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in Intra-Sectoral Wage
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Role of Artificial Intelligence in Intra-Sectoral Wage Inequality in an Open Economy: A Finite Change Approach

Abstract

Artificial Intelligence (AI) has the potential to significantly impact the income of individuals. Cross-country data shows that introduction of AI is inequality enhancing in developing and less developed countries. In this paper, we attempt to understand the reason for increase in wage inequality across labourers due to introduction of AI, in a finite change General Equilibrium (GE) set up which allows for emergence of a new activity. AI-induced technological shock is introduced in the non-traded sector of an open economy with heterogeneous skills. We show how the advent of AI (which was initially non-existent) in the non-traded sector separates the skills of the once homogenous workers, thus, creating an intra-sectoral wage gap. What proportion of the low-skilled workers can move to the higher wage paying sector depends on an adaptability factor that acts as an eligibility criterion in fragmenting the erstwhile homogenous labourers and also works towards rising intra-group wage gap.

JEL-Codes: O330, J310, D500.

Keywords: artificial intelligence, finite change, sectoral wage gap.

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1. Introduction

When artificial intelligence or machine learning is added to the internet of things it augments the way things are done. AI-driven technology has been transforming the non-traded service sector in the entire world, challenging the traditional way of doing business. When a section of the society is unable to exploit or access the technologies or digital information, the economy experiences a digital divide. In the 20th-century cab booking, food delivery, or for that matter robotic customer services are performed by AI-induced technologies or applications. A certain fraction of the workers is impacted due to the AI-induced tech shock. They are mostly low-income groups, with little or no education, or the elderly section of the society. In this paper, we try to find out the impact on welfare and inequality of labourers due to the emergence of a new artificial intelligence (AI)-induced sector in an open economy with heterogeneous skills. The emergence of a new sector, a novel feature of the model is expressed as a Finite Change in a general equilibrium (GE) setup. To motivate our paper, we have also performed a simple empirical exercise to understand whether the introduction of AI in any economy is actually affecting the wage inequality. If so, what are the factors that might be contributing to the inequality due to AI?

Economists and policy-makers have studied the impact of adoption of computers on wage inequality. But an analysis to understand the wage inequality consequences of AI is yet at a nascent stage. The erstwhile work in AI and the welfare have looked at the relationship more descriptively. This led to contrasting conclusions. Most economists think that AI is a labour-saving technology. Apprehension is that the demand for labour would come down and as a result it will negatively affect their return.

The core idea of this paper is how introduction of AI driven technological change or for that matter any new technology not only encourages inequality to rise across sectors, a well-known consequence touched upon by many, but also generates intra-group inequality among erstwhile homogeneous workers. Those in the group who can better adapt the new technology come out winners. This implies that along with interesting change in inter-sectorial wage gap in favour the new technology sector, there would be winners and losers within the AI driven sector. After the survey of the literature we try to provide a motivational empirical background before we

engage in a proper theoretical work. We also demonstrate how emergence of a new sector can be modelled in a standard GE structure in terms of an approach related to finite change.

2. Literature

In the beginning, labourers who lack AI-induced skills, will have a disadvantage as their income can decline, due to lowering of demand for other competing services (Korinek and Stiglitz, 2018). AI-induced machines or machine learning technologies are more efficient and hence can substitute workers performing routine jobs (Autor, 2003, Korinek and Stiglitz, 2019, Berg et al., 2018,). Researchers such as Freensta & Hanson (1997), Bustos (2005), and Wendy Duong (2007) have on the other hand pointed out that the introduction of AI will make a third-world country poorer but will help semi-skilled laborers access better-paying jobs (Ernst, Merola & Samaan, 2018). Again, Autor and Salomons, (2018) and Gregory et al. (2019) pointed out that the medium skilled labourers are susceptible to routine replacing technical change (RRTC) and will either be replaced by AI machines or witness a drastic wage drop. Baranay & Siegel (2018) and Author, Levy & Murnane (2003) show that the emergence of AI would kill middle-skilled jobs and thus, hollowing the middle class. However, it is possible that artificial intelligence might be intelligence assisting (IA) (Korinek and Stiglitz, 2021) or efficiency improving. Thus, contributing to enhancing welfare in an economy. As Acemoglu and Autor (2011) pointed out that AI innovated self-driving trucks does not necessarily mean unemployment of truck drivers, as they will be needed to monitor the trucks, load, offload, complete orders etc. If we take a look at our current scenario, AI-induced cab services such as Uber and Ola have not retrenched traditional cab drivers, but rather helped them access a large customer base. Owing to its convenience, safety & security, easy availability, and time management AI-induced cab services are preferred by most travellers. The AI cabs are capable of identifying the shortest and fastest route based on live traffic updates helping a customer to receive hassle-free services. AI-induced technological progress may enhance the prospects of labourers. Apart from AI-induced transportation services, AI is also used in micro-surgeries, smart agriculture, hospitality services, e-marts, e-banking, etc., It is also used for mundane jobs such as janitor services. So, AI is not a substitute of labour (yet), it helps in increasing the efficiency. But one needs capital resources and skill-enhancing activities to use AI technologies for their betterment. Hence the impact on inequality of such a change should be studied clearly.

Acemoglu and Restrepo (2016, 2018) found that automation of high-skilled jobs decreases inequality whereas as that of low-skilled ones increases inequality. To add to the inequality

concern. AI technological revolution will make it more difficult for the less developed and developing economies to catch up with the developed economies (Korinek and Stiglitz, 2021). The conflicting results thus creates a ground for further analysis to find out the impact of AI on inequality given certain characteristics of the economy and sectors. To understand how AI has actually affected the inequality globally we glance through the relevant data initially.

3. Motivation

The approach we take here is to look at internet penetration data across countries to proxy for AI usage. An endogenous structural break analysis (Perron, 2008) was conducted for each country to find out the time at which series exhibited a change in slope. The year so obtained was taken to be the proxy for introduction of AI. Among the factors which support the introduction of AI, internet connections are an important enabler (UNESCAP, 2020). Data on ‘Individuals using the Internet (% of population)’ was obtained from the World Development Indicators (World Bank Data Catalogue). The time ranged from the year 1990 to 2020. To analyse the inequality due to the emergence of AI in a less developed or developing economy we used labour real-value-added per worker data¹. Wage data was not available, sector and year-wise for such a long time series. Since our theory focuses on skilled-unskilled income inequality, we worked with sectoral (eight sectors) data which revealed the gap between earnings of labourers with different skill-levels. The sectors being (i) Agriculture (ii) Mining, (iii) Manufacturing, (iv) Utilities, (v) Construction, (vi) Wholesale, Accommodation and food service activities, (vii) Transportation, information and communication, Finance, insurance, real estate and business services, and (viii) Government services, Community, social and personal services. The sectors are defined as per International Standard Industrial Classification of all Economic Activities (ISIC) definitions².

The Gini coefficient was calculated for eight sectors, using labour real-value-added per worker data, across countries for the time period 1990-2020³. Change in the coefficient pre and post structural break year has been taken as measure for change in inequality pre and post introduction of AI. After finding out the structural break year, two-tailed paired sample t-tests were run to verify the significance of change in inequality pre and post introduction of AI. 70.59% of the developing and less developed countries with significant t-values had higher inequality post introduction of the AI¹. On the other hand, only 30.43% of the developed countries with significant t-values had higher inequality post introduction of the AI.¹ Therefore, we can conclude that the emergence of AI may lead to a rising income gap among labourers

¹ Table 2 in appendix

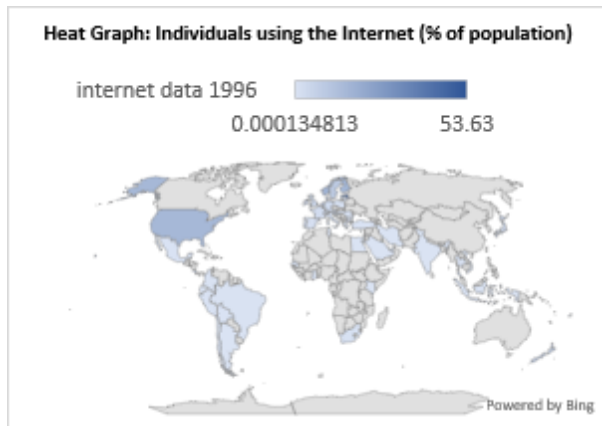
² Table 5 in appendix – detailed description of the 8 sectors

³ Table 6 in appendix

with differing skill-sets mostly in less developed or developing economies. Also, we have run a simple regression to analyse the impact of internet penetration on the wage gap across countries. The negative value of the coefficient (-.32) clearly indicates a rise in wage gap with increasing internet or AI penetration. Heat maps are used to explain the results visually.

