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Abstract

We consider infinite-horizon bargaining in which an uninformed seller sequentially makes a price offer to a privately informed buyer who decides whether to accept or reject it in every bargaining round. Existing theories suggest that the presence (absence) of an arbitrarily small outside option available to the buyer in this dynamic screening problem leads the seller (buyer) to enjoy a substantial surplus, and we examine the validity of the differences in the share of the surplus theoretically and experimentally. We first show that the theoretical differences collapse if an arbitrarily small fraction of optimistic buyers would believe that the sellers sometimes ask for a lower price in the subsequent rounds. We then present experimental evidence that the earnings of both the buyers and the sellers when the buyer has an outside option are not significantly different from those without the outside option. We find supporting evidence that some buyers reject the current-round offers, optimistically believing that the next offer would be more favorable to them.

JEL-Codes: C780, C910, D030.

Keywords: positive selection, outside options, laboratory experiments.

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1 Introduction

The Coase conjecture, one of the most fundamental ideas in bargaining theory, durable-good monopoly, and dynamic screening problem, proposes that the uninformed seller eventually benefits not at all from inter-temporal price discrimination among different buyer types. Consider a bargaining game between a buyer whose willingness to pay (type) is privately informed and an uninformed seller who only knows the prior distribution of buyer types. The remaining buyers at any offered price are more likely to be of low types, which leads to a *negative selection* in the demand pool. The seller then responds to cut the offering price over time. Anticipating such a price cut, even a high-type buyer tends to delay her purchase, which pushes the seller to lower the price even in the early stages to induce any purchase. As a result, the seller will charge, in effect, the lowest price all the time and earn the lowest possible expected profit in equilibrium. The idea of negative selection has been theoretically examined and confirmed by, among others, Fudenberg et al. (1985) and Gul et al. (1986).[1]

Board and Pycia (2014) (henceforth, BP) extend the bargaining game by considering an outside option available to buyers. One can think of the outside option as an alternative product from a third party. Low-type buyers tend to exercise the outside option and exit the market more quickly rather than haggling with the seller, so the remaining demand pool consists of high-type buyers. BP show that introducing the buyer's outside option overturns the Coase conjecture: The *positive selection* in the remaining demand pool drives the seller to earn profit substantially higher than what the Coase conjecture predicts. This result is surprisingly robust in the sense that the buyer's value of the outside option being positive (however close to zero) suffices to result in the (qualitatively) same bargaining outcome.[2]

Positive selection provides a channel through which the monopoly seller in the market overcomes a lack of commitment power and turns the entire consumer surplus into part of its profit. Thus, BP's result has a significant implication for the market design and regulatory policy in various markets, including durable-good monopoly, sequential auction, and lemon market, of which the dynamic screening problem is at the core. If the market designer's goal is to protect consumer surplus, then BP's result, together with the Coase conjecture, suggests that it is sufficient for the designer to prevent buyers from accessing any outside options. However, this policy implication seems

[1] The Coase conjecture was initially discussed in the context of durable-good monopoly (Coase, 1972). However, a durable-good monopoly is mathematically equivalent to the bargaining game between an uninformed seller and an informed buyer.

[2] The stark contrast led by the absence/presence of an outside option has a solid theoretical ground. BP show that the Coase conjecture fails in the unique equilibrium if the buyer's outside option is bounded away from zero. Catonini (2022) strengthens the BP's result by proving that the outcome in this unique equilibrium is the only strongly rationalizable one. For the case without an outside option, Gul et al. (1986) prove that the Coase conjecture holds in the unique equilibrium whenever the buyer's lowest possible value is strictly positive. Cho (1994) shows that the Coase conjecture holds in all rationalizable strategy profiles with the restriction that the buyer's acceptance rule is weakly stationary.

contrary to the conventional wisdom that restricting monopoly power usually makes the market more competitive and increases consumer surplus. Hence, it is crucial to obtain empirical validity of the positive and negative selection before discussing the policy implications, which justifies our approach of using controlled laboratory experiments.

The sharp contrast in theoretical predictions and its importance on the practical market design inspire our research. Consider two similar markets (A) without an outside option and (B) with a near-zero outside option. According to the Coase conjecture, Market A involves the negative selection leading to the smallest profit of the seller, while BP predict that Market B involves the positive selection leading to the substantial profit of the seller. Would this stark difference be empirically valid, even when some players are not entirely rational? To answer this question, we provide a theoretical extension that could mitigate the stark contrast and examine those theoretical predictions with experimental data.

The main driving force behind the positive selection is that the market *unravels* with the low-type buyers leaving earlier (by taking the outside option) than the high-type ones.[3] However, unraveling may not take place perfectly if (a small fraction of) players 1) lack first-order rationality such that some low-type buyers do not leave the market early, 2) lack higher-order rationality such that the seller is unsure about whether the lower-type buyers leave the market early, or both. Any of these scenarios leads the buyer to believe (either wrongly or correctly) that the seller's price in the subsequent rounds may be lower than that in the current round. To capture this intuition parsimoniously, we consider the bargaining game with a small fraction of buyers who *optimistically* believe that the seller would occasionally offer a low price in a subsequent round. We show that even when the fraction of the optimistic buyers is arbitrarily close to zero, the unraveling fails and the positive selection breaks down in an epsilon-Perfect Bayesian equilibrium that allows for a small mistake in sellers' best responses. The equilibrium predicts that the negotiation takes multiple periods with positive probability, and the price declines over time. It exhibits some features reflecting both positive and negative selection: Any rational low-type buyers exercise the outside option immediately, and any rational higher-type buyers trade with the seller, possibly after a delay. Among these buyer types, one with a higher valuation trades earlier than others.

The primary purpose of our laboratory experiment is to examine the treatment effect of the outside option. More specifically, we consider the random-termination bargaining game between an uninformed seller and an informed buyer with and without an outside option. Since the literature has provided mixed evidence for negative selection (e.g., Güth et al., 1995; Rapoport et al., 1995; Reynolds, 2000; Srivastava, 2001; Cason and Reynolds, 2005), it is crucial to note that our main objective is neither to obtain an empirical validity of the Coase conjecture nor to test the predictions

[3] More precisely, unraveling in BP begins with the observation that the lowest buyer type (with the lowest gains from trade) never receives an offer strictly more favorable than the outside option *in any equilibrium*; hence, the lowest type takes the outside option immediately. Given this, the same argument applies to the next lowest type, who has the minimum gains from trade among the remaining types.

of Board and Pycia (2014) per se, but to examine the difference in predictions with respect to the presence/absence of an outside option. We thus consider three experimental treatments, one without an outside option and two with outside options of different sizes. Our main interests are on (1) the price dynamics, (2) seller and buyer profits, and (3) the seller's beliefs about the matched buyer's valuation. If the differences of those with and without the outside option are consistent with the differences in theoretical predictions, we could further leverage our findings into the design of market policies, as it implies that the mere existence/absence of the outside option can determine who will take the lion's share of the gains from trade. However, if the differences are not present, as our model with a small fraction of optimistic buyers predicts, we would want the theoretical predictions and the policy implications thereof to be considered with caveats.

We found that the overall behaviors observed in the two experimental treatments with an outside option are qualitatively the same with each other. Thus, we combine our data from these two treatments (jointly called OutYes) and compare them with the data from the treatment without an outside option (OutNo). We have two main observations. First, the average number of bargaining round was significantly higher in OutNo than in OutYes. On one hand, this result is consistent with the predictions from the theory without buyer's optimism. On the other hand, a substantial degree of delay was also found in OutYes. It was evident that some fraction ($> 15\%$) of the buyers remains by rejecting the offer causing some delay in OutYes. This observation is inconsistent with the standard predictions but consistent with those with buyer's optimism. Second, we found that the seller's initial price offer was significantly higher in OutNo than OutYes. In both OutNo and OutYes, we observed weakly negative trends in the initial price offers, but the gradients were essentially the same.

Given that one of our main interests is to understand whether sellers believe lower-type buyers exit the market earlier or not in the presence of an outside option, we asked our seller participants to report their beliefs about the minimum buyer type after each round when the price offer was rejected. Our data reveal that the average minimum belief that the seller participants reported in OutYes was marginally larger than that in OutNo. However, the individual-level reports on the minimum buyer type in OutYes and OutNo were, by and large, the same. Moreover, we found that a substantial fraction of the reported values were even below the maximum buyer type whose dominant strategy is to take the outside option immediately and exit the market. That is, these seller participants were indeed not sure about whether the lower-type buyers would leave the market earlier or not.

The standard theory predicts that the absence of an outside option results in the negative selection in the demand pool yielding the smallest profit of the seller, while the presence of an outside option leads to the positive selection with the substantially larger profit of the seller. In our experiment, the seller's average profit turned out to be substantially and significantly higher in OutNo than in OutYes, which sharply contradicts to the main prediction from the positive selection.

Furthermore, we observed pervasive rejections in OutYes, the observation that is never predicted by the positive selection in the demand pool, nor by the inequity aversion (Fehr and Schmidt, 1999).[4] As mentioned above, however, the existence of the outside option might still be desirable because it enhances the bargaining efficiency as bargainers reach an agreement more quickly.

The observed seller's profit ranking reversal between OutNo and OutYes is not only due to the failure of positive selection in OutYes but also due to incomplete execution of negative selection in OutNo. On one hand, our data from OutNo exhibit several features qualitatively consistent with negative selection; first, the average price offers declined over rounds in OutNo whereas no such trend was found in OutYes; second, the maximum belief that the sellers reported after the price offer was rejected in OutNo was steadily decreasing, implying that the posterior belief of the seller was a right-truncation of the prior in each round. On the other hand, the average initial price offer in OutNo was substantially higher than what negative selection predicts, and both the price offer and the reported maximum belief declined too slowly compared to the theoretical predictions. In aggregate, we observed that the average seller profit in OutYes was significantly lower than the commitment profit while that in OutNo was not statistically different from the theoretical prediction from negative selection. The empirical seller's profit in OutNo being not statistically different from the theoretical level was not a consequence of the seller fully exercising the inter-temporal price discrimination but more of a combination between higher price offers and higher rejection rates than the theoretical predictions.

The key driver making the positive selection fragile is the belief that some low-type buyers may remain in the market. We acknowledge that the buyer's optimism, although consistent with what we observed in the experiment, is not the only way to explain our observations. For example, introducing an obstinate buyer type generates equilibria with inefficient delay where rational buyers mimic obstinate types to increase their payoff (Myerson, 1991; Abreu and Gul, 2000). Insufficient skepticism, frequently observed in the information disclosure experiments (Jin et al., 2021) and in the field setup (Brown et al., 2012), will also lead to a failure of unraveling in our environment. Perhaps a more direct way of explaining our data is to consider the failure of first-order rationality as mentioned earlier.[5] Thus, we view our setup with the optimistic buyers as a parsimonious workhorse model visualizing the main intuition that seller's belief about low-type buyers staying in the market will lead to a failure of the inductive process of unraveling and thus of the positive selection.

The rest of this paper is organized as follows. In the following subsection, we discuss the closely related literature. Section 2 describes the theoretical environment. In Section 3, we present a model

[4] A formal model with an inequity averse buyer and its predictions are presented in Appendix E.

[5] We explore this idea formally and present an alternative model in Appendix D in which the buyer lacks first-order rationality and may choose to delay sub-optimally only if the payoff-consequence of the sub-optimal decision is not larger than $\epsilon > 0$. We show that the ϵ-irrationality results in the outcome qualitatively identical to that from our main model with buyer's optimism even when ϵ tends to 0.

of bargaining with buyer's optimism. Section 4 describes the experimental design and procedure. The results are reported in Section 5. Other possible modelling approaches are discussed in Section 6. Section 7 concludes.

1.1 Literature Review

The Coase conjecture is originally proposed by Coase (1972), and more formal theoretical treatments of the conjecture are provided by Fudenberg et al. (1985), Gul et al. (1986), Ausubel and Deneckere (1989), among others. The idea of negative selection has played an important role from the very beginning of the development of this literature; see for example, Fudenberg et al. (1985, Lemma 1) and Ausubel et al. (2002, Lemma 1). The idea of negative selection is also the basis of not only the Coase conjecture but also dynamic screening problems in other contexts, including dynamic lemon markets (Evans, 1989; Vincent, 1989; Deneckere and Liang, 2006) and sequential auctions (McAfee and Vincent, 1997; Liu et al., 2019).

The idea of positive selection has received attention recently. Board and Pycia (2014) show that the introduction of the buyer's outside option, even if the value of the outside option is arbitrarily close to zero, overturns the Coase conjecture in the sense that a seller can earn profit substantially higher than what the Coase conjecture predicts. In the similar vein, Tirole (2016) shows that a principal can implement the outcome of profit-maximizing mechanism even without commitment power for a large class of dynamic screening problems, while the Coase conjecture implies the least profit to the principal. These results demonstrate the contrasting effect of positive selection from negative selection. While negative selection is generally harmful to the interest of the principal, positive selection leads to the best outcome for the principal.

Empirical and experimental evidence for the Coasean dynamics is, by and large, mixed. Vast majority of the previous experiments have reported evidence that contradicts the Coasean dynamics in various environments (e.g., Güth et al., 1995; Rapoport et al., 1995; Reynolds, 2000; Srivastava, 2001; Cason and Reynolds, 2005). They commonly found that initial prices are increasing in the discount factor and substantially above the static monopoly price level. Cason and Sharma (2001) and Güth et al. (2004) are two exceptions that obtain partial support for the Coase conjecture. More recently, Fanning and Kloosterman (2022) propose an experimental design that relies on subjects' private information about preferences for fairness to test the Coase conjecture. Considering two settings, an infinite horizon bargaining game and an ultimatum game, they find that, consistent with the Coase conjecture, initial offers, minimum acceptable offers, responder payoffs, and efficiency are significantly larger in the infinite horizon environment. To the best of our knowledge, no empirical or experimental evidence for positive selection has been provided, and we are the first to give a systematic experimental investigation on it.

It is not new to consider optimistic buyers in the negotiation process. Yildiz (2011) reviews the drivers of costly delays in negotiations and claims that the deadline effect and the learning about bargaining power would make optimistic players delay the agreement to a later period. The second-order optimism—belief about the other party's optimism toward her prospects—could also play a role in causing a delay (Friedenberg, 2019). Li and Wong (2009) show that mutual optimism is a source of delayed agreement and substantial efficiency loss in the Rubinstein bargaining environment. The role of optimism in negotiation process is also well supported by empirical evidence. In the context of medical malpractice lawsuits, patients are more optimistic than doctors when the injury is severer (Merlo and Tang, 2019). For pretrial settlement negotiations, the involving parties' optimism about the judge's decision may lead to a substantial delay (Vasserman and Yildiz, 2019). We exploit the idea of optimism to the assumption that some buyers expect to receive a better offer in a later bargaining round.

2 Theoretical Background

2.1 Model

Environment Consider a price negotiation between a seller (she) and a buyer (he) over infinite-horizon discrete time (period) $n = 0, 1, 2, \ldots$. The seller holds an indivisible good for sale, whose value to himself is normalized to zero. The buyer's value of the good (i.e., the buyer's type) $v \in V = \{v_1, v_2, \ldots, v_N\}$ is his private information, and the seller holds the prior belief that $v = v_j$ with probability $f(v_j) > 0$. Let $F(v) := \sum_{v' \leq v} f(v')$ denote the cumulative distribution function. We order the buyer types in V by $0 < v_1 < v_2 < \ldots < v_N$ and denote the lowest and the highest possible buyer type as $\underline{v} \equiv v_1$ and $\overline{v} \equiv v_N$, respectively.

The buyer also has an outside option that he can exercise anytime during the negotiation. Throughout this paper, we assume that the value of the outside option is *type-independent* and worth w to all buyer types. Further, we focus on the two special cases such that $w > 0$ and $w = -\infty$.

In each period $n \geq 0$, the seller offers a price $p_n \geq 0$, and then the buyer decides to accept p_n (trade), exercise the outside option (opt-out), or delay. If the buyer accepts p_n, the negotiation ends with final payoffs $e^{-rn\Delta}p_n$ and $e^{-rn\Delta}(v - p_n)$ for the seller and the buyer respectively, where $r > 0$ and $\Delta > 0$ are the common discounting rate and the time duration between consecutive periods. If the buyer exercises the outside option in period n, the negotiation ends with final payoffs $e^{-rn\Delta}w$ for the buyer and zero for the seller. If the buyer chooses to delay, the negotiation moves on to the next period, and then the two parties repeat the same bargaining protocol. Both parties obtain zero payoff if they fail to reach any agreement forever.

If $w = -\infty$, the buyer will never exercise the outside option as he can guarantee himself at least zero payoff by delaying the negotiation indefinitely; hence, the assumption $w = -\infty$ is equivalent to the assumption that the buyer has no outside option at all. Define each buyer type's net-value, denoted by $u(v)$, as the difference between v and the autarky payoff that the buyer can guarantee himself regardless of which strategy the seller employs:

$$u(v) := \begin{cases} v & \text{if } w = -\infty \\ v - w & \text{if } w > 0. \end{cases} \tag{2.1}$$

We may interpret $u(v)$ as the gains from trade, and also note that the buyer will never accept p_n higher than her net-value $u(v)$. We assume $u(v) \geq 0$ for all $v \in V$ without loss.[6]

Strategies and Equilibrium The equilibrium concept is *perfect Bayesian equilibrium (PBE)*. Here, we introduce some notations useful to describe equilibrium strategies and beliefs. Suppose that the buyer has rejected the seller's offers $p_0, p_1, \ldots, p_{n-1}$ and continues the negotiation in period n. We generically denote such a history of the game by $h_n = (p_0, p_1, \ldots, p_{n-1})$, while h_0 refers to the null history. For any $n \geq 1$, let $H_n = [0, \infty)^n$ denote the set of possible histories up to the beginning of period n, and let $H = \bigcup_{n \in \mathbb{N}} H_n \cup \{h_0\}$ denote the set of all possible histories.

For any $p \geq 0$, $h_n \in H$, and $v \in V$, $\sigma^B(p|h_n, v)$ generically denotes the behavioral strategy of a buyer type v in response to the seller's offer p at h_n, where $\sigma^B(p|h_n, v)[T]$, $\sigma^B(p|h_n, \theta)[O]$, and $\sigma^B(p|h_n, v)[D] \in [0, 1]$ refer to probabilities that the buyer of type v chooses T (accept to trade at p), O (exercise the outside option), and D (delay), respectively. $\sigma^S(h_n) \in \triangle(\mathbb{R}_+)$ denotes the seller's behavioral strategy at h_n. $f^S(v|h_n) \in [0, 1]$ denotes the seller's posterior belief that the buyer's type is v, and $F^S(v|h_n) = \sum_{v' \leq v} f^S(v'|h_n)$ denotes the corresponding distribution function.

$\sigma = (\sigma^B, \sigma^S, f^S)$ generically denotes a PBE assessment, and let $\mathcal{E}_w(\Delta)$ denote the set of all PBEs.[7] We also follow the convention that $\text{supp}(\sigma^S(h_n))$ and $\text{supp}(f^S(h_n))$ denote the supports of $\sigma^S(h_n)$ and $f^S(h_n)$. Finally, $\overline{v}^\sigma(h_n) := \max\{v \in V : f^S(v|h_n) > 0\}$ and $\underline{v}^\sigma(h_n) := \min\{v \in V : f^S(v|h_n) > 0\}$ denote the highest and the lowest buyer types in $\text{supp}(f^S(h_n))$.

Full Commitment Benchmark For a benchmark, suppose that the seller can fully commit to any price path before the negotiation commences. Following the standard lines, we can show that it is optimal for the seller to commit to the following single price indefinitely:

$$p_w^* \in \arg\max_{p \geq 0} \underbrace{\left\{ p \sum_{v: u(v) \geq p} f(v) \right\}}_{:= \Pi(p)}. \tag{2.2}$$

[6]For any buyer type with $u(v) < 0$ (if any), it is a strictly dominant strategy to exercise the outside option immediately. Thus, the presence of such a buyer type does not make any difference on the equilibrium strategies of other players. In our experiment, we consider a treatment (Out60) in which some buyer types indeed have negative net-value. Theory predicts that those buyer types exercise the outside option in period 0, while all other players behave as discussed in this section.

