = 1$. In the model, women without children work for the entire youth period, which gives them high experience when old. Thus, we assume that they earn the same wage as old men. This assumption will only affect the computation of the average wage of old women, which will be higher than that observed in the data, without any other implication.[12] We express the other average wages as ratios of $w^h(x)$. Thus, the wage of a young man is set to $w_m(x) = 15,021/42,953 = 0.349$. The wage of an old woman with intermediate experience is $w^i(x) = 32,158/42,953 = 0.749$. As to the remaining wages, that of an old woman without job experience is $w^n(x) = 12,657/42,953 = 0.295$. The wage of a young woman is $w_f(x) = 13,186/42,953 = 0.307$. Note that the average unadjusted gender wage gap of young workers $(w_m(x) - w_f(x))/w_f(x)$ is equal to 13.9 per cent.

Table 2: Calibrated parameters and variables for Spain

Parameters	Value	Source/Target
Proportion of women with children, ρ	0.704	National Institute of Statistics 2018
Average duration of parental leave, α	0.050	(4 months $\times$ 2 children) / (12 months $\times$ 19 years $\times \rho$)
Wage of old men and women without children, $w^h(x)$	1	Normalized
Wage of young men, $w_m(x)$	0.349	2018 Spanish Wages Structure Survey
Wage of old women without job experience, $w^n(x)$	0.295	2018 Spanish Wages Structure Survey
Wage of old women with intermediate job experience, $w^i(x)$	0.749	2018 Spanish Wages Structure Survey
Firm's cost when mothers are on leave or quit, q	0.147	Solves equation (16)
Minimum consumption level c	0.312	Spanish Institute of Statistics (INE)
Shape of the Weibull distribution cost function, η_{shape}	0.229	Matches $F(\eta^*)$ and $F(\eta^c)$
Scale of the Weibull distribution cost function, η_{scale}	0.112	Matches $F(\eta^*)$ and $F(\eta^c)$
Variables		
Labour force participation of young women, equation (19)	0.810	2018 Spanish Labour Force Survey
Wages of young women, $w_f(x)$	0.307	2018 Spanish Wages Structure Survey
Average wage of old women, $\bar{\omega}_f(x)$	0.732	Solves equation (20)
Proportion of constrained households with children, $F(\eta^*) - F(\eta^c)$	0.064	2010 Spanish Labour Force Survey
Proportion of mothers returning to work after child-birth, $F(\eta^c)$	0.717	Solves (19)
Unconstrained threshold level of child care costs η^*	0.782	Solves equation (10)
Constrained threshold level of child care costs η^c	0.357	Solves equation (12)
Lump-sum tax, τ	0.0029	Solves equation (25)

Minimum consumption level Using data from the Spanish Institute of Statistics 2018, we set the minimum level of consumption equal to the average expenditure in food, housing, water and

[11] Unfortunately, the Spanish Wages Structure Survey has no direct information on accumulated years of experience across different jobs. We assume that male and female workers with job experience have more than 11 years of job seniority, because this value corresponds to the average of job seniority for both groups of workers. In turn, we assume that workers without job experience have less than one year of job seniority since, due to the intensive use of temporary contracts, the average duration of a contract in Spain is 49 days.

[12] Our main aim is to isolate the effect of career interruptions on the average earnings of old women; we thus neglect other possible sources of gender differences in wages in old age.

energy per household, which equals 13,403 euros in 2018. Thus, $c = 13,403/42,953 = 0.312$.

Labour supply and proportion of mothers returning to work after leave By assumption, all young men, old men, and old women work. Only young women can be inactive, if they have children and do not go back to work after the leave. Using data from the Spanish Labour Force Survey, we target the labour force participation rate of young women aged between 30 and 49 years old in Spain in 2018 at 81.0 per cent. In the model, the share of women with children who return to work is $F(\eta^c)$. Then, using Eq. (19) and the female labour force participation rate (81.0 per cent), we obtain the proportion of mothers who go back to work after the leave is over $F(\eta^c) = 0.717$. Plugging this value in (20), we also obtain the calibrated average wage of an old woman: $\bar{\omega}_f(x) = 0.732$.