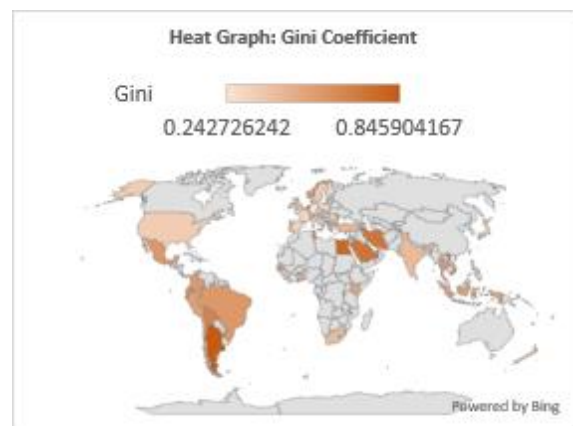
Dependent Variable	Base Model
Gini	OLS
Independent Variable	Coefficients
Individuals using the Internet (% of population)	-0.32** (0.13)
Constant	0.58
R-squared	0.09
<i>Note:</i> *p<0.1; **p<0.05; ***p<0.01	

Table 1



Darker shade represents higher internet penetration

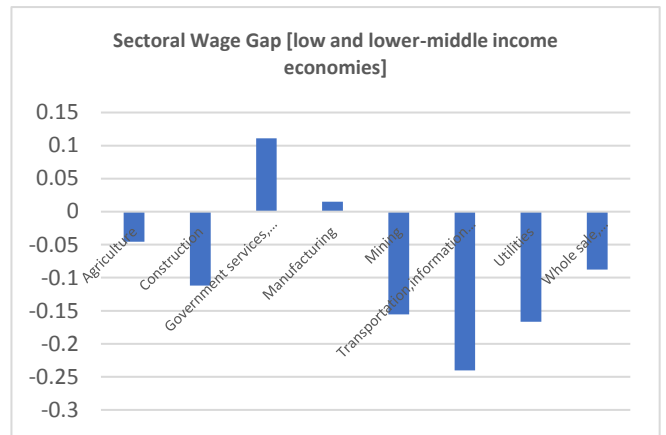
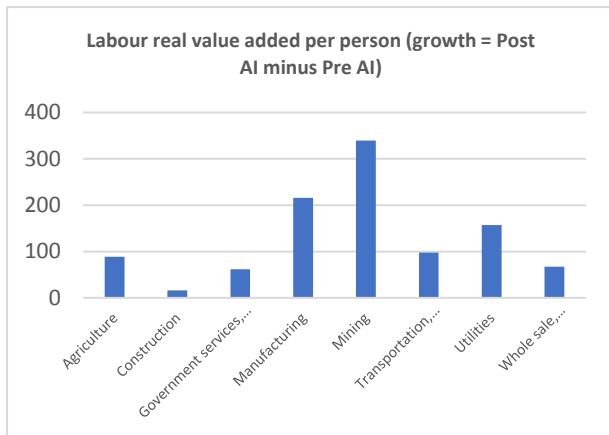
Graph 1



Darker shade represents higher wage gap.

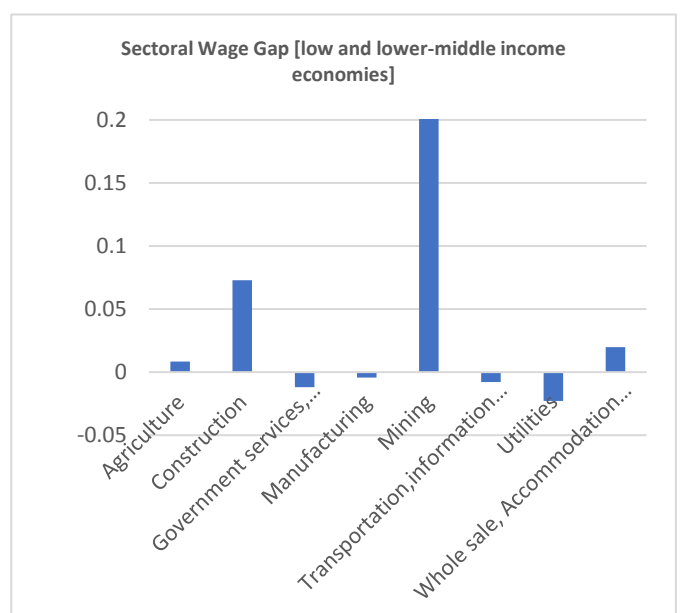
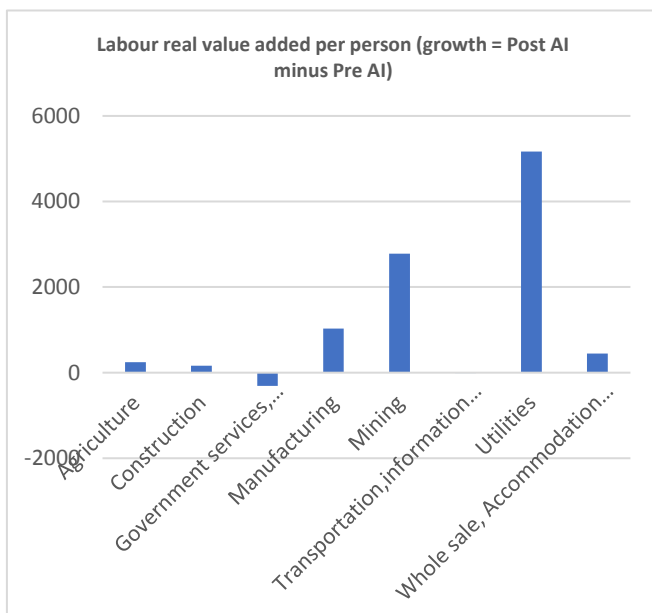
Graph 2

Empirically we have tried to study the inter-sectoral wage gap in the economies as well. The countries are divided into three groups based on WDI classification, namely, low & lower-middle-income economies, upper-middle-income economies, and high-income economies. The growth rate (%) of the labour-value added per employee⁴ is maximum in the low & lower-middle-income economies for the skilled-intensive sectors (Mining) and least for the relatively unskilled sectors (Construction, Government services). Additionally, on average, the sectoral wage gap is increasing in Government services and the Manufacturing industries that are semi-skilled and skilled intensive, respectively.



Graph 3 (a), (b)

On the other hand, in the upper-middle-income economies, the growth of labour-value added per employee is maximum in the semi-skilled-intensive sectors (Utilities sector) and least for the Government sector which can be assumed to be a semi-skilled sector as per the definitions of ISIC. However, the rise in the wage gap is similar to the low & lower-middle-income economies. The Gini coefficient worsens implying a wage gap in the Mining sector which is a skilled-intensive⁴.