[7]Our notation suppresses the dependence of $\mathcal{E}_w(\Delta)$ on other parameters (e.g., F and r) which we keep fixed throughout the analysis.

Faced with this price path, the buyer accepts p_w^* in period zero if and only if $u(v) \geq p_w^*$. The buyer types with $u(v) < p_w^*$ are excluded by the seller despite positive gains from trade. If $w > 0$, these excluded buyer types exercise the outside option in period zero; if $w = -\infty$, they continue the negotiation indefinitely by rejecting $p_{-\infty}^*$ in all periods. In either case, the seller exercises no inter-temporal price discrimination. We call this outcome *the full-commitment benchmark*.

In what follows, we additionally impose the following regularity assumption. Because the type space $V = \{v_1, v_2, \ldots, v_N\}$ is discrete, the seller can restrict herself to choose a price from $\{u(v_1), u(v_2), \ldots, u(v_N)\}$ in solving the optimization problem (2.2). After we restrict the domain to $\{u(v_1), u(v_2), \ldots, u(v_N)\}$ without loss, it is required that the objective function in (2.2) is single-peaked around $p = p_w^*$.

Assumption 1. *There is $u^* \in \{u(v) : v \in V\}$ such that $\Pi(u(v_{j-1})) < \Pi(u(v_j))$ for all j such that $u(v_j) \leq u^*$ and $\Pi(u(v_j)) > \Pi(u(v_{j+1}))$ for all such that $u(v_j) \geq u^*$.*

Assumption 1 holds, for example, if V is equally gridded in the sense that $V = \{\underline{v} + g(k-1) : k = 1, 2, \ldots, N\}$ for some $g > 0$, and the seller's prior f is the uniform distribution over V. Note that Assumption 1 also guarantees that the optimization problem admits a unique solution. Let

$$\Pi_w^* := \max_{p \geq 0} \Pi(p) = p_w^* \sum_{v : u(v) \geq p_w^*} f(v)$$

denote the seller's profit in the full-commitment benchmark outcome. Clearly, the seller cannot earn strictly more than Π_w^* in any PBE.

Proposition 1. *The following holds in the full-commitment benchmark outcome:*

(i) No Inter-temporal Pricing: The seller offers the same price p_w^* in all periods.

(ii) No Delay: All trade (if any) must occur in period zero.

(iii) Exclusion: The seller does not trade with the buyer types whose $u(v)$ is smaller than p_w^*.

To exclude trivial cases, we further assume $u(\underline{v}) < p_w^*$, so the exclusion occurs with non-zero probability.[8] Without this assumption, (i) the full benchmark outcome coincides with the first best, and (ii) the seller can easily achieve this outcome by offering $p_0 = u(\underline{v})$ in period zero even without the full commitment power; hence, the bargaining problem becomes trivial with or without the seller's commitment power.[9]

[8] This assumption holds if, for example, f is the uniform distribution such that $u(\underline{v}) < 2u(\overline{v})$.

[9] All buyer types will accept $p_n = u(\underline{v})$ after any history in any PBE; see, for example Fudenberg et al. (1985, Lemma 2) and Board and Pycia (2014, Lemma 1). Hence, the seller's expected profit is necessarily no less than $u(\underline{v})$ after any history in any PBE.

2.2 Bargaining with No Outside Option

Now we turn to the case in which the seller cannot commit to future prices. First, consider the case that the buyer has no outside option (i.e., $w = -\infty$). It is intuitively appealing to conjecture that the seller *moves down the demand curve* $1 - F$ in equilibrium. Higher buyer types enjoy a higher payoff from consuming the good, hence a delay of purchase is more costly for them. As a result, high buyer types are more eager to purchase and end the negotiation earlier, inducing *negative selection* in the demand pool.

This intuition is indeed correct. Fix an arbitrary PBE σ, and suppose that the seller offers p_n in period n after a history h. Also, suppose that a buyer type v is willing (at least indifferent) to accept this offer, given his expectation about the future offers. One can prove that any buyer type $v' > v$ also *strictly* prefers to accept p_n; hence, all buyer types higher than v must trade. This observation, known as Skimming Property in the literature, implies that (i) $\overline{v}^\sigma(h)$ monotonically decreases over time, and (ii) the seller's posterior f^S is always a right-truncation of the prior belief f. Its proof can be found in Fudenberg et al. (1985).

Proposition 2 (Negative Selection)**.** *Suppose $w = -\infty$. The following holds in any $\sigma \in \mathcal{E}_w(\Delta)$:*

(i) $\overline{v}^\sigma(h,p) \leq \overline{v}^\sigma(h)$ after any history h and a price p rejected immediately after h.

(ii) f^S is always a right-truncation of f: After any history h on the equilibrium path, there is $M \in (0,1]$ such that $f^S(v|h) = f(v)/M$ for any $v < \overline{v}^\sigma(h)$.[10]

Negative selection in the demand pool keeps pushing $\overline{v}^\sigma(h)$ downward. The seller's posterior belief becomes more pessimistic (about the gains from trade) and hence lowers her offer period after period. In other words, the seller exercises inter-temporal price discrimination in equilibrium. Under the assumption that $u(\underline{v}) = \underline{v} < p^*_{-\infty}$, the negotiation takes multiple periods with positive probability. The proof of the following proposition can be found in, for example, Gul et al. (1986).

Proposition 3. *Suppose $w = -\infty$ and $u(\underline{v}) = \underline{v} < p^*_{-\infty}$.*

(i) Delay: In any PBE, the negotiation takes over multiple periods with positive probability.

(ii) Inter-temporal Pricing and Negative-selection: The seller's offer strictly declines toward $\underline{v}$ over time on the path of any PBE. All buyer types trade with the seller eventually, where higher buyer types trade earlier at a higher price than low buyer types.

[10] More precisely, after any history $h_n = (\tilde{p}_0, \ldots, \tilde{p}_{n-1})$,

$$f^S(v|h_n) = \frac{f(v)}{\sum_{v' < \overline{v}^\sigma(h)} f(v') + f(\overline{v}^\sigma(h)) \prod_{k=0}^{n-1} \sigma^B(\tilde{p}_k | h_k, \overline{v}^\sigma(h_n))[D]} \quad \forall v < \overline{v}^\sigma(h_n)$$

where h_0 denotes the null history, and $h_k = (\tilde{p}_0, \ldots, \tilde{p}_{k-1})$ denotes a truncation of h_n, respectively. In the denominator, $\prod_{k=0}^{n-1} \sigma^B(\tilde{p}_k | h_k, \overline{v}^\sigma(h_n))[D]$ stands for the probability that the buyer type $\overline{v}^\sigma(h)$ has rejected all $\tilde{p}_0, \ldots, \tilde{p}_{n-1}$.

(iii) *Coase Conjecture: There is an integer $L \in \mathbb{N}_0$ such that, in any $\sigma \in \mathcal{E}_{-\infty}(\Delta)$, all buyer types trade with the seller in period L or before, and the seller's offer p_n on the equilibrium path is bounded by*

$$\underline{v} \leq p_n \leq (1 - e^{-rL\Delta})\overline{v} + e^{-rL\Delta}\underline{v} \quad \forall n \leq L.$$

Moreover, L is independent of Δ.

The Coase conjecture implies that the bargaining process ends in the "twinkling of an eye" as $\Delta \to 0$. Consequently, the seller's benefit from inter-temporal price discrimination is completely undermined. Indeed, the upper bound in Proposition 3-(iii) converges to $\underline{v}$ as $\Delta \to 0$, hence, the seller's equilibrium profit as well as equilibrium offers converge to $\underline{v}$ in the limit. Furthermore, the duration of the bargaining also converges to zero, and thus the first-best outcome is achieved in the limit.

The bargaining game generically admits a unique PBE in the following sense. Fix an arbitrary probability mass function f over V. For any $-1 < \epsilon < 1$, let f_ϵ denote the perturbation of f such that

$$f_\epsilon(\overline{v}) = (1 - \epsilon)f(\overline{v}) + \epsilon \quad \text{and} \quad f_\epsilon(v) = (1 - \epsilon)f(v) \quad \forall v \neq \overline{v}.$$

Then, there is $\bar{\epsilon} > 0$ such that all PBEs in $\mathcal{E}_{-\infty}(\Delta)$ induce the identical outcome whenever the seller's prior belief belongs to $\{f_\epsilon : -\bar{\epsilon} < \epsilon < \bar{\epsilon}\}\backslash\{f\}$. Moreover, even when the bargaining game admits multiple PBEs, all buyer types play the same strategies on the path and the seller's expected equilibrium payoff is identical across all PBEs (Fudenberg et al., 1985; Gul et al., 1986).

2.3 Bargaining with Outside Option

Now suppose that $w > 0$. Proposition 2 does not hold in this case; in particular, $\underline{v}^\sigma(h)$ is not necessarily $\underline{v}$. Lower buyer types have smaller gains from trade, and hence they tend to exit the market earlier than higher buyer types by exercising the outside option, especially when the seller is expected to insist on high prices during the negotiation. The possibility of such *positive selection* changes the PBE outcome dramatically. For example, consider the following strategies profile.

- The seller offers p_w^* in every period after any history, with the posterior belief $f^S(h) = f$.
- Suppose that the seller offers p_n in period n (after any history). Buyer types $v < p_w^* + \frac{w}{e^{-r\Delta}}$ accept p_n iff $v - p_n \geq w$ and exercises the outside option otherwise. Buyer types $v \geq p_w^* + \frac{w}{e^{-r\Delta}}$ accept p_n iff $v - p_n \geq e^{-r\Delta}(v - p_w^*)$ and delay otherwise.

It is straightforward to check that this strategy profile is a PBE. In this PBE, the seller could *exactly* implement the full-commitment benchmark outcome, starkly different from the case without the buyer's outside option. Moreover, Board and Pycia (2014) prove that this is an essentially unique PBE.[11]

Proposition 4 (Board and Pycia 2014)**.** *Suppose $w > 0$. The full-commitment benchmark outcome is induced in the essentially unique equilibrium.*

We can understand this result with the language of positive and negative selection. In the essentially unique PBE, the possibility of positive selection allows the seller to resist any temptation of price cut, holding sufficiently optimistic belief about the remaining gains from trade. More formally, note that the lowest possible gains from trade is $\underline{v} - w$ among the buyer types in V. As a result, the seller will never offer strictly lower than $\underline{v} - w$ after any history in any PBE (Board and Pycia, 2014, Lemma 1).

This observation implies that the lowest buyer type's payoff from delaying the negotiation is at most

$$e^{-r\Delta} \max\{\underline{v} - (\underline{v} - w), w\} = e^{-r\Delta} w,$$

where $\underline{v} - (\underline{v} - w)$ captures the case that the seller offers $\underline{v} - w$ in the next period, and w captures the case that the buyer exercises the outside option in the next period. But $e^{-r\Delta} w \leq w$ hence it is always better for the lowest type to exercise the outside option in period zero than any delay. Consequently, positive selection would occur immediately, at least for the low type, whenever the seller attempts any price cut.

But this is not the end of the story. Suppose the buyer types $[\underline{v}, \underline{v} + \xi]$ have exited in the first period by the argument above, where $\xi > 0$ is a small positive number. In the next period, $v = \underline{v} + \xi$ becomes the lowest buyer type in the seller's posterior belief. The same argument shows that the seller never offers lower than $p_n = (\underline{v} + \xi) - w$ in the continuation game, which means that the buyer type $v = \underline{v} + \xi$ can never hope for a payoff larger than w from continuing the negotiation; this buyer type will also find it optimal to exercise the outside option immediately. We can repeat the same argument to obtain the following conclusion: For any $\tilde{v} \in V$, if the seller trades with the buyer types in $[\tilde{v}, \overline{v}]$ in period zero, all buyer types in $[\underline{v}, \tilde{v})$ will find it optimal to exercise the outside option immediately. Hence, neither price cut nor delay occurs in any PBE, which makes the above PBE unique.

Proposition 4, especially its prediction of no delay, is robust in several ways. Note that the above argument—buyers either accept the offer or exercise the outside option immediately—applies to the equilibrium play in any period (not only period zero) after any negotiation history (both on and off the equilibrium paths). Therefore, all buyer types never choose to delay in response to *any price offer* by the seller *after any history*; see the proof of Proposition 1 in BP. An immediate

[11] Board and Pycia (2014) consider a more general case with type-dependent outside options. An equilibrium is said to be essentially unique if each type of players achieves the exactly same expected payoff in all other equilibria (if any).

implication is that the negotiation always ends in period zero with probability 1, even in the case that the seller trembles hands and mistakenly chooses a non-equilibrium offer.

Proposition 4 also remains to hold under a more flexible solution concept. Recently, Catonini (2022) proves that the full-commitment benchmark outcome is induced in every *strongly rationalizable* strategy profile. In other words, the assumption of rationality and common *strong* belief in rationality (RCSBR) suffices to predict that neither price cut nor delay ever occurs in negotiation with the buyer's outside option $w > 0$. It implies that we can dispense with the assumption that all the players (i.e., both the buyer and the seller) hold rational expectations about the opponents' strategies.

We close this section with the discussion on how the bargaining outcome changes as we gradually change w. First, note that a small change in $w > 0$ does not result in a qualitative change in the essentially unique equilibrium outcome as long as w remains strictly positive (however small it is); the only change is that the seller's offer p_w^* continuously varies with w. Next, for any negative but finite $w \in (-\infty, 0)$,[12] the buyer will never exercise the outside option as he can obtain a strictly higher payoff from delaying the negotiation forever. Hence, the case with a negative outside option effectively equivalent to the case with no outside option (however small its magnitude is); the generically unique equilibrium outcome does not respond to any change in $w \in (-\infty, 0)$. What if w is exactly zero? In this case, there are multiple PBEs including the two PBEs described in Propositions 3 and 4. In this sense, there is a discontinuity regarding the uniqueness of equilibrium but a continuity regarding existence of each equilibrium at $w = 0$ from both directions.

3 Optimism in Bargaining with Outside Option

3.1 Bargaining Model with Optimistic Buyer Types

In this section, we show that the stark contrast between the two cases with and without the buyer's outside option disappears when we introduce the buyer's optimism about the bargaining process. To illustrate this idea, we modify the negotiation game as follows. First, extend the buyer's type space to

$$\Theta := V \times \{\tau_r, \tau_o\},$$

where a finite set $V \subset (0, \infty)$ represents the support of the buyer's valuation of the seller's good as in the last section. We respectively denote the highest and the lowest valuation in V by $\overline{v}$ and $\underline{v}$ and also denote a generic element in Θ by $\theta = (v, \tau)$.

Each buyer type with $\tau = \tau_r$ holds *correctly specified (rational) model* about the seller's behavior in the negotiation: The seller will play as predicted by a standard equilibrium notion (which will

[12] The payoff from exercising the outside option can be negative if it takes non-negligible search and/or transportation cost to identify and exercise the outside option, even though the value of the outside option itself is positive.

be defined shortly). On the other hand, each buyer type with $\tau = \tau_o$ has the following *misspecified model*: In every period,

- the seller offers $p^\dagger \in [0, \infty)$ with probability $\eta \in (0,1)$, regardless of which equilibrium strategy the seller prepares, and
- the seller plays her equilibrium strategy with probability $1 - \eta \in (0,1)$.[13]

We focus on the case that $p^\dagger$ is sufficiently small (see Assumption 3 for the exact condition), so that this misspecified model makes more optimistic predictions about the future bargaining process than the correctly specified one. For simplicity, we assume that both η and $p^\dagger$ are common across all buyer types with $\tau = \tau_r$.

The buyer types with $\tau = \tau_o$ are called *optimistic types* while the buyer types with $\tau = \tau_r$ are called *rational types*. The set of all rational buyer types and the set of all optimistic buyer types are denoted by $\Theta_r = \{(v, \tau) \in \Theta : \tau = \tau_r\}$ and $\Theta_o = \{(v, \tau) \in \Theta : \tau = \tau_o\}$, respectively. The realizations of v and τ are assumed to be stochastically independent. For any finite support V being fixed, let $f_V(v)$ denote the seller's prior probability that the buyer's valuation is v, and let $\phi \in [0,1]$ denote the prior probability that the buyer is an optimistic type. We write the joint probability mass function as $f_\Theta(\theta) = f_\Theta(v, \tau)$, hence $f_\Theta(v, \tau_o) = f_V(v)\phi$ and $f_\Theta(v, \tau_r) = f_V(v)(1 - \phi)$.

The negotiation procedure is identical to that presented in the previous section. The buyer still has a type-independent outside option. In this section, we focus on the case $\underline{v} \geq w > 0$ without loss.[14] Each optimistic buyer type $\theta = (v, \tau_o)$ (mistakenly) believes that he can always guarantee a continuation payoff

$$\sum_{k=1}^{\infty} e^{-rk\Delta}(1-\eta)^k \eta (v - p^\dagger) = \frac{e^{-r\Delta}\eta}{1 - e^{-r\Delta}(1-\eta)}(v - p^\dagger)$$

by waiting for the seller to offer $p^\dagger$. Hence, we may define the net-value of each optimistic buyer type as

$$u(\theta) := v - \max\left\{\frac{e^{-r\Delta}\eta}{1 - e^{-r\Delta}(1-\eta)}(v - p^\dagger), w\right\} \quad \forall \theta = (v, \tau_o) \in \Theta_o.$$

We continue to define the net-value of each rational buyer type as (2.1) in the last section.

Provided that $\phi \geq 0$ is sufficiently close to zero, the presence of the optimistic buyer type does not alter the full-commitment benchmark qualitatively. With full commitment power, it is optimal for the seller to ignore the negligible portion of the optimistic type and to insist on a single price

[13] In other words, all buyer types with $\tau = \tau_o$ incorrectly believe the following. At the beginning of each period $n \geq 0$, nature draws number u from a uniform random variable $U[0,1]$. If $u \leq \eta$ the seller is *forced* to offer $p_n = p^\dagger$. If $u > \eta$, the seller may choose any price p_n as she wishes.

[14] The main result in this section (Proposition 6) easily extends to the case of $w = -\infty$ with minor changes in the proof.

that maximizes the profit from the rational buyer type, which is given as follows:

$$\Pi_r(p) := p \sum_{\theta \in \Theta_r : u(\theta) \geq p} f_\Theta(\theta).$$

We continue to denote the benchmark price that maximizes $\Pi_r(p)$ as p_w^* and the seller's full-commitment profit level (i.e., the payoff from insisting on p_w^*) as Π_w^*, respectively. We also maintain the following two assumptions.

Assumption 2. *There is $u^* \in \{u(v, \tau_r) : v \in V\}$ such that the following two conditions hold:*

(i) Π_r is single-peaked: For any u', u'' with $u' < u'' \leq u^*$, $\Pi_r(u') < \Pi_r(u'')$. For any u', u'' with $u^* \leq u' < u''$, $\Pi_r(u') > \Pi_r(u'')$.

(ii) $u(\underline{v}, \tau_r) < u^*$.

Assumption 3. $0 \leq p^\dagger < p_w^*$.

Assumption 2-(i) requires Π_r to be single-peaked in the essentially same way we assumed Π in the previous section (Assumption 1), and hence p_w^* is well-defined whenever ϕ is sufficiently small. Assumption 2-(ii) allows us to avoid the case that the full-commitment benchmark outcome induces trading with all rational buyer types. Assumption 3 requires that the optimistic buyer type finds $p^\dagger$ strictly more favorable than the full-commitment benchmark price p_w^*. Note that, once other parameters such as $p^\dagger$, w, η, Δ, and r being fixed, p_w^* and Π_r are determined by the seller's prior (V, f_V, ϕ). In this sense, Assumptions 2 and 3 are the conditions imposed on the seller's prior belief.