Taxes We calibrate the lump-sum tax by calculating the revenues necessary to cover the cost of paid maternity leave per taxpayer as a fraction of $w^h(x) = 1$. Using equation (25), the tax τ required to finance the maternity leave is equal to $(\rho\alpha w_f(x))/(3+(1-\rho)+(1-\alpha)\rho F(\eta^c)) = 0.0029$. This implies an annual amount of 124 euros per taxpayer, which is not far from the average expenditure in parental leave per employee observed in Spain in 2018 (94 euros).

Liquidity constrained households The 2010 Spanish special module on reconciliation between work and family life from the Labour Force Survey shows that 6.4 per cent of the mothers with children below 15 years old do not work because child care services are too expensive. We assume that this percentage matches that of households who are liquidity constrained. Then, $F(\eta^*) - F(\eta^c) = 0.064$. Thus, we obtain $F(\eta^*) = 0.781$.

Child care costs Each household needs to spend a different amount on child care for the mother to be able to return to work. These costs depend on a large variety of elements, for example: whether the household can get help from relatives, and how much; availability of public or private child care facilities nearby; working schedules; commuting time; whether the child gets sick often (needing a different arrangement, like a baby sitter who takes care of him/her at home); the age distribution of children, as older children can take care of younger ones, or other special needs.

Calibrating the distribution of these costs is not an easy task. The distribution of actual expenditure on child care can be a good measure of the distribution of child care costs only for those households that buy child care on the market, but it is not informative of the costs faced by those households, that decide to rely on household provision of child care. Since the costs of the latter type of households are not observed, we assume that the overall distribution of child care costs is of the Weibull type and calibrate the parameters of the distribution to match the values of $F(\eta^*)$ and $F(\eta^c)$ that we have obtained before.[13] To calibrate this distribution we first need the thresholds η^* and η^c, which are calibrated using (10) and (12). We obtain $\eta^* = 0.782$

[13]We use the Weibull distribution because it is flexible and can capture the characteristics of many different types of distributions without further assumptions.

and $\eta^c = 0.357$. Then, having two parameters to calibrate in each case (the scale parameter and the shape parameter), and using the targets $F(\eta^*)$ and $F(\eta^c)$ as well as the Weibull distribution function, we obtain $\eta_{shape} = 0.229$ and $\eta_{scale} = 0.112$. The median of this distribution is 0.029, approximatively 10 per cent of the young woman wage at the benchmark scenario corresponding to a fully paid leave of 4 months per child ($\alpha = 0.050$).

Firm's adjustment costs The costs incurred by the firm when mothers are on leave or quit are obtained using the wage equation (16) with $\hat{\eta} = \eta^c$. We get $q = 0.147$.

5.2 Simulations

We first explore the effect on gender inequality of increasing the duration of paid maternity leave when some households are liquidity constrained. We then study the effects of a proportional subsidy and a loan. We know that both instruments reduce gender inequality, and that a loan can eliminate the liquidity constraint. Therefore, in the simulation, we calculate the subsidy rate that eliminates the liquidity constraint so that the subsidy and the loan can be compared on equal terms.

5.2.1 Modifying the length of fully paid maternity leave

We change the duration of the fully paid maternity leave α when households are liquidity constrained. We maintain the assumption that households have $2/\rho$ children. Besides the benchmark scenario ($\alpha = 0.050$), we consider two additional scenarios. The first one assumes that there is no paid leave and mothers work for the entire first period. Thus, we set $\alpha = 0$. In the second, the paid leave increases to 12 months per child ($\alpha = 0.15$), which is near to the average paid leave duration in OECD countries in 2020 according to Table PF2.1.A in the Family Database. Note that, according to our strategy of calibration, all these scenarios imply adjusting the lump-sum tax to finance the change in the leave duration. Table 3 shows the simulated scenarios.

Table 3: Simulated effects of changes to paid maternity leave duration α when households are constrained (Benchmark $\alpha = 0.05$)

Variable	1. $\alpha = 0.00$	2. $\alpha = 0.05$	3. $\alpha = 0.15$
Young female participation rate (%)	79.90	81.00	83.22
$w_f(x)$ as fraction of $w^h(x) = 1$	0.3127	0.3070	0.2943
$\bar{\omega}_f(x)$ as fraction of $w^h(x) = 1$	0.731	0.732	0.733
Constrained households (%)	6.40	6.40	6.41
Gender wage gap (%), young	11.83	13.91	18.84
Lump sum tax τ as fraction of $w^h(x) = 1$	0.0000	0.0029	0.0084

