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9840
2022

July 2022

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
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CESifo *Working Paper No. 9840*

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Abstract

In this paper, we revisit the fiscal theory of the price level (FTPL) within the New Keynesian (NK) model. We show in which cases the average maturity of government debt matters for the transmission of policy shocks. The central task of this paper is to shed light on the theoretical predictions of the maturity structure on macro dynamics with an emphasis on model-implied expectations. In particular, we address the transmission channels of monetary and fiscal policy shocks on the interest rate and inflation dynamics. Our results illustrate the role of the maturity of existing debt in the wake of skyrocketing debt-to-GDP ratios and increasing government expenditures. We highlight our results by quantifying the effects of the large-scale US fiscal packages (CARES) and predict a surge in inflation if the deficits are not sufficiently backed by future surpluses.

JEL-Codes: E320, E120, C610.

Keywords: NK models, FTPL, government debt, maturity structure, CARES.

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July 2022
We are grateful to John Cochrane, Galo Nuño, Sebastian Merkel, Ricardo Reis and Nora Traum for helpful comments. We thank Josie Oetjen for excellent research assistance. We appreciate financial support from the German Research Foundation (DFG, Project No. 446166239).

1 Introduction

In response to the global coronavirus pandemic, governments around the world tried to cushion the economic downturn by financing large-scale fiscal support and relief packages such as the US Coronavirus Aid, Relief, and Economic Security (CARES) Act, with unprecedented volumes. For example, when including loan guarantees, the CARES Act amounts to about $2 trillion (or 10% of US GDP) with substantial budgetary effects. The Congressional Budget Office (CBO) projects CARES to add $1.7 trillion to deficits over the next decade.[1] In order to alleviate a deep recession, policy makers have implemented further stimulus packages (e.g., the American Rescue Plan, the Next Generation EU fund, NGEU). The funding of these unprecedentedly large fiscal programs drastically increased debt levels with yet unknown consequences (e.g., accounting for distributional effects, CARES increases the debt-to-GDP ratio by 12% in Kaplan, Moll, and Violante, 2020).

In the macroeconomic literature, there are, however, open questions and ongoing debates about the effects of sovereign debt on macro aggregates, inflation, the term structure, and inflation expectations where no consensus has been reached. One central question here is how the structure of outstanding government debt affects the transmission channels of fiscal and monetary policy. Clearly, governments face a challenging task to maintain a sustainable level and maturity structure of outstanding sovereign debt. On the one hand, fiscal policy faces a financing decision on whether to either increase the level of public debt or to raise taxes today. On the other hand, fiscal policy needs to decide on whether to issue bonds with longer maturities, or to simply roll-over maturing debt with short-term bonds. What will be the effect of those large-scale fiscal programs, in particular, how does the maturity structure of outstanding debt affect those outcomes? This paper fills this gap in the macroeconomic analysis of fiscal and monetary policy.

In this paper we address the transmission of fiscal and monetary policy shocks on interest rates and inflation dynamics in a framework which combines the fiscal theory of the price level (FTPL) with the traditional New Keynesian (NK) model of inflation. Our central aims are the theoretical predictions of transitory and permanent policy shocks, which offer empirical testable implications for the role of the maturity structure of debt on the transmission of fiscal and monetary policy. Our application studies the effects of the recent CARES Act trough the lens of fiscal theory. We depart from the existing literature on the effects of the maturity structure of government debt in three dimensions. First, our formulation allows us to link the macro model easily to term-structure models in finance (Vasicek, 1977; Cox, Ingersoll, and Ross, 1985) and model-implied inflation expectations. Our approach allows us to compute the term structure of interest rates and inflation expectations by solving a partial differential equation, which is easily extended to nonlinear solutions, default risk, and term premia. Second, in contrast to existing

[1] Congressional Budget Office, CARES Act, https://www.cbo.gov/publication/56334

approaches[2], we directly compute zero-coupon bond prices for arbitrary maturities and states and then show bounds for the effects of the maturity structure of government debt on macro dynamics and inflation decomposition. Finally, we show that the fiscal theory in the continuous-time version works through two distinct channels: (i) a direct FTPL effect through a discrete jump in the price of existing bonds and (ii) an indirect effect through changing the path of future real interest rates. While the first channel is a pure asset pricing channel, the second channel is the traditional effect present in forward-looking rational expectations models. Hence, even in the model with short-term debt, the fiscal theory has implications on the future path of the real interest rate, in particular, the term structure of interest rate, inflation expectations, and the real economy.

We calibrate a simple FTPL-NK model to match the average maturity of outstanding US government debt and study aggregate dynamics. We find that the average maturity of debt has important implications for the transmission channels of both monetary and fiscal policy. Our results show how the maturity of existing sovereign debt significantly shapes the inflation response to fiscal and monetary policy shocks. First, following a transitory monetary policy shock, a longer maturity structure translates to a larger response in the real interest rate. In cases where outstanding government debt consists solely of short-term debt, the traditional negative correlation of the nominal interest rate and current inflation is reversed and term structure and inflation expectations are more sensitive to shocks. Similarly, based on the underlying maturity structure of government debt, expansionary fiscal policy leads to higher inflation and more accumulation of debt with short-term debt. Our inflation decomposition shows that with perpetuities, the inflation response to transitory shocks is dictated solely by future fiscal policy with changes in future monetary policy being soaked up by an immediate asset pricing effect. Second, we illustrate how inflation expectations and the term structure helps in identifying permanent policy shocks. Here, the maturity structure often produces some unpleasant short-term side effects. For example, a permanently lower inflation target increases current inflation and interest rates, but reduces long-term bond yields due to the re-evaluation of existing bonds.

Our findings confirm the hypothesis that the CARES Act with its unprecedented large-scale fiscal stimulus programs, i.e., the large cuts in primary surplus and hikes in government debt, has generated a market response with strong inflationary effects but effectively helped stimulating the real economy. However, the recent surge in inflation and medium-term inflation expectations indicate that markets do *not* expect that the newly issued debt is backed by subsequent higher future surpluses. This seems in contrast to the aftermath of the global financial crisis and raises cautionary flags as hyperinflations are widely believed to have fiscal origins (cf. Leeper and Leith, 2016).

In line with the existing literature on the fiscal theory, we confirm a prominent role

[2] Among others see Leeper, Leith, and Liu (2019), Lustig, Sleet, and Yeltekin (2008), Faraglia, Marcet, Oikonomou, and Scott (2013) or Faraglia, Marcet, Oikonomou, and Scott (2019).

of those ideas in the FTPL-NK model with a plausible maturity structure of sovereign debt (cf. Cochrane, 2001; Leeper and Leith, 2016).[3] Most theoretical studies, such as Sims (2011, 2013), Leeper and Leith (2016), and Cochrane (2018), highlight important insights, e.g., the role of long-term bonds in the simple NK model causing a 'boomerang inflation' response to monetary policy shocks. In these models, long-term bonds are used to offset an otherwise initial positive co-movement of the inflation and the interest rates.[4] Other studies focus on the low-frequency relationship between the fiscal stance and inflation in a model with long-term debt (see Kliem, Kriwoluzky, and Sarferaz, 2016) or the government spending multiplier (see Leeper, Traum, and Walker, 2017). We are not aware of a comprehensive study on the effects of fiscal and monetary policy shocks on inflation and inflation expectations, or generally about the role of fiscal theory in the NK model with an empirically calibrated average maturity of existing sovereign debt. Unfortunately, an inflation decomposition into a direct FTPL effect and an indirect effect is tricky and less clear-cut in the discrete-time model because the price level can jump (which in the continuous-time version is determined by past inflation). Hence, a continuous-time version of the FTPL-NK model (see also Sims, 2011; Cochrane, 2018) helps identifying the effects of the maturity structure because in the model with short-term debt, as in traditional NK models with fiscal policy and sovereign debt, the direct bond pricing effect is zero and the fiscal theory would work solely through the indirect effect.

Many theoretical and empirical studies recognize an important effect of the maturity structure of government in a broader context of optimal monetary and fiscal policies.[5] Leeper et al. (2019) show how high sovereign debt levels and the debt maturity structure can increase the 'inflationary bias'. In this setup, higher debt levels and shorter maturities increase the temptation of the policy maker to use surprise inflation and to decrease the real value of government debt. Similarly, Lustig et al. (2008) study the optimal policy if the fiscal authority is constrained by its ability to lend and only issues non-contingent nominal debt. In this case, optimal policy is achieved by almost the exclusive use of long-term debt. Even though the holding return on long-term debt is more volatile in contrast to short-term debt, it offers a hedge against fiscal shocks. Faraglia et al. (2013) analyze how inflation is affected by the maturity of sovereign debt and debt levels when fiscal and monetary policy are coordinated. They conclude that higher debt levels cause higher inflation, while a longer maturity structure increases its persistence.

More recently, Kaplan et al. (2020) and Bayer, Born, and Luetticke (2021) also evaluate

[3]In this paper we focus on the fiscal regime and neglect potential fiscal-monetary coordination problems which may arise in a regime-switching model as in Bianchi (2012) or Bianchi and Melosi (2019).

[4]Cochrane (2022c) and Liemen (2022) discuss alternative ideas and show that long-term debt is not necessary to address this counterfactual response for short-term debt in the FTPL-NK model.

[5]Other papers study the optimal debt-maturity management (cf. Buera and Nicolini, 2004; Shin, 2007; Faraglia, Marcet, and Scott, 2010; Debortoli, Nunes, and Yared, 2017; Bigio, Nuño, and Passadore, 2019). For example, Bigio et al. (2019) show how liquidity costs can prevent an instantaneous re-balancing across maturities and identify different forces that ultimately shape the optimal debt-maturity distribution.

the role of skyrocketing debt levels, following the large-scale fiscal stimulus programs within the NK models with heterogeneous agents (HANK). Focusing on the role of public debt as private liquidity, Bayer et al. (2021) find that the expansionary stimulus programs decreased the liquidity premium of government bonds over less liquid assets.

The rest of the paper is organized as follows. First, in Section 2 we formalize the simple perfect-foresight FTPL-NK model and study dynamics of transitory and permanent structural zero-probability shocks. In Section 3 we provide a thorough analysis and simulation of the CARES Act of 2020 and discuss the recent surge in inflation and differences to the aftermath of the global financial crisis in 2008. Section 4 concludes.

2 The Model

In this section, we show how the FTPL mechanism outlined in Sims (2011) and Cochrane (2018) is embedded in the continuous-time NK model (cf. Posch, 2020). For reasons of clarity, we shortly discuss the main channels of FTPL in the linear NK framework and abstract from the effects of uncertainty and nonlinearities.

2.1 Monetary policy or fiscal theory of monetary policy

As shown in Cochrane (2018), the presence of longer-term debt has effects on both the real economy and on how monetary policy is conducted, and more generally how government policies affect inflation. Consider the three-equation perfect-foresight NK model

$$\mathrm{d}x_t = (i_t - \rho - \pi_t)\mathrm{d}t \tag{1}$$

$$\mathrm{d}\pi_t = (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t \tag{2}$$

$$\mathrm{d}i_t = \theta(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_t^*))\mathrm{d}t, \tag{3}$$

in which x_t is the output gap, y_t is output, i_t is the nominal interest rate, ρ the rate of time preference, π_t is inflation, where κ controls the degree of price stickiness with $\kappa \to \infty$ as the frictionless (flexible price) and $\kappa \to 0$ perfectly inelastic (fixed price) limits, θ controls interest rate smoothing with $\theta \to \infty$ implying the traditional feedback rule, $i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1)$, and with π_t^* and i_t^* being parametric values.

Following Cochrane (2018) we implement the fiscal theory of the price level (FTPL) by closing the system with a fiscal block

$$\mathrm{d}a_t = ((i_t - \pi_t)a_t - s_t)\mathrm{d}t \tag{4}$$

$$\mathrm{d}s_t = f(s_t, y_t, a_t)\mathrm{d}t, \tag{5}$$

in which a_t is the real value of sovereign debt (held by households), and $s_t \equiv T_t - g_t$ is the primary surplus, where T_t denotes lump-sum tax revenues and g_t government spending other than interest payments. It represents the net payments to holders of bonds, both through interest and retirement of outstanding debt (cf. Sims, 2011). In what follows, we use the notion of 'sovereign debt' and 'government bonds' interchangeably, which after all can be considered as a medium of exchange (paper money).

The central equation in the FTPL-NK model links the primary surpluses to the real value of sovereign debt. In fact, solving forward (4), the future path of primary surpluses imposes a 'constraint' for fiscal policy (government budget constraint), because

$$a_t \equiv \frac{n_t p_t^b}{p_t} = \mathbb{E}_t \int_t^\infty e^{-\int_t^u (i_v - \pi_v) \mathrm{d}v} s_u \mathrm{d}u, \tag{6}$$

where n_t denotes the number of outstanding bonds, p_t^b the bond price, and p_t the price level, which must equal its (expected) real present value.[6] In this paper, we focus on bounded solutions and $\lim_{T\to\infty} e^{-\int_t^T (i_v - \pi_v)\,\mathrm{d}v} a_T = 0$.[7] Rather than being a budget constraint or limiting fiscal capacity, equation (6) should be thought of as being a valuation formula as it asserts a value p_t^b to the supply of government bonds n_t and a given price level p_t.

Similar to assuming perfectly flexible prices, it is unrealistic assuming that government debt is either floating debt or perpetual debt (cf. Sims, 2011). In what follows, we refer to floating debt as short-term and to long-term debt as perpetuities. We introduce bonds with decaying coupon payments (similar to Woodford, 2001), and assume that longer-term bonds (average duration) are amortized at rate δ and pay a nominal coupon $\chi+\delta$ such that at steady state the bonds sell at par and results compare to Sims (2011). No-arbitrage requires (see PDE approach Cochrane, 2005, chap. 19.4),

$$\mathrm{d}p_t^b = (i_t - ((\chi+\delta)/p_t^b - \delta))p_t^b \mathrm{d}t + \mathrm{d}\delta_{p_t^b}, \quad \mathbb{E}_t(\mathrm{d}\delta_{p_t^b}) = 0 \tag{7}$$

in which $\mathrm{d}\delta_{p_t^b}$ captures discrete changes in the bond price due to zero-probability structural shocks, with $\chi = i_{ss}$ such that $p_{ss}^b = 1$ is identical to floating debt. Note that (7) is *not* a stochastic differential equation (SDE) because the 'shocks' have zero probability. Following the literature, $\mathrm{d}\delta_{p_t^b}$ reminds us that the variable p_t^b can jump (forward-looking). In theory, we can issue floating debt which pays at $\chi = i_t$ and with $\delta \to \infty$ average duration approaches zero such that $p_t^b \equiv 1$. In contrast, for long-term bond we set $\delta = 0$ (cf. Sims, 2011). By integrating the linear approximation of equation (7), we obtain

$$p_t^b = 1 - \mathbb{E}_t \int_t^\infty e^{-(\chi+\delta)(u-t)}(i_u - i_{ss})\mathrm{d}u, \tag{8}$$

[6] Cochrane (2018) as well as Sims (2011) abstract from government consumption, g_t, in their framework, such that primary surpluses correspond to taxes, $s_t = T_t$.

[7] Hence, we focus on the standard no-bubble solution (e.g., Sims, 2011; Cochrane, 2018). There is a literature showing that a 'bubble term' can be important for the budget constraint (cf. Reis 2021).

which shows that the initial response of the bond price is determined entirely by the discounted and maturity-adjusted path of the nominal interest rate. If we use the average duration of 6.8 years from the central bank's Security Open Market Account (SOMA), we calibrate $\delta = 1/6.8$ and $\chi = 0.03$ (see Del Negro and Sims, 2015).[8]

In contrast to the discrete-time model, the price level p_t cannot jump and is given by past price quotations (Calvo's insight).[9] Because the number of outstanding bonds in (6) is fixed and cannot jump either, only the bond price p^b_t, which is determined in general equilibrium, can jump due to changes in either future surplus s_u or the future discount rate $i_u - \pi_u$ for $u \geq t$ (direct FTPL effect). Because with short-term debt $p^b_t \equiv 1$, the direct FTPL requires the presence of longer-term debt. The bond price effect then passes on to the value of debt, inducing a jump in a_t (market value), i.e., a forward-looking variable. Hence, the average duration δ of the maturity structure of government debt determines the strength of the direct FTPL effect, such that $\delta \to \infty$ eliminates jumps in p^b_t.

The path of the primary surplus on the right-hand side of equation (6) is determined by fiscal policy, so by assumption, surpluses typically do not jump if the value of sovereign debt changes (we discuss different scenarios below). Hence, changes in fiscal policy are accommodated by the real interest rate (indirect FTPL effect) such that (6) is not violated. So even without the presence of long-term debt, monetary policy must accommodate future changes in fiscal policy. Although households are indifferent with respect to the maturity of government debt because of arbitrage, the bottom line of this paper is to show that it has important implications for inflation dynamics, the term structure, inflation expectations, and the real economy. Thus, for ease of illustration, we focus on a fiscal regime (or fiscal dominance) throughout the paper, while the insights are useful for a more realistic regime-switching approach, as in Bianchi and Melosi (2019).

2.2 Simple fiscal policy rules versus policy inertia

There seems to be a consensus among economists that there is a systematic response of fiscal policy to the state of the economy. While theoretical papers often assume contemporaneous responses using simple fiscal policy rules (Sims, 2011; Cochrane, 2018), most empirical studies suggest that there is a time lag (inertia) between the relevant variables and the policy response, such as changes in the tax code or a revised public expenditure budget plan (cf. Kliem et al., 2016; Bianchi and Melosi, 2019). In this paper, we provide a general framework, where the specifications can be coherently studied and which allows us to investigate the effects of temporary and permanent shocks. Starting with the central

[8]Below we use a zero-coupon bond with time-to-maturity of $1/\delta$ years interchangeably.

[9]Because no mass of firms can change prices instantaneously, the NK Phillips curve allows a jump in the inflation rate but not in the price level (cf. Cochrane, 2018, Online Appendix). Here, the price-level jump of the discrete-time model rather translates into a smooth change by affecting inflation.

FTPL-NK equation in (5), $s_t \equiv T_t - g_t$, and specifying a tax rule as

$$\mathrm{d}T_t = \rho_\tau \left(\tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss}) - (T_t - T_t^*)\right) \mathrm{d}t, \tag{9}$$

where ρ_τ controls the degree of inertia with $\rho_\tau \to \infty$ as the flexible limit (feedback rule), in which $T_t = T_t^* + \tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss})$. For $\rho_\tau \to 0$ we obtain the inelastic limit where $T_t \equiv T_t^*$. This fiscal policy is accompanied by a rule for government spending

$$\mathrm{d}g_t = \rho_g \left(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_t^*)\right) \mathrm{d}t, \tag{10}$$

where ρ_g controls the degree of inertia with $\rho_g \to \infty$ as the flexible limit (feedback rule), in which $g_t = g_t^* + \varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss})$. For $\rho_g \to 0$ we obtain the inelastic limit where $g_t \equiv g_t^*$. In what follows, we refer to the model parameters, or more generally, to the levels of government expenditures, taxes, and debt as 'fiscal policy', such that

$$\begin{aligned} \mathrm{d}s_t &= \rho_\tau \left(\tau_y(y_t/y_{ss} - 1) + \tau_a(a_t - a_{ss}) - (T_t - T_t^*)\right) \mathrm{d}t \\ &\quad -\rho_g \left(\varphi_y(y_t/y_{ss} - 1) + \varphi_a(a_t - a_{ss}) - (g_t - g_t^*)\right) \mathrm{d}t. \end{aligned}$$

Note that we could add others variables such as the inflation rate, π_t, which will be a function of the relevant state variables.[10] In a linearized version, such addition of variables gives rise to different parametrization of the responses to the state variables. Our results thus shed light on reasonable fiscal policy rules, which ultimately is an empirical question and beyond the scope of our analysis (e.g., Kliem and Kriwoluzky, 2014).

