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Abstract

We explore the intertwined dynamics of asset prices and the macroeconomy in a Behavioural model of Credit Cycles (BCC) characterized by a credit friction à la Kiyotaki and Moore and heterogeneous expectations cum heuristic switching à la Brock and Hommes. This behavioural approach allows to better understand and replicate the effects of shocks. In the absence of actual defaults, following a positive productivity shock, our behavioural model (BCC Mark I) generates hump-shaped impulse-response functions that are more realistic than those generated by the same shock in a corresponding model with rational expectations (RCC). When the behavioural model allows also for defaults (BCC Mark II), a productivity shock triggers ample and persistent fluctuations (if the intensity of choice of the lender is sufficiently high), a feature that is absent in BCC Mark I (and of course in RCC).

JEL-Codes: E320, E440, D840.

Keywords: credit market, collateral constraints, heterogeneous expectations, bankruptcy, boom bust cycles.

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1 Introduction

Expectations play a key role in shaping "market sentiment", which in turn drives the booms and busts of asset prices. This is a well known fact, explored both empirically and theoretically in the finance literature. One important strand of this literature adopts the behavioural approach pioneered by Brock and Hommes (1997, 1998) (BH hereafter) in conceiving the asset market as populated by investors who adopt heterogeneous heuristics[1] to form expectations and switch from one rule to the other depending upon the relative performance of each one in forecasting the future asset price.[2] In the simplest setting, each investor can be either a "fundamentalist" or a "naive" extrapolator. When the majority of investors switch to naive expectations large swings in asset prices are likely to occur. If the population consisted of rational fundamentalists only, in fact, any departure of the asset price from the fundamental value would be short lived and would not develop into a large fluctuation. As Bernanke and Gilchrist (2018) put it "Purely fundamentals-based models have difficulty accounting for the boom and then subsequent bust in [asset] prices. This opens up the possibility for a behavioural approach to explain how a wave of optimism turned to pessimism."

In the last decade, "animal spirits" have been advocated as important drivers of large macroeconomic fluctuations.[3] We conceive animal spirits as the equivalent – in a macroeconomic context – of market sentiment in a model of the asset market. There is nothing conceptually new in this idea. Already in 1927, Pigou focused on "wave-like swings in the mind of the business world between errors of optimism and

[1]In the following we will use the terms *rule of thumb* or *predictor* as synonimous with *heuristic*.

[2]There is abundant empirical evidence, both in surveys and in the lab, that agents hold heterogeneous expectations. See, for instance, Branch (2004), Pfajfar & Santoro (2010) and Hommes (2013, 2021).

[3]According to Dai et al. (2017) over 40% of US output fluctuations during the Great Recession can be traced back to animal spirits.

errors of pessimism" as drivers of business fluctuations. A few years later Keynes (1936) famously wrote: "Most of our decisions to do something positive...can only be taken as the result of animal spirits — a spontaneous urge to action rather than inaction."

In contemporary macroeconomics different approaches have been proposed to embed the idea of animal spirits in an analytical and computational setting. For instance Angeletos and La'O (2013) augment a canonical macroeconomic model with shocks they label "sentiments" that capture sudden changes in expectations of economic activity. Angeletos, Collard and Dellas (2018) introduce a shock to higher-order beliefs that yields waves of optimism and pessimism. Beaudry and Portier (2014) propose a model of news driven business cycles in which agents' information set is continuously updated by "news" that provide an incentive to re-formulate expectations on the fundamentals of the macroeconomy.

An important alternative strand of macroeconomic literature departs from the rational expectations paradigm and makes use of the heuristic switching approach to model expectations formation in New Keynesian (NK) behavioural macroeconomic models. For instance De Grauwe (2011, 2012) proposes a NK model in which agents form expectations of inflation and output using only two heuristics: a fundamentalist rule – according to which expected inflation or output are pinned down to the respective steady state levels (the central bank's inflation target for inflation and potential GDP for output) – or a naive rule according to which inflation or output expected in t for t+1 is anchored to the levels of the same variables recorded in t-1.

