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9913
2022

August 2022

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CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
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CESifo *Working Paper No. 9913*

Viable Nash Equilibria: An Experiment

Abstract

This paper examines the usefulness of Kalai (2020)'s measure of the viability of Nash equilibrium. We experimentally study a class of participation games, which differ in the number of players, the success threshold, and the payoff to not participating. We find that Kalai's measure captures well how the viability of the everyone-participates (eP) equilibrium depends on the success threshold; the measure does not capture other elements of the game which affect the likelihood that the eP equilibrium is played.

JEL-Codes: C000, C700, C920, D900.

Keywords: Nash equilibrium, viability, laboratory experiments, coordination game.

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August 23, 2022
We deeply thank Ehud Kalai for his comments, encouragement, and support for this project. Duk Gyoo Kim thanks Christian Hilscher for research assistance. John Wooders gratefully acknowledges financial support from Tamkeen under the NYU Abu Dhabi Research Institute Award CG005.

1 Introduction

Nash equilibrium is a fundamental solution concept in game theory. However, Nash equilibria differ in the likelihood that they are played. In some games a Nash equilibrium is played robustly, i.e., it is viable, while in other games no Nash equilibria are viable. Kalai (2020) proposes two simple indices—a "formation" index and a "defection" index—to measure the *viability* of an equilibrium in a n-player strategic form game. The formation index of a Nash equilibrium is the minimum number of players, called loyalists, who, when committed to playing their part of the equilibrium, make it a dominant strategy for each of the remaining players to play their part as well. In other words, the formation index measures the number of loyalists required to form an equilibrium. An equilibrium is less viable as the formation index is larger.

The present paper tests the empirical usefulness of Kalai's viability measure. Is a Nash equilibrium indeed less likely to be played as the formation index is larger? We also investigate whether the viability of an equilibrium depends on aspects of the game which are not accounted for in Kalai (2020)'s viability measure.

Our experiment studies n-player "participation" games, in which players simultaneously choose whether or not to participate. A participation game is characterized by three parameters: the number of players (n), a success threshold (t), and a probability (k) which governs the payoffs to players who choose not to participate. There are two possible payoffs: if $t+1$ or more players participate, then each participating player gets the "success" payoff; otherwise, each participating player gets the "failure" payoff; players who do not participate obtain a lottery which gives the success payoff with probability k and the failure payoff with probability $1-k$. A participation game has two pure strategy Nash equilibria: everyone-participates (eP) and no-one-participates (nP). The formation index of the eP equilibrium is t. Thus, according to Kalai (2020)'s measure, the eP equilibrium is less viable as t increases.

We find that the formation index t of the eP equilibrium is empirically a useful measure of equilibrium viability, holding other aspects of the game fixed: subjects participate with lower probability in participation games with higher values of t. However, we also find that the formation index does not capture all aspects of the viability of the eP equilibrium. In particular, the viability of the eP equilibrium depends on n and k, which do not enter into the formation index.

Related Literature

Kalai (2020) proposes two viability indices that can be obtained from the characteristics of a game, the formation and defection indices, which are dual. There are papers that study notions closely related to the defection index: e.g., fault tolerance (Ben-Or et al., 1988; Eliaz, 2002; Gradwohl and Reingold, 2014) and resilience (Abraham et al., 2006). We experimentally study the empirical usefulness of the formation index which is indirectly related to the notions above via duality.

To the best of our knowledge, few studies have directly investigated how the characteristics of a game would affect the viability of its Nash equilibria. Li (2017) introduces the notion of an obviously dominant strategy, and shows experimentally that if a game has an equilibrium in obviously dominant strategies, then that equilibrium is more likely to be played than other equilibria. Bartling and Netzer (2016) study robustness of Nash equilibrium to the presence of externalities not captured by the description of a game. While both notions above are related to the defection index, these studies do not directly test the empirical usefulness of the viability indices.

Our paper is related to the literature on coordination games. Following Harsanyi and Selten (1988)'s introduction of the notion of Pareto-dominant and risk-dominant equilibria, many studies such as Carlsson and van Damme (1993), Cooper et al. (1990), and Van Huyck et al. (1990) investigate whether the Pareto-dominant or risk-dominant equilibrium is more likely to be played. More recent contribution to this literature includes Rankin et al. (2000), Van Huyck et al. (2018), and Duffy and Fehr (2018). Our focus is different: we study the viability of the Pareto-dominant equilibrium as the formation index varies.

Participation games are also related to the weakest-link public goods game (Hirshleifer, 1983) and the threshold public goods game (Palfrey and Rosenthal, 1984). In particular, a participation game with threshold $n-1$ is a weakest-link public goods game. Unlike a participation game, the weakest-link game lacks a parameter which varies the formation index. In the threshold public goods game all the player enjoy the public good if the threshold is reached, whereas in a participation game only those players who participate benefit when the threshold is reached.