Kliem and Kriwoluzky (2014) show that the standard fiscal policy rules, in which tax rates respond to the level of output, are not supported by the data. Most contributions in the FTPL literature, such as Sims (2011) and Cochrane (2018), study models with an output response only.[11] Kliem et al. (2016) find that there is only weak empirical evidence in favor of output in fiscal policy rules, but rather evidence in favor of responses with respect to the fiscal stance (such as the level of debt or debt-to-GDP ratios). We follow the conventional approach and focus on (locally) determinate solutions only. As shown in Leith and von Thadden (2008), this has important implications for the admissible parameter set for a particular regime, in particular the size of parameters τ_a and φ_a.

More generally, because the discussion for the appropriate fiscal policy rules applies to both tax rates and government expenditures, we conclude that no consensus has emerged yet about $f(a_t, s_t, y_t)$ in the surplus equation (5). In contrast to most central banks with a clear mandate, the fiscal policy parameters may depend on political orientation and/or

[10]With a fiscal policy rule responding to inflation, a higher interest rate may produce lower inflation even with short-term debt (cf. Cochrane, 2022c, Chap. 5.7).

[11]Note that Sims (2011) and Cochrane (2018) impose $\rho_\tau \to \infty$ (feedback rule), and the fiscal policy rule $g = s_g(y/y_{ss} - 1)$ can be replicated for $\rho_g \to \infty$ (feedback rule) and by setting $\varphi_y = s_g$.

institutional details. But this choice is far from being innocuous: To see the role of τ_a in determining active/passive fiscal policy, abstract from inflation dynamics, $r_t \equiv i_t - \pi_t$, and consider a simple feedback rule $s_t = s_{ss} - \tau_a(a_t - a_{ss})$. A linearized version is

$$\mathrm{d}a_t = (a_{ss}(r_t - r_t^*) + (\rho - \tau_a)(a_t - a_{ss}))\mathrm{d}t. \tag{11}$$

If $\tau_a > \rho$ in (11), the real debt dynamics would be non-explosive for bounded solutions. Following Leeper (1991), this corresponds to *passive* fiscal policy and *vice versa* for the case of $\tau_a < \rho$. As soon as fiscal policy turns passive, the fiscal policy block no longer affects other variables of the model, and the model dynamics for non-fiscal-block variables coincide with the ones of the three-equation NK model. While fiscal-block variables still respond to shocks, they remain completely decoupled from the underlying NK model. Since our focus is on the recent surge in debt levels in times with monetary policy facing an effective lower bound (ELB), we focus on the fiscal regime with $\tau_a < \rho$.

Our benchmark parametrization closely follows Kliem and Kriwoluzky (2014), which allows for inertia in the fiscal policy rule for tax revenues. Since our focus is on the effects of maturity on the transmission of shocks, we abstract from introducing distortionary taxes. In the main text, we focus on a tax rule (9) with an output response $\tau_y > 0$ and an inelastic fiscal expenditure target such that $g_t \equiv g_t^*$ with $\rho_g \to 0$, and a corresponding T_t^* to match the US debt-to-GDP ratio of about 108% right before the pandemic (2020Q1).[12] We follow Bilbiie, Monacelli, and Perotti (2019) and set the steady-state government consumption-to-output ratio equal to 15.34%. A higher share of government consumption-to-output of about 20%, similar to Justiniano, Primiceri, and Tambalotti (2013) and Eichenbaum, Rebelo, and Trabandt (2020), only slightly affects the model dynamics.

Our benchmark parametrization is summarized in Table 1 such that the implied fiscal rule $f(s_t, y_t, a_t)$, in the law of motion for primary surplus (5), takes the form

$$f(s_t, y_t, a_t) \equiv y_t/y_{ss} - 1 - (s_t - s_t^*). \tag{12}$$

Market clearing and the fiscal policy rule then imply (cf. Appendix A.1.3):

$$y_t/y_{ss} - 1 = (1 - s_g)x_t. \tag{13}$$

[12] U.S. Office of Management and Budget and Federal Reserve Bank of St. Louis, Federal Debt: Total Public Debt as Percent of Gross Domestic Product [GFDEGDQ188S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GFDEGDQ188S, January 13, 2022.

Table 1: Parametrization 1 (benchmark, similar to Kliem and Kriwoluzky 2014).

ρ	0.03	subjective rate of time preference
κ	0.4421	degree of price stickiness
y_{ss}	1	normalized steady state output
ϕ_π	0.6	inflation response Taylor rule (fiscal regime)
ϕ_y	0	output response Taylor rule
θ	1	inertia Taylor rule
π_{ss}	0	inflation target rate
τ_y	1	output response fiscal tax rule (Sims, 2011; Cochrane, 2018)
τ_a	0	debt response fiscal tax rule
ρ_τ	1	inertia of fiscal tax rule
φ_y	0	output response fiscal expenditure rule
φ_a	0	debt response fiscal expenditure rule
ρ_g	0	inertia of fiscal expenditure rule
s_g	0.1534	government consumption to output ratio (Bilbiie et al., 2019)
s_{ss}	0.0324	steady-state surplus (to match US debt/GDP 2020Q1)
χ	0.03	net coupon payments (Del Negro and Sims, 2015)
$1/\delta$	6.8	average duration of government bonds (Del Negro and Sims, 2015)

Hence, the equilibrium dynamics can be summarized as

$$\mathrm{d}x_t = (i_t - \rho - \pi_t)\mathrm{d}t \tag{14a}$$

$$\mathrm{d}\pi_t = (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t \tag{14b}$$

$$\mathrm{d}i_t = (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\mathrm{d}t \tag{14c}$$

$$\mathrm{d}a_t = ((i_t - \pi_t)a_t - s_t)\mathrm{d}t \tag{14d}$$

$$\mathrm{d}s_t = ((1 - s_g)x_t - (s_t - s_t^*))\mathrm{d}t \tag{14e}$$

in which x_t, π_t are forward-looking (jump) variables, and a_t satisfies (6).[13]

2.3 Solution to the linearized equilibrium dynamics

Following the FTPL literature, we solve a linearized system around the steady state for the initial values π_0 and x_0 given the state variables i_0, a_0, and s_0.[14] To this end, we use an eigenvalue-decomposition on the Jacobian matrix of the set of differential equations and study the local dynamics induced by an unexpected (zero-probability) shock on the stable manifold back to a steady state. Technically, we solve the system using the stable eigenvalues in order to find the unique (backward) solution. The jumps in forward-looking

[13] For an alternative parametrization, $f(s_t, y_t, a_t) \equiv (\tau_a - \varphi_a)(a_t - a_{ss}) - (s_t - s_t^*)$ together with a slightly changed Phillips curve (14b), our results can be found in Appendix C.1 (cf. Table D.1).

[14] Alternative approaches, which can account for non-linearities and risk, either solve the boundary value problem for a grid of state variables to approximate the policy function (cf. Posch, 2020), or use perturbation (cf. Parra-Alvarez, Polattimur, and Posch, 2021) to obtain the policy functions.

variables π_t and x_t, together with zero-probability shocks to the state variables i_t, a_t, and s_t, determine the initial values of the endogenous model variables.

In case of long-term debt, we use the bond price equation (7) and the dependence of a_t on the price in p_t^b from the valuation equation (6). Note that we need the bond price equation (7) only to pin down the initial price jump (direct FTPL effect), which translates to a shock to a_t. For example, consider a monetary policy shock $\mathrm{d}\varepsilon_i \equiv i_t - i_{t-}$ in the model with longer-term debt and store the implied initial price jump $\mathrm{d}\delta_{p_t^b} \equiv p_t^b - p_{t-}^b$. Consider then the same monetary policy shock $\mathrm{d}\varepsilon_i$ in the model with short-term debt, without bond price effects (no direct FTPL effect), and a contemporaneous shock $\mathrm{d}\varepsilon_a \equiv a_t - a_{t-} = \mathrm{d}\delta_{p_t^b}$, i.e., use the stored price jump as an additional structural shock to a_t, the short-term debt model has exactly the same solution as the model with long-term debt.

Proposition 1 (Linear solution) *The linear approximation to the system of the model's equilibrium dynamics (14) implies a set of functions for given states (i_t, a_t, s_t)*

$$\begin{aligned} x_t &= \bar{x}_i(i_t - i_{ss}) + \bar{x}_a(a_t - a_{ss}) + \bar{x}_s(s_t - s_{ss}), && \text{(15a)} \\ \pi_t &= \pi_{ss} + \bar{\pi}_i(i_t - i_{ss}) + \bar{\pi}_a(a_t - a_{ss}) + \bar{\pi}_s(s_t - s_{ss}), && \text{(15b)} \\ p_t^b &= p_{ss}^b + \bar{p}_i^b(i_t - i_{ss}) + \bar{p}_a^b(a_t - a_{ss}) + \bar{p}_s^b(s_t - s_{ss}), && \text{(15c)} \end{aligned}$$

where bars denote the partial derivatives (slopes), evaluated at (i_{ss}, a_{ss}, s_{ss}):

$$\begin{aligned} \bar{x}_i &= x_i(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_i^b v_{ss}\bar{x}_a/(1 - v_{ss}\bar{p}_a^b), \\ \bar{x}_a &= x_v(i_{ss}, v_{ss}, s_{ss}) p_{ss}^b(1 - v_{ss}\bar{p}_a^b)/(1 - v_{ss}\bar{p}_a^b + p_{ss}^b v_{ss}\bar{p}_a^b), \\ \bar{x}_s &= x_s(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_s^b v_{ss}\bar{x}_a/(1 - v_{ss}\bar{p}_a^b), \\ \bar{\pi}_i &= \pi_i(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_i^b v_{ss}\bar{\pi}_a/(1 - v_{ss}\bar{p}_a^b), \\ \bar{\pi}_a &= \pi_v(i_{ss}, v_{ss}, s_{ss}) p_{ss}^b(1 - v_{ss}\bar{p}_a^b)/(1 - v_{ss}\bar{p}_a^b + p_{ss}^b v_{ss}\bar{p}_a^b), \\ \bar{\pi}_s &= \pi_s(i_{ss}, v_{ss}, s_{ss}) - \bar{p}_s^b v_{ss}\bar{\pi}_a/(1 - v_{ss}\bar{p}_a^b), \\ \bar{p}_i^b &= p_i^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b), \\ \bar{p}_a^b &= p_v^b(i_{ss}, v_{ss}, s_{ss})/(1 + v_{ss}p_n^b(i_{ss}, v_{ss}, s_{ss})/p_{ss}^b), \\ \bar{p}_s^b &= p_s^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b). \end{aligned}$$

Here, $v_t \equiv n_t/p_t$ defines the real number of bonds because the partial derivatives in terms of a_t (market value) reflect the indirect effects only, keeping fixed the price of government debt, p_t^b, while the total effects are visible only in terms of v_t (face value).

Proof. Appendix A.4 ■

Our linearized solution (15) thus gives the policy functions in terms of v_t in Figure 1. For illustration, we also show the policy functions in terms of a_t (cf. Figure 2). Except for the bond price p_t^b, the policy functions coincide for different maturity structures and

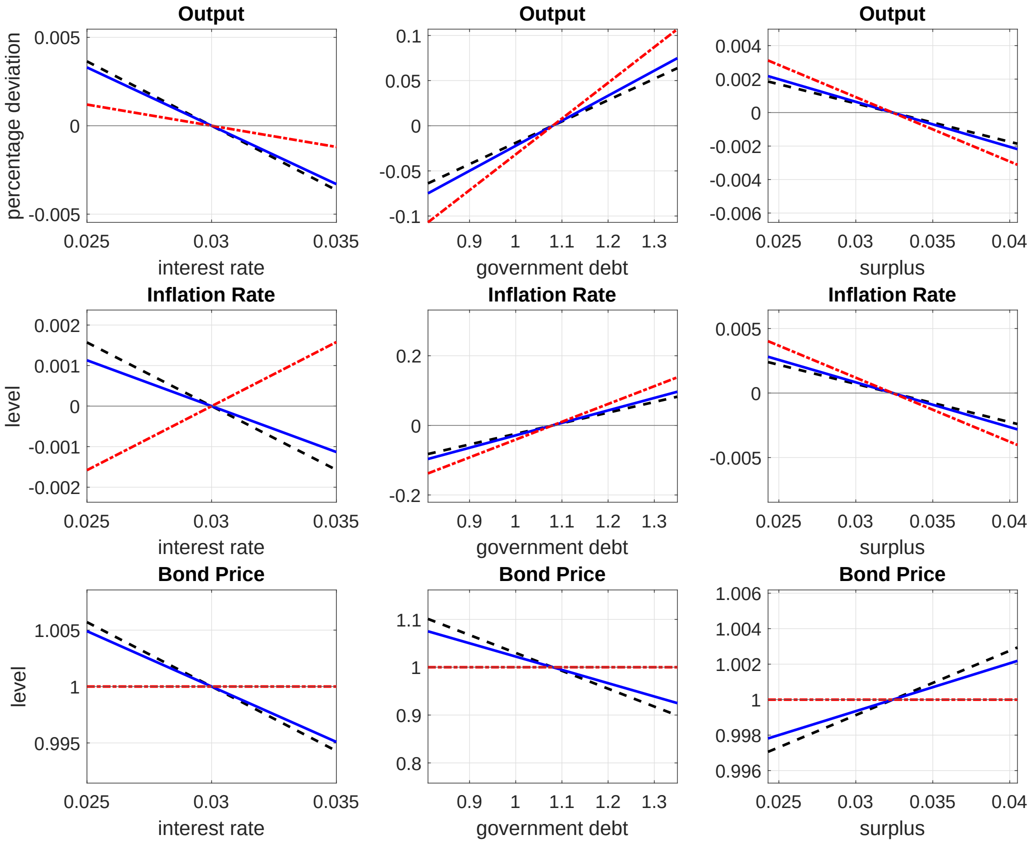

Figure 1: Policy functions for the parametrization in Table 1, showing the total response in terms of v_t (indirect and direct effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

correspond in terms of a_t to the short-term debt case in terms of v_t. Figure 1 sheds light on how the maturity structure of government debt matters for the responses of macro aggregates with changes in the state variables. Probably the most striking result is the link between inflation and interest rates: For the average duration of government bonds in the data (blue solid), we obtain the traditional negative link between interest rates and current inflation rates. This shows that the fiscal regime is crucial to the traditional effect of monetary policy. A knife-edge case exists in which the direct FTPL effect offsets the indirect effect and interest rates would have no contemporaneous effect on inflation.

2.4 Term structure of interest rates

The term structure of interest rate, defined as the yield of zero-coupon bonds as a function of their maturity, reveals important insights on expectations about the future path of macro aggregates and inflation. Given the equilibrium prices, we can price any asset. The no-arbitrage condition implies that the asset prices adjust such that the households will be indifferent in their portfolio decision. Let us consider a nominal (zero-coupon) bond

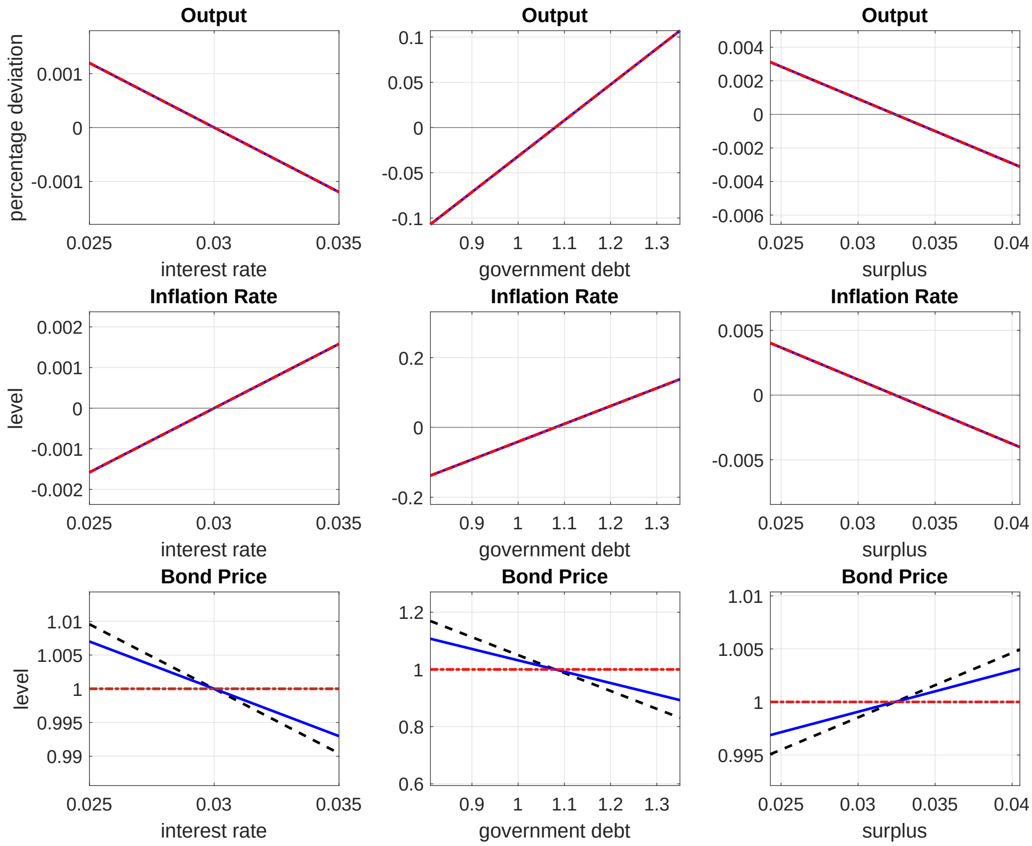

Figure 2: Policy functions for the parametrization in Table 1, showing the partial response in terms of a_t (indirect effects). Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

with unity payoff at maturity N:

$$p_t^{(N)} = \mathbb{E}_t \left(e^{-\rho N} \lambda_{t+N} / \lambda_t e^{-\int_t^{t+N} \pi_u du} \right), \tag{16}$$

where λ_t is the marginal value of wealth, or the current value shadow price, consistent with equilibrium dynamics of macro aggregates. Note that the equilibrium price p_t^b can be computed along the same lines (because the maturity distribution is approximately exponential with a duration of $1/\delta$, the average-maturity bonds will share the same properties as zero-coupon bonds at maturity $1/\delta$). The equilibrium bond price can be obtained from the fundamental pricing equation for the price $p_t^{(N)}$ (Cochrane, 2005, chap. 19.4):

$$\mathbb{E}_t \left((\,\mathrm{d}p_t^{(N)}) / p_t^{(N)} \right) - \left(1/p_t^{(N)} (\partial p_t^{(N)} / \partial N) + i_t \right) \mathrm{d}t = 0. \tag{17}$$

Observe that in equilibrium, the bond price $p_t^{(N)}$ is a function of the state variables, so $p_t^{(N)} = p_t^{(N)}(i_t, a_t, s_t)$, where from (14c), (14d), and (14e) we get

$$\begin{aligned}\mathrm{d}p_t^{(N)} &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t)\,\mathrm{d}t \\ &\quad +(\partial p_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t + ((1 - s_g)x_t - (s_t - s_t^*))\,\mathrm{d}t\end{aligned}$$

together with the solution (15) and thus the PDE (henceforth *PDE approach*) reads:

$$\begin{aligned}&(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial p_t^{(N)}/\partial i_t) + ((1 - s_g)x_t - (s_t - s_t^*))(\partial p_t^{(N)}/\partial s_t) \\ &+((i_t - \pi_t)a_t - s_t)(\partial p_t^{(N)}/\partial a_t) = (\partial p_t^{(N)}/\partial N) + i_t p_t^{(N)}.\end{aligned} \tag{18}$$

The solution to the pricing equation implies the complete term structure of interest rate for any given interest rate and maturity:

$$y_t^{(N)} \equiv y^{(N)}(i_t, a_t, s_t) = -\log p_t^{(N)}(i_t, a_t, s_t)/N. \tag{19}$$

Our strategy is to use collocation to approximate the function $p_t^{(N)} \approx \Phi(N, i_t, a_t, s_t)v$, in which v is an n-vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from a *linear* interpolation equation:

$$\begin{aligned}&(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2'(N, i_t, a_t, s_t)v + ((i_t - \pi_t)a_t - s_t)\Phi_3'(N, i_t, a_t, s_t)v \\ &+((1 - s_g)x_t - (s_t - s_t^*))\Phi_4'(N, i_t, a_t, s_t)v = \Phi_1'(N, i_t, a_t, s_t)v + i_t\Phi(N, i_t, a_t, s_t)v,\end{aligned}$$

or

$$\begin{aligned}&\big((1 - s_g)x_t - (s_t - s_t^*))\Phi_4' + ((i_t - \pi_t)a_t - s_t)\Phi_3' \\ &\quad +(\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1' - i_t\Phi\big)v = 0_n,\end{aligned} \tag{20}$$

where $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4$ with boundary condition $\Phi(0, i_t, a_t, s_t)v = 1_n$. So we concatenate the two matrices and solve the linear system for the unknown coefficients. While in this paper, we focus on the expectation channel and abstract from other determinants such as risk premia and liquidity, an extension to include risk and term premia in the analysis is straightforward (cf. Posch, 2020). In particular we want to study the effects of temporary and permanent shocks on the term structure of interest rates.