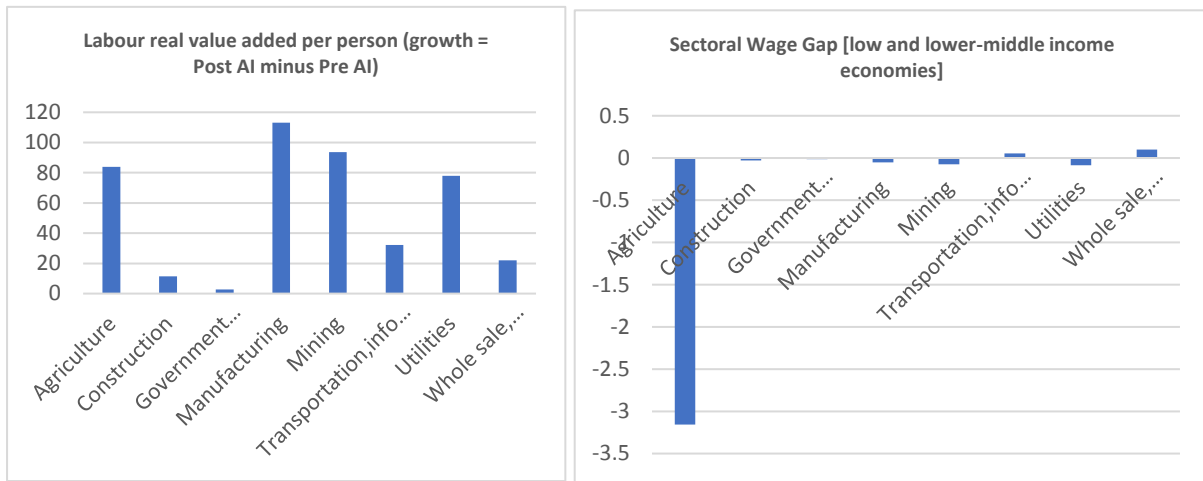


Graph 4 (a), (b)

Again, glancing through the data of the high-income economies, we find that the growth of labour-real-value-added per employee is maximum for the Manufacturing sector and least for the Government sector. Both can be assumed to be semi-skilled to skilled sectors as per the definitions of ISIC. However, it is interesting to see that even in these economies, the wage gap

⁴ Table 6 in appendix

worsens in the skilled-intensive sector (i.e., the Wholesale, Transportation, information and communication, Finance, insurance, real estate and business services etc).



Graph 5(a), (b)

On running a few correlations with the negative of Gini difference (Table 3), we find that education (0.3245) and GDP per capita (0.6217) showed moderately positive correlation among developing and less developed countries. This shows that a minimum level of educational attainment and income is required for introduction of AI in such countries.

Table 2: Correlation with Gini difference

	<u>Correlation with Gini_diff</u>
Old_Pop	0.11
Edu	0.33
GDP	0.12
Infl	0.15
GDP_per_cap	0.62

Where,

Post structural break Gini - Pre structural break Gini	Gini_dif
RATIO = Old population/total population	Old_Pop
The percentage of population ages 25 and over that attained or completed at least Bachelor's or equivalent.	Edu
GDP (constant 2015 US\$)	GDP
Inflation, consumer prices (annual %)	Infl
GDP per capita (constant 2015 US\$)	GDP_per_cap

⁴Table 6 in the appendix

Summing up, we find that the Individuals using the Internet (as a % of the population) and Gini-coefficients are negatively correlated, with a significant p-value. The result implies that countries with higher internet penetration (which can proxy for AI applications) will face higher wage inequalities. 70.59% of the low- and middle-income economies with significant t-values had witnessed rise in inequality post introduction of the AI. In comparison, only 30.43% of the developed countries with significant t-values witnessed rise in inequality post introduction of the AI.

On the sectoral level, service-oriented sectors such as transportation, information, communication, Finance and Manufacturing, Mining, Government services, etc experienced a widening wage gap post introduction of AI. Plausible reasons for the rising wage gap can be attributed to a threshold education and income required to attain the AI training that gradually helps in adapting to the evolving AI-induced technological applications.

The broader literature on trade vs. technology looks into the effect of these two on wage inequality. In case of developed countries empirical evidence is inconclusive (Singh and Dhumale, 2000). However, an overwhelming number of papers attributes the causes of variation in income distribution to factors other than trade or technology (Katz and Autor, 1999 and Atkinson, 2000). Inflation, unemployment, exchange rate, social norms etc. are found to be more potent forces impacting wage inequality (Burtless, 1990, Card, 1992, Harrison and Bluestone, 1990 and Galbraith, 1998). In case of developing countries trade, technology and factors like liberalisation of financial flows and social factors are found to have caused wage inequality (Slaughter, 1998, Lustig, 2000 and World Bank, 2000).

At the firm-level the size of appropriation of capital resources can impact skilled-unskilled wage gap. Yu and Chao (2021) show that if size of such appropriation is high the wage inequality increases in the short run given the number of firms. Chao, Nguyen and Yu (2019) relate tariff reduction to wage inequality. Wage inequality can reduce due to tariff liberalisation if the import substituting manufacturing sector experience capital flight to competitive agricultural sector. Such impact can however be negated if the reduction in capital cost result in entry of firms in the manufacturing sector. In a recent work (Yu and Chao, 2022) shows that given the dualistic manufacturing sector in China a land-rent policy support to urban informal sector can reduce wage inequality in the short run. However, a land-supply policy can have opposite effects. Thus, the wage inequality can be impacted by very subtle mechanisms which can be related to trade, technology, policy etc.

Past studies and empirical exercise so far thus show that technology might have played a role in impacting wage inequality especially in developing and less developed countries. What is the mechanism through which this happens? In order answer this question we have looked at how AI can induce higher wage inequality. A theoretical model using a finite change GE approach has been built in the context of a developing economy to un earth the process through which technology infusion might cause changes in wage inequality.

4. Theoretical Model

Consequently, we have built a finite change GE model to systematically capture the emergence of AI in a developing economy with heterogeneous skills. We have developed the model with traded and non-traded sectors and showed how AI technology helps in separating the workers on the basis of their skill levels thus contributing to wage inequality. The emergence of a new sector without the disappearance of the old sectors, leads to an intra-sectoral wage gap. Our model, meticulously shows how a 2X3 (two sector and three specific factor) structure in a GE setup transforms into to a 3X3 structure. Thus, the setup is very different from a conventional tech-shock found in past work.

4.1 Competitive Price Conditions

We begin with a 2X3 sector X (traded) and Y (non-traded) using skilled and unskilled labour and capital as inputs. Capital being the freely mobile input between the two sectors. Gradually we show the emergence of another non-traded sector Z which replicates the traditional Y sector but uses artificial intelligence. This is an addition to other sectors already present.

$$w_s a_{sx} + r a_{Kx} = P_x \dots \dots \dots (1)$$

$$w a_{Ly} + r a_{Ky} = P_y \dots \dots \dots (2)$$

$$w(\beta) a(\beta) + r a_{Kz} = P_z - P_A \dots \dots \dots (3)$$

4.2 Full Employment Conditions

$$a_{sx} X = \bar{S} \dots \dots \dots (4)$$

$$a_{Ly} Y + L(1 - \bar{\beta}) = \bar{L} \dots \dots \dots (5)$$

$$a_{Kx} X + a_{Ky} Y + a_{Kz} \int_{\bar{\beta}}^1 \frac{L}{a(\beta)} d\beta = \bar{K} \dots \dots \dots (6) \text{ with } \int_0^1 L d\beta = L, \quad \beta \in [0,1)$$

β is an adaptability factor. A low β indicates poor adaptability in terms of skills among the unskilled labourers where as a high β indicates higher adaptability. Thus, if the Z sector

contracts, the labourers with higher β can move back to the Y sector to which they originally belonged. But, an unskilled labour with low β , cannot move to the new AI induced sector if sector Y shrinks. In either case, the Y sector workers will suffer. Thus, we can say that only the labourers below the cut-off point of $\bar{\beta}$ are specific to the Y sector (down side mobility of the semi-skilled labourers who can move between Z and Y).