2 Theory and Hypotheses

In this section, we introduce basic definitions and examples from Kalai (2020).

2.1 Basic Theory

Let $\Gamma = (N, (A_i)_{i\in N}, (u_i)_{i\in N})$ be a game in strategic form, where N is the set of players and for each $i \in N$, A_i is player i's set of actions and u_i is player i's utility function. Let $\alpha = (\alpha_1, \dots, \alpha_N) \in A_1 \times \cdots \times A_N$ be a strategy profile. We say α is a *best response* to strategy profile β if, for each player $i \in N$ we have that $u_i(\alpha_i, \beta_{-i}) \geq u_i(\alpha'_i, \beta_{-i})$ for all $\alpha'_i \in A_i$.

Let $\pi = (\pi_1, \dots, \pi_N)$ denote an arbitrary fixed focal profile. The strategy profile α is a *d-defector profile from* π if

$$|\{i \in N | \alpha_i \neq \pi_i\}| = d,$$

i.e., if there are exactly d players in α whose actions are different from π.

Kalai (2020) proposes two indices to measure the viability of an equilibrium, the defection index and (its dual) the formation index.

Definition 1. *The* ***defection index of*** π*, denoted by* $D(\pi)$*, is the smallest integer* $d \in \{0,\ldots,N\}$ *such that* π *is not a best response to some* d*-defector profile* α *from* π*. If* π *is the best response to any profile* α*, then define* $D(\pi) = n$*.*

If $D(\pi) = 0$, then π is not a best response to itself and thus π is not a Nash equilibrium. If $D(\pi) \geq 1$, then π is a best response to the 0-defector profile α from π, i.e., it is Nash. If $D(\pi) = 1$, then π is not a best response to some 1-defector profile α from π, i.e., there is a deviation by a single player such that at least one other player would then find it optimal to deviate as well. While such a strategy profile π is a Nash equilibrium, it is not highly viable. At the other extreme, if $D(\pi) = n$, then π is an equilibrium in dominant strategies—for any deviation by $n-1$ players, it remains optimal for the remaining player to follow his part of π. Such an equilibrium profile is highly viable.

As $D(\pi)$ increases, an equilibrium profile π is increasingly viable. It is more resistant to known deviations by the other players and to uncertainty about whether the other players will follow their part of equilibrium.

We illustrate the defection index with two examples. The first example, the Party Line Game, is due to Kalai (2020). The second example, the Participation Game, is due to Kalai and Kalai (2021).

Example 1. The Party Line Game: Three Democrats and five Republicans simultaneously select one of two choices, E and F. The payoff of a player is the number of opposite-party players whose choice she *mismatches*. In the divisive equilibrium, Div, all the Democrats choose F and all the Republicans choose E. Democrats and Republicans obtain payoffs of 5 and 3, respectively.

In the Party Line Game, $D(Div) = 2$. In particular, if two Democrats deviate to E it is no longer optimal for Republicans to choose F.[1] By contrast, if only a single player—Democrat or Republican—deviates from Div, it remains optimal for each player to play their part of Div.

Example 2. The Participation Game: In the n-player Participation Game with threshold t, the players simultaneously choose whether to participate (P) or not (N). The payoff

[1] Formally, $\pi = (F,F,F;E,E,E,E,E)$ is not a best response to the 2-defector profile $(F,E,E;E,E,E,E,E)$ from π.

of each participant is 1 if the number of participants exceeds t and is 0 otherwise. The payoff of each non-participant is k, where $0 < k < 1$. In the everyone-participates equilibrium, eP, each player obtains a payoff of 1.

A participation game is defined by a triple (n,t,k). In a n-player participation game, the defection index of the eP equilibrium is $D(eP) = n - t$. This is the smallest number of players who can deviate from eP such that eP is not a best response to the deviation.[2] If $n-t-1$ or fewer players deviate from eP to N (leaving $t+1$ or more players choosing P), it remains optimal for every player to choose P.

Proposition 1 of Kalai (2020) shows that $D(\pi)$ can equivalently be viewed as the maximum number of defectors that π deters. In particular, if one fixes the actions of any coalition $S \subset N$ consisting of $n - D(\pi)$ players to $\pi_{(S)} := \{\pi_i \in \pi | i \in S\}$, then in the game induced on the remaining $D(\pi)$ players the action π_i is a dominant strategy for each player $i \in N \setminus S$.[3]

This result is easy to see for participation games. Imagine fixing the actions of t players in the Participation Game (n,t,k) to P. This induces a participation game on the remaining $n-t$ players. In the induced game, P is a dominant strategy for every player since a player obtains a payoff of 1 choosing P and obtains k choosing N, regardless of the actions of the other $n-t-1$ players.

Kalai (2020)'s second index, the formation index, is defined as follows.