If we start from the benchmark calibration with $\alpha = 0.05$ – see column 2 – and eliminate maternity leave by setting $\alpha = 0$ – see column 1 –, the female labour force participation rate decreases from 81.0 per cent to 79.9 per cent, while $w_f(x)$ increases from 0.307 to 0.3127. As a result, the gender wage gap of young workers falls from 13.91 per cent to 11.83 per cent. In contrast, when maternity leave duration increases from four months to one year ($\alpha = 0.15$) – see column 3 –, the female labour force participation rate increases from 81.0 per cent to 83.22 per cent, while $w_f(x)$ falls from 0.307 to 0.2943. Thus, the gender wage gap of young workers increases from 13.91 per cent to 18.84 per cent. Since participation of young women increases when the duration of the leaves goes up, more old women have intermediate experience and the average wage of old women increases.

According to our model, the reduction in the wage of young mothers takes place because the negative effect of a higher α on $w_f(x)$ dominates the positive one due to a higher labour force participation (case b in Proposition 2). Note that the share of constrained households does not fall in response to higher α. In fact, while more households can afford to pay child care (η^c shifts to the right) it is also the case that more households find it optimal to do so (η^* shifts to the right). For example, the percentage of constrained households increases from 6.40 per cent to 6.41 per cent when the paid leave parameter increases from $\alpha = 0.05$ to $\alpha = 0.15$. Finally, note that the increase in the maternity leave duration from four months ($\alpha = 0.05$) to one year ($\alpha = 0.15$) increases the lump sum tax from 124 ($\tau = 0.0029$) to 360 ($\tau = 0.0084$) euros.

5.2.2 Introducing child care subsidies

In section 4.2 we saw that a proportional subsidy on child care costs can reduce the proportion of households that are liquidity constrained and, thus, gender inequality. We now compare two different scenarios of child care subsidies. The first one corresponds to our benchmark scenario where the proportion of the child care cost subsidised by the government is equal to $s = 0$. In the second scenario, we introduce a proportional subsidy s and set the rate so that the percentage of households that cannot pay child care costs but would be better off if they could is set to zero. This happens when $s = 0.543$. We adjust the lump-sum tax to finance the change in s, which implies increasing τ from 0.0029 (124 euros) to 0.0088 (378 euros).

Table 4: Simulated effects of removing liquidity constraints with a proportional child care subsidy

Variable	1. $s = 0$	2. $s = 0.543$
Young female participation rate (%),	81.00	85.27
$w_f(x)$ as fraction of $w^h(x) = 1$	0.3070	0.3169
$\bar{\omega}_f(x)$ as fraction of $w^h(x) = 1$	0.732	0.752
Constrained households (%)	6.40	0.00
Gender wage gap (%), young	13.91	10.35
Lump sum tax τ as fraction of $w^h(x) = 1$	0.0029	0.0088

As expected, the female labour force participation increases and so do wages, with an ensuing reduction of gender wage gaps. This happens because more households can afford for women to participate in the labour market, thus reducing the firm's expected cost of quitting, with positive effects on female wages and labour force participation. Specifically, the participation rate of women increases from 81.0 per cent to 85.27 per cent and the wage of young women increases from 0.3070 to 0.3169. As a result, the average wage of old women goes up, reducing the gender wage gap in old age.

5.2.3 Introducing loans

We now explore the effect of removing the liquidity constraint through the provision of a loan. The loan (see equation (35)) covers the difference between the child care cost the household faces if the mother returns to work once the maternity leave is over, $(1-\alpha)\eta$, and household income in the first period $w_m(x) - c - \tau + (1-\alpha)(w_f(x) - \tau)$. In other words, the child care cost $(1-\alpha)\eta^c$, which they can afford with this income. In our simulated scenario, the average loan provided by the government amounts to 0.173 as a fraction of $w^h(x) = 1$ (7,430 euros, 2,653 per child).

