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Aggregate Properties of Open Economy Models with Expanding Varieties

Abstract

We present a unified dynamic framework to study the interconnections between international trade and business cycle models. We prove an aggregate equivalence between a competitive, representative firm model that has aggregate production externalities and dynamic trade models that feature monopolistic competition, endogenous entry, and heterogeneous firms. The production externalities in the representative firm model have to be introduced in the intermediate and final good sectors so that the model is isomorphic to dynamic trade models that embody love-of-variety and selection effects. In a quantitative exercise with multiple shocks, we show that to improve the fit of the dynamic trade models with the data, the most important ingredient is negative capital externality in the intermediate good sector. We conclude that this presents a puzzle for the literature as standard dynamic trade models provide micro-foundations for positive capital externality.

JEL-Codes: F120, F410, F440, F320.

Keywords: international business cycle, dynamic trade models, heterogeneous firms, production externalities, monopolistic competition, export costs, entry costs.

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1 Introduction

The standard international business cycle model, the IRBC model (e.g., Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2002)), has been used extensively to answer quantitative questions. While successful on many fronts, the model has difficulty matching some important second moments such as a higher international correlation of output compared to consumption, positive cross-country correlations of investment and hours, a high volatility of the trade balance, and a low cyclicality of the real exchange rate.

The basic IRBC model features a representative firm and perfectly competitive product markets. One can alternatively consider environments used in the modern trade literature, as developed in Krugman (1980) and Melitz (2003), which were introduced in the business cycle literature by Ghironi and Melitz (2005). A natural question then is whether these alternative environments — primarily, monopolistic competition, endogenous entry, and heterogeneous firms — lead to a better fit with the data in terms of aggregate international moments. Even more importantly, how precisely do these alternative environments affect the transmission mechanisms in response to aggregate shocks, and how do they impact international business cycle dynamics?

We provide a unified model of international business cycles and trade that can address these questions, both theoretically and quantitatively. On the theoretical front, our main result establishes an *isomorphism* between an *IRBC model extended with production externalities* in particular sectors — our unified model — and *generalized* versions of *dynamic Krugman and Melitz models*.[1] On the quantitative front, the theoretical results first enable us to pin-point how trade features affect the transmission of aggregate shocks. Second, they allow us to flexibly explore how the fit with the data can be improved. We find that the most important ingredient is negative capital externality in the intermediate good sector.

Let us now explain in detail the key components of our models and the results. In the basic IRBC model, each country uses capital and labor in a Constant

[1] In Section 1.1 of Bhattarai and Kucheryavyy (2022), we consider a dynamic Eaton and Kortum (2002) model with capital accumulation. The equivalence of that model with the IRBC model is immediate as there are no externalities, and we do not discuss this result in the current paper.

Returns to Scale technology to produce a unique and traded intermediate good. Intermediate goods originating from different countries are combined into a final good using a Constant Elasticity of Substitution technology. The final good is used for consumption and investment into capital. Our unified model extends the basic IRBC model by introducing external economies of scale in production of intermediate (in terms of both labor and capital) and final goods. In addition to the unified model, we also formulate *generalized* dynamic versions of the Krugman (1980) and Melitz (2003) models by extending their *standard* counterparts. These set-ups then allow us to prove that the *generalized* dynamic versions of the Krugman and Melitz models are isomorphic to the *unified* model in their aggregate predictions.

Aggregate externalities introduced in the unified model are key to establishing the isomorphism, and these externalities do arise even in *standard* versions of the Krugman and Melitz models. These standard versions, however, allow only a *one-way mapping* to the unified model. This is because the standard models imply tight relationships between technological parameters in the corresponding unified model: Cobb-Douglas share of capital in the intermediate good technology, elasticity of substitution between intermediate goods in the final good technology, and strengths of external economies of scale. In particular, all of these parameters of the unified model are determined by only one structural parameter in the Krugman model — the elasticity of substitution between varieties, and by two structural parameters in the Melitz model — the elasticity of substitution between varieties and the shape of Pareto distribution. The essence of our generalizations of the Krugman and Melitz models is in breaking the implied tight relationships between technological parameters in the corresponding unified model, which is needed to establish a *two-way mapping* (that is, an isomorphism).

One building block for our results is that the measures of firms in the Krugman and Melitz models play the role of capital in the unified model. Thus, even though labor is the only factor of production in both the Krugman and Melitz models, the corresponding unified model features an aggregate production function for intermediate goods that uses both capital and labor. The *total* capital's exponent in this function is determined by the love-of-variety effect in the Krugman model and the selection effect in the Melitz model.

Next, under the usual assumption that in the Krugman and Melitz models technology of production of differentiated varieties is linear in labor, the aggregate production function for intermediate goods in the corresponding unified model is also linear in labor. Only a part of capital and a part of labor used in this function are internalized by the representative firm, while the remaining parts induce (positive) externalities. The internalized part of capital is equal to the share of firms' revenues accrued as profits in both the Krugman and Melitz models. Similarly, the internalized part of labor is equal to the share of firms' revenues that is paid as wages to labor used in production of varieties.

Besides externalities in intermediate goods production, the Krugman model does not generate other externalities. The Melitz model however, additionally generates a (positive) externality in final good production. This externality arises due to the selection effect that works through the importer's total demand for varieties: greater demand for varieties increases the number of exporters entering the market, which lowers importer's price index due to love-of-variety. The representative producer of the final good does not internalize this entry effect on the price index.

Our generalization of the Krugman model introduces correction for the love-of-variety effect in the production function for the final good, a labor externality in the production function for varieties, and an externality in the production function for the final good. The generalization of the Melitz model introduces correction of the selection effect in fixed costs of serving markets and a labor externality in the production function for varieties. Thus, to achieve the isomorphism with the unified model, we target the sources of externalities directly (love-of-variety and selection) when possible, or introduce externalities precisely into the production functions, thereby freeing the tight relationships among externalities implied by the standard Krugman and Melitz models.

Given our theoretical results, we undertake a quantitative exercise. First, we show that standard dynamic Krugman and Melitz models do not resolve the key empirical puzzles related to cross-country correlations. Our theoretical result offers the explanation: standard formulations and calibrations of these models lead to relatively *small, tightly restricted, and positive* externalities. This then leads to transmission mechanisms that are very similar to the IRBC model, as the endogenous cyclical movements in productivity introduced by the externalities are minor.

Second, we pinpoint what is needed to achieve a better fit with the data. We consider two types of shocks, intermediate good and final good productivity shocks, and show that an essential feature to improve fit with the data is a *negative* capital externality in the intermediate goods technology for both shocks. This exercise is possible only because our unified model perspective allows us to isolate different types of shocks and, more importantly, to vary the strengths of externalities independently from each other and from other parameters of the model.[2]

To understand why negative capital externality in the intermediate goods technology helps improve fit with the data, it is helpful to first review the empirical puzzles associated with the two-country IRBC model and their underlying source. In the model, the international correlation of consumption is counterfactually higher than that for output, while the international correlations of labor hours and investment are lower than in the data.[3] A common source behind all these anomalies is the tendency in the IRBC model for a positive intermediate good shock in the home country to lead to a substantial rise in factor use at home, while inducing a cut in factor use abroad. A negative capital externality makes the transmission of both the intermediate and final good productivity shocks endogenously negative. This limits the persistent rise in factor use at home, while limiting the fall in factor use abroad and, thus, helps with improving international correlations.[4]

Let us first discuss the standard intermediate good productivity shock. With negative capital externality, from the *perspective of individual firms*, it is *as if the aggregate intermediate good productivity shock is less persistent*, but has the same initial impact. This is because due to higher capital accumulation in future, the productivity faced by the firms is lower than the exogenous shock. The less persistent productivity increase leads to less persistent increase in hours, investment, and output at home. This endogenous decrease in persistence of productivity at home

[2]Given the two-way mapping between the unified model and the generalized trade models, we can interpret the underlying source of negative capital externality from either model perspectives.

[3]In fact, labor hours and investments across countries often co-move negatively in the model while they co-move positively in the data. The high cross-country correlation in consumption is not entirely due to perfect risk-sharing and we make this clear by presenting results for complete financial markets, bond economy, and financial autarky.

[4]Negative labor or final good externalities, in contrast, do not help improve the fit uniformly.

also acts against the reallocation of factors away from the foreign country. Moreover, with consumption smoothing motives, in the face of a less persistent rise in income at home, consumption increases by less initially and more importantly, its dynamic response changes non-trivially due to change in the path of investment.

Overall, these changes to the transmission help increase output, investment, and hours correlation across countries while decreasing consumption correlation. In addition, negative capital externality also leads to an endogenous positive correlation of home productivity with the foreign, as typically the foreign country would decumulate capital. This generates, compared to the no externality case, an increase in hours, output, and investment in the foreign country.

For the final good productivity shock, a similar mechanism holds. While this shock does not directly affect the intermediate good production function, endogenously however, intermediate good productivity declines as there is higher future capital accumulation in response to a positive final good shock. This negative effect on intermediate good productivity acts in an opposite direction to the positive effect of the final good productivity shock on the home country, thereby limiting the persistent rise in factors use at home and the cut in factors use abroad.

To complete our quantitative exercise, we estimate our unified model with the intermediate and final good productivity shocks, which are not exogenously imposed to be correlated across countries, by matching several second moments from the data.[5] In particular, we match not just cross-country correlations, but also volatility and cyclicality of both domestic and open economy variables. We estimate significant negative capital externality and find that the final good productivity shock drives the international business cycle.[6] These quantitative conclusions hold for either complete markets or the bond economy.

Our results on negative capital externality pose a puzzle for the quantitative literature that studies the interactions of international trade and business cycles

[5]Two key components of our estimation are that shocks across countries are not exogenously correlated and the trade elasticity is positive. Heathcote and Perri (2014) show that the standard IRBC model with intermediate good productivity shocks, even under complete markets, can match several key international correlations if the exogenous shock correlation is calibrated to match cross-country output correlation, and if the trade elasticity is negative.

[6]While both the productivity shocks lead to a domestic business cycle, for some open economy variables, the final good productivity shock enables a better fit with the data.

models. They point towards two possible implications for future research. One is to take a stance that existing models feature "missing negative capital externality" and move towards fully micro-founding it in existing models. Our theoretical results show one possible avenue as our generalized Melitz model can generate negative capital externality through congestion effects in serving markets: the larger is the number of firms operating in a particular market, the higher is the per-period fixed cost of serving this market.[7] It is a potentially interesting area for future research to try to assess whether such congestion effects — estimated at business cycles frequencies — are large enough to improve the fit with international moments.[8]

The other implication for future research is to modify the core structure of the models beyond those often considered in the literature such that positive capital externality, as embedded and micro-founded in dynamic trade models, can help improve the fit on international moments. As one extension, we introduce sticky prices in a standard dynamic Krugman model, but find that both the theoretical and quantitative conclusions remain largely the same as in our baseline model with flexibe prices. This suggests that further modifications are necessary to generate significant quantitative differences between the IRBC and dynamic trade models and for positive capital externality to help improve the fit on international moments.

Our paper is related to several strands of the literature. In particular, Ghironi and Melitz (2005) and Jaef and Lopez (2014), extend the set-up in Melitz (2003) to a dynamic setting and develop models most similar to the standard dynamic trade models we present. They assess how important international trade features are for real exchange rate and business cycle dynamics.[9] Alessandria and Choi (2007)

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[7] One advantage of formulating the generalized Melitz model, such that there is a two-way mapping between the generalized IRBC and Melitz models, is that it allows us to see through which channel, within the context of the Melitz model, we can generate negative capital externality.

[8] Another possiblity (outside of the model we present here) for the negative capital exernality to arise is through similar mechanisms as have been used to motivate decreasing returns to scale in technology such as the Lucas (1978) limited span of control set-up or pehaps even financial frictions. We should note however, that negative externality in capital and decreasing returns to scale that is internalized lead to different propagation mechanisms. The current pandemic has provided an example of a relevant negative externality, but such examples are arguably not easy to find to explain general and historical international business cycle phenomena.

[9] Eaton *et al.* (2016) add physical capital accumulation to the competitive Eaton and Kortum

address these questions in an environment that features sunk costs of entry into exporting markets, which create exporters hysteresis — the feature absent in the setup of the generalized Melitz model of the current paper. Alessandria and Choi (2019) extend the model in Alessandria and Choi (2007) to generate procylical entry and importer dynamics together with high volatility in number of exporters and importers. They show how open economy aspects play a key role in generating procyclical and volatile entry. Their paper however, does not feature a evaluation with respect to the international comovement puzzle, which constitutes our main focus. Cook (2002) presents a model where procyclical entry helps generate international comovement. The key mechanism operates through countercyclical markups in a model with Cournot competition and further requires incomplete markets and variable capacity utilization. The model environment and necessary additional propagation mechanisms are therefore quite different from what we focus on in our paper.

Head (2002) presents a variant of the dynamic Krugman model with externality in production of the consumption aggregate. This externality is modelled as a weighted sum of the measures of home and foreign varieties. Head (2002) shows that with large enough strength of this externality the model can generate positive international comovements in output, investment, and hours. From the point of view of our generalized IRBC model, instead of working directly with externalities generated by the Krugman model, and focusing on their role in business cycle dynamics, Head (2002) is introducing a new externality.

Our result on isomorphism is related to a similar result in a static environment in Kucheryavyy *et al.* (2022), who present a version of the competitive model with multiple manufacturing sectors that feature external economies of scale in production. They show that their model is isomorphic to generalized static versions of multi-industry Krugman and Melitz models. Here, we focus on dynamic versions of Krugman and Melitz models that have only one manufacturing sector and other

(2002) set-up while Jaef and Lopez (2014) add physical capital accumulation on top of the entry of varieties present in Ghironi and Melitz (2005). In Sections 1.1 and 1.2 of Bhattarai and Kucheryavyy (2022), we develop both the dynamic Eaton-Kortum and the Krugman models with physical capital accumulation and show that our isomorphism result applies. In the current paper, we do not consider trade models with physical capital in order to emphasize how the aggregate production function changes in these models even though the firm-level production function only uses labor.

"non-manufacturing" sectors: final aggregate, investment and consumption.

Extension of the isomorphism from static to dynamic environments is non-trivial as it adds several new features due to capital accumulation and endogenous trade deficits. Most important are the split of externalities between labor and capital, which plays an important role both qualitatively and quantitatively, and the final good externality that appears in the Melitz model because of endogenous trade balance.[10] The general formulation of externalities that can be used in both dynamic and static contexts constitutes one of our theoretical contributions. We then use the general model for a quantitative evaluation of business cycle statistics and transmission mechanisms in response to multiple aggregate shocks.

Our paper is also related to the closed economy endogenous growth literature (e.g., Romer, 1986), in which growth is generated by increasing returns in production, and where production externalities are modeled with respect to the capital input.[11] In our unified open economy model, production externalities exist with respect to both capital and labor. In a closed-economy business cycle analysis, Benhabib and Farmer (1994) introduced production externalities in the RBC model to generate the possibility of multiple, bounded equilibria. In a closed economy set-up as well, Bilbiie *et al.* (2012) discuss how firm dynamics and firm entry in a model with monopolistic competition and sunk cost of entry look similar to capital stock dynamics and investment in the standard competitive model. Our general model provides a similar interpretation, while additionally showing formally how a competitive open economy set-up with different levels and types of production externalities is in fact isomorphic to various generalized versions of monopolistic competition trade models with firm heterogeneity.

[10]Even in the static context, we show that, whether entry costs are paid in terms of labor (standard assumption in the trade literature) or final good (more in line with the business cycle literature's assumption for investment), has important implications for the formulation of externalities.

[11]In this respect, our results on needing negative capital externality pose a further puzzle if we were to unify business cycles, trade, and growth all together in one framework. One possible resolution is that congestion effects are large in the short run, but are overpowered by increasing returns in production in the long run. Exploring the short versus long run dichotomy in this context might be a possible area of future research. Another possible resolution is to modify models such that productivity shocks endogeneously spillover from one country to another, while ensuring that such a model generates appropriate co-movement, cyclicalty, and volatility of domestic and international variables.

2 Unified Model of Business Cycles and Trade

The world consists of $N = 2$ countries.[12] Each country has four production sectors: intermediate, final aggregate, consumption, and investment. Capital and labor are the primary factors of production. All markets are perfectly competitive.

2.1 Intermediate Goods and International Trade

Output of a country-n's intermediate good producer that in period t employs $k_{X,nt}$ units of capital and $l_{X,nt}$ units of labor is given by $S_{X,nt} k_{X,nt}^{\alpha_{X,K}} l_{X,nt}^{\alpha_{X,L}}$, where $\alpha_{X,K} \geq 0$ and $\alpha_{X,L} \geq 0$ with $\alpha_{X,K} + \alpha_{X,L} = 1$, and

$$S_{X,nt} \equiv \Theta_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}} \tag{1}$$

is aggregate productivity. The aggregate productivity consists of two parts: exogenous productivity, $\Theta_{X,n} Z_{X,nt}$, and endogenous productivity, $K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}$. The term $Z_{X,nt}$ in the exogenous productivity part is an aggregate shock, while the term $\Theta_{X,n}$ is a normalization constant introduced to later show isomorphisms between different models. The endogenous productivity part captures external economies of scale in production of intermediates, and it is taken by firms *as given*. The terms $K_{X,nt}$ and $L_{X,nt}$ are the total amounts of country n's capital and labor used in production of intermediates. Parameters $\psi_{X,K}$ and $\psi_{X,L}$ drive the strength of external economies of scale. Perfect competition in production of intermediates implies that the total output of intermediates in country n in period t is given by $X_{nt} = S_{X,nt} K_{X,nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}$.

Let $P_{X,nt}$ denote the price of country n's intermediate good, and W_{nt} and R_{nt} denote the wage and capital rental rate in country n. Due to perfect competition, $K_{X,nt} = \alpha_{X,K} \left(P_{X,nt} X_{nt}\right)/R_{nt}$ and $L_{X,nt} = \alpha_{X,L} \left(P_{X,nt} X_{nt}\right)/W_{nt}$. Moreover,

$$P_{X,nt} = \frac{R_{nt}^{\alpha_{X,K}} W_{nt}^{\alpha_{X,L}}}{\widetilde{\Theta}_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}}, \quad \text{where } \widetilde{\Theta}_{X,n} \equiv \alpha_{X,K}^{\alpha_{X,K}} \alpha_{X,L}^{\alpha_{X,L}} \Theta_{X,n}. \tag{2}$$

[12]Even though we consider only the case of $N = 2$ countries, we keep N in the notation as it makes sums over country indices more transparent. Moreover, all our theoretical results are true for the general case of $N \geq 2$ countries.

Intermediate goods are the only traded goods, and trade in these goods is subject to iceberg costs: in order to deliver one unit of intermediate good to country n, country i needs to ship $\tau_{ni,t} \geq 1$ units of this good. The price of country i's intermediate good sold in country n is then given by $P_{ni,t} \equiv \tau_{ni,t} P_{X,it}$.

2.2 Final Aggregates and Consumption Goods

Final aggregate is produced by combining intermediate goods imported from different counties. Let $X_{ni,t}$ denote the amount of intermediate good that country n buys from country i in period t. The total output of final aggregate in country n at time t is given by $Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left(\omega_{ni} X_{ni,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$, where $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, $\sigma > 0$ is an Armington elasticity of substitution between intermediate goods produced in different countries, and

$$S_{Y,nt} \equiv \Theta_{Y,n} Z_{Y,nt} \left(P_{Y,nt} Y_{nt} / W_{nt} \right)^{\psi_Y} \tag{3}$$

is aggregate productivity with $P_{Y,nt}$ being the price of the final aggregate.

As in production of intermediates, productivity in production of the final aggregate has exogenous and endogenous parts given, correspondingly, by $\Theta_{Y,n} Z_{Y,nt}$ and $\left(P_{Y,nt} Y_{nt} / W_{nt} \right)^{\psi_Y}$, with ψ_Y driving the strength of external economies of scale in production of the final aggregate. The term $Z_{Y,nt}$ is an aggregate productivity shock. We do not put any restrictions on its correlation with the shock $Z_{X,nt}$ in the intermediate goods sector. The term $\Theta_{Y,n}$ is a normalization constant introduced to later show isomorphisms between different models. The endogenous part of $S_{Y,nt}$ captures external economies of scale in production of the final aggregate, and it is taken by firms *as given*. $(P_{Y,nt} Y_{nt}) / W_{nt}$ is the number of country-n's workers that produce the same value as the value of the final aggregate.[13]

[13] $Z_{Y,nt}$ is new to the IRBC model and plays an important role quantitatively. The particular form in which the externality in production of the final aggregate is introduced is chosen to later show isomorphism with the dynamic Melitz model.

Perfect competition in production of the final aggregate implies that $P_{Y,nt}$ is

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\Theta_{Y,n} Z_{Y,nt} \left(P_{Y,nt} Y_{nt}/W_{nt}\right)^{\psi_Y}}, \tag{4}$$

and country n's share of expenditure on country i's intermediate good, $\lambda_{ni,t}$, is

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{1-\sigma}}. \tag{5}$$

Final aggregate in country n is used directly as the consumption good in country n as well as in the production process of the investment good, as described next.

2.3 Investment Goods

Let I_{nt} denote the total output of the investment good in country n in period t, and $P_{I,nt}$ the price of this good. Investment good is produced from labor and the final aggregate with the production technology given by

$$I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \tag{6}$$

where $0 \leq \alpha_I \leq 1$. Here $L_{I,nt}$ and $Y_{I,nt}$ are the total amounts of labor and final aggregate used in production of the investment good, $Z_{I,nt}$ is an exogenous aggregate productivity shock, and $\Theta_{I,n}$ is a normalization constant introduced to later show isomorphisms between different models. We do not put any restrictions on correlation of $Z_{I,nt}$ with the shocks $Z_{X,nt}$ and $Z_{Y,nt}$.[14]

Perfect competition in production of the investment good implies that $L_{I,nt} = \alpha_I \left(P_{I,nt} I_{nt}\right)/W_{nt}$ and $Y_{I,nt} = (1-\alpha_I)\left(P_{I,nt} I_{nt}\right)/P_{Y,nt}$. Moreover,

$$P_{I,nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}, \quad \text{where } \widetilde{\Theta}_{I,n} \equiv \alpha_I^{\alpha_I} \left(1-\alpha_I\right)^{1-\alpha_I} \Theta_{I,n}. \tag{7}$$

[14] In the IRBC model, investment is made directly from the final good. This technology can be obtained from (6) by setting $\Theta_{I,n} = 1$, $Z_{I,nt} = 1$, and $\alpha_I = 0$. As we will see later, the technology for producing the investment good in the standard versions of dynamic Krugman and Melitz models corresponds to setting $\alpha_I = 1$ and having $\Theta_{I,n} Z_{I,nt} \neq 1$. These differing choices can have non-trivial implications for the cyclicality of net exports, and, therefore, we take a general approach.

2.4 Households

Each country n has a representative household with the period-t utility function given by $U\left(C_{nt}, L_{nt}\right)$, where C_{nt} and L_{nt} are the household's consumption and supply of labor in period t. The household chooses consumption, supply of labor, investment, and holdings of financial assets (if allowed) so as to maximize the expected lifetime utility, $E_t \sum_{s=0}^{\infty} \beta^s U\left(C_{n,t+s}, L_{n,t+s}\right)$, subject to the budget constraint and the law of motion of capital, where $\beta \in (0,1)$ is the discount factor, and E_t denotes the expectation over the states of nature taken in period t. The law of motion of capital is given by $K_{n,t+1} = (1-\delta) K_{nt} + I_{nt}$, where I_{nt} is investment in period t and $\delta \in [0,1]$ is the depreciation rate.

Depending on the international financial markets structure, households face different budget constraints. We consider three standard alternatives for international financial markets: complete markets, bond economy, and financial autarky. In the case of financial autarky the budget constraint is given by $P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} = W_{nt}L_{nt} + R_{nt}K_{nt}$. In the case of the bond economy and complete markets the budget constraints can be written by adding the expenditure and income from financial assets. Since those budget constraints are standard, to conserve space, we delegate their formal description to Appendix A.1. Also, in the same appendix we provide the first-order conditions associated with the household problem.

In the case of complete financial markets and bond economy, international trade in assets allows unbalanced trade. For future use, we define country n's real trade balance TB_{nt} as the value of net exports of intermediate goods in terms of the final good,

$$\mathrm{TB}_{nt} \equiv \left(P_{X,nt}X_{nt} - P_{Y,nt}Y_{nt}\right) / P_{Y,nt}\ , \tag{8}$$

and define country n's real current account CA_{nt} as the change in this country's net financial assets position in terms of the final good.[15]

[15]The definition of CA_{nt} is standard, but as it requires additional notation, it is in Appendix A.1.