Notice that it is perfectly rational to choose a non fundamentalist rule because during a boom or a recession, an extrapolator makes a forecast error which is generally smaller than the one made by a fundamentalist, who blindly expects the variable to revert to the fundamental level. Fundamentalists know the "true

model" of the economy and therefore can compute the steady state levels of inflation and output but their expectations are not model-consistent because they do not take into account the presence of extrapolators.
In a behavioural NK model, the current level of each variable (inflation and output) depends linearly on the expected (aggregate) future levels of the same variables. The aggregate expectation of a variable in turn is the weighted average of the expectations held by fundamentalists and naive extrapolators, the weights being the fractions of agents who adopt each rule. These fractions in turn depend non-linearly on the past levels of each variable. Therefore, in the end, the deep deterministic skeleton of the model is highly non linear.

De Grauwe's simple measure of animal spirits is the fraction of the population that uses the naive rule to form output expectations. In a boom, extrapolators are optimist and their optimism feeds back into economic activity and inflation. In a recession, they are pessimist and their pessimism precipitates the macroeconomy into a deep recession and possibly deflation.

In parallel with the development of the New Keynesian framework, a large literature has emphasized the role of financial frictions in determining the financial accelerator, a magnifying mechanism by which relatively mild departures from the long run level of GDP turn into large fluctuations. Prominent among them is the model of credit cycles pioneered by Kiyotaki and Moore (1997) (KM hereafter). In this framework, borrowers face a financing constraint because lenders extend credit up to the present value of borrowers' collateralizable wealth, that is land (a real asset). KM study the emergence of a financially magnified fluctuation generated by a temporary shock to productivity in a rational expectations setting. The appealing feature of this model is the relation of asset price changes and borrowing constraints: following the shock the asset price – i.e., the price of land in KM – increases and the borrowing constraint becomes less stringent, enhancing the borrowers' investment and economic activity; the upswing, in turn, affects asset prices; when the shock disappears the economy goes back to the original steady state. The literature that sprung from this pioneering work has followed in the footsteps of KM. In a nutshell, this is a *Rational* model of Credit Cycles (RCC).

The framework proposed by KM is the perfect setting to explore the intertwined dynamics of market sentiment on the asset market and animal spirits in macroeconomic activity but in order to pursue this line of research in the most fruitful way, in our opinion it is necessary to depart from full information rational expectations and adopt a heterogeneous expectations perspective. In this paper we contribute to the literature by studying how heterogeneous expectations à la Brock and Hommes affect the asset price, the borrowing constraint and macroeconomic activity in a *Behavioural* model of *Credit Cycles* (BCC).

We consider two variants of this model. When *only lenders* form heterogeneous expectations (BCC Mark I), from simulations it turns out that the amplitude of the impulse-response functions generated by a shock to productivity is enhanced but the persistence is mitigated (relative to the corresponding RCC).

When both lenders and borrowers form heterogeneous expectations (BCC Mark II) and the (average) lender's expectation differ from the (average) borrower's expectation, default can occur. In the presence of default, and a sufficiently large *intensity of choice* (i.e., a measure of the agent's "level of rationality"), after a shock variables do not go back to the steady state. The shock, albeit temporary, triggers ample and frequent oscillations. Defaults, in fact, drive an alternation between booms and busts. For very high levels of the intensity of choice, there can be chaotic dynamics.

The paper is organized as follows. Section 2 presents a concise review of the literature. Section 3 explores a Rational Credit Cycles (RCC) model, which essentially is a KM framework modified by the introduction of limited pledgeability. In this model, contrary to KM, only a fraction of land is collateralized. In this context we examine the emergence of the financial accelerator with rational expectations. The RCC framework will play the role of rational benchmark for the subsequent analysis. In section 4 we model the expected price of land according to the heuristic switching model. In section 5 we explore the Behavioural Credit Cycles (BCC) model Mark I, characterized by the incorporation of heterogeneous expectations à la BH *only among lenders* into the KM framework with limited pledgeability. Section 6 presents BCC Mark II in which *both lenders and borrowers* hold heterogeneous expectations. This generalized heterogeneity makes default possible, with interesting macro-dynamic consequences. Section 7 concludes.

2 Related literature

In this section we overview the state of the art on models of the credit cycle à la Kiyotaki & Moore (1997) and models of heterogeneous expectations à la Brock & Hommes (1997), the foundational frameworks of our model of behavioral credit cycles.