Table 5: Simulated effects of removing liquidity constraints with loans

Variable	1. $B = 0$	2. $B > 0$
Young female participation rate (%),	81.00	85.30
$w_f(x)$ as fraction of $w^h(x) = 1$	0.3070	0.3170
$\bar{\omega}_f(x)$ as fraction of $w^h(x) = 1$	0.732	0.752
Constrained households (%)	6.40	0.00
Gender wage gap (%), young	13.91	10.32
Lump sum tax τ as fraction of $w^h(x) = 1$	0.0029	0.0029

Table 5 shows the benchmark calibration with constrained households ($F(\eta^*) - F(\eta^c) > 0$, column 1) and the results of simulating the removal of household liquidity constraints ($F(\eta^*) - F(\eta^c) = 0$, column 2). Removing liquidity constraints increases female labour force participation and reduces gender wage gaps for both young and old. Specifically, the participation rate of women increases from 81.0 per cent to 85.3 per cent. This effect is slightly larger than the one obtained with the proportional subsidy because taxes remain unchanged in this case. Young women's wages increase from 0.3070 to 0.3170 and old women's wages increase from 0.732 to 0.752.

6 Concluding comments

We develop a model to illustrate the impact that liquidity constraints associated with child care costs have on gender inequality and study the effect of several policies on gender inequality in this scenario. In particular, we investigate an expansion in the duration of the paid maternity leave, a proportional subsidy on child care costs, and a simple loan as policy levers. Maternity leaves and child care subsidies are widely used around the world to guarantee mothers a job-protected leave and promote work-life balance. They are also a sizeable fraction of overall family policy expenditure in OECD countries. The potential benefits of loans in the context of family policy, instead, have been put forward by Chapman and Higgins (2009), but their role in addressing gender inequality in the labour market has not been considered in the literature, as far as we know.

We find that increasing the duration of paid maternity leave has ambiguous effects on gender inequality when women in some households cannot work due to a liquidity constraint that prevents them from buying child care. In our numerical example based on Spanish data the effect is positive on female participation, but negative on female wages. Subsidising child care costs unambiguously reduces gender inequality, and so does a loan given out in the form of a child care voucher. The tax per worker required to fund the subsidy is relatively small.

Future work can assess the effectiveness of these policies in reducing gender inequality in more complex environments, where uncertainty about future earnings plays a role. Note also that we have studied the effects of these policies on gender gaps in participation and wages rather than on overall welfare, taking a positive rather than a normative approach. We leave the analysis of welfare effects for future research.

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Appendix 1

We show what conditions are necessary and sufficient for the existence of the equilibrium in wages and participation, both for the case of binding and non-binding liquidity constraints.

We report the equilibrium wage condition (16) and we rewrite it as $w_f(x, \alpha) = \kappa(\hat{\eta})$. The equilibrium threshold $\hat{\eta} = \{\eta^*(x, \alpha), \eta^c(x, \alpha)\}$ is a function of the wage $w_f(x, \alpha)$, so we write $\hat{\eta} = h(w_f)$. Then $w_f = \kappa(h(w_f))$. This function will have a fixed point w_f^* provided that $\kappa(h(w_f^{\min})) > 0$, where $w_f^{\min}$ is the lowest possible wage, and $0 < \kappa' h' < 1$.

Our $w_f^{\min}$ corresponds to the case where no mother returns to work after the maternity leave: $F(\eta) = F(\eta^*) = 0$. Substituting this in (3) we get

$$w_f = x - \frac{\rho}{1-\rho} q(x)$$

which is positive by Assumption 1. Thus, $\kappa(h(w_f^{\min})) > 0$.

The derivative $\kappa'(\hat{\eta})$ is

$$\kappa'(\hat{\eta}) = \frac{\rho(1-\alpha) f(\hat{\eta}) q(x)}{\left(1 - \rho + \rho\,(1-\alpha)\,F(\hat{\eta})\right)^2} > 0$$

Now we calculate the derivatives $h'(\hat{\eta})$ for $\hat{\eta} = \eta^*(x, \alpha)$ and $\hat{\eta} = \eta^c(x, \alpha)$, respectively.

1. The equilibrium threshold of child care costs at the interior solution is, for given w_f, (10), that we write $\eta^*(x, \alpha) = h_*(w_f^*(x))$, with:

$$h'_*(w_f(x)) = 1$$

   Then, the necessary and sufficient condition for the existence of an equilibrium when there are no liquidity constraints is $\kappa'(\eta^*(x, \alpha)) h'_*(w_f(x)) < 1$ or (17).