2.5 Market Clearing Conditions

The labor market clearing condition for country n is given by

$$W_{nt}L_{X,nt} + W_{nt}L_{I,nt} = W_{nt}L_{nt} + aP_{Y,nt} \cdot \mathrm{TB}_{nt}, \tag{9}$$

where a is a constant. When $a = 0$, we have a standard labor market clearing condition. The extra term $aP_{Y,nt} \cdot \mathrm{TB}_{nt}$ is introduced to show isomorphism with the dynamic Melitz model, for which $a > 0$, and for which this term appears *only* if trade is unbalanced. Next, since capital is used only in production of intermediate goods, we have $K_{X,nt} = K_{nt}$ for each n. The final aggregate is used in consumption and production of the investment good, $C_{nt} + Y_{I,nt} = Y_{nt}$ for each n. Demand for intermediate goods is equal to supply, $\sum_{n=1}^{N} \tau_{ni,t} X_{ni,t} = X_{it}$ for each i. In the case of the bond economy and complete markets we also have the sets of market clearing conditions for financial assets.

The full set of equilibrium conditions is provided in Appendix A.2.

3 Generalized Krugman and Melitz Models

We next present the key elements of generalized dynamic versions of the Krugman and Melitz models, focusing on elements of these models that differ from their standard expositions in the literature. Anticipating isomorphisms between the unified, Krugman, and Melitz models, we use the same notation for parameters and variables of these models that map into each other. To mark some of the parameters and variables as being specific to a particular model, we use superscripts "K" for the Krugman model and "M" for the Melitz model.[16]

[16]Our presentation omits all the detailed derivations, which can be found in Sections 1.2 and 1.3 of Bhattarai and Kucheryavyy (2022).

3.1 Generalized Dynamic Version of the Krugman Model

3.1.1 Production of Varieties, International Trade, and Final Aggregate

Each country i produces a unique set of varieties Ω_{it}, which is endogenously determined in every period t. Let M_{it} be the measure of this set. All varieties can be internationally traded. Let $p_{ni,t}(\nu)$ denote the price of variety $\nu \in \Omega_{it}$ produced by country i and sold in country n. Assuming iceberg trade costs and no arbitrage in international trade implies that $p_{ni,t}(\nu) = \tau_{ni,t} p_{ii,t}(\nu)$.

Countries use varieties to produce non-traded final aggregates. Technology of production of the final aggregate is given by the nested CES production function

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left[M_{it}^{\phi_{Y,M} - \frac{1}{\sigma^K - 1}} \left[\int_{\nu \in \Omega_{it}} \left(\omega_{ni} x_{ni,t}(\nu) \right)^{\frac{\sigma^K - 1}{\sigma^K}} d\nu \right]^{\frac{\sigma^K}{\sigma^K - 1}} \right]^{\frac{\eta^K - 1}{\eta^K}} \right]^{\frac{\eta^K}{\eta^K - 1}}, \quad (10)$$

where $x_{ni,t}(\nu)$ is the amount of variety $\nu \in \Omega_{it}$ that country n buys from country i in period t, $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, and $S_{Y,nt} \equiv \Theta_{Y,n} Z_{Y,nt} \left(P_{Y,nt} Y_{nt} / W_{nt} \right)^{\psi_Y}$. All terms of $S_{Y,nt}$ have the same meaning as in the corresponding definition (3) in the unified model.

The nested CES structure of (10) implies that the elasticity of substitution between varieties produced in one country, given by σ^K, is different from the elasticity of substitution between varieties produced in different countries, given by η^K. We assume that $\sigma^K > 1$ and $\eta^K > 1$. The term $M_{it}^{\phi_{Y,M} - \frac{1}{\sigma^K - 1}}$ introduces correction for the love-of-variety effect, which is the only source of externalities in the standard Krugman model with CES preferences. As is discussed in Benassy (1996), parameter $\phi_{Y,M}$ governs the taste for variety in the Krugman model (the standard Krugman model implies that the strength of the taste for variety is $1/\left(\sigma^K - 1\right)$). At the same time, in the unified model, parameter $\phi_{Y,M}$ governs the strength of economies of scale induced by capital in production of intermediate goods. Having this parameter is critical for showing the isomorphism with the unified model.

Perfect competition in production of the final aggregate gives CES demand,

$$x_{ni,t}(\nu) = S_{Y,nt}^{\eta^K-1} M_{it}^{(\sigma^K-1)\left(\phi_{Y,M}-\frac{1}{\sigma^K-1}\right)} \omega_{ni}^{\eta^K-1} \left(\frac{p_{ni,t}(\nu)}{P_{ni,t}}\right)^{-\sigma^K} \left(\frac{P_{ni,t}}{P_{Y,nt}}\right)^{-\eta^K} Y_{nt}, \tag{11}$$

$$P_{ni,t} = M_{it}^{-\left(\phi_{Y,M}-\frac{1}{\sigma^K-1}\right)} \left[\int_{\nu\in\Omega_{it}} p_{ni,t}(\nu)^{1-\sigma^K} d\nu\right]^{\frac{1}{1-\sigma^K}}, \tag{12}$$

$$P_{Y,nt} = S_{Y,nt}^{-1} \left[\sum_{i=1}^{N} \left(P_{ni,t}/\omega_{ni}\right)^{1-\eta^K}\right]^{\frac{1}{1-\eta^K}}. \tag{13}$$

Production of variety $\nu \in \Omega_{nt}$ requires only labor and is given by

$$x_{nt}(\nu) = S_{X,nt}^{K} l_{nt}(\nu), \tag{14}$$

where $l_{nt}(\nu)$ is the amount of labor used in production of variety ν, and $S_{X,nt}^{K} \equiv \Theta_{X,n} Z_{X,nt} L_{X,nt}^{\phi_{X,L}}$ is the aggregate productivity in production of varieties.[17] The aggregate productivity $S_{X,nt}^{K}$ consists of two parts: exogenous productivity, $\Theta_{X,n} Z_{X,nt}$, and endogenous productivity, $L_{X,nt}^{\phi_{X,L}}$. Here $\Theta_{X,n}$ is a normalization constant, $Z_{X,nt}$ is an exogenous shock, and $L_{X,nt}$ is the total amount of labor allocated to production of varieties in country n. The endogenous part of the aggregate productivity is an additional source of external economies of scale (on top of the love-of-variety effect) and is taken by firms as given. Having this additional source of externality is critical for showing the full isomorphism with the unified model.

Producers of varieties ν are engaged in monopolistic competition. Hence, the price of variety $\nu \in \Omega_{it}$ is $p_{ni,t}(\nu) = \frac{\sigma^K}{\sigma^K-1}\left(\tau_{ni,t} W_{it}/S_{X,it}^{K}\right)$ and the bilateral price

[17] In Section 1.2 of Bhattarai and Kucheryavyy (2022) we consider a more general (nonlinear) technology that features physical capital in addition to labor, $x_{nt}(\nu) = S_{X,nt}^{K}\left[l_{nt}(\nu)^{\gamma_X} k_{nt}(\nu)^{1-\gamma_X}\right]^{\gamma}$ with $0 \le \gamma_X \le 1$ and $\gamma > 0$. This generalization first allows us to demonstrate clearly the difference between internal versus external economies of scale in labor in the Krugman model, without conceptually changing them. Second, it allows us to show that our key result on isomorphism continues to apply even with physical capital used to produce varieties. We discuss the economic implications of the nonlinear technology and physical capital in Sections 2.1 and 2.2 of Bhattarai and Kucheryavyy (2022). One conclusion concerning the nonlinear technology is that, while adding flexibility, it does not help us with matching data moments because it is still restrictive.

index is $P_{ni,t} = \tau_{ni,t} P_{X,it}$, where

$$P_{X,it} \equiv \frac{\sigma^K}{\sigma^K - 1} \cdot \frac{W_{it}}{\Theta_{X,i} Z_{X,it} M_{it}^{\phi_{Y,M}} L_{X,it}^{\phi_{X,L}}}, \tag{15}$$

which can be interpreted as the price of the output of varieties in country i in period t. Substituting expression for $P_{ni,t}$ into (13), we get

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\eta^K}\right]^{\frac{1}{1-\eta^K}}}{\Theta_{Y,n} Z_{Y,nt} \left(P_{Y,nt} Y_{nt} / W_{nt}\right)^{\psi_Y}}. \tag{16}$$

Next, the share of expenditure of country n on country i's varieties is

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\eta^K}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj}\right)^{1-\eta^K}}. \tag{17}$$

The total expenditure of country n on country-i's varieties is given by $\mathcal{X}_{ni,t} = \lambda_{ni,t} P_{Y,nt} Y_{nt}$. Let $\mathcal{X}_{nt}$ denote the value of total output of varieties in country n and D_{nt} denote the average profit of country n's producers of varieties Ω_{nt}. We have

$$\mathcal{X}_{nt} = \frac{\sigma^K}{\sigma^K - 1} W_{nt} L_{X,nt} \quad \text{and} \quad D_{nt} = \frac{1}{\sigma^K} \cdot \frac{\mathcal{X}_{nt}}{M_{nt}}. \tag{18}$$

3.1.2 Entry and Exit of Producers of Varieties

In order to enter the economy, producer of a variety in country n in period t needs to pay sunk cost equal to $\dfrac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$, where $0 \leq \alpha_I \leq 1$, and $\widetilde{\Theta}_{I,n} Z_{I,nt}$ is an exogenous cost shifter. Paying this sunk cost involves hiring $L_{I,nt} = \alpha_I \, V_{nt} / W_{nt}$ units of labor and using $Y_{I,nt} = (1 - \alpha_I) \, V_{nt} / P_{Y,nt}$ units of the final aggregate, where V_{nt} is the value of a variety in country n in period t.[18]

In every period t, each country has an unbounded mass of prospective entrants (firms) into the production of varieties. Entry into the economy is free, and, there-

[18] In Section 1.2 of Bhattarai and Kucheryavyy (2022) we derive the sunk cost by introducing an R&D sector and specifying an invention process for new varieties. Labor and final aggregate needed to pay the sunk cost are interpreted as the production factors used in the R&D sector.

fore, the value of a variety is equal to the sunk cost of entry:

$$V_{nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}. \tag{19}$$

Timing is as follows. Firms entering in period t start producing in the next period. At the end of each period t, an exogenous fraction δ of the total mass of firms (i.e., a fraction δ of M_{nt}) exits. The probability of exit is the same for all firms regardless of their age. Since exit occurs at the end of a period, any firm that entered into the economy produces for at least one period. Let $M_{I,nt}$ denote the number of producers of varieties that enter into the country n's economy in period t. Given the described process of entry and exit of firms, the law of motion of varieties is

$$M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}. \tag{20}$$

Producers of varieties are owned by households. We turn to their problem next.

3.1.3 Households

In each country n, a representative household maximizes the expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \beta^t U(C_{nt}, L_{nt})$, by choosing consumption C_{nt}, supply of labor L_{nt}, the number of new varieties $M_{I,nt}$, and holdings of financial assets (if allowed). Constraints faced by the households are the budget constraint and the law of motion of varieties given by (20). The specification of the budget constraint depends on the financial markets structure. In the case of financial autarky it is given by

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt}. \tag{21}$$

The left-hand side of (21) contains household's expenditure in period t: it spends its budget on consumption and entry of new firms. The right-hand side contains household's income in period t: sum of labor income and profits of firms. In the case of the bond economy and complete markets the budget constraints can be written by adding the expenditure and income from financial assets in the same manner as it is done in the unified model in Appendix A.1.

3.1.4 Markets Clearing Conditions

Labor is used for production and invention of varieties, $L_{x,nt} + L_{I,nt} = L_{nt}$, demand for varieties is equal to supply, $\sum_{n=1}^{N} \mathcal{X}_{ni,t} = \mathcal{X}_{it}$, and the final aggregate is used for consumption and invention of varieties, $C_{nt} + Y_{I,nt} = Y_{nt}$. The complete set of equilibrium conditions is provided in Appendix B.1.

3.2 Generalized Dynamic Version of the Melitz Model

Production side of the Melitz model is similar to that of the Krugman model in using only labor in production of intermediate goods, featuring monopolistic competition, and having sunk costs of entry into the economy.[19] Additional features of the Melitz model are heterogeneous firms with Pareto distribution of efficiencies of production and the requirement that firms pay fixed costs of serving markets.

3.2.1 Production of Varieties, International Trade, and Final Aggregate

In every period t, country i can produce any of the varieties from an endogenously determined set of varieties Ω_{it} with measure M_{it}. All varieties from the set Ω_{it} can be internationally traded, but not all of them are available in a particular country n. The subset of country-i's varieties available in country n is denoted by $\Omega_{ni,t}$ (with $\Omega_{ni,t} \subseteq \Omega_{it}$), and its measure is denoted by $M_{ni,t}$. Subsets of varieties $\Omega_{ni,t}$ are endogenously determined. Importantly, only a subset $\Omega_{ii,t}$ of the whole set of varieties Ω_{it} is available in the domestic market i, and, generally, some varieties from Ω_{it} are not available in any country. Moreover, in general it can happen that some varieties from Ω_{it} are available in country $n \neq i$, but not in country i.

In order to sell in the country-n's market, a country-i's producer of a variety has to pay two types of costs: the usual per-unit iceberg trade costs $\tau^{\mathrm{M}}_{ni,t}$ and fixed

[19] The version of the Melitz model considered here is close to the framework described in Ghironi and Melitz (2005), which does not have physical capital. This is different from other similar models considered in Alessandria and Choi (2007, 2019) or Jaef and Lopez (2014), which feature physical capital. The key reason we do not include physical capital in our generalized trade models is to show clearly in the theory how endogenous entry decisions lead to dynamics of varieties that is similar to standard dynamics of capital. Similarly to what we mentioned before with regards to the Krugman model, adding physical capital to the generalized Melitz model does not change our isomorphism results. The only difference is that in the isomorphic competitive model we have "two" types of capital stocks: physical capital and varieties.

cost $\Phi_{ni,t} > 0$, which are paid in terms of country-n's labor. The fixed cost $\Phi_{ni,t}$ is an endogenous object. Its formal definition is given later.

As in the Krugman model, countries combine varieties to produce non-traded final aggregates using the nested CES technology,

$$Y_{nt} = \left[\sum_{i=1}^{N} \left[\int_{\nu \in \Omega_{ni,t}} \left(\omega_{ni} x_{ni,t} \left(\nu \right) \right)^{\frac{\sigma^{\text{M}}-1}{\sigma^{\text{M}}}} d\nu \right]^{\frac{\sigma^{\text{M}}}{\sigma^{\text{M}}-1} \cdot \frac{\eta^{\text{M}}-1}{\eta^{\text{M}}}} \right]^{\frac{\eta^{\text{M}}}{\eta^{\text{M}}-1}} . \tag{22}$$

Differently from the Krugman model, we do not add correction for the love-of-variety effect in (22).[20] Also, (22), differently from (10), does not have an exogenous shock and external economies of scale. The reason for this is that the structure of the Melitz model endogenously generates both the exogenous shock and externalities in production of the final aggregate — both of these components of production function come from the fixed costs of serving markets, which are introduced below.

Perfect competition in production of the final aggregate implies the usual expressions for the CES demand that are almost the same as the corresponding expressions (11)-(13) in the Krugman model, except that there is no term correcting for the love of variety, and in the definition of $P_{ni,t}$ integration is over $\Omega_{ni,t}$ instead of Ω_{it}. For future reference, we provide the definition of $P_{ni,t}$,

$$P_{ni,t} = \left[\int_{\nu \in \Omega_{ni,t}} p_{ni,t} \left(\nu \right)^{1-\sigma^{\text{M}}} d\nu \right]^{\frac{1}{1-\sigma^{\text{M}}}} . \tag{23}$$

Production technology of variety $\nu \in \Omega_{it}$ is given by

$$x_{it} \left(\nu \right) = S^{\text{M}}_{X,it} z_i \left(\nu \right) l_{it} \left(\nu \right) , \tag{24}$$

where $l_{it} \left(\nu \right)$ is the amount of labor used in production of ν, $z_i \left(\nu \right)$ is the efficiency of production of ν, and $S^{\text{M}}_{X,it} \equiv \Theta^{\text{M}}_{X,i} Z_{X,it} \left[L^{\text{M}}_{X,it} \right]^{\phi_{X,L}}$ is the aggregate productivity in production of varieties, with $L^{\text{M}}_{X,it}$ being the total amount of labor used in production of varieties in country i.[21] As in the Krugman model, $S^{\text{M}}_{X,it}$ features external economies of scale and is taken by firms as given. Monopolistic competition

[20] In Section 1.3 of Bhattarai and Kucheryavyy (2022) we introduce correction for the love-of-variety and discuss the implications in Sections 2.3 and 2.4 of Bhattarai and Kucheryavyy (2022).

[21] Differently from the Krugman model, a more general (nonlinear) technology $x_{it} \left(\nu \right) =$

in production of varieties implies that the price of variety $\nu \in \Omega_{ni,t}$ is given by $p_{ni,t}(\nu) = \frac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}}-1}\left(\tau^{\mathrm{M}}_{ni,t}W_{it}\right)\Big/\left(S^{\mathrm{M}}_{X,it}z_i(\nu)\right)$.

3.2.2 Entry and Exit of Producers of Varieties

This part of the Melitz model is almost the same as the corresponding part of the Krugman model with one important difference that, upon entry, producer of a new variety in country n gets an idiosyncratic draw of efficiency of production, $z_n(\nu)$, from the Pareto distribution given by its cumulative distribution function with shape θ^{M} and minimal efficiency $z_{\min,n}$, $G_n(z) \equiv \text{Prob}\left[z_n(\nu) \leq z\right] = 1 - \left(z_{\min,n}/z\right)^{\theta^{\mathrm{M}}}$. For technical reasons, we need to have an assumption that $\theta^{\mathrm{M}} > \sigma^{\mathrm{M}} - 1$ — this ensures that all integrals over efficiencies of production z are finite.

As in the Krugman model, the expected value of entry (before drawing the efficiency of production) is denoted by V_{nt}. The sunk cost of entry is equal to $\frac{W_{nt}^{\alpha_I}P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n}Z_{I,nt}}$. Assuming that entry is free, the sunk cost of entry is equalized with the expected value of entry in equilibrium,

$$V_{nt} = \frac{W_{nt}^{\alpha_I}P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n}Z_{I,nt}}. \tag{25}$$

The number of producers of varieties entering into country n in period t is denoted by $M_{I,nt}$. The law of motion of varieties is $M_{n,t+1} = (1-\delta)M_{nt} + M_{I,nt}$. Since the probability of exit is the same for all varieties $\nu \in \Omega_{nt}$, the distribution of efficiencies of production of varieties $\nu \in \Omega_{nt}$ in any period t is given by $G_n(z)$.

Under the assumption that efficiencies of production of varieties are distributed Pareto, the set of country-i's varieties available in country n is given by $\Omega_{ni,t} = \left\{\nu \in \Omega_{it} \,\middle|\, z_i(\nu) \geq z^*_{ni,t}\right\}$, where $z^*_{ni,t}$ is given by

$$\left(\frac{z_{\min,i}}{z^*_{ni,t}}\right)^{\theta^{\mathrm{M}}} = \frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt}\Phi_{ni,t}M_{it}},$$

$S^{\mathrm{M}}_{X,it}z_i(\nu)\,l_{it}(\nu)^{\gamma}$, with $\gamma > 0$, breaks the isomorphism of the Melitz model with the unified model. In particular, a combination of such nonlinear technology with fixed costs of entry into markets generates variable trade elasticity that is a complicated function of other variables of the model.

with $\mathcal{X}_{ni,t}$ being the total value of varieties that country n buys from i in period t.

3.2.3 Fixed Costs of Serving Markets

We now introduce the formal definition of the fixed costs of serving market n by firms from market i, $\Phi_{ni,t}$. Let $L_{F,nt}$ be the total amount of country n's labor that is used to pay the fixed costs of serving its market. We posit that

$$\Phi_{ni,t} \equiv \left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}} \right]^{\frac{1}{\vartheta}} F_{ni,t}, \tag{26}$$

where $F_{ni,t}$ is an exogenous part of the fixed costs, $\left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}} \right]^{\frac{1}{\vartheta}}$ is an endogenous part of the fixed costs that is taken by firms as given, and $\vartheta \equiv \frac{1}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}}$. Under the assumption that $\theta^{\mathrm{M}} > \sigma^{\mathrm{M}} - 1$, we have that $\vartheta > 0$. The term $\left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}} \right]^{\frac{1}{\vartheta}}$ corrects for the externalities that arise due to the selection effects.[22] In the corresponding unified model, $\phi_{F,M}$ governs the strength of capital externality in production of intermediate goods while $\phi_{F,L}$ governs the strength of externality in production of the final aggregate.

Under the assumption (26) on the form on fixed costs of serving markets, the bilateral price index is $P_{ni,t} = \tau^{\mathrm{M}}_{ni,t} P_{X,it}$, where

$$P_{X,it} = \frac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}}-1} \cdot \frac{W_{it}}{z_{\min,i} \Theta^{\mathrm{M}}_{X,i} Z_{X,it} M_{it}^{\phi_{F,M}} \left[L^{\mathrm{M}}_{X,it} \right]^{\phi_{X,L}}} \tag{27}$$

is interpreted as the price of the output of varieties in country i in period t. The price of the final aggregate is

$$P_{Y,nt} = \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}} \right)^{-\frac{1}{\sigma^{\mathrm{M}}-1}+\phi_{F,L}} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^{\mathrm{M}} W_{nt}} \right)^{-\phi_{F,L}} \left[\sum_{i=1}^{N} \left(F^{\vartheta}_{ni,t} \tau^{\mathrm{M}}_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^{\mathrm{M}}\xi} \right]^{-\frac{1}{\theta^{\mathrm{M}}\xi}}, \tag{28}$$

[22] Selection effects are the changes in the decomposition of country i's firms serving country n in response to changes in market conditions in country n. A detailed explanation is in Section 4.2.

where

$$\xi \equiv \frac{1}{\left(\frac{1}{\eta^{\mathrm{M}}-1} - \frac{1}{\sigma^{\mathrm{M}}-1}\right)\theta^{\mathrm{M}} + 1}, \tag{29}$$

and the share of expenditure of country n on country i's varieties is

$$\lambda_{ni,t} = \frac{\left(F_{ni,t}^{\vartheta}\tau_{ni,t}^{\mathrm{M}}P_{X,it}/\omega_{ni}\right)^{-\theta^{\mathrm{M}}\xi}}{\sum_{l=1}^{N}\left(F_{nl,t}^{\vartheta}\tau_{nl,t}^{\mathrm{M}}P_{X,lt}/\omega_{nl}\right)^{-\theta^{\mathrm{M}}\xi}}. \tag{30}$$

The total expenditure of country n on country-i's varieties is given by $\mathcal{X}_{ni,t} = \lambda_{ni,t}P_{Y,nt}Y_{nt}$. The value of total output of varieties in country n, $\mathcal{X}_{nt}$, and total average profits of country n's producers of varieties, D_{nt}, are given by

$$\mathcal{X}_{nt} = \frac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}}-1}W_{nt}L_{X,nt}^{\mathrm{M}} \quad \text{and} \quad D_{nt} = \frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{nt}}{M_{nt}}. \tag{31}$$

The total amount of country n's labor used to serve its market is $L_{F,nt} = \frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \times \left(P_{Y,nt}Y_{nt}/W_{nt}\right)$. Defining trade deficit as the real value of net exports of varieties in terms of the final good, $\mathrm{TB}_{nt} \equiv (\mathcal{X}_{nt} - P_{Y,nt}Y_{nt})/P_{Y,nt}$, we get

$$L_{F,nt} = \left(\frac{1}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}}\right)L_{X,nt}^{\mathrm{M}} - \frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{P_{Y,nt}\cdot \mathrm{TB}_{nt}}{W_{nt}}. \tag{32}$$

3.2.4 Household's Problem and Markets Clearing Conditions

The household's problem is identical to the one in the Krugman model. Labor market clearing condition is different to account for labor used for serving markets,

$$L_{X,nt}^{\mathrm{M}} + L_{F,nt} + L_{I,nt} = L_{nt}. \tag{33}$$

The other market clearning conditions are the same as in the Krugman model. The complete set of equilibrium conditions is provided in Appendix B.2.

4 Theoretical Results

In this section and in the rest of the paper, for brevity, when there is no risk of confusion, we refer to the generalized dynamic international trade models of Section 3 simply as "the Krugman model" and "the Melitz model".