2.5 Inflation decomposition and expected inflation

Inflation and expected inflation are key determinants of monetary policy. In what follows we decompose the total effects of structural shocks on those key variables from their theoretical impulse response functions (IRFs). By the decompositions we answer the question how much such shocks contribute to the observed response.

For our decomposition based on the IRFs, we start with the linearized debt evolution using $r \equiv i_{ss} - \pi_{ss} = \rho$ and $s_{ss} = \rho a_{ss}$ (our decomposition follows Cochrane, 2022a,c)

$$\mathrm{d}(a_t/a_{ss} - 1) = (i_t - \pi_t + r(a_t/a_{ss} - 1) - s_t/a_{ss})\mathrm{d}t$$

and

$$a_t/a_{ss} - 1 = \mathbb{E}_t \int_t^\infty e^{-r(u-t)} s_u/a_{ss}\mathrm{d}u - \mathbb{E}_t \int_t^\infty e^{-r(u-t)}(i_u - \pi_u)\mathrm{d}u,$$

which is the linearized present value formula corresponding to (6). The real value of debt is the present value of surpluses, discounted at the real interest rate.

From the linearized definition (6), the real value of sovereign debt (market value) can be decomposed into

$$a_t/a_{ss} - 1 = v_t/v_{ss} - 1 + p_t^b/p_{ss}^b - 1, \tag{21}$$

either by changes in debt issued or valuation (direct effects). Hence, we get the identity

$$\begin{aligned} \int_t^\infty e^{-r(u-t)}\pi_u\mathrm{d}u \quad &= \quad \int_t^\infty e^{-r(u-t)} i_u\mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u/a_{ss}\mathrm{d}u \\ &\quad + p_t^b/p_{ss}^b - 1 + v_t/v_{ss} - 1 \end{aligned} \tag{22}$$

in the perfect-foresight model, which allows us, for example, to decompose the effects of zero-probability shocks on present values of future inflation into changes in the present value of future interest rates (monetary policy), the present value of changes in future surpluses (fiscal policy), and the direct effects (real debt decomposition).

Moreover, from (8) and with $\chi \equiv r$ and $v_t \equiv v_{ss}$ in the perfect-foresight model

$$p_t^b = 1 - \int_t^\infty e^{-(r+\delta)(u-t)}(i_u - i_{ss})\mathrm{d}u,$$

we conclude that the strength of the direct FTPL bond price effect depends on both the average maturity $1/\delta$ and the expected future path of monetary policy, at $t = 0$,

$$\int_0^\infty e^{-ru}(\pi_u - \pi_{ss})\mathrm{d}u = \int_0^\infty e^{-ru}\left(1 - e^{-\delta u}\right)(i_u - i_{ss})\mathrm{d}u - \int_0^\infty e^{-ru}(s_u - s_{ss})/a_{ss}\mathrm{d}u.$$

The effect is strongest for perpetuities with $\delta \to 0$, where all changes in future interest rates (monetary policy) will be soaked up in an initial re-evaluation of sovereign debt, and fiscal policy fully determines inflation. In contrast, in the short-term model with $\delta \to \infty$, changes in future monetary policy affect future expected inflation most.

Similarly, inflation expectations are at the core of monetary policy, often considered even as a separate variable. Hence, we can study the effects of monetary and fiscal policy shocks on the model-implied expected inflation, e.g., to confront the rational expectation forecast results with survey data. From the Phillips curve in (14b) it follows

$$\pi_t - \pi_t^* = \kappa \int_t^\infty e^{-\rho(v-t)} x_u \mathrm{d}u.$$

The inflation rate, π_t, denotes *current* expected inflation measured as deviation from its policy target rate π_t^*. Multiplying the differential equation for the inflation rate by the integrating factor and evaluating from t to $t+N$, we obtain

$$\pi_t^{(N)} \equiv \mathbb{E}_t(\pi_{t+N}) = \pi_t^* + e^{\rho N}(\pi_t - \pi_t^*) - \kappa e^{\rho N} \int_t^{t+N} e^{-\rho(u-t)} x_u \, \mathrm{d}u. \tag{23}$$

Intuitively, the model-implied inflation forecast is a forward contract to inflation, which can be more informative than using forward rates (Gürkaynak, Sack, and Wright, 2007). We compute the rational expectation forecast π_{t+N} as a function of the current state variables (i_t, a_t, and s_t) and the fixed forecasting horizon N. Hence, for the N-year ahead future expected inflation rate, we compute $\pi_t^{(N)}$ from (using Feynman-Kac)

$$\begin{aligned} \partial\pi_t^{(N)}/\partial N &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))(\partial\pi_t^{(N)}/\partial i_t)\,\mathrm{d}t \\ &\quad + (\partial\pi_t^{(N)}/\partial a_t)((i_t - \pi_t)a_t - s_t)\mathrm{d}t + (\partial\pi_t^{(N)}/\partial s_t)((1-s_g)x_t - (s_t - s_t^*))\,\mathrm{d}t \end{aligned}$$

together with the known solution (15) and by imposing the boundary condition $\pi_t^{(0)} = \pi_t$. Similar to the term structure of interest rates, the solution to the PDE then implies the N-years ahead inflation expectations for a given state variable as

$$\pi_t^{(N)} = \pi^{(N)}(i_t, a_t, s_t). \tag{24}$$

Our strategy is to use collocation to approximate the function $\pi_t^{(N)} \approx \Phi(N, i_t, a_t, s_t)v$. The n-vector v is a vector of coefficients and Φ denotes the known $n \times n$ basis matrix, and can compute the unknown coefficients from the *linear* interpolation equation

$$\begin{gathered} \big(((1-s_g)x_t - (s_t - s_t^*))\Phi_4' + ((i_t - \pi_t)a_t - s_t)\Phi_3' \\ + (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\Phi_2' - \Phi_1'\big)v = 0_n, \end{gathered}$$

where $n = n_1 \cdot n_2 \cdot n_3 \cdot n_4$ with the boundary condition $\Phi(0, i_t, a_t, s_t)v = 1_n \cdot \pi_t$. So we concatenate the two matrices and solve the linear system for the unknown coefficients.

Because the model time unit is years, the N-year ahead inflation forecast $\pi_t^{(N)}$ refers to the empirical NY1Y measure. As a simple approximation, we may define the weighted sum of N-year ahead inflation forecast for the successive k years $\pi_t^{(N,k)}$ as

$$\pi_t^{(N,k)} \approx (1/k) \ln\Big(\sum_{i=N}^{k} \big(1 + \pi_t^{(i)}\big)\Big). \tag{25}$$

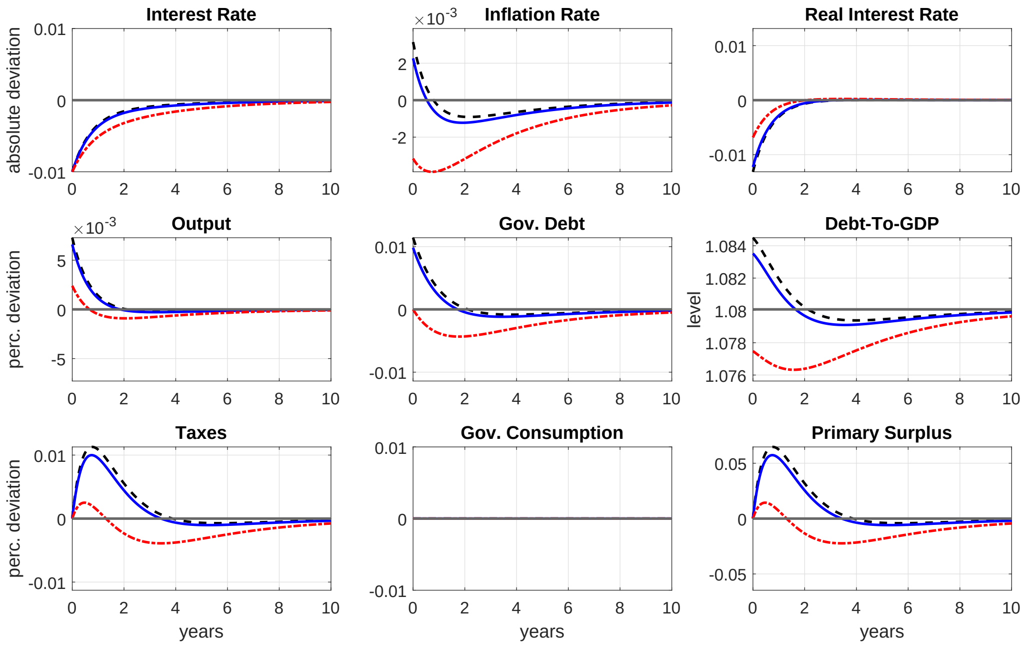

Figure 3: Transitory monetary policy shock for the parametrization in Table 1. Decrease in nominal interest rate by 1 percentage point. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 2: Inflation decomposition (22) for the monetary policy shock in Figure 3.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_v du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-rv}s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
Long-Term	-0.29	-1.14	0.29	1.14
Average	-0.48	-1.25	0.21	0.98
Short-Term	-1.62	-1.91	-0.29	0

2.6 Monetary and/or fiscal policy and transitional dynamics

Defining monetary policy shocks as changes in monetary policy with no exogenous changes in surplus (cf. Cochrane, 2018), we can answer the question of how maturity matters in the model for the transition of unexpected (zero-probability) shocks. Similarly, we consider unexpected changes in fiscal policy without changing the nominal interest rate.

2.6.1 Transitory shocks

Consider an expansionary transitory monetary policy shock of 100 basis points (bp), i.e., the policy rate i_t decreases unexpectedly by 1 percentage point. That unexpected decrease

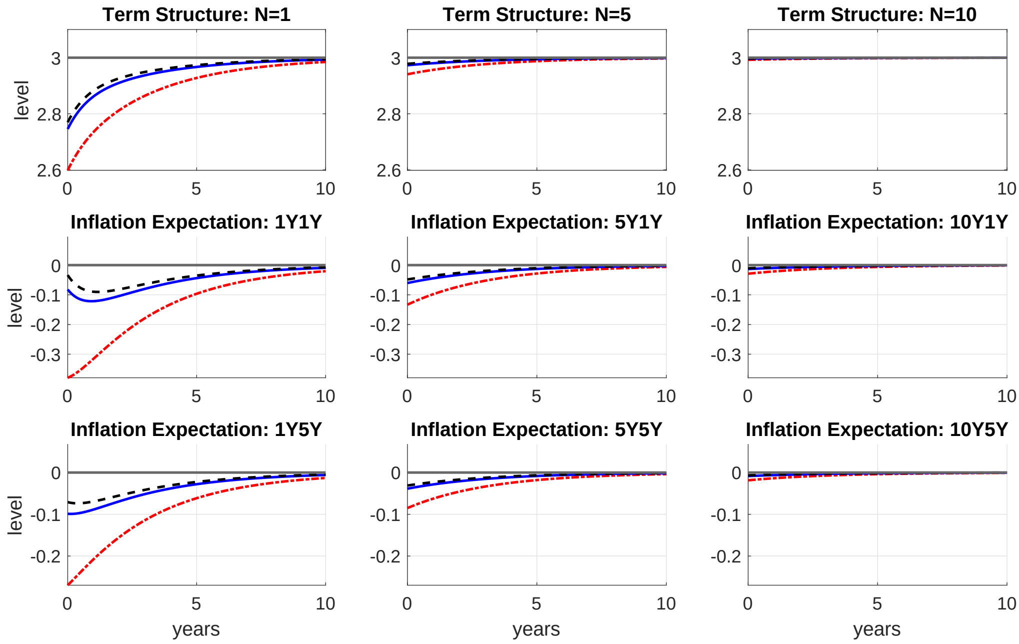

Figure 4: Transitory monetary policy shock for the parametrization in Table 1. Decrease in nominal interest rate by 1 percentage point. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

in nominal interest rates i_t initially has expansionary effects on output because the real interest rate decreases (cf. Figure 3). This effect is larger the longer the average maturity of government debt (i.e., 'stepping on a rake effect of inflation' for perpetuities). Here, the maturity structure matters because the monetary policy shock decreases the real interest rate even more for long-term bonds (black dashed) than with only short-term debt (red dotted). Because with short-term debt the direct FTPL effect is missing, the real debt does not respond immediately and we are left with the indirect FTPL effect, which unambiguously lowers inflation on impact (cf. Cochrane, 2018).

Fiscal authorities now habitually react following the specified fiscal rule and respond to the increased output by higher surpluses from increased tax receipts. A higher surplus then lowers inflation (cf. Figure 1), which again slowly increases the real interest rate. While the sign of the initial response of inflation depends on the maturity structure, which is basically dictated by the policy functions, future expected inflation turns negative for all maturities (as shown in Figure 4). In fact, the net present value of future expected inflation is negative, ranging from -0.29 to -1.62 percentage points depending on the maturity of government debt (cf. Table 2). Here, the negative effect on inflation can be attributed to either fiscal policy (black dashed), where future monetary policy is soaked up by higher bond prices, or a mix of monetary and fiscal policy, which is buffered by *lower* net present value of future tax receipts (solid blue and red dotted).

The direct FTPL effect increases the value of government debt as bonds appreciate,

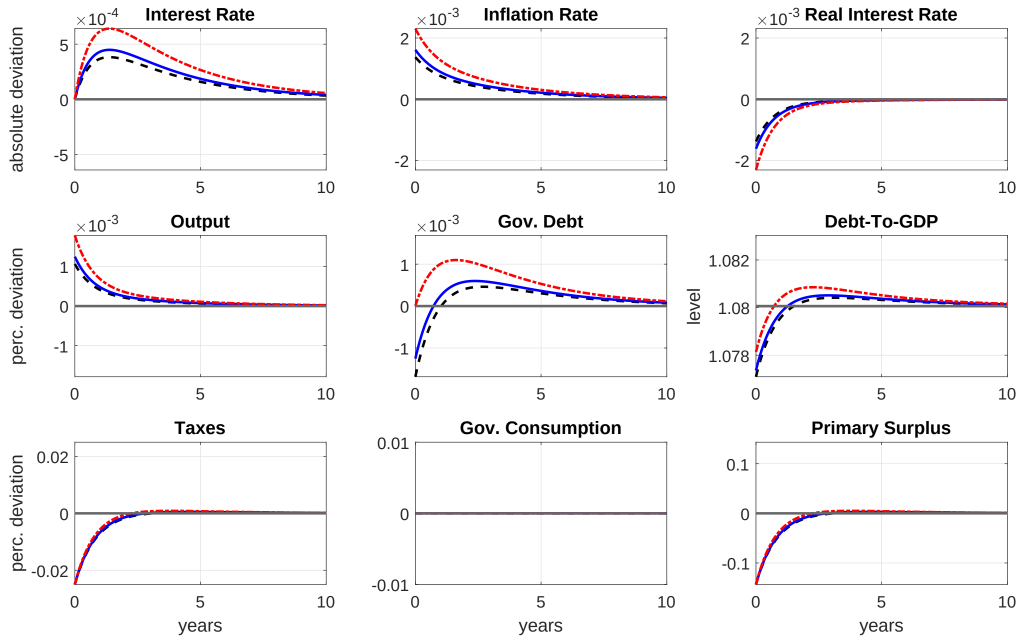

Figure 5: Transitory fiscal policy shock for the parametrization in Table 1. Decrease in taxes (surplus) by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 3: Inflation decomposition (22) for the fiscal policy shock in Figure 5.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect
Long-Term	0.29	0.17	-0.29	-0.17
Average	0.34	0.20	-0.27	-0.12
Short-Term	0.48	0.28	-0.20	0

even more than output in the case of perpetuities such that lower interest rates initially lead to a higher debt-to-GDP ratio. With short-term debt only, essentially the picture is reversed: government debt initially is reduced because of higher output, which in turn leads to a substantially lower debt-to-GDP ratio.