Differently from the standard real business cycle model that requires large and persistent aggregate productivity shocks in order to replicate major fluctuations of economic variables, in KM the dynamic interaction between credit limits and asset prices allows even small and transitory shocks to persist, amplify, and spread out. The framework has undergone major modifications and adaptations starting form the relaxation of basic assumptions. Mendicino (2012) has proved that it is sufficient to introduce inefficiency in the debt enforcement procedure to have the same results as KM, even under standard assumptions. By replacing lumpy investments with the neo-classical input accumulation, Pintus (2011) was able to rule out the trade-off between amplification and persistence that arises in KM and shows how they actually go hand in hand. First Iacoviello (2005) and then Guerrieri and Iacoviello (2017) moved the observational lens on the housing market, and by enriching the model with New-Keynesian nominal frictions, they replicated some empirical results, including that financial frictions matter disproportionately more in a recession than in a boom. Assenza and Berardi (2009) have worked on a credit economy à la KM introducing for the first time heterogeneous learning dynamics in the formation of expectations about the future price of the collateral which can lead to bankruptcy and hysteresis. Finally Pintus and Suda (2019) have identified greater effects of financial shocks when economic agents update their beliefs according to adaptive learning.

Let's now turn to Brock and Hommes (1997). This heuristics switching model has inspired several works and laid the foundations for a behavioural approach in macroeconomics. De Grauwe and Ji (2019) and De Grauwe (2011, 2012) present a deep investigation of the New-Keynesian model with heterogeneous expectations à la BH and show interesting results on optimal monetary policy that cannot be taken into consideration if rational expectations are assumed. See Branch and McGough (2018) for an extensive survey. Bofinger et al. (2013) contribute to this strand of literature with a New-Keynesian model à la Iacoviello (2005) and conclude by suggesting the adoption of an augmented Taylor rule that incorporates house prices. Also asset pricing models have been further explored in light of behavioural heterogeneity. Boswijk et al. (2007) offer an explanation of yearly S&P 500 dynamics in terms of alternation between fundamentalists and trend followers, namely agents expecting the stock price to go back towards its fundamental value and agents who are optimistic and expect the price trend to continue. Lastly, Hommes et al. (2017) reinforce this analysis showing how such a dual-regime assumption is able to explain the amplification of booms and busts during the dot-com bubble and the Great Recession.

3 Rational Credit Cycles

In this section we present a model of credit cycles when agents hold rational expectations (Rational Credit Cycles for short) which is a variant of Kiyotaki and Moore (1997) seminal paper. In KM the borrower's durable asset (land) is fully pledgeable so that the (gross) Loan-to-Value ratio is unity. We explore a variant of KM in which only a fraction of the land owned by the borrower is pledgeable (so that $0 < LTV < 1$). The KM model with partial pledgeability is the rational counterpart of the models of behavioural credit cycles that we will present in sections 5 and 6.

As in KM we consider an agrarian economy populated by two classes of agents, farmers and gatherers. For the sake of comparability among models, let's assume that there is a continuum of unit mass of (identical) farmers and a continuum of unit mass of (identical) gatherers. Variables pertaining to the latter are characterized by an accent. Both farmers and gatherers produce a non-storable good ("fruit") by combining their work effort and a durable, non-reproducible input, land whose total size is fixed at $\overline{K}$. The main difference between the two classes is the rate of time preference (*intertemporal preference heterogeneity*). By assumption the farmer is less patient than the gatherer. In symbols:

Assumption 1 $\gamma < \gamma'$

where γ (resp. γ') is the farmer's (gatherer's) discount factor. For simplicity, in the following we will refer to the farmer as the impatient agent. Agents are infinitely lived and maximize expected utility, i.e., the expected discounted sum of period utilities, which in turn coincide with period consumption flows. In symbols:

$$E_t\left(\sum_{s=0}^{\infty}\gamma^s x_{t+s}\right) \qquad \text{and} \qquad E_t\left(\sum_{s=0}^{\infty}\gamma'^s x'_{t+s}\right),$$

where x (resp. x') is the farmer's (gatherer's) consumption flow.

The farmer produces output y (fruit) cultivating land k according to the constant returns technology

$$y_t = (a + c)k_{t-1},$$

where ak_{t-1} is tradable output and ck_{t-1} is non-tradable output (e.g., bruised fruit). The gatherer picks fruit from land according to the production function:

$$y'_t = G(k'_{t-1}).$$

where $G(.)$ is increasing and concave and satisfies Inada conditions.