4.1 Formal Characterization

In order to formulate our main theoretical result, we need to introduce an additional assumption for the Melitz model:

Assumption 1. ***(Melitz)***

(i) $\left(F_{ni,t}/F_{nn,t}\right)^{\vartheta}\tau^{\mathrm{M}}_{ni,t} \geq 1$ *for all n, i and all t;*

(ii) $\left(F_{nl,t}F_{li,t}\right)^{\vartheta}\tau^{\mathrm{M}}_{nl,t}\tau^{\mathrm{M}}_{li,t} \geq \left(F_{ni,t}F_{nn,t}\right)^{\vartheta}\tau^{\mathrm{M}}_{ni,t}$ *for all n, l, i and all t.*

Our main theoretical result is given in the following proposition with its proof provided in Appendix C.

Proposition 1. *By an appropriate relabeling of variables and parameters in the Krugman and Melitz models, and by making an additional Assumption 1 for the Melitz model, we can write the equilibrium system of equations in both models in a form identical to the equilibrium system of equations in the unified model. Thus, these models are isomorphic to each other in their aggregate predictions.*

Proposition 1 says that, up to relabeling, the generalized versions of the Krugman and Melitz models are essentially the same. Moreover, under certain parameterizations, these models are identical to the standard IRBC model extended to allow for external economies of scale in production and iceberg trade costs, despite having very different microfoundations. Informally, the average firms' profit in country n and the measure of country n's varieties in the Krugman and Melitz models play the role of, correspondingly, return on capital in country n and the stock of country n's capital in the unified model.[23] Mappings between parameters of the Krugman, Melitz, and unified models are summarized in Table 1.

[23]In Section 1.2 of Bhattarai and Kucheryavyy (2022), we show that our theoretical result on isomorphism applies even when physical capital is used to produce varieties. For simplicity, we

Model	$\alpha_{X,K}$	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	α_I	Trade elasticity
Standard Krugman	$\frac{1}{\sigma^K}$	$\frac{1}{\sigma^K - 1} - \frac{1}{\sigma^K}$	$\frac{1}{\sigma^K}$	0	1	$\sigma^K - 1$
Standard Melitz	$\frac{\sigma^M - 1}{\sigma^M\theta^M}$	$\frac{1}{\sigma^M\theta^M}$	$\frac{\sigma^M - 1}{\sigma^M\theta^M}$	$\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$	1	θ^M
Generalized Krugman	$\frac{1}{\sigma^K}$	$\phi_{Y,M} - \frac{1}{\sigma^K}$	$\phi_{X,L} + \frac{1}{\sigma^K}$	ψ_Y	α_I	$\eta^K - 1$
Generalized Melitz	$\frac{\sigma^M - 1}{\sigma^M\theta^M}$	$\phi_{F,M} - \frac{\sigma^M - 1}{\sigma^M\theta^M}$	$\phi_{X,L} + \frac{\sigma^M - 1}{\sigma^M\theta^M}$	$\phi_{F,L}$	α_I	$\theta^M\xi$

Notes: $\alpha_{X,K}$ is the capital share in production of intermediates in the unified model. $\psi_{X,K}$ and $\psi_{X,L}$ are the scale elasticities of capital and labor in production of intermediates in the unified model. ψ_Y is the scale elasticity of real output of the final aggregate in production of the final aggregate in the unified model. σ^K and σ^M are the elasticities of substitution between varieties in the Melitz and Krugman models. θ^M is the shape of Pareto distribution in the Melitz model. $\phi_{Y,M}$ is the correction for the love-of-variety effect in the generalized Krugman model. $\phi_{X,L}$ is the scale elasticity of labor in production of varieties in the generalized Krugman and Melitz models. $\phi_{F,M}$ and $\phi_{F,L}$ are the scale elasticities of total measure of varieties and total amount of labor in fixed costs of serving markets in the generalized Melitz model. α_I is the labor share in production of the investment good in the unified model as well as the labor share in the cost of entry into the economy in the Krugman and Melitz models. Trade elasticity in the unified model is given by the exponent of $\tau_{ni,t}$ in expression (5). η^K is the elasticity of substitution between varieties produced by different countries in the Krugman model. $\xi = \left[\left(\frac{1}{\eta^M - 1} - \frac{1}{\sigma^M - 1}\right)\theta^M + 1\right]^{-1}$.

Table 1: Parameter mappings between models

4.2 Economic Explanation: Static versus Dynamic Environments

We now provide economic explanation of and detailed intuition for Proposition 1. Previous literature (e.g., Kucheryavyy *et al.*, 2022) has shown that static versions of the Krugman and Melitz models are isomorphic to a static Armington model of trade with external economies of scale. Externalities in the corresponding static Armington model are induced by labor only and appear in the technology of production of intermediate goods. As explained in, for example, Kucheryavyy *et al.* (2022), the source of externalities in the static Krugman model is the love-of-variety

illustrate this only in the Krugman model, where the externality appears in both labor and physical capital inputs and from the perspective of the isomorphic unified model, it is as if there are two types of capital. These two types of capital in turn become indistinguishable if technology for investment in the physical capital is identical to technology for creation of new varieties.

effect (with its strength given by $1/(\sigma^{\mathrm{K}}-1)$), while the source of externalities in the static Melitz model is the selection effect (with its strength given by $1/\theta^{\mathrm{M}}$).[24]

Compared to the static environment, it needs to be explained why the dynamic Krugman and Melitz models feature the capital externality in addition to the labor externality in production of intermediate goods, and why the dynamic Melitz model features the final good externality. For this, we focus on the standard versions of these models, which can be obtained in a straightforward way from their generalized counterparts by setting parameters according to Table 1.

Consider first the standard dynamic Krugman model. As we show in Appendix B.1, we can write the price of intermediate goods as $P_{X,nt} = \left[\widetilde{\Theta}^{\mathrm{K}}_{X,n} Z_{X,nt}\right]^{-1} \times M_{nt}^{-\left(\frac{1}{\sigma^{\mathrm{K}}-1}-\frac{1}{\sigma^{\mathrm{K}}}\right)} L_{X,nt}^{-\frac{1}{\sigma^{\mathrm{K}}}} D_{nt}^{\frac{1}{\sigma^{\mathrm{K}}}} W_{nt}^{1-\frac{1}{\sigma^{\mathrm{K}}}}$. The split between W_{nt} and D_{nt} in this expression for $P_{X,nt}$ takes that form because labor gets $\left(1-\frac{1}{\sigma^{\mathrm{K}}}\right)$ share of total revenue, while the remaining $\frac{1}{\sigma^{\mathrm{K}}}$ share of revenue are profits of firms. These shares are constant as firms charge constant markups over costs, due to CES preferences. Since labor is the only factor of production of varieties, and the technology of production is linear in labor, from the perspective of the unified model it is as if the representative firm uses only $\left(1-\frac{1}{\sigma^{\mathrm{K}}}\right)$ log-share of labor in its technology, while the remaining $\frac{1}{\sigma^{\mathrm{K}}}$ log-share of labor induces a technological externality. Next, from the perspective of the unified model, the technology of production of intermediate goods uses capital to the power of $\frac{1}{\sigma^{\mathrm{K}}-1}$ (which is the love-of-variety effect). Part of this capital is internalized by firms in terms of $\frac{1}{\sigma^{\mathrm{K}}}$ share of revenue, while the remaining part — equal to $\left(\frac{1}{\sigma^{\mathrm{K}}-1}-\frac{1}{\sigma^{\mathrm{K}}}\right)$ — induces an externality.

Now consider the standard dynamic Melitz model. Again, as we show in Appendix B.2, we can write the price of intermediate goods as $P_{X,nt} = \left[\widetilde{\Theta}^{\mathrm{M}}_{X,n} Z_{X,nt}\right]^{-1} \times M_{nt}^{-\frac{1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}}} L_{X,nt}^{-\frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}}} D_{nt}^{\frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}}} W_{nt}^{1-\frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}}}$. Similar to the Krugman model, the split between capital and labor in production technology for intermediate goods in the Melitz model arises because labor gets $\left(1-\frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}}\right)$ share of total revenue with the remaining $\frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}}$ share accruing as profits of firms. Again, since the technology of production is linear in labor, from the perspective of the unified model, the remaining $\frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}}$ log-share of labor induces a technological externality. The technology of

[24] See also Section 2.4 of Bhattarai and Kucheryavyy (2022) for more details.

production of intermediate goods uses capital to the power of $\frac{1}{\theta^{M}}$ (which is the selection effect). Out of this capital, the $\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}$ log-share is internalized by firms, while the remaining part — equal to $\frac{1}{\theta^{M}} - \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}} = \frac{1}{\sigma^{M}\theta^{M}}$ — induces an externality.

The capital externality is usually absent from the isomorphism in the static environments as they are described in, e.g., Kucheryavyy *et al.* (2022), because of the typical assumption that the cost of entry per firm is paid in terms of labor only. This assumption implies that the number of firms in each country is proportional to the total labor used in production of varieties, and, thus, in such environments the number of firms is simply replaced by labor in all relevant expressions. However, if the costs of entry are paid in terms of the final good — as we assume in most of our quantitative, and all estimation, exercises in Section 5 — then even the static environments would feature a split between capital and labor.

An additional reason why the dynamic environment features the capital externality is that the number of firms is a state variable, while the costs of entry are paid only by the firms entering the economy in the current period, $M_{I,nt}$. Thus, at a given time period, there is no fixed relationship between the total number of firms in the economy and the total amount of labor used in production of varieties, even when entry costs are paid in terms of labor.

Let us now consider the final good externality in the Melitz model. While this externality in principle can arise even in the static environment featuring endogenous (elastic) labor supply (due to the selection effect), it has a non-trivial behavior in the dynamic environment due to trade imbalances and, in the case of investment done in terms of labor, due to period-by-period firm entry decisions (or, equivalently, capital accumulation decisions). To understand these points, let us write the expression for the final good externality as

$$\frac{P_{Y,nt}Y_{nt}}{W_{nt}} = \frac{\theta^{M}\sigma^{M}}{\theta^{M}\sigma^{M}+1-\sigma^{M}}\left(L_{nt}-L_{I,nt}\right) - \frac{\theta^{M}\left(\sigma^{M}-1\right)}{\theta^{M}\sigma^{M}+1-\sigma^{M}} \cdot \frac{\mathrm{TB}_{nt}\cdot P_{Y,nt}}{W_{nt}}, \tag{34}$$

where we used $P_{Y,nt}Y_{nt} = \mathcal{X}_{nt} - P_{Y,nt}\cdot \mathrm{TB}_{nt}$, and additionally used (31) for $\mathcal{X}_{nt}$, (32) for $L_{F,nt}$, and the labor market clearing condition (33).

Consider then the steady state of the dynamic version of the Melitz model. In the steady state, $\mathrm{TB}_{nt} = 0$. If investment is done in terms of final good only, then $L_{I,nt} = 0$, and (34) implies that the final good externality is proportional to L_{nt}.

Alternatively, if investment is done in terms of labor only, then in the steady state (assuming $\beta = 1$ and $\delta = 1$ for simplicity), $L_{I,nt} = \frac{1}{\theta^{M}} L^{M}_{X,nt}$, which, using (34), allows us to show that $P_{Y,nt} Y_{nt} / W_{nt} = \frac{\sigma^{M}}{\sigma^{M}-1} L^{M}_{X,nt}$. Thus, again, the final good externality is proportional to L_{nt}. Therefore, the final good externality would matter in the static Melitz model only if labor supply is endogenous. Otherwise the final good externality would be absorbed by a constant term in the production function for the final aggregate in the isomorphic static competitive model.

In the dynamic setting with borrowing and lending, $\mathrm{TB}_{nt} \neq 0$ and, thus, the final good externality has a non-trivial behavior even if labor supply is inelastic and investment is done in terms of the final good only. If investment is done in terms of labor, then there is no fixed relationship between $L_{I,nt}$ and L_{nt}. Then, even under balanced trade (as in the case of financial autarky), and irrespective of whether labor supply is endogenous or inelastic, period-by-period firm entry decisions generate a potentially non-trivial behavior of the final good externality.

The upshot is that the precise formulations of the externalities depend on four factors: dynamic vs. static setting; endogenous vs. fixed labor supply; endogenous international borrowing vs. balanced trade; and whether investment is done in terms of final good or labor. Our presentation on isomorphism in terms of various externalities is the most general as it nests all these various cases (as well as those that have appeared in the literature previously in static settings). Moreover, our formulation, in particular the split between capital and labor externality, is also substantively important for quantitative questions, as shown in Section 5.

4.3 Relation to the Literature

The unified model described in Section 2 is a generalization of the standard IRBC model studied in the previous literature. For example, Heathcote and Perri (2002)'s model can be obtained as a special case of the unified model by shutting down externalities, requiring that capital investment uses the final aggregate only, leaving exogenous shocks only in production of intermediate goods, dropping the additional term $aP_{Y,nt} \cdot \mathrm{TB}_{nt}$ in the labor market clearing condition, and removing iceberg trade costs. Formally, this requires setting $\psi_{X,K} = \psi_{X,L} = \psi_{Y} = 0$, $\alpha_{I} = 0$, $Z_{Y,nt} = Z_{I,nt} = 1$, $\Theta_{X,n} = \Theta_{Y,n} = \Theta_{I,n} = 1$, $a = 0$, and $\tau_{ni,t} = 1$.

There are no direct analogs in the existing literature of the generalized Melitz model of Section 3.2 (or even to its standard version described in Table 1). The closest to this model are the dynamic versions of the Melitz model introduced in Ghironi and Melitz (2005) and Jaef and Lopez (2014), with two important differences. First, fixed costs of serving markets in the Melitz model of Section 3.2 are paid in terms of the destination-country labor, while in the existing dynamic Melitz models they are paid in terms of the source-country labor. Second, the Melitz model of Section 3.2 features fixed costs of serving domestic markets, which are absent in the existing dynamic Melitz models. Quantitatively, the effects of these different assumptions are small in a model with two symmetric countries, which is traditionally the focus of the international business cycles literature (as in our paper).

If we shut down external economies of scale in production of varieties and in the fixed costs of serving markets ($\phi_{X,L} = 0$, $\phi_{F,M} = \frac{1}{\theta^M}$, and $\phi_{F,L} = \vartheta$), and if we require that the sunk costs of entry into the economy are paid in terms of labor only ($\alpha_I = 1$), then the only essential differences between the Melitz model of Section 3.2 and the models in Ghironi and Melitz (2005) and Jaef and Lopez (2014) are the assumptions about fixed costs of serving markets described above. Other differences are that Ghironi and Melitz (2005) assume exogenous (aggregate) labor supply and do not have a nested CES production structure of the final aggregate. Both of these features matter for impulse-responses and business cycle moments. Jaef and Lopez (2014) assume that production technology for intermediate varieties uses capital together with labor, but, as we have said before, this does not conceptually change the isomorphism result.

Another relevant model related to the generalized Melitz model of Section 3.2 is described in Alessandria and Choi (2007). Their model does not have period-by-period firm entry decisions, which is an important feature of our environment. At the same time, the environment in Alessandria and Choi (2007) features sunk costs of entry into exporting markets — the feature absent in the setup of the generalized Melitz model of the current paper. Using their model, Alessandria and Choi (2007) check the consequences of altering the love-of-variety effect and find that increasing love-of-variety results in increased consumption correlations and decreased output correlations. Since the environment in Alessandria and Choi

(2007) is not exactly comparable to ours, it is hard to direclty relate to their results. Neverthless, the consequences of altering the love-of-variety effect in the generalized Melitz model is an interesting question. In Section 1.3 of Bhattarai and Kucheryavyy (2022) we introduce correction for the love-of-variety in the generalized Melitz model and discuss the implications in Sections 2.3 and 2.4 of Bhattarai and Kucheryavyy (2022). The result is that the love-of-variety correction simultaneously changes the trade elasticity and the final good externality (from the point of view of the unified model), which makes it hard to isolate and understand the effects of this change.

5 Quantitative Results

We now quantitatively assess the international business cycle implications of dynamic trade models by first comparing the fit of the IRBC and dynamic Krugman and Melitz models. Next, use the unified model of Section 2 to explore the ingredients needed to achieve better fit with the data, while explaining in detail the transmission mechanisms that change when we vary externalities. We end with a estimation exercise where we match a comprehensive set of moments.

5.1 Calibration

We focus on a world economy with two symmetric countries for direct comparison with the business cycle literature. Periods are interpreted as quarters. We use balanced-growth preferences that are widely used in the literature (e.g., Heathcote and Perri, 2002): $U\left(C_{nt}, L_{nt}\right) = \dfrac{1}{1-\gamma}\left[C_{nt}^{\mu}\left(1-L_{nt}\right)^{1-\mu}\right]^{1-\gamma}$. Our benchmark calibration is consistent with a standard calibration in the literature and is summarized in Table 2 and explained in detail in Appendix E. For this calibration, we choose three sets of parameter values of the unified model that correspond to the IRBC, Krugman, and Melitz models. The calibration of the IRBC model follows Heathcote and Perri (2002), while the calibrations of the Krugman and Melitz models follow Bilbiie *et al.* (2012) and Ghironi and Melitz (2005).

We choose parameter values of the unified model corresponding to the Krug-

Common Parameters	$\beta = 0.99, \gamma = 2, \delta = 0.025, \mu = 0.34, \sigma = 2, \tau_{ni,t} = 5.67, \omega_{ni} = 0.5,$ $\Theta_{X,n} = \Theta_{I,n} = 1, \Theta_{Y,n} = 2.069, b_{adj} = 0.0025$ When allowed, productivity process in the intermediate goods sector is given by $\begin{bmatrix}\log(Z_{X,1t})\\ \log(Z_{X,2t})\end{bmatrix} = \begin{bmatrix}\rho_{X,11} & 0\\ 0 & \rho_{X,22}\end{bmatrix} \times \begin{bmatrix}\log(Z_{X,1,t-1})\\ \log(Z_{X,2,t-1})\end{bmatrix} + \begin{bmatrix}\varepsilon_{X,1t}\\ \varepsilon_{X,2t}\end{bmatrix},$ $\begin{bmatrix}\varepsilon_{X,1t}\\ \varepsilon_{X,2t}\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\sigma^2_{X,1} & 0\\ 0 & \sigma^2_{X,2}\end{bmatrix}\right),$ with $\rho_{X,11} = \rho_{X,22} = 0.97, \sigma_{X,1} = \sigma_{X,2} = 0.0073$ Similarly, when allowed, productivity process in the final goods sector, $Z_{Y,nt}$, has the same structure as $Z_{X,nt}$ above with the same autocorrelation parameter values, $\rho_{Y,11} = \rho_{Y,22} = 0.97$, and normally distributed uncorrelated shocks with standard deviations $\sigma_{Y,1} = \sigma_{Y,2} = 0.0073$.
IRBC	$\alpha_{X,K} = 0.36, \quad \psi_{X,K} = \psi_{X,L} = \psi_Y = 0, \quad \alpha_I = 0, \quad a = 0, \quad Z_{I,nt} = 1$
Krugman	$\alpha_{X,K} = \frac{1}{3.8} \approx 0.26, \quad \psi_{X,K} = \frac{1}{3.8-1} - \frac{1}{3.8} \approx 0.094, \quad \psi_{X,L} = \frac{1}{3.8} \approx 0.26,$ $\psi_Y = 0, \quad \alpha_I = 1, \quad a = 0, \quad Z_{I,nt} = Z_{X,nt}$
Melitz	$\alpha_{X,K} = \frac{3.8-1}{3.8*3.4} \approx 0.22, \ \psi_{X,K} = \frac{1}{3.8*3.4} \approx 0.077, \ \psi_{X,L} = \frac{3.8-1}{3.8*3.4} \approx 0.22,$ $\psi_Y = \frac{1}{3.8-1} - \frac{1}{3.4} \approx 0.063, \ \alpha_I = 1, \ a = \frac{3.4+1-3.8}{3.8*3.4} \approx 0.046,$ $Z_{I,nt} = Z_{X,nt}$

Table 2: Benchmark calibrations of models.

man and Melitz models so that all generalizations are shut down in these models except that we allow for the nested CES production technology of the final aggregate.[25] Formally, the parameterization for the Krugman model is $\phi_{Y,M} = \frac{1}{\sigma^K - 1}$ and $\phi_{X,L} = 0$, but allowing for $\eta^K \neq \sigma^K$, while the parameterization for the Melitz model is $\phi_{F,M} = \frac{1}{\theta^M}$, $\phi_{F,L} = \vartheta$, and $\phi_{X,L} = 0$, but allowing for $\eta^M \neq \sigma^M$. As we explain in Appendix E, the standard parameterization of the Melitz model implies that

[25] With a slight abuse of terminology compared to the previous section, we will refer to these models as standard Krugman and Melitz models in this section.

$Z_{Y,nt} = [Z_{X,nt}]^{\vartheta}$. Thus, the shock from the intermediate goods sector spills over to the final goods sector. Given this, in our quantitative exercises with the standard Melitz model where we allow for only one fundamental shock $Z_{X,nt}$, we set this shock according to Table 2 while setting $Z_{Y,nt} = [Z_{X,nt}]^{\vartheta}$. At the same time, in our quantitative exercises where we shut down the shock to the intermediate goods sector while allowing for the shock to the final goods sector, we set $Z_{X,nt} = 1$ while setting $Z_{Y,nt}$ according to Table 2.

Finally, in the main text we present results for the complete markets only. Results for the bond economy and financial autarky for the benchmark parameterization are presented in Tables G.1 and G.2 in Appendix G and briefly discussed in Appendix H.1.

5.2 Comparison Across IRBC and Standard Trade Models

Moments, both domestic and international, across models under the calibration in Table 2 are presented in Table 3. Column (1) provides data moments from Heathcote and Perri (2002). In column (2) we present results for the standard IRBC model and in columns (3) and (4) for standard versions of the Krugman and Melitz models respectively. These standard models feature only one exogenous shock — the intermediate sector productivity shock. In columns (5)-(7) we present results for the same models but with the final sector productivity shock only. We note that for both shocks, the domestic moments show the presence of a business cycle, that is a positive co-movement of within-country output, consumption, investment, and labor.[26] They both are thus natural candidates for a study of international business cycle moments.[27]

[26]While we will use the same investment correlation data for comparison across models, the investment expenditure in the Krugman and Melitz models is best interpreted as R&D expenditure. As we mentioned before, theoretically, allowing for physical capital in the dynamic trade models does not affect our results. Moreover, even in the IRBC model, the theoretical construct of investment is much simpler compared to the data, not distinguishing, e.g, between structures and equipment or between residential and non-residential investment. These various sub-components of investment might have different international correlations, which is an interesting topic, but studying which is outside the scope of our paper.

[27]Later in our estimation exercise, as well as in our discussion of transmission using impulse responses, we will discuss how the two shocks affect some international moments differently.

		Benchmark calibration						Investment final good			
		Int. sector shock			Final sector shock			Int. sector shock		Final sector shock	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	0.58	−0.03	−0.10	−0.09	−0.17	−0.06	−0.06	−0.09	−0.11	−0.17	−0.20
$\text{Corr}\,(C_1, C_2)$	0.36	0.47	0.45	0.45	−0.10	0.10	0.11	0.41	0.34	−0.03	−0.07
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.39	−0.27	−0.26	−0.62	−0.13	−0.13	−0.41	−0.45	−0.59	−0.61
$\text{Corr}\,(L_1, L_2)$	0.42	−0.30	−0.45	−0.46	−0.22	−0.69	−0.70	−0.40	−0.42	−0.27	−0.33
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.49	0.58	0.61	−0.69	0.72	0.72	−0.25	−0.26	−0.63	−0.63
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.32	0.36	0.85	0.88	−0.18	0.98	0.98	0.60	0.57	0.11	0.12
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.81	0.93	0.25	0.25	0.92	0.97	0.97	0.86	0.86	0.93	0.93
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.13	0.61	0.64	0.67	0.74	0.73	0.73	0.68	0.70	0.75	0.75
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.21	0.16	0.17	0.53	0.08	0.08	0.19	0.23	0.44	0.45
Domestic moments:											
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.86	0.94	0.93	0.94	0.98	1.00	1.00	0.95	0.95	0.98	0.98
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.87	0.98	0.96	0.96	0.99	0.93	0.93	0.98	0.98	0.99	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.97	0.97	0.97	0.95	1.00	0.99	0.96	0.95	0.95	0.94
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.81	0.37	0.45	0.47	0.43	0.83	0.84	0.42	0.43	0.48	0.49
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.66	0.46	0.51	0.47	0.40	0.14	0.13	0.42	0.41	0.37	0.36
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	2.84	3.33	3.80	4.20	3.90	1.56	1.65	3.94	4.64	4.48	5.20

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. $\alpha_I = 0$ for columns (2), (5), and (8)-(11); and $\alpha_I = 1$ for columns (3)-(4) and (6)-(7). $Z_{I,nt} = 1$ for columns (2) and (5)-(11); and $Z_{I,nt} = Z_{X,nt}$ for columns (3) and (4). $Z_{X,nt}$ is set according to Table 2 for columns (2)-(4) and (8)-(9); and $Z_{X,nt} = 1$ for columns (5)-(7) and (10)-(11). $Z_{Y,nt} = 1$ for columns (2), (3), and (8); $Z_{Y,nt} = [Z_{X,nt}]^{\frac{1}{3.8-1} - \frac{1}{3.4}}$ for columns (4) and (9); and $Z_{Y,nt}$ is set according to Table 2 for columns (5)-(7) and (10)-(11).