Along the same line, defining fiscal policy as a change in the surplus (or its components), with no change in monetary policy, we can answer the question of how maturity matters in the model for the transition of zero-probability fiscal policy shocks. Consider an expansive fiscal policy shock (cut T_t by 2.5 percent). That unexpected cut in taxes (decreases surplus s_t) has expansionary effects on output and thus unambiguously increases inflation and leads to higher inflation expectations, such that for a given short-term rate,

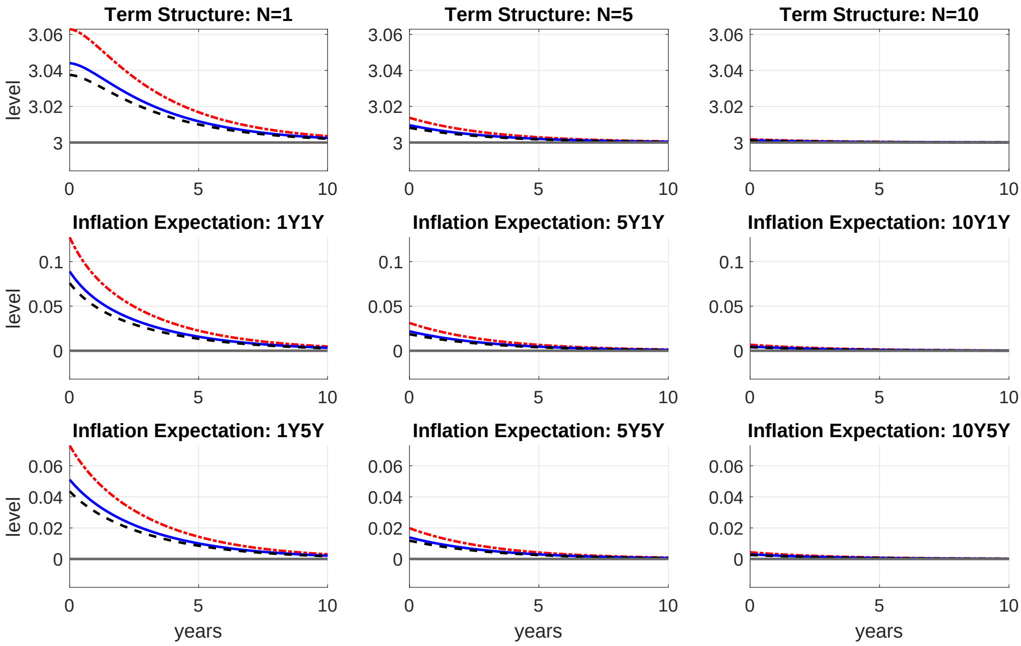

Figure 6: Transitory fiscal policy shock for the parametrization in Table 1. Decrease in taxes (surplus) by 2.5 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

the real interest rate is lower (cf. Figures 5 and 6).

Hence, expansive fiscal policy (decreased surplus) leads to more inflation and lowers the real interest rate (cf. Figure 1). This in turn causes the monetary authority, following a Taylor rule, to slightly increase nominal rates, whereas the effects on 5-year bond yields are being driven mainly by higher inflation expectations. Lower primary surpluses, after an initial devaluation of real government debt, lead to further accumulation of debt and are accompanied by higher future inflation. In fact, the net present value of future inflation is positive, ranging from 0.29 to 0.48 percentage points depending on the maturity structure of government debt (cf. Table 3). Again, the total effect on inflation can be attributed to either fiscal policy (black dashed), where future monetary policy is soaked up by lower bond prices, or a mix of monetary and fiscal policy (blue solid and red dotted).

After all, the maturity structure of government debt matters most for the direct FTPL effect, which dampens the effects on interest rates, inflation, and output dynamics. The direct FTPL effect decreases the real value of government debt as bonds depreciate and output increases, which initially leads even to a lower debt-to-GDP ratio. Here, the initial deficits are not repaid by subsequent surpluses or output growth but at the cost of higher inflation and more nominal debt, which is inflated away by subsequent *unexpected* inflation with no permanent changes in the real value of debt. This in fact is like a ‘partial default’ on nominal debt. For the case of short-term debt, higher output leads after a decrease in the debt-to-GDP ratio to more debt accumulation because the direct effect is missing,

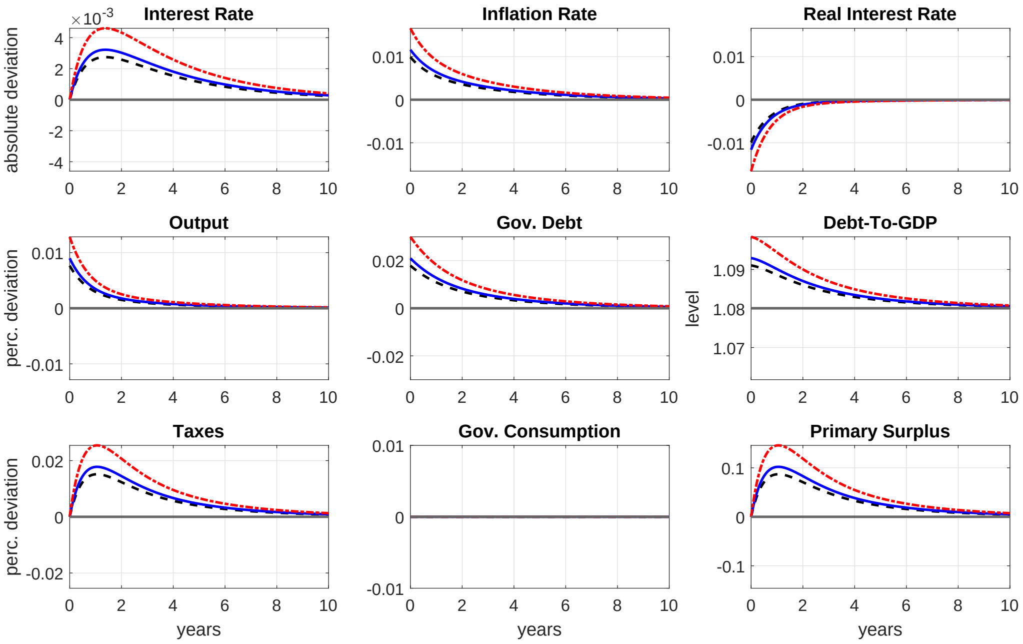

Figure 7: Transitory fiscal policy shock for the parametrization in Table 1. Increase in government debt by 3 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 4: Inflation decomposition (22) for the fiscal policy shock in Figure 7.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	2.08	1.21	0.92	-1.21	3.00
Average	2.44	1.42	1.08	-0.90	3.00
Short-Term	3.49	2.03	1.54	0	3.00

all deficits are being inflated away. What may seem like a deal, "the trick is to convince people that sinning once does not portend a dissolute life; that this is a once-and-never-again devaluation or at best a rare state-contingent default, not the beginning of a bad habit." (p.245 Cochrane, 2022c).

Finally, consider a fiscal policy shock of issuing new debt (increase n_t by 3 percent). Suppose that this increase in government debt leaves the average maturity unchanged, and that this unexpected change is without changes in long-run surpluses. Then, the newly issued debt creates unexpected inflation and higher inflation expectations because the debt is not fully paid back by subsequent surpluses (inflate away the debt) and has expansionary effects through a lower real interest rate (cf. Figures 7 and 8). In fact, the

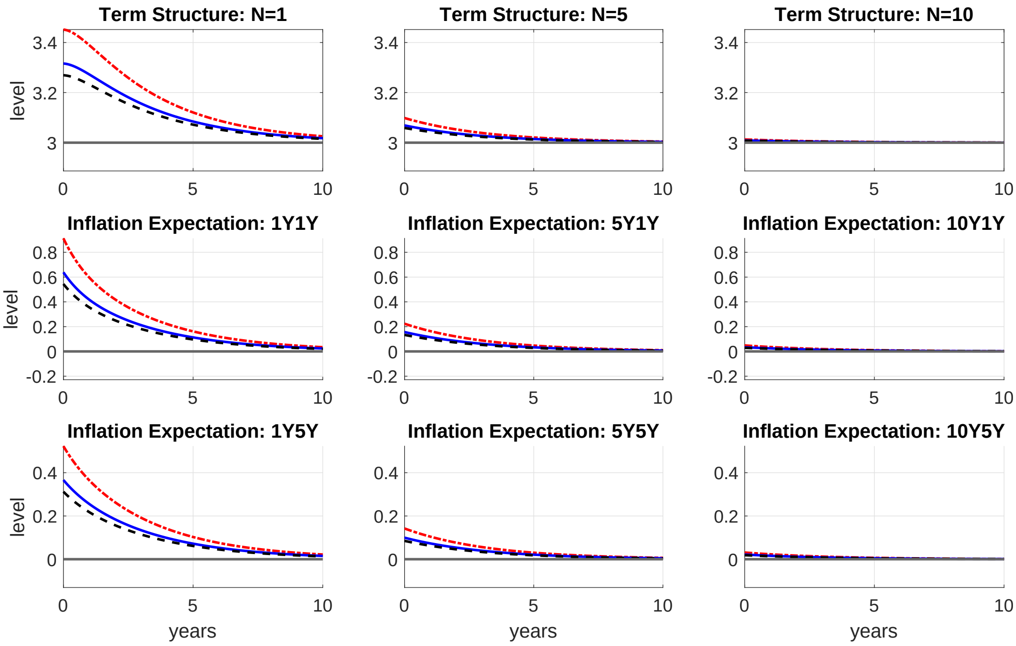

Figure 8: Transitory fiscal policy shock for the parametrization in Table 1. Increase in government debt by 3 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

net present value of future expected inflation ranges from 2.08 to 3.49 percentage points depending on the maturity structure of government debt (cf. Table 4). It is most striking for long-term debt, where the total effect on inflation and on inflation expectations is smallest as one third of the initial debt shock is repaid by higher surpluses. Only the remainder creates unexpected future inflation, and future monetary policy is soaked up by lower bond prices (black dashed). For the case of short-term debt, the direct effect does not offset monetary policy, which results in the highest net present value of future inflation, even higher than the initial debt shock (red dotted).

Again, the maturity structure of government debt matters because the direct FTPL effect devaluates long-term debt such that the initial increase in real debt (market value) is lower and the effect on inflation is largest for short-term debt. The indirect effect rises inflation and inflation expectations, which forces the monetary authority to increase nominal interest rates. Though the higher output also leads to higher tax receipts and implies a larger future primary surplus, the stimulus only partially accounts for the increased liabilities. Eventually, the unexpected increase in real debt (face value) is inflated away by unexpected future inflation and is only partially repaid by higher surpluses. However, the number of outstanding bonds increases permanently to $n_{ss} = v_{ss} e^{\int_t^\infty \pi_u \mathrm{d}u}$.

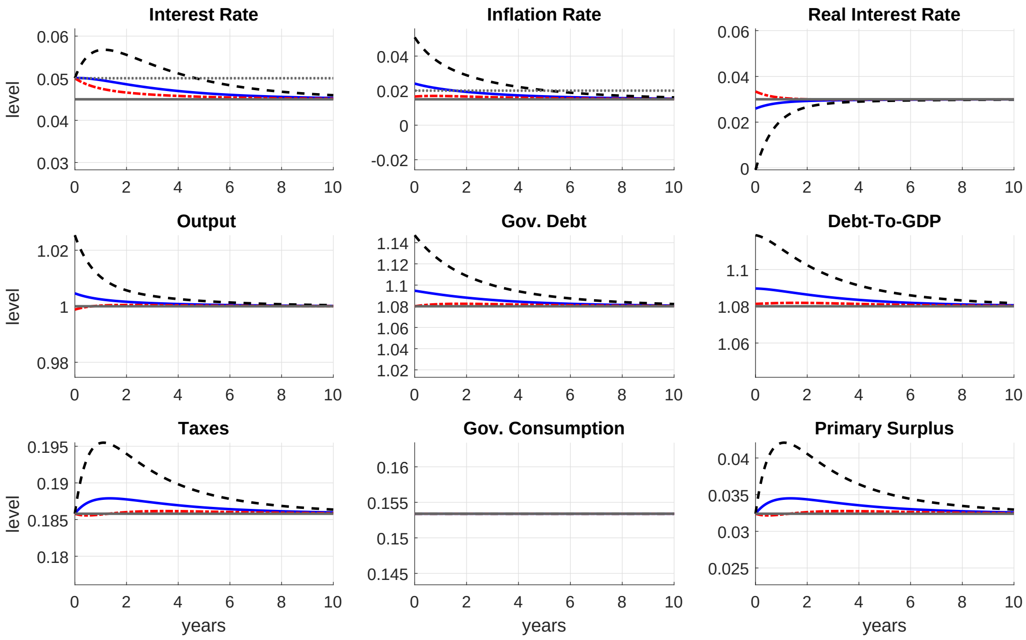

Figure 9: Permanent monetary policy shock for the parametrization in Table 1. Decrease $\pi_{ss} = 0.02$ by 50 bp to $\pi_{ss}^{new} = 0.015$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 5: Inflation decomposition (22) for the monetary policy shock in Figure 9.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^{b,new} - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	8.02	5.16	3.34	-4.91	11.11
Average	2.39	1.88	0.85	-1.24	2.60
Short-Term	0.81	0.96	0.15	0	0

2.6.2 Permanent shocks

Consider a monetary policy shock decreasing the inflation target by 50 bp, or equivalently, the policy interest rate target (which is isomorphic to the inflation target), $i_{ss}^{new} = \rho + \pi_{ss}^{new}$, decreases by 0.5 percentage points. Suppose for the moment that the policy change is fully credible and fully observed, i.e., does not require learning and filtering. An unexpected lower long-term interest rate or inflation target then has an expansionary effect on output because it creates inflation and the real interest rate decreases (cf. Figure 9, solid blue).

In all models, independent of the maturity structure, the permanent shock clearly shows up in the 10-year ahead inflation expectations and bond yields (cf. Figure 10).

While the permanent shock increases the 1-year bond yields up to 50 bp, it decreases 10-year bond yields by 50 bp (cf. Figure 10, dashed black). However, in the model with short-term debt only, the permanent lower inflation target would be even contractionary because lower current inflation increases the real interest rate. Most importantly, the maturity structure matters because the permanent shock even *increases* current expected inflation and decreases the real interest rate (solid blue and black dashed). Because the direct FTPL effect is missing in the model with short-term debt, real debt does not respond immediately and we are left with the indirect effect. However, the direct FTPL effect substantially increases the real value of existing long-term government debt such that the lower inflation target leads to a higher debt-to-GDP ratio, higher tax receipts and thus higher primary surpluses. With short-term debt, the picture is different: initially lower tax revenues (primary surpluses) and lower output with only small changes in real debt lead to negligible effects on the debt-to-GDP ratio. Hence, the maturity effect is more pronounced the longer the average maturity of government debt (cf. Table 5). In fact, current inflation increases by more than 300 bp in the model with perpetuities with net present value of future inflation of about 8 percent. How can we understand this dramatic response for inflation dynamics in the model with long-term debt?

The simple answer is that the response of inflation is due to a price or valuation effect on existing longer-term bonds, which (still) pay a nominal coupon $\chi + \delta$. Hence, a monetary policy shock in form of a lower inflation target $\pi_t^* \equiv \pi_{ss}^{new} = \pi_{ss} - 0.005$ translates into a higher price $p_{ss}^{b,new}$, and with no change in fiscal surplus results into a lower steady-state value of sovereign debt v_{ss}^{new}. From the decomposition (22), we get

$$\begin{aligned}\int_t^\infty e^{-r(u-t)}(\pi_u - \pi_{ss}^{new})\mathrm{d}u &= \int_t^\infty e^{-r(u-t)}(i_u - i_{ss}^{new})\mathrm{d}u - \int_t^\infty e^{-r(u-t)}(s_u - s_{ss})/a_{ss}\mathrm{d}u \\ &\quad + p_t^b/p_{ss}^{b,new} - 1 + v_t/v_{ss}^{new} - 1,\end{aligned}$$

or

$$\int_t^\infty e^{-r(u-t)}\pi_u \mathrm{d}u = \int_t^\infty e^{-r(u-t)} i_u \mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u/a_{ss}\mathrm{d}u + p_t^b/p_{ss}^{b,new} - 1 + v_t/v_{ss}^{new} - 1,$$

with a new

$$p_{ss}^{b,new} = \frac{\chi + \delta}{i_{ss}^{new} + \delta}, \quad \text{and} \quad v_{ss}^{new} = a_{ss}/p_{ss}^{b,new}. \tag{26}$$

Hence, a permanent monetary policy shock leads to a debt shock $v_t/v_{ss}^{new} - 1$ because of existing longer-term bonds do no longer sell at par in steady state. Relative to the lower new steady state level of government debt v_{ss}^{new} (face value), the current debt level v_t now is above its steady-state level – because debt v_t does not jump, which thus can be interpreted as an 'implicit' expansionary fiscal policy shock (compare to Figure 7). This shock is inflationary and the shock size depends on the maturity structure (cf. Table 5).

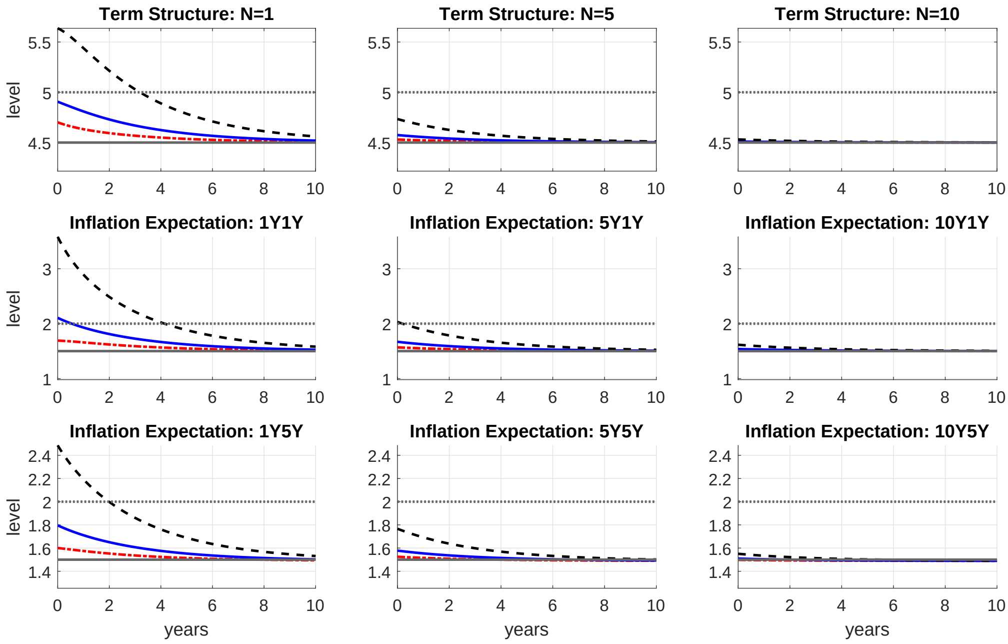

Figure 10: Permanent monetary policy shock for the parametrization in Table 1. Decrease $\pi_{ss} = 0.02$ by 50 bp to $\pi_{ss}^{new} = 0.015$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

The effect is already sizable with average maturity (by 2.60 percent), and is substantial with longer maturities (up to more than 11 percent for perpetuities). Both direct effects give the change in the market value of government debt. Even the price effect is negative of about -1.24 percent (p_0^b increases, but p_{ss}^b increases even more), the implied debt shock by 2.60 percent leads to an increase of the market value by 1.36 percent.

Along the same line, consider an expansive fiscal policy shock (cut T_t^* by 1 percent).[15] An unexpected change in future tax revenues (decreases surplus s_t^*) has expansionary effects on output today and thus increases current inflation and inflation expectations, which lowers real interest rates (cf. Figures 11 and 12). The stimulus to output quickly leads to higher tax revenues in the short run at the cost of higher inflation. In this case, the net present value of future inflation is positive, ranging from 4.02 to 6.66 percentage points depending on the maturity structure of government debt (cf. Table 6). Our fiscal policy shock leads to an instantaneous devaluation of long-term debt and dampens the effects on interest rate and inflation dynamics. Again, the total effect on inflation can be attributed either to fiscal policy (black dashed), where future monetary policy is soaked up by lower bond prices, or to a mix of monetary and fiscal policy (solid blue and red dotted). The indirect effect unambiguously rises inflation (decreases the real interest rate), which causes the monetary authority to adjust the nominal interest rates. Temporarily higher

[15] A contemporaneous fiscal policy shock $T_t = 0.99T_{t-}$ with permanent effects, $T_{ss}^{new} = 0.99T_{ss}$ has a similar decomposition and would create more unexpected inflation (cf. Figure A.2 and Table A.2).