4.3 Mobility Condition

$$w(\bar{\beta}) = w \dots \dots \dots (7)$$

From (3) and (7)

$$wa(\bar{\beta}) + ra_{Kz} = P_z - P_A \dots \dots \dots (8)$$

From (4), (5) and (6)

$$\frac{a_{Ky}}{a_{Ly}} \bar{\beta} L + a_{Kz} \int_{\bar{\beta}}^1 \frac{L}{a(\beta)} d\beta = \bar{K} - \frac{a_{Kx}}{a_{sx}} S$$

$$\text{or, } \left[\frac{a_{Ky}}{a_{Ly}} \left(\frac{r}{w} \right) \bar{\beta} L + a_{Kz} \int_{\bar{\beta}}^1 \frac{L}{a(\beta)} d\beta \right] = \bar{K} - \frac{a_{Kx}}{a_{sx}} \left(\frac{r}{w_s} \right) \dots \dots \dots (9)$$

LHS of (9) is demand for K given Y and Z and (r/w) , call it K_d

$$\frac{\partial K_d}{\partial \bar{\beta}} = \frac{a_{Ky}}{a_{Ly}} L - \frac{a_{K(z)}}{a(\bar{\beta})} L < 0$$

If $\frac{a_{Ky}}{a_{Ly}} < \frac{a_{K(z)}}{a(\bar{\beta})}$ then as $\bar{\beta}$ goes up K_d falls. So, we can write

$$K_d \left(\bar{\beta}, \frac{r}{w} \right) = \bar{K} - \frac{a_{Kx}}{a_{sx}} \left(\frac{r}{P_x - r a_{Kx}} \right) \dots \dots \dots (10)$$

Now consider (2) and (8) if Z is K-intensive, as $\bar{\beta}$ goes up $a(\bar{\beta})$ falls. One can show r will go up and w will fall due to Stolper-Samuelson result. Also, w_s will fall from (1).

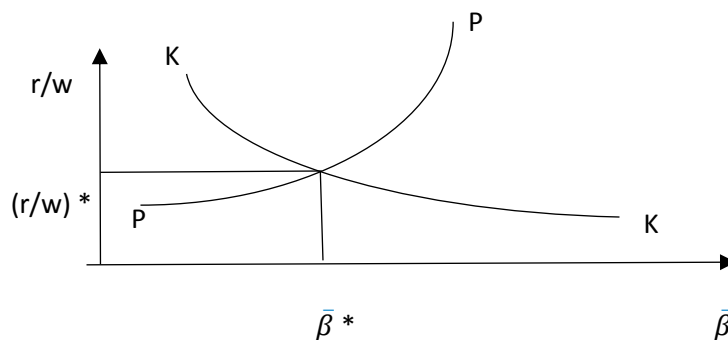


Figure 1

From (10), if $\bar{\beta}$ goes up K_d falls we have excess supply of K given (w, r, w_s) . So, we need to reduce r or r/w and hence r/w_s . That will make (10) valid. The KK curve represents the capital

market equilibrium for the non-traded sector and the PP curve represents the competitive price equilibrium of the non-traded sector. Thus, the curves KK and PP gives us the equilibrium values $(r/w)^*$ and $(\bar{\beta}^*)$. Then we can derive all other values given commodity prices and factor endowments. This completes the general equilibrium of the system.

One can use Figure 1 for all comparative static results. Two interesting results are the effects of a decline in the price of AI input and a rise in stock of K. We also demonstrate the case of a FINITE CHANGE, a key contribution of this paper. Whereby due to a fall in the price of the technology input i.e., AI (working possibly via a technology application), capital leaves in bulk from Y and X to set up the NEW sector Z which was non-existent.

- a) A decline in P_A : This will increase r/w at given $\bar{\beta}$ by the Stolper-Samuelson (SS) result. PP shifts to the left. KK shifts to the left because as skilled wage falls due to a rise in r extra K is available for Y and Z and to absorb that r/w must fall and that reduces $\bar{\beta}$. More people move to Z sector. r/w can go either way with opposing effects of SS result and excess supply of K flowing out of X into Y and Z. So, Z must expand and if X does not release much K, r/w must rise.

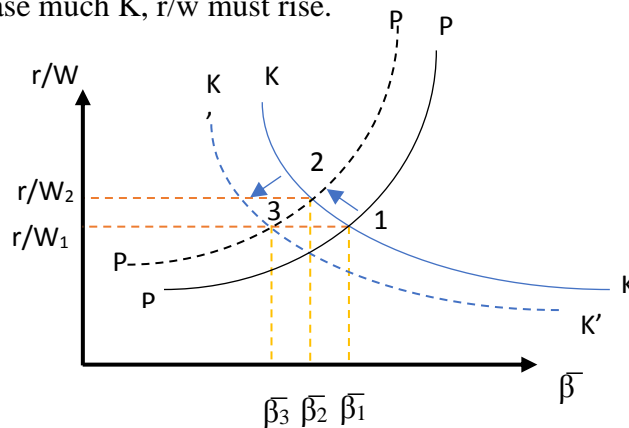


Figure 2

- b) An increase in K: This will mean excess supply of K. At the given r/w , $\bar{\beta}$ must fall. So, KK will shift to the left reducing both r/w and $\bar{\beta}$. Excess capital allows more people to go to Z by reducing r and increasing w .

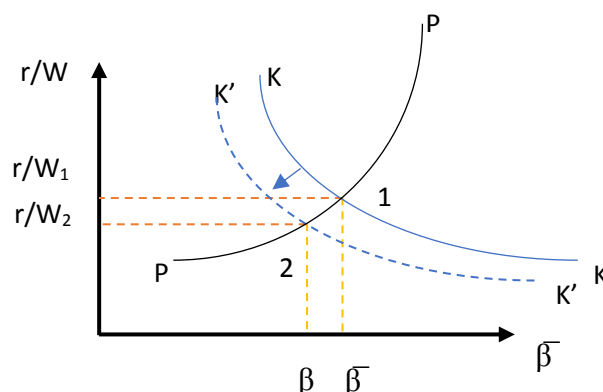


Figure 3

c) Finite change refers to a situation where there is a finite amount of decline in Y, not infinitesimal decline as captured mathematically by calculus via a jump in the process. Initially P_A is very high and Z does not exist. PP must be intersecting KK beyond point to the right. Only Y will exist. This means that beyond A, the equilibrium r which is determined by the 2x3 specific factor model that prevails in X and Y greater than what Z can offer. Once price of the intermediate input falls, PP shifts to the left and ultimately the equilibrium moves to the left of A, drawing K away from Y and X and Z jumps to a positive value from zero. Now the 2x3 structure is transformed from the specific factor model to a 3x3 structure, a finite change model (Jones and Marjit (1992), Findlay and Jones (2000), Marjit and Mandal (2014), Marjit and Gupta (2022)) etc.

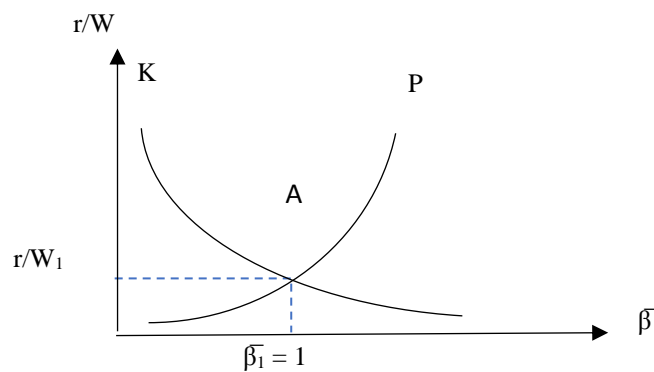


Figure 4

Proposition 1: Whenever $\bar{\beta}$ drops intra-group wage inequality denoted by $\frac{w(\beta)}{w(\bar{\beta})} = \frac{a(\bar{\beta})}{a(\beta)}$ must increase.

Proof: Follows from $a'(\bar{\beta}) < 0$.

QED.