Table 3: Moments from benchmark calibrations of models. Complete Markets.

Performance of Krugman vs. Melitz model. Table 3 shows that there is not much qualitative or quantitative difference between the Krugman and Melitz models, for either shocks and for either domestic and international moments. From the point of view of the unified model, the standard Melitz model has three different features relative to the standard Krugman model: external economies of scale, shocks in production of the final aggregate, and the additional term $a\text{TB}_{nt}$ in the labor market clearing condition. The standard calibration used for the Melitz model (column (4) in Table 3), however, implies that these features have a small impact quantitatively as the relevant parameters are small: $\psi_Y \approx 0.063$, $Z_{Y,nt} \approx [Z_{X,nt}]^{0.063}$, and $a \approx 0.046$. In the calibration for the Melitz model model, three parameters — $\alpha_{X,K}$, $\psi_{X,K}$, and $\psi_{X,L}$ — have values different from the calibration for the Krugman model. But again, this difference is small given our parameterization, which was tailored to be in line with the literature. Table 2 in fact shows clearly how the calibration implies small differences between the Krugman and Melitz models.

Performance of Krugman and Melitz models vs. standard IRBC model. Table 3 shows that *broadly, for both shocks, the Krugman and Melitz models perform well and fail in the same cross-country moments as the standard IRBC model.*[28] The only important difference between performance of the IRBC model versus the Krugman and Melitz models is the cyclicality of the trade balance: the correlation of trade balance with output is counterfactually positive for the Krugman and Melitz models.

From the point of view of the standard IRBC model, the Krugman and Melitz models have several key modifications that could potentially have opposite or hard to understand effects on second moments and transmission of shocks. Most interesting among them are external economies of scale in production of intermediate and final aggregate goods. Before we focus on the role played by these externalities, we note that the counterfactual procyclicality of trade balance in the Krugman and Melitz models is due to another important difference of these models from the standard IRBC model: the standard Krugman and Melitz models imply that the investment good (from the perspective of the unified model) is produced using domestic labor ($\alpha_I = 1$), while in the standard IRBC model the investment good is

[28]Similarity between IRBC and a Melitz-type dynamic model for the case of the intermediate good productivity shock was also reported in a numerical analysis by Jaef and Lopez (2014).

produced using the final aggregate ($\alpha_I = 0$).

To show that this indeed is the reason for opposite cyclicality of the trade balance, in columns (8)-(11) of Table 3 we present results of an exercise with investment done in terms of the final good only ($\alpha_I = 0$) in otherwise standard Krugman and Melitz models. We see that trade balance is countercyclical and similar to the IRBC model. In addition to this exercise, Table G.3 in Appendix G presents results of an exercise where we change the canonical IRBC model with the intermediate good productivity shock such that investment is done in terms of labor only (by setting $\alpha_I = 1$). An outcome of this exercise is that the trade balance is pro-cyclical, unlike in Table 3.[29] Importantly, these exercises show that the Krugman and Melitz models still perform similarly to the standard IRBC model.

What then is the main reason for the similar performance of the IRBC and the Krugman and Melitz models? Our result on isomorphism provides the answer: Even though the Krugman and Melitz models feature external economies of scale, their magnitudes implied by the parameterization from the literature (see Table 2) are not large enough to make a quantitative difference. Moreover, these standard parameterizations imply positive capital externalities, whereas, as we show next, negative capital externalities are needed to improve over the IRBC model.

5.3 Changing Production Externalities

From now on, we assume that investment is done in terms of the final good. We then use our unified model of Section 2 to explore if it is possible to achieve a better fit with the data. The unified model perspective is critical as we can vary each externality independently. This allows for a clean inspection of the transmission mechanisms. We do comparative statics for all three externalities, but focus on the role of capital externality as it turns out to be most crucial quantitatively.

[29] Thus it is not the case that adding investment to an international business cycles model ensures a countercyclical trade balance by acting against the consumption smoothing force. It is crucial how the investment good is produced. We discuss the intuition behind this result in Appendix H.2. Raffo (2008) additionally points out that even with investment good being produced with the final good, the IRBC mechanism for generating a countercyclical trade balance relies on a strong response of the terms-of-trade, which is not observed in the data.

5.3.1 Role of Negative Capital Externality

	Int. sector shock			Final sector shock		
$\psi_{X,K} =$	0	0.3	-1	0	0.3	-1
	(1)	(2)	(3)	(4)	(5)	(6)
International moments:						
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	-0.03	-0.07	0.08	-0.17	-0.21	-0.08
$\text{Corr}\,(C_1, C_2)$	0.47	0.55	0.34	-0.10	0.03	-0.33
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	-0.39	-0.47	-0.26	-0.62	-0.68	-0.53
$\text{Corr}\,(L_1, L_2)$	-0.30	-0.52	0.00	-0.22	-0.42	0.01
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.49	-0.40	-0.57	-0.69	-0.66	-0.73
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.36	0.52	0.17	-0.18	-0.06	-0.29
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.93	0.90	0.96	0.92	0.91	0.94
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.61	0.67	0.46	0.74	0.77	0.58
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.21	0.15	0.28	0.53	0.46	0.60
Domestic moments:						
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.94	0.93	0.92	0.98	0.99	0.91
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.98	0.97	0.99	0.99	0.99	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.97	0.96	0.97	0.95	0.94	0.96
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.37	0.44	0.25	0.43	0.50	0.33
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.46	0.42	0.53	0.40	0.35	0.49
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	3.33	3.01	3.81	3.90	3.57	4.40

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 4: Capital externalities in the unified model. Complete markets.

Table 4 shows that an essential feature to improve fit with the data is negative capital externality in intermediate goods production.[30] In this table, we provide

[30] From the point of the generalized Melitz model of Section 3.2, negative capital externality can

moments from the model without any externality, as well as with a positive and a negative capital externality, for both the intermediate good and the final good productivity shocks.

As a starting point, we note that the main empirical puzzles of the IRBC model are associated with co-movement across countries in output, consumption, hours, and investment. That is, as is clear from Table 3, in the IRBC model, the co-movement of consumption is counterfactually higher than that of GDP. Moreover, while in the data labor hours and investment co-move positively, in standard models they co-move negatively. Additionally, the canonical IRBC model with the intermediate good productivity shocks leads to a more procyclical real exchange rate and a less volatile trade balance compared to the data.

Table 4 then shows that negative capital externality helps bring the model closer to the data on these important international moments. This is seen from comparing column (3) with (1) for the intermediate good productivity shock, and column (6) with (4) for the final good productivity shock. For both shocks, compared to the standard IRBC model with no externalities, negative capital externality leads to higher cross-country output, investment, and labor correlations and a lower consumption correlation. Moreover, it leads to a more volatile trade balance. Let us discuss the mechanisms behind this result for each shock.

Intermediate good productivity shock We now provide an economic interpretation for the results in columns (1)-(3) of Table 4 by analyzing the transmission mechanisms using impulse-response functions, in which a 1% exogenous technology shock in the intermediate goods sector hits the home country. Figure 1 shows the results under complete markets, where we vary the externality in capital, $\psi_{X,K}$.

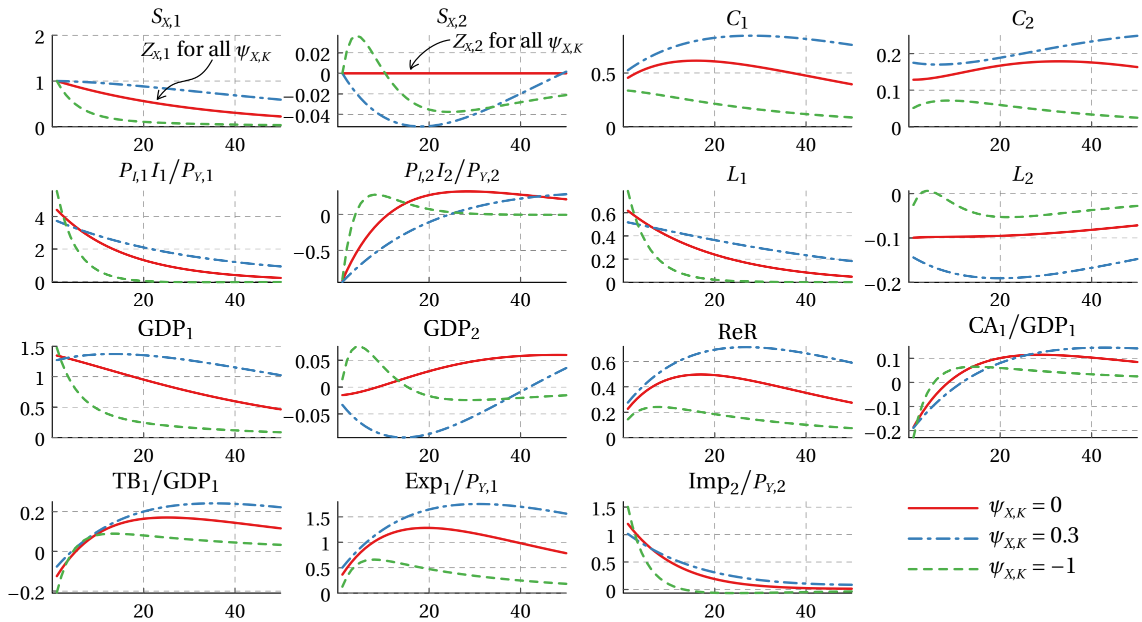
To set the stage, let us discuss the transmission mechanism under no externality, that is, for the IRBC model. When a positive intermediate good productivity shock, which is persistent but mean-reverting, hits home, the substitution effect of increased wage dominates the income effect, and the household supplies more labor. Moreover, given increased productivity, there is an increase in investment

be generated by a negative value of parameter $\phi_{F,L}$ in the fixed costs of serving markets given by (26). This would imply that fixed costs of serving a market endogenously increase with the increase in the number of firms serving this market, which can be interpreted as a congestion effect.

at home. With higher income currently and in the future, consumption also increases, and it is smoothed over time as the usual permanent income hypothesis intuition applies. The flip side of consumption smoothing is the large response of investment. These positive domestic correlations lead to a standard business cycle at home. Given the increased productivity at home, the home country finances increased investment by running a current account deficit. In the foreign country, as consumption increases due to risk-sharing, the wealth effect leads to a decrease in labor supply. Moreover, the foreign country cuts down on investment as it is optimal to concentrate production in the more productive home country. Over time, the foreign country runs a current account surplus, using the saving to rebuild the depleted capital stock. These transmission mechanisms underlie the failure of the model to match the cross-country correlations in output, investment, and hours.

Let us now turn to the explanation of why negative capital externality helps move the model closer to the data in terms of these international correlations. The key observation is that, *in the presence of negative capital externalities in production of intermediate goods, individual firms perceive the aggregate country-specific shock as being less persistent with the same initial impact*. This is because, in future, due to positive capital accumulation, the productivity increase faced by the firms — which is measured by $S_{x,1}$ — is lower than the exogenous productivity shock — which is measured by $Z_{x,1}$. We can now analyze what happens to labor supply at home when the productivity increase has the same initial size but is more transient (compared to the no-externality case). Under a more transient shock, the substitution effect of wage increase is even stronger than the income effect. This implies that households supply more labor today. Given the same initial capital stock, a larger initial supply of labor leads to a larger initial response of output.

Next, while the initial effect on income is higher, in future, as the productivity process is more transient, income will be lower than in the model without externalities. This lack of a persistent rise in home output helps mitigate the failure of the IRBC model to generate output co-movement across countries. Moreover, due to consumption smoothing, consumption rises by less than in the no-externality case. The smaller rise of consumption at home, as well as an altered path of consumption due to change in investment, help reduce international correlation in



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediate good sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,K} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,K}$.

Figure 1: Impulse-response functions for $Z_{X,1}$. Capital externalities in the intermediate goods sector in the unified model. Complete markets.

consumption. While the smaller rise in consumption at home implies that home investment increases more on impact, it does not worsen international correlation in investment.[31] Over time, both investment and labor at home follow the more transient path of productivity, leading to higher cross-country correlations in investment and labor. Overall, endogenous productivity being less persistent than the productivity shock at home decreases the extent of productivity differences across countries that plagues the IRBC model and improves cross-country correla-

[31]The initial impact effect of investment at home is higher even though the productivity faced by firms is less persistent. Unless the trade elasticity is very high, this result holds. The higher initial impact on hours however, is independent of trade elasticity.

tions in both factors and output.[32]

To understand the dynamic responses of foreign variables in Figure 1, observe that while the country-specific productivity shocks are uncorrelated in our experiments, negative capital externality leads to an endogenous positive correlation in the productivity faced by the two countries. In particular, from the foreign country's perspective, starting from the next period, there is a positive effect on productivity, as typically there would be a negative effect on investment in the foreign country following a positive home productivity shock. This positive effect on productivity faced by the foreign country then leads to increased labor hours and increased investment for very standard reasons. The resulting increase in output in the foreign country helps further with increasing output co-movement across countries. Moreover, note how foreign output and labor supply completely track the dynamics of $S_{x,2}$, as is common in RBC models, but which would have been very difficult to interpret without the perspective of the unified model that leads us to follow the dynamics of $S_{x,2}$ for intuition, instead of $Z_{x,2}$.[33]

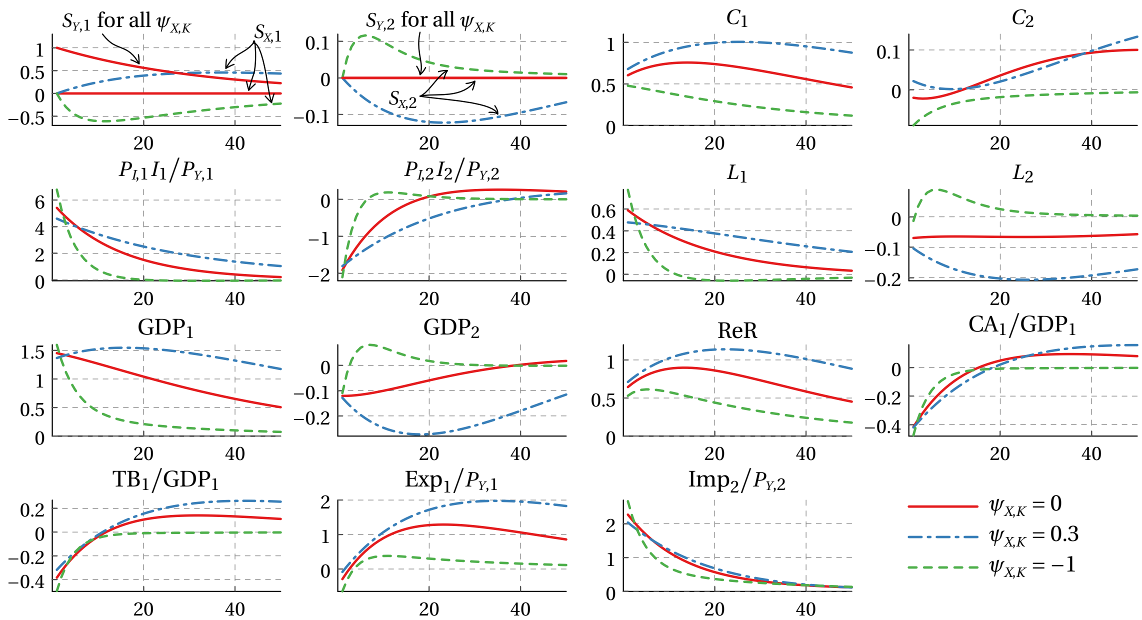
We close with a discussion of two other important open economy variables: the cyclicality of the real exchange rate and volatility of the trade balance. First, as negative capital externalities lead to a larger initial increase in home investment, there is a sharper response of trade balance. This helps with increasing the volatility of the trade balance. Second, the change in the dynamic path of output that we discussed above reduces the cyclicality of the real exchange rate with output. Finally, negative capital externality also helps decrease the cyclicality of exports, which is affected by the change in path of investment in the foreign country.[34]

[32]To make this even clearer, in Appendix I.2 we perform two estimation exercises. First, we show that the canonical IRBC model requires a very low persistence in productivity shocks to fit the data. Second, we show that if the canonical IRBC model is augmented just with capital externalities, and estimated to match the same moments while fixing the persistence parameter to a high value, it requires negative capital externalities. The fit on cross-country correlations is very similar across these two cases, which illustrates that the channel through which negative capital externality improves fit is by endogenously decreasing the persistence of the shock.

[33]As noted earlier, with negative capital externality and high trade elasticity, home investment does not increase on impact following a less persistent productivity process. In this case, the margin affected most is the cut in foreign investment, which gets reduced significantly compared to no externality. This in turn generates higher international correlations in output and factors.

[34]One exception here is that negative capital externalities lead to more procyclical imports, which makes the fit worse with the data. The reason imports become more procyclical is that the behavior of imports closely follows that of investment (as can be clearly seen in Figure 1). Given that

Final good productivity shock We now consider the productivity shock in the final good sector. For economic explanations, we again turn to an analysis of impulse response functions in which a 1% exogenous productivity shock in the final goods sector hits the home country. Figure 2 shows the results under complete markets where we vary only the externality in capital input, $\psi_{X,K}$.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the final good sector in country 1, $Z_{Y,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$).

Figure 2: Impulse-response functions for $Z_{Y,1}$. Capital externalities in the intermediate goods sector in the unified model. Complete markets.

As this shock is new to the IRBC model, the transmission even under no externality requires an explanation. The first result to note is that when the final good productivity shock hits, domestic output, labor, investment, and consumption all co-move, thereby, generating a home business cycle. In RBC models, such

investment and output increase more sharply initially with negative capital externalities, imports follow a similar pattern, thereby increasing its pro-cyclicality.

a business cycle pattern is notoriously difficult to generate for shocks other than the intermediate good productivity shock. What explains the new result here?

Following a final good shock at home, productivity of the final good increases, which leads to a decrease in its relative price. Naturally, this leads to an increase in both consumption and investment, as they both are produced using the final good. Importantly, labor supply also increases, even though the intermediate good production function, where labor is used, has not experienced a shock. In a closed economy model, one would expect the positive wealth effect due to increased consumption to lead to a decrease in labor supply, with real wages not getting affected much. The open economy situation here is different, because the price of consumption/final good and the price of the home good are not the same. To understand the implications, consider the optimal labor supply condition of the household

$$-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}} = \frac{W_{nt}}{P_{X,nt}} \cdot \frac{P_{X,nt}}{P_{Y,nt}}.$$

For intuition, hold $\frac{W_{nt}}{P_{X,nt}}$ constant, as the intermediate good productivity shock that affects the marginal product of labor is not being considered here. When the final good productivity shock hits, the first-order effect is an increase in the relative price $\frac{P_{X,nt}}{P_{Y,nt}}$, driven by the fall in the final good price. Then the household finds it optimal to supply more labor, and both consumption and hours at home increase.

Let us now consider the international transmissions of this shock under no externality. The home country finances increased investment and consumption by running a current account deficit. Moreover, the real exchange rate depreciates as the final good produced by the home country is cheaper. The depreciation of the real exchange rate is high enough that it requires foreign consumption to fall to ensure that relative consumption is equated with the real exchange rate. This happens even though the financial markets are complete, which is a unique aspect of the final good shock. This explains why in Table 4, for the case of no externality, there is a negative cross-country correlation in consumption. Next, like with the intermediate good productivity shock, the foreign country cuts down on its labor supply and investment, which leads to a negative cross-country correlation in hours and investment, with the wealth effect on labor supply muting a bit the negative co-movement in labor across countries. Finally, because of the large effect on relative prices, two key moments get affected more compared to the intermediate good shock. First, the trade balance response is stronger, as a decline in relative price leads to an increased trade balance response. And, second, exports are much less cyclical. These two features will play an important role in the estimation.

Overall, while the final good productivity shock also leads to a domestic business cycle like the intermediate good productivity shock, for the IRBC model, Table 3 shows that international correlations are still hard to match. Thus, while consumption correlation across countries is reduced, it is still higher than output correlation, and labor and investment do not co-move positively. At the same time, as shown in columns (4)-(6) of Table 4, negative capital externality does move the model closer to the data on international correlations for the final good shock, similarly to the intermediate good shock, by increasing cross-country output, investment, and hours correlations and decreasing consumption correlation.

For international correlations, a mechanism similar to the intermediate good shock holds. As shown in Figure 2, while the final good productivity shock does not directly affect intermediate good productivity, with negative externality, endogenously and dynamically, intermediate good productivity declines as there is typically higher capital accumulation in response to this shock. This negative endogenous effect on intermediate good productivity then negates the positive effects of the final good shock: it is as if now there are two aggregate productivity shocks, one off-setting the other. The home country's overall increase in productivity is thus muted over time, thereby leading to less persistent effects on home variables, combined with a larger effect on hours, investment, and output initially, as shown in Figure 2. These less persistent effects at home act against the cut in factors of production in the foreign country, improving international correlations.

Figures 1 and 2 show that for these two different shocks, the changes in transmission due to negative capital externality are quite close. Like with the intermediate good productivity shock case with negative externality, here as well, the foreign country's endogenous productivity increases, which further helps with co-movement. Finally, the fit is also improved in terms of generating a less cyclical real exchange rate as well as a more volatile trade balance thanks to the same underlying mechanisms as in the intermediate good productivity shock case.

At the end, we note that our main claims above also generally hold for the bond economy and financial autarky. We discuss these financial markets arrangements in Section 3.1.3 of Bhattarai and Kucheryavyy (2022).

5.3.2 Role of Negative Labor and Final Good Externalities

In a dynamic model, labor and capital externalities play very different roles in affecting the transmission of shocks. The effect of final good externality, in turn, is distinct from labor externality, as they operate through completely different channels. Most importantly, while negative capital externality helps move the model closer to the data in terms of various international moments, negative labor and final good externalities do not uniformly do so. We illustrate this point for the intermediate good productivity shock under complete markets in Table G.4 in Appendix G and provide a detailed discussion of the transmission mechanisms in Appendix H.3.[35] Intuitively, with labor externality, the productivity process looks like it has shifted downwards at every point in time, and the response of other variables then follows the path of productivity. For negative final good externality, the transmission is as if there was a negative final good productivity shock.

5.4 Estimation Exercise

Motivated by our findings above on how varying production externalities affects important international moments, we now undertake a more formal exercise to show carefully the need for negative capital externality to improve fit with the data. In particular, we now match a comprehensive list of moments, a larger set than the one presented above, while estimating the parameters governing the shock processes and all three externalities. Our criterion for model fit is the equally-weighted mean squared error.[36]

[35] In Section 3.3 of Bhattarai and Kucheryavyy (2022) we discuss the transmission mechanisms for labor and final good externalities with the final good productivity shock.

[36] We still do not use any data moments based on autocorrelations. This is deliberate as we want to emphasize that persistence of shocks can be identified from cross-country correlations. In a grid search estimation method, we construct a 7-dimensional grid for parameters $\psi_{X,K}$, $\psi_{X,L}$, ψ_Y, σ_X, σ_Y, ρ_X, and ρ_Y, and compute moments in each point of the grid. We then calculate the loss

Moment	Data	Model	Moment	Data	Model
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	0.50	$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.33
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.37	$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.96
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	0.22	$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	1.00
$\text{Corr}\left(L_1, L_2\right)$	0.42	0.50	$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.99
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.49	-0.50	$\text{Std}\left(\text{GDP}_1\right)$	1.67	1.85
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	0.45	$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.15
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.99	$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.59
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.15	$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	3.86

Parameter estimates:

$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	σ_X	σ_Y	ρ_X	ρ_Y
-2.70	0.91	-0.06	0.000	0.002	0.00	0.99

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = \left(W_n L_n + R_n K_n\right)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$, $\text{TB}_1 = \left(P_{X,1}X_1 - P_{Y,1}Y_1\right)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 5: Results of the estimation of the unified model for $\sigma = 2$. Complete markets.