2. The equilibrium threshold of child care costs at the constrained solution is, for given w_f, (12) that we write $\eta^c = h_c(w_f)$ with:

$$h'_c(w_f(x)) = \frac{1}{1-\alpha} > 0$$

   Then, the necessary and sufficient condition for the existence of an equilibrium in the presence of liquidity constraints is: $\kappa'(\eta^c(x, \alpha)) h'_c(w_f(x)) < 1$, or (18).

Appendix 2

In this Appendix we provide several comparative statics results reported in the body of the paper. First, the effect of changing productivity on equilibrium wages and participation. Second, the effect of extending the maternity leave. Third, the effect of increasing taxes in the absence and in the presence of subsidies.

Changing productivity

We start with the effect of changing household productivity on gender inequality. To explore this, we perform the comparative statics analysis of changing x on the equilibrium. The aim of this exercise is to study whether gender inequality is reduced when households are more productive.

The analysis focuses on the constrained equilibrium, but it can be done following the same steps for the unconstrained equilibrium, with similar results.

When some households are constrained, the equilibrium results from the simultaneous determination of the threshold determining the number of women going back to work after the leave $\eta^c(x,\alpha)$, given by (12), and the wage of young women (16), i.e., by the system of implicit equations that we denote by $H(w_f,\eta^c)$ and $W(w_f,\eta^c)$:

$$H(w_f,\eta^c) \;:\; \eta^c - \frac{w_m(x) - \tau + w_f - (1-\alpha)\tau - c}{1-\alpha} = 0 \tag{A.1}$$

$$W(w_f,\eta^c) \;:\; w_f - x + \rho\frac{1-(1-\alpha)F(\eta^c)}{1-\rho+\rho\,(1-\alpha)\,F(\eta^c)}q(x) = 0 \tag{A.2}$$

Differentiation of these implicit equations with respect to η^c, w_f and x allows us to derive the effect of x on the endogenous variables η^c, w_f by applying Cramer's rule.

We start differentiating the above equations with respect to η^c, w_f and x. We find:

$$H_{\eta^c} = 1 \tag{A.3}$$

$$H_{w_f} = -\frac{1}{1-\alpha} \tag{A.4}$$

$$H_x = -\frac{1}{1-\alpha}\frac{dw_m}{dx} \tag{A.5}$$

$$W_{\eta^c} = -\frac{\rho\,(1-\alpha)\,f\,(\eta^c)\,q(x)}{\left(1-\rho+\rho\,(1-\alpha)\,F(\eta^c)\right)^2} \tag{A.6}$$

$$W_{w_f} = 1 > 0 \tag{A.7}$$

$$W_x = -1 + \frac{\rho\,(1-(1-\alpha)F(\eta^c))}{1-\rho+\rho\,(1-\alpha)\,F(\eta^c)}q'(x) \tag{A.8}$$

We use Cramer's rule to identify the effect of changing x on wages:

$$\frac{dw_f}{dx} = \frac{\begin{vmatrix} H_{\eta^c} & -H_x \\ W_{\eta^c} & -W_x \end{vmatrix}}{\begin{vmatrix} H_{\eta^c} & H_{w_f} \\ W_{\eta^c} & W_{w_f} \end{vmatrix}} = \frac{\begin{vmatrix} 1 & \frac{1}{1-\alpha}\frac{dw_m}{dx} \\ -\frac{\rho(1-\alpha)f(\eta^c)q(x)}{(1-\rho+\rho(1-\alpha)F(\eta^c))^2} & 1-\frac{\rho(1-(1-\alpha)F(\eta^c))}{1-\rho+\rho(1-\alpha)F(\eta^c)}q'(x) \end{vmatrix}}{1-\frac{\rho f\,(\eta^c)\,q(x)}{\left(1-\rho+\rho\,(1-\alpha)\,F(\eta^c)\right)^2}} \tag{A.9}$$

The determinant of the Hessian matrix of derivatives in the denominator of (A.9) is

$$H_{\eta^c}W_{w_f} - H_{w_f}W_{\eta^c} = 1 - \frac{\rho f\,(\eta^c)\,q(x)}{\left(1-\rho+\rho\,(1-\alpha)\,F(\eta^c)\right)^2}$$

which is positive, given the condition required for the existence of an equilibrium when households are constrained (18). Then, we can conclude from (A.9) after some simplifications that:

$$sign\frac{dw_f}{dx} = sign\left(1 - \frac{\rho\,(1-(1-\alpha)F(\eta^c)}{(1-\rho+\rho\,(1-\alpha)\,F(\eta^c))}q'(x) + \frac{\rho f\,(\eta^c)\,q(x)}{\left(1-\rho+\rho\,(1-\alpha)\,F(\eta^c)\right)^2}\frac{dw_m}{dx}\right)$$