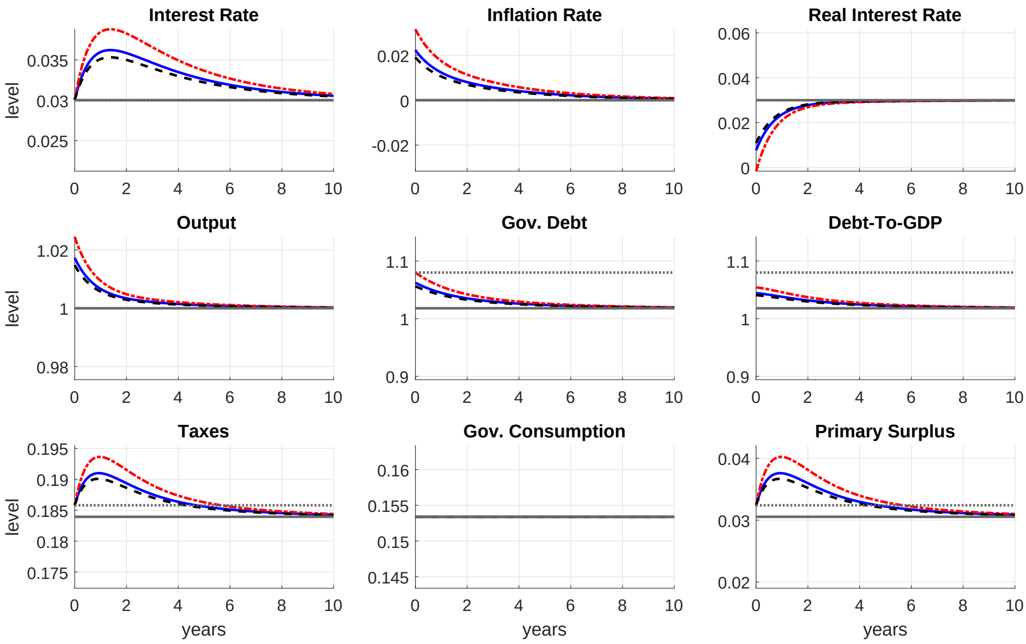

Figure 11: Permanent fiscal policy shock for the parametrization in Table 1. Decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 6: Inflation decomposition (22) for the fiscal policy shock in Figure 11.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u/a_{ss}^{new}\mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	4.02	2.34	2.07	−2.34	6.08
Average	4.70	2.74	2.38	−1.74	6.08
Short-Term	6.66	3.88	3.31	0	6.08

tax revenues (higher surplus) then lead to a further decline of government debt, and the debt-to-GDP ratio converges to its lower steady-state level.

In particular, the change in the target tax receipts, $T_t^* \equiv T_{ss}^{new} = 0.99T_{ss}$ translates into changes in the steady-state values of primary surplus, $s_{ss}^{new} = T_{ss}^{new} - g_{ss}$, and sovereign debt, $a_{ss}^{new} = s_{ss}^{new}/\rho$ or $v_{ss}^{new} = a_{ss}^{new}/p_{ss}^b$, and from the identity (22),

$$\int_t^\infty e^{-r(u-t)}(\pi_u - \pi_{ss})\mathrm{d}u = \int_t^\infty e^{-r(u-t)}(i_u - i_{ss})\mathrm{d}u - \int_t^\infty e^{-r(u-t)}(s_u - s_{ss}^{new})/a_{ss}^{new}\mathrm{d}u$$
$$+p_t^b/p_{ss}^b - 1 + v_t/v_{ss}^{new} - 1$$

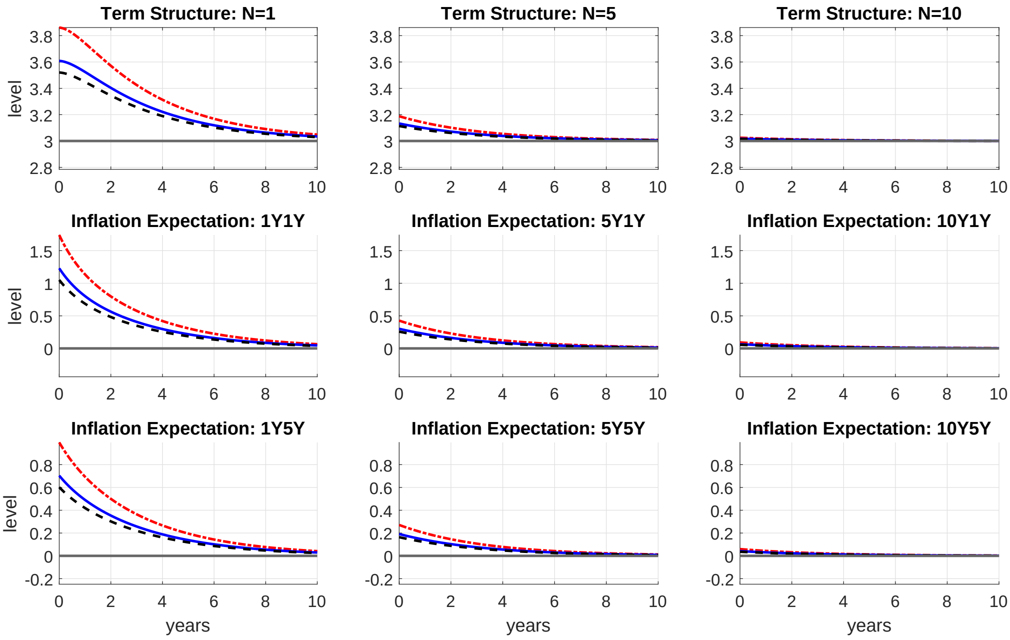

Figure 12: Permanent fiscal policy shock for the parametrization in Table 1. Decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

or

$$\begin{aligned}\int_t^\infty e^{-r(u-t)}\pi_u \mathrm{d}u &= \int_t^\infty e^{-r(u-t)} i_u \mathrm{d}u - \int_t^\infty e^{-r(u-t)} s_u / a_{ss}^{new} \mathrm{d}u \\ &\quad + p_t^b / p_{ss}^b - 1 + v_t / v_{ss}^{new} - 1\end{aligned}$$

such that our permanent fiscal policy shock leads to an 'implicit' debt shock $v_t/v_{ss}^{new} - 1$, because debt v_t does not jump and is 'too high' relative to the new and lower v_{ss}^{new}. More generally, with similar arguments – because of government debt being backed by taxes – any (austerity) measure leading to *higher* tax receipts, T^*, and/or *lower* government consumption, g_t^*, such that the steady-state primary surplus, $s_t^* = T_t^* - g_t^*$, increases, eventually need to *increase* the long-run real bond supply and the real value of government debt (increase the market and face value debt-to-GDP ratio).

3 The CARES Act

The Coronavirus Aid, Relief, and Economic Security (CARES) Act is an extensive US economic stimulus package that was signed into law on March 27, 2020, in response to the COVID-19 pandemic. Its central objective was a direct and fast assistance for the real economy in order to keep it afloat and as functioning as possible. The unprecedented volume of the act is estimated to be more than \$2 trillion (10% of US GDP). However,

Table 7: Upper Part: Predictions of the CARES Act by the Congressional Budget Office (CBO), the Joint Committee on Taxation (JCT), and estimated effect on debt-to-GDP ratio from Kaplan et al. (2020). Lower part: Translation to FTPL-NK model.

CARES Act: Empirical Figures

		Billions of Dollars	as % of GDP	as % of Outlays (receipts) 2019
A	Increased Mandatory Outlays	988	4.6%	22.2%
B	Increased Discretionary Outlays	326	1.5%	7.3%
C	Decreased Revenues	408	1.9%	11.8%
D	Estimated Increase of debt-to-GDP Ratio: 12% (cf. Kaplan et al., 2020)			

CARES Act: FTPL-NK Model

			abs. Change	as % of GDP	as % of Steady State Value
A + B	$\equiv$	Shock g_t	0.061	6.1%	39.8%
C	$\equiv$	Shock T_t	-0.019	-1.9%	-10.2%
D	$\equiv$	Shock v_t by 12% (either temporary and/or permanent)			

Sources: Congressional Budget Office (2020).

since CARES includes loan guarantees, the Congressional Budget Office (CBO) projects smaller budgetary effects. Still, the CBO estimates that CARES will add $1.7 trillion to deficits between 2020 and 2030, but most effects take place until 2022.

3.1 Taking the model to the data

In this section, we translate the empirical data to model variables and assume them to arrive as (structural) zero-probability shocks. Table 7 shows the CBO's breakdown of the $1.7 trillion into outlays and receipts. The size of the budgetary relevant part of the CARES Act exceeds more than 8% of US GDP. Following Kaplan et al. (2020), we presume that the increased outlays (6.1% of GDP) together with decreased revenues (1.9% of GDP) are going to increase the US debt-to-GDP ratio by 12% in the first eighteen months. The lower part of Table 7 shows how we transfer the CARES Act into zero-probability shocks in the FTPL-NK model. We attribute the increase in outlays to an unexpected rise in g_t by 6.1% of GDP (cf. Table 7). Here, the shock in g_t corresponds to an increase in government consumption by about 39.8%. In the empirical data, the rise in mandatory and discretionary outlays amounts to 29.5% of total expenditures in 2019. Analogously we attribute the decrease in revenues as a revenue shock by 1.9% of GDP, which translates to a decrease in tax receipts by 10.2%. Empirically, the decrease in revenues was about 11.8% of total receipts in 2019. It shows that the order of magnitude of shocks in our stylized model is roughly in line with the empirical figures.

For the simulation (see Section 3.2 below), we employ our benchmark parametrization in Table 1, except for the government expenditures (and thus surplus dynamics). Because we want to model a persistent shock to government consumption with own dynamics, we set $\rho_g \equiv 1$ and assume a counter-cyclical output response of $\varphi_y = -s_g$,

$$\mathrm{d}g_t = (\varphi_y(y_t/y_{ss} - 1) - (g_t - g_t^*))\,\mathrm{d}t, \tag{27}$$

e.g., example policies like food stamps, unemployment insurance, or predictable stimulus programs, such that surplus reacts pro-cyclically (cf. Sims, 2011; Cochrane, 2022c).

Moreover, keep in mind that monetary policy was not silent in response to the global coronavirus pandemic, but responded to the large drop in output growth and fears of deflationary pressures. In March 2020, the Federal Reserve decreased the federal funds rate in two steps from 1.58% to 0.05%. Since the timing of the rate cuts and the introduction of the CARES Act was about the same time, we study the additional effects of a temporary expansionary monetary policy shock by 150 bp (see Section 3.3). Finally, we consider the case where the unprecedented value of newly issued debt – at least to some degree – permanently increases the debt-to-GDP ratio in both face value v_{ss}^{new}/y_{ss} and market value a_{ss}^{new}/y_{ss} because $p_{ss}^b = 1$ (see Section 3.4). Our experiment sheds light on the debate of permanent vs. temporary changes in the debt-to-GDP ratio and gives important insights into the predictions of the FTPL-NK model. Recall that debt is backed by taxes such that a higher level of real debt requires a higher future surplus. Hence, we assume that tax receipts ultimately have to rise in the future, while future government consumption remains unchanged (higher value of surplus s_{ss}^{new}). We set T_{ss}^{new} to match a fraction α of the 12% projected increase (face value) in the current *and* the permanent debt-to-GDP ratio. Subsequently, we compute the predicted responses and also analyze a combination of the fiscal shocks together with the contemporaneous monetary policy shock.

3.2 The CARES Act shock

We are mainly interested in quantifying the effects of the large scale fiscal policy operation to which we refer as the CARES Act shock (cf. Table 7). Suppose that the economy is at steady state. Without a contemporaneous response of the monetary authority we now study the effects of the shocks to government consumption ($\mathbf{A} + \mathbf{B} = 6.1\%$ of GDP), and to tax receipts ($\mathbf{C} = -1.9\%$ of GDP), such that the steady state primary surplus turns

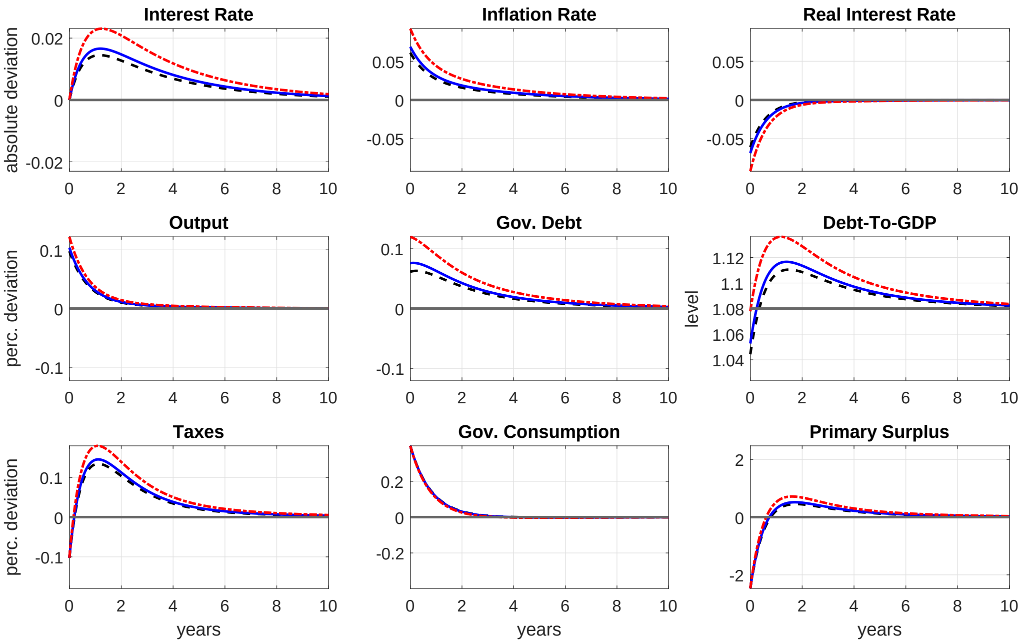

Figure 13: Transitory CARES Act shock for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table 8: Inflation decomposition (22) for the CARES Act shock in Figure 13.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u du$ inflation	$\int_0^\infty e^{-ru}i_u du$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} du$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	10.05	5.85	1.95	-5.85	12.00
Average	11.68	6.81	2.71	-4.41	12.00
Short-Term	16.68	9.71	5.03	0	12.00

into a large deficit of roughly $s_t = -8.0\%$ of GDP and amounts to nearly -250%. Finally, the CARES Act is projected to increase the debt-to-GDP ratio ($\mathbf{D} = 12\%$ of GDP). In our model, the initial increase in debt also increases output on impact. We define $\mathbf{D}$ as a shock to debt (or equivalently v_t/y_{ss}) rather than a shock to the debt-to-GDP ratio.

Both shocks to the primary surplus and to the debt-to-GDP ratio are expansionary and create unexpected current inflation between 6 and 8 percent, and increase, e.g., the 5-year ahead inflation expectations about 1 percent, such that for a given short-term rate, the real interest rate drops substantially (cf. Figures 13 and 14).

Hence, the CARES Act shock (decreased surplus and increased debt) unambiguously

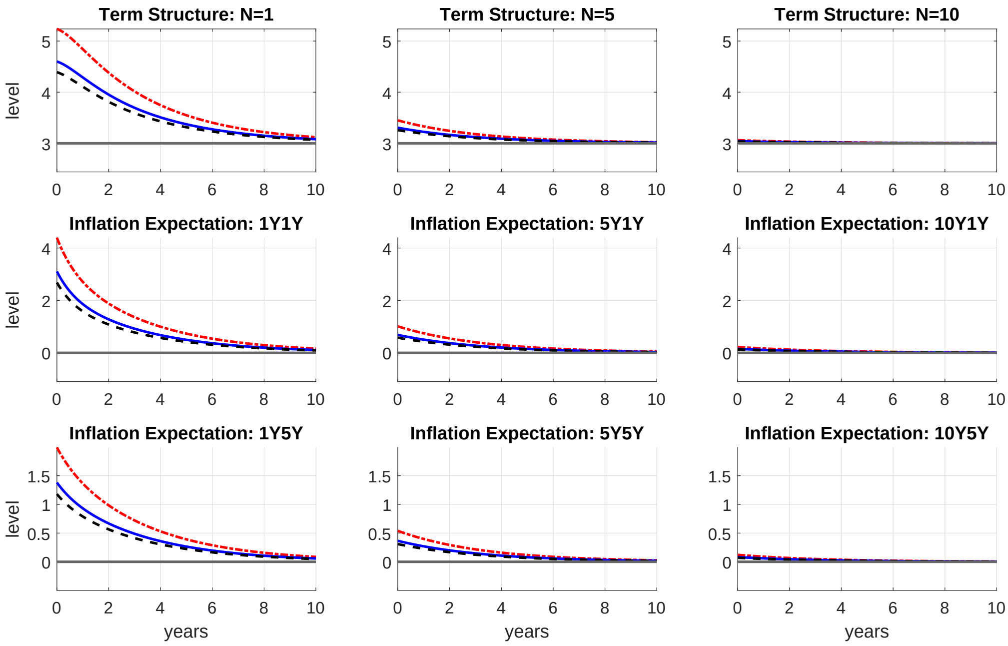

Figure 14: Transitory CARES Act shock for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

leads to higher inflation, inflation expectations, short-term bond yields, and lowers the real interest rate, which forces the monetary authority to increase nominal rates. Through the lens of fiscal theory, this unprecedented large-scale fiscal program, which is not followed by sufficiently higher subsequent surpluses, is expected to spur inflation and inflation expectations. In particular, the net present value of future inflation is even about the same size of the increase in the debt-to-GDP ratio (11.68 percentage points), and depending on the maturity structure of government debt ranges from 10 to more than 16 percentage points (cf. Table 8). Some of the newly issued debt v_t thus can be repaid by a higher net present value of future surpluses between 1.95 and 5.03 percentage points, but most of it will be deflated away by future inflation. Hence, the total effect on inflation can be fully attributed to fiscal policy and the large build-up of government debt (black dashed), where future monetary policy is soaked up by lower bond prices of -5.85 percentage points, or a mix of monetary and fiscal policy with either a slightly smaller response of bond prices (blue solid), or no response of bond prices (red dotted). Here, the shorter the maturity of government debt, the larger the effect on future inflation.

The main take-away from this experiment is a predicted surge in inflation because of the large unexpected build-up of government debt and the expansionary increase of outlays (and decrease of taxes). In the next section, we contrast these results to a situation in which the fiscal policy shock is accompanied by a monetary policy shock.

3.3 The CARES Act and monetary policy shock

In this section we quantify the effects of the CARES Act shock (cf. Table 7) together with an expansionary monetary policy shock decreasing nominal rates by 150 bp. While this shock typically increases current inflation, the net present value of future inflation is negative (cf. Section 2.6.1). Hence, the contemporaneous monetary policy shock accompanying the CARES Act might help reducing the large inflationary effects.