Table 5 reports data moments, model moments, and the parameter estimates under the best fit for the complete markets.[37] The main estimate of interest is $\psi_{X,K} = -2.70$. This highlights the key quantitative role of negative capital externality in accounting for international business cycle moments.

In addition, we see that the shock driving the business cycle now is the final good productivity shock. In particular, there is no need for the intermediate good productivity shock. While our discussion previously highlighted how the two shocks lead to similar domestic and international business cycle dynamics, the

function $\mathcal{L} = \sqrt{\sum_{m=1}^{M}\left(\left[\text{Mom}^{\text{model}}\right]_m / \left[\text{Mom}^{\text{data}}\right]_m - 1\right)^2}$, where $\left[\text{Mom}^{\text{model}}\right]_m$ and $\left[\text{Mom}^{\text{data}}\right]_m$ are moments calculated in the model and data, and find the point on the grid with the lowest value of $\mathcal{L}$. See Appendix F for more details.

[37] In Section 3.4 of Bhattarai and Kucheryavyy (2022) we present results of the estimation exercise for the bond economy. The estimates and fit are generally quite similar for the complete markets and the bond economy.

final good productivity shock is estimated to be dominant here as it helps to reduce export cyclicality and increase trade balance volatility, for reasons discussed in Section 5.3.1.

A moment that is hard to match is the relative volatility of consumption, which is lower than the data. While even the model without externality, that is, the canonical IRBC model, also had this issue (as shown in Table 3), here it gets worse as negative capital externality improves fit on several dimensions simultaneously by endogenously reducing the persistence of the shock.[38] This then means that consumption is very smooth, for standard consumption smoothing reasons.[39]

5.5 Sensitivity Analysis

In Bhattarai and Kucheryavyy (2022) we perform several sensitivity analyses. We find that our two key quantitative results, that dynamic trade extensions of the IRBC model behave similarly over the business cycle and that negative capital externality improves the fit of the unified model, are robust to alternate model environments and parameterizations. These results hold under the bond economy and financial autarky, under both a higher ($\sigma = 6$) and a lower ($\sigma = 0.9$) trade elasticity compared to our baseline calibration ($\sigma = 2$), and even when we allow for spillovers and correlated shocks as in Heathcote and Perri (2002).

6 Dynamic Krugman Model with Sticky Prices

In this section, we revisit our most important theoretical and quantitative results in the context of a dynamic version of the Krugman model with sticky prices. In particular, first, we show how the isomorphisms results of Section 4 can be extended to such an environment. Second, we evaluate quanitatively whether endogenous

[38] Raffo (2008) pointed out this excess smootheness of consumption in the basic IRBC model and proposes changing preferences to generate enough consumption volatility. To fully account for relative volatility of consumption and generate an even better fit, a new shock might be needed. While we already find a role in this estimated model for a novel shock compared to the standard intermediate good productivity shock, to generate higher consumption volatility a preference shock as in Stockman and Tesar (1995) might be useful. The challenge with introducing such a shock would be to obtain similar fit for the other moments, including generating a domestic business cycle.

[39] In Appendix I.1 we provide and discuss results in terms of some untargeted moments.

entry affects dynamics and second-moments significantly in a model with sticky prices. Finally, we explore the potential role of negative capital exernality in affecting business cycle co-movement across countries.

To this end, we consider a dynamic version of the generalized Krugman model with nominal rigidities generated by Rotemberg (1982) pricing of firms.[40] We discuss only the key model elements and variables here and present the details in Appendix D. Each country's nominal variables are denominated in that country's currency. For nominal bilateral variables ($p_{ni,t}(\nu)$, $P_{ni,t}$, $\mathcal{X}_{ni,t}$, etc.) we use an asterisk to denote the denomination of this variable in terms of the importer country's currency (e.g., $p^*_{ni,t}(\nu)$ denotes the price of variety ν produced in country i and sold in country n and denominated in country n's currency), while the same variable without an asterisk is used to denote denomination in terms of the exporter country's currency. Country n's currency can be converted to the "reserve" currency using the nominal exchange rate $\mathcal{E}_{nt}$. In this notation, $p_{ni,t}(\nu) = \mathcal{E}_{nt} p^*_{ni,t}(\nu) / \mathcal{E}_{it}$.[41]

We assume that the producer of variety $\nu \in \Omega_{it}$ sold in country n pays a quadratic price adjustment cost. Furthermore, we assume that country i's producers pay this adjustment cost in country n (as opposed to paying this cost locally in country i) in terms of the basket of goods country n purchases from country i. Formally, the real price adjustment cost, $\mathrm{pac}_{ni,t}(\nu)$, is equal to

$$\mathrm{pac}_{ni,t}(\nu) \equiv \frac{\kappa}{2} \cdot \left[\frac{p^*_{ni,t}(\nu)}{p^*_{ni,t-1}(\nu)} - 1 \right]^2 \frac{p^*_{ni,t}(\nu)\, x_{ni,t}(\nu)}{P^*_{x,ni,t}}, \tag{35}$$

where $\kappa \geq 0$ is a cost-of-adjustment parameter, $p^*_{ni,t}(\nu)\, x_{ni,t}(\nu)$ is the revenue from sales in country n of variety ν produced in country i, and $x_{ni,t}(\nu)$ is the quantity (net of iceberg costs) of country i's variety ν produced for sale in country n.[42] The above specificaiton for the price adjustment cost implies that prices are fixed in terms of the importer's currency.

The rest of the setup is as in Section 3.1. It can then be shown that the system

[40] Bilbiie *et al.* (2014) develop a closed-economy model with endogenous entry and sticky prices as in Rotemberg (1982).

[41] The reserve currency convention allows a straight-forward presentation of the model with an arbitrarry number of countries, which helps to save on notation.

[42] Taking into account iceberg trade costs, the total quantity of country i's variety ν produced for sale in country n is $\tau_{ni,t} x_{ni,t}(\nu)$.

of equilibrium conditions in this model is isomorphic to the system of equilibrium conditions of a competitive model with a particular production technology for intermediate goods, and thus a version of Proposition 1 can again be stated.[43] Namely, for each pair of countries i and n, country i has a perfectly competitive (n,i)-producer that manufactures an intermediate good specific to country n with production technology given by

$$X_{ni,t} = \tau_{ni,t}^{-1} S_{\text{X},ni,t} K_{\text{X},it}^{\alpha_{\text{X,K},ni,t}} L_{\text{X},ni,t}^{\alpha_{\text{X,L},ni,t}}, \tag{36}$$

where

$$S_{\text{X},ni,t} \equiv \Theta_{\text{X},i} \left(\alpha_{\text{X,L},ni,t} \mu_{ni,t}\right)^{-1} Z_{\text{X},it} K_{\text{X},it}^{\psi_{\text{X,K}} - \alpha_{\text{X,K},ni,t}} L_{\text{X},ni,t}^{\psi_{\text{X,L}} + 1 - \alpha_{\text{X,L},ni,t}} \tag{37}$$

is an endogenous productivity shifter taken by the (n,i)-producer as given. Here $K_{\text{X},it}$ and $L_{\text{X},ni,t}$ are capital and labor used in production of the intermediate good. Capital is assumed to be non-rival across (n,i)-producers and so all (n,i)-producers rent the same amount of capital, while any worker can work for only one (n,i)-producer. Capital and labor shares in production are endogenous and are given by

$$\alpha_{\text{X,K},ni,t} \equiv 1 - \frac{\mu_{ni,t}^{-1}}{1 - \frac{\kappa}{2}\left[\pi_{ni,t}^{*}\right]^2} \quad \text{and} \quad \alpha_{\text{X,L},ni,t} \equiv \frac{\mu_{ni,t}^{-1}}{1 - \frac{\kappa}{2}\left[\pi_{ni,t}^{*}\right]^2},$$

where

$$\pi_{ni,t}^{*} \equiv \frac{K_{it}^{\psi_{\text{X,K}}} P_{\text{X},ni,t}^{*}}{K_{i,t-1}^{\psi_{\text{X,K}}} P_{\text{X},ni,t-1}^{*}} - 1$$

is the importer price inflation, $P_{\text{X},ni,t}^{*}$ is the nominal price of the intermediate good (n,i) in the country-n's currency, K_{it} is the total amount of capital in country i ($K_{it} = K_{\text{X},it}$ in equilibrium),

$$\mu_{ni,t} \equiv \frac{\sigma^{\text{K}}}{(\sigma^{\text{K}} - 1) + \kappa \Psi_{ni,t}}$$

is the quantity taken by (n,i)-producers as given (this is the monopolistic markup

[43]See Appendix D.1 for the details and Proposition 2 there for a precise statement. In the text here, we focus on describing how the theoretical results show the role for externalities in generating this isomorphism and on the new aspects due to nominal rigidities.

in the Krugman model) with

$$\Psi_{ni,t} \equiv \left(1+\pi^*_{ni,t}\right)\pi^*_{ni,t} - \frac{\sigma^{\kappa}-1}{2}\left[\pi^*_{ni,t}\right]^2$$
$$- E_t\left[\Lambda_{i,t,t+1}\left(1+\pi^*_{ni,t+1}\right)\pi^*_{ni,t+1}\cdot\frac{1-\frac{\kappa}{2}\left[\pi^*_{ni,t}\right]^2}{1-\frac{\kappa}{2}\left[\pi^*_{ni,t+1}\right]^2}\cdot\frac{K_{it}}{K_{i,t+1}}\cdot\frac{P_{X,ni,t+1}X_{ni,t+1}}{P_{X,ni,t}X_{ni,t}}\right]$$

and $\Lambda_{i,t,t+1}$ is the country-i-household's stochastic discount factors at date t for the future dates $t+1$ given by

$$\Lambda_{i,t,t+1} = \beta\cdot\frac{U_{1,i,t+1}}{U_{1,it}}\cdot\frac{P_{Y,it}}{P_{Y,i,t+1}}.$$

Observe that the production technology (36) features the iceberg trade cost $\tau_{ni,t}$. Also, note that — as in the unified model of Section 2 — the endogenous productivity shifter $S_{X,ni,t}$ given by (37) is subject to capital and labor externalities. However, differently from the unified model of Section 2, here the scale parameters depend on time and do so in a very specific way: we are free to choose only time-independent parameters $\psi_{X,K}$ and $\psi_{X,L}$. This time-variation in scale parameters comes about due to sticky prices. Also, differently from the unified model of Section 2, the productivity shifter in (37) is subject to an endogenous shock $\left(\alpha_{X,L,ni,t}\mu_{ni,t}\right)^{-1}$ in addition to the exogenous shock $Z_{X,it}$. Finally, observe that if $\kappa = 0$ then $\mu_{ni,t}$ is just the standard constant Dixit-Stiglitz markup, and we are back to the generalized Krugman model of Section 3.1.[44]

The rest of this competitve model setup is similar to the one in Section 2. We now turn to quantitative explorations of this setup, where we model monetary policy using a price level targeting regime. First, in Table 6 we present international and domestic second moments for this model, analogous to our presentation in Table 3 earlier. As before, we focus on complete markets. Also, we consider only

[44]Strictly speaking, when $\kappa = 0$, we get a *version* of the generalized Krugman model of Section 3.1, in which each country i has a perfectly competitive (n,i)-producer that manufactures an intermediate good specific to country n. This setup is generally different from the setup in Section 3.1 where the intermediate good producer in any country i produces a good that is not specific to any country n. These two setups are equivalent if the production technology for the intermediate good — from the perspective of the social planner — is linear in labor, which is the case in basic Krugman setup with $\psi_{X,L} = 0$.

	Data	IRBC	Krugman: Entry, No Capital $\kappa=0$	Krugman: Entry, No Capital $\kappa=35$	Krugman: Entry, No Capital $\kappa=77$	Krugman: No Entry, Capital $\kappa=0$	Krugman: No Entry, Capital $\kappa=35$	Krugman: No Entry, Capital $\kappa=77$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
International moments:								
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	0.58	−0.03	−0.09	−0.13	−0.13	−0.03	−0.09	−0.09
$\text{Corr}\,(C_1, C_2)$	0.36	0.47	0.41	0.37	0.35	0.51	0.42	0.37
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.39	−0.41	−0.54	−0.55	−0.38	−0.52	−0.52
$\text{Corr}\,(L_1, L_2)$	0.42	−0.30	−0.40	−0.45	−0.48	−0.39	−0.25	−0.13
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.49	−0.25	−0.35	−0.35	−0.19	−0.40	−0.44
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.32	0.36	0.60	0.44	0.50	0.65	0.37	0.37
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.81	0.93	0.86	0.85	0.87	0.85	0.87	0.89
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.13	0.61	0.68	0.72	0.73	0.66	0.70	0.71
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.21	0.19	0.25	0.21	0.15	0.27	0.27
Domestic moments:								
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.86	0.94	0.95	0.95	0.95	0.94	0.95	0.95
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.87	0.98	0.98	0.88	0.71	0.97	0.99	0.99
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.97	0.96	0.94	0.94	0.96	0.94	0.94
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.81	0.37	0.42	0.44	0.46	0.42	0.42	0.42
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.66	0.46	0.42	0.39	0.45	0.48	0.42	0.39
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	2.84	3.33	3.94	4.12	3.97	3.89	4.29	4.28

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_nL_n + R_nK_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 6: Moments for the sticky price model. Complete markets. Investment in terms of the final good.

the version of the Krugman model with investment in terms of the final good. For reference, column (1) presents data moments, while column (2) presents the IRBC model moments.[45] Then, columns (4) and (5) present the Krugman model with sticky prices (as described above in this section). We consider two values for the sticky price parameter: $\kappa = 77$ is as in Bilbiie *et al.* (2014) and is not chosen to match any moments; and $\kappa = 35$ is roughly the midway between the flexible price setup with $\kappa = 0$ and the standard value $\kappa = 77$. We see from columns (4) and (5) of Table 6 that broadly, *the Krugman model with sticky prices performs well and fails in the same cross-country and open-economy moments as the standard IRBC model and the Krugman model with flexible prices.*

In column (3) of Table 6, for comparsion, we present the results under flexible prices to isolate the role played by sticky prices in the Krugman model.[46] We see that sticky prices help improve the fit in one dimension by decreasing consumption correlations across countries.[47] This feature that sticky prices decrease cross-country correlations in consumption is not due to endogenous entry as embedded in the Krugman model. To make this clear, columns (6)-(8) of Table 6, present results for the model with physical capital but no endogenous entry. Comparing flexible prices with sticky prices, we see that in this set-up as well, sticky prices decrease consumption correlation across countries. Another upshot of Table 6 then is that endogenous entry and physical capital accumulation interact with sticky prices similarly in our quantitative example.[48]

Second, in Table 7 we provide moments from the competitive model without any externality, as well as with a positive and a negative capital externality. That is, this exercise is analogous to the one in Table 4 earlier for the case of flexible prices. Our main finding continues to hold even under sticky prices: negative capital externality in intermediate goods production helps improve fit with the data. We reach this same conclusion given Table 7 as cross-country correlation in outut, hours, and investment increase with negative capital externality. Moreover, negative capital externality also decreases the procyclicality of the real exchange rate while increasing the volatility of the trade balance.[49]

[45] Column (2) in Table 6 is the same as column (2) in Table 3.

[46] Column (3) in Table 6 is the same as column (8) in Table 3.

[47] Fit worsens however for output, investment, and hours correlations across countries.

[48] That endogenous entry and physical capital accumulation affect aggregate dynamics similarly is a recurring theme of our theoretical results.

[49] Compared to Table 4, one difference here is that consumption correlation across countries

	$\kappa = 35$			$\kappa = 77$		
$\psi_{X,K} =$	0	0.3	−0.3	0	0.3	−0.3
	(1)	(2)	(3)	(4)	(5)	(6)
International moments:						
Corr $(\text{GDP}_1, \text{GDP}_2)$	0.14	0.03	0.25	0.12	−0.00	0.24
Corr (C_1, C_2)	0.63	0.63	0.66	0.58	0.55	0.69
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	−0.38	−0.52	−0.29	−0.46	−0.57	−0.40
Corr (L_1, L_2)	−0.11	−0.58	0.50	−0.27	−0.60	0.57
Corr $\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.22	−0.04	−0.35	−0.27	−0.06	−0.43
Corr $(\text{Exp}_1, \text{GDP}_1)$	0.56	0.69	0.41	0.50	0.68	0.25
Corr $(\text{Imp}_1, \text{GDP}_1)$	0.84	0.77	0.89	0.85	0.79	0.90
Corr $(\text{ReR}, \text{GDP}_1)$	0.58	0.66	0.43	0.58	0.68	0.33
Std $\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.26	0.21	0.35	0.28	0.20	0.43
Domestic moments:						
Corr (C_1, GDP_1)	0.92	0.92	0.90	0.90	0.91	0.87
Corr (L_1, GDP_1)	0.84	0.64	0.97	0.65	0.44	0.95
Corr $\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.91	0.89	0.92	0.89	0.88	0.91
$\frac{\text{Std}(C_1)}{\text{Std}(\text{GDP}_1)}$	0.43	0.52	0.35	0.45	0.54	0.34
$\frac{\text{Std}(L_1)}{\text{Std}(\text{GDP}_1)}$	0.37	0.42	0.40	0.43	0.58	0.40
$\frac{\text{Std}(P_{I,1}I_1/P_{Y,1})}{\text{Std}(\text{GDP}_1)}$	4.29	3.71	4.91	4.43	3.72	5.36

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. Calibration is as implied by the baseline Krugman model with investment in terms of the final good and sticky prices, with two differences: (i) the steady-state labor externality is shut down by setting $\psi_{X,L} = 0$; and (ii) $\psi_{X,K}$ is varied.

Table 7: Changing capital externality in the sticky price model. Complete markets.

does not decrease with negative capital externality. Thus, given the results in Tables 6 and 7, one consistent finding is that it is in comsumption correlation across countries that sticky prices lead to qualitatively different results compared to flexible prices.

7 Conclusion

We present a unified framework to fully study the interconnections between international trade and business cycle models. We prove an aggregate equivalence between a competitive, representative firm open economy model that has production externalities and dynamic trade models that feature monopolistic competition, heterogeneous firms, and costs of entry and exporting. Our theoretical results shed light on why the business cycle implications of the IRBC and the standard dynamic trade models that appear in the literature are similar: the implied externalities are small, positive, and tightly restricted across factors. In a quantitative exercise with multiple shocks, we show that to resolve some well known empirical puzzles in the literature, the most important ingredient is negative capital externality.

A main upshot of our results on the need for negative capital externality is that it poses a puzzle for the quantitative literature that studies the interactions of international trade and business cycles models, as those models generate positive capital externality. This has two possible implications for future research. One is to move towards fully micro-founding negative capital externality in existing models. Our theoretical results already provide some guidance on that front as our generalized Melitz model can generate negative capital externality through congestion effects in serving markets: the larger is the number of firms operating in a particular market, the higher is the per-period fixed cost of serving this market. The other is to modify the core structure of the models beyond those often considered in the literature such that positive capital externality, as embedded in dynamic trade models, can help improve the fit on international moments. While we find that an extension with sticky prices and productivity shocks does not lead to large quantitative diferences, additional frictions and shocks, together with some mechanisms for intenational spillovers of technology might lead to different conclusions.

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Appendix

A Unified Model

A.1 Households Budget Constraints and First-Order Conditions

A.1.1 Financial Autarky

Under financial autarky, there is no international trade in financial assets. Households in country n face the flow budget constraint $P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} = W_{nt}L_{nt} + R_{nt}K_{nt}$. First-order conditions for the household's optimization problem are

$$P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1 \left(C_{n,t+1}, L_{n,t+1}\right)}{U_1 \left(C_{nt}, L_{nt}\right)} \left[R_{n,t+1} + (1-\delta) P_{I,n,t+1}\right] \right\}, \tag{38}$$

$$-\frac{U_2 \left(C_{nt}, L_{nt}\right)}{U_1 \left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \tag{39}$$

where $U_1(\cdot, \cdot)$ and $U_2(\cdot, \cdot)$ are derivatives of the utility function with respect to consumption and labor, correspondingly. Here (38) is the standard Euler equation, while (39) is the standard labor supply equation.

A.1.2 Bond Economy

We consider a bond economy where each country issues a non-state-contingent bond denominated in its consumption units. Holdings of country i's bond by country n are denoted by $B_{ni,t}$. The household's budget constraint is given by

$$\begin{aligned} P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} + \sum_{i=1}^{N} P_{Y,it} \left(B_{ni,t} + \frac{b_{adj}}{2} B_{ni,t}^2 \right) \\ = W_{nt}L_{nt} + R_{nt}K_{nt} + \sum_{i=1}^{N} P_{Y,it} \left(1 + r_{i,t-1}\right) B_{ni,t-1} + T_{nt}^B, \end{aligned}$$

where $r_{i,t-1}$ is period-t return on country-i's bond, and $T_{nt}^B \equiv \frac{b_{adj}}{2} \sum_{i=1}^{N} P_{Y,it} B_{ni,t}^2$ is the bond fee rebate, taken as given by the household. Here b_{adj} is the adjustment cost of bond holdings, which is introduced to ensure stationarity. First-order conditions are given by (38) and (39) and an additional set of Euler equations:

$$P_{Y,it}\frac{U_1\left(C_{nt},L_{nt}\right)}{P_{Y,nt}}\left(1+b_{adj}B_{ni,t}\right)=\beta E_t\left\{\frac{U_1\left(C_{n,t+1},L_{n,t+1}\right)}{P_{Y,n,t+1}}P_{Y,i,t+1}\left(1+r_{it}\right)\right\}.$$

International trade in bonds allows unbalanced trade in intermediate goods. Country n's real trade balance TB_{nt} was defined in (8) in Section 2. Define country n's real current account CA_{nt} as the change in this country's net financial assets position in terms of the final good, $\mathrm{CA}_{nt}\equiv\sum_{i=1}^{N}\frac{P_{Y,it}}{P_{Y,nt}}\left(B_{ni,t}-B_{ni,t-1}\right)$.

A.1.3 Complete Financial Markets

Under complete markets, we employ notation for the states of nature in period t, denoted by s_t, and history of states in period t, denoted by s^t. In each state with history s^t, countries trade a complete set of state-contingent nominal bonds denominated in the numeraire currency. Let $\mathcal{B}_{n,t+1}\left(s^t,s_{t+1}\right)$ denote the amount of the nominal bond with return in state s_{t+1} that country n acquires in the state with history s^t. Assuming that there are no costs of trading currency or securities between countries, we can denote by $P_{B,t}\left(s^t,s_{t+1}\right)$ the international price of this bond in the state with history s^t. Country n's budget constraint is given by

$$\begin{aligned}P_{Y,nt}\left(s^t\right)C_{nt}\left(s^t\right)+P_{I,nt}\left(s^t\right)I_{nt}\left(s^t\right)+\mathcal{A}_{nt}\left(s^t\right)\\=W_{nt}\left(s^t\right)L_{nt}\left(s^t\right)+R_{nt}\left(s^t\right)K_{nt}\left(s^t\right)+\mathcal{B}_{nt}\left(s^t\right),\end{aligned}$$

where

$$\mathcal{A}_{nt}\left(s^t\right)\equiv\sum_{s_{t+1}}P_{B,t}\left(s^t,s_{t+1}\right)\mathcal{B}_{n,t+1}\left(s^t,s_{t+1}\right)$$

is country n's net foreign assets position in period t. First-order conditions in the case of complete markets are given by conditions (38) and (39) (with the state-dependent notation added to them), plus an additional set of conditions:

$$P_{B,t}\left(s^t,s_{t+1}\right)=\beta\frac{\pi_{t+1}\left(s^{t+1}\right)}{\pi_t\left(s^t\right)}\cdot\frac{P_{Y,nt}\left(s^t\right)}{P_{Y,n,t+1}\left(s^{t+1}\right)}\cdot\frac{U_1\left(C_{n,t+1}\left(s^{t+1}\right),L_{n,t+1}\left(s^{t+1}\right)\right)}{U_1\left(C_{nt}\left(s^t\right),L_{nt}\left(s^t\right)\right)},$$

$$Q_{ni,t}\left(s^t\right) = \kappa_{ni} \frac{U_1\left(C_{nt}\left(s^t\right), L_{nt}\left(s^t\right)\right)}{U_1\left(C_{it}\left(s^t\right), L_{it}\left(s^t\right)\right)}, \quad \text{for each } i, \tag{40}$$

where $\pi_t\left(s^t\right)$ is the probability of history s^t occurring in period t, $Q_{ni,t}\left(s^t\right) \equiv P_{Y,nt}\left(s^t\right)/P_{Y,it}\left(s^t\right)$ is the real exchange rate, and

$$\kappa_{ni} \equiv \left(\frac{U_1\left(C_{n0}\left(s^0\right), L_{n0}\left(s^0\right)\right)/P_{Y,n0}\left(s^0\right)}{U_1\left(C_{i0}\left(s^0\right), L_{i0}\left(s^0\right)\right)/P_{Y,i0}\left(s^0\right)}\right)^{-1}.$$

In what follows, we drop the state-dependent notation for brevity. Condition (40) is the standard Backus-Smith condition that says that the real exchange comoves with the ratio of marginal utilities. As in the case of the bond economy, trade balance is defined by (8), and current account is defined as the change in net foreign assets position, $\text{CA}_{nt} = (\mathcal{A}_{nt} - \mathcal{A}_{n,t-1})/P_{Y,nt}$.