For the effect of changing x on the participation threshold η^c, we need to calculate:

$$\frac{d\eta^c}{dx} = \frac{\begin{vmatrix} -H_x & H_{w_f} \\ -W_x & W_{w_f} \end{vmatrix}}{\begin{vmatrix} H_{\eta^c} & H_{w_f} \\ W_{\eta^c} & W_{w_f} \end{vmatrix}} = \frac{\begin{vmatrix} \frac{1}{1-\alpha}\frac{dw_m}{dx} & -\frac{1}{(1-\alpha)} \\ 1-\frac{\rho(1-(1-\alpha)F(\eta^c))}{1-\rho+\rho(1-\alpha)F(\eta^c))}q'(x) & 1 \end{vmatrix}}{1-\frac{\rho f\,(\eta^c)\,q(x)}{\left(1-\rho+\rho\,(1-\alpha)\,F(\eta^c)\right)^2}} \tag{A.10}$$

hence,

$$sign\frac{d\eta^c}{dx} = sign\left(\frac{dw_m}{dx} + 1 - \frac{\rho\left(1-(1-\alpha)F(\eta^c)\right)}{\left(1-\rho+\rho\left(1-\alpha\right)F(\eta^c)\right)}q'(x)\right)$$

Consider first the case in which $q'(x) < 0$, i.e., the cost for firms of female leaves/quits is decreasing in the productivity of women. Women's wages unambiguously increase in productivity, and so does participation. To explore the effects on gender gaps, recall that men work all the time. Then, men's participation does not change, women's participation increases and, thus, the gender gap in participation decreases. The average wage of old men does not change, while the average wage of old women increases because they work more when young. Hence, the gender gap in wages decreases for old workers. Finally, young men's wages are given by $w_m = x$ and a marginal increase in productivity translates in a one-to-one increase in wages. In contrast, for young women, a marginal increase in productivity translates into a more than proportional increase in wages, thus reducing gender gaps in young worker's wages. To see this last point, consider equation (A.9) and note that the numerator is larger than 1 when $q'(x) < 0$, and the denominator is smaller than 1 from (18). As a result, gender gaps in participation and wages are lower, the higher the productivity.

If $q'(x) > 0$, higher productivity women impose a higher cost of leaves/quits on firms, limiting the positive effect of productivity on wages and participation. It is sensible to assume that the negative effect of productivity on wages via the costs $q(x)$ is never large enough to make women's wages decrease in productivity. To guarantee this, it is sufficient that $1-\frac{\rho(1-(1-\alpha)F(\eta^c))}{(1-\rho+\rho(1-\alpha)F(\eta^c))}q'(x) > 0$. Then, $sign\frac{d\eta^c}{dx} > 0$ and gender gaps in participation and old workers' wages decrease. In contrast, the gender gap in young workers' wages could increase or decrease depending on whether women's wages grow more or less than men's.

Increasing the duration of maternity leave

To study the effect of extending the duration of the leave α on participation and wages we proceed as before. We now need the derivatives of of (A.1) and (A.2) with respect to α:

$$H_\alpha = -\frac{w_m(x) - \tau + w_f - (1-\alpha)\tau - c}{(1-\alpha)^2} < 0 \tag{A.11}$$

$$W_\alpha = \frac{\rho F(\eta^c) q(x)}{\left(1-\rho+\rho\left(1-\alpha\right)F(\eta^c)\right)^2} > 0 \tag{A.12}$$

Using Cramer's rule to identify the effect of changing α on wages, we have that:

$$sign\left(\frac{dw_f}{d\alpha}\right) = sign\left(-H_{\eta^c}W_\alpha + W_{\eta^c}H_\alpha\right) \tag{A.13}$$