3.2 Equilibrium Concept

In this section, we employ *a version of ϵ-PBE* as the equilibrium concept. A formal definition of ϵ-PBE requires additional notations. First of all, we inherit all the notations for equilibrium strategies and beliefs (i.e., σ^B, σ^S, and f^S) from Section 2.1. For any assessment $\sigma = (\sigma^B, \sigma^S, f^S)$, history $h_n \in H$, and offer $p \geq 0$, define $V_S^\sigma(p; h_n)$ as the seller's continuation payoff from a one-shot deviation to p at h_n. Also, define

$$P^D(h_n; \sigma) := \{p \geq 0 : \sigma^B(p|h_n, \theta)[D] > 0 \text{ for some } \theta \in \operatorname{supp}(f^S(\theta|h_n))\}$$

as the collection of offers in response to which some remaining buyer types will choose to delay. We call the offers in $P^D(h_n; \sigma)$ *delay-inducing* (at history h_n).

Next, a seller's strategy σ^S is called *ϵ-best at $h_n \in H$*, where $\epsilon \geq 0$, if and only if the following inequality holds:

$$V_S^\sigma(p'; h_n) \leq V_S^\sigma(p; h_n) + \epsilon \mathbb{1}\{p \text{ or } p' \in P^D(h_n; \sigma)\} \quad \forall p \in \operatorname{supp}(\sigma^S(h_n)) \text{ and } p' \geq 0. \tag{B}$$

A strategy σ^S is called ϵ-best if it is ϵ-best at any histories.[15] Note that the condition (B) allows a margin of error *only at a subclass of histories*, and in this sense, our notion of ϵ-best strategy is closer to a fully optimal strategy than alternatives that allow a small error *at any history.* Precisely, it allows a small margin of improving the seller's continuation payoff for the case that either p or p' is delay-inducing; the condition coincides with the standard best-response condition for all other cases. This asymmetric treatment to the delay-inducing price offers is motivated by the observation that a precise assessment of $V_S^\sigma(p; h_n)$ requires a more demanding cognitive capacity (e.g., a prediction of the buyer's future behaviors) when the seller expects further delay and haggling.

Now we are ready to define our notion of ϵ-PBE. The definition of ϵ-PBE is identical to the definition of the standard PBE except that the seller is only required to play an ϵ-best strategy. In any ϵ-PBE, as in the standard PBE, all buyer types exactly play the optimal strategies after all histories.[16] Finally, the seller's posterior belief is also rationally obtained by Bayes rule whenever possible. For any $\epsilon > 0$, $\Delta > 0$, and the seller's prior belief (V, f_V, ϕ), let $\mathcal{E}(\epsilon|\Delta, V, f_V, \phi)$ denote the set of all ϵ-PBEs. Note that our notion of ϵ-PBE includes the standard PBE as a special case.[17]

3.3 Equilibrium Analysis

In this subsection, we investigate the ϵ-PBEs under Assumptions 2 and 3. Fix $p^\dagger \geq 0$, $\eta \in [0,1]$, $w > 0$, and $r > 0$ throughout the section, and let $\mathcal{F}$ denote the collection of the seller's prior beliefs, generically denoted by (V, f_V, ϕ), such that (i) $|V| < \infty$, (ii) $\phi \in [0,1]$, and (iii) Assumptions 2 and 3 hold. We also maintain the assumption $\underline{v} = \min V \geq w$ without loss (see footnote 6).

We first consider the case with $\phi = 0$. Note that the model coincides with the one discussed in the last section in this case (i.e., the model with all the buyer types being rational). The following proposition shows that all the key theoretical predictions discussed in the last section continue to hold. Recall that h_0 generically denotes the null history of the game.

Proposition 5. *Fix $\epsilon > 0$, $\Delta > 0$ and $(V, f_V, \phi) \in \mathcal{F}$ such that $\phi = 0$. Then, there is a unique ϵ-PBE*

[15] Recall that the left-hand side of the inequality in Condition (B) represents the payoff from a *one-shot deviation*. Instead, we may require the right-hand side of the inequality is no less than the payoffs from *multi-shot deviations*. Both Propositions 5 and 6 remain to hold with such an alternative formulation.

[16] More precisely, the optimistic types choose the optimal strategies (i.e., the best response to the seller's strategy) under the misspecified model as discussed above. All rational types choose the optimal strategies under the correct model.

[17] ϵ-PBE is employed to resolve a technical issue due to the multi-dimensional type space. If all buyer types are rational (hence, the type space is one-dimensional) as in the previous section, the order of purchase is monotonic in buyer's type regardless of the seller's strategy in the sense that a high-value buyer type *always* accept the seller's offer earlier than lower buyer types. In contrast, with the two-dimensional type space $\Theta = V \times \{\tau_r, \tau_o\}$, the order of purchase among buyer types is endogenous to seller's strategy. The full equilibrium analysis thus requires to inspect the effect of each deviation by the seller on the order of purchase, which imposes substantial technical challenges. The use of ϵ-PBE as the equilibrium concept eases this issue by allowing us to disregard the optimistic type's purchase as its impact on the seller's profit is limited when ϕ is small. Also note that a similar technical issue arises in the multi-dimensional mechanism design literature where the set of binding incentive-compatibility constraints depends on the mechanism's rule. See, e.g., Rochet and Stole (2003).

in $\mathcal{E}(\epsilon|\Delta, V, f_V, \phi)$. Furthermore, in this unique ϵ-PBE:

(i) The seller earns the full-commitment benchmark profit Π^*_w.

(ii) There is a cutoff $v^* \in V$ such that all buyer types with $v \geq v^*$ trade, and all other buyer types exercise the outside option in period 0.

(iii) No delay occurs after any offer $p_0 \in [0, \infty)$ by the seller in period 0 (i.e., $P^D(h_0; \sigma) = \varnothing$ in the unique ϵ-PBE σ).

Now we turn our attention to the case with $\phi > 0$. Our main result shows that, in the presence of optimistic buyer types, there is an ϵ-PBE such that the seller practices the inter-temporal price discrimination as in the case of no outside option (Section 2.2). We refer to such equilibrium as *quasi-Coasean equilibrium.*

Definition 1. *An ϵ-PBE is called a quasi-Coasean equilibrium if the following outcome is induced on its path:*

- *Delay and Inter-temporal Pricing: The negotiation takes multiple periods with positive probability, and p_n declines over time on the equilibrium path.*
- *There is $v^* \in V$ such that the following holds on the equilibrium path:*
  - *(i) Positive selection: any rational buyer type $\theta = (v, \tau_r)$ such that $v < v^*$ exercises the outside option immediately, and*
  - *(ii) Negative selection: any rational buyer type $\theta = (v, \tau_r)$ such that $v \geq v^*$ trades with the seller (possibly after a delay). Among these buyer types, one with a higher valuation v trades earlier than others.*

The observed negotiation process in a quasi-Coasean equilibrium is a mixture of the two equilibrium plays described in Propositions 3 and 4, featuring both positive and negative selection as well as inter-temporal price discrimination and exclusion. A chunk of buyer types in $\{(v, \tau_r) : v < v^*\}$ exercise the outside option in period zero. In addition, the seller practices inter-temporal price discrimination for the remaining buyer types in $\{(v, \tau_r) : v \geq v^*\}$, where high buyer types trade earlier among these buyer types. Note that the PBE discussed in Section 2.2 (for the case $w = -\infty$) satisfies all the conditions in the definition of quasi-Coasean equilibrium. On the other hand, quasi-Coasean equilibria differ from the equilibrium discussed in Section 2.3 (for the case $w > 0$) in which no price discrimination occurs between the non-excluded buyer types. Furthermore, any quasi-Coasean equilibrium fails to achieve the full-commitment benchmark outcome.

The next proposition identifies the condition under which a quasi-Coasean equilibrium exists for the case with $w > 0$. For any $(V, f_V, \phi) \in \mathcal{F}$, let $\overline{f}_V := \max\{f_V(v) : v \in V\} > 0$ denote the maximum value that the probability mass function $f_V(v)$ may take.

Proposition 6. *Fix* $\epsilon > 0$. *Then, there is* $\bar{\phi}(\epsilon) \in (0,1)$ *such that the following holds for any* (V, f_V, ϕ) *with* $0 \leq \overline{f}_V p^\dagger < \epsilon$ *and* $0 < \phi < \bar{\phi}(\epsilon)$*:* $\mathcal{E}(\epsilon|\Delta, V, f_V, \phi)$ *includes a quasi-Coasean equilibrium whenever* Δ *is sufficiently small.*

The intuition for Proposition 6 is straightforward. As they believe that the seller will concede to the low price $p^\dagger$ with some probability in the future, the optimistic buyer types will refuse to end the negotiation by exercising the outside option; these optimistic buyer types are the ones who are tough to deal with from the seller's perspective. Then, rational buyer types can leverage the seller's fear that the buyer is of an optimistic type by mimicking those optimistic types and refusing to exercise the outside option. As a result, positive selection among buyer types do not occur sufficiently, and hence the seller still has an incentive to practice inter-temporal price discrimination for the remaining buyer types.

4 Experimental Design and Hypotheses

4.1 Experimental Design

We consider an infinite horizon bargaining game between a seller and a buyer with one-sided private information in our experiment. The seller has an indivisible good for sale. It is common knowledge that the seller has zero intrinsic value to the good. The buyer's value of the good v is drawn uniformly from the support $[50, 400]$ prior to the negotiation, and it is private information of the buyer.

In each round[18] $n = 1, 2, \ldots$, the seller offers a price $p_n \in (0, 400)$, and the buyer decides whether to accept, reject the price offer, or to take the outside option. The buyer's value of the outside option $w \in \{\varnothing, 50, 60\}$ is commonly known. To implement an infinitely repeated game in the lab, we introduce the random termination of a supergame (Roth and Murnighan, 1978) with fixed continuation probability of $c = 0.8$. The expected length of each supergame (called a match) is $\frac{1}{1-c} = 5$ rounds. When the buyer accepts p_n in round n, the negotiation ends, and the seller and the buyer receive the respective payoffs of p_n and $v - p_n$. When the buyer rejects p_n, the negotiation proceeds to the next period with probability c. In case of termination, both the seller and the buyer receive payoff of zero. When the buyer decides to pursue the outside option, the negotiation ends, and the seller and the buyer receive the respective payoffs of 0 and w. The experiment consists of seven matches, and participants are reshuffled to form new pairs after each match so that there are no strategic dynamics between matches.

Our experimental design is summarized in Table 1. Three treatment conditions differ in the size, if exists, of the outside option. Out50 and Out60 are treatments where the buyers have an outside option whose value is 50 and 60, respectively. Out0 is the control condition where the buyers do

[18] Round n in the experimental setting refers to period $n - 1$ in Sections 2 and 3.

Table 1: Experimental Design

Out0 (*OutNo*)	*Out50* (*OutYes*)	*Out60* (*OutYes*)
No outside option	Outside option 50	Outside option 60

* Each participant has seven newly paired supergames (matches).
* Continuation probability to the next round is 0.8.
* Buyer's value v is drawn from $U[50, 400]$.

not have an outside option. It is worth noting that we consider both Out50 and Out60, although theoretical differences between them are not substantial. We believe that equilibrium reasoning or cognitive loads for the positive selection could be easier and faster when the outside option is 60, as the probability that the buyer draws a value lower or equal to the outside option is distinctively larger than the case of the outside option 50. Thus, the comparison between Out50 and Out60 could help us examine whether our observations are driven by cognitive loads of the participants. Otherwise, we collectively call Out50 and Out60 as OutYes, and accordingly Out0 as OutNo.

After each rejection of the price offer, the sellers were asked to report their beliefs about the paired buyer's types before making another price offer. More precisely, we asked them to report the range of the possible values within which the buyer's value falls. This report enables us to measure the seller's belief about how much positive selection and negative selection have occurred in the current round. Connecting this measurement with the seller's offer in the next round, we can study how the seller respond to positive and negative selection. To make it as incentive compatible as possible for seller participants to report their beliefs truthfully, we presented the minimum and maximum values for the buyer types reported in Round n in the decision screen of the same player in Round $(n+1)$ so that the player can potentially utilize the information to make a better decision.[19]

Our experiment was conducted by oTree (Chen et al., 2016) at the HKUST. A total of 196 subjects were recruited from the graduate and undergraduate population of the university. We had 4 sessions each for treatments Out0 and Out50 and 5 sessions for treatment Out60. Each session consisted of 12 to 18 participants and we had 58, 72, and 66 participants in treatments Out0, Out50, and Out60, respectively. In all sessions, subjects participated in seven matches of the bargaining game described above under one treatment condition. Sample experimental instructions can be found in Appendix A.

[19] Although this belief reporting is not financially incentivized, we argue no substantial reasons to believe that subjects misreport their beliefs. Incentive compatible mechanisms at the end of every bargaining round could have been considered, but Burdea and Woon (2021) show that the quality of reported beliefs may depend less on the formal incentive compatibility properties of the elicitation procedure than on the difficulty of comprehending the elicitation task and how well incentives induce cognitive effort. By explicitly stating that the purpose of the belief reporting is to help the subjects make non-erroneous decisions in the subsequent round, we believe that our elicitation is simple yet reasonable enough to induce cognitive effort. Moreover, the belief reporting could function as a suggestion or a guideline for the future self, as similarly done in Halevy et al. (2021). If we regard this belief reporting as communication between team partners–the current self and the future self– then the truthful reporting is more convincing (Burchardi and Penczynski, 2014).

All sessions were conducted via the real-time online mode using Zoom and oTree. Upon arrival at the designated Zoom meeting, subjects were instructed to turn on their video. Each received a web-link of the experimental instructions. To ensure that the information contained in the instructions is induced as public knowledge, the instructions were presented and read aloud by the experimenter via Zoom. All questions were privately addressed via the chat function in Zoom.

One of the seven matches was randomly selected for each subject's payment. The payoffs a subject earned in the selected match were converted into Hong Kong dollars at a fixed and known exchange rate of HK$1 per token. In addition to these earnings, subjects received HK$40 as a show-up payment. Subjects on average earned HKD 115 ($\approx$ USD 16) by participating in a session that lasted 1.2 hours. They were paid electronically via the autopay system of HKUST into the bank account he or she has registered with the Student Information System (SIS).

4.2 Hypotheses

We compare theoretical predictions under the assumption that there are no optimistic buyers. In other words, we set the differences between theoretical predictions with and without the outside option under the assumption of $\phi = 0$ as our null hypotheses, so the qualitative support for the null hypotheses is closely related to what we summarized in Section 2.[20]

Although less crucial from the theoretical perspective, our first hypothesis is that the size of the outside option does not affect the delay in bargaining. Whenever with an outside option, no delay occurs in equilibrium. This prediction of no practical difference also holds when we take into account some optimistic buyers.

Hypothesis 1 (Irrelevance of the Size of Outside Option)**.** *No significant differences between Out50 and Out60 exist regarding the bargaining length.*

Although the seller's equilibrium price offer and the buyer's response would depend on the value of the outside option as stated in Proposition 4, the differences between 50 and 60 are meager compared to the support of the type distribution. So, the qualitative difference should be minimal in equilibrium. If Hypothesis 1 is rejected, it might imply that some crucial factors are not accounted for in any of the proposed theories, or some assumptions we postulate would not hold. Perhaps it could mean that the cognitive loads of the experiment participants play an important role, or some other unintended aspects become salient. We present the following hypotheses focusing on the difference between OutYes and OutNo, assuming that rejecting Hypothesis 1 is not the case.

The next hypothesis regards the bargaining efficiency in terms of the delay. When an outside option is unavailable to buyers, theory predicts some delay (although the delay shrinks to zero as the continuation probability approaches 1). On the other hand, when there is an outside option, the

[20] Rejections of the null hypotheses may not directly imply that the model we introduce in Section 3 is correct. With this caveat, we address how the experimental findings are along with the predictions of our model in Section 5.

positive selection takes place such that no delay is expected in equilibrium. We say that bargaining ends either when they reach an agreement or when the buyer takes the outside option.

Hypothesis 2 (Outside Option and Delay)**.** *The average number of bargaining rounds in OutNo is strictly larger than 1, while that in OutYes is 1.*

Hypothesis 2 contains two testable statements. We examine (1) whether the average length of bargaining in OutNo is longer than OutYes and (2) whether bargaining in OutYes ends in round 1.

The next three hypotheses are about the seller's profits, actions, and beliefs.

Hypothesis 3 (Seller Profits)**.** *The seller's profits are higher in OutYes than in OutNo.*

Assuming $\phi = 0$, theory predicts that without an outside option, negative selection arises so that the seller's profit is effectively the lowest level, while the seller achieves the monopoly profit with the presence of an outside option. Thus, we expect that the seller's profit is higher in OutYes than in OutNo. However, if $\phi > 0$, i.e., some optimistic buyers exist, the negative selection would dominate, and the seller's profit would not differ significantly by the presence of the outside option.

Hypothesis 4 (Offering Prices)**.** *The average offer price is smaller in OutNo than in OutYes.*

The following elaborates more on Hypothesis 4. Without the outside option, the seller believes that the higher buyer types trade earlier, and thus, the price offer for the remaining buyers decreases over time until it reaches $p = \underline{v}$. Meanwhile, with the outside option (and unless assuming $\phi > 0$), it is hard to tell the price dynamics because the equilibrium predicts that the seller offers p^* and bargaining ends in round 1. One testable implication from these two predictions is that the average offered price would be smaller in OutNo than in OutYes.

After observing a buyer's rejection, the way the seller updates her belief differs by the presence of the outside option. Without the outside option, the buyer whose type is higher would be more likely to accept the previous price offer, so the minimum of the belief distribution (the lowest possible type remaining in the market) would be unaffected. With the outside option, however, positive selection occurs to the seller's belief in any history of any equilibrium. That is, the seller would believe (correctly) that all buyer types lower than a history-dependent cutoff would end bargaining by immediately exercising the outside option. This reasoning leads to the following hypothesis.

Hypothesis 5 (Belief about the Type Distribution)**.** *When the seller's offer is rejected, her belief about the minimum type of the buyer is an increasing function of the size of the outside option in OutYes, but it is constant at $\underline{v}$ in OutNo.*

Regarding Hypothesis 5, it is worth mentioning that examining the players' actions after the second round is challenging because when the outside option exists, the standard theory (i.e.,

$\phi = 0$) does not provide a concrete testable implication. The equilibrium predicts that bargaining ends instantly, and a delay arises only if the buyer deviates from the equilibrium play. As the standard definition of PBE does not restrict the seller's posterior belief after such a deviation, the equilibrium analysis does not make any testable prediction for the seller's belief after any non-null history. However, one concrete argument we can make is that buyers whose type is below w/c (62.5 in Out50 and 75 in Out60) should have taken the outside option for any belief on the price offer in round 2. From the perspective of such a buyer, even when the price in round 2 is 0, taking the outside option to earn w is better than the expected gains from trade in round 2, $c(v - 0)$. Unless the sellers have the optimistic buyers in mind, they must have taken this observation into account when stating their belief about the buyer's type. Thus, we mainly examine whether the minimum of the stated belief after being rejected in round 1 is greater than w/c.

The next hypothesis regards the buyer's behavior.

Hypothesis 6 (Buyer Actions)**.** *In OutYes, low-type buyers take the outside option, and high-type buyers accept the seller's offer in round 1. In OutNo, some buyers reject the seller's price offer in round 1.*

The first part of Hypothesis 6 restates the theoretical prediction by BP (Propositions 1 and 4 in Section 2): For the case that the buyer has an outside option, there is a cutoff value such that the buyer types with valuation higher than this cutoff (i.e., high-value buyers) trade with the seller immediately, while other buyer types (i.e., low-value buyers) exercise the outside option immediately. Then, a testable implication is that the buyer never rejects the seller's equilibrium offer in round 1. On the other hand, when the buyer has no outside option, we would observe some rejections and delay in round 1 in equilibrium (Proposition 3).[21]

5 Results

In this section, we report experimental findings in the corresponding order of our hypotheses. Note again that our hypotheses are based on the assumption that the optimistic buyers do not exist, that is, $\phi = 0$. When experimental evidence is inconsistent with the hypotheses, which is the case for most of the time, we explain whether and to what extent it is consistent with our theory under $\phi > 0$.