As a result, an accompanying monetary policy shock of 150 bp creates slightly less inflation for all maturities with similar dynamics (cf. Figure A.3 and Table A.3). Now the net present value of future interest rates is smaller because of initially lower interest rates, which translate to even more negative real interest rates and even more expansionary effects. Moreover, the net present value of future surpluses (fiscal policy) is higher for average maturity and perpetuities but slightly decreases for short-term bonds. In contrast, the direct FTPL effect is smaller because it offsets a smaller net present value of future interest rates (monetary policy). Overall, the picture does not change dramatically when assuming that the CARES Act shock was accompanied by an expansionary monetary policy shock. Though a profound analysis, which requires estimating the structural parameters and potentially latent state variables, is beyond the scope of the paper, the experiment mimics a low interest rates environment, a situation which seems more plausible for the US at the outset of the great pandemic. It shows that fiscal theory identifies the large-scale fiscal packages as the source of the recent surge in inflation.

3.4 A permanent shock scenario?

A key question is whether agents 'believe' that the observed large-scale fiscal operations will be backed by subsequent higher future surpluses. What do responses to inflation and inflation expectations tell us about such beliefs at the core of the fiscal theory? From the fiscal theory point of view, this question translates to whether the increase in debt is followed by a subsequent higher future surplus. While the higher future surplus does not necessarily have to be permanent, possibly the cleanest analysis is to ask whether the CARES Act shock is considered permanent or transitory. In what follows, we consider a scenario in which the CARES Act shock does have a permanent component causing a permanently higher debt-to-GDP ratio. Because the debt level is ultimately determined by future surpluses, a permanently higher debt level $a_{ss}^{new} \equiv s_{ss}^{new}/\rho$ requires higher surpluses s_{ss}^{new}. Put differently, the *real* debt level or debt-to-GDP ratio can increase permanently only if economic agents presume that additional debt is financed by either higher revenues and/or lower government consumption (i.e., backed by higher future surpluses).

Suppose that a fraction α of the newly issued debt is followed by permanently higher tax revenues, so that $v_{ss}^{new} = v_{ss} + \alpha(v_0 - v_{ss})$. Hence, we may interpret α as the fraction of the newly issued debt $v_0 - v_{ss} = \mathbf{D}v_{ss}$ that is backed by higher future surpluses. If the observed shock to debt v_t (face value) was permanent, i.e., the fiscal expansion was backed by higher future surpluses, we set $\alpha = 1$. If only a fraction of the newly issued debt $\alpha\mathbf{D}$ is backed by higher future surpluses, we may set $0 \leq \alpha < 1$. Here, the case of $\alpha = 1$ shows that from the fiscal theory point of view, an initial shock to v_t which is fully backed by higher future surpluses does not lead to an unexpected 'debt shock'. In fact, the effective 'debt shock' size in our inflation decomposition (22) is $(1 + \mathbf{D})/(1 + \alpha\mathbf{D}) - 1 \geq 0$.

For illustration, suppose for the moment that half of the newly issued debt are backed by subsequent higher future surpluses, $\alpha = 0.5$, which for $\mathbf{D} = 0.12$ implies a debt shock of $((1 + 0.12)/(1 + 0.5 \cdot 0.12) - 1) \cdot 100 = 5.66$ percent (cf. Figure A.4 and Table A.4). We then contrast our results to both a permanent CARES Act shock scenario with $\alpha = 1$ (cf. Figure A.5 and Table A.5) and the transitory scenario with $\alpha = 0$ (see Section 3.2). Comparing Table 8 to the permanent scenarios highlights that only the CARES Act shock in which the newly issued debt is *not* sufficiently backed by higher future surpluses leads to a surge in future expected inflation similar to the observed response.

In particular, the effects of the CARES Act on future discounted inflation with $\alpha = 1$ would be moderate between 1.72 and 2.92 percentage points. Here, the debt component of the CARES Act shock (an increase in v_t by 12 percentage points) is soaked up by higher future tax revenues such that $v_{ss}^{new} = v_0$ and $v_0/v_{ss}^{new} - 1 = 0$. Without this permanent shock, the debt shock directly would add up to 12 percentage points for $\alpha = 0$ to the net present value of future inflation, as shown in Table 8. The maturity structure matters because longer maturities dampen the response of the real value of debt through the direct effect (changes in bond prices). Similar to the temporary case with $\alpha = 0$, even in the case of $\alpha = 1$ the permanent CARES Act scenario would be expansionary and thus temporarily increases output. Consequently, the debt-to-GDP ratios (market value) for all maturities initially only increase by roughly 3.5 percentage points before gradually approaching the higher steady state value of about 120 percent.

3.5 Further discussion

Similar to Sims (2011), Leeper and Leith (2016), and Cochrane (2018), our benchmark parametrization in Table 1 with policy functions in Figures 1 and 2 suggests that sovereign debt with average maturities $1/\delta > 0$ is crucial for obtaining the traditional negative relationship between (current) inflation and the interest rate in the FTPL-NK model. It should be clarified, however, that long-term debt is useful but neither a necessary nor a sufficient condition. Cochrane (2022c) shows that a contractionary monetary policy shock can initially decrease the inflation rate even in the presence of short-term debt when we allow for a direct inflation response in the fiscal policy rule. While this specification might be controversial within empirically estimated fiscal policy rules, the consequences are intriguing and point toward the need to intensify research on fiscal policy rules.

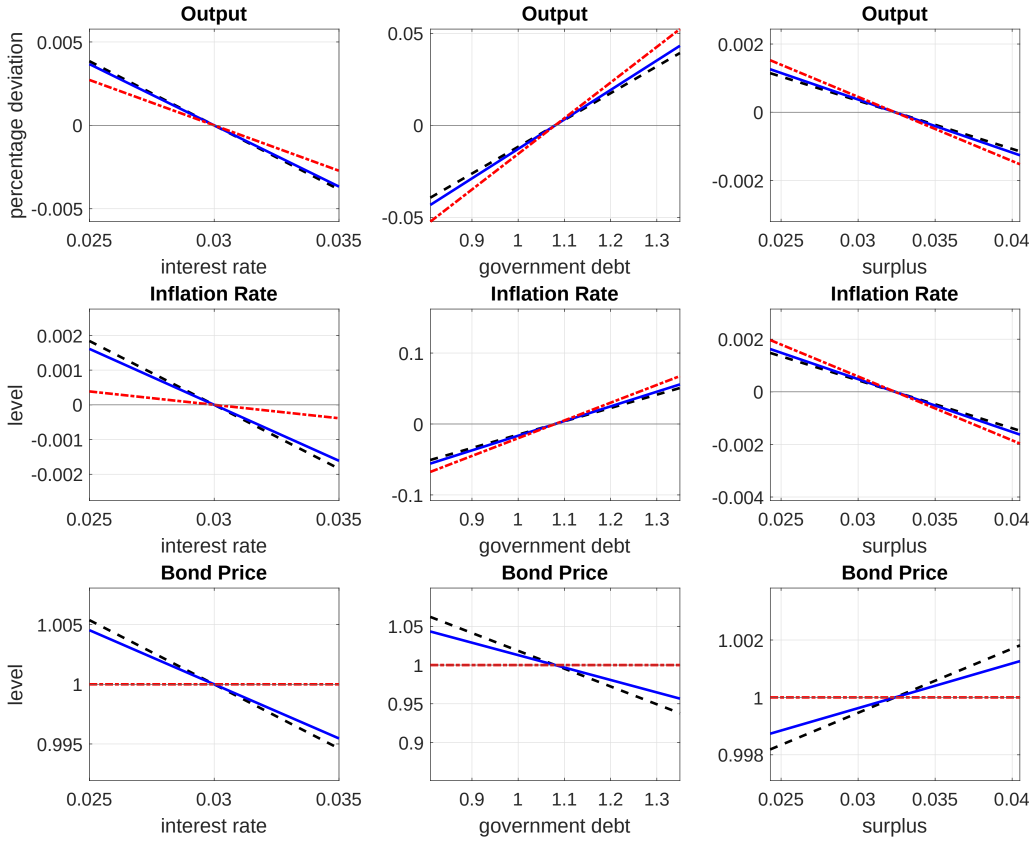

Figure 15: Policy functions for the parametrization in Table 1 together with an explicit inflation response in (28) in terms of v_t. Solid blue lines show policy functions with average duration, dashed black for perpetuities, and dotted red for short-term debt.

We may replicate Cochrane (2022c) for the parametrization in Table 1, but extending either fiscal policy rule by allowing for an explicit inflation response, e.g., replacing (9) by

$$\mathrm{d}T_t \quad = \quad \rho_\tau(\tau_y(y_t/y_{ss}-1)+\tau_a(a_t-a_{ss})+\tau_\pi(\pi_t-\pi_t^*)-(T_t-T_{ss}))\mathrm{d}t \qquad (28)$$

which is yet another specification of $f(s_t, y_t, a_t)$ in the dynamics of primary surplus (5) as long as for $x_i \neq 0$. Figure 15 shows the corresponding policy functions for $\tau_\pi \equiv 1$. In fact, a negative slope $\bar{\pi}_i$ is obtained not only for longer-term debt but also for short-term debt. Otherwise, an inflation response $\tau_\pi = 1$ does not qualitatively change the policy functions. Liemen (2022) shows how to obtain the negative inflation response with short-term debt in a FTPL-NK model with capital. In either way, the average maturity plays a role, and the introduction of longer-term bonds shapes model dynamics, as discussed in the previous sections. In other words, the maturity structure matters for macro dynamics.

Somewhat different to the CARES Act shock, fiscal policy does not routinely inflate away debt but largely consists of borrowing and credibly promising future surpluses to repay debt (see Cochrane, 2022c, p.12). Hence, a today's surplus decline must turn around and rise later on: a particular function $f(s_t, y_t, a_t)$ to which Cochrane refers an "s-shaped" surplus response. As discussed in Section 3.4, the degree to which debt is backed by higher future surpluses determines the degree to which the net present value of primary surpluses dampen or magnify the present value of future inflation. Clearly, in order to pay back the newly issued debt the primary surplus has to follow an s-shape. If a fiscal shock creates stimulus to output through unexpected inflation, it typically creates higher tax revenues and larger primary surpluses following the shock (cf. Figure 13). Such 'built-in' s-shaped dynamics are not sufficient, however, because they simply reflect that fiscal policy was *not* fully backed by subsequent higher future surpluses. As long as agents 'believe' that a fiscal policy is not fully backed by higher future surpluses, a policy shock will create unexpected inflation (compare Table 8 to Tables A.4 and A.5). Alternatively, Cochrane (2022c) introduces a latent state variable to replicate a 'typical' fiscal policy. While it may be useful for matching the empirical patterns, it should be clarified that it is not the specific timing or shape of the surplus dynamics but rather the change in the net present value which creates unexpected inflation (cf. Figure A.1 for different scenarios).[16]

A more subtle issue is the assumption of perfect foresight. Thus, the absence of risk implies that there is no term premium and/or default risk premium. In particular, our analysis neglects a potential feedback of the fiscal stance on risk premia. Though it goes beyond the scope of the present analysis, the insights from both our term structure and inflation expectation analysis are limited (cf. Posch, 2020, for a more realistic NK model). In crisis periods, governments can only 'devalue' via inflation rather than default explicitly. Because sovereign bonds are valued by the present value formula, changes of default risk due to fiscal shocks may have substantial effects on the price of existing bonds.

4 Conclusion

We revisit the fiscal theory and extend the simple NK model with a fiscal block in order to analyze the role of the maturity structure of sovereign debt on interest rates and inflation dynamics. Our results suggest that the average maturity of existing debt has a prominent role for the propagation of transitory and permanent policy shocks in the FTPL-NK model. We show how the effects translate to the term structure of interest rate and to model-implied inflation expectations. Our finding justifies a critical assessment of neglecting the direct FTPL effect in the traditional NK framework. Through the lens of the fiscal theory, we decompose the present value of future inflation into indirect effects (changes in future monetary policy and fiscal policy) and a direct FTPL effect, which basically is an asset pricing re-evaluation of existing bonds. In particular, we highlight that sovereign debt, with an empirically plausible average maturity for the US, largely

[16]See also Cochrane (2022b) for a simple discrete-time version with partially-repaid debt.

offsets the impact of monetary policy on the present value of future inflation.

Our application simulates the CARES Act of 2020, which we translate to shocks to the primary surplus of about 8 percent of GDP and to the debt (face value) by 12 percent. Without a credible future (s-shaped) policy change, the FTPL-NK model predicts a surge in inflation, which amounts to an increase of the net present value of future inflation about the same size as the increase of newly issued debt. We show how this dramatic inflation response not only depends on the average maturity of existing bonds, but also primarily on the perception of agents whether the large-scale fiscal operations are ultimately backed by a higher future surplus or not. In contrast to the aftermath of the global financial crisis of 2008, where the inflation response was not as strong or inflation even declined, the recent surge in inflation and medium-term inflation expectations indicates that the newly issued debt is not considered as being backed by subsequent higher surpluses.

We believe that this paper is a promising starting point for the fiscal theory in more elaborate models, including regime-switching, nonlinearities, and stochastic shocks. First, our results for the term structure of interest rates and inflation expectations would be much more informative. Our setup is a natural starting point and benchmark for models with term premia (cf. Posch, 2020), convenience yield, or default risk. Second, more research is needed for the surplus dynamics, e.g., estimating parameters of the fiscal policy rule (cf. Kliem et al., 2016). Third, we need to study the effects of maturity in medium-size NK models including regime switches (see Bianchi and Melosi, 2019), financial frictions (cf. Brunnermeier and Sannikov, 2014), and productive capital (cf. Brunnermeier, Merkel, and Sannikov, 2021; Liemen, 2022), and to study the effects and transmission in models with heterogeneous agents (cf. Kaplan, Moll, and Violante, 2018; Bayer et al., 2021). This opens the path toward a more profound fiscal policy evaluation and to address questions of fiscal limits and sovereign defaults (fiscal sustainability).

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A Appendix

A.1 Technical details FTPL model

In this paper, we use a linear version of the micro-founded NK model (cf. Posch, 2020). The basic structure of the model is as follows. A representative household consumes, saves, and supplies labor. The final output is assembled by a final good producer, who uses as inputs a continuum of intermediate goods manufactured by monopolistic competitors. The intermediate good producers rent labor to manufacture their good and face the constraint that they can only adjust the price following Calvo's pricing rule (Calvo, 1983). Finally, there is a monetary authority that fixes the short-term nominal interest rate through open market operations following a Taylor rule and a detailed government sector with a fiscal authority that issues debt, taxes, and consumes following fiscal policy rules.

A.1.1 Households

Let the reward function of the households be given as

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left\{ \log c_t - \psi \frac{l_t^{1+\vartheta}}{1+\vartheta} \right\} dt, \qquad \psi > 0, \tag{A.1}$$

where ρ denotes the subjective rate of time preference, ϑ is the inverse of the Frisch labor supply elasticity, and ψ scales the disutility from working by supplying labor in terms of hours l_t (we use ψ to normalize $l_{ss} = 1$). Let n_t denote the number of shares of government bonds; assuming that each bond has a nominal value of one unit, whereas p_t^b is the equilibrium price of bonds. Suppose the household earns a disposable income of

$$\delta^c n_t + p_t w_t l_t - p_t T_t + p_t \digamma_t$$

where δ^c are coupon payments, p_t is the price level (or price of the consumption good), w_t is the real wage, T_t are lump-sum taxes, and $\digamma_t$ are the profits of the firms in the economy. Hence, the household's budget constraint reads

$$\mathrm{d}n_t = \big((\delta^c n_t - p_t c_t + p_t w_t l_t - p_t T_t + p_t \digamma_t) / p_t^b - \delta n_t \big) \, \mathrm{d}t, \tag{A.2}$$

in which p_t^b denotes the bond price. Each bond pays a proportional coupon χ per unit of time and is amortized at the rate δ.

The first-order condition for households to maximize (A.1) subject to (A.2) is

$$\psi l_t^\vartheta c_t = mc_t, \tag{A.3}$$

which is the standard static optimality condition between labor and consumption. Hence, for the given preferences (A.1), the Euler equation for consumption reads (cf. Posch, 2020)

$$\mathrm{d}c_t = (i_t - \pi_t - \rho)c_t \,\mathrm{d}t, \tag{A.4}$$

or the linearized version

$$\mathrm{d}c_t \approx (i_t - \rho - \pi_t)c_{ss} \,\mathrm{d}t, \tag{A.5}$$

with π_t being determined in general equilibrium.

A.1.2 The final good producer

There is one final good, produced using intermediate goods with

$$y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \,\mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.6}$$

where ε is the elasticity of substitution.

Final good producers are perfectly competitive and maximize profits subject to the production function (A.6), taking as given all intermediate goods prices p_{it} and the final good price p_t. Hence, the input demand functions associated with this problem are:

$$y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\varepsilon} y_t \qquad \forall i,$$

and

$$p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}} \tag{A.7}$$

is the (aggregate) price level.

A.1.3 Intermediate good producers

Each intermediate firm produces differentiated goods out of labor using:

$$y_{it} = l_{it}, \tag{A.8}$$

where l_{it} is the amount of the labor input rented by the firm. Therefore, the marginal cost of the intermediate good producer is the same across firms:

$$mc_t = w_t. \tag{A.9}$$

The monopolistic firms engage in price setting à la Calvo, which then gives rise to the NK Phillips curve (see, e.g., Leith and von Thadden, 2008; Posch, 2020)

$$\mathrm{d}(\pi_t - \pi_{ss}) \approx (\rho(\pi_t - \pi_{ss}) - \kappa_0(mc_t/mc_{ss} - 1)) \,\mathrm{d}t. \tag{A.10}$$

Note that from (A.3) $\psi y_t^{\vartheta} c_t = mc_t$ such that a linearized version is

$$mc_t/mc_{ss} - 1 \approx (c_t/c_{ss} - 1) + \vartheta(y_t/y_{ss} - 1).$$

Moreover, for the parametrization in Table 1, we have that $g_t \equiv g_{ss}$ and thus

$$\begin{aligned} \mathrm{d}(\pi_t - \pi_{ss}) &= (\rho(\pi_t - \pi_{ss}) - \kappa_0((c_t/c_{ss} - 1) + \vartheta(y_t/y_{ss} - 1)))\,\mathrm{d}t \\ &= (\rho(\pi_t - \pi_{ss}) - \kappa_0((y_t/y_{ss} - 1)y_{ss}/c_{ss} + \vartheta(y_t/y_{ss} - 1)))\,\mathrm{d}t \\ &\equiv (\rho(\pi_t - \pi_{ss}) - \kappa x_t)\,\mathrm{d}t \end{aligned} \tag{A.11}$$

as in (2), where $x_t \equiv (y_t/y_{ss} - 1)/(1 - s_g)$ is the output gap and $\kappa \equiv \kappa_0(1 + \vartheta(1 - s_g))$ captures 'price stickiness'. Our definition of the output gap is to formulate the benchmark model as close as possible to the one used in the literature, where typically $s_g \equiv 0$.

Note that with this definition of the output gap, we obtain (1) from (A.5) as

$$\begin{aligned} \mathrm{d}(y_t - g_{ss}) &= (i_t - \rho - \pi_t)(y_{ss} - g_{ss})\,\mathrm{d}t \\ &= (i_t - \rho - \pi_t)(1 - s_g)y_{ss}\,\mathrm{d}t \end{aligned}$$

after inserting our definition $x_t \equiv (y_t/y_{ss} - 1)/(1 - s_g)$.