Expansion in Z must imply that $\frac{w(\beta)}{w}$ must be higher for all β . Since w is paid to workers in Y sector. New technology separates the better ones from the rest.

The trade literature was focussed as an instrument for separating the better from worse and hence a source for increasing productivity and inequality (Chaudhuri and Marjit, 2017). In this paper we have proved it mathematically.

4.4 Demand Issues

So far, we have assumed (P_Y, P_Z) do not change [choose X as the numeraire, so that $P_X = 1$]. As Z expands, initially through finite changes, P_Z cannot fall as Z was not there. One can safely assume a Cobb-Douglas utility function with (α) fraction spent on X and $(1 - \alpha)$ spent on Y and Z.

$$\alpha[X + P_Y Y + P_Z Z] = P_Y Y + P_Z Z \dots \dots \dots (17)$$

(Y, Z) are non-traded goods so (P_Y, P_Z) change with demand-supply shifts.

$$\alpha x = (1 - \alpha)[P_Y Y + P_Z Z] \dots \dots \dots (18)$$

To start with (α) was spent on x and $(1 - \alpha)$ was spent on Y. Now expenditure on Y will go down. As in a standard model we know initially supply of Y will be rising in P_Y and demand for Y will be declining in P_Y .

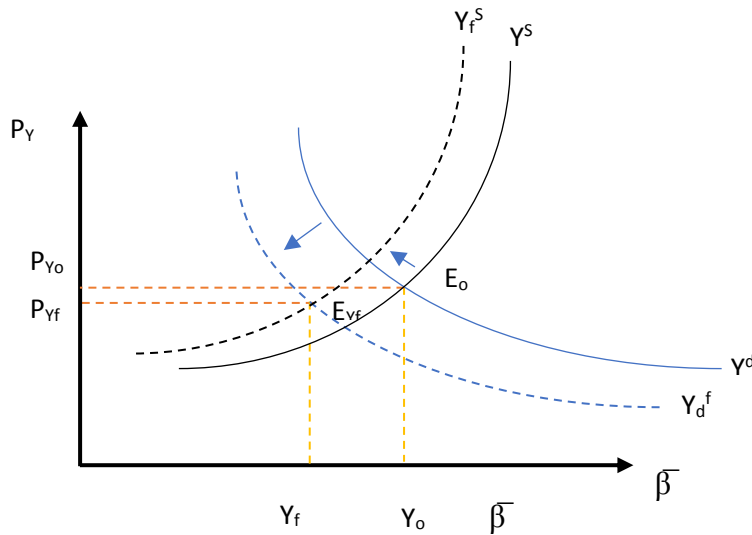


Figure 5

Finite Change

E_{Yf} is the equilibrium after Finite Change. After finite change demand and supply both contract for Y (Y_f^d, Y_f^s) and emerge for Z.

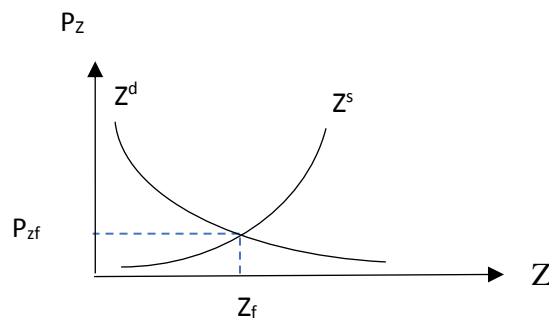


Figure 6

Note that $P_{zf}Z_f = [P_oY_o - P_{Yf}Y_f]$. E_o is the initial equilibrium and E_{Yf} is after the finite change. It is possible that P_Y may increase after the change, if supply contraction is significant but that does not happen normally as Y_S tends to be much steeper so P_Y can fall substantially. That itself will crash w , as Y is labour intensive.

Many people will like to come into Z as w falls. But $a(\beta)$ function is critical.

$$w_z(\beta) = \frac{P_z - r a_{Kz} - P_A}{a(\beta)} = w$$

After finite change w falls to a new level such that $w_z(\bar{\beta}) = w$. It must be the case that for some workers $w_z(\beta) > w$, they will be able to come in.

So for them $\frac{P_z - r a_{Kz} - P_A}{a(\beta)} > w \dots\dots\dots(19)$

Numerator in (19) is independent of β , call it w_m . [$a' < 0$]

For any β , if $a(\beta)$ is really low we have point 2. If it is really high, we have point 3. In case of point 2 ($1 - \bar{\beta}_2$) comes into Z, in point 3 ($1 - \bar{\beta}_3$) comes into Z. So, for same fall in w , Z can increase more or less. So, P_z can be quite high as Z may not increase much.

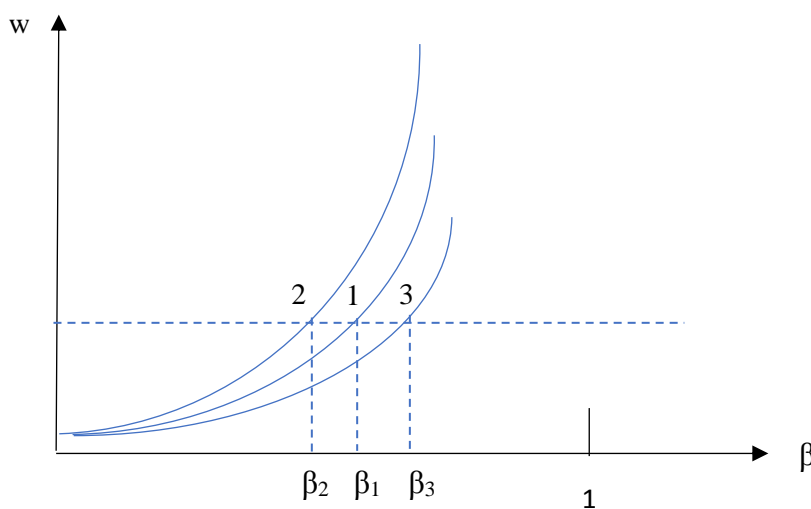


Figure 7

One exercise or thought experiment that can be inferred from the above analysis includes:

1. P_A falls so K moves out of Y into Z, w falls, r goes up, P_Y drops, Y falls.
2. As w falls, $\bar{\beta}$ is decided by the diagram (Finite Change Figure 2). Z is higher from $Z = 0$. Given demand – supply P_z is determined.
3. Final equilibrium determines w , r , P_Y , P_z , etc.

5. Conclusion and Policy Implication

Artificial Intelligence (AI) and big data resources have opened the possibility of 4th-dimensional trade opportunities. From virtual assistance to recommendation search engines, AI has helped to increase productivity and efficiency in many sectors. AI-induced technological up-gradation such as Siri, Google search engine, Alexa, and the autopilot Tesla cars are creating a new era in the research arena.

To motivate the paper, a suggestive empirical exercise was performed wherein we found that 70.59% of the developing countries and 30.43% of the developed countries experienced an increase in wage-gap post introduction of artificial intelligence. Further, to unearth the reasons for such empirical observation we have built a finite change GE model. Where we saw that the emergence of AI in the non-traded sector in an open economy increases intra-occupational wage inequality. Only those unskilled labourers who can upgrade their skills can move to the higher-wage paying sector. The movement to the higher-paying sector is determined by $\bar{\beta}$ (required adaptability skills) which acts as a cut-off.

Thus, such AI-induced technological progress will have winners and losers. Even though, AI will create losers, the gains to the winners exceed the total loss (as PPF shifts outwards due to the emergence of AI as for any other technological change). Since it is possible to mitigate the losses and achieve a Pareto improvement, the policymakers need to implement policies to help the losers. As our model suggests, only those who can adapt to the new technology can move to the higher wage paying sector.

$1 - \bar{\beta}$ = workers who can adapt to AI and

$\bar{\beta}$ = workers who cannot adapt to AI

So, 0 to $\bar{\beta}$ can consist of two groups of people; one who need skill enhancement trainings to adapt to the new AI economy and the others who cannot adapt and adopt AI per se, senior citizens. The challenges can be addressed by the policy makers in a number of ways. Policies related to education, redistribution of income through taxations, subsidies etc. can be thought of.