First of all, we compare the observations from Out50 with those from Out60. As described in Hypothesis 1, all the proposed theories predict no differences between the two treatments in terms

[21] Although Hypothesis 6 focuses on the buyer's action on the path, particularly in round 1, we can also extend the same hypothesis to the play off the path. For example, by generalizing the proof of Proposition 1 in Board and Pycia (2014, p.660), one can prove that a rejection never occurs after any history when the buyer has an outside option. On the other hand, when the buyer has no outside option, trading at $p > \underline{v}$ is not acceptable to the lowest buyer type $v = \underline{v}$. Hence, a rejection should occur with positive probability after any history unless the standing offer is lower than $\underline{v}$.

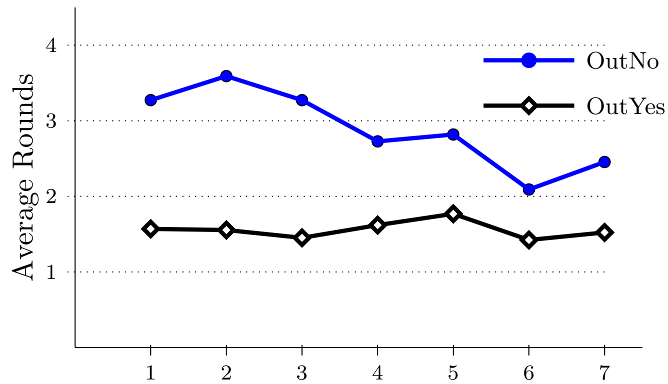


Figure 1: Average Length of Bargaining across Match

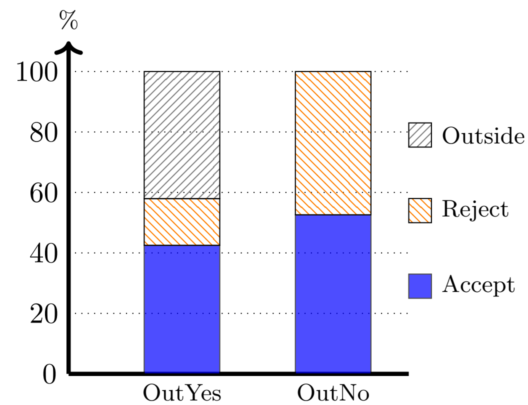


Figure 2: End-of-Bargaining States

of the length of bargaining and negligible differences in terms of buyer's and seller's earnings and seller's initial offers. As predicted, the average length of bargaining (Out60=1.53, Out50=1.59) is almost identical (t-test,[22] p=0.558), and the bargaining length distributions cannot reject the null hypothesis that those come from the same population distribution (Kolmogorov-Smirnov two-sample (KS) test, p=0.354). Moreover, the seller's average earnings (Out60=59.03, Out50=68.42) and the seller's average initial offers (Out60=194.45, Out50=211.48) are different in a statistically insignificant manner (p-values are 0.213 and 0.167, respectively.) The buyer's average earning in Out50 (78.72) and that in Out60 (94.82) are significantly different (p=0.025). However, this is mostly due to the fact that the exercise of the outside option in Out60 (46.52% of the entire cases) renders an additional earning of 10 relative to Out50. If the buyer's earning from the outside option were 50 instead of 60, the difference in the buyer's average earnings should become insignificant (p=0.116). Accordingly, the distributions of the seller's earnings are not statistically different between Out50 and Out60 (KS test, p=0.207). From now on, unless otherwise necessary, we pool Out50 and Out60 as OutYes and focus on the differences between OutYes and OutNo.[23]

Result 1. *No significant differences between Out50 and Out60 are found.*

Regarding Hypothesis 2, we compare the length of bargaining in OutNo with that in OutYes. Figure 1 shows the average number of rounds by match. On average, it takes 2.89 rounds to end the bargaining process in OutNo (either in the form of agreement or termination), while it takes 1.56 rounds in OutYes. This difference is statistically significant ($p<0.001$,) and it is primarily driven by the exercise of the outside option, not by different degrees of acceptance. Figure 2 shows the proportion of bargaining matches that end with an acceptance of the offer (in blue), a random termination after a rejection (in orange), and the exercise of the outside option (in gray),

[22]Unless otherwise stated, p-value reported in the parentheses is the result of a two-tailed t-test comparing the means, and the standard error of the mean difference is clustered at the individual level as each individual played the same role for the entire seven matches.

[23]In Appendix C, some figures in the main context reappear with Out50 and Out60 being separated from OutYes.

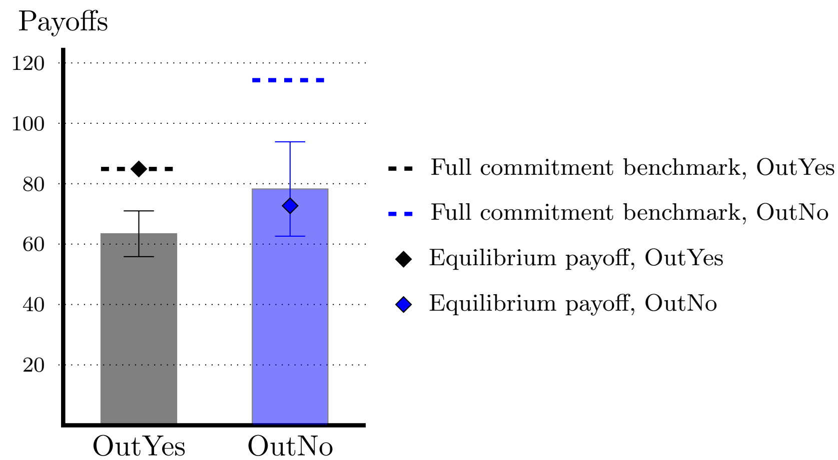


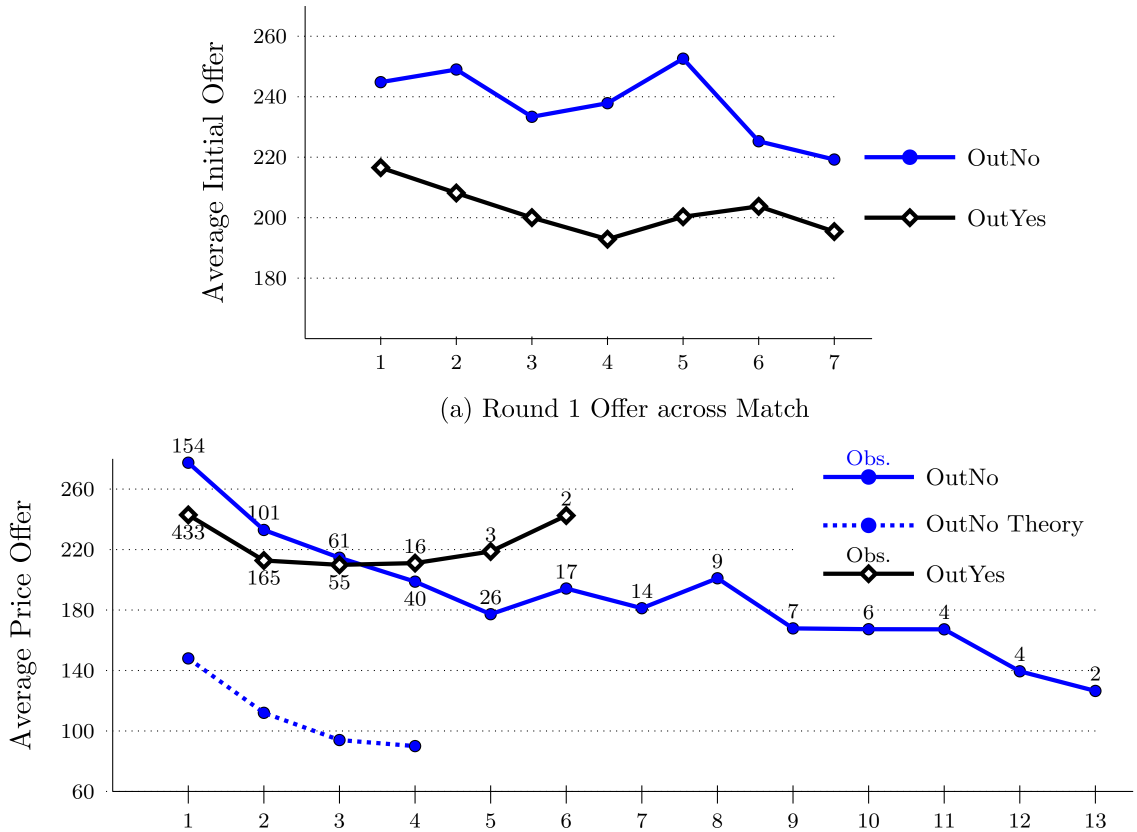
Figure 3: Seller's Earnings

respectively. In OutNo, 47.40% of the bargaining matches end with random termination after the buyer rejects an offer, but in OutYes, that proportion plummets to 15.46%. It is worth noting that a substantial proportion (42.04%) of the bargaining matches end by the buyer exercising the outside option in OutYes, and the proportion of matches ending with the acceptance of an offer (42.50%) is still comparable to that in OutNo (52.60%). These observations support our claim that the difference in the length of bargaining is driven by the exercises of the outside option, which otherwise may end up with the random termination after a rejection.

The observed difference in length of bargaining between OutYes and OutNo does not imply that we should entirely confirm Hypothesis 2. In OutYes, where the delay should not be observed in theory, the average length of bargaining rounds is larger than 1 in a statistically significant manner ($p<0.001$). While we reject Hypothesis 2, our observations regarding the length of bargaining are consistent with the predictions under $\phi > 0$. Even with the outside option, where many low-type buyers leave the market by taking it, some fraction of the (optimistic) buyers remains by rejecting the offer, which causes some delay.

Result 2. *The average number of bargaining rounds in OutNo is strictly larger than that in OutYes, and the average number of bargaining rounds in OutYes is strictly larger than 1.*

Figure 3 shows the seller's average earnings in OutYes and OutNo, along with the aggregate-level standard deviations, the full-commitment benchmarks, and the equilibrium payoffs. Hypothesis 3 states that the seller's profits are higher in OutYes than in OutNo. We find the opposite from the lab: The seller's average earning in OutYes (63.43) is smaller than that in OutNo (78.25). The difference is statistically significant at the 5% level of significance (one-sided, $p=0.039$). We thus reject Hypothesis 3. However, the observed reversal of the seller's earnings is consistent with the theoretical prediction from the model with optimistic buyers: The sellers effectively achieve the



(a) Round 1 Offer across Match

(b) Offer across Round

Figure 4: Average Price Offers

lowest level of profit and lose the entire gains from trade when the buyers exercise the outside option. Another noticeable observation is that the equilibrium payoff level in OutNo falls inside the confidence interval of the seller's average payoff. However, this "coincidence" shall not be interpreted as supporting evidence that the negative selection precisely worked in the lab because 47% of seller payoffs is zero in OutNo, implying that a substantial proportion of the bargaining matches in OutNo ended by random termination after the buyers rejected a price offer. We shortly revisit this observation when examining the dynamics of the sellers' price offers.

Result 3. *The seller's average profit in OutYes is smaller than that in OutNo.*

The seller's initial offers are also inconsistent with Hypothesis 4. Figure 4a shows the average initial offers by match. Overall, the average initial offer in OutYes (202.44) is smaller than that in OutNo (237.45), and the difference is statistically significant (p=0.003). Both in OutYes and in OutNo, we can observe a small but significant downward trend of the average initial offer over time.[24] This small learning effect is perhaps due to the experience of random termination or the

[24] After controlling for the treatment effect, the average initial offer decreases by 2.8853 tokens in the next match

outside option being exercised. However, the learning effect does not depend on the availability of an outside option, as the trend is insignificantly different between treatments.[25] Even in the first match, the average initial offers (OutNo=244.82, OutYes=216.55) are different in a statistically significant manner (p=0.023). This observation implies that the sellers, who consider the possibility for the buyers to exercise their outside option, proactively lower the offering price to avoid zero gains from trade. Thus, it strongly rejects Hypothesis 4.

Result 4. *The seller's initial offer in OutYes is strictly smaller than that in OutNo.*

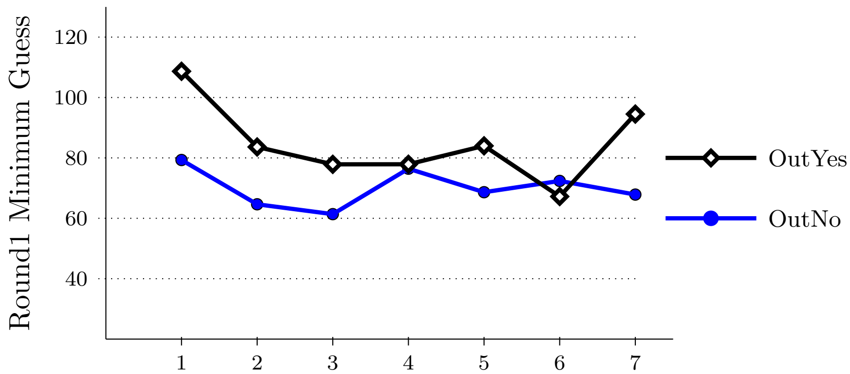
Figure 4b shows how the average price offers change by round. It is clear that the average price offer tends to decline over round in OutNo. The declining trend appears similar to the theoretical prediction of the Coasean equilibrium (presented with a dotted line). Yet the price offers are much larger in level than what the negative selection predicts, implying that negative selection is in force but only partially executed in the lab. No such trend is found in OutYes. We have to emphasize that the result should be interpreted with a caveat because the data from the second round and beyond are collected only when the first round price offer is rejected and the bargaining process is not terminated. Hence, the number of observations, as shown in the figure, sharply decreases with round.

Hypothesis 5 regards the seller's belief after observing the rejection of the initial offer. At the end of every unfinished bargaining round, we asked the seller to guess the range of the possible valuations that the buyer draws. Figure 5a shows the minimum of the reported range, averaged by match. As explained in Hypothesis 5, if the sellers assume no optimistic buyers, i.e., $\phi = 0$, the minimum of the guess should have increased, at least, by w/c (62.5 in Out50 and 75 in Out60). The average minimum of the guess in OutYes (81.58) is larger than that in OutNo (71.36), and the difference is statistically significant only at the 10% level (p=0.059).[26] The average minimum of the guess in OutYes is also significantly larger than w/c (p=0.016), so we cannot reject Hypothesis 5 directly. Yet it is too hasty to take the observed difference as suggestive evidence that the sellers in OutYes reflect on the positive selection. Since some sellers move on to Round 2 more frequently than others, the (unweighted) averages tend to overweight such sellers' reports. To correct the misinterpretation due to the heterogeneous individual weights, we also examine the individual's average minimum of the reported range. Almost half of the sellers (29 out of 59 sellers) in OutYes report it below w/c on average, and we cannot reject the null hypothesis that the sample distribution of such averages in OutYes is from the same population distribution for those in OutNo (KS, p-value=0.328). In other words, the individual-level reports on the minimum of the guess in OutYes and those in OutNo are, by and large, the same.

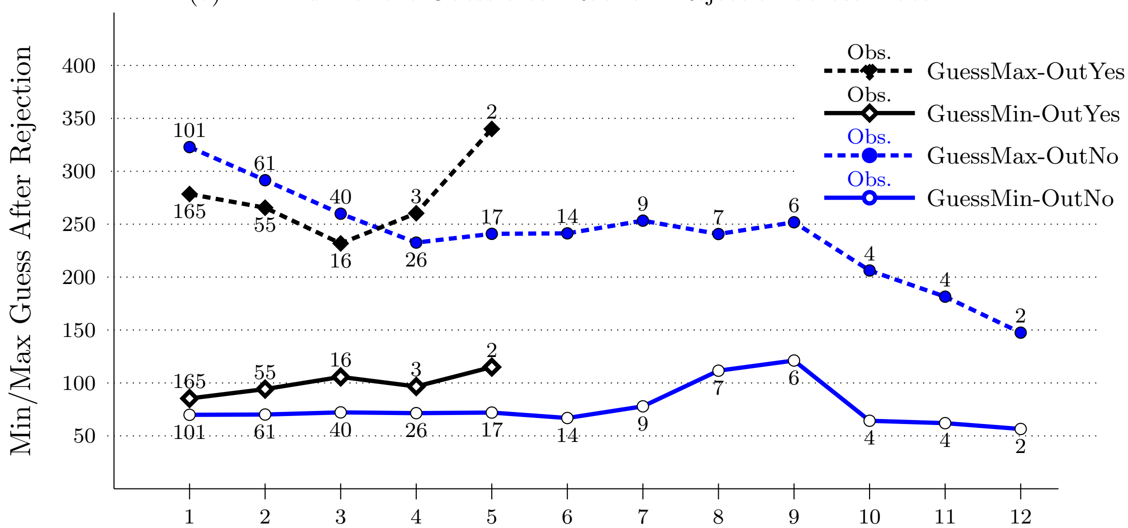
(p=0.032).

[25] After controlling for the treatment effect and the learning effect, the interaction effect of the treatment and learning is insignificant (p=0.668).

[26] When looking at the last four matches only, we still observe that the average minimum of the guess in OutYes is larger than that in OutNo, but the difference is statistically insignificant (p=0.248).



(a) Minimum of the Guess after Round 1 Rejection across Match



(b) Mimimum/Maximum of the Guess across Round

Figure 5: Seller's Reported Beliefs

Figure 5b presents how the average (min and max) guesses evolve over round. The decreasing trend of the maximum guess in OutNo provides supplementary evidence that the negative selection takes place, albeit partially, as the Coasean equilibrium predicts. The minimum of the guess is overall stable over time in OutNo, while the maximum of it decreases steadily, especially for the first four rounds, which implies that the sellers understand that the buyers with the high valuations gradually leave the market when outside options are unavailable. A similar pattern of negative selection is also observed in OutYes, especially for the first three rounds, but it is not as distinctive as in OutNo. We emphasize that the same caveat for Figure 4b applies when interpreting Figure 5b as the observations for the later rounds are conditional on previous rejections.

Meanwhile, the observation is consistent with the prediction under the assumption of $\phi > 0$. When some buyers are optimistic, the seller would believe that all buyers who reject the initial offer do not necessarily draw the lower valuation, and thus, the lower bound of the belief remains unaffected regardless of the availability of the outside option.