For the parametrization in Table D.1 in the online appendix, with variable government consumption,

$$\begin{aligned} mc_t/mc_{ss} - 1 &= (1 + \vartheta(1 - s_g))(y_t/y_{ss} - 1)/(1 - s_g) - (g_t/g_{ss} - 1)s_g/(1 - s_g) \\ &= (1 + \vartheta(1 - s_g))x_t - (g_t/g_{ss} - 1)s_g/(1 - s_g) \end{aligned}$$

and thus the Phillips curve in the generalized version obeys

$$\mathrm{d}(\pi_t - \pi_{ss}) = (\rho(\pi_t - \pi_{ss}) - \kappa x_t + \kappa_0 s_g/(1 - s_g)(g_t/g_{ss} - 1))\,\mathrm{d}t. \tag{A.12}$$

A.1.4 Government

We assume that the monetary authority sets the nominal interest rate i_t of short-term bonds through open market operations according to either the feedback model,

$$i_t - i_t^* = \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1), \quad \phi_\pi > 0,\ \phi_y \geq 0, \tag{A.13a}$$

or the partial adjustment model (cf. Posch, 2020):

$$\mathrm{d}i_t = \theta(\phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t/y_{ss} - 1) - (i_t - i_t^*))\mathrm{d}t, \quad \theta > 0, \tag{A.13b}$$

which includes a response to inflation and output, and a desire to smooth interest rates.

The fiscal authority trades a nominal non-contingent bond. Let n_t be the outstanding stock of nominal government bonds, i.e., the total nominal value of outstanding debt (alternative assets are priced using arbitrage arguments but are in net zero supply). The government incurs a real primary surplus $s_t \equiv T_t - g_t$ where revenues T_t and expenditure g_t rules are given in (9) and (10). Each bond pays a proportional coupon χ per unit of time and is amortized at the rate δ. Hence, the government faces the constraint that the newly issued debt must cover amortization plus coupon payments of outstanding debt, net of the primary surplus such that the nominal value of outstanding debt follows

$$\mathrm{d}n_t = \big(((\delta + \chi)n_t - p_t s_t)\,/p_t^b - \delta n_t\big)\ \mathrm{d}t, \tag{A.14}$$

where p_t^b is the bond price.

A.1.5 Aggregation

First, market clearing demands:

$$y_t = c_t + g_t = c_t + T_t - s_t, \tag{A.15}$$

and suppose aggregate output is produced according to (e.g., in the linearized model)

$$y_t = l_t$$

in which we normalized to $y_{ss} = l_{ss} \equiv 1$ in the benchmark parametrization, and the income is generated through

$$y_t = w_t l_t + \digamma_t.$$

All outstanding sovereign debt is owned by households, so (A.2) and (A.14) yield

$$(\delta + \chi)n_t - p_t s_t = \delta^c n_t - p_t c_t + p_t w_t l_t - p_t T_t + p_t \digamma_t.$$

Recall that the real value of sovereign debt is defined as in (6), $a_t = n_t p_t^b / p_t$. In equilibrium,

$$i_t \mathrm{d}t = ((\chi + \delta)/p_t^b - \delta)\mathrm{d}t + (1/p_t^b)\,\mathrm{d}p_t^b$$

such that the bond price follows (7). We define the inflation rate π_t such that

$$\mathrm{d}p_t = \pi_t p_t \mathrm{d}t \tag{A.16}$$

and the (realized) rate of inflation is locally non-stochastic.

Hence, the budget constraint of the fiscal authority (6) can be written as

$$\begin{aligned}
\mathrm{d}a_t &= (p_t^b \, \mathrm{d}n_t + n_t \, \mathrm{d}p_t^b - n_t p_t^b / p_t \, \mathrm{d}p_t)/p_t \\
&= ((\delta + \chi)n_t/p_t - s_t) \; \mathrm{d}t - \delta n_t p_t^b/p_t + n_t \, \mathrm{d}p_t^b/p_t - n_t p_t^b/p_t(1/p_t) \, \mathrm{d}p_t \\
&= ((\delta + \chi)n_t/p_t - s_t) \; \mathrm{d}t - \delta a_t \, \mathrm{d}t + a_t i_t \, \mathrm{d}t - ((\delta + \chi)n_t/p_t - \delta a_t) \, \mathrm{d}t - a_t \pi_t \, \mathrm{d}t,
\end{aligned}$$

which is equation (4) in the fiscal block.

Similarly, the household's budget constraint (A.2) can be written as

$$\begin{aligned}
\mathrm{d}a_t &= (p_t^b \, \mathrm{d}n_t + n_t \, \mathrm{d}p_t^b - n_t p_t^b / p_t \, \mathrm{d}p_t)/p_t \\
&= \big((\delta + \chi)a_t/p_t^b - s_t\big) \; \mathrm{d}t - \delta a_t + a_t(1/p_t^b) \, \mathrm{d}p_t^b - a_t \pi_t \, \mathrm{d}t \\
&= \big((\delta + \chi)a_t/p_t^b - s_t\big) \; \mathrm{d}t - \delta a_t + (-((\delta + \chi)/p_t^b - \delta) + i_t)a_t \mathrm{d}t - a_t \pi_t \, \mathrm{d}t \\
&= -s_t \, \mathrm{d}t + i_t a_t \mathrm{d}t - a_t \pi_t \, \mathrm{d}t \\
&= ((i_t - \pi_t)a_t + w_t l_t - c_t - T_t + \digamma_t) \, \mathrm{d}t,
\end{aligned}$$

which again shows that the household's budget constraint coincides with the government budget constraint. Using (A.2) and (A.14), together with market clearing (A.15), the coupon payments cover payouts and amortization such that $\delta^c \equiv \delta + \chi$.

A.1.6 Steady-state values

From (1), (4), and (7), we obtain $i_{ss} = \rho + \pi_{ss}$, $a_{ss} = s_{ss}/\rho$, and $p_{ss}^b = 1$. In this model

$$mc_{ss} = w_{ss} = \frac{\varepsilon - 1}{\varepsilon},$$

where ε is the elasticity of substitution between intermediate goods. Moreover, condition (A.3) implies together with the market clearing condition (A.15) that

$$\psi l_{ss}^{\vartheta} c_{ss} = w_{ss}.$$

Observe that $c_{ss} = y_{ss} - g_{ss} = l_{ss} - g_{ss}$, defining $s_g = g_{ss}/y_{ss}$ such that

$$\psi l_{ss}^{1+\vartheta}(1 - s_g) = w_{ss}.$$

Hence, we parameterize

$$\psi \equiv w_{ss} l_{ss}^{-(1+\vartheta)}/(1 - s_g)$$

to normalize the steady-state output $y_{ss} = l_{ss} = 1$, such that $\digamma_{ss} = 1/\varepsilon$, $c_{ss} = 1 - g_{ss}$, $T_{ss} = s_{ss} + g_{ss}$ (s_{ss} and s_g are calibrated using US targets).

A.2 Reformulation in terms of real debt in face value

Recall equation (A.14)

$$\mathrm{d}n_t = \left(\left((\delta+\chi)n_t - p_t s_t\right)/p_t^b - \delta n_t\right)\,\mathrm{d}t.$$

With the price level following

$$\mathrm{d}p_t = p_t \pi_t \mathrm{d}t.$$

Define,

$$v_t \equiv n_t/p_t \tag{A.17}$$

so that v_t is the value of debt (face value) in real terms. Differentiating,

$$\mathrm{d}v_t = \mathrm{d}\left(\frac{n_t}{p_t}\right) = \frac{\mathrm{d}n_t}{p_t} - \frac{n_t}{p_t}\frac{\mathrm{d}p_t}{p_t}.$$

or

$$\mathrm{d}v_t = \left(\left((\delta+\chi)/p_t^b - \delta - \pi_t\right)v_t - s_t/p_t^b\right)\mathrm{d}t. \tag{A.18}$$

Thus, we can rewrite our baseline model as

$$\begin{aligned}
\mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && \text{(A.19a)}\\
\mathrm{d}\pi_t &= (\rho(\pi_t - \pi_t^*) - \kappa x_t)\mathrm{d}t && \text{(A.19b)}\\
\mathrm{d}i_t &= (\phi_\pi(\pi_t - \pi_t^*) - (i_t - i_t^*))\mathrm{d}t && \text{(A.19c)}\\
\mathrm{d}v_t &= \left(\left((\delta+\chi)/p_t^b - \delta - \pi_t\right)v_t - s_t/p_t^b\right)\mathrm{d}t && \text{(A.19d)}\\
\mathrm{d}s_t &= ((1-s_g)x_t - (s_t - s_t^*))\mathrm{d}t. && \text{(A.19e)}
\end{aligned}$$

Sims (2011) and Cochrane (2018) utilize the real value of debt, a_t, as relevant state variable in their models, which can jump due to changes in the bond price. In contrast to real market value debt a_t, real face value debt, v_t, does not jump. Thus, we can use v_t together with the bond price, p_t^b, to obtain the real debt (market value) as

$$a_t \equiv v_t p_t^b. \tag{A.20}$$

This formulation makes the reasons for jumps in a_t clearer. Furthermore, it simplifies the interpretation of shocks to debt (we can directly shock v_t). However, keep in mind that both model formulations imply the same dynamics and refer to the same model.

A.3 Linearized dynamics

In this paper use the linearized NK model, so we need to linearize the equations (A.4), (4), and (7). Let us summarize the equilibrium dynamics for our parametrization. Alternative equilibrium dynamics are summarized in the online appendix.

A.3.1 Benchmark parametrization (Table 1)

Using $\pi_t^* = \pi_{ss}$, $i_t^* = i_{ss} = \rho + \pi_{ss}$, and $s_t^* = s_{ss}$, together with the parametrization of the benchmark model (cf. Table 1), the linearized equilibrium dynamics can be written as

$$\begin{aligned}
\mathrm{d}x_t &= (i_t - \rho - \pi_t)\mathrm{d}t && \text{(A.21)}\\
\mathrm{d}\pi_t &= (\rho(\pi_t - \pi_{ss}) - \kappa x_t)\,\mathrm{d}t && \text{(A.22)}\\
\mathrm{d}i_t &= (\phi_\pi(\pi_t - \pi_{ss}) - (i_t - i_{ss}))\mathrm{d}t && \text{(A.23)}\\
\mathrm{d}a_t &= (a_{ss}(i_t - \pi_t - \rho) + \rho(a_t - a_{ss}) - (s_t - s_{ss}))\mathrm{d}t && \text{(A.24)}\\
\mathrm{d}s_t &= ((y_t/y_{ss} - 1) - (s_t - s_{ss}))\,\mathrm{d}t && \text{(A.25)}\\
\mathrm{d}p_t^b &= \big((i_t - i_{ss}) + (\chi + \delta)(p_t^b - 1)\big)\,\mathrm{d}t, && \text{(A.26)}
\end{aligned}$$

where

$$y_t/y_{ss} - 1 = (c_t - c_{ss} + g_t - g_{ss})/y_{ss}$$

such that with $g_t = g_{ss}$ we get $\kappa \equiv (1 + \vartheta(1 - s_g))\kappa_0$, and

$$x_t = (y_t/y_{ss} - 1)/(1 - s_g) = (c_t/c_{ss} - 1)(c_{ss}/y_{ss})/(1 - s_g) = (c_t/c_{ss} - 1),$$

i.e., the consumption Euler equation can be written in terms of the output gap.

A.4 Proof of Proposition 1

Recall that in the model with long-term debt, a proper predetermined state variable (which does not jump) is v_t rather than a_t, hence, we linearize

$$a_t - a_{ss} \;=\; p_{ss}^b(v_t - v_{ss}) + v_{ss}(p_t^b - p_{ss}^b)$$

such that the real value of government debt changes due to two channels

$$\mathrm{d}a_t = p_{ss}^b\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p_t^b. \tag{A.27}$$

The partial derivatives of the policy function $x(i_t, a_t, s_t)$ show the indirect FTPL effect for a given bond price, p_t^b, such that we need to isolate the direct FTPL effect due to the re-evaluation of sovereign debt. Now, evaluating the effect of a change to i_t at some reference point, say $\bar{x}_i = x_i(i_{ss}, a_{ss}, s_{ss})$, the slope of the policy function in terms of a_t would only include the indirect effect, keeping fix the price of government debt. Note that our solution implies both $p_t^b = p^b(i_t, v_t, s_t)$ or $p_t^b = p^b(i_t, a_t, s_t)$ such that

$$\mathrm{d}p_t^b = p_i^b(i_t, v_t, s_t)\,\mathrm{d}i_t + p_v^b(i_t, v_t, s_t)\,\mathrm{d}v_t + p_s^b(i_t, v_t, s_t)\,\mathrm{d}s_t \tag{A.28}$$

and $\mathrm{d}p_t^b = p_i^b(i_t, a_t, s_t)\,\mathrm{d}i_t + p_a^b(i_t, a_t, s_t)\,\mathrm{d}a_t + p_s^b(i_t, a_t, s_t)\,\mathrm{d}s_t$ and thus using (A.27)

$$\mathrm{d}p_t^b = p_i^b(i_{ss}, a_{ss}, s_{ss})\,\mathrm{d}i_t + p_a^b(i_{ss}, a_{ss}, s_{ss})(p_{ss}^b\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p_t^b) + p_s^b(i_{ss}, a_{ss}, s_{ss})\,\mathrm{d}s_t$$

or equivalently

$$\begin{aligned}\mathrm{d}p_t^b \quad = \quad & \frac{p_i^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}i_t + \frac{p_{ss}^b p_a^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}v_t \\ & + \frac{p_s^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})}\,\mathrm{d}s_t \end{aligned} \tag{A.29}$$

and by matching coefficients with (A.28)

$$\begin{aligned} p_i^b(i_t, v_t, s_t) \quad &= \quad \frac{p_i^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})} \\ p_v^b(i_t, v_t, s_t) \quad &= \quad \frac{p_{ss}^b p_a^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})} \\ p_s^b(i_t, v_t, s_t) \quad &= \quad \frac{p_s^b(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p_a^b(i_{ss}, a_{ss}, s_{ss})}, \end{aligned}$$

we can conclude that

$$\begin{aligned} \bar{p}_i^b \quad &\equiv \quad p_i^b(i_{ss}, a_{ss}, s_{ss}) = p_i^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b) \\ \bar{p}_a^b \quad &\equiv \quad p_a^b(i_{ss}, a_{ss}, s_{ss}) = \frac{p_v^b(i_{ss}, v_{ss}, s_{ss})}{1 + v_{ss}p_n^b(i_{ss}, v_{ss}, s_{ss})/p_{ss}^b} \\ \bar{p}_s^b \quad &\equiv \quad p_s^b(i_{ss}, a_{ss}, s_{ss}) = p_s^b(i_{ss}, v_{ss}, s_{ss})(1 - v_{ss}\bar{p}_a^b). \end{aligned}$$

Similarly, for the inflation rate we can utilize

$$\mathrm{d}\pi_t = \pi_i(i_t, v_t, s_t)\,\mathrm{d}i_t + \pi_n(i_t, v_t, s_t)\,\mathrm{d}n_t + \pi_s(i_t, v_t, s_t)\,\mathrm{d}s_t \tag{A.30}$$

or, equivalently,

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)\,\mathrm{d}a_t + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t. \tag{A.31}$$

We substitute equation (A.27)

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)(p_{ss}^b\,\mathrm{d}v_t + v_{ss}\,\mathrm{d}p_t^b) + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t$$

or

$$\mathrm{d}\pi_t = \pi_i(i_t, a_t, s_t)\,\mathrm{d}i_t + \pi_a(i_t, a_t, s_t)p^b_{ss}\,\mathrm{d}v_t + \pi_s(i_t, a_t, s_t)\,\mathrm{d}s_t + v_{ss}\pi_a(i_t, a_t, s_t)\,\mathrm{d}p^b_t.$$

Substitute by equation (A.29)

$$\begin{aligned}
\mathrm{d}\pi_t &= \left(\pi_i(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}i_t \\
&\quad + \left(\pi_a(i_{ss}, a_{ss}, s_{ss})p^b_{ss} + \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}v_t \\
&\quad + \left(\pi_s(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}\right)\mathrm{d}s_t
\end{aligned}$$

and matching coefficients with equation (A.30)

$$\begin{aligned}
\pi_i(i_{ss}, v_{ss}, s_{ss}) &= \pi_i(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_i(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\
\pi_v(i_{ss}, v_{ss}, s_{ss}) &= \pi_a(i_{ss}, a_{ss}, s_{ss})p^b_{ss} + \frac{p^b_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})} \\
\pi_s(i_{ss}, v_{ss}, s_{ss}) &= \pi_s(i_{ss}, a_{ss}, s_{ss}) + \frac{p^b_s(i_{ss}, a_{ss}, s_{ss})v_{ss}\pi_a(i_{ss}, a_{ss}, s_{ss})}{1 - v_{ss}p^b_a(i_{ss}, a_{ss}, s_{ss})}.
\end{aligned}$$

Rearranging terms we arrive at

$$\begin{aligned}
\bar{\pi}_i &\equiv \pi_i(i_{ss}, a_{ss}, s_{ss}) = \pi_i(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_i v_{ss}\bar{\pi}_a}{1 - v_{ss}\bar{p}^b_a} \\
\bar{\pi}_a &\equiv \pi_a(i_{ss}, a_{ss}, s_{ss}) = \pi_v(i_{ss}, v_{ss}, s_{ss})\frac{p^b_{ss}(1 - v_{ss}\bar{p}^b_a)}{1 - v_{ss}\bar{p}^b_a + v_{ss}p^b_{ss}\bar{p}^b_a} \\
\bar{\pi}_s &\equiv \pi_s(i_{ss}, a_{ss}, s_{ss}) = \pi_s(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_s v_{ss}\bar{\pi}_a}{1 - v_{ss}\bar{p}^b_a}.
\end{aligned}$$

We proceed analogously for the output gap, $x(i_t, v_t, s_t)$ and $x(i_t, v_t, s_t)$. Except for notation the derivations are identical to the inflation rate. Thus,

$$\begin{aligned}
\bar{x}_i &\equiv x_i(i_{ss}, a_{ss}, s_{ss}) = x_i(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_i v_{ss}\bar{x}_a}{1 - v_{ss}\bar{p}^b_a} \\
\bar{x}_a &\equiv x_v(i_{ss}, a_{ss}, s_{ss}) = x_v(i_{ss}, v_{ss}, s_{ss})\frac{p^b_{ss}(1 - v_{ss}\bar{p}^b_a)}{1 - v_{ss}\bar{p}^b_a + v_{ss}p^b_{ss}\bar{p}^b_a} \\
\bar{x}_s &\equiv x_s(i_{ss}, a_{ss}, s_{ss}) = x_s(i_{ss}, v_{ss}, s_{ss}) - \frac{\bar{p}^b_s v_{ss}\bar{x}_a}{1 - v_{ss}\bar{p}^b_a},
\end{aligned}$$

which closes the proof (inflation rates and output gap analogously).