Education policies aiming to provide quality education, and increase the number of graduates has the potential to reduce inequality in the long run. Pareto improvement can be achieved in the long run, even if a section of the workers is hurt in the short and medium run. Such skill development policies can be promoted via a public-private tie-up. This can be seen in many countries. Paraguay is a country where such a policy was implemented where a small group of farmers took advantage of a similar partnership endorsed by the German Agency for Technical Cooperation (GTZ) and the non-governmental organization Action against Hunger (ACF) (Masi, Fernando, 2011). The PM-DAKSH Yojana (India, 2020-21) focuses on upskilling artisans who have become marginalized due to better technologies in market. They are trained to adopt to newer processes and increase their incomes. A similar scheme for urban areas may

be thought of. Some Free Coaching Schemes (a public-private tie-up) can help young individuals learn new skills at zero or low cost.

Since, the winners will gain more than the loss of the losers, redistribution policies can be adopted. A fraction of the gain from the skilled and semi-skilled workers can be transferred to the unskilled ones in the form of wage subsidies, earned tax credits, or direct benefit transfers systems which can directly impact the income inequality through effective targeting of doles. Skilled and semi-skilled workers can be taxed progressively (e.g., A skilling CESS) and it can be transferred to the losers as a wage subsidy or income tax credits. A direct benefit transfer or pension schemes can be arranged for those unskilled workers who lack the ability to adopt AI. The redistributive impact though varies across countries such as Latin America, Eastern Europe, parts of Asia have been successful, where they are actively using pension funds to dampen the effects of higher income inequality (World social report 2020).

If redistribution is feasible, and can be implemented, AI induced tech growth is desirable and recommended. However, if the transfer costs are high, redistribution of income might not be possible and hence, it will be difficult to compensate the unskilled workers. If $\bar{\beta}$ is low, compensation is easy, on the other hand, if $\bar{\beta}$ is high compensating will be difficult, since a huge amount would be required to compensate them.

Technological growth and innovations are non-rivalry but excludable. In the long run such AI-induced growth is desirable and can benefit the society. Hence, it is possible that in the near future the traditional non-traded sectors will be replaced by AI non-traded sectors. Under such a scenario, the authorities should formulate policies to expand substitute sectors where AI specific skills are not required, to absorb the unskilled workers who are not able to adapt to the new technology. Credit schemes should be initiated to help low skilled workers re-skill themselves and earn higher income. A prevailing scheme where this is possible is the Credit Enhancement Guarantee Scheme where individuals can take loans to train themselves to develop entrepreneur skills to create value for themselves and the society. The Government also helps them with credit to set up their businesses.

AI-technological revolution will increase the productivity of an open economy but with a rising wage inequality. Initially, emergence of AI might be portrayed as pareto-inferior or sub-optimal equilibrium, but with the adequate policies and adjustments, the economy can march towards pareto improvement as AI in the non-traded sector of an open economy is labour

augmenting rather than labour saving. The sooner the economy adjusts to the AI shock, the faster the society can move towards improvement in welfare.

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Appendix

Definition

$$\text{Wage per labour} = \frac{\text{Labour real value added}}{\text{Labour count in each sector}}$$

The empirical part uses two datasets namely (i) Individuals using the Internet (% of population) and (ii) sectoral labour productivity. Both are downloaded from World Development Indicators (World Bank Data Catalogue). Homogenizing both data sets across countries and employment sectors we received 59 countries to work with.

Table 3: Country classification as mentioned by world bank income report (2021-22)

	Low-Income Economies	Lower Middle-Income Economies	Upper-Middle-Income Economies	High-Income Economies	Total
Total countries	1	9	14	35	59

12 out of 17 countries (less developed + developing) with significant t tests were found to have higher Gini Coefficients post structural break. $12/17 * 100\% = 70.59\%$. On the other hand, 7 out of 23 developed countries with significant t tests were found to have higher Gini Coeff post structural break. $7/23 * 100\% = 30.43\%$. Thus, clearly stating the introduction of AI contributes to wage gap in developing economy but not always in a developed economy.

Table 4: Across Country - Inequality post emergence of Artificial Intelligence in economies.

Economies	Countries	Structural break year	t value	Significance - 2 tail	Mean difference pre and post structural break (Post - pre) (a negative value indicated welfare worsening)	GINI Coeff before structural break	Gini Coeff after structural break	Inequality = Post structural break Gini - Pre structural break Gini (a negative value indicates improvement)
Low-income economies	Viet Nam	2001	-7.27	0.00	194.20	0.62	0.66	0.04
Lower Middle-Income Economies	Bolivia	1999	0.90	0.37	-13.85	0.68	0.66	-0.02
	Egypt	2000	-4.76	0.00	798.63	0.74	0.83	0.10
	Ghana	2016	-2.37	0.03	42.15	0.40	0.63	0.23
	India	2007	-10.03	0.00	0.39	0.40	0.45	0.05

	Indonesia	2016	-0.49	0.63	28.42	0.64	0.46	-0.18
	Iran	2012	2.53	0.01	-3868.40	0.80	0.65	-0.15
	Kenya	2000	12.02	0.00	-677.56	0.50	0.55	0.05
	Senegal	2013	4.39	0.00	-6000.41	0.66	0.43	-0.24
	Tunisia	1999	0.33	0.74	-1.18	0.61	0.59	-0.02
Upper-Middle-Income Economies	Argentina	1998	-1.60	0.11	252.19	0.67	1.00	0.33
	Brazil	1996	-1.97	0.05	2.70	0.41	0.74	0.33
	Bulgaria	1998	-3.16	0.00	1.22	0.33	0.32	-0.01
	Colombia	1998	-6.49	0.00	42297.31	0.55	0.60	0.05
	Costa Rica	1997	-7.08	0.00	6559.79	0.33	1.00	0.67
	Ecuador	2006	0.44	0.66	-0.30	0.51	0.75	0.24
	Fiji	2002	-4.57	0.00	2.26	0.29	0.30	0.01
	Mauritius	2010	0.29	0.77	-53.67	0.58	0.26	-0.32
	Mexico	1997	-4.40	0.00	357.39	0.53	0.62	0.09
	Peru	2016	-3.41	0.00	35.20	0.55	0.52	-0.02
	Romania	1998	-0.51	0.61	0.85	0.33	0.35	0.02
	South Africa	2010	-6.11	0.00	140.07	0.39	0.45	0.05
	Thailand	1998	-7.03	0.00	813.74	0.48	0.65	0.16
	Turkey	1999	-3.35	0.00	14.20	0.51	0.37	-0.14
High-Income Economies	Austria	1996	-15.22	0.00	30.81	0.31	0.32	0.01
	Bahrain	2012	2.06	0.04	-439.13	0.90	0.80	-0.10
	Belgium	1996	-15.85	0.00	32.89	0.32	0.26	-0.06
	Chile	1998	-8.67	0.00	28557.82	0.54	0.54	0.00
	Croatia	1998	-4.60	0.00	32.40	0.29	0.28	-0.02
	Cyprus	1997	1.10	0.29	-0.88	0.31	0.29	-0.03
	Denmark	1995	-5.02	0.00	1099.14	0.51	0.66	0.15
	Estonia	1997	-1.26	0.22	1.31	0.35	0.30	-0.05
	Finland	2006	-11.86	0.00	43.28	0.33	0.28	-0.05
	France	1995	-12.11	0.00	24.64	0.33	0.29	-0.05
	Germany	1995	-15.88	0.00	16.23	0.23	0.25	0.02
	Greece	1996	-1.28	0.24	0.87	0.34	0.40	0.06
	Hungary	1996	-0.75	0.48	85.64	0.29	0.24	-0.05
	Ireland	1996	-1.34	0.22	2.47	0.36	0.41	0.05
	Israel	1995	-3.19	0.00	37.42	0.31	0.47	0.16
	Italy	1995	-6.46	0.00	16.18	0.39	0.35	-0.04
	Japan	1995	-9.55	0.00	2.46	0.35	0.32	-0.03
	Korea	1997	-9.25	0.00	75.98	0.48	0.55	0.07
	Latvia	2002	-11.58	0.00	6.97	0.31	0.24	-0.08
	Lithuania	2000	-4.03	0.00	3.89	0.33	0.26	-0.07
	Luxembourg	2013	-3.21	0.00	17.49	0.25	0.26	0.01
	Netherlands	2012	-1.73	0.09	76.70	0.72	0.64	-0.08
New Zealand	2009	-3.94	0.00	17.87	0.43	0.41	-0.03	
Norway	2014	-3.79	0.00	1259.28	0.65	0.50	-0.14	
Poland	1996	0.14	0.89	-3.77	0.34	0.29	-0.05	