Result 5. *The seller's reported belief about the lowest type of the buyer who rejects the initial offer in OutYes is weakly higher than that in OutNo on average. However, the difference is not substantially large in its magnitude.*

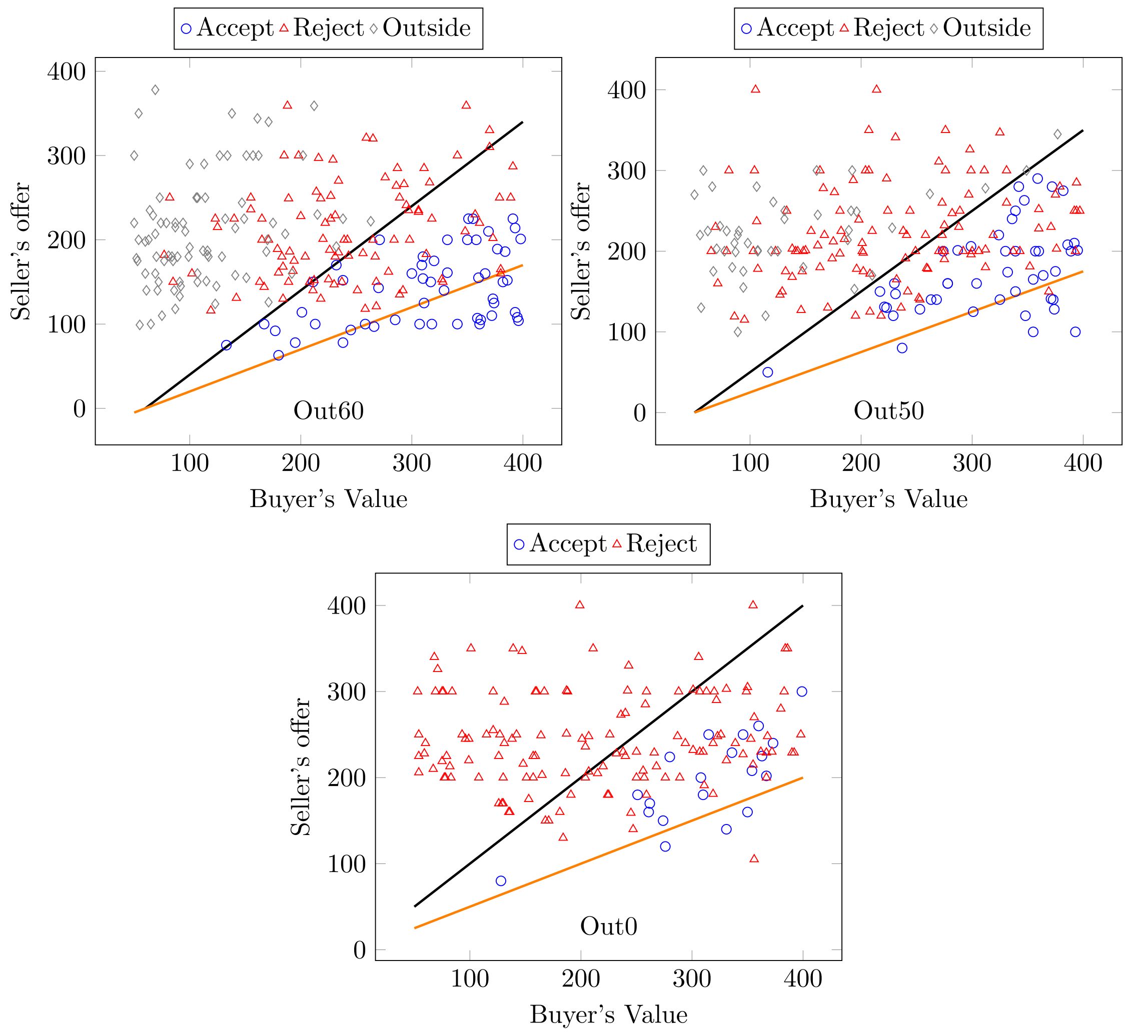


Figure 6: Buyer's Action in Round 1, by Treatment

Our last set of results regards the buyers' decisions. Figure 6 shows the buyers' actions in Round 1 on the plane of the buyer's value and the seller's offer. Each scatterplot point (x, y) reads that the buyer's value is x and the seller's offer is y. Blue circles, red triangles, and gray diamonds respectively indicate that the buyers accept the offer, reject the offer, and take the outside option. We draw two auxiliary lines to grasp the overall patterns. The black 45-degree lines show the buyer's value minus the outside option, which means that when the value–offer coordinate is on the right-hand side of the line, accepting the offer is strictly better than taking the outside option. The orange lines show half of the buyer's value plus the outside option, which means that the value–offer coordinate on the orange line equally splits the gains from trade between the buyer and the seller. Both in Out60 and Out50, given the seller's offer, we observe a clear tendency that the buyers are more likely to accept the offer if they draw a higher value: See the blue circles are mostly on the lower-right area of the scatterplot. However, the rejections are pervasive, which means that the first implication of Hypothesis 6—no rejection with the presence of the outside option—is not supported. With the absence of the outside option, the majority (87.01%) of the action is to reject the price offer. The observation that the buyer frequently rejects the initial offer in both OutYes and OutNo implies that the buyers expect a lower price offer in the next round attractive enough to compensate for the potential loss due to random termination. It is worth noting that the model with $\phi > 0$ predicts that rejections would be observed even with the presence of the outside option, which is consistent with our observation.

Another possible story could go along with the orange lines, the equal-split lines of the gains from trade. Below the orange lines, we observe a dominant fraction of acceptance while we observe a substantial fraction of rejection on the opposite area. Although one may interpret that the buyers' inequity aversion would have played a role to make their accept/reject decisions, it is challenging to explain the overall observations using the inequity aversion only. Even if we explicitly consider an inequity-averse player (where a formal model and its results are presented in Appendix E), the buyer's decision with the presence of the outside option ought to be either accepting the offer or exercising the outside option, so it does not help us explain the massive rejections. Moreover, the inequity aversion does not explain why the seller's average initial offer in OutYes is smaller than that in OutNo.

6 Discussions

We acknowledge that the buyer's optimism, although consistent with what we observed in the experiment, is not the only way to rationalize our findings. Although we have enunciated that our primary goal is neither to identify where the buyers' optimism is originated from nor to claim how the assumption of the optimistic buyers is persuasive, we firmly believe that it is worth discussing relevant issues. For this purpose, we review some studies from various literature to grasp shared features, including obstinacy in bargaining, uncertainty about the outside option, insufficiently-updated skepticism in information disclosure, and lack of first-order rationality. We claim that a key driver making the positive selection fragile is the belief that some low-type buyers may remain in the market. Thus, we view our setup with the optimistic buyers as a parsimonious workhorse model facilitating such a belief.

6.1 Obstinacy in reputational bargaining

The reputational bargaining literature has demonstrated that an inefficient delay may arise when a small fraction of bargainers are known to be (irrationally) obstinate. Rational bargainers pretend to be obstinate to derive more favorable deal, hence an inefficient delay may arise in equilibrium (Myerson, 1991; Abreu and Gul, 2000). Embrey et al. (2017) report experimental evidence consistent with the theoretical predictions about inefficient delay in reputational bargaining. Compte and Jehiel (2002) show that the existence of an outside option may cancel such effect of obstinacy. The bargaining counterpart's outside option limits the rational bargainer's benefit from mimicking the obstinate type, because the counterpart would exercise the outside option immediately rather than haggling with the obstinate type.

The standard assumption in the reputational bargaining literature is that the gains from trade (net-values) are common knowledge, hence whether a bargainer is an obstinate type is the only source of information asymmetry. Fanning (2021) considers a bargaining problem such that the buyer's obstinacy as well as his value of the seller's good and outside option are the buyer's private information. Fanning shows that, in contrast to the BP's result, the Coase conjecture is partially restored in when the buyer is possibly of an obstinate type. A rational buyer type may benefit from mimicking an obstinate type rather than exercising the outside option immediately, and such a possibility undermines the effect of positive selection.

Although the precise logical steps for reaching the above-mentioned theoretical conclusions differ, one shared driver behind the results seems to be that the buyers' belief that the sellers might make more favorable offers could lead to the inefficient delay. In the sense that our model with optimistic buyers directly postulates such a belief, we might say that the optimism de facto captures the obstinate type behaviors.

6.2 Uncertainty on the outside option

Unravelling in BP takes off when the seller makes a price offer such that the buyer type whose net value is lowest cannot enjoy any positive gains from trade and decides to leave the market immediately. Thus, it is not difficult to imagine that positive selection may break down when the seller is sometimes unable to make such a price offer. Such situations naturally arise when the seller has uncertainty about the buyer's outside option. The uncertainty could be about the timing of arrival of the outside option as in Hwang and Li (2017). They show that the Coase conjecture is restored if the buyer's outside option arrives at a random time. The positive selection may occur only after the outside option arrives, hence, the uncertainty in the availability of the outside option erodes the positive selection. Lomys (2020) considers a bargaining problem in which both parties are symmetrically uninformed of the existence of the buyer's outside option as well as its arrival time. He discusses how the equilibrium outcome (e.g., whether the efficient outcome occurs) varies with the nature of the outside option.

In our environment, it is not the uncertainty imposed on the outside option that prevents the seller from making a price offer to induce the lower type buyers to take the outside option immediately. It is (the seller's belief about) the buyer's optimism that makes the lower type buyers to haggle. No matter what it is to make (the seller believe that) the lower type buyers remain in the market, the consequence is the same, in that positive selection cannot take place from the beginning of the inductive unraveling process.

6.3 Information disclosure and its inference

In the context of information disclosure where a privately-informed party discloses some information and an uninformed party infers the type of the informed party, an inductive process of information unraveling leads to the full information disclosure (Grossman, 1981; Milgrom, 1981). The key driver of this result is skepticism, i.e., the uninformed party attributes any incomplete disclosure to the informed party concealing unfavorable information. However, the full information disclosure based on the skepticism is hardly supported by empirical and experimental data. For example, some film studios withhold movies from critics before their release, and some moviegoers do not infer low quality from such cold opening (Brown et al., 2012). In the experimental setting, information receivers are insufficiently skeptical about non-disclosed information (Jin et al., 2021). This implies that the informed party may be able to exploit the insufficient inference of the uninformed party.

In our context with the reputational bargaining with an outside option, the sellers should have inferred that those who take the outside option in the first round must be the lowest type buyers such that remaining demand pool in the second round consists of higher types. However, some sellers might not have fully inferred it perhaps due to similar reasons for why the uninformed party in the information disclosure environment does not fully update the belief about non-disclosed information. The optimistic buyers may be one way to parsimoniously capture those who try to exploit the insufficient inference of the seller.

6.4 Failure of rationality

Perhaps a more direct way of explaining the departure of experimental observations from the theoretical prediction is to consider the failure of rationality. In our experimental setup, if the seller worries that low-type buyers, who are supposed to immediately take the outside option, suboptimally continue the negotiation, the key mechanism of Board and Pycia (2014) would not work. The positive selection cannot take off as strongly as theory predicts at the start of unraveling, and hence the stark contrast in the presence/absence of an outside option may disappear.

We explore this idea formally in Appendix D. We consider the ϵ-PBEs such that the buyer may choose to delay sub-optimally, provided that the payoff loss relative to the best response is less than

ϵ. In turns out that, even in the absence of any optimistic buyer types, such ϵ-irrationality results in the outcome qualitatively identical to the quasi-Coasean equilibrium studied in Section 3. The basic intuition is essentially identical to the one for Proposition 6. The seller's belief that some buyers may sub-optimally remain in the negotiation undermines positive selection among buyer types.[27]

7 Conclusion

In this paper, we experimentally consider positive and negative selection in an infinite-horizon bargaining with one-sided, incomplete information. Consistent with theory, we find that the bargaining lasts longer when an outside option is unavailable than otherwise. Inconsistent with theory, however, a substantial proportion of price offers are rejected even when the outside option is available. Inconsistent with theory, seller's initial price offer is strictly higher in the absence of outside option than in the presence. Similarly, the seller's average profit is larger when the outside option is unavailable. The reported beliefs from the sellers after a price offer is rejected further confirm that positive selection does not take place in the lab when the outside option is available, while we find partial evidence of negative selection when the outside option is unavailable.

Our main findings are attributed to the failure of inductive process of unraveling. We explicitly consider a model with a fraction of optimistic buyer types, and we show that a quasi-Coasean equilibrium exists and is essentially unique in the bargaining with an outside option. When (the seller believes that) optimistic buyer types exist, the seller is unable to make a price offer to induce the lower type buyers to take the outside option and leave the market immediately. Thus, positive selection fails. In that equilibrium, the rational buyer type with lower values take the outside option immediately while the seller engages in the inter-temporal price discrimination with the remaining demand pool such that negative selection takes place. This equilibrium successfully describes the qualitative features of the observed behavior in the lab. However, we emphasize that what allows us to explain our data well does not need to be the model with the buyer optimism. As we discussed in the previous section, any other models that make it impossible for unraveling to be executed completely will generate qualitatively the same set of predictions.

We provide the first evidence in the literature to suggest that the distributional effect of an outside option available to buyers may not be as drastic in real world as in theory. However, the existence of an outside option in a market might still be desirable because it enhances the bargaining efficiency as bargainers reach an agreement more quickly. On one hand, we strongly believe that it is important and valuable to re-confirm our main findings obtained in the controlled laboratory environment in the field setup in order for the literature to fully understand the impact of outside

[27] Board and Pycia (2014, Section I.B) discuss that, without assuming buyer's optimism nor ϵ-irrationality, a similar outcome may arise if some buyer types have zero outside options.

option to the market participants. On the other hand, due to the fact that the main environment of interests involves private information of one side of market, it would not be straightforward to obtain a reasonable data set in the field to tackle this question. We leave it for future research.

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A Sample Experimental Instructions

Welcome to the experiment. Please read these instructions carefully. There will be a quiz around the end of the instructions, to make sure you understand this experiment. The payment you will receive from this experiment depends on your decisions.

Your Role and Match

At the beginning of the experiment, one-half of the participants will be randomly assigned to the role of a **seller** and the other half the role of a **buyer**. Your role will remain fixed throughout the experiment.

The experiment consists of 7 **matches**. At the beginning of each match, one seller participant and one buyer participant are randomly paired. The pair is fixed **within the match**. After each match, participants will be reshuffled to form new pairs. You will not learn the identity of the participant you are paired with, nor will that participant learn your identity—even after the end of the experiment.

In each match, the seller holds an asset. The value of the asset is 0 for the seller. However, the buyer has a strictly positive value of the asset. Let B denote the buyer's value of the asset. At the beginning of a match, a computer randomly and independently draws B between 50 and 400. Every integer in $[50, 400]$ has an equal chance to be drawn. The value B is fixed within each match, and a new B is independently drawn for a new match. **The buyer knows the value B, but the seller does not.**

Your Decisions in Each Match

Each match consists of at least one round of bargaining. In a round, a seller offers a price to sell the asset, and the buyer responds. If the offer is rejected, the match may move on to the next round of bargaining. The details follow.

Your Task as a Buyer: Suppose your role is a buyer. At the beginning of Round 1, you will see the following figure. The horizontal position of the dark blue line represents B, your value of the asset. (B is 330

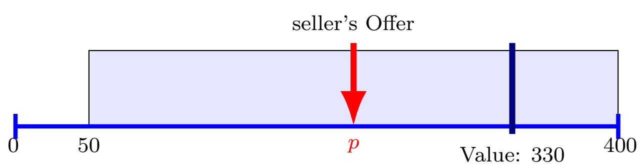


Figure 7: Buyer's Screen in Each Round

in this example, but your value will vary.) Once the seller in your pair makes a price offer, p, a red vertical arrow will appear on the figure. The position of the red arrow represents p. After that, decide whether to

- **accept** the offer and earn $(B - p)$,
- **reject** it and move on to the next round with an 80% chance, or
- **take an outside option** to earn 50 tokens.

Your task is the same for every round. It is important to understand that when you reject the offer, **there is a 20% chance that the match is terminated, and both of you and the seller in your pair earn 0 tokens.**

Beware that you cannot accept the offered price p if it is strictly greater than your value B, otherwise your payoff becomes negative.

<u>**Your Task as a Seller:**</u> Suppose your role is a seller. At the beginning of Round 1, you will see the following figure. The blue shaded area between 50 and 400 represents the range of all possible buyer's values.

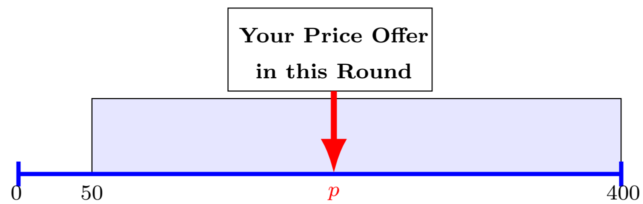
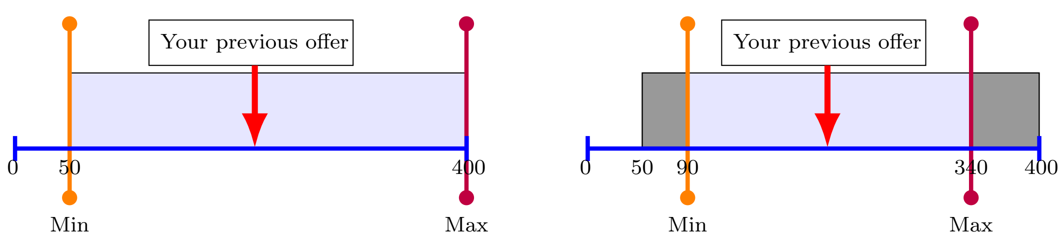


Figure 8: Seller's Screen: Round 1

Choose your price offer by clicking on the line. A red vertical arrow, whose position represents your price offer, p, will move to the point you click. You are free to choose any point in the range **[0, 400]** for your price offer, and you can adjust it as much as you wish. After that, click the submit button, and wait for the buyer's decision. You expect one of three possible outcomes.

- If the buyer **accepts** the offer, you earn p tokens.
- If the buyer **takes an outside option**, you earn 0 tokens.
- If the buyer **rejects** the offer, then the match moves to Round 2 with an 80% chance. Note that if the match is terminated with a 20% chance, both you and the buyer earn 0 tokens.

If the match moves to Round 2 and beyond, then you will see the following figure below. The red vertical arrow represents your (rejected) previous offer.



(a) Before

(b) After

Figure 9: Reporting Beliefs in Round n

Before submitting a new offer price, adjust an orange slider and a purple slider to indicate the updated range of possible values B in your mind. **The reported range will appear in your decision screen but**

will not be shared with the buyer. Its sole objective is to help you think about an appropriate price offer. There is nothing to gain by indicating a range that differs from what you actually believe, so please report your belief as accurately as possible. To indicate the range, move the orange and purple sliders. The horizontal positions of the sliders respectively represent the minimum and the maximum of the range in your mind. The minimum can't exceed the maximum.

Note that the buyer's value of the asset (B) and the value of the outside option (50 tokens) will remain the same across rounds within a match.

Probability of Match Termination

As described, after a buyer rejected a price offer, the match continues to the next round with an 80% chance. Your screen presents a spinning wheel that consists of red area (20%) and green area (80%) as illustrated below. Once you click the "Spin" button, the wheel starts spinning. If the spinning wheel stops at the green area, the match continues. Otherwise, the match terminates. Note that the seller and buyer in the same match always see the same outcome from the wheel for each round.

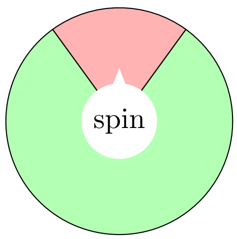


Figure 10: Spinning Wheel

Information Feedback

- At the end of each **round**, you will know the seller's price offer and the buyer's decision. If the buyer rejects the offer, you will know whether the match is continued to the next round or terminated.
- At the end of each **match**, you will know how many tokens you receive from the match.

Your Monetary Payments

At the end of the experiment, a computer will randomly select one match out of 7 for your payment. Every match has an equal chance to be selected for your payment, so it is in your best interest to take each match equally seriously. Participants will receive the amounts of tokens according to the outcome from the selected match with the exchange rate of 1 token = 1 HKD. Also, every participant will receive a **show-up fee of HKD 40**.

Completion of the Experiment

After the 7th match, the experiment will be over. You will be instructed to fill in the receipt for your payment. The amount you earn will be paid **electronically via the HKUST Autopay System to the bank account you provide to the Student Information System (SIS)**. The Finance Office of HKUST will arrange the auto-payment. An email notification will be sent to your HKUST email address on the pay date under the name of the sender "FOPSAP" (Finance Office Payment System Auto Payment).

Comprehension Check

To ensure your comprehension of the instructions, you will answer four multiple-choice questions. You can proceed only with all correct answers. Afterwards, you will participate in a practice match.

Q1 Suppose you are a seller. Which of the followings is NOT TRUE? (a) I do not know how much the buyer values the asset. (b) If the buyer takes an outside option, I earn 50 tokens. (c) If I offer 200 tokens, and the buyer accepts it, then I earn 200 tokens. (d) If the buyer rejects my offer, then I can make a new offer with an 80% chance.

Q2 Suppose you are a buyer, and the value of the asset is 300. Which of the followings is TRUE? (a) If I accept a price offer of 200 tokens, I earn 200 tokens. (b) If I take an outside option, I earn 250 tokens. (c) If I accept a price offer of 200 tokens, I earn 100 tokens. (d) In Round 2 of this match, the value of the asset will be different from 300.

Q3 Suppose the price offer in Round 1 is rejected. Which of the followings CAN HAPPEN? (a) The match is terminated, and both the seller and the buyer earn 0 tokens. (b) The match is continued forever, even after continuous rejections. (c) The match is terminated, and each participant in the pair earns a half of value B. (d) The match initiates an open chat to negotiate.

Q4 Suppose the first match is done. Which of the followings is TRUE? (a) It is almost sure that I will be paired with the same participant in the first match. (b) I may play another role different from what I did in the first match. (c) The buyer's value of the asset in the second match will be the same as the one in the first match. (d) My previous actions do not affect the value of the asset in the new match.