Definition 2. *The **formation index** of π is defined by $F(\pi) = n - D(\pi)$.*

Kalai (2020) shows that the formation index $F(\pi)$ has a natural interpretation as the minimal number of players required to form π. In particular, if one fixes the actions of any coalition S consisting of $F(\pi)$ players to $\pi_{(S)}$, then in the game induced on the remaining $n - F(\pi)$ players the action π_i is a dominant strategy for each player $i \in N \setminus S$. This result is easy to see for the Participation Game (n,t,k), where $F(eP) = n - D(eP) = t$. If the actions of t players are fixed to P, then P is a dominant strategy for each of the $n-t$ players in the induced game.

To sum up, an equilibrium π is more viable as $D(\pi)$ increases or as $F(\pi)$ decreases. A higher $D(\pi)$ implies that π is more resistant to deviations, while a lower $F(\pi)$ implies that π is more easily formed.

2.2 The Experimental Game

To evaluate the empirical usefulness of these indices, we study participation games experimentally. This is an attractive class of games to study since the viability of the eP

[2] If $n-t$ players deviate to N, then it is no longer optimal for the remaining t players to participate.

[3] See Kalai (2020) for a formal description of the result, which we describe only informally here.

equilibrium is determined by a single parameter t, for n fixed. However, we also study how the viability of equilibrium varies in n, in which case $D(\pi)$ and $F(\pi)$ are no longer dual. Further, we investigate whether the viability of equilibrium depends on k.

The Participation Game (n,t,k) has two-pure strategy—everyone-participates (eP) and no-one-participates (nP)—equilibria. There is also a unique symmetric mixed-strategy equilibrium in which each player participates with probability p_m which solves

$$\sum_{i=t}^{n-1} \binom{n-1}{i} p_m^i (1-p_m)^{n-i-1} = k.$$

The formation index of the mixed-strategy equilibrium is $n-1$, suggesting that it is unlikely to be formed.[4] Hence, we focus on pure-strategy equilibria. While we frame our results in terms of the viability of eP we could, equivalently, frame them in terms of the viability of nP since $D(eP) = F(nP)$ and $F(eP) = D(nP)$.

2.3 Hypotheses

Let $p(n,t,k)$ be the true but unknown probability that a randomly selected player in the Participation Game (n,t,k) plays P. We use $p(n,t,k)$ as the measure of the viability of eP, with the equilibrium being more viable as $p(n,t,k)$ increases. It would be equivalent to measure the viability of eP by the probability that the threshold is reached or by the probability that eP is played, i.e., everyone participates.

We now describe our hypotheses.

Hypothesis 1. *The eP equilibrium is less viable as t increases, i.e., $t < t'$ implies $p(n,t,k) > p(n,t',k)$.*

Note that $F(eP)$ does not depend on n or k. The next hypotheses concern whether the empirical viability of eP depends on n or k.

Hypothesis 2. *The viability of the eP equilibrium does not vary with n, i.e., $p(n,t,k) = p(n',t,k)$ for $n \neq n'$.*

If we reject Hypotheses 2, it means either $F(eP)$ should be understood to be a measure of viability for a fixed number of players, or a richer measure of viability is required for comparing equilibria in games with different n.

An alternative intuitive measure of equilibrium viability is the fraction of players who must participate to reach the threshold: the eP equilibrium is less viable as the fraction increases.

[4]The defection index of the symmetric mixed-strategy Nash equilibrium is 1 since if a single player deviates from the equilibrium mixture, the best response of each remaining player is a pure strategy. Furthermore, it is easy to verify that p_m is increasing in t and k, but decreasing in n. As we will see later, our data are inconsistent with these comparative statics.

Hypothesis 3. *The eP equilibrium is less viable as t/n increases, i.e., $t/n < t'/n'$ implies $p(n,t,k) > p(n',t',k)$.*

Kalai (2020)'s measure of viability of the eP equilibrium does not depend on k. Our last hypothesis concerns whether the empirical viability of eP depends on k.

Hypothesis 4. *The viability of the eP equilibrium does not depend on k, i.e., $p(n,t,k) = p(n,t,k')$ for $k \neq k'$.*

3 The Experiment

The experiment had three treatments, each with four rounds. The *Main* treatment proceeded as follows: In each round, a threshold t of either 4, 8, 12, or 16, was announced, and then 20 subjects simultaneously decided to participate (P) or not (N). We call a subject who chose P a participant and a subject who chose N a non-participant. The payment of each participant was €8 if the number of participants exceeded t and €0 otherwise. Each non-participant received the lottery that gives €8 with probability 1/2 and €0 otherwise. At each round, they played the same game but with a different threshold. Subjects received no feedback between rounds, and each subject engaged in only one treatment.[5]

This experiment implements the participation game described in Example 2. It is without loss of generality to assign a utility of 1 to receiving €8 and a utility of 0 to receiving €0. The expected utility of the lottery is 1/2. Since there are only two outcomes, €8 or €0, there is no scope for risk preferences to affect equilibrium play.