Portugal	2004	-3.55	0.00	4.39	0.36	0.38	0.02
Qatar	2002	2.10	0.04	-108.81	0.71	0.64	-0.07
Saudi Arabia	1999	1.87	0.07	-86.44	0.75	0.71	-0.05
Slovenia	2001	-8.20	0.00	4.27	0.30	0.23	-0.07
Spain	1995	-11.25	0.00	18.74	0.34	0.30	-0.03
Sweden	2008	-0.31	0.76	10.58	0.38	0.28	-0.10
Switzerland	1997	-1.29	0.22	4.39	0.37	0.30	-0.06
United Kingdom	1995	-8.31	0.00	48.29	0.36	0.47	0.11
United States	2002	-8.60	0.00	28.57	0.36	0.33	-0.03
United Arab Emirates	2012	1.78	0.08	-933.09	0.80	0.60	-0.21

Table 5: Sector definition as per ISIC

Sectors	Sector Description
Agriculture	Crop and animal production, hunting and related service activities, Forestry and logging, Fishing and aquaculture
Mining	Mining of coal and lignite, Extraction of crude petroleum and natural gas, Mining of metal ores, Mining support service activities
Manufacturing	Manufacture of food products, beverages, tobacco products, textiles, wearing apparel, leather and related products, wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials, paper and paper products, Printing and reproduction of recorded media, coke and refined petroleum products, chemicals and chemical products, pharmaceuticals, medicinal chemical and botanical products, rubber and plastics products, non-metallic mineral products, basic metals, fabricated metal products, except machinery and equipment, fabricated metal products, except machinery and equipment, electrical equipment, motor vehicles, trailers and semi-trailers, repair, installation, furniture, jewellery etc.
Utilities	Basic amenities, such as water, sewage services, electricity, dams, and natural gas
Construction	Construction of buildings, Civil engineering, Specialized construction activities etc.
Whole sale, Accommodation and food service activities	Wholesale and retail trade and repair of motor vehicles and motorcycles, Accommodation, Division 56 Food and beverage service activities, etc.
Transportation, information and communication, Finance, insurance, real estate and business services	Land transport and transport via pipelines, water, air transportation, Warehousing and support activities for transportation, Postal and courier activities, Publishing activities, Motion picture, video and television programme production, sound recording and music publishing activities, Programming and broadcasting activities, Telecommunications, Computer programming, consultancy and related activities, Information service activities, Financial service activities, except insurance and pension funding, Insurance, reinsurance and pension funding, except compulsory social security, Activities auxiliary to financial service and insurance activities, Real estate activities etc.
Government services, Community, social and personal services	Public administration and defence; compulsory social security, Human health and social work activities, etc.

Sectoral Wage improvement along with reduced wage gap (Welfare Growth and Gini improvement)

38.98%	% of countries with improved welfare and Gini (22 out of 59)
0.1%	% of less and low middle-income countries with improved welfare and Gini (1 out of 10)
14.28%	% of less and low middle-income and upper-middle income countries with improved welfare and Gini (2 out of 14)
51.43%	% High income countries with improved welfare and Gini (18 out of 35)

There are 24 countries whose Gini has deteriorated i.e., overall wage gap has increased.

Table 6: Sectoral Labour real value-added growth rate and Gini-Coefficient.

Economies	Countries	Variables	Ag	Con	Gov	Man	Mining	Trans	Util	Whlsl
Low-Income Economies	VietNam	RLVA*	101.16	54.76	57.42	146.30	191.76	98.83	180.43	53.06
		Gini_diff	-0.01	-0.10	0.01	-0.04	-0.25	-0.79	-0.11	0.05
Lower Middle - Income Economies	Kenya	RLVA*	-1.66	-73.88	-38.90	-70.33	-73.69	-47.08	-15.70	-61.17
		Gini_diff	0.02	-0.22	0.88	-0.11	-0.19	-0.07	-0.08	-0.12
	Egypt	RLVA*	95.62	58.12	163.32	169.52	513.42	24.96	24.74	33.07
		Gini_diff	-0.10	-0.10	-0.11	-0.19	-0.13	-0.17	-0.21	-0.12
	India	RLVA*	80.93	16.11	34.79	223.79	123.45	226.55	282.12	173.78
		Gini_diff	-0.02	0.00	-0.12	-0.05	-0.05	-0.13	-0.12	-0.10
	Ghana	RLVA*	78.08	-65.79	-7.25	322.63	529.86	41.61	140.32	8.00
		Gini_diff	-0.12	-0.13	-0.10	0.47	-0.17	-0.05	-0.32	-0.15
Upper-Middle - Income Economies	Argentina	RLVA*	60.57	-6.82	-2.81	30.76	-16.88	-1.50	59.40	3.49
		Gini_diff	0.02	-0.05	0.01	-0.06	0.13	0.01	-0.01	-0.01
	Brazil	RLVA*	309.82	38.99	87.86	146.79	397.12	19.25	185.95	149.72
		Gini_diff	0.38	0.13	0.24	0.21	17.82	0.13	0.17	0.29
	Colombia	RLVA*	74.10	-24.86	51.68	25.51	30.49	29.46	78.76	-53.61
		Gini_diff	-0.08	-0.08	-0.05	-0.11	-0.02	-0.13	-0.20	-0.13
	Costa Rica	RLVA*	159.04	53.46	-6.39	39.85	117.65	-0.29	40.94	-47.58
		Gini_diff	-0.10	0.84	-0.05	-0.06	-0.11	-0.01	-0.09	-0.04
	Ecuador	RLVA*	159.04	53.46	-6.39	39.85	117.65	-0.29	40.94	-47.58
		Gini_diff	0.03	0.03	-0.01	0.01	0.01	-0.03	0.08	-0.10