1 Practice Match and 7 Actual Matches

[After passing the quiz] Thank you for paying attention to the instructions. Before you will play the 7 actual matches, you will have one practice match (Match #0) which is not relevant to your payment. Its objective is to get you familiar with the computer interface and the flow of the decisions in each round of a match. Once the practice match is over, it moves to the actual matches.

B Omitted Proofs

B.1 Proof of Proposition 5

The proof is essentially identical to the proof of Proposition 1 in BP, which we sketch here.

Step I: Fix any ϵ-PBE $\sigma = (\sigma^B, \sigma^S, f^S)$. The seller never offers $p_n < \underline{u}^\sigma(h_n) := \min\{u(\theta) : \theta \in \text{supp} f^S(h_n)\}$ at any h_n. Suppose for contradiction $G := \sup_{h_n \in H} \underline{u}^\sigma(h_n) - \underline{p}^\sigma(h_n) > 0$, where $\underline{p}^\sigma(h_n) := \inf(\text{supp} f^S(h_n))$. Choose h_n and $0 < \beta < (1 + e^{-r\Delta})/2$ such that $\underline{u}^\sigma(h_n) - \underline{p}^\sigma(h_n) > (1-\beta)G$. By the argument identical to Lemma 1 in BP, any deviation to $p \in [\underline{p}^\sigma(h_n), \underline{p}^\sigma(h_n) + \beta G)$ would induce an immediate trade with the buyer, yielding $V_S^\sigma(p; h_n) = p$ as the seller's final payoff. Hence, $\sigma^S(h_n)$ never chooses any price in $[\underline{p}^\sigma(h_n), \underline{p}^\sigma(h_n) + \beta G)$, contradicting the supposition $G > 0$.

Step II: By the argument identical to Proposition 1 in BP, the observation in Step I implies the following: in any ϵ-PBE, any offer p_0 at h_0 never induces a delay. Hence, the seller will charge $p_0 = p_w^*$ to achieve the full-commitment benchmark profit in any ϵ-PBE.

B.2 Proof of Proposition 6

Throughout the proof, denote the common discounting factor by $\delta \equiv e^{-r\Delta} \in (0,1)$. Define

$$v_c := \min\{v \in V : \delta\eta(v - p^\dagger - w) > (1-\delta)w\}, \quad u_c := v_c - w, \quad \text{and} \quad p_c := \frac{(1-\delta)v_c + \delta\eta p^\dagger}{1 - \delta + \delta\eta}.$$

Assumptions 2 and 3 guarantee that v_c is well-defined. In particular, v_c is well-defined as V is finite. Also,

$$v^\dagger := \min\{v \in V : v - p^\dagger - w > 0\}.$$

B.2.1 Step I: Preliminary Observations

First, we make several preliminary observations on v_c and p_c. Note that v_c converges to $v^\dagger$ as $\delta \to 1$. Furthermore, given that $|V| < \infty$, there is a cutoff $\bar{\delta} \in (0,1)$ such that $v_c = v^\dagger$ and $v_c - p^\dagger - w = v^\dagger - p^\dagger - w > \frac{1-\delta}{\delta\eta} w > 0$ whenever $\delta \in (\bar{\delta}, 1)$. Finally, $p_c > p^\dagger$ for all $\delta \in (0,1)$ and $p_c \downarrow p^\dagger$ as $\delta \to 1$.

Lemma 1. *There is $\delta^* \in (0,1)$ such that the following properties hold whenever $\delta \in (\delta^*, 1)$.*

(i) $v - w > p_c$ *for* $v \geq v_c$.

(ii) $v_c - p_c = \delta(1-\eta)(v_c - p_c) + \delta\eta(v_c - p^\dagger) = \frac{\delta\eta}{1-\delta+\delta\eta}(v_c - p^\dagger) > w$.

(iii) $v - p_c > \delta(1-\eta)(v - p_c) + \delta\eta(v - p^\dagger) > \frac{\delta\eta}{1-\delta(1-\eta)}(v - p^\dagger) > w$ *for any* $v > v_c$.

(iv) $v - p_c < \delta(1-\eta)(v - p_c) + \delta\eta(v - p^\dagger) < \frac{\delta\eta}{1-\delta(1-\eta)}(v - p^\dagger) \leq w$ *for any* $v < v_c$.

Proof. Throughout the proof, we assume without loss $v_c = v^\dagger$ and $v^\dagger - p^\dagger - w > \frac{1-\delta}{\delta\eta} w > 0$. The part (i) directly follows the following observation:

$$v - w - p_c = \frac{\delta\eta(v^\dagger - p^\dagger - w) - (1-\delta)w}{1 - \delta(1-\eta)} + v - v_c > v - v_c \geq 0 \quad \forall v \geq v_c.$$

The part (ii) follows the definition of v_c and p_c. To show (iii), recall from (ii) that the following equation exactly holds at $v = v_c$:

$$v - p_c = \delta(1-\eta)(v - p_c) + \delta\eta(v - p^\dagger) = \frac{\delta\eta}{1-\delta(1-\eta)}(v - p^\dagger). \tag{B.1}$$

The comparison between the coefficients of v on each side of (B.1) reveals that the first two inequalities in (iii) hold strictly whenever $v > v_c$. The last inequality $\frac{\delta\eta}{1-\delta(1-\eta)}(v - p^\dagger) > w$ in (iii) follows the definition of v_c. The proof of (iv) is similar to the proof of (iii). □

Next, we make observation on the seller's static profit:

$$\Pi(p) := \sum_{\theta\in\Theta:u(\theta)\ge p} p f_\Theta(\theta) = (1-\phi)\Pi_r(p) + \phi \sum_{\theta\in\Theta_o:u(\theta)\ge p} p f_\Theta(\theta).$$

$\Pi(p)$ is the seller's payoff from insisting on $p_n = p$ in all periods. In the next lemma, u^* denotes the argument maximizing Π_r (see Assumption 2). Note that $\Pi(p)$ is also maximized at $p = u^*$ (hence, $p_w^* = u^*$), provided that ϕ is sufficiently small; we choose $\bar{\phi}(\epsilon)$ as a cutoff such that u^* indeed coincides with p_w^* for all $0 < \phi < \bar{\phi}(\epsilon)$.

Lemma 2. *Suppose $0 \le \overline{f}_V p^\dagger < \epsilon$. There are $\bar{\phi}(\epsilon) \in (0,1)$ and $\bar{\delta}(\epsilon) \in (0,1)$ such that the following holds true whenever $0 < \phi < \bar{\phi}(\epsilon)$ and $\bar{\delta}(\epsilon) < \delta < 1$: For any $\underline{v} - w \le p' \le p'' \le p_c$, $\Pi(p') \le \Pi(p'') + \epsilon$.*

Proof. $\{u(\theta) : \theta \in \Theta_r\}$ is a finite set, and hence we may enumerate its elements by $u_1 < u_2 < \ldots < u_{|V|}$. Note that $\Pi_r(p)$ is linearly increasing in p over each interval (u_k, u_{k+1}) and has a downward jump at each u_k. Also, let M be the integer such that $p^\dagger \in [u_M, u_{M+1})$. Recall that p_c decreases in δ and converges to $p^\dagger$ as $\delta \to 1$. Hence, we may choose $\bar{\delta}(\epsilon)$ such that $p_c \in (u_M, u_{M+1})$ whenever $\bar{\delta}(\epsilon) < \delta < 1$, which we will indeed suppose throughout the proof.

Fix any p' and p'' such that $\underline{v} - w \le p' \le p'' \le p_c$. The inequality $\Pi(p') \le \Pi(p'') + \frac{\epsilon}{2}$ trivially holds if both p' and $p'' \in (u_k, u_{k+1}]$ for some k. Hence, we may assume without loss that there is u_k such that $p' \le u_k < p''$; if there are multiple such u_k's, pick the largest one among them. Then, $p' \le u_k < p'' \le p_c < u_{M+1}$, and therefore,

$$\Pi_r(p'') - \Pi_r(p') \ge \Pi_r(u_k+) - \Pi_r(u_k) = -u_k f_V(v_k) \ge -p_c \overline{f}_V,$$

where $\Pi_r(u_k+)$ stands for the right-limit of Π_r at $p = u_k$ and $v_k = u_k + w$. Hence,

$$\Pi_r(p'') - \Pi_r(p') > -p_c\overline{f}_V = -\left[p^\dagger - \frac{1-\delta}{1-\delta+\delta\eta}(v_c - p^\dagger)\right]\overline{f}_V > -\epsilon + \overline{f}_V \frac{1-\delta}{1-\delta+\delta\eta}(v_c - p^\dagger) > -\epsilon.$$

Finally,

$$\Pi(p'') - \Pi(p') \ge (1-\phi)\left[\Pi_r(p'') - \Pi_r(p')\right] - \phi\overline{v} > (1-\phi)\left[-\epsilon + \overline{f}_V \frac{1-\delta}{1-\delta+\delta\eta}(v_c - p^\dagger)\right] - \phi\overline{v}.$$

This bound for $\Pi(p'') - \Pi(p')$ is independent of p' and p''. Hence, there is $\overline{\phi}(\epsilon)$ such that, whenever $\overline{\phi}(\epsilon) < \phi < 1$, $\Pi(p'') - \Pi(p') \ge -\epsilon$ for all $p' \le p'' \le p_c$. □

B.2.2 Step II: Quasi-Coasean Equilibrium Assessment

Fix $\epsilon > 0$, and choose ϕ and Δ sufficiently small (i.e., $0 < \phi < \bar{\phi}(\epsilon)$ and $\overline{\delta}(\epsilon) < \delta = e^{-r\Delta} < 1$) so that Lemmas 1 and 2 hold. In particular, we may assume

$$(1-\delta)\overline{v} < \epsilon \quad \text{and} \quad \delta(v - p_c) \begin{cases} > w & \text{if } v \geq v_c, \\ < w & \text{if } v < v_c. \end{cases} \tag{B.2}$$

Also, from Lemma 1-(ii), (iii) and (iv),

$$u(\theta) = v - \max\left\{\frac{\delta\eta}{1-\delta(1-\eta)}(v - p^\dagger), w\right\} = \begin{cases} \frac{(1-\delta)v+\delta\eta p^\dagger}{1-\delta(1-\eta)} & \text{for } \theta \in \Theta_o \text{ such that } v \geq v_c, \\ v - w & \text{for } \theta \in \Theta_o \text{ such that } v < v_c. \end{cases} \tag{B.3}$$

From Lemma 1-(i) and (iv),

$$u(\theta) = v - w \begin{cases} > p_c & \text{for } \theta \in \Theta_r \text{ such that } v \geq v_c, \\ < p_c & \text{for } \theta \in \Theta_r \text{ such that } v < v_c. \end{cases} \tag{B.4}$$

We construct an assessment $\sigma_c = (\sigma_c^B, \sigma_c^S, f_c^S)$ as follows. Later, we will show that this assessment constitutes a quasi-Coasean equilibrium.

The Buyer's Strategy: Suppose that the seller offers p_n at $h_n \in H$. Any buyer type $\theta = (v, \tau)$ with $v \geq v_c$ accepts p_n if $p_n \leq P(\theta)$ and delays if $p_n > P(\theta)$, where

$$P(\theta) = P(v,\tau) \coloneqq \begin{cases} (1-\delta)v + \delta p_c & \text{if } v \geq v_c \text{ and } \tau = \tau_r, \\ (1-\delta)v + \delta p_c - \delta\eta(p_c - p^\dagger) & \text{if } v \geq v_c \text{ and } \tau = \tau_o. \end{cases}$$

We call $P(\theta)$ the reservation price of a buyer type θ. Any buyer type $\theta = (v, \tau)$ with $v < v_c$ accepts p_n if $p_n \leq u(\theta) = v - w$ and exercises the outside option if $p_n > u(\theta) = v - w$.

In what follows, let $\Theta_c \coloneqq \{(v,\tau) : v \geq v_c\}$ denote the buyer types with valuation weakly larger than v_c. Note that the reservation price $P(\theta)$ is defined only for buyer types in Θ_c. Note that $P(v_c, \tau_o) = p_c$ and $P(v,\tau) > p_c$ for all $\theta \in \Theta_c \backslash \{(v_c, \tau_o)\}$ by Lemma 1-(ii) and (iii).

The Seller's Strategy and Posterior Beliefs: Let p_c^+ denote the second lowest element in $\{P(v,\tau) : (v,\tau) \in \Theta_c\}$ (recall the p_c is the lowest element in this set). At any history h_n, the seller offers

$$p_n = p(h_n) \coloneqq \begin{cases} p_c^+ & \text{at the null history } h_n = h_0, \\ p_c & \text{at all other histories.} \end{cases} \tag{B.5}$$

The seller never randomizes at any history. At any non-null history $h_n = (p_0, \ldots, p_{n-1})$ such that $p_k > P(v_c, \tau_o) = p_c$ for all k, the buyer type (v_c, τ_o) still remains in the negotiation, and thus, the seller's posterior belief $f_c^S(h_n)$ is well-defined via the Bayes' rule. Any other non-null histories are off the path, and we assume that the seller's posterior belief assigns probability 1 to (v_c, τ_o) at all such histories. Note that $f_c^S(h_n)$ assigns zero probability to the buyer types with $v < v_c$ at any non-null history both on and off the path.

Equilibrium Outcome: σ_c induces the following play on the path. In period 0, the seller offers $p_0 = p(h_0) > p_c$ and all buyer types in $\Theta_c \backslash \{(v_c, \tau_o)\}$ accept it, while the buyer type (v_c, τ_o) chooses to delay. All other buyer types exercise the outside option in period 0. In period 1, the seller offers $p_1 = p_c < p(h_0)$ and the remaining buyer type (v_c, τ_o) accepts it.

The outcome of σ_c is consistent of the definition of quasi-Coasean equilibrium. In the remaining part of the proof, we show that σ_c is indeed an ϵ-PBE. We first prove the optimality of the buyer's strategy σ_c^B. Suppose that the seller offers p_n at $h_n \in H$ (in period n). If the buyer rejects p_n, the seller will offer $p_k = p_c$ in all subsequent periods. We analyze the responses of the rational and optimistic buyer types separately. First, suppose that the buyer is of a rational type. The buyer's payoff from rejecting p_n is $\max\{\delta(v-p_c), w\}$, where $\delta(v-p_c)$ is the payoff from accepting $p_{n+1} = p_c$ in the next period, and w is the payoff from exercising the outside option in the current period. Recall from (B.2) that $\delta(v-p_c) \geq w$ for any $\theta \in \Theta_c$ with $v \geq v_c$, hence these buyer types find it optimal to accept p_n iff $v - p_n \geq \delta(v-p_c)$, or equivalently, $p_n \leq P(v, \tau_r)$. On the other hand, $\delta(v-p_c) \leq w$ for all rational buyer types with $v \leq v_c$, and hence these buyer types find it optimal to accept p_n iff $u(v, \tau_r) = v - w \geq p_n$.

Next, suppose that the buyer is of an optimistic type. The buyer's payoff from rejecting p_n is

$$W_o(v) := \max\left\{\delta(1-\eta)(v-p_c) + \delta\eta(v-p^\dagger), \frac{\delta\eta}{1-\delta(1-\eta)}(v-p^\dagger), w\right\},$$

where $\delta(1-\eta)(v-p_c)+\delta\eta(v-p^\dagger)$ is the buyer's expected payoff from trading in the next period (an optimistic buyer believes that $p_{n+1} = p_c$ with probability $1-\eta$ and $p_{n+1} = p^\dagger$ with probability η), and $\frac{\delta\eta}{1-\delta(1-\eta)}(v-p^\dagger)$ is the payoff from waiting indefinitely until the seller trembles hands and offers $p^\dagger$. By Lemma 1-(iii) and (iv),

$$W_o(v) = \begin{cases} \delta(1-\eta)(v-p_c) + \delta\eta(v-p^\dagger) & \text{if } v \geq v_c \\ w & \text{if } v < v_c \end{cases}$$

Hence, each optimistic buyer type with $v \geq v_c$ indeed finds it optimal to accept p_n iff $v - p_n \geq \delta(1-\eta)(v - p_c) + \delta\eta(v-p^\dagger)$, or equivalently, $p_n \leq P(v, \tau_o)$. On the other hand, any optimistic buyer type with $v < v_c$ finds it optimal to accept p_n iff $u(v, \tau_o) = v - w \geq p_n$.

Next, we prove that $\sigma_c^S(h_n)$ is ϵ-best at any history $h_n \in H$, which will be a direct consequence of the observations in Claims 1–3.

Claim 1. *(i) $P(\theta) \leq u(\theta)$ for any $\theta = (v, \tau)$ such that $v \geq v_c$. (ii) $u(\theta) < p_c$ for any $\theta = (v, \tau)$ such that $v < v_c$.*

Proof. The part (i) follows the following observation: For any $\theta = (v, \tau)$ such that $v \geq v_c$,

$$P(\theta) = (1-\delta)v + \delta p_c = v - \delta(v-p_c) < v - w = u(\theta) \qquad \text{if } \tau = \tau_r,$$
$$P(\theta) = v - \delta(1-\eta)(v-p_c) - \delta\eta(v-p^\dagger) < v - \max\left\{\frac{\delta\eta}{1-\delta(1-\eta)}(v-p^\dagger), w\right\} = u(\theta) \qquad \text{if } \tau = \tau_o,$$

where the two inequalities follow (B.2) and Lemma 1, respectively. The part (ii) follows (B.3) and (B.4). □

Claim 2. *(i) $P^D(h_n; \sigma_c) = (p_c, \infty)$. (ii) $V_S^{\sigma_c}(p_c; h_n) + \epsilon \geq V_S^{\sigma_c}(p'; h_n)$ for any $h_n \in H$ and $p' \leq p_c$.*

Proof. The buyer type (v_c, τ_o) always belongs to $\text{supp}(f_c^S(h_n))$ and chooses to delay in response to any $p_n > p_c = P(v_c, \tau_o)$; hence, $(p_c, \infty) \subset P^D(h_n; \sigma_c)$ at any h_n. On the other hand, by construction of σ_c, all buyer types $\theta \in \Theta$ never choose to delay in response to any $p_n \le p_c$. Thus, $P^D(h_n; \sigma_c) = (p_c, \infty)$ at any h_n.

To show the part (ii) of the claim for the case $h_n = h_0$, suppose that the seller offers $p' \le p_c$ at h_0. Because $p' \notin P^D(h_0; \sigma_c)$, all buyer types accept p' iff $u(\theta) \ge p'$, and thus, $V_S^{\sigma_c}(p'; h_0) - V_S^{\sigma_c}(p_c; h_0) = \Pi(p') - \Pi(p_c) \le \epsilon$, where the last inequality holds due to Lemma 2. For the case $h_n \ne h_0$, note that the seller believes that the buyer's valuation v is weakly larger than v_c for any non-null history. Hence, any price $p' \le p_c$ is accepted at $h_n \ne h_0$, yielding $V_S^{\sigma_c}(p'; h_n) - V_S^{\sigma_c}(p_c; h_n) = p' - p_c \le 0 \le \epsilon$. □

Claim 3. *$|V_S^{\sigma_c}(p; h_n) - V_S^{\sigma_c}(p'; h_n)| \le \epsilon$ for any $p, p' \ge p_c$ and $h_n \in H$.*

Proof. Fix a history $h_n \in H$. Suppose that the seller offers $p \ge p_c$ in period n at h_n. By construction, the seller will offer $p_{n+1} = p_c$ in the next period. All the buyer types with $v \ge v_c$ will accept either p_n or $p_{n+1} = p_c$ for sure, while all buyer types with $v < v_c$ will exercise the outside option in period n. Hence,

$$V_S^{\sigma_c}(p; h_n) \ge \delta p_c \sum_{\theta \in \Theta : v \ge v_c} f_\Theta(v, \tau) \quad \forall p \ge p_c.$$

On the other hand, because $P(v, \tau) \le (1 - \delta)\overline{v} + \delta p_c$ for all buyer types with $v \ge v_c$, hence,

$$V_S^{\sigma_c}(p'; h_n) \le ((1 - \delta)\overline{v} + \delta p_c) \sum_{\theta \in \Theta : v \ge v_c} f_\Theta(v, \tau) \quad \forall p' \ge p_c.$$

Combining these two inequalities,

$$|V_S^{\sigma_c}(p; h_n) - V_S^{\sigma_c}(p'; h_n)| \le |\delta p_c - (1 - \delta)\overline{v} - \delta p_c| \le (1 - \delta)\overline{v} < \epsilon \quad \forall p, p' \ge p_c,$$

where the last inequality holds due to (B.2). □

Finally, we are ready to prove that σ_c^S is ϵ-best. First, consider the seller's strategy at the null history. By construction of σ_c^S, the seller will offer $p(h_0) \in (p_c, \infty) = P^D(h_0; \sigma_c)$. Claims 2 and 3 jointly imply that any deviation cannot increase the seller's payoff by more than ϵ, and hence σ_c^S is indeed ϵ-best at h_0.