The *Small* treatment was the same as the Main treatment except that the group size and the thresholds were reduced by one half. The *Scale* treatment differed from the Main treatment in that each non-participant received €8 with probability 3/4 and €0 otherwise. Table 1 summarizes the three treatments.

Treatment	Group Size	NP Lottery	P Payment	Thresholds
Main	20	(8,0.50)	8 or 0	{4, 8, 12, 16}
Small	10	(8,0.50)	8 or 0	{2, 4, 6, 8}
Scale	20	(8,0.75)	8 or 0	{4, 8, 12, 16}

Table 1: Treatments

[5] To control for order effects, we counterbalanced the order of the thresholds. In the first Main session, the threshold order was 8–16–12–4. In the second and third sessions, the threshold orders were 16–12–4–8 and 12–4–8–16, respectively. The same thresholds and orders were used in the Scale and Small treatments, except that the thresholds were reduced by one half in Small. Since subjects were unaware of the threshold until it was announced, from a subject's perspective the thresholds were randomly ordered.

In the experiment, subjects chose between P or N by clicking a button. We set P as the default, as illustrated in Figure 1. In particular, P was initially selected and a subject could choose P by merely clicking the Continue button. Thus, the equilibrium eP is focal, and we test the viability of eP.[6]

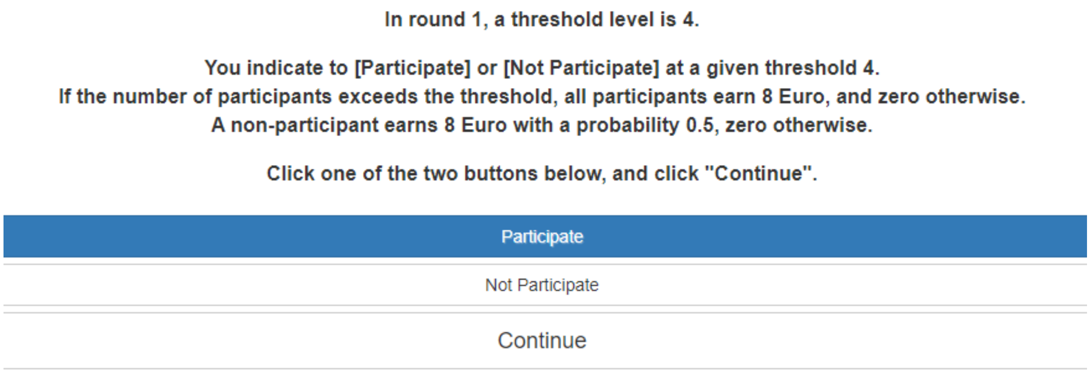


Figure 1: Screenshot of the Game

One round was randomly selected for payment. In addition to their earning, subjects also received €5 for participating in the experiment.

4 Results

4.1 Data Summary

Let $Y_i(n,t,k)$ denote subject i's decision when the group size is n, the threshold is t, and the winning probability of the non-participation lottery is k, where $Y_i(n,t,k) = 1$ if the subject chooses P and 0 otherwise. We had three sessions for each treatment, and hence there are $3n$ observations for each Participation Game (n,t,k). Let $Y(n,t,k)$ denote the random variable for the number of participation decisions in (n,t,k) over the three sessions, and let $\overline{Y}(n,t,k) = Y(n,t,k)/3n$ be the participation rate. Table 2 summarizes the realized values of these random variables for each (n,t,k).

4.2 Results

In this section, we show the formation index $F(eP)$ is a good measure of the viability of the eP equilibrium. In particular, the participation probability increases as t decreases, holding n and k fixed. We also show that the participation probability depends on n and k, which is not captured by the formation index.

[6] Since P was the default, it had "primary salience" according to Mehta et al. (1994). Subjects were not told that P was the default, and hence P did not have "secondary salience."

Treatment (n,k)	Obs. $3n$	Participation Decision $Y(n,t,k)$				Participation Rate $\overline{Y}(n,t,k)$			
Main	60	t=4	t=8	t=12	t=16	t=4	t=8	t=12	t=16
(20, 0.5)		57	54	43	26	0.95	0.90	0.72	0.43
Small	30	t=2	t=4	t=6	t=8	t=2	t=4	t=6	t=8
(10, 0.5)		29	28	21	11	0.97	0.93	0.70	0.37
Scale	60	t=4	t=8	t=12	t=16	t=4	t=8	t=12	t=16
(20, 0.75)		59	49	30	17	0.98	0.82	0.50	0.29