A.2 Equilibrium Conditions

Equilibrium conditions of the unified model are given by

$$\begin{aligned}
&P_{I,nt} = \beta E_t\left\{\frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)}\left[R_{n,t+1} + (1-\delta) P_{I,n,t+1}\right]\right\}, \\
&-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&K_{n,t+1} = (1-\delta) K_{nt} + I_{nt}, \\
&X_{nt} = \left(\Theta_{X,n} Z_{X,nt} K_{nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}\right) K_{nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}, \\
&Y_{nt} = \Theta_{Y,n} Z_{Y,nt}\left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y}\left[\sum_{i=1}^{N}\left(\omega_{ni} \frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \\
&I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
&L_{X,nt} + L_{I,nt} = L_{nt} + \frac{a P_{Y,nt} \cdot \text{TB}_{nt}}{W_{nt}}, \\
&C_{nt} + Y_{I,nt} = Y_{nt}, \\
&\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it},
\end{aligned}$$

$$
\begin{aligned}
\lambda_{ni,t} &= \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj}\right)^{1-\sigma}}, \\
K_{nt} &= \alpha_{X,K} \frac{P_{X,nt} X_{nt}}{R_{nt}}, \\
L_{X,nt} &= \alpha_{X,L} \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
L_{I,nt} &= \alpha_{I} \frac{P_{I,nt} I_{nt}}{W_{nt}}, \\
Y_{I,nt} &= \left(1 - \alpha_{I}\right) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}.
\end{aligned}
$$

The household's budget constraint in the case of financial autarky is given by

$$
P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} = W_{nt} L_{nt} + R_{nt} K_{nt},
$$

in the case of the bond economy it is given by

$$
P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} + \sum_{i=1}^{N} P_{Y,it} B_{ni,t} = W_{nt} L_{nt} + R_{nt} K_{nt} + \sum_{i=1}^{N} P_{Y,it} \left(1 + r_{i,t-1}\right) B_{ni,t-1},
$$

and in the case of complete markets it is given by

$$
P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} + \mathcal{A}_{nt} = W_{nt} L_{nt} + R_{nt} K_{nt} + \mathcal{B}_{nt},
$$

with

$$
\mathcal{A}_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1 \left(C_{n,t+1}, L_{n,t+1}\right)}{U_1 \left(C_{nt}, L_{nt}\right)} \mathcal{B}_{n,t+1} \right\}.
$$

Additional conditions in the case of the bond economy are $\sum_{n=1}^{N} B_{ni,t} = 0$ and

$$
P_{Y,it} \left(1 + b_{adj} B_{ni,t}\right) = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1 \left(C_{n,t+1}, L_{n,t+1}\right)}{U_1 \left(C_{nt}, L_{nt}\right)} P_{Y,i,t+1} \left(1 + r_{it}\right) \right\};
$$

while in the case of complete markets they are $\sum_{i=1}^{N} \mathcal{A}_{it} = 0$ and

$$
\frac{P_{Y,it}}{P_{Y,jt}} = \kappa_{ij} \frac{U_1 \left(C_{it}, L_{it}\right)}{U_1 \left(C_{jt}, L_{jt}\right)},
$$

where $\kappa_{ij} \equiv \left(\frac{U_1 \left(C_{i0}, L_{i0}\right) / P_{Y,i0}}{U_1 \left(C_{j0}, L_{j0}\right) / P_{Y,j0}} \right)^{-1}$ is found in the steady state.

B Equilibrium Systems of Equations for the Generalized Versions of the Standard Trade Models

In this section we present the full systems of equilibrium conditions for the generalized Krugman and Melitz models in the forms isomorphic to the equilibrium system of equations in the unified model.

B.1 Generalized Dynamic Version of the Krugman Model

Most of the equilibrium conditions for the Krugman model are provided in the main text. These are the expenditure share on varieties (17) together with the total expenditure of country n on country-i's varieties given by $\mathcal{X}_{ni,t} = \lambda_{ni,t} P_{Y,nt} Y_{nt}$; market clearing conditions for labor, varieties, and final aggregates described in Section 3.1.4; the law of motion of varieties (20); households budget constraints (21). Expressions for the total amount of labor allocated to production of varieties, $L_{X,nt}$, and the total number of firms in country n, M_{nt}, can be obtained from equations (18). Expressions for the amounts of labor and final aggregate allocated to "production" of varieties, $L_{I,nt}$ and $Y_{I,nt}$, are given in Section 3.1.2.

Expression (15) for $P_{X,nt}$ can be written as

$$P_{X,nt} = \left(1 - \frac{1}{\sigma^K}\right)^{-1} \frac{D_{nt}^{\frac{1}{\sigma^K}} W_{nt}^{1-\frac{1}{\sigma^K}}}{\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M}} L_{X,nt}^{\phi_{X,L}} D_{nt}^{\frac{1}{\sigma^K}} W_{nt}^{-\frac{1}{\sigma^K}}},$$

while (18) can be written as $W_{nt} = \left(1 - \frac{1}{\sigma^K}\right) \mathcal{X}_{nt} / L_{X,nt}$ and $D_{nt} = \frac{1}{\sigma^K} \mathcal{X}_{nt} / M_{nt}$. Substituting W_{nt} and D_{nt} into the denominator of expression for $P_{X,nt}$, we get

$$P_{X,nt} = \frac{D_{nt}^{\frac{1}{\sigma^K}} W_{nt}^{1-\frac{1}{\sigma^K}}}{\widetilde{\Theta}_{X,n}^K Z_{X,nt} M_{nt}^{\phi_{Y,M}-\frac{1}{\sigma^K}} L_{X,nt}^{\phi_{X,L}+\frac{1}{\sigma^K}}}, \tag{41}$$

where $\widetilde{\Theta}^{\kappa}_{X,n}$ is a constant. Substituting (18) for D_{nt} and W_{nt} into (41) gives

$$X_{nt} = \left[\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M} - \frac{1}{\sigma^{\kappa}}} L_{X,it}^{\phi_{X,L} + \frac{1}{\sigma^{\kappa}}}\right] M_{nt}^{1 - \frac{\sigma^{\kappa}-1}{\sigma^{\kappa}}} L_{X,it}^{\frac{\sigma^{\kappa}-1}{\sigma^{\kappa}}},$$

where $X_{nt} \equiv \mathcal{X}_{nt}/P_{X,nt}$ is the real output of varieties. Next, we can use (16) for the final aggregate price to write the final aggregate production function as

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\eta^{\kappa}-1}{\eta^{\kappa}}}\right]^{\frac{\eta^{\kappa}}{\eta^{\kappa}-1}}.$$

First-order conditions for the household's problem imply that

$$\begin{aligned}
&V_{nt} = \beta E_t \left\{\frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[D_{n,t+1} + (1-\delta) V_{n,t+1}\right]\right\}, \\
&-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}.
\end{aligned}$$

And, finally, the costs of entry of new varieties specified in Section 3.1.2 imply that the "production function" for new varieties is given by $M_{I,nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}$.

Combining all expressions and definitions, we get the equilibrium system in isomorphic form (for the case of financial autarky),

$$\begin{aligned}
&V_{nt} = \beta E_t \left\{\frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[D_{n,t+1} + (1-\delta) V_{n,t+1}\right]\right\}, \\
&-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}, \\
&X_{nt} = \left[\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M} - \frac{1}{\sigma^{\kappa}}} L_{X,it}^{\phi_{X,L} + \frac{1}{\sigma^{\kappa}}}\right] M_{nt}^{1 - \frac{\sigma^{\kappa}-1}{\sigma^{\kappa}}} L_{X,it}^{\frac{\sigma^{\kappa}-1}{\sigma^{\kappa}}}, \\
&Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\eta^{\kappa}-1}{\eta^{\kappa}}}\right]^{\frac{\eta^{\kappa}}{\eta^{\kappa}-1}}, \\
&M_{I,nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
&L_{X,nt} + L_{I,nt} = L_{nt}, \\
&C_{nt} + Y_{I,nt} = Y_{nt},
\end{aligned}$$

$$\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it},$$

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}\right)^{1-\eta^{K}}}{\sum_{l=1}^{N} \left(\tau_{nl,t} P_{X,lt}\right)^{1-\eta^{K}}},$$

$$M_{nt} = \left(1 - \frac{\sigma^{K} - 1}{\sigma^{K}}\right) \cdot \frac{P_{X,nt} X_{nt}}{D_{nt}},$$

$$L_{X,nt} = \frac{\sigma^{K} - 1}{\sigma^{K}} \cdot \frac{P_{X,nt} X_{nt}}{W_{nt}},$$

$$L_{I,nt} = \alpha_{I} \frac{V_{nt} M_{I,nt}}{W_{nt}},$$

$$Y_{I,nt} = (1 - \alpha_{I}) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}},$$

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt}.$$

B.2 Generalized Dynamic Version of the Melitz Model

The first-order conditions and constraints for the households problem are the same as in the Krugman model. Also, the market clearing condition for varieties and final aggregates, expressions for the amounts of labor and final aggregate allocated to "production" of varieties, $L_{I,nt}$ and $Y_{I,nt}$, and the "production" function for new varieties are the same as in the Krugman model.

Expressions for the total amount of labor allocated to production of varieties, $L_{X,nt}$, and the total number of firms in country n, M_{nt}, can be obtained from equations (31). Using the definition (44) of $L_{X,nt}$ and expression (32) for $L_{F,nt}$, the labor market clearing condition (33) is $L_{X,nt} + L_{I,nt} = L_{nt} + \dfrac{\theta^{M} + 1 - \sigma^{M}}{\theta^{M}\sigma^{M}} \times \dfrac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{W_{nt}}$.

Expression (27) for $P_{X,nt}$ can be written as

$$P_{X,nt} = \frac{D_{nt}^{\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} W_{nt}^{1-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}}}{\widetilde{\widetilde{\Theta}}{}^{M}_{X,n} Z_{X,nt} M_{nt}^{\phi_{F,M}-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}} \left[L^{M}_{X,nt}\right]^{\phi_{X,L}+\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}}},$$

where $\widetilde{\widetilde{\Theta}}{}^{M}_{X,n}$ is a constant. Using definition (44) of $L_{X,nt}$ provided in Section 4.1, we get that $P_{X,nt}$ can be written as

$$P_{X,nt} = \frac{D_{nt}^{\frac{\sigma^M - 1}{\sigma^M \theta^M}} W_{nt}^{1 - \frac{\sigma^M - 1}{\sigma^M \theta^M}}}{\widetilde{\Theta}_{X,n}^M Z_{X,nt} M_{nt}^{\phi_{F,M} - \frac{\sigma^M - 1}{\sigma^M \theta^M}} L_{X,nt}^{\phi_{X,L} + \frac{\sigma^M - 1}{\sigma^M \theta^M}}}, \tag{42}$$

where $\widetilde{\Theta}_{X,n}^M$ is a constant. Substituting (31) for D_{nt} and W_{nt} into (42) gives

$$X_{nt} = \left(\Theta_{X,n} M_{nt}^{\phi_{F,M} - \frac{\sigma^M - 1}{\sigma^M \theta^M}} L_{X,nt}^{\phi_{X,L} + \frac{\sigma^M - 1}{\sigma^M \theta^M}} \right) M_{nt}^{\frac{\sigma^M - 1}{\sigma^M \theta^M}} L_{X,nt}^{1 - \frac{\sigma^M - 1}{\sigma^M \theta^M}},$$

where $X_{nt} \equiv \mathcal{X}_{nt} / P_{X,nt}$ is real output of varieties, and
$\Theta_{X,n} \equiv \left(\frac{\sigma^M}{\sigma^M - 1} - \frac{1}{\theta^M} \right)^{-1-\phi_{X,L}} \Theta_{X,n}^M z_{\min,n}$. Using expression (28) for $P_{Y,nt}$, we can write

$$Y_{nt} = \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M} \right)^{\frac{1}{\sigma^M - 1} - \phi_{F,L}} \left[\sigma^M \right]^{-\phi_{F,L}} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{F_{ni,t}^{\vartheta} \tau_{ni,t}^M P_{X,it} / \omega_{ni}} \right)^{\frac{\theta^M \xi}{1 + \theta^M \xi}} \right]^{\frac{1 + \theta^M \xi}{\theta^M \xi}},$$

with $\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{-\theta^M \xi}}{\sum_{l=1}^{N} \left(\tau_{nl,t} P_{X,lt} / \omega_{nl} \right)^{-\theta^M \xi}}$. Redefining iceberg trade costs as $\tau_{ni,t} \equiv \left(F_{ni,t} / F_{nn,t} \right)^{\vartheta} \tau_{ni,t}^M$ and writing $F_{nn,t}^{-\vartheta} = \Theta_{Y,n}^M Z_{Y,nt}$, we get

$$Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it} / \omega_{ni}} \right)^{\frac{\theta^M \xi}{1 + \theta^M \xi}} \right]^{\frac{1 + \theta^M \xi}{\theta^M \xi}},$$

where

$$\Theta_{Y,n} \equiv \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M} \right)^{\frac{1}{\sigma^M - 1} - \phi_{F,L}} \left[\sigma^M \right]^{-\phi_{F,L}} \Theta_{Y,n}^M. \tag{43}$$

Combining all expressions and definitions, we get the equilibrium system in isomorphic form (for the case of financial autarky),

$$\begin{aligned}
&V_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1 \left(C_{n,t+1}, L_{n,t+1} \right)}{U_1 \left(C_{nt}, L_{nt} \right)} \left[D_{n,t+1} + (1 - \delta) V_{n,t+1} \right] \right\}, \\
&- \frac{U_2 \left(C_{nt}, L_{nt} \right)}{U_1 \left(C_{nt}, L_{nt} \right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&M_{n,t+1} = (1 - \delta) M_{nt} + M_{I,nt},
\end{aligned}$$

$$
\begin{aligned}
X_{nt} &= \left(\Theta_{X,n} M_{nt}^{\phi_{F,M}-\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{\phi_{X,L}+\frac{\sigma^M-1}{\sigma^M\theta^M}}\right) M_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}}, \\
Y_{nt} &= \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^M\xi}{1+\theta^M\xi}}\right]^{\frac{1+\theta^M\xi}{\theta^M\xi}}, \\
M_{I,nt} &= \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
L_{X,nt} + L_{I,nt} &= L_{nt} + \frac{\theta^M + 1 - \sigma^M}{\theta^M\sigma^M} \cdot \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{W_{nt}}, \\
C_{nt} + Y_{I,nt} &= Y_{nt}, \\
\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} &= P_{X,it} X_{it}, \\
\lambda_{ni,t} &= \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^M\xi}}{\sum_{l=1}^{N} \left(\tau_{nl,t} P_{X,lt}/\omega_{nl}\right)^{-\theta^M\xi}}, \\
M_{nt} &= \frac{\sigma^M - 1}{\sigma^M\theta^M} \cdot \frac{P_{X,nt} X_{nt}}{D_{nt}}, \\
L_{X,nt} &= \left(1 - \frac{\sigma^M - 1}{\sigma^M\theta^M}\right) \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
L_{I,nt} &= \alpha_I \frac{V_{nt} M_{I,nt}}{W_{nt}}, \\
Y_{I,nt} &= (1-\alpha_I) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}}, \\
P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} &= D_{nt} M_{nt} + W_{nt} L_{nt}.
\end{aligned}
$$

C Proof of Proposition 1

Most of the work required to prove Proposition 1 is done in Appendices B.1 and B.2, where we write the equilibrium systems of equations for the Krugman and Melitz models in forms isomorphic to the equilibrium system of equations for the unified model shown in Appendix A.2. A line-by-line comparison of the equilibrium systems of equations across these models shows that they are identical up to relabeling of variables and parameters. Namely, we need to relabel variables M_{nt} as $K_{X,nt}$, D_{nt} as R_{nt}, and V_{nt} as $P_{I,nt}$; and relabel parameters according to Table 1.

Achieving the isomorphism between the Melitz and unified models requires some extra work relative to the Krugman model. Namely, we need to make two redefinitions. First, we need to redefine iceberg trade costs as $\tau_{ni,t} \equiv \left(\frac{F_{ni,t}}{F_{nn,t}}\right)^{\vartheta} \tau^{\mathrm{M}}_{ni,t}$. Assumption 1 guarantees that $\tau_{ni,t}$ defined this way are, indeed, iceberg trade costs that satisfy the no-arbitrage condition. Second, we need to redefine labor used in production of intermediate goods as

$$L_{X,nt} \equiv \left(\frac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}}\right) L^{\mathrm{M}}_{X,nt}. \tag{44}$$

This redefinition of labor is necessary because in the Melitz model there is an extra use of labor to pay fixed costs of serving markets, which can be written as

$$L_{F,nt} = \left(\frac{1}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}}\right) L^{\mathrm{M}}_{X,nt} - \frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{P_{Y,nt} \cdot \mathrm{TB}_{nt}}{W_{nt}}. \tag{45}$$

The sum of $L^{\mathrm{M}}_{X,nt}$ and the first term on the right-hand side of (45) give $L_{X,nt}$ in (44). The second term on the right-hand side of (45) is mapped into the additional term on the right-hand side of (9) in the unified model with $a = \frac{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}}$.

D Dynamic Version of the Krugman Model with Rotemberg Pricing

Since producers of varieties are homogeneous, for brevity of notation, we can drop the dependence on ν of all variables pertaining to these producers. Country i's variety producer chooses at each date t prices $\{p_{ni,s}\}_{s=t}^{\infty}$ so as to maximize the discounted flow of profits from sales to country n,

$$\max_{p_{ni,s}, y_{ni,s}, l_{ni,s}} E_t \sum_{s=t}^{\infty} \Lambda_{i,ts} \cdot \left[p_{ni,s} x_{ni,s} - W_{is} l_{ni,s} - P_{X,ni,s} \cdot \mathrm{pac}_{ni,s}\right]$$

s.t.

$$x_{ni,s} = S_{Y,ns}^{\eta^{\mathrm{K}}-1} M_{is}^{(\sigma^{\mathrm{K}}-1)\left(\phi_{Y,M} - \frac{1}{\sigma^{\mathrm{K}}-1}\right)} \omega_{ni}^{\eta^{\mathrm{K}}-1} p_{ni,s}^{-\sigma^{\mathrm{K}}} P_{X,ni,s}^{\sigma^{\mathrm{K}}-1} \mathcal{X}_{ni,s},$$

$$\tau_{ni,s} x_{ni,s} = S^{\mathrm{K}}_{X,is} l_{ni,s}, \tag{46}$$

$$\mathrm{pac}_{ni,s} = \frac{\kappa}{2} \cdot \left[\frac{\mathcal{E}_{is} p_{ni,s}/\mathcal{E}_{ns}}{\mathcal{E}_{i,s-1} p_{ni,s-1}/\mathcal{E}_{n,s-1}} - 1\right]^2 \frac{p_{ni,s} x_{ni,s}}{P_{X,ni,s}}. \tag{47}$$

Here $\mathcal{X}_{ni,s}$ is the total (nominal) expenditure of country n on varieties produced by country i, and $\Lambda_{i,ts}$ are country-i-households' stochastic discount factors at date t for all future dates s given by

$$\Lambda_{i,ts} = \frac{[\beta(1-\delta)]^{s-t}}{1-\delta} \cdot \frac{U_{1,is}}{U_{1,it}} \cdot \frac{P_{Y,it}}{P_{Y,is}} \quad \text{for } s > t,$$

and $\Lambda_{i,tt} = 1$. First-order conditions of this problem imply

$$p_{ni,t} = \mu_{ni,t} \cdot \frac{\tau_{ni,t} W_{it}}{S^{K}_{X,it}} \;\text{ with }\; \mu_{ni,t} \equiv \frac{\sigma^{K}}{(\sigma^{K}-1) + \kappa \Psi_{ni,t}}, \tag{48}$$

where

$$\begin{aligned} \Psi_{ni,t} \equiv & \left(1+\pi^{*}_{ni,t}\right)\pi^{*}_{ni,t} - \frac{\sigma^{K}-1}{2}\left[\pi^{*}_{ni,t}\right]^2 \\ & - E_t\left[\Lambda_{i,t,t+1} \cdot \frac{\mathcal{E}_{it}/\mathcal{E}_{nt}}{\mathcal{E}_{i,t+1}/\mathcal{E}_{n,t+1}} \cdot \left(1+\pi^{*}_{ni,t+1}\right)^2 \pi^{*}_{ni,t+1} \cdot \frac{x_{ni,t+1}}{x_{ni,t}}\right] \end{aligned}$$

and $\pi^{*}_{ni,t} \equiv p^{*}_{ni,t} \big/ p^{*}_{ni,t-1} - 1$ is the importer price inflation. Here $\mu_{ni,t}$ is the markup over maginal cost. Observe that if $\kappa = 0$ then $\mu_{ni,t}$ is just the standard constant Dixit-Stiglitz markup.

Combining (46) and (48), we get

$$W_{it} l_{ni,t} = \frac{\tau_{ni,t} W_{it}}{S^{K}_{X,it}} x_{ni,t} = \mu^{-1}_{ni,t} p_{ni,t} x_{ni,t}.$$

Integrating the above expression across all firms gives

$$W_{it} L_{X,ni,t} = \mu^{-1}_{ni,t} \mathcal{X}_{ni,t}. \tag{49}$$

This also allows us to find the average nominal profit per country-i-firm from sales in country n at time t,

$$D_{ni,t} \equiv \left(1 - \mu^{-1}_{ni,t} - \frac{\kappa}{2}\left[\pi^{*}_{ni,t}\right]^2\right) \frac{\mathcal{X}_{ni,t}}{M_{it}}. \tag{50}$$

Next, let $X_{ni,t} \equiv \mathcal{X}^*_{ni,t} \Big/ P^*_{X,ni,t} = \mathcal{X}_{ni,t}/P_{X,ni,t}$ be the aggregate basket of varieties that country n imports from i. A part of this basket is spent on paying the price adjustment costs for producers of varieties from country i. Denote

$$\widetilde{X}_{ni,t} \equiv X_{ni,t} - M_{it} \cdot \mathrm{pac}_{ni,t}.$$

Using the definition (35) of $\mathrm{pac}_{ni,t}$, we can find $X_{ni,t} = \left(1 - \frac{\kappa}{2}\left[\pi^*_{ni,t}\right]^2\right)^{-1} \widetilde{X}_{ni,t}$. Substituting this into (49) and (50), we get

$$D_{ni,t} = \gamma_{M,ni,t} \frac{P_{X,ni,t}\widetilde{X}_{ni,t}}{M_{it}} \quad \text{and} \quad W_{it} = \gamma_{L,ni,t} \frac{P_{X,ni,t}\widetilde{X}_{ni,t}}{L_{X,ni,t}} \tag{51}$$

where

$$\gamma_{M,ni,t} \equiv \frac{1 - \mu^{-1}_{ni,t} - \frac{\kappa}{2}\left[\pi^*_{ni,t}\right]^2}{1 - \frac{\kappa}{2}\left[\pi^*_{ni,t}\right]^2} \quad \text{and} \quad \gamma_{L,ni,t} \equiv \frac{1}{\left(1 - \frac{\kappa}{2}\left[\pi^*_{ni,t}\right]^2\right)\mu_{ni,t}}. \tag{52}$$

Observe that $\gamma_{M,ni,t} + \gamma_{L,ni,t} = 1$.