Next, consider any non-null history $h_n = (p_0, p_1, \ldots, p_{n-1})$. There are two subcases.

- First, suppose that $p_k \le p_c^+$ for some k. In this case, $f_c^S(h_n)$ assigns probability 1 to the buyer type (v_c, τ_o), and hence, the equilibrium offer $p(h_n) = p_c = P(v_c, \tau_o)$ is exactly optimal for the seller.
- Second, suppose that $p_k > p_c^+$ for any k. In this case, the seller offers $p(h_n) = p_c \notin P^D(h_n; \sigma_c)$. By construction, all buyer types (v, τ) with $v < v_c$ already have exercised the outside option, while any remaining buyer type will accept any price weakly lower than p_c; hence, $V_S^{\sigma_c}(p'; h_n) = p' \le V_S^{\sigma_c}(p(h_n); h_n) = p_c$ for any $p' \notin P^D(h_n; \sigma_c)$. Finally, Claim 3 guarantees that the seller cannot increase her continuation payoff by more than ϵ by deviating to $p' \in (p_c, \infty) = P^D(h_n; \sigma_c)$.

Hence, the seller's strategy σ_c^S is indeed ϵ-best.

C Supplementary Figures

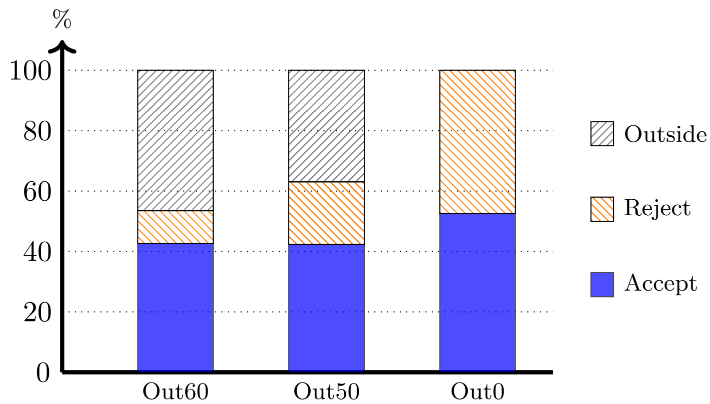


Figure 11: End-of-Bargaining States by Treatment

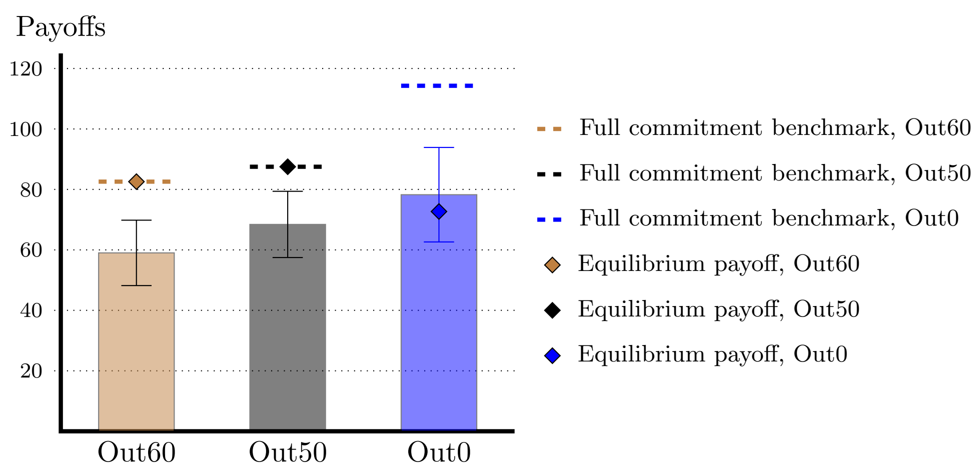


Figure 12: Seller's Earnings by Treatment

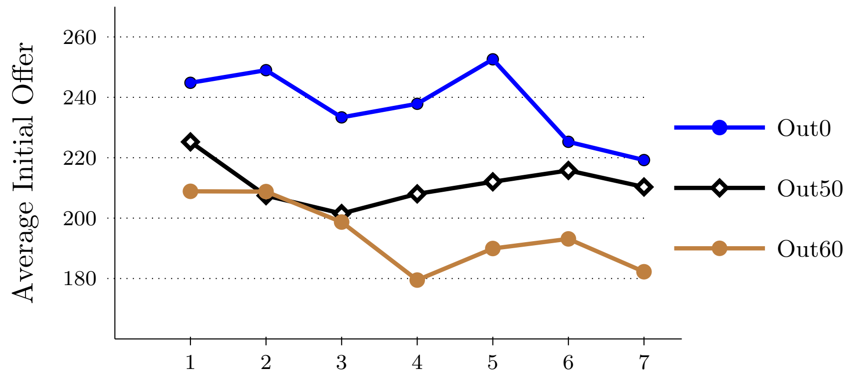


Figure 13: Round 1 Offer across Match

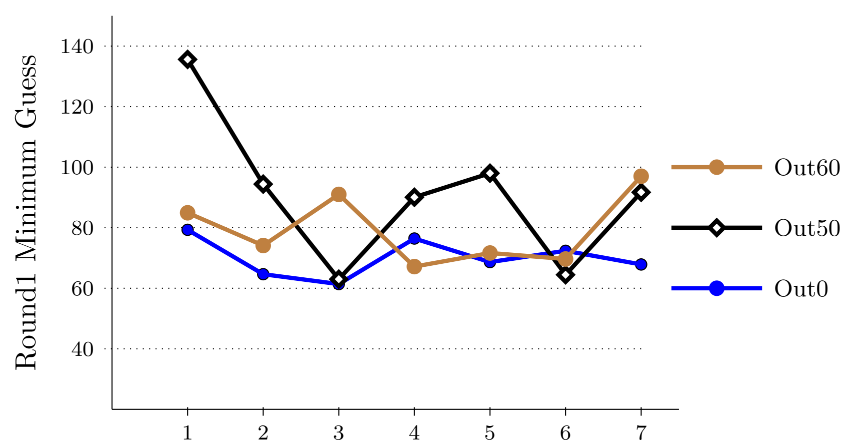


Figure 14: Minimum of the Guess after Round 1 Rejection across Match, by Treatment

D Bargaining with First-Order ϵ-Irrationality

D.1 Model

In this section, we consider the case that a small proportion of buyer types are possibly not fully rational. Suppose that the buyer's type space is given as follows:

$$\Theta = [\underline{v}, \overline{v}] \times \{\tau_r, \tau_{nr}\}.$$

A generic element of Θ is denoted by $\theta = (v, \tau)$, where v represents the buyer's valuation of the good, and τ indicates whether the buyer is rational ($\tau = \tau_r$) or non-rational ($\tau = \tau_{nr}$). The realizations of v and τ are stochastically independent and revealed only to the buyer before the negotiation begins. We denote the marginal probability of $\tau = \tau_{nr}$ by $\rho \in (0,1)$. The marginal density and distribution functions of v are denoted by $g : [\underline{v}, \overline{v}] \to \mathbb{R}$ and $G : [\underline{v}, \overline{v}] \to [0,1]$, respectively.[28] We assume that the hazard rate $g(v)/(1 - G(v))$ increases in v and $\underline{v}g(\underline{v}) < w$.[29]

The negotiation procedure is identical to the bargaining game in Section 2.1, including information structure, timing of moves, and payoff functions. In particular, the buyer has the identical payoff function and the set of feasible moves as specified in Section 2.1, regardless of whether he is rational or non-rational.[30] We also maintain the assumptions that (i) all buyer types have the same outside option w, with the focus on the case that $w \in (0, \infty)$, and (ii) all players have the common discounting factor $e^{-r\Delta} \in (0,1)$, where $r > 0$ and $\Delta > 0$ denotes the discounting rate and the time duration between two consecutive periods, respectively.

The non-rational buyer types differ from the rational types only in that they may make ϵ-optimal decisions in equilibrium, while the rational types always make the exactly optimal decision. Formally, we employ *ϵ-equilibrium* as our equilibrium concept in this section. Its definition is identical to the standard definition PBE except that the behavioral strategies of non-rational buyer types are required to be only ϵ-optimal; the seller and all rational buyer types play exactly optimal behavioral strategies after any history, and the seller's posterior belief is obtained by Bayes rule whenever possible. For any $\epsilon > 0$, $\Delta > 0$, and $\rho \in (0,1)$, let $\mathcal{E}^*(\epsilon|\Delta, \rho)$ denotes the set of all ϵ-equilibrium.[31]

Remark 1. *The above model aims to capture the potential irrationality of some buyer types in a parsimonious way. Note that we keep the model's deviation from the standard model (such that all players are fully rational) as minimally as possible. Formally, we will focus on the case that (i) the fraction of non-rational buyer types is small (i.e., $\rho \approx 0$), and moreover, (ii) the degree of those non-rational types' irrationality is also small (i.e., $\epsilon > 0$ is small).*

Note that the model degenerates to the bargaining model in Section 2.1 if $\rho = 0$ or $\epsilon = 0$. In this case, the result in BP applies, and hence, the full-commitment benchmark outcome is achieved in the essentially

[28] Departing from Section 2.1, we assume here that v is drawn from continuous probability distribution. This assumption simplifies the proof of the results in this section. All results remain valid if we assume that v is drawn from a discrete probability distribution.

[29] This assumption guarantees that the full commitment benchmark (2.2) in Section 2 admits a unique interior solution.

[30] Hence, the non-rational buyer types should *not* be equated with commitment types (who blindly stick to certain strategies) in reputational bargaining games.

[31] Note that ϵ-equilibrium differs from ϵ-PBE in Section 3 which allows the seller plays an ϵ-optimal strategy.

unique equilibrium; in particular, Coase conjecture fails and neither inter-temporal price discrimination nor delay occurs when the seller does not have full commitment power. The BP's result fails if both ρ and ϵ are positive.

Definition 2. *For any $\epsilon > 0$, an ϵ-equilibrium is called a* quasi-Coasean ϵ-equilibrium *if the following outcome is induced on its path.*

- *Delay and Inter-temporal Pricing: The negotiation takes multiple periods with positive probability, and p_n declines over time on the equilibrium path.*
- *There is v_r^* and $v_{nr}^* \in [\underline{v}, \overline{v}]$ such that the following holds on the equilibrium path:*
  - *(i) Positive selection: Any rational buyer type with $v < v_r^*$ exercises the outside option immediately. Similarly, any non-rational buyer type with $v < v_{nr}^*$ exercises the outside option immediately.*
  - *(ii) Negative selection: Any rational buyer type with $v > v_r^*$ trades with the seller (possibly after a delay); among these buyer types, one with a higher valuation v trades earlier than others. Similarly, any non-rational buyer type with $v > v_{nr}^*$ trades with the seller (possibly after a delay); among these buyer types, one with a higher valuation v trades earlier than others.*

Proposition 7. *Suppose $w > 0$. For any $\epsilon > 0$ and $\rho \in (0, 1)$, there is $d_{\rho,\epsilon} > 0$ such that $\mathcal{E}^*(\epsilon|\Delta, \rho)$ admits a quasi-Coasean ϵ-equilibrium whenever $\Delta \in (0, d_{\rho,\epsilon})$.*

The proof of Proposition 7 is constructive. In this proof, we adopt the notations from Section 2. Fix $\epsilon > 0$ and $\rho > 0$. Choose $v^* \in (\underline{v}, \overline{v})$ and $\Delta > 0$ such that

$$\frac{1 - e^{-r\Delta}}{e^{-r\Delta}} w < \epsilon \quad \text{and} \quad v_\Delta^* := v^* + \frac{1 - e^{-r\Delta}}{e^{-r\Delta}} w < \overline{v} \tag{D.1}$$

$$v_\Delta^* - \frac{1 - G(v_\Delta^*)}{g(v_\Delta^*)} = w \quad \iff \quad (v - w)(1 - G(v - w)) \text{ is maximized at } v = v_\Delta^*. \tag{D.2}$$

v_Δ^* and v^* will play the role of v_r^* and v_{nr}^* in Definition 2. In what follows, we will construct an ϵ-equilibrium such that the following play is observed on its path:

(i) The seller offers

$$p_0^C := (1 - e^{-r\Delta})v_\Delta^* + e^{-r\Delta}(v^* - w) = v_\Delta^* - w \quad \text{and} \quad p_1^C := v^* - w < p_0^C,$$

in periods 0 and 1, respectively.

(ii) The rational buyer types with $v \geq v_\Delta^*$ trades with the seller in period 0.

(iii) The non-rational buyer types with $v \in [v_\Delta^*, \overline{v}]$ trade with the seller in period 0.

(iv) The non-rational buyer types with $v \in (v^*, v_\Delta^*)$ trades with the seller in period 1.

(v) All other buyer types exercise the outside option in period 0.

Equilibrium Assessment Consider the following strategy profile. Define

$$p_0^C := (1 - e^{-r\Delta})v_\Delta^* + e^{-r\Delta}(v^* - w) = v_\Delta^* - w \quad \text{and} \quad p_1^C := v^* - w,$$

so that

$$\max\{v - p_0, w\} \geq e^{-r\Delta}(v - p_1^C) \quad \text{if} \quad v \in [\underline{v}, \overline{v}] \quad \text{and} \quad p_0 \in [0, p_0^C], \quad \text{and} \tag{D.3}$$

$$v - p_0 = w = e^{-r\Delta}(v - p_1^C) \quad \text{if} \quad v = v_\Delta^* \quad \text{and} \quad p_0 = p_0^C. \tag{D.4}$$

We divide all non-null histories into the following two groups:

$$H_A := \{(p_0, p_1, \ldots, p_{n-1}) \in H \backslash \{h_0\} : p_k \in (p_1^C, p_0^C] \text{ for all } k \leq n-1\},$$
$$H_B := \{(p_0, p_1, \ldots, p_{n-1}) \in H \backslash \{h_0\} : p_k \notin (p_1^C, p_0^C] \text{ for some } k \leq n-1\}.$$

(C1) The seller offers

$p_0 = p_0^C$	at the null history,
$p_n = p_1^C$	in any period n after $h_n \in H_A$,
$p_n = \overline{v} - w$	in any period n after $h_n \in H_B$.

(C2) Suppose that the seller offers $p_n \in (p_1^C, p_0^C]$ after any history $h_n \in H_A \cup \{h_0\}$.

(C2-a) All the rational buyer types and the non-rational buyer types with valuation $v \in [\underline{v}, v^*] \cup [v_\Delta^*, \overline{v}]$ employ the following behavioral strategy:

Accept p_n	(i.e., $\sigma^B(p_n; h_n, \theta)[T] = 1$)	if $v - p_n \geq w$,
Exercise the outside option	(i.e., $\sigma^B(p_n; h_n, \theta)[O] = 1$)	if $w > v - p_n$.

(C2-b) All non-rational buyer types with $v \in (v^*, v_\Delta^*)$ delay the negotiation to the next period.

(C3) Suppose that the seller offers $p_n \notin (p_1^C, p_0^C]$ after any history $h_n \in H_A \cup \{h_0\}$. Then, all buyer types accept p_n if $v - p_n \geq w$ and exercise the outside option in all other cases.

(C4) Suppose that the seller offers $p_n \geq 0$ after any history in $h_n \in H_B$. Then, all buyer types accept p_n if $v - p_n \geq w$ and exercise the outside option in all other cases.

Next, we specify the seller's posterior beliefs. Note that, on the equilibrium path, the seller offers $p_0 = p_0^C$ and then p_1^C in the first two periods, and all buyer types either exercise the outside option or trade with the seller by the end of period 1. In particular, any history in H_B lies off the equilibrium path, and hence we may assign any posterior belief for those cases.

(C5) According to (C1)–(C4), non-rational buyer types with $v \in (v^*, v_\Delta^*)$ choose to delay any offer in $(p_1^C, p_0^C]$ after any history in $H_A \cup \{h_0\}$. Hence, the Bayes rule is applicable to pin down the seller's posterior belief after any history in $H_A \cup \{h_0\}$.

(C6) After any history in H_B, we assume that the seller believes $(v, \tau) = (\overline{v}, \tau_r)$ with probability 1.

(Approximate) Optimality of the Buyer's Behavioral Strategy We first show (C2-a) and (C2-b) are exactly optimal and ϵ-optimal, respectively. By (C1), the seller will offer $p_{n+1} = p_1^C$ in the next period because (h_n, p_n) still belongs to H_A. If the buyer rejects $p_{n+1} = p_1^C$ again, the seller will offer $p_{n+k} = \overline{v} - w$ in all future periods. Hence, the buyer's highest payoff from delaying in period n is

$$e^{-r\Delta} \max\{w, v - p_1^C, e^{-r\Delta}(v - \overline{v} + w)\},$$

which is less than $\max\{v - p_n, w\}$ by (D.3). Hence, it is optimal for the buyer to accept p_n if $v - p_n \geq w$ and exercises the outside option if $v - p_n < w$. This shows that (C2-a) is optimal for the buyer.

To show ϵ-optimality of (C2-b) for the buyer types with $v \in (v^*, v_\Delta^*)$, it suffices to show

$$v - p_n \leq e^{-r\Delta} \max\{v - p_1^C, w\} + \epsilon \quad \text{and} \quad w \leq e^{-r\Delta} \max\{v - p_1^C, w\} + \epsilon, \tag{D.5}$$

where $e^{-r\Delta} \max\{v - p_1^C, w\}$ represents the payoff from delaying the negotiation in period n. To show (D.5), first note that

$$v - p_n - e^{-r\Delta}(v - p_1^C) < (1 - e^{-r\Delta})(v_\Delta^* - p_1^C) = \frac{1 - e^{-r\Delta}}{e^{-r\Delta}} w < \epsilon, \tag{D.6}$$

where the first inequality in (D.6) holds because $v < v_\Delta^*$, and the last inequality in (D.6) follows (D.1). We also have $w \leq e^{-r\Delta} w + \epsilon$ by (D.1). Summing up all the observations so far, (D.5) holds and therefore (C2-b) is indeed ϵ-optimal for the buyer with $v \in (v^*, v_\Delta^*)$.

Next, to show the optimality of (C3), suppose that the buyer rejects $p_n \notin (p_1^C, p_0^C]$ after $h_n \in H_A \cup \{h_0\}$. Then, $(h_n, p_n) \in H_B$, and hence, the seller will offer $p_k = \overline{v} - w$ in all subsequent periods $k \geq n$. Note that the offer $p_k = \overline{v} - w$ is not acceptable to almost all buyer types, and therefore, the buyer's continuation payoff from delaying is dominated by the payoff from exercising the outside option in period n. This shows (C3) is optimal for all buyer types. Similarly, (C4) is also optimal for all buyer types.

Optimality of the Seller's Behavioral Strategy On the equilibrium path, the seller offers $p_0 = p_0^C$ and then $p_1 = p_1^C$. The negotiation ends by the end of period 1 for sure. We first check the seller has no incentive to deviate from the equilibrium after any non-null history in H_A. To see this, note that only the non-rational buyer types with $v \in (v^*, v_\Delta^*)$ still remain active after any $h_n \in H_A$. By (C2)–(C4), in all subsequent periods, those buyer types never accept any offer strictly higher than p_1^C; hence, it is indeed optimal for the seller to offer $p_n = p_1^C$ after h_n.