Table 2: Summary of Data

To test Hypothesis 1, we conduct a one-tailed Fisher's exact test for every pair of thresholds within each treatment. In particular, for any two thresholds t and t' such that $t < t'$, we consider the null and alternative hypotheses that

$$H_0 : p(n,t,k) = p(n,t',k) \quad vs. \quad H_1 : p(n,t,k) > p(n,t',k).$$

Rejecting the null provides evidence that the participation probability decreases as t increases from t to t'. There are 60 participation decisions made for each threshold. Under the null hypothesis, the probability that $Y(n,t,k) = y$, conditional on z participation decisions in total for thresholds t and t', is

$$P[Y(n,t,k) = y | Y(n,t,k) + Y(n,t',k) = z] = \frac{\binom{60}{y}\binom{60}{z-y}}{\binom{120}{z}}$$

for $y \in \{\max\{0, z-60\}, \ldots, \min\{60, z\}\}$ according to Siegel and Castellan (1988).[7] For $y \geq \max\{0, z-60\}$, the p-value associated with an outcome $Y(n,t,k) = y$ is

$$\sum_{w=y}^{\min\{60,z\}} P[Y(n,t,k) = w | Y(n,t,k) + Y(n,t',k) = z].$$

We reject the null hypothesis at the 5 percent significance level if the p-value is less than 0.05. In the Main treatment, for example, we have $Y(20,4,0.5) = 57$ and $Y(20,12,0.5) = 43$. Hence for the null hypothesis $p(20,4,0.5) = p(20,12,0.5)$, the associated p-value is

$$\sum_{w=57}^{60} \frac{\binom{60}{w}\binom{60}{100-w}}{\binom{120}{100}} \approx 0.0005.$$

[7]Given that $Y(n,t,k) + Y(n,t',k) = z$, then $Y(n,t,k)$ must take an integer value between $\max\{0, z-60\}$ and $\min\{60, z\}$.

We reject this null hypothesis at even the 1 percent significance level.

Figure 2 summarizes the data, showing the empirical participation rates for each threshold and treatment. It also shows the results of tests of the hypothesis that participation probabilities are equal at adjacent thresholds. The null hypothesis that these participation probabilities are the same is rejected for all but two comparisons: the pair $t = 2$ and $t' = 4$ in the Small treatment and the pair $t = 4$ and $t' = 8$ in the Main treatment. This null is rejected for every comparison of non-adjacent thresholds.

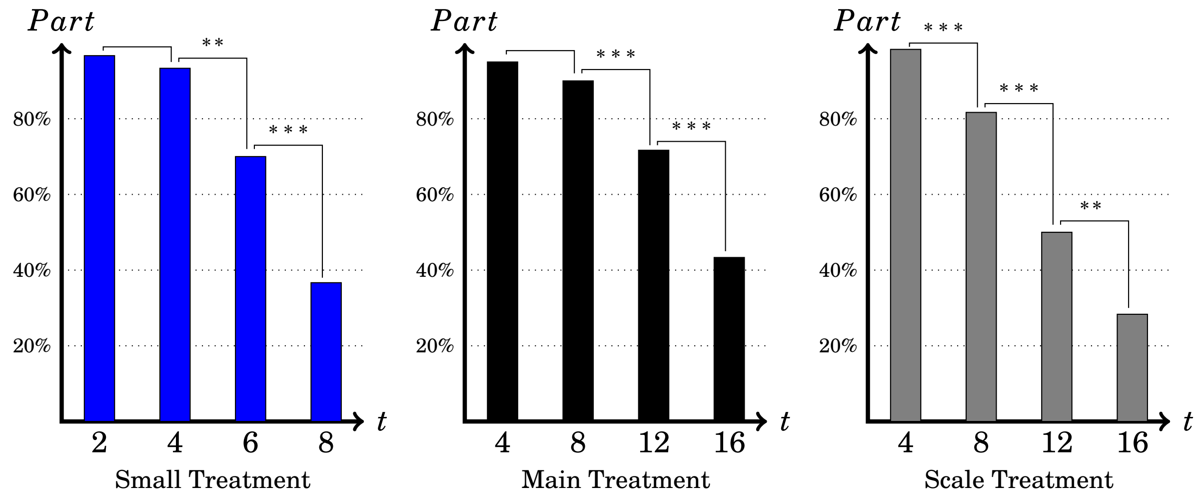


Figure 2: Participation Rates by Treatment

$**$ and $***$ indicate statistical significance of Fisher's exact test at the 5% and 1% level, respectively.

We obtain the following result.

Result 1. *Participation probabilities decrease in t.*

Result 1 shows that $F(eP)$ is a good measure of viability for changes in the threshold. In particular, as $F(eP)$ increases, the eP equilibrium becomes less viable.