We can then write the bilateral price index as

$$\begin{aligned}
P_{X,ni,t} &= M_{it}^{-\left(\phi_{Y,M} - \frac{1}{\sigma^K - 1}\right)} \left[\int_{\nu \in \Omega_{it}} p_{ni,t}(\nu)^{1-\sigma^K} d\nu\right]^{\frac{1}{1-\sigma^K}} \\
&= M_{it}^{-\phi_{Y,M}} p_{ni,t} \\
&= \mu_{ni,t} \cdot \frac{\tau_{ni,t} W_{it}}{\Theta_{X,i} Z_{X,it} M_{it}^{\phi_{Y,M}} L_{X,ni,t}^{\phi_{X,L}}} \\
&= \mu_{ni,t} \cdot \frac{\tau_{ni,t} D_{ni,t}^{\gamma_{M,ni,t}} W_{it}^{\gamma_{L,ni,t}}}{\Theta_{X,i} Z_{X,it} D_{ni,t}^{\gamma_{M,ni,t}} W_{it}^{\gamma_{L,ni,t}-1} M_{it}^{\phi_{Y,M}} L_{X,ni,t}^{\phi_{X,L}}} \\
&= \mu_{ni,t} \cdot \frac{\tau_{ni,t} D_{ni,t}^{\gamma_{M,ni,t}} W_{it}^{\gamma_{L,ni,t}}}{\gamma_{M,ni,t}^{\gamma_{M,ni,t}} \gamma_{L,ni,t}^{\gamma_{L,ni,t}-1} \Theta_{X,i} Z_{X,it} M_{it}^{\phi_{Y,M} - \gamma_{M,ni,t}} L_{X,ni,t}^{\phi_{X,L} - \gamma_{L,ni,t} + 1}}.
\end{aligned}$$

Substituting (51) into the above and after some manipulations, we get

$$\tau_{ni,t}\widetilde{X}_{ni,t} = \left[\Theta_{X,i} \gamma^{-1}_{L,ni,t} \mu^{-1}_{ni,t} Z_{X,it} M_{it}^{\phi_{Y,M} - \gamma_{M,ni,t}} L_{X,ni,t}^{\phi_{X,L} + 1 - \gamma_{L,ni,t}}\right] M_{it}^{\gamma_{M,ni,t}} L_{X,ni,t}^{\gamma_{L,ni,t}}.$$

The rest of the model is as in Section 3.1. The full equilibrium system of equations (for the case of financial autarky) is

$$V_{it} = \beta E_t \left\{ \frac{P_{Y,it}}{P_{Y,i,t+1}} \cdot \frac{U_1\left(C_{i,t+1}, L_{i,t+1}\right)}{U_1\left(C_{it}, L_{it}\right)} \left[D_{i,t+1} + (1-\delta) V_{i,t+1}\right] \right\}, \tag{53}$$

$$-\frac{U_2\left(C_{it}, L_{it}\right)}{U_1\left(C_{it}, L_{it}\right)} = \frac{W_{it}}{P_{Y,it}}, \tag{54}$$

$$M_{i,t+1} = (1-\delta) M_{it} + M_{I,it},$$

$$\tau_{ni,t}\widetilde{X}_{ni,t} = \left[\Theta_{X,i}\gamma_{L,ni,t}^{-1}\mu_{ni,t}^{-1}Z_{X,it}M_{it}^{\phi_{Y,M}-\gamma_{M,ni,t}}L_{X,ni,t}^{\phi_{X,L}+1-\gamma_{L,ni,t}}\right] M_{it}^{\gamma_{M,ni,t}}L_{X,ni,t}^{\gamma_{L,ni,t}},$$

$$Y_{nt} = \Theta_{Y,n}Z_{Y,nt}\left(\frac{P_{Y,nt}Y_{nt}}{W_{nt}}\right)^{\psi_Y}\left[\sum_{i=1}^{N}\left(\omega_{ni}\widetilde{X}_{ni,t}\right)^{\frac{\eta^K-1}{\eta^K}}\right]^{\frac{\eta^K}{\eta^K-1}},$$

$$M_{I,it} = \Theta_{I,i}Z_{I,it}L_{I,it}^{\alpha_I}Y_{I,it}^{1-\alpha_I},$$

$$\sum_{n=1}^{N} L_{X,ni,t} + L_{I,it} = L_{it},$$

$$C_{it} + Y_{I,it} = Y_{it},$$

$$\mathcal{E}_{it}P_{X,ni,t}\widetilde{X}_{ni,t} = \frac{\left(\mathcal{E}_{it}P_{X,ni,t}/\omega_{ni}\right)^{1-\eta^K}}{\sum_{l=1}^{N}\left(\mathcal{E}_{lt}P_{X,nl,t}/\omega_{nl}\right)^{1-\eta^K}}\mathcal{E}_{nt}P_{Y,nt}Y_{nt},$$

$$M_{it} = \gamma_{M,ni,t}\frac{P_{X,ni,t}\widetilde{X}_{ni,t}}{D_{ni,t}},$$

$$L_{X,ni,t} = \gamma_{L,ni,t}\frac{P_{X,ni,t}\widetilde{X}_{ni,t}}{W_{it}},$$

$$L_{I,it} = \alpha_I\frac{V_{it}M_{I,it}}{W_{it}},$$

$$Y_{I,it} = \left(1-\alpha_I\right)\frac{V_{it}M_{I,it}}{P_{Y,it}},$$

$$D_{it} = \sum_{n=1}^{N} D_{ni,t},$$

$$P_{Y,it}C_{it} + V_{it}M_{I,it} = W_{it}L_{it} + D_{it}M_{it};$$

with $\gamma_{M,ni,t}$ and $\gamma_{L,ni,t}$ given by (52), $\mu_{ni,t}$ given by (48), and

$$\pi_{ni,t}^{*} = \frac{M_{it}^{\phi_{Y,M}}P_{X,ni,t}^{*}}{M_{i,t-1}^{\phi_{Y,M}}P_{X,ni,t-1}^{*}} - 1 = \frac{M_{it}^{\phi_{Y,M}}\ \mathcal{E}_{it}P_{X,ni,t}/\mathcal{E}_{nt}}{M_{i,t-1}^{\phi_{Y,M}}\ \mathcal{E}_{i,t-1}P_{X,ni,t-1}/\mathcal{E}_{n,t-1}} - 1,$$

and

$$\Psi_{ni,t} = \left(1+\pi^{*}_{ni,t}\right)\pi^{*}_{ni,t} - \frac{\sigma^{\kappa}-1}{2}\left[\pi^{*}_{ni,t}\right]^{2} \\ - E_t\left[\Lambda_{i,t,t+1}\left(1+\pi^{*}_{ni,t+1}\right)\pi^{*}_{ni,t+1}\cdot\frac{1-\frac{\kappa}{2}\left[\pi^{*}_{ni,t}\right]^{2}}{1-\frac{\kappa}{2}\left[\pi^{*}_{ni,t+1}\right]^{2}}\cdot\frac{M_{it}}{M_{i,t+1}}\cdot\frac{P_{X,ni,t+1}\widetilde{X}_{ni,t+1}}{P_{X,ni,t}\widetilde{X}_{ni,t}}\right].$$

The bond economy and complete financial markets setups are similar to the ones described for the unified model in Appendices A.1.2 and A.1.3 with the only difference that we need to write the budget constraints using the right currency. In the bond economy, country n's household's budget constraint is given by

$$P_{Y,nt}C_{nt} + V_{nt}M_{I,nt} + \sum_{i=1}^{N}\frac{\mathcal{E}_{it}P_{Y,it}}{\mathcal{E}_{nt}}\left(B_{ni,t}+\frac{b_{adj}}{2}B^{2}_{ni,t}\right) \\ = W_{nt}L_{nt} + D_{nt}M_{nt} + \sum_{i=1}^{N}\frac{\mathcal{E}_{it}P_{Y,it}}{\mathcal{E}_{nt}}\left(1+r_{i,t-1}\right)B_{ni,t-1} + T^{B}_{nt},$$

with $T^{B}_{nt} \equiv \frac{b_{adj}}{2}\sum_{i=1}^{N}\frac{\mathcal{E}_{it}P_{Y,it}}{\mathcal{E}_{nt}}B^{2}_{ni,t}$. Under complete financial markets, country n's household's budget constraint is given by

$$P_{Y,nt}\left(s^{t}\right)C_{nt}\left(s^{t}\right) + V_{nt}\left(s^{t}\right)M_{I,nt}\left(s^{t}\right) + \mathcal{A}_{nt}\left(s^{t}\right)/\mathcal{E}_{nt}\left(s^{t}\right) \\ = W_{nt}\left(s^{t}\right)L_{nt}\left(s^{t}\right) + D_{nt}\left(s^{t}\right)M_{nt}\left(s^{t}\right) + \mathcal{B}_{nt}\left(s^{t}\right)/\mathcal{E}_{nt}\left(s^{t}\right),$$

where

$$\mathcal{A}_{nt}\left(s^{t}\right) \equiv \sum_{s_{t+1}} P_{B,t}\left(s^{t},s_{t+1}\right)\mathcal{B}_{n,t+1}\left(s^{t},s_{t+1}\right)$$

is country n's net foreign assets position in period t.

D.1 Corresponding Competitive Model Setup

Consider now a competitive model setup similar to the one described in Section 2 with the only difference in the intermediate good production function. Namely, as we described in Section 6, for each pair of countries i and n, country i has a perfectly competitive (n,i)-producer that manufactures an intermediate good specific to country n with production technology given by

$$X_{ni,t} = \tau_{ni,t}^{-1} S_{X,ni,t} K_{X,it}^{\alpha_{X,K,ni,t}} L_{X,ni,t}^{\alpha_{X,L,ni,t}}, \tag{55}$$

where

$$S_{X,ni,t} \equiv \Theta_{X,i} \left(\alpha_{X,L,ni,t}\mu_{ni,t}\right)^{-1} Z_{X,it} K_{X,it}^{\psi_{X,K}-\alpha_{X,K,ni,t}} L_{X,ni,t}^{\psi_{X,L}+1-\alpha_{X,L,ni,t}} \tag{56}$$

The definitions of all parameters of this technology are given in the main text in Section 6.

One can readily verify that this setup results in the equilibrium system of equations that — up to relabeling of variables and parameters — is the same as the one described above for the Krugman model with sticky prices. The relabeling of variables of the Krugman model with sticky prices is almost the same as in the case of the Krugman model with flexible prices: we need to relabel M_{nt} as $K_{X,nt}$, D_{nt} as R_{nt}, and V_{nt} as $P_{I,nt}$. In additon to that, we need to relabel $\widetilde{X}_{ni,t}$ as $X_{ni,t}$ and $D_{ni,t}$ as $R_{ni,t}$, where $R_{ni,t}$ is the competitive model's rate of return on capital used in country i's production of the intermediate good sold in country n. As for the parameters, we need to relabel η^{κ} as σ, $\gamma_{M,ni,t}$ as $\alpha_{X,K,ni,t}$, $\gamma_{L,ni,t}$ as $\alpha_{X,L,ni,t}$, $\phi_{Y,M}$ as $\psi_{X,K}$, and $\phi_{X,L}$ as $\psi_{X,L}$. We summarize this result in the following proposition.

Proposition 2. *By an appropriate relabeling of variables and parameters in the Krugman model with Rotemberg pricing of firms, we can write the equilibrium system of equations of this model in a form identical to the equilibrium system of equations in the competitive model with the intermediate goods technology given by* (55)-(56) *and the rest of the setup as in the unified model of Section 2.*

E Description of Calibration in Section 5.1

We now explain the calibration in Table 2. We first choose a set of common parameter values for our three models. Most of these values are taken from the literature. Values of parameters β, γ, δ, and μ are the same as in, for example, Heathcote and Perri (2002) and Ghironi and Melitz (2005). We follow the macro literature (as apposed to the international trade literature) and set the elasticity of substitution between intermediate goods in production of the final good to 2, i.e., we set

$\sigma = 2$. This implies that the trade elasticity is equal to 1.[50] We choose the level of iceberg trade costs $\tau_{ni,t} = 5.67$ for $n \neq i$ to match the steady-state share of imports of intermediate goods of 0.15. Differently from Heathcote and Perri (2002), we do not have home bias in production of the final aggregate and set $\omega_{ni} = 0.5$ for all n and i. Values of autocorrelations $\rho_{X,11}$ and $\rho_{X,22}$ of the productivity process in the intermediate goods sector, $Z_{X,nt}$, as well as volatilities of shocks $\sigma_{X,1}$ and $\sigma_{X,2}$ to $Z_{X,nt}$ are taken from Heathcote and Perri (2002).[51] The productivity process in the final goods sector, $Z_{Y,nt}$, has the same parameterization as $Z_{X,nt}$.

We set the normalization constants in the intermediate goods and investment sectors to 1, $\Theta_{X,n} = \Theta_{I,n} = 1$. In order to match the value of fixed costs of serving foreign markets in Ghironi and Melitz (2005) (which is discussed below), we set the normalization constant in the final aggregates sector to 2.069, $\Theta_{Y,n} = 2.069$. Finally, for the case of the bond economy, we choose a relatively low value of the bond holdings adjustment cost, $b_{adj} = 0.0025$.

The values of the remaining parameters are different between the IRBC, Krugman, and Melitz models. For the IRBC model, we set the same share of capital in production of intermediate goods as in Heathcote and Perri (2002), $\alpha_{X,K} = 0.36$, and require that investment is made in terms of the final good only (i.e., set $\alpha_I = 1$). The IRBC model does not have any externalities ($\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$), it does not have a productivity shock in the investment sector ($Z_{I,nt} = 1$), and it does not have the additional term $a\mathrm{TB}_{nt}$ in the labor market clearing condition ($a = 0$). The standard IRBC model does not have a productivity shock in the final goods sector ($Z_{Y,nt} = 1$), but we introduce this shock in certain quantitative exercises.

For the parameterization corresponding to the standard Krugman model, we use the value of $\sigma^K = 3.8$ from Bilbiie *et al.* (2012). This choice immediately implies values for all key parameters specific to the Krugman model: $\alpha_{X,K} = \frac{1}{\sigma^K} \approx 0.26$, $\psi_{X,K} = \frac{1}{\sigma^K - 1} - \frac{1}{\sigma^K} \approx 0.094$, and $\psi_{X,L} = \frac{1}{\sigma^K} \approx 0.26$ (see Table 1 for parameter

[50] See, for example, Hillberry and Hummels (2013) on the choice between "macro" versus "micro" trade elasticity. In Section 3.1.1 of Bhattarai and Kucheryavyy (2022) we do a sensitivity analysis with lower (than 1) and higher elasticities of substitution.

[51] Differently from Heathcote and Perri (2002), we do not allow either for spillovers in the process for $Z_{X,nt}$ or for correlation of shocks to $Z_{X,nt}$. We do this to ensure that the dynamics are driven by endogenous propagation mechanisms in the models. In Section 3.1.2 of Bhattarai and Kucheryavyy (2022) we allow for spillovers and correlation of shocks in a sensitivity analysis.

mappings between the models). The standard Krugman model has neither externalities nor productivity shocks in the final goods sector ($\psi_Y = 0$ and $Z_{Y,nt} = 1$; but we introduce the productivity shock in the final goods sector in certain exercises). Also, the standard Krugman model does not have the additional term $a\mathrm{TB}_{nt}$ in the labor market clearing condition ($a = 0$). Investment is made in terms of labor only ($\alpha_I = 0$). We follow Bilbiie *et al.* (2012) in setting the productivity shock in production of investment goods identical to the productivity shock in production of intermediate goods ($Z_{I,nt} = Z_{X,nt}$). The choice of the investment-sector normalization constant $\Theta_{I,n} = 1$ implies that the sunk entry cost into the economy in the Krugman model — given by $\widetilde{\Theta}_{I,n}^{-1}$ — is equal to 1.[52] Finally, trade elasticity equal to 1 in the unified model implies that the elasticity of substitution between varieties from different countries in the Krugman model is equal to $\eta^{\mathrm{K}} = 2$.

Turning to the standard Melitz model, let us first consider fixed and variable costs of serving markets. We assume that in the Melitz model $F_{12,t} = F_{11,t}$ and $F_{21,t} = F_{22,t}$ for all t. This implies that $\tau^{\mathrm{M}}_{ni,t} = \tau_{ni,t} = 5.67$. Following Ghironi and Melitz (2005), we further assume that the fixed costs of serving markets in the Melitz model are subject to the same shock as the production technology of varieties. Formally, we assume that $F_{nn,t} = f_{nn}/Z_{X,nt}$, where f_{nn} is a time-independent constant (defined below). The derivations of the isomorphic equilibrium system for the Melitz model in Appendix B.2 imply that $Z_{Y,nt} = F_{nn,t}^{-\vartheta}/\Theta^{\mathrm{M}}_{Y,n}$ with $\vartheta = \frac{1}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}}$, and, hence, $Z_{Y,nt} = [Z_{X,nt}]^{\vartheta}$ and $f_{nn} = \left[\Theta^{\mathrm{M}}_{Y,n}\right]^{-\frac{1}{\vartheta}}$. Using mapping (43) in Appendix B.2, we find that the fixed costs of serving markets are

$$f_{nn} = \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}\right)^{\frac{\sigma^{\mathrm{M}}-1}{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}} \frac{1}{\sigma^{\mathrm{M}}} \left[\Theta_{Y,n}\right]^{-\frac{1}{\vartheta}}. \tag{57}$$

Next, following Ghironi and Melitz (2005), we choose $\sigma^{\mathrm{M}} = 3.8$ (which is also the same as σ^{K}) and $\theta^{\mathrm{M}} = 3.4$. The choices of σ^{M} and θ^{M} imply that $\alpha_{X,K} = \frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}} \approx 0.22$, $\psi_{X,K} = \frac{1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}} \approx 0.077$, $\psi_{X,L} = \frac{\sigma^{\mathrm{M}}-1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}} \approx 0.22$, $\psi_Y = \frac{1}{\sigma^{\mathrm{M}}-1} - \frac{1}{\theta^{\mathrm{M}}} \approx 0.063$, and $Z_{Y,nt} \approx [Z_{X,nt}]^{0.063}$. Using expression (57) we get that the implied value of the fixed costs of serving markets in the Melitz model is $f_{nn} \approx 0.0084$, which is the same

[52] Bilbiie *et al.* (2012) also have the value of the sunk costs of entry into the economy equal to 1. As Bilbiie *et al.* (2012) note, this value does not affect any impulse-responses under CES preferences.

as the fixed cost of serving *foreign* markets in Ghironi and Melitz (2005). The labor market clearing condition now features the additional term $aP_{Y,nt} \cdot \text{TB}_{nt}$ with $a = \frac{\theta^M + 1 - \sigma^M}{\sigma^M \theta^M} \approx 0.046$. As in the calibration corresponding to the Krugman model, $Z_{I,nt} = Z_{X,nt}$ and $\alpha_I = 1$. The implied sunk entry cost into the economy is equal to 1. Finally, the choice of $\sigma = 2$ in the unified model implies that in the Melitz model the elasticity of substitution between varieties from different countries is equal to $\eta^M = 1 + \left(\frac{1}{\sigma - 1} + \vartheta\right)^{-1} \approx 1.94$.

F Grid Search

In the grid search, we consider the following values for our parameters: $\psi_{X,K} \in [-10, 10]$ with step 0.1, $\psi_{X,L} \in [-5, 5]$ with step 0.1, $\psi_Y \in \{-0.35, -0.3, -0.2, -0.1, 0, 0.1\}$, $\sigma_X \in [0, 0.01]$ with step 0.001, $\sigma_X \in [0, 0.008]$ with step 0.001, $\rho_X, \rho_Y \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.92, 0.94, 0.96, 0.98, 0.99\}$. For some parameter combinations the steady state does not exists, or the Blanchard-Kahn conditions are not satisfied, and thus moments cannot be calculated. For each point on the grid where we can compute moments, we calculate the loss function

$$\mathcal{L} = \sqrt{\sum_{m=1}^{M} \left(\left[\text{Mom}^{\text{model}}\right]_m / \left[\text{Mom}^{\text{data}}\right]_m - 1 \right)^2},$$

where $\left[\text{Mom}^{\text{model}}\right]_m$ and $\left[\text{Mom}^{\text{data}}\right]_m$ are moments calculated in the model and data. With then find the point on the grid with the minimal value of $\mathcal{L}$.

For the case of complete markets, after finding the point with the minimal value of $\mathcal{L}$, we additionally consider a finer grid around the set of points with the value of $\mathcal{L}$ within 1% of the the minimal value of $\mathcal{L}$. In the finer grid, we use step 0.01 for $\psi_{X,K}$, $\psi_{X,L}$, and ψ_Y, and step 0.01 for ρ_X and ρ_Y, while we leave the same grid for σ_X and σ_Y. We calculate the moments on this finer grid and the associated loss functions. We then, again, find the set of points with the value of $\mathcal{L}$ within 1% of the the minimal value of $\mathcal{L}$ on this finer grid. We repeat this procedure until such set of points stops changing. For the result for complete markets in Table 5, we report the result for the point with the minimal value of $\mathcal{L}$ in this set.

G Additional Tables with Moments

		Benchmark calibration						Investment final good			
		Int. sector shock			Final sector shock			Int. sector shock		Final sector shock	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	0.58	0.02	−0.02	−0.02	−0.15	−0.03	−0.03	0.01	−0.04	−0.12	−0.17
$\text{Corr}\,(C_1, C_2)$	0.36	0.11	0.19	0.21	−0.19	0.05	0.05	0.15	0.13	−0.14	−0.15
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.35	−0.19	−0.18	−0.60	−0.04	−0.04	−0.34	−0.40	−0.56	−0.59
$\text{Corr}\,(L_1, L_2)$	0.42	−0.04	−0.21	−0.23	−0.11	−0.44	−0.48	−0.11	−0.19	−0.08	−0.19
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	−0.60	−0.01	0.19	−0.69	0.71	0.71	−0.53	−0.52	−0.66	−0.66
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.32	0.13	0.64	0.77	−0.18	0.99	0.99	0.32	0.33	−0.00	0.01
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.81	0.96	0.78	0.76	0.92	0.99	0.99	0.94	0.93	0.93	0.93
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.13	0.50	0.52	0.60	0.73	0.72	0.72	0.60	0.65	0.73	0.74
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.27	0.08	0.08	0.54	0.04	0.05	0.24	0.28	0.49	0.49
Domestic moments:											
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.86	0.98	0.98	0.98	0.98	1.00	1.00	0.98	0.98	0.99	0.99
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.87	0.99	0.97	0.97	0.99	0.97	0.96	0.99	0.98	0.99	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.97	0.97	0.97	0.95	1.00	1.00	0.96	0.95	0.95	0.94
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.81	0.43	0.52	0.54	0.46	0.87	0.87	0.48	0.49	0.52	0.53
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.66	0.40	0.44	0.41	0.38	0.11	0.10	0.36	0.36	0.33	0.33
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	2.84	3.29	3.75	4.13	3.83	1.51	1.59	3.91	4.61	4.46	5.18

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. $\alpha_I = 0$ for columns (2), (5), and (8)-(11); and $\alpha_I = 1$ for columns (3)-(4) and (6)-(7). $Z_{I,nt} = 1$ for columns (2) and (5)-(11); and $Z_{I,nt} = Z_{X,nt}$ for columns (3) and (4). $Z_{X,nt}$ is set according to Table 2 for columns (2)-(4) and (8)-(9); and $Z_{X,nt} = 1$ for columns (5)-(7) and (10)-(11). $Z_{Y,nt} = 1$ for columns (2), (3), and (8); $Z_{Y,nt} = [Z_{X,nt}]^{\frac{1}{3.8-1} - \frac{1}{3.4}}$ for columns (4) and (9); and $Z_{Y,nt}$ is set according to Table 2 for columns (5)-(7) and (10)-(11).

Table G.1: Moments from benchmark calibrations of models. Bond economy.