There are two markets: a competitive spot market in which land is exchanged for fruit at the price q_t and a credit market in which one unit of fruit is lent in period t in exchange for the promise of R units of fruit in period t+1, where $R > 1$ is the (gross) rate of interest. In equilibrium, the farmer will be the borrower and the gatherer will play the role of lender.

By assumption farmers are "specialists", i.e., they are skilled peasants and cannot be costlessly replaced. In the words of Hart and Moore (1994) they are characterized by *inalienable human capital.* Since the farmer can withdraw labour and cannot be replaced, the gatherer runs the risk of not being reimbursed. She protects herself by collateralizing the farmer's land. Following Mendicino (2012) and Iacoviello (2005) we depart from the original KM framework by assuming that only a fraction $(1 - m)$ of the durable asset is pledgeable, $m \in (0, 1)$. The market value of collateral at maturity therefore is $q_{t+1}(1 - m)k_t$. The gatherer extends credit for an amount b_t that should not be greater than the discounted expected value of the collateral at maturity: $b_t \leq \frac{1-m}{R} E_t q_{t+1} k_t$ where $E_t q_{t+1}$ is the expected value, taken in t, of the price of land in t+1. This inequality is the *borrowing constraint.*

The farmer is also subject to a *flow of funds constraint.* For any period t, the following must hold:

$$q_t(k_t - k_{t-1}) + Rb_{t-1} + x_t = (a+c)k_{t-1} + b_t$$

where $q_t(k_t - k_{t-1})$ is the farmer's "investment" in land.

Given a unit increase in tradable output, the farmer decides how much to consume/save comparing the marginal utility of three alternative heuristics ("strategies" in KM wording): (1) save tradable output and use it as downpayment to get credit and increase landholding; (2) save tradable output and lend it to get a return that will be used to increase landholding in the future; (3) consume tradable output. We assume the following:

Assumption 2 $c > a\left(\frac{R-1+m}{\gamma(R-1)(1-m)} - 1\right)$

In appendix A we prove that from Assumption 1 follows that strategy (1) dominates strategy (2) and from Assumption 2 follows that strategy (1) dominates strategy (3). In fact, from these assumptions follows that, in the neighborhood of the steady state, strategy (1) yields the highest discounted sum of period marginal utilities. Hence the farmer consumes as little as possible (consumption consists only of non-tradable output: $x_t = ck_{t-1}$), saves as much as possible (all the tradable output), chooses to be a borrower (instead of lending his savings) and aims at the maximum level of credit that he can get. In this setting the borrowing constraint is binding:

$$b_t = \frac{1-m}{R} E_t q_{t+1} k_t. \tag{1}$$

Plugging $x_t = ck_{t-1}$ and the borrowing constraint into the flow of funds constraint of the farmer, we get

$$\left[q_t - \frac{E_t q_{t+1}}{R}(1-m)\right] k_t = (a+q_t)k_{t-1} - Rb_{t-1},$$

where $d_t := q_t - \frac{E_t q_{t+1}}{R}(1-m)$ is the down payment (per unit of land). The right hand side of the equation above is the farmer's net worth at the beginning of period t. Hence the law of motion of the farmer's land is:

$$k_t = \frac{1}{q_t - \frac{E_t q_{t+1}}{R}(1-m)} \left[(a+q_t)k_{t-1} - Rb_{t-1}\right]. \tag{2}$$

The gatherer's problem consists in maximizing expected utility subject to the gatherer's flow of funds constraint. In symbols:

$$\max_{x'_{t+s}, k'_{t+s}, b_{t+s}} \quad E_t\left(\sum_{s=0}^{\infty} \gamma'^s x'_{t+s}\right)$$
$$\text{s.t.} \quad q_{t+s}(k'_{t+s} - k'_{t+s-1}) + b_{t+s} + x'_{t+s} = G(k'_{t+s-1}) + Rb_{t+s-1},$$

Focusing on t, singling out x'_t from the flow of funds constraint and plugging it into the utility function we get:

$$\begin{aligned} x'_t + \gamma' E_t x'_{t+1} + ... =& G(k'_{t-1}) + Rb_{t-1} - q_t(k'_t - k'_{t-1}) - b_t + \\ &+ \gamma'[G(k'_t) + Rb_t - E_t q_{t+1}(k'_{t+1} - k'_t) - E_t b_{t+1}] \end{aligned}$$