Hypothesis 2 is that the viability of eP does not depend on n. We test this hypothesis using data from the Main ($n = 20$) and Small ($n = 10$) treatments. We consider the null and alternative hypotheses that

$$H_0 : p(20, t, 0.5) = p(10, t, 0.5) \quad vs. \quad H_1 : p(20, t, 0.5) \neq p(10, t, 0.5),$$

for $t = 4$ and $t = 8$. Under the null hypothesis that $p(20, t, 0.5) = p(10, t, 0.5)$, the Pearson goodness of fit test statistic is

$$Q = \sum_{n \in \{10,20\}} \left\{ \frac{(Y(n,t,0.5) - 3n\hat{p})^2}{3n\hat{p}} + \frac{(3n - Y(n,t,0.5) - 3n(1-\hat{p}))^2}{3n(1-\hat{p})} \right\},$$

where

$$\hat{p} = \frac{Y(20,t,0.5) + Y(10,t,0.5)}{3(20) + 3(10)},$$

and it is asymptotically distributed chi-square with 1 degree of freedom. The p-value is the probability that Q is greater or equal to an observed value.

Figure 3 depicts participation rates in the Main and Small treatments for $t = 4$ and $t = 8$. We cannot reject the null that $p(20,t,0.5) = p(10,t,0.5)$ for $t = 4$ (p-value of 0.745), while we reject the null at the 1 percent significance level for $t = 8$ (p-value<0.001). We reject the joint null hypothesis that $p(20,4,0.5) = p(10,4,0.5)$ and $p(20,8,0.5) = p(10,8,0.5)$ at the 1 percent significance level.

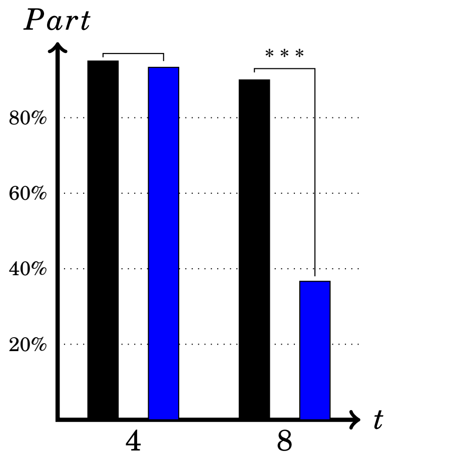


Figure 3: Participation Rates for Main and Small
$***$ indicates statistical significance of Pearson's χ^2 test at the 1% level.

Result 2. *Participation probabilities depend on n.*

Results 1 and 2 established that participation probabilities vary not only with t but also with n. A natural conjecture consistent with these results is that the participation probability decreases as the fraction of participants required to reach the threshold increases. Hypothesis 3 is that the eP equilibrium is less viable as t/n increases. We consider the null and alternative hypotheses that

$$H_0 : p(n,t,0.5) = p(n',t',0.5) \quad vs. \quad H_1 : p(n,t,0.5) > p(n',t',0.5),$$

for $t/n < t'/n'$. Rejecting the null provides evidence in favor of Hypothesis 3.

We conduct a one-tailed Fisher's exact test for every pair of games such that $n \neq n'$ and $t/n \neq t'/n'$. For example, the Participation Game $(10,4,0.5)$ has $t/n = 0.4$ and it is paired with the Participation Games $(20,4,0.5)$, $(20,12,0.5)$, and $(20,16,0.5)$ from the Main treatment. Table 3 shows for the null $p(10,4,0.5) = p(20,4,0.5)$ and alternative $p(10,4,0.5) < p(20,4,0.5)$ the p-value of the Fisher exact test is 0.543. We do not reject the null. For the null $p(10,4,0.5) = p(20,12,0.5)$ and the alternative $p(10,4,0.5) >$

$p(20,12,0.5)$, the p-value is 0.014. In this case we reject the null in favor of the alternative. In Table 3, except for two pairs, the null is rejected for each pair-wise comparison.

			$n = 10$			
			$t = 2$	$t = 4$	$t = 6$	$t = 8$
		t/n	0.2	0.4	0.6	0.8
$n = 20$	$t = 4$	0.2	-	0.543	0.002^{***}	0.000^{***}
	$t = 8$	0.4	0.253	-	0.020^{**}	0.000^{***}
	$t = 12$	0.6	0.003^{***}	0.014^{**}	-	0.002^{***}
	$t = 16$	0.8	0.000^{***}	0.000^{***}	0.015^{**}	-

Table 3: p-values for Fisher Exact Tests
$**$ and $***$ indicate statistical significance of Fisher's exact test at the 5% and 1% level, respectively

Figure 4 compares participation rates in the Main and the Small treatments ordered by the value of t/n. We cannot reject that participation probabilities are the same in games with the same ratio t/n, i.e., we cannot reject the null hypothesis that $p(n,t,0.5) = p(n',t',0.5)$ for $t/n = t'/n'$ against the two-sided alternative. Specifically, for the nulls (i) $p(10,2,0.5) = p(20,4,0.5)$, (ii) $p(10,4,0.5) = p(20,8,0.5)$, (iii) $p(10,6,0.5) = p(20,12,0.5)$, and (iv) $p(10,8,0.5) = p(20,16,0.5)$, the associated p-values are 1.000, 0.714, 1.000, and 0.651.