		RLVA *	71.79	-13.33	-6.52	13.29	7.20	4.02	17.02	4.20
	Fiji	Gini_diff	-0.08	-0.07	-0.07	-0.06	-0.02	-0.01	-0.02	-0.04
		RLVA *	0.61	-0.48	-0.21	0.25	0.80	-0.31	1.72	-0.37
	Mexico	Gini_diff	-0.08	-0.07	-0.05	-0.08	-0.08	-0.03	-0.06	-0.08
		RLVA *	48.22	48.65	0.14	109.81	47.57	61.15	32.67	63.26
	Romania	Gini_diff	0.17	0.10	0.04	0.21	0.14	0.13	0.13	0.20
		RLVA *	1281.54	1467.87	-2889.75	8803.10	23983.16	-244.08	45388.18	3933.24
	South Africa	Gini_diff	-0.17	-0.09	-0.07	-0.06	-0.11	-0.06	-0.12	3933.24
		RLVA *	115.23	-42.43	41.00	93.75	354.23	77.40	515.01	-1.01
	Thailand	Gini_diff	0.00	-0.01	-0.11	-0.06	-0.34	-0.07	-0.11	0.00
High-Income Economies		RLVA *	112.46	20.95	0.75	129.42	67.32	31.37	58.00	30.10
	Austria	Gini_diff	0.00	0.00	0.01	-0.06	0.07	-0.06	-0.02	-0.05
		RLVA *	116.13	13.28	3.73	65.07	77.82	46.03	46.24	20.30
	Chile	Gini_diff	-27.47	-0.05	-0.01	-0.18	-0.14	-0.09	-0.19	-0.03
		RLVA *	281.77	20.50	2.89	96.12	339.57	19.89	46.22	46.83
	Denmark	Gini_diff	-0.17	-0.03	-0.01	0.02	-0.30	0.95	-0.11	-0.04
		RLVA *	66.67	-1.92	4.65	65.85	51.11	20.78	47.19	12.12
	Germany	Gini_diff	-0.06	-0.01	-0.01	-0.01	0.02	-0.05	0.00	0.03
		RLVA *	49.14	-3.19	-3.67	54.18	152.77	-3.87	72.14	18.98
	Israel	Gini_diff	0.86	0.01	0.02	0.05	0.22	0.01	0.10	0.08
		RLVA *	223.59	29.50	19.41	469.01	203.77	126.49	372.32	88.53
	Korea	Gini_diff	-0.46	-0.08	-0.05	-0.14	-0.20	-0.17	-0.28	-0.03
		RLVA *	-56.25	-3.13	-2.38	23.44	-25.36	15.33	5.92	9.80
	Luxembourg	Gini_diff	-0.11	-0.03	-0.06	-0.11	-0.16	-0.13	-0.09	-0.02
		RLVA *	22.46	-7.21	-2.77	36.01	6.64	1.26	22.92	2.76
	Portugal	Gini_diff	-0.91	-0.02	0.01	0.00	0.02	0.02	-0.06	0.98
		RLVA *	73.33	34.29	8.33	110.34	163.93	55.32	155.71	29.17
	UK	Gini_diff	-0.06	-0.05	-0.01	-0.04	-0.20	0.01	-0.14	-0.01

*RLVA = Real Labour Value Added per person growth rate

Where:

Ag: agriculture

Con: construction

Gov: Government services, Community, social and personal services

Man: Manufacturing

Trans: Transportation, information and communication, Finance, insurance, real estate and business services

Util: Utilities

Whsl: Whole sale, Accommodation and food service activities

Mathematical Derivation with P_A changing.

From (2) and (8)

$$\theta_{Ly}\widehat{w} + \theta_{Ky}\widehat{r} = 0 \dots \dots \dots (11)$$

$$\theta_{Lz}\widehat{w} + \theta_{Kz}\widehat{r} = -\theta_{Az}\widehat{P}_A + \theta_{Lz} \in \widehat{\beta}, \in = a'(\widehat{\beta}) \cdot \frac{\widehat{\beta}}{a(\widehat{\beta})} > 0 \dots \dots \dots (12)$$

$$\widehat{r} = \frac{\begin{vmatrix} \theta_{Ly} & 0 \\ \theta_{Lz} & -\theta_{Az}\widehat{P}_A + \theta_{Lz} \in \widehat{\beta} \end{vmatrix}}{|\theta|} = \frac{\theta_{Ly}(-\theta_{Az}\widehat{P}_A + \theta_{Lz} \in \widehat{\beta})}{|\theta|}$$

$$\widehat{w} = \frac{\begin{vmatrix} 0 & \theta_{Ky} \\ -\theta_{Az}\widehat{P}_A + \theta_{Lz} \in \widehat{\beta} & \theta_{Kz} \end{vmatrix}}{|\theta|} = \frac{-\theta_{Ky}(-\theta_{Az}\widehat{P}_A + \theta_{Lz} \in \widehat{\beta})}{|\theta|}$$

.....(12)

Hence,

$$\widehat{r} - \widehat{w} = \left[\frac{(\theta_{Ly} + \theta_{Ky})(-\theta_{Az}\widehat{P}_A + \theta_{Lz} \in \widehat{\beta})}{|\theta|} \right]$$

$$= \frac{(\theta_{Lz} \in \widehat{\beta} - \theta_{Az}\widehat{P}_A)}{|\theta|} \dots \dots \dots (13)$$

(12) is the PP line shifted up when $\widehat{P}_A < 0$.

From (10)

$$\widehat{K}_d = -\eta_1\widehat{\beta} - \eta_2 \left(\frac{\widehat{r}}{\widehat{w}} \right) = \sigma_x(\widehat{r} - \widehat{w}_s) \dots \dots \dots (14)$$

Note that if r goes up, w_s must fall given $\widehat{P}_x = 0$,

$$\theta_{sx}\widehat{w}_s + \theta_{Kx}\widehat{r} = 0$$

$$\text{or, } \theta_{sx}\widehat{w}_s + (1 - \theta_{sx})\widehat{r} = 0$$

$$\text{or, } \theta_{sx}(\widehat{w}_s - \hat{r}) + \hat{r} = 0$$

$$(\hat{r} - \widehat{w}_s) = \frac{\hat{r}}{\theta_{sx}} \dots \dots \dots (15)$$

(12), (14) and (15) imply

$$-\eta \left(\frac{r}{w} \right) = \eta_1 \hat{\beta} + \sigma_x \frac{\theta_{Ly}(\theta_{Lz} \hat{\beta} - \theta_{Az} P_A)}{|\theta|} \dots \dots \dots (15)$$

When $\widehat{P}_A = 0$ it is the KK curve. Note that when $\hat{\beta} = 0$, a drop in P_A , $\widehat{P}_A < 0$ will mean $\frac{r}{w_s}$ rising leading to excess flow of K away from X into Y and Z. That means $\frac{r}{w}$ must fall. So, KK also shifts to the left as in Figure 2.

A drop in P_A definitely reduces $\bar{\beta}$. As $\frac{r}{w}$ rises it also releases K from K_x into Y and Z requiring a drop in $\frac{r}{w}$ to generate extra demand. Overall $\frac{r}{w}$ may not change much. But Z will expand.

In the standard 2X2 model since factor prices are uniquely determined by commodity prices, a drop in P_A must increase $\frac{r}{w}$ as Z is K-intensive. Y should contract and Z should expand. But here the additional effect is because $\frac{r}{w_s}$ is also rising K_x will fall and more K is available. So, initial adjustment in Y and Z is not enough. Hence given $\bar{\beta}$ the extra K needs to be demanded by a further increase in K_d . K_Z increases by further decline in $\bar{\beta}$.

In figure 1 from point 1 to point 2 is the standard impact $\frac{r}{w}$ increasing leading to excess supply of K and extra released K increases Z by lowering $\bar{\beta}$. Lower $\bar{\beta}$ tends to lower $\frac{r}{w}$. We reach point 2. At point 2 some K_x has been released. To absorb them $\frac{r}{w}$ drops shifting KK down. K_Z starts increasing by a further decline in $\bar{\beta}$. $\frac{r}{w}$ adjusts upwards.

If X does not release much K_x , the equilibrium is more likely to be at point 2 not at point 3 with higher $\frac{r}{w}$ and lower $\bar{\beta}$. Every time $\bar{\beta}$ adjusts it is like a productivity change in Z for the most skilled worker.

When only K rises and we absorb the entire K by lowering $\bar{\beta}$ at a given $\frac{r}{w}$ the price system is disturbed because lower $\bar{\beta}$ is like a bad productivity shock for the worst group of workers so $\frac{r}{w}$ must drop. As $\frac{r}{w}$ drops K_d rises, hence $\bar{\beta}$ increases a bit. So $\frac{r}{w}$ drops and $\bar{\beta}$ drops as well.

(16) and (13) can be solved for $(\hat{r} - \widehat{w}_s)$ and $\hat{\beta}$ in equilibrium with a change in P_A . Similarly change in K can be accommodated in terms of changes in $(\hat{r} - \widehat{w}_s)$ and $\hat{\beta}$.