From the FOC with respect to b_t we infer that the interest rate is pinned down by the rate of time preference of the lender:

$$R = 1/\gamma'$$

From the FOC with respect to k_t' we infer that in the optimum the gatherer equates the discounted marginal productivity of land to the the opportunity cost of holding land $u_t = q_t - \frac{E_t q_{t+1}}{R}$:

$$\frac{1}{R} G'(k_t') = u_t$$

Since $\overline{K} = k_t + k_t'$, we can rewrite the LHS of this equality as $\frac{1}{R} G'(\overline{K} - k_t) = u(k_t)$. Thanks to the concavity of the gatherer's production function, $u(k_t)$ is increasing in k_t: $\frac{du}{dk_t} > 0$. Note that, differently from KM, in our setting the down payment d_t does not coincide with the opportunity cost u_t. In fact $d_t = u_t + m \frac{E_t q_{t+1}}{R}$.
This FOC can be written as an asset price equation:

$$q_t = u(k_t) + \frac{E_t q_{t+1}}{R} \tag{3}$$

In this setting, only the gatherer/lender form expectations (on the future price of land). In this Rational Credit Cycles model, as in KM, by assumption the lender has rational expectations: The forecasting error $\varepsilon_{t+1} = q_{t+1} - E_t q_{t+1}$ is a a white noise. In the absence of shocks the expected and future price of land coincide (perfect foresight): $q_{t+1} = E_t q_{t+1}$. Imposing perfect foresight in (1),(2),(3) we obtain the *deterministic skeleton* of the model, i.e., a system of difference equations in the state variables b, k, q:

$$b_t = \frac{1-m}{R} q_{t+1} k_t \tag{4}$$

$$k_t = \frac{1}{q_t - \frac{q_{t+1}}{R}(1-m)} \left[(a + q_t) k_{t-1} - R b_{t-1} \right] \tag{5}$$

$$q_t = u(k_t) + \frac{q_{t+1}}{R} \tag{6}$$

From the first equation follows that the (gross) LTV ratio is: $\dfrac{R b_t}{q_{t+1} k_t} = m$. Since the lender will repossess only a fraction of the defaulting farmer's land, the loan the farmer can obtain is smaller than in the case of full pledgeability. On the other hand, if the farmer defaults, she will still own land (mk_t) that may be employed to produce, consume and increase landholding. In other words, limited pledgeability allows a "fresh start" after default. This institutional feature is not crucial in the present setting because actual defaults never occur. On the contrary, it will play an important role in our model with heterogeneous expectations when defaults can actually occur (see section 6).

Retrieving b_{t-1} from (4) with a one period lag and substituting it into the expression in brackets of (5), we can write the equation of net worth at the beginning of period t as follows: $n_t = (a + mq_t)k_{t-1}$. Given the quantity of land the farmer owns in t-1, net worth with limited pledgeability is bigger than in the case of full pledgeability because interest payments on previous debt are smaller. Hence the resources for downpayment are bigger and the farmer's demand for land is also bigger.

Imposing the conditions $q_t = q_{t-1}; k_t = k_{t-1}; b_t = b_{t-1}$ we obtain the steady-state values:[4]

$$q^* = \frac{aR}{(R-1)(1-m)} \tag{7}$$

$$k^* = u^{-1}\left(\frac{a}{1-m}\right) \tag{8}$$

$$b^* = \frac{a}{R-1}k^* \tag{9}$$

Notice that in the steady state $d^* = a\frac{R-1+m}{(R-1)(1-m)}$. In the steady state the resources available for downpayment $n^* = (a + mq^*)k^*$ are bigger than in the case of full pledgeability ($m = 0$). Limited pledgeability leads to an increase in the farmer's demand for land and this yields also an increase of collateralizable assets

[4]The steady state allocation of land to the farmer k^* is implicitly defined by the condition $u(k) = \dfrac{a}{1-m}$.

and of credit extended.