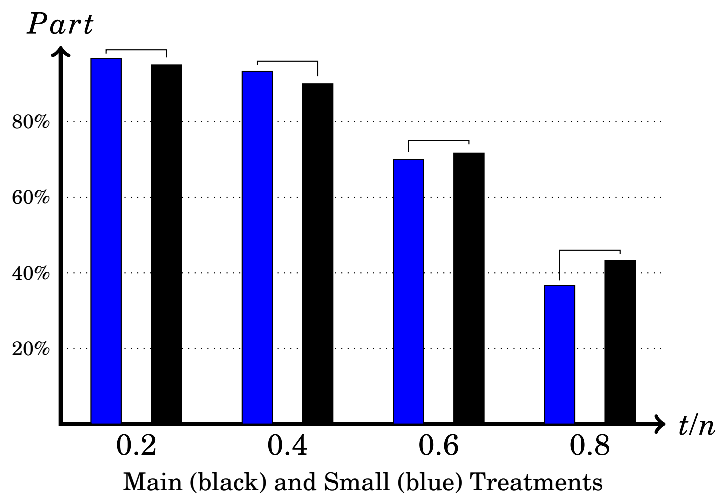


Figure 4: Comparison between Main and Small
Two-sided Fisher's exact tests

Taken together, these results establish that participation probabilities are determined by t/n and are decreasing in t/n.

Result 3. *Participation probabilities are determined by t/n and decreasing in t/n.*

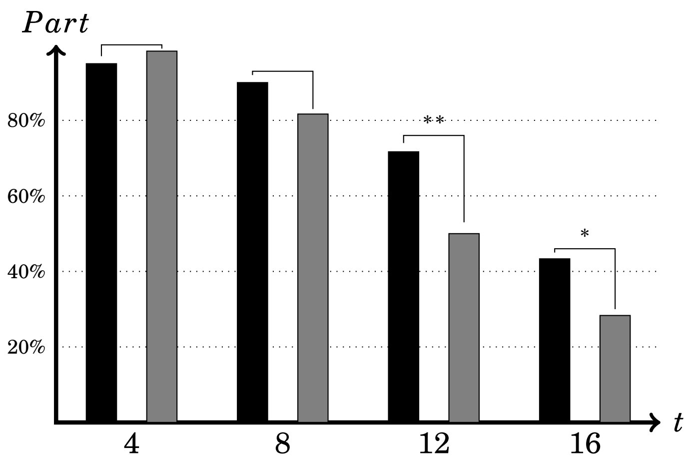
Hypothesis 4 is that the viability of eP does not depend on k, i.e., the null and alternative hypotheses are

$$H_0 : p(n,t,0.5) = p(n,t,0.75) \quad vs. \quad H_1 : p(n,t,0.5) \neq p(n,t,0.75).$$

We test this hypothesis using the data from the Main and Scale treatments. As in the tests for Hypotheses 2, we use the Pearson goodness of fit test.

Figure 5 below depicts participation rates in the Main and Scale treatments for each $t \in \{4,8,12,16\}$. We reject the null hypothesis that $p(20,t,0.5) = p(20,t,0.75)$ for $t = 12$ at the 5 percent significance level (p-value of 0.015) and $t = 16$ at the 10 percent significance level (p-value=0.087). We reject the joint null that $p(20,t,0.5) = p(20,t,0.75)$ for all $t \in \{4,8,12,16\}$ at the 5 percent significance level (p-value of 0.021). Hence we obtain the following result.

Result 4. *Participation probabilities depend on k, the winning probability of the non-participation lottery.*



Main ($k = 0.5$, black) and Scale ($k = 0.75$, gray) Treatments

Figure 5: Participation Rates for Main and Scale
$*$ and $**$ indicate statistical significance of Pearson's χ^2 test at the 10% and 5%, respectively.

The measure $F(eP)$ does not depend on k. We find that the empirical viability of the eP equilibrium varies with k. Figure 5 shows that the participation probabilities tend to decrease in k. In other words, the formation index does not capture the greater attractiveness of the strategy N that results from an increase in the winning-probability of the non-participation lottery.