The first term $(-H_{\eta^c}W_\alpha)$ accounts for the reduction in the wage because mothers who return to work do so for a shorter time when α increases. The second term $(W_{\eta^c}H_\alpha)$ is the increase in the wage coming from the fact that more mothers go back to work when α increases, reducing firms's costs for their absence. Substituting (A.3), (A.12), (A.6) and (A.11) in (A.13) and simplifying, we can write:

$$sign\left(\frac{dw_f}{d\alpha}\right) = sign\left(\frac{w_m(x) - \tau + w_f(x) - (1-\alpha)\tau - c}{(1-\alpha)} - \frac{F(\eta^c)}{f\left(\eta^c\right)}\right). \tag{A.14}$$

We now turn to the effects of α on the decision to go back to work after the leave:

$$sign\frac{d\eta^c}{d\alpha} = sign\left(-H_\alpha W_{w_f} + W_\alpha H_{w_f}\right)$$

that is

$$sign\frac{d\eta^c}{d\alpha} = sign\left(\frac{w_m(x) - \tau + w_f(x) - (1-\alpha)\tau - c}{1-\alpha} - \frac{\rho F(\eta^c) q(x)}{(1-\rho(1-F(\eta^c)(1-\alpha)))^2}\right) \quad \text{(A.15)}$$

To determine this sign, we note that more women go back to work (first term), but each of them works less time and this reduces the wage, making fewer women willing to go back to work.

Summing up, we know from the previous analysis that both young women's wages and the proportion of young women going back to work after the leave will increase if both (A.14) and (A.15) are positive. This happens when (26) is satisfied (case a in Proposition 2) since, by (18), it is always the case that

$$\frac{F(\eta^c)}{f(\eta^c)} > \frac{\rho F(\eta^c) q(x)}{(1-\rho(1-F(\eta^c)(1-\alpha)))^2}$$

In case b, (A.14) is negative, but (A.15) is positive. And in case c both are negative.

With respect to the effect of increasing the duration of the leave α on the young female labour force participation (19), this will be:

$$\frac{dFLF}{d\alpha} = \rho(1-F(\eta^c)) + \rho(1-\alpha) f(\eta^c)\frac{d\eta^c}{d\alpha} \quad \text{(A.16)}$$

A longer duration of the leave keeps women attached to the labour force longer and can increase or decrease the number of mothers going back to work after the leave. Then, $d\eta^c/d\alpha > 0$ is sufficient for female participation to increase. This explains why young workers' participation gaps decrease in a and b. When $d\eta^c/d\alpha < 0$, as in case c, the impact of a change in the duration of the leave on female participation is ambiguous (see A.16).

Finally, for the effect of increasing α on average wages of old women when some households are liquidity constrained, differentiating (20) with $\hat{\eta} = \eta^c(x,\alpha)$, we have:

$$\frac{d\bar{\omega}_f}{d\alpha} = \rho f(\eta^c(x,\alpha))\frac{d\eta^c(x,\alpha)}{d\alpha}\left(w^i(x,\alpha) - w^n(x)\right) \quad \text{(A.17)}$$

Hence, old workers' gender wage gaps decrease in cases a and b, since $\frac{d\eta^c(x,\alpha)}{d\alpha} > 0$ but increase in case c.

Summing up these pieces of information, we write Proposition 3.

Increasing taxes

We compute the derivatives of (A.1) and (A.2) with respect to τ:

$$H_\tau = -\frac{1+1-\alpha}{1-\alpha} < 0 \quad \text{(A.18)}$$

$$W_\tau = 0 \quad \text{(A.19)}$$

and apply Cramer's rule as in (A.9) and (A.10), to obtain

$$sign\frac{dw_f}{d\tau} = sign\left(-H_{\eta^c} W_\tau + W_{\eta^c} H_\tau\right)$$

and

$$sign\frac{d\eta^c}{d\tau} = sign\left(-H_\tau W_{w_f} + W_\tau H_{w_f}\right)$$

Using (A.3), (A.4), (A.6), (A.7), (A.18), and (A.19), we obtain (29) and (30).

The analysis differs slightly when we consider a subsidy rate on child care costs s. Now, instead of (A.1), we have

$$H(w_f, \eta^s) : \eta^s - \frac{w_m(x) - \tau + w_f - (1-\alpha)\tau - c}{(1-s)(1-\alpha)} = 0$$

This does not change the sign of the effect of taxes on young women's wages and participation rates. We obtain (33) and (34).