Notice that the LTV parameter m behaves consistently: as $m \to 0$, $d^* \to u^*$ and we are back to the original steady-states in KM. In their original paper KM show the effects of an unexpected temporary positive shock to the productivity of the farmer's land (a) in a rational expectations setting. The dual role of land, that is both a factor of production and collateralizable wealth, is the ultimate source of a self-reinforcement mechanism that creates a persistent and magnified departure from the steady state. This is, in a nutshell, the "financial accelerator". We have explored the same type of shock in our RCC model with limited pledgeability (once again with rational expectations). For the sake of comparison with the behavioural BCC model, however, we postpone the discussion of the results of this experiment to section 5.

4 Heuristic switching and the expected price of land

KM adopt the rational expectation perspective: The expected future price of land may deviate from the actual future price only in the presence of a white noise shock. In our opinion this reductionist perspective may not capture the effects of expectations on the macroeconomy in the real world. In this section we propose an alternative method to model the expectation of the future price of land based on heuristic switching.

As Hommes (2021) has pointed out "The real economy is too complex to fully understand and agents use simple ... heuristics" (or rules) to form expectations. In their 1997 seminal paper BH assume that a finite set of rules to form expectations compete in the mind of each agent. The competition is won by the rule that yields the best forecasting performance, i.e., the smallest forecast error. Each agent switches from a predictor to another depending on the precision of the results obtained in forecasting.

In our model, for simplicity, the set of available heuristics consists only of two elements.

1. According to the *extrapolative* or “naive” rule the expectation formed in period t (before the current price q_t is observed) of the price of land in t+1 coincides with the most recent observation of the price of land, namely q_{t-1}. In symbols $q^{e,1}_{t,t+1} = q_{t-1}$.

2. The *fundamentalist* rule pins down the expected future price of land to the long run (steady state) level: $q^{e,2}_{t,t+1} = q^*$.

Fundamentalists know the “true model” of the economy (i.e., the model that we will present in section 5); compute the steady state price and implicitly assume mean reverting behavior of the price. Their expectations are not model consistent, however, because they do not take into account the presence of extrapolators. In other words they form expectations that would be rational only in a representative agent economy in the absence of shocks while the economy they live in is characterized by heterogeneous expectations.

Extrapolators, on the other hand, are, by definition, myopic. In De Grauwe’s wording, they are driven by “animal spirits”. In a boom, they are optimist because their expectations follow the increasing trajectory of the price of land. In a recession, for the same trend-following behaviour, they are pessimist. Along an out of equilibrium monotonic trajectory (a boom or a bust), as soon as the price of land deviates from the steady state, the naive predictor outperforms the fundamentalist one. In this situation, therefore, the best strategy is the naive heuristic because an extrapolator makes a forecast error which is generally smaller than the one made by a fundamentalist. From this follows that the higher asset price volatility, the more frequent are waves of optimism and pessimism.

The *average expectation* of the future price of land is the weighted average of the expectations held by fundamentalists and extrapolators, the weights being the fractions of agents who adopt each rule:

$$q^e_{t,t+1} = n_{1,t-1}q_{t-1} + n_{2,t-1}q^* \tag{10}$$

where the weights $n_{1,t-1}$ and $n_{2,t-1}$ are the fractions of agents that use predictor 1 (naive) and predictor 2 (fundamentalist) in t-1 (known in t).[5] These fractions are endogenously determined as follows:

$$n_{1,t-1} = \frac{\exp[\beta(U_{1,t-1})]}{\exp[\beta(U_{1,t-1})] + \exp[\beta(U_{2,t-1})]}$$

$$n_{2,t-1} = \frac{\exp[\beta(U_{2,t-1})]}{\exp[\beta(U_{1,t-1})] + \exp[\beta(U_{2,t-1})]},$$

where β is the intensity of choice and $U_{i,t-1}$ measures the "fitness" of the i-th predictor (in period t-1), $i = 1, 2$. The fitness, in turn, is negatively correlated with the (squared) size of the forecasting error made (in t-1) using that predictor: $U_{i,t-1} = -(\varepsilon_{i,t-1})^2$ where $\varepsilon_{i,t-1} = q_{t-1} - q^{e,i}_{t-2,t-1}$. In our setting:

$$U_{1,t-1} = -(q_{t-1} - q^{e,1}_{t-2,t-1})^2$$

$$U_{2,t-1} = -(q_{t-1} - q^{e,2}_{t-2,t-1})^2,$$

with $q^{e,1}_{t-2,t-1} = q_{t-3}$ and $q^{e,2}_{t-2,t-1} = q^*$. Since the current price is not known when agents form expectations on the future price, the expectation formed in t-2 for t-1