5 Discussion and Conclusion

The hypotheses we study can be viewed as hypotheses about the nature of the "iso-probability" curves that govern the probability that a player chooses to participate (P). The iso-probability curve for entry probability $\bar{p}$ in Participation Game (n,t,k) can be defined as

$$\{(n,t)|p(n,t,k) = \bar{p}\}.$$

Figure 6 below illustrates two examples of iso-probability curves, for a fixed k.[8]

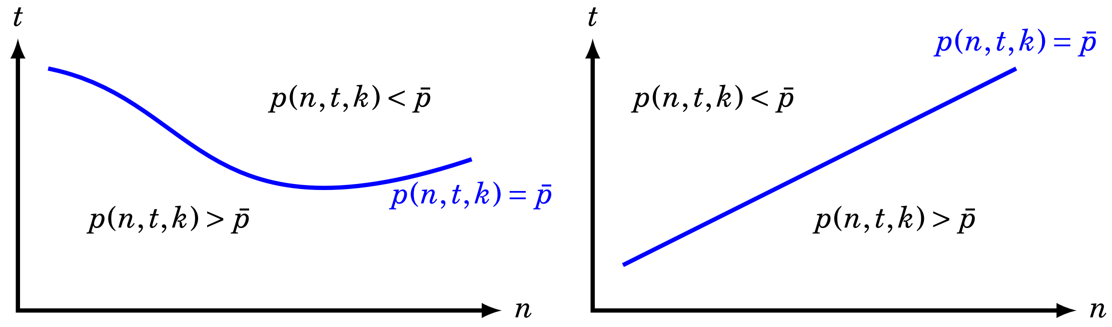


Figure 6: Examples of Iso-Probability Curves

Hypothesis 1, if true, rules out that iso-probability curves are anywhere vertical. It also implies, as shown in Figure 6, that $p(n,t,k) > \bar{p}$ for (n,t) that lie below the iso-probability curve $p(n,t,k) = \bar{p}$. Result 1 finds support for Hypothesis 1. Hypothesis 2 is that iso-probability curves are flat, i.e., $p(n,t,k)$ in participation games does not depend on n. Result 2 rejects Hypothesis 2, thereby ruling out that iso-probability curves are anywhere flat. Hypothesis 3, if true, implies that iso-probability curves are upward sloping. Result 3 finds support for Hypothesis 3. It finds, in addition, that the data is consistent with linear iso-probability curves. Result 4 shows that $p(n,t,k)$ indeed depends on k and thus a change in k leads to a shift of the iso-probability curves. The data is consistent with the iso-probability curve for entry probability $\bar{p}$ shifting down as k increases.

We find that Kalai (2020)'s formation index is a useful measure of the viability of the everyone-participates equilibrium as t varies, holding n and k fixed (Result 1). We also find that Kalai (2020)'s viability indices do not capture all the aspects of a participation game that affect the empirical viability of equilibrium, e.g., changes in the group size n (Result 2) or in the winning probability k of the non-participation lottery (Result 4).

[8] Changes in k potentially shift the iso-probability curves.

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Appendix A. Instructions

Sample Instructions (Main treatment)

Welcome to this experiment. Please read these instructions carefully.

In this experiment you will play the Participation Game for several rounds. Your earnings will depend on the decisions you make, as well as the decisions of the other players.

Task: Participation Game

You will play the Participation Game with 19 other players. At the start of each round, a threshold number T is announced to all the players. You and the other players then each choose an action: **Participate** or **Not Participate**.

Your earnings in the round are determined as follows:

- If you choose **Not Participate**, then you earn a lottery ticket that gives you 8 Euro with a probability 0.5 and gives you 0 Euro otherwise.
- If you choose **Participate**, then you earn 8 Euro if the number of players who chose Participate, including yourself, is *strictly greater* than the threshold number T. Otherwise, you earn 0 Euro.

Payment

At the end of the experiment, one round will be randomly chosen for payment. For that round, you will learn the number of players who chose Participate and your earning in the round. Your total payment will be these earnings plus the show-up fee of 5 Euro. Since each round is equally likely to be selected for payment, it is in your best interest to take each round seriously.

Comprehension check

Suppose that the threshold is 12, and you chose Participate. If there are 10 subjects who chose Participate, what is your payoff?

Appendix B. Procedure

The experimental sessions were conducted in English using subjects recruited from the Mannheim Laboratory for Experimental Economics of the University of Mannheim. Due to COVID-19, we did not bring subjects to the lab. Instead, we invited them to join an online meeting to get announcements from the experimenter and report technical issues to the experimenter during the session, distributed the unique link for participating in the online experiment, and paid them via online transfers (either PayPal or bank transfer) afterward.

Three sessions were conducted for each treatment, and a total of 150 subjects participated in one of the nine ($= 3 \times 3$) sessions. We used an interactive online platform called LIONESS (Live Interactive Online Experimental Server Software, Arechar et al., 2018). After the subjects joined an online meeting, the experimenter asks them to turn off the webcam, remove their profile photos, if any, and rename their displayed names to two alphabet letters they arbitrarily chose so that their identities, hence decisions, remain anonymous to the experimenter as well as other subjects. Subjects were asked to read the instructions displayed on their screens carefully and to pass a comprehension quiz.

The average payment per subject was €10.12. The payments were made via online transfer after receiving the personal payment code generated at the end of the experiment. Each session lasted less than 30 minutes.