Next, we show that the seller has no incentive to deviate in period 0 (at the null history). We first characterize the seller's expected payoff from offering p_0 in period 0, assuming that all players will follow the equilibrium strategy thereafter. Let $V_S(p_0)$ denote this payoff. First, consider the case $p_0 \in [\underline{v} - w, p_1^C] \cup (p_0^C, \overline{v} - w]$. By (C3) and (C4), all the buyer types accept p_0 iff $v - p_0 \geq w$ and exercise the outside option iff $v - p_0 < w$. Hence,

$$V_S(p_0) = p_0[1 - G(p_0 + w)] \quad \forall p_0 \in [\underline{v} - w, p_1^C] \cup (p_0^C, \overline{v} - w]. \tag{D.7}$$

Next, consider the case $p_0 \in (p_1^C, p_0^C]$. By (C1), the seller will offer $p_1 = p_1^C$ in the next period if p_0 is rejected in period 0. Let $v_0 \in [v^*, v_\Delta^*]$ denote the buyer's value such that $v_0 - p_0 = w$.

- By (C2-a), each rational buyer type $\theta = (v, \tau_r)$ trades with the seller at p_0 iff

$$v - p_0 \geq w \quad \Longleftrightarrow \quad v \geq v_0$$

and exercises the outside option in period 0 in all other cases.

- By (C2-b), each non-rational buyer type with $v \notin (v^*, v^*_\Delta)$ responses to the seller's offer p_0 exactly identically to the rational buyer types with the same valuation (as described in the last bullet point). In particular, all non-rational buyer types with $v > v^*_\Delta$ accept p_0, while all non-rational buyer types with $v < v^*$ exercise the outside option. On the other hand, each non-rational buyer type with $v \in (v^*, v^*_\Delta)$ rejects p_0 and then trades with the seller at p_1^C in the next period.

Hence, the seller's expected payoff from offering $p_0 \in (p_1^C, p_0^C]$ is bounded as follows:

$$\begin{aligned} V_S(p_0) &= (1-\rho)p_0[1 - G(p_0 + w)] + \rho p_0[1 - G(v^*_\Delta)] + e^{-r\Delta}\rho[G(v^*_\Delta) + G(v^*)]p_1^C \\ &\leq (1-\rho)p_0[1 - G(p_0 + w)] + \rho p_0[1 - G(v_0)] + e^{-r\Delta}\rho[G(v_0) - G(v^*)]p_1^C \\ &= p_0[1 - G(p_0 + w)] + e^{-r\Delta}\rho[G(v_0) - G(v^*)]p_1^C \end{aligned} \tag{D.8}$$

for any $p_0 \in (p_1^C, p_0^C]$, and

$$V_S(p_0^C) = p_0^C[1 - G(p_0^C + w)] + e^{-r\Delta}\rho[G(v^*_\Delta) + G(v^*)]p_1^C. \tag{D.9}$$

Finally, we are ready to prove that the seller has no incentive to deviate in period 0. By (D.7)–(D.9), it suffices to show

$$p_0^C[1 - G(p_0^C + w)] \geq \max_{p_0 \in [\underline{v} - w, \overline{v} - w]} p_0[1 - G(p_0 + w)] = \max_{v \in [\underline{v}, \overline{v}]} (v - w)(1 - G(v)).$$

Because $p_0^C = (1 - e^{-r\Delta})v^*_\Delta + e^{-r\Delta}(v^* - w) = v^*_\Delta - w$ by definition of v^*_Δ and p_0^C, the left-hand side of the last inequality equals $(v^*_\Delta - w)(1 - G(v^*_\Delta))$. Hence, the last inequality directly follows (D.2).

It remains to check the seller's incentive after any non-null history in H_B. By (C1), the seller insists on $p_n = \overline{v} - w$ in all subsequent periods after $h_n \in H_B$. Additionally, by (C6), the seller believes that the seller believes $v = \overline{v}$ with probability 1 after any deviation from the equilibrium play; hence, the seller's insisting on $p_n = \overline{v} - w$ is clearly optimal for the seller.

E Bargaining Between Inequity-Averse Buyer and Seller

In this section, we show that the inequity aversion (with a reasonable choice of parameters) does not make a qualitative change in the bargaining model in Section 2.1. Suppose the same bargaining environment, except that the buyer and the seller have the inequity aversion à la Fehr and Schmidt (1999).

- Suppose that the buyer exercises the outside option in period $n \geq 0$. In this case, the buyer and the seller obtain the final discounted utilities

$$U_B = e^{-rn\Delta}\Big[w - \alpha_B \max\{0-w,0\} - \beta_B \max\{w-0,0\}\Big] = e^{-rn\Delta}(1-\beta_B)w \tag{E.1}$$

$$U_S = e^{-rn\Delta}\Big[0 - \alpha_S \max\{w-0,0\} - \beta_S \max\{0-w,0\}\Big] = e^{-rn\Delta}(-\alpha_S w), \tag{E.2}$$

respectively, where

(i) $e^{-r\Delta}$ represents the common discounting factor;

(ii) The pecuniary payoff of the buyer is w in (E.1), and that of the seller is 0 in (E.2);

(iii) for both $k = B$ and S, α_k and β_k respectively capture each player's distaste for disadvantageous and advantageous inequities.

- Suppose that the buyer accepts the seller's offer p_n in period n. The seller obtains pecuniary outcome p_n, and the buyer does $v - p_n$. Hence, the buyer's and the seller's final discounted utilities are

$$U_B = e^{-rn\Delta}\Big[v - p_n - \alpha_B \max\{p_n - (v-p_n), 0\} - \beta_B \max\{(v-p_n) - p_n, 0\}\Big] \tag{E.3}$$

$$U_S = e^{-rn\Delta}\Big[p_n - \alpha_S \max\{(v-p_n) - p_n, 0\} - \beta_S \max\{p_n - (v-p_n), 0\}\Big] \tag{E.4}$$

respectively.

The negotiation procedure is identical to the bargaining game in Section 2.1, including information structure, timing of moves, and payoff functions. As in Section 2.1, the buyer's value of the good (i.e., the buyer's type) $v \in V = \{v_1, v_2, \ldots, v_N\}$ is his private information, where $0 < \underline{v} := v_1 < v_2 < \ldots < \overline{v} := v_N$. The seller holds the prior belief that $v = v_j$ with probability $f(v_j) > 0$. Let $F(v) := \sum_{v' \le v} f(v')$ denote the cumulative distribution function. We also maintain the assumptions that all buyer types have the same outside option w and $0 < w < \underline{v}$. The solution concept is PBE.

Assumption We impose the following assumption throughout this section.

$$\alpha_k \ge 0 \text{ and } \beta_k < 1/2 \text{ for both } k = B \text{ and } S. \tag{A^*}$$

The condition $\beta_k < 1/2$ in Assumption (A^*) guarantees that both (E.3) and (E.4) are monotone in p_n. If $\beta_B > 1/2$, for example, the buyer's distaste for advantageous inequality becomes so strong that the buyer's overall utility increases as the buyer makes more payment to the seller; the condition $\beta_k < 1/2$ excludes such extreme cases. Note that $\beta_k < 1/2$ still allows β_k to be moderately positive or even negative.

Notations Throughout this section, we adopt the notations from Section 2. Additionally, define

$$\widetilde{U}_B(v,p) := v - p - \alpha_B \max\{p-(v-p),0\} - \beta_B \max\{(v-p)-p,0\} = \begin{cases} (1-\beta_B)v - (1-2\beta_B)p & \text{if } p \le v/2 \\ (1+\alpha_B)v - (1+2\alpha_B)p & \text{if } p \ge v/2 \end{cases}$$

as the buyer's utility from trading at price p (in period 0). Similarly,

$$\widetilde{U}_S(v,p) := p - \alpha_S \max\{(v-p)-p,0\} - \beta_S \max\{p-(v-p),0\}$$

denotes the seller's utility from trading with the buyer type v at price p (in period 0). Finally, define

$$P^* := \arg\max_{p \geq 0} \sum_{k=1}^{N} \mathbb{1}\{\widetilde{U}_B(v_k, p) \geq (1 - \beta_B)w\}\widetilde{U}_S(v_k, p)f(v_k)$$

P^* is the solution of the seller's profit maximization problem if she is endowed with the full commitment power.

Now, we are ready to state and prove our main proposition in this section. For any $p^* \in P^*$, there is a PBE, denoted by $\sigma(p^*)$, such that

(i) the seller offers p^* in period zero, and

(ii) the buyer accepts it iff $\widetilde{U}(v, p) \geq (1 - \beta_B)w$ or, otherwise, exercises the outside option.

Proposition 8. *All PBEs are payoff-equivalent to one of PBE in* $\{\sigma(p^*) : p^* \in P^*\}$.

This proposition directly follows the following three preliminary lemmas.

Lemma 3. $\widetilde{U}_B(v, p)$ *increases in* v *and decreases in* p. $\widetilde{U}_S(v, p)$ *increases in* p.

Proof. The monotonicity of $\widetilde{U}_S(v, p)$ with respect to p is straightforward, hence we focus on $\widetilde{U}_B(v, p)$. Assumption (A^*) guarantees that $\widetilde{U}_B(v, p)$ clearly decreases in p for any fixed $v \in V$. To show that $\widetilde{U}_B(v, p)$ increases in v, first note that the case $\alpha + \beta < 0$ is trivial, because in this case

$$\widetilde{U}_B(v, p) = \max\{(1 - \beta_B)v - (1 - 2\beta_B)p, (1 + \alpha_B)v - (1 + 2\alpha_B)p\}$$

and both $(1 - \beta_B)v - (1 - 2\beta_B)p$ and $(1 + \alpha_B)v - (1 + 2\alpha_B)p$ increases in v, and so does their maximum $\widetilde{U}_B(v, p)$. Hence, let us focus on the case

$$1 - 2\beta_B \leq 1 + 2\alpha_B \quad \iff \quad \alpha_B + \beta_B \geq 0,$$

and therefore

$$\widetilde{U}_B(v, p) = \min\{(1 - \beta_B)v - (1 - 2\beta_B)p, (1 + \alpha_B)v - (1 + 2\alpha_B)p\} \quad \forall v, p.$$

Fix $p \geq 0$ and consider two buyer types v_H and v_L such that $v_H \geq v_L$. The only non-trivial case arises when $\frac{v_L}{2} \leq p \leq \frac{v_H}{2}$. In this case,

$$\widetilde{U}(v_H, p) = (1 - \beta_B)v_H - (1 - 2\beta_B)p \quad \text{and} \quad \widetilde{U}(v_L, p) = (1 + \alpha_B)v_L - (1 + 2\alpha_B)p$$

and therefore, $\widetilde{U}(v_H, p) \geq \widetilde{U}(v_L, p)$ iff $(1 - \beta_B)v_H - (1 + \alpha_B)v_L \geq -2(\alpha_B + \beta_B)p$. Indeed,

$$-2(\alpha_B + \beta_B)p \leq -(\alpha_B + \beta_B)v_L = (1 - \beta_B)v_L - (1 + \alpha_B)v_L \leq (1 - \beta_B)v_H - (1 + \alpha_B)v_L \tag{E.5}$$

where the first and second inequalities hold due to conditions $\frac{v_L}{2} \leq p$ and $1 - \beta_B > 0$, respectively. $\square$

For any $v \geq w$, $\widetilde{U}_B(v, 0) = (1 - \beta_B)v \geq (1 - \beta_B)w$ and $\widetilde{U}_B(v, v) = -\alpha_B v \leq 0 \leq (1 - \beta_B)w$. Hence, there is a unique p_v such that

$$\widetilde{U}_B(v, p_v) = (1 - \beta_B)w$$

where $(1-\beta_B)w$ is the buyer's utility from exercising the outside option. That is, the buyer type v is indifferent between accepting p_v and taking the outside option. Lemma 3 implies that p_v increases in v. For any buyer type $v \geq w$, denote such price level $u^*(v)$ (i.e., $u^*(v) \equiv p_v$) and call this *net value*. The buyer never accepts a price higher than $u^*(v)$.

Fix any PBE $\sigma = (\sigma^B, \sigma^S, f^S)$. For any non-null history $h_n \in H$ and price $p_n \geq 0$, let $\widetilde{\Gamma}^\sigma(h_n, p_n)$ denote the continuation game after (i) the buyer has rejected $p_0, p_1, \ldots, p_{n-1}$ in the first n periods, and then, (ii) the seller charges p_n in period n (but the buyer has not responded in period n yet). $\widetilde{\Gamma}^\sigma(h_n, p_n)$ begins with the buyer's decision node at which the buyer decides whether to accept p_n, exercise the outside option, or delay. Similarly, define $\widetilde{\Gamma}^\sigma(h_0, p_0)$ as the continuation game after the seller charges p_0 in period 0 (but the buyer has not responded yet). For any $(h_n, p_n) \in H \times [0, \infty)$, a history $h_m = (h_n, p_n, p_{n+1}, \ldots, p_{m-1})$, where $m \geq n+2$, is called *reachable in* $\widetilde{\Gamma}^\sigma(h_n, p_n)$ if it lies on the path of σ. Formally:

(i) $p_{n+1} \in \operatorname{supp} \sigma^S(h_n, p_n)$ and $p_k \in \operatorname{supp} \sigma^S(h_n, p_n, p_{n+1}, \ldots, p_{k-1})$ for all $k = n+2, \ldots, m-1$.

(ii) $\sum_{v \in V} f^S(v; h_n) \sigma^B(p_n; h_n, v)[D] > 0$ and $\sum_{v \in V} f^S(v; h_n, p_n, \ldots, p_{k-1}) \sigma^B(p_k; (h_n, p_n, \ldots, p_{k-1}), v)[D] > 0$ for all $k = n+1, \ldots, m-1$.

Lemma 4. *In any PBE $\sigma = (\sigma^B, \sigma^S, f^S)$, $\inf \operatorname{supp} \sigma(h_n) \geq u^*(\underline{v}^\sigma(h_n))$ for all history $h_n \in H$.*

Proof. For any history h_n, define $\underline{p}^\sigma(h_n) := \inf \operatorname{supp} \sigma^B(h_n)$ and $G := \sup_{h_n \in H} u^*(\underline{v}^\sigma(h_n)) - \underline{p}^\sigma(h_n)$. Suppose for contradiction $G > 0$. Pick a positive number ϵ and a history $h_m \in H$ such that

$$u^*(\underline{v}^\sigma(h_m)) - \underline{p}^\sigma(h_m) > G - \epsilon > 0 \quad \text{and} \quad \epsilon < \frac{(1-\delta)\underline{v}}{2} \frac{\min\{1+\alpha_B, 1-\beta_B\}}{\max\{1+2\alpha_B, 1-2\beta_B\}}. \tag{E.6}$$

Also, pick $p_m \in \operatorname{supp} \sigma^S(h_m)$ such that $p_m \in [\underline{p}^\sigma(h_m), \underline{p}^\sigma(h_m) + \epsilon)$.

We claim that all buyer types in $\operatorname{supp} f^S(h_m)$ accept p_m with probability 1. It suffices to show that all these buyer types strictly prefer accepting p_m to any other alternatives. First of all, because $p_m < u^*(\underline{v}^\sigma(h_m))$, all buyer types in $\operatorname{supp}(f^S(h_m))$ strictly prefer accepting p_m to exercising the outside option in period m or any future period. Next, we show that all of these buyer types also strictly prefer accepting p_m to any delayed purchase. Suppose for contradiction that there are $v^\diamond \in \operatorname{supp} f^S(h_m)$ and $h_\ell = (h_m, p_m, p_{m+1}, \ldots, p_\ell)$, where $\ell > m$, such that (i) h_ℓ is reachable in $\widetilde{\Gamma}^\sigma(h_m, p_m)$, (ii) $\underline{v}^\sigma(h_m) \leq \underline{v}^\sigma(h_\ell) \leq v^\diamond$, and (iii) $\widetilde{U}_B(v^\diamond, p_m) \leq \delta^{\ell-m} \widetilde{U}_B(v^\diamond, \underline{p}^\sigma(h_\ell))$. By the definition of G and (E.6),

$$p_m \leq \underline{p}^\sigma(h_m) + \epsilon \leq u^*(\underline{v}^\sigma(h_m)) - G + 2\epsilon \quad \text{and} \quad \underline{p}^\sigma(h_\ell) \geq u^*(\underline{v}^\sigma(h_\ell)) - G \geq u^*(\underline{v}^\sigma(h_m)) - G.$$

Hence, because $\widetilde{U}_B(v^\diamond, p)$ decreases in p,

$$\widetilde{U}_B(v^\diamond, u^*(\underline{v}^\sigma(h_m)) - G + 2\epsilon) \leq \widetilde{U}_B(v^\diamond, p_m) \leq \delta^{\ell-m} \widetilde{U}_B(v^\diamond, \underline{p}^\sigma(h_\ell)) \leq \delta^{\ell-m} U_B(v^\diamond, u^*(\underline{v}^\sigma(h_m)) - G).$$

and therefore,

$$(1-\delta^{\ell-m}) \widetilde{U}_B(v^\diamond, u^*(\underline{v}^\sigma(h_m)) - G) \leq \widetilde{U}_B(v^\diamond, u^*(\underline{v}^\sigma(h_m)) - G) - \widetilde{U}_B(v^\diamond, u^*(\underline{v}^\sigma(h_m)) - G + 2\epsilon).$$

The right-hand side on the last inequality is bounded from above by $2\epsilon \max\{1+2\alpha_B, 1-2\beta_B\}$, while the left-hand side is bounded from below by $(1-\delta)\underline{v}\min\{1+\alpha_B, 1-\beta_B\}$. Hence, the last inequality implies

$$\epsilon \geq \frac{(1-\delta)\underline{v}}{2} \frac{\min\{1+\alpha_B, 1-\beta_B\}}{\max\{1+2\alpha_B, 1-2\beta_B\}}.$$

This contradicts (E.6).

By the essentially same argument, any deviation to a price in $(p_m, \underline{p}^\sigma(h_m)+\epsilon)$ also leads to an immediate trade with probability 1. This means that p_m is not a best response at h_m, contradicting the supposition that $p_m \in \operatorname{supp} f^S(h_m)$. □

Finally, the next lemma shows that all buyer types never delay the negotiation in response to any seller's offer in period 0.

Lemma 5. *In any PBE, all buyer types never choose to delay in response any seller's offer p_0 in period 0.*

Proof. This lemma easily follows the previous lemma. Fix a PBE σ, and suppose that the seller offers p_0 in period 0. Suppose for contradiction that a positive measure of buyer types choose to delay in response to p_0. Let $\underline{v}^\sigma(p_0)$ denote the lowest buyer type among them. Then, the last lemma shows that the seller never offers lower than $u(\underline{v}^\sigma(p_0))$ in the continuation game. Hence, each buyer's payoff from delaying is bounded from above by

$$e^{-r\Delta}\max\{\widetilde{U}_B(v, u^*(\underline{v}(p_0))), (1-\beta_B)w\}.$$

The buyer type $\underline{v}^\sigma(p_0)$ would choose to delay in period 0 only if

$$\max\{\widetilde{U}_B(\underline{v}^\sigma(p_0), p_0), (1-\beta_B)w\} \leq e^{-r\Delta}\max\{\widetilde{U}_B(\underline{v}^\sigma(p_0), u^*(\underline{v}(p_0))), (1-\beta_B)w\} \tag{E.7}$$

where the left-hand side is the payoff from concluding the negotiation in period 0. By the definition of $u^*(\cdot)$, the right-hand side of the last inequality equals

$$e^{-r\Delta}\max\{\widetilde{U}_B(\underline{v}^\sigma(p_0), u^*(\underline{v}(p_0))), (1-\beta_B)w\} = e^{-r\Delta}\widetilde{U}_B(\underline{v}^\sigma(p_0), u^*(\underline{v}(p_0))) = e^{-r\Delta}(1-\beta_B)w > 0.$$

Hence, (E.7) holds only if

$$(1-\beta_B)w \leq \max\{\widetilde{U}_B(\underline{v}^\sigma(p_0), p_0), (1-\beta_B)w\} \leq e^{-r\Delta}(1-\beta_B)w.$$

This is impossible given that $w > 0$. □

Given that all buyer types never choose to delay, the seller's equilibrium offer in period 0 must be chosen from P^*. This completes the proof of Proposition 8.