		Benchmark calibration						Investment final good			
		Int. sector shock			Final sector shock			Int. sector shock		Final sector shock	
	Data	IRBC	Krug	Mel	IRBC	Krug	Mel	Krug	Mel	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
International moments:											
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	0.58	0.15	0.06	0.06	0.03	-0.01	-0.00	0.16	0.16	0.05	0.05
$\text{Corr}\,(C_1, C_2)$	0.36	0.17	0.12	0.11	0.05	-0.00	-0.00	0.18	0.17	0.07	0.06
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	0.14	0.01	0.01	0.02	-0.01	-0.01	0.15	0.15	0.03	0.04
$\text{Corr}\,(L_1, L_2)$	0.42	0.13	-0.01	-0.00	0.01	-0.02	-0.02	0.14	0.14	0.02	0.03
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.49										
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.32	0.89	0.86	0.88	1.00	1.00	1.00	0.90	0.91	0.99	0.99
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.81	0.89	0.86	0.88	1.00	1.00	1.00	0.90	0.91	0.99	0.99
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.13	0.65	0.67	0.67	0.70	0.71	0.71	0.65	0.65	0.69	0.69
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45										
Domestic moments:											
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.86	0.98	0.98	0.98	0.98	1.00	1.00	0.98	0.98	0.98	0.98
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.87	0.99	0.97	0.96	0.99	0.98	0.98	0.98	0.98	0.98	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	1.00	0.98	0.98	1.00	1.00	1.00	0.99	0.99	0.99	0.99
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.81	0.45	0.56	0.58	0.45	0.91	0.91	0.51	0.53	0.51	0.53
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.66	0.39	0.41	0.37	0.39	0.08	0.08	0.34	0.33	0.34	0.33
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	2.84	2.64	3.50	3.86	2.64	1.51	1.59	3.18	3.66	3.18	3.66

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. $\alpha_I = 0$ for columns (2), (5), and (8)-(11); and $\alpha_I = 1$ for columns (3)-(4) and (6)-(7). $Z_{I,nt} = 1$ for columns (2) and (5)-(11); and $Z_{I,nt} = Z_{X,nt}$ for columns (3) and (4). $Z_{X,nt}$ is set according to Table 2 for columns (2)-(4) and (8)-(9); and $Z_{X,nt} = 1$ for columns (5)-(7) and (10)-(11). $Z_{Y,nt} = 1$ for columns (2), (3), and (8); $Z_{Y,nt} = [Z_{X,nt}]^{\frac{1}{3.8-1} - \frac{1}{3.4}}$ for columns (4) and (9); and $Z_{Y,nt}$ is set according to Table 2 for columns (5)-(7) and (10)-(11).

Table G.2: Moments from benchmark calibrations of models. Financial autarky.

Moment	Data	Compl. markets IRBC $Z_{I,n}=1$	Compl. markets IRBC $Z_{I,n}=Z_{X,n}$	Bond economy IRBC $Z_{I,n}=1$	Bond economy IRBC $Z_{I,n}=Z_{X,n}$	Fin. autarky IRBC $Z_{I,n}=1$	Fin. autarky IRBC $Z_{I,n}=Z_{X,n}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
International moments:							
$\text{Corr}\,(\text{GDP}_1, \text{GDP}_2)$	0.58	0.07	−0.06	0.13	−0.00	0.11	0.06
$\text{Corr}\,(C_1, C_2)$	0.36	0.58	0.47	0.36	0.17	0.11	0.12
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	0.12	−0.19	0.34	−0.11	0.11	0.02
$\text{Corr}\,(L_1, L_2)$	0.42	−0.84	−0.38	−0.56	−0.16	0.10	−0.00
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	−0.49	0.68	0.63	0.66	0.07		
$\text{Corr}\,(\text{Exp}_1, \text{GDP}_1)$	0.32	0.93	0.91	0.98	0.77	0.89	0.86
$\text{Corr}\,(\text{Imp}_1, \text{GDP}_1)$	0.81	0.10	0.31	0.44	0.81	0.89	0.86
$\text{Corr}\,(\text{ReR}, \text{GDP}_1)$	0.13	0.68	0.67	0.66	0.62	0.67	0.67
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.23	0.14	0.14	0.05		
Domestic moments:							
$\text{Corr}\,(C_1, \text{GDP}_1)$	0.86	0.96	0.95	0.99	0.98	1.00	0.98
$\text{Corr}\,(L_1, \text{GDP}_1)$	0.87	0.86	0.97	0.93	0.98	0.99	0.98
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	1.00	0.99	0.99	0.99	1.00	0.99
$\frac{\text{Std}\,(C_1)}{\text{Std}\,(\text{GDP}_1)}$	0.81	0.74	0.44	0.82	0.51	0.90	0.54
$\frac{\text{Std}\,(L_1)}{\text{Std}\,(\text{GDP}_1)}$	0.66	0.26	0.55	0.16	0.49	0.10	0.46
$\frac{\text{Std}\,(P_{I,1}I_1/P_{Y,1})}{\text{Std}\,(\text{GDP}_1)}$	2.84	1.37	3.02	1.29	2.97	1.41	2.85

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table G.3: Standard IRBC model with investment in terms of labor.

	Interm. sector shock				Final sector shock			
	$\psi_{X,L}$		ψ_Y		$\psi_{X,L}$		ψ_Y	
	0.7	-1	0.2	-1	0.7	-1	0.2	-1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
International moments:								
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	-0.17	0.10	-0.31	0.12	-0.21	-0.18	-0.50	0.21
$\text{Corr}\left(C_1, C_2\right)$	0.25	0.62	0.19	0.74	-0.06	-0.19	-0.34	0.19
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	-0.48	-0.31	-0.70	0.01	-0.57	-0.69	-0.83	-0.20
$\text{Corr}\left(L_1, L_2\right)$	-0.35	-0.25	-0.52	-0.30	-0.28	-0.16	-0.59	0.24
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.55	-0.45	-0.66	0.62	-0.68	-0.71	-0.79	-0.56
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.20	0.47	-0.21	0.94	-0.09	-0.29	-0.53	0.42
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.92	0.93	0.91	0.49	0.94	0.90	0.92	0.95
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.64	0.58	0.68	0.44	0.72	0.75	0.79	0.62
$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.46	0.12	0.56	0.08	0.76	0.45	1.03	0.18
Domestic moments:								
$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.94	0.94	0.94	0.91	0.97	0.99	0.97	0.99
$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.99	0.98	0.99	0.97	0.99	0.99	0.99	0.99
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.97	0.96	0.95	0.99	0.96	0.94	0.95	0.97
$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.32	0.41	0.34	0.39	0.36	0.49	0.38	0.49
$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.49	0.44	0.48	0.46	0.46	0.36	0.44	0.36
$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	3.54	3.17	4.04	2.63	3.83	4.09	4.64	3.09

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,1}$, $\text{Exp}_1 = P_{X,21}X_{21}/P_{Y,1}$, $\text{Imp}_1 = P_{X,12}X_{12}/P_{Y,1}$ $\text{TB}_1 = (P_{X,1}X_1 - P_{Y,1}Y_1)/P_{Y,1}$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table G.4: Labor and final good externalities in the unified model. Compete markets.

H Discussion of Additional Results

H.1 Bond Economy and Financial Autarky

To conserve on space, in the main text we presented results for complete markets only. Here we discuss results for the bond economy and financial autarky.

Comparing Table G.1 for the bond economy with Table 3 for complete markets, we see that the results for are qualitatively very similar. The only difference is that the cyclicality is negative for the Krugman model in the case of the bond economy and the shock to the intermediate goods sector (column (3) in Table G.1). Still, the extent of countercyclicality is dramatically reduced compared to the IRBC model.

Heathcote and Perri (2002) show that compared to complete markets or the bond economy, the IRBC model under financial autarky leads to international correlations closer to the data. Financial autarky, by construction, however, cannot account for trade balance dynamics and the differential cyclicality of exports and imports. Still, for completeness, in Table G.2, we show results under financial autarky. First, while under financial autarky the IRBC model does lead to more positive international correlations in output, investment, and hours, they are still lower than the data and additionally, consumption correlation is still higher than output.[53] More importantly, and what constitutes our main point, is that the IRBC model and the Krugman and Melitz models lead to very similar moments for both shocks, even under financial autarky.

H.2 IRBC with Investment Using Labor

In Table G.3 in Appendix G, we change the canonical IRBC model with the intermediate good productivity shock such that investment is done in terms of labor only (by setting $\alpha_I = 1$). For completeness, we show two cases under that specifica-

[53] The fact that international correlations in investment and labor are lower here than in Heathcote and Perri (2002) for the IRBC model with intermediate good productivity shock is the different calibration of the elasticity of substitution between intermediate goods in production of the final good (we use $\sigma = 2$ while Heathcote and Perri (2002) use $\sigma = 0.90$). When domestic and foreign goods are complements, as with $\sigma = 0.90$, the international transmission of shocks changes nontrivially in the IRBC model irrespective of whether one considers financial autarky or complete markets.

tion. First, one in which there is no shock in the investment technology ($Z_{I,nt} = 1$), which leads to the most direct comparison with the IRBC model. Second, one in which the intermediate good productivity shock also perturbs the investment production function ($Z_{I,nt} = Z_{X,nt}$), which is what is implied by the dynamic Krugman and Melitz models. It is clear there that the trade balance is now pro-cyclical, unlike in Table 3 in the main text.

The intuition behind the results in Table G.3 is the following. If investment good is produced with labor input only, while investment certainly increases with a positive productivity shock, investment is less volatile, and it does not render net exports countercyclical because the rise in imports is much more muted. This is because imports now follow consumption closely, which is smoothed over time due to consumption smoothing incentives. This plays the key role in making net exports procyclical.

H.3 Labor and Final Good Externalities with Intermediate Sector Shock: Complete Markets

In the discussion in Section 5.3, we focused mostly on comparative statics related to capital externality, as they are the most important quantitatively in our estimation exercise. Moreover, as is clear from Table G.4, even qualitatively, while negative capital externality helps move the model closer to the data, negative labor and final good externalities do not uniformly do so. In this appendix section we discuss the transmission mechanisms for these two externalities with the intermediate good productivity shock under the complete financial markets arrangement, emphasizing the differences from capital externality.[54]

From Table G.4 we see that negative labor externality increases consumption correlation across countries, while making trade balance less volatile and also less countercyclical. Figure H.1 shows the impulse responses to this shock under complete markets where we vary only the externality in labor input, $\psi_{X,L}$. The key to

[54] In Section 3.2 of Bhattarai and Kucheryavyy (2022) we provide results for labor and final good externalities with the intermediate good productivity shock for bond economy (Figures 5 and 6). The transmission mechanisms are similar to the compete markets case. In Section 3.3 of Bhattarai and Kucheryavyy (2022) we discuss the transmission mechanisms for labor and final good externalities with the final good productivity shock.

understanding the transmission is that with negative labor externality, while the productivity process faced by the home country is also less transient, as typically there would be an increase in labor hours in future, the initial impact also shifts down. This is because, unlike the capital stock which is pre-determined, labor hours respond positively today as well. This then looks like a productivity process for the home country that has shifted downwards at every point in time, as can be seen from the path of $S_{x,1}$. Then, unlike the case of negative capital externality, the home household does not increase hours initially, which in turn means that the initial increase in investment and output also does not happen. The effect of negative labor externality is thus not as strong as that of negative capital externality in moving the co-movement of hours and investment towards positive.

Given lower GDP both on impact and in future, consumption smoothing implies that consumption drops uniformly at home with negative externality, compared to the case of no externality. This lower response of investment and consumption means that, unlike the case of negative capital externality, net exports do not become more volatile or more countercyclical.

Furthermore, as typically there would be a negative response of foreign labor hours in response to this shock, there is an endogenous correlation of home and foreign productivities.[55] This helps, at least qualitatively, with generating a less negative response of foreign investment and hours. For consumption response in the foreign country, the effects are less clear overall, because of the combination of perfect risk-sharing and the different response of hours at home when labor externality is negative compared to when capital externality is negative. Overall, consumption in the foreign country does not change its dynamic response and in fact, changes in a non-monotonic way across various levels of labor externality, as there is relatively less difference in its investment and output paths. This contributes to an increase in cross-country consumption correlation.

Next, we vary the final good externality. We see in Table G.4 that for the intermediate good productivity shock, negative final good externality increases consumption correlation across countries while making trade balance both less volatile and less countercyclical. We show detailed transmission mechanisms in

[55] The dynamic positive correlation of productivity across countries that occurs with negative capital externality however, does not happen, as is clear in Figure H.1.

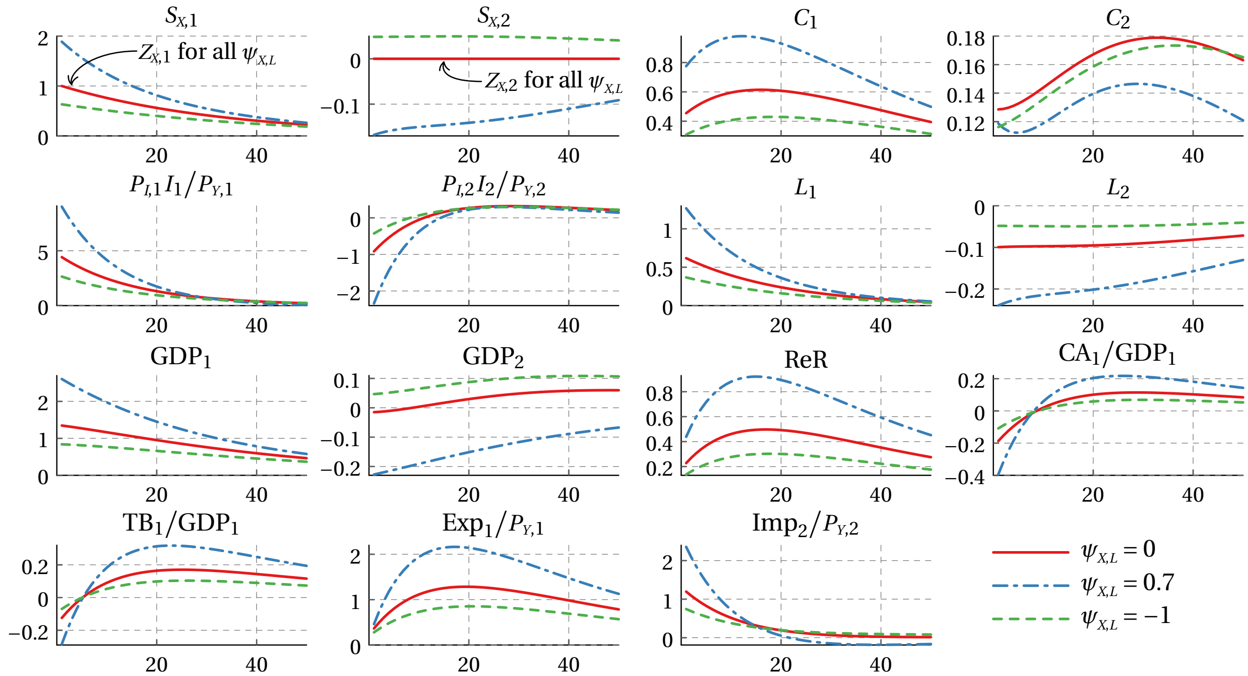
Figure H.2, where we vary only the externality in the final good aggregator technology, ψ_Y. While the overall patterns appear similar to labor externality , the channel is however, different. The reason is that this externality does not affect at all the path of productivity in the intermediate goods sector, unlike labor externality. Instead, this externality only affects endogenously the productivity in the final good sector, as can be seen in Figure H.2 from the path of $S_{Y,1}$.

To understand the direction of effects, note that the final good externality acts in terms of $(P_{Y,nt}Y_{nt})/W_{nt}$, the number of country-n's workers that produce the same value as the value of the final aggregate. Thus, we need to understand the effects of home and foreign country GDP.[56] When this externality is negative, since output increases at home with a productivity increase in the intermediate good sector, it means that productivity in the final good sector at home endogenously decreases. This effect holds both on impact and dynamically. Then the transmission that follows is as if there were a negative final good productivity shock.[57] Thus, it drives both consumption and investment at home down, compared to the case of no externality. This lower demand for the aggregate final good translates to lower production of the home intermediate good and lower home labor supply, given the low import share. Like with negative labor externality, this lower effect on investment plays an important role in making net exports less countercyclical (in this example, trade balance is positive under negative externality), and perhaps more importantly, less volatile.

In the foreign country, again unlike labor externality, there is no impact on productivity in the intermediate good sector, and the effect is only on the final good productivity. In particular, as foreign GDP increases, it endogenously has a negative effect on final good productivity. Critically, however, this negative productivity effect is much stronger for the home country compared to the foreign country. Thus, relative to no externality, this is still a positive effect for the foreign country compared to the home country. As a result, there is an increase in foreign hours and investment, compared to the no externality case.

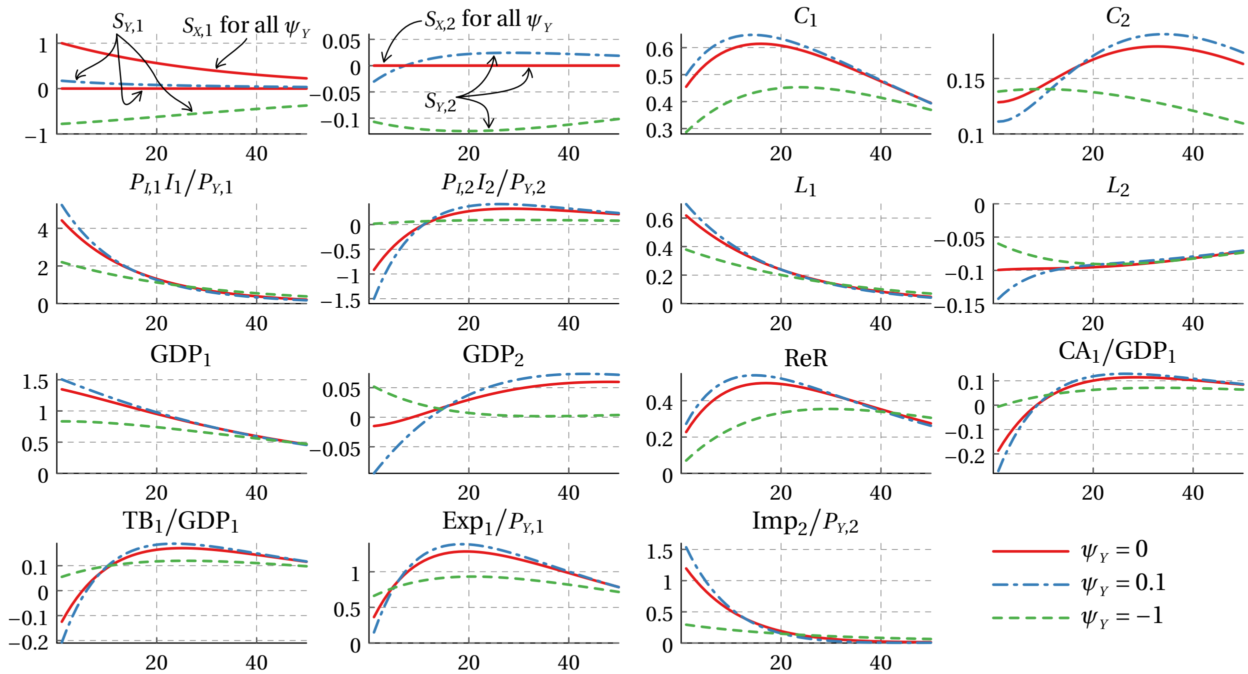
[56] For the particular case of financial autarky only, as we pointed out before, this externality term is proportional to total hours.

[57] It would thus be the inverse of the transmission for the shock we described in Figure 2.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_{X,L} = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_{X,L} = 0.7$ and $\psi_{X,L} = -1$ differ from the case with $\psi_{X,L} = 0$ only in having labor externality in the production of intermediates (with the corresponding value for $\psi_{X,L}$). The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,L} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,L}$.

Figure H.1: Impulse-response functions for $Z_{X,1}$. Labor externalities in the intermediate goods sector. Complete markets.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure the number of quarters after the shock. Vertical axes on the figures for the current account and trade balance measure the number of percentage points. Vertical axes on the rest of the figures measure percent deviation from steady state. The case with $\psi_Y = 0$ corresponds to the benchmark calibration of the unified model with no externalities. Calibrations for the cases with $\psi_Y = 0.1$ and $\psi_Y = -1$ differ from the case with $\psi_Y = 0$ only in having externality in production of the final aggregates (with the corresponding value for ψ_Y).

Figure H.2: Impulse-response functions for $Z_{X,1}$. Externality in the final aggregates sector. Complete markets.

I Additional Estimation Results

I.1 Untargeted Moments Under Best Fit

Table I.1 contains results in terms of some untargeted moments for the estimated model. Our estimated model underpredicts the volatility of exports and imports, as well as that of the real exchange rate. While the volatility of these variables is still higher than what would be obtained in the baseline IRBC model, future work can address mechanisms to further improve fit along these dimensions.

Moment	Data	Model
Std (Exp_1)	3.94	1.92
Std (Imp_1)	5.42	1.97
$\frac{\text{Std (ReR)}}{\text{Std }(\text{GDP}_1)}$	2.23	0.23

Notes: See notes for Table 5.

Table I.1: Results of the estimation of the unified model for $\sigma = 2$: untargeted moments. Complete markets.

I.2 Best Fit for IRBC

Throughout the paper, we have emphasized the key role played by negative capital externality in improving the fit of the model with the data. The channel we have highlighted is that negative capital externality endogenously decreases the persistence of productivity shocks hitting the economy, with the impact effect unchanged. To show that this channel is in fact in operation in all international business cycle models, we do two additional estimation exercises.

First, we estimate the canonical IRBC model with the intermediate good productivity shock by matching the same set of moments as in our baseline exercise. The results are in Table I.2, which show that the estimated persistence of the shock, $\rho_X = 0.37$, is much lower than the values often calibrated in the literature.

Second, for the canonical IRBC model augmented just with capital externalities,

Moment	Data	Model	Moment	Data	Model
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	0.02	$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.53
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.10	$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.94
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	-0.32	$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	1.00
$\text{Corr}\left(L_1, L_2\right)$	0.42	0.01	$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.98
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.49	-0.67	$\text{Std}\left(\text{GDP}_1\right)$	1.67	1.99
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	-0.05	$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.18
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.98	$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.58
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.35	$\frac{\text{Std}\left(P_{I,1}I_1/P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	4.20

Parameter estimates:

σ_X	ρ_X
0.013	0.37

Notes: Estimated parameters are σ_X and ρ_X. Other parameters are fixed at zero: $\psi_{X,K} = 0$, $\psi_{X,L} = 0$, $\psi_Y = 0$, $\sigma_Y = 0$, $\rho_Y = 0$. Also see notes for Table 5.

Table I.2: Results of the estimation for $\sigma = 2$: Canonical IRBC model. Complete markets.

we match the same set of moments as in our baseline exercise while fixing the persistence parameter to 0.97, the standard IRBC calibration. The results are in Table I.3 and show a negative estimate for capital externalities.

The fit on cross-country correlations is very similar across Tables I.2 and I.3, which again illustrates that the key channel through which negative capital externality improves international correlations is by endogenously decreasing the persistence of the shock. The fit is slightly better for the model with negative capital externality, as it leads to more complicated dynamics, and in particular, higher international co-movement via endogenously influencing foreign productivity.

Moment	Data	Model	Moment	Data	Model
$\text{Corr}\left(\text{GDP}_1, \text{GDP}_2\right)$	0.58	0.10	$\text{Std}\left(\frac{\text{TB}_1}{\text{GDP}_1}\right)$	0.45	0.54
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.26	$\text{Corr}\left(C_1, \text{GDP}_1\right)$	0.86	0.92
$\text{Corr}\left(\frac{P_{I,1} I_1}{P_{Y,1}}, \frac{P_{I,2} I_2}{P_{Y,1}}\right)$	0.30	-0.25	$\text{Corr}\left(L_1, \text{GDP}_1\right)$	0.87	1.00
$\text{Corr}\left(L_1, L_2\right)$	0.42	0.08	$\text{Corr}\left(\frac{P_{I,1} I_1}{P_{Y,1}}, \text{GDP}_1\right)$	0.95	0.98
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \text{GDP}_1\right)$	-0.49	-0.63	$\text{Std}\left(\text{GDP}_1\right)$	1.67	2.18
$\text{Corr}\left(\text{Exp}_1, \text{GDP}_1\right)$	0.32	0.06	$\frac{\text{Std}\left(C_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.81	0.19
$\text{Corr}\left(\text{Imp}_1, \text{GDP}_1\right)$	0.81	0.97	$\frac{\text{Std}\left(L_1\right)}{\text{Std}\left(\text{GDP}_1\right)}$	0.66	0.57
$\text{Corr}\left(\text{ReR}, \text{GDP}_1\right)$	0.13	0.36	$\frac{\text{Std}\left(P_{I,1} I_1 / P_{Y,1}\right)}{\text{Std}\left(\text{GDP}_1\right)}$	2.84	4.07

Parameter estimates:

$\psi_{X,K}$	σ_X
-3.05	0.014

Notes: Estimated parameters are $\psi_{X,K}$ and σ_X. Persistence of $Z_{X,nt}$ is from the standard IRBC calibration, $\rho_X = 0.97$. Other parameters are fixed at zero: $\psi_{X,L} = 0$, $\psi_Y = 0$, $\sigma_Y = 0$, $\rho_Y = 0$. Also see notes for Table 5.

Table I.3: Results of the estimation for $\sigma = 2$: IRBC model with capital externality and fixed persistence of productivity shock. Complete markets.