A.5 Figures and Tables

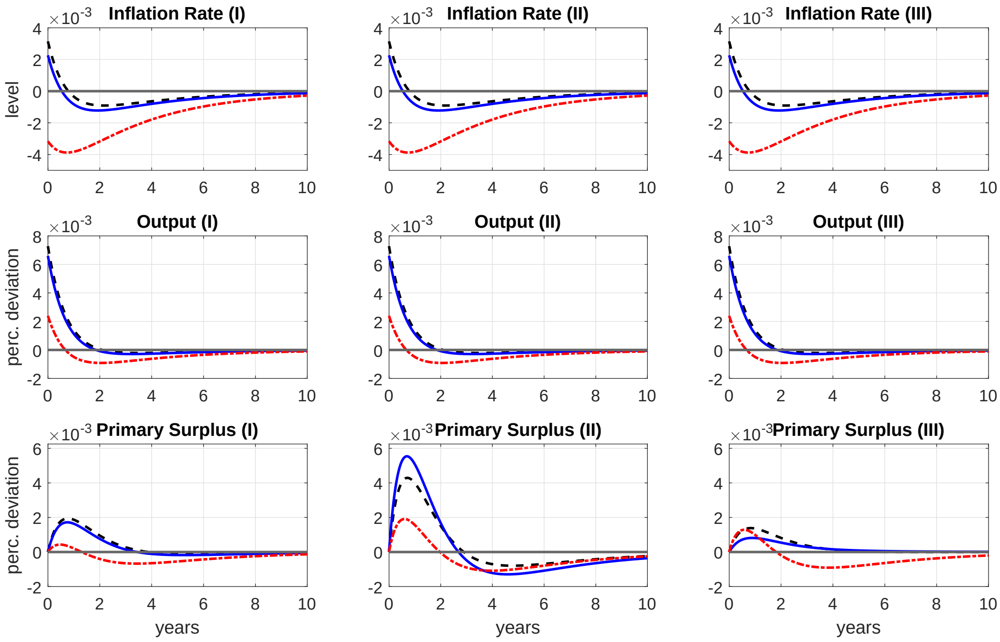

Figure A.1: Transitory monetary policy shock for the parametrization in Table 1 and different surplus dynamics. Decrease nominal interest rate by 1 percentage point. Left-hand panel: Baseline scenario, $\tau_\pi = 0$ and $\tau_y = 1$. Middle panel: $\tau_\pi = 1.02$ and $\tau_y = 3.08$. Right-hand panel: $\tau_\pi = 0.5$ and $\tau_y = -0.25$. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.1: Inflation decomposition (22) for the monetary policy shock in Figure A.1.

Surplus Rule	Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru} i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru} s_u / a^{new}_{ss} \mathrm{d}u$ surplus	$p^b_0/p^b_{ss} - 1$ direct effect
I	Average	-0.48	-1.25	0.21	0.98
II	Average	-0.48	-1.25	0.21	0.98
III	Average	-0.48	-1.25	0.21	0.98

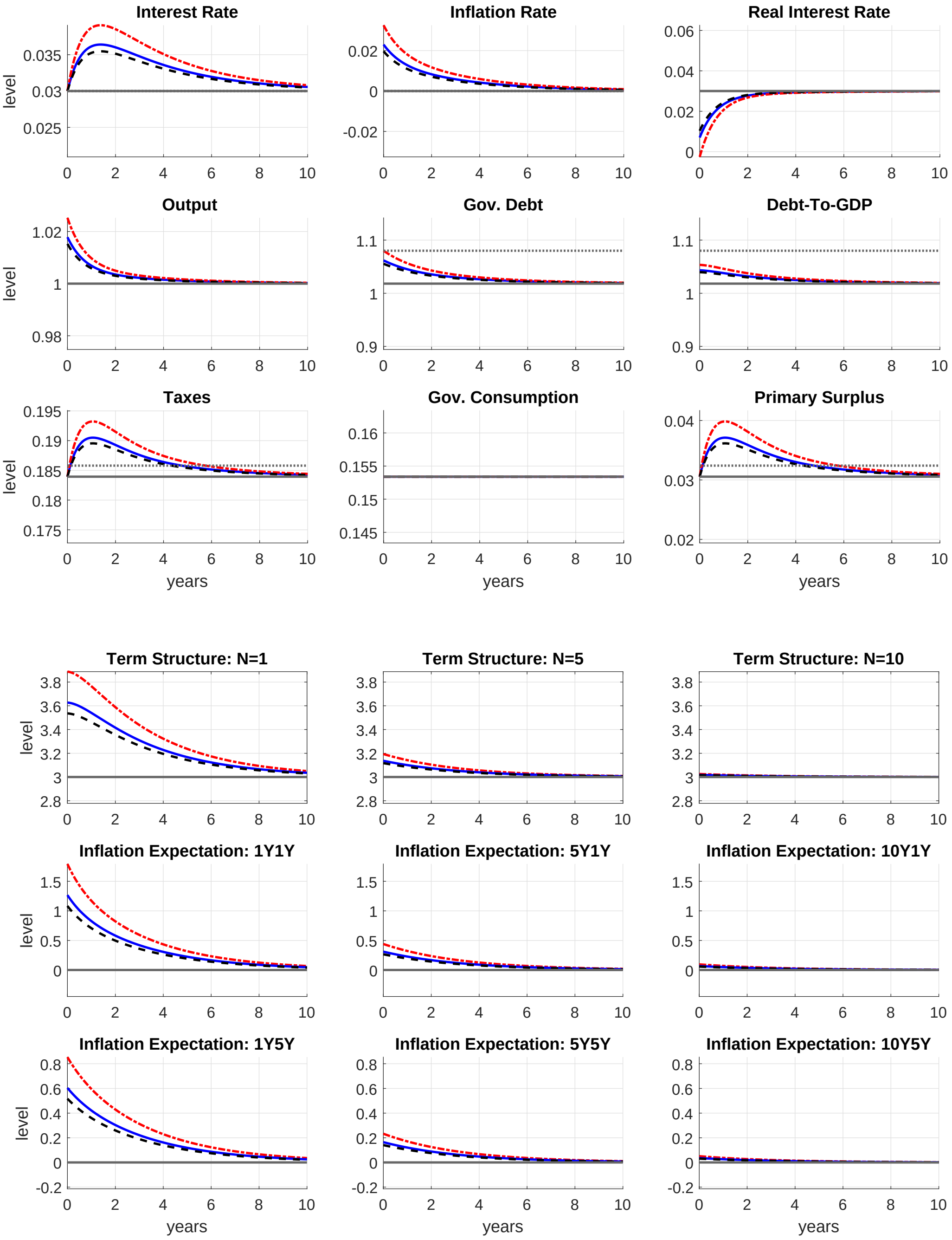

Figure A.2: Permanent fiscal policy shock for parametrization in Table 1. Permanent decrease of T_{ss} by 1 percent to $T_{ss}^{new} = 0.99T_{ss}$, together with a transitory shock that decreases taxes by 1 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

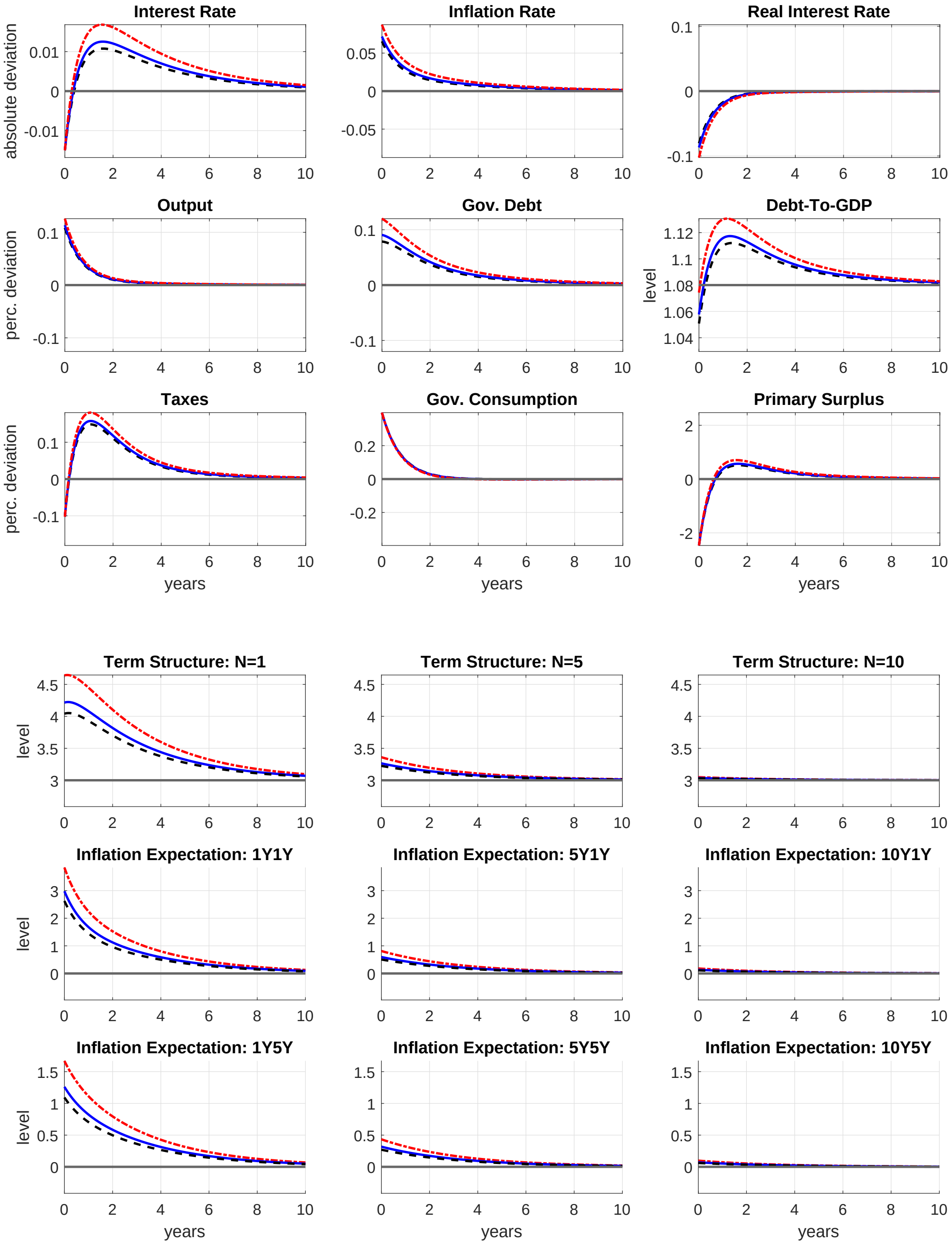

Figure A.3: CARES Act and monetary policy shock using parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP, and increase in debt (face value) by 12 percent, and decrease interest rates by 150 bp. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

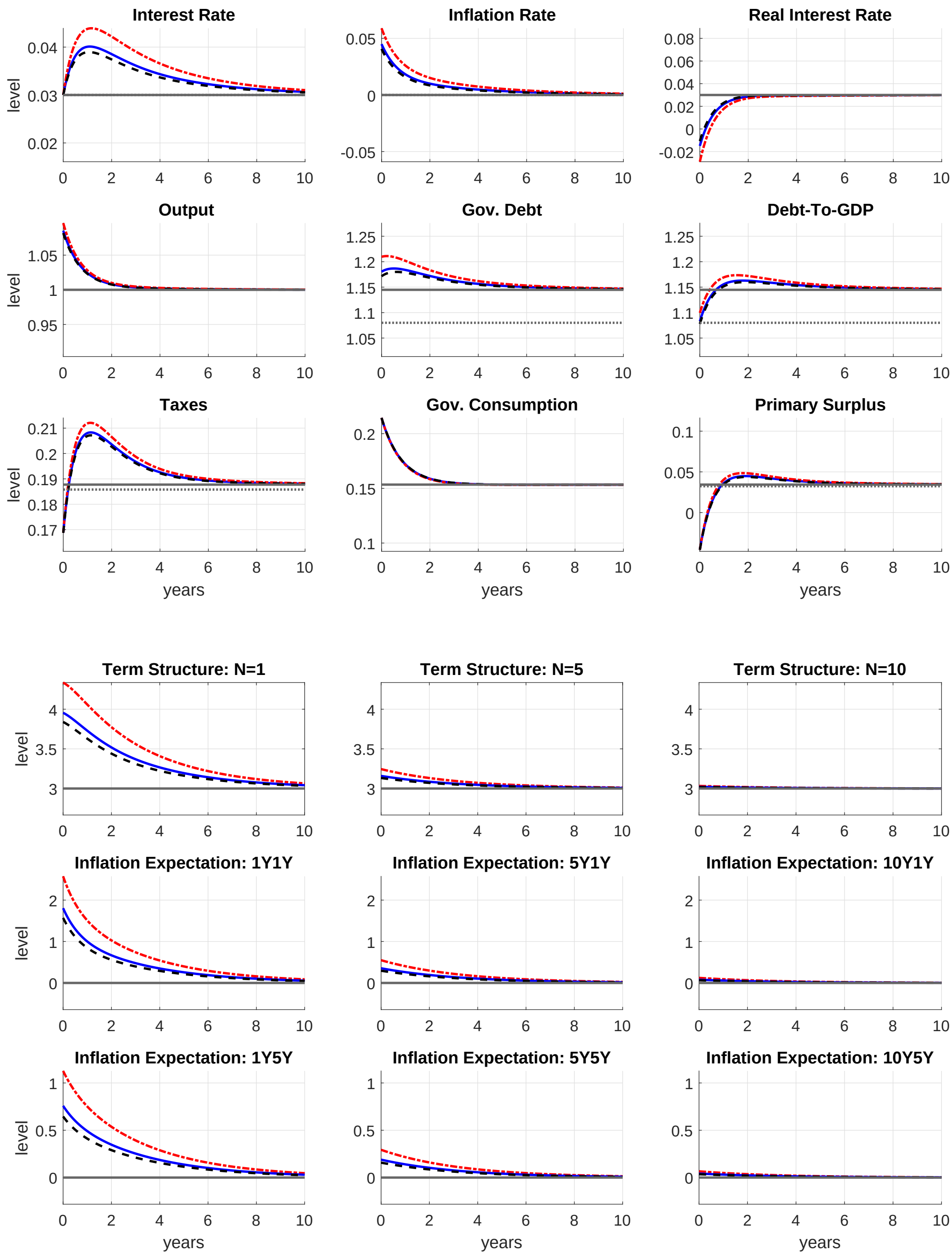

Figure A.4: CARES Act shock with permanent increase of v_{ss} by 6 percent ($\alpha = 0.5$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

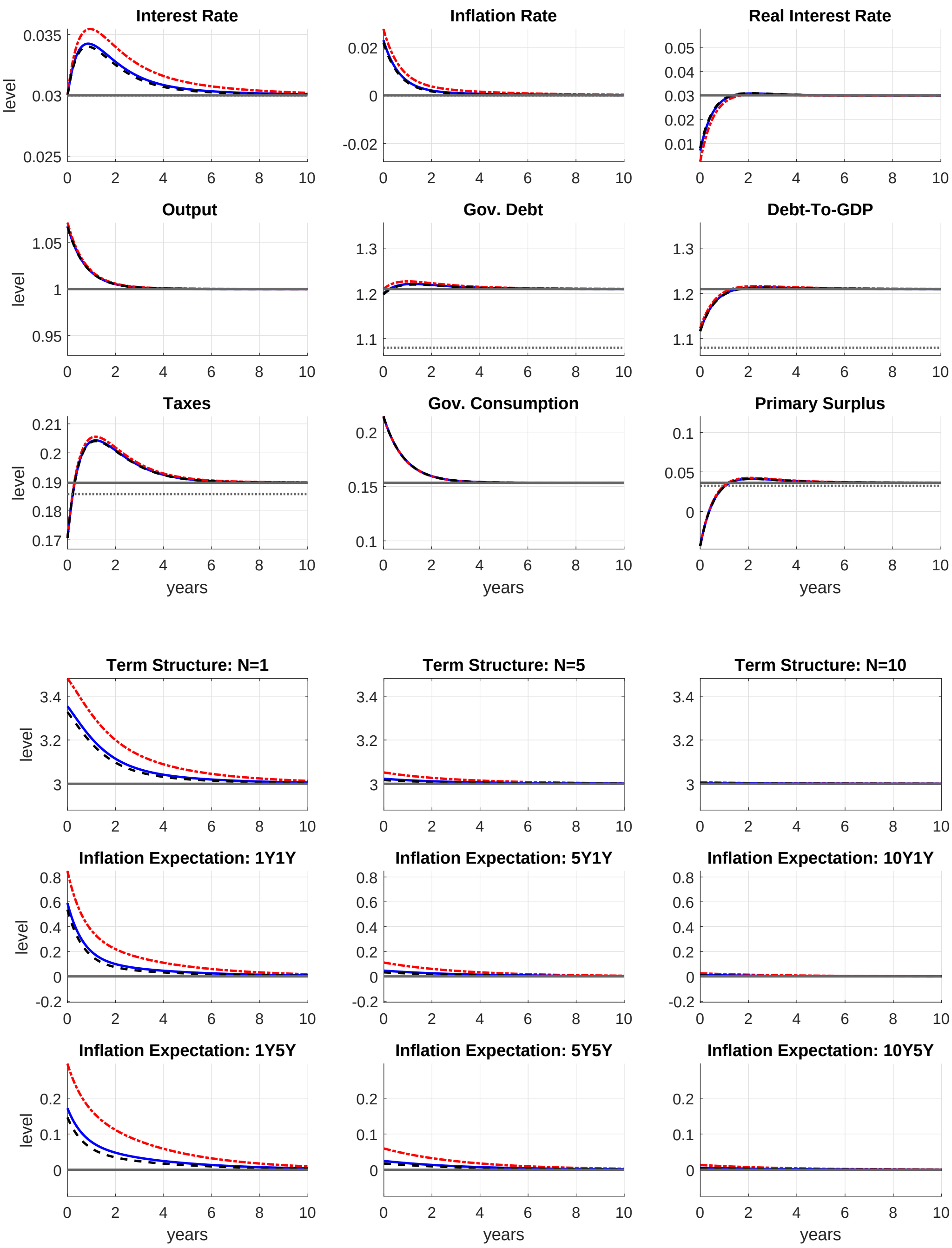

Figure A.5: CARES Act shock with permanent increase of v_{ss} by 12 percent ($\alpha = 1$) for the parametrization in Table 1 with $\rho_g = 1$ and $\varphi_y = -s_g$. Decrease in surplus by 8 percent of GDP and increase in debt (face value) by 12 percent. Solid blue lines show the responses matching average duration, dashed black for perpetuities, and dotted red for short-term debt.

Table A.2: Inflation decomposition (22) for the fiscal policy shock in Figure A.2.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	4.14	2.41	1.94	−2.41	6.08
Average	4.84	2.82	2.28	−1.79	6.08
Short-Term	6.86	3.99	3.22	0	6.08

Table A.3: Inflation decomposition (22) for the CARES Act in Figure A.3.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss} - 1$ debt shock
Long-Term	9.60	4.14	2.40	−4.14	12.00
Average	10.96	4.93	3.04	−2.93	12.00
Short-Term	14.28	6.86	4.58	0	12.00

Table A.4: Inflation decomposition (22) for the CARES Act shock in Figure A.4.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	5.72	3.33	−0.06	−3.33	5.66
Average	6.63	3.86	0.34	−2.56	5.66
Short-Term	9.61	5.60	1.65	0	5.66

Table A.5: Inflation decomposition (22) for the CARES Act shock in Figure A.5.

Debt Maturity	$\int_0^\infty e^{-ru}\pi_u \mathrm{d}u$ inflation	$\int_0^\infty e^{-ru}i_u \mathrm{d}u$ interest rate	$\int_0^\infty e^{-ru}s_u/a_{ss}^{new} \mathrm{d}u$ surplus	$p_0^b/p_{ss}^b - 1$ direct effect	$v_0/v_{ss}^{new} - 1$ debt shock
Long-Term	1.72	1.00	−1.72	−1.00	0
Average	1.93	1.12	−1.63	−0.83	0
Short-Term	2.92	1.70	−1.22	0	0