[5] In the following we will denote the operation of averaging across types of agents defined by the heuristic they use with the symbol E^H_t. By definition, therefore: $q^e_{t,t+1} = E^H_t(q_{t+1})$.

is the price in t-3. Hence $U_{1,t-1} = -(q_{t-1} - q_{t-3})^2$ and $U_{2,t-1} = -(q_{t-1} - q^*)^2$. Taking into account these definitions, in the end, each fraction turns out to be a non linear function $n_i(.)$ of the price of land in t-1 and in t-3 and of the fundamental value, given the intensity of choice:

$$n_{1,t-1} = n_1(q_{t-1}, q_{t-3}, q^*; \beta) := \frac{\exp[-\beta(q_{t-1} - q_{t-3})^2]}{\exp[-\beta(q_{t-1} - q_{t-3})^2] + \exp[-\beta(q_{t-1} - q^*)^2]}$$

$$n_{2,t-1} = n_2(q_{t-1}, q_{t-3}, q^*; \beta) := \frac{\exp[-\beta(q_{t-1} - q^*)^2]}{\exp[-\beta(q_{t-1} - q_{t-3})^2] + \exp[-\beta(q_{t-1} - q^*)^2]}$$

Substituting these expressions in (10) we end up with:

$$q^e_{t,t+1} = q(q_{t-1}, q_{t-3}, q^*; \beta) := n_1(q_{t-1}, q_{t-3}, q^*; \beta)q_{t-1} + n_2(q_{t-1}, q_{t-3}, q^*; \beta)q^* \quad (11)$$

It is straightforward to generalize this argument.

Definition 1 ***Average expectation***

The average expectation taken in t+s of the future price of land in t+s+1, with $s \in \mathbb{Z}$ *is* $q^e_{t+s,t+s+1} := n_{1,t+s-1}q_{t+s-1} + n_{2,t+s-1}q^*$ *with* $n_{1,t+s-1} = n_1(q_{t+s-1}, q_{t+s-3}, q^*; \beta)$ *and* $n_{2,t+s-1} = 1 - n_{1,t+s-1} = n_2(q_{t+s-1}, q_{t+s-3}, q^*; \beta)$.
Hence $q^e_{t+s,t+1} := q(q_{t+s-1}, q_{t+s-3}, q^*; \beta)$

The average expectation formed in t-1 of the price of land in t is derived from the definition above setting $s = -1$:

$$q^e_{t-1,t} = q(q_{t-2}, q_{t-4}, q^*; \beta) := n_1(q_{t-2}, q_{t-4}, q^*; \beta)q_{t-2} + n_2(q_{t-2}, q_{t-4}, q^*; \beta)q^* \quad (12)$$

The intensity of choice β plays a key role in agents' expectation formation. It can take on any value in the interval $0 \leq \beta < \infty$. The size of this parameter measures the rapidity with which an agent switches to the best heuristic: the higher is β, the faster is the agent in adopting the rule with the lowest error.[6] At the lower bound $\beta = 0$ agents are incapable of switching: they stick to the chosen predictor whatever the forecasting performance of the heuristic they use. By construction, when $\beta = 0$ the population is split in half: $n_1 = n_2 = 1/2$. In the opposite polar case $\beta \to \infty$ (so called "neoclassical deterministic choice model"), agents switch immediately to the best strategy.

5 Behavioural Credit Cycles (Mark I)

In this section and in the following one, we model *Behavioural Credit Cycles* (BCC hereafter) by embedding the heuristic switching mechanism presented above in a credit cycle model à la KM with limited pledgeability. In this section we present and discuss a simple setting - labelled BCC-Mark I - in which only lenders hold heterogeneous expectations.

Consider an agrarian economy in which there are two classes of agents: a continuum of unit mass of identical (impatient) farmers/borrowers and a continuum of unit mass of heterogeneous (patient) gatherers/lenders. We assume the following:

Assumption 3 ***Heuristic switching among lenders***
Only gatherers/lenders form expectations on the future price of land and the mechanism of expectation formation is based on switching between two heuristics: the extrapolator or naive rule (predictor of type 1) and the fundamentalist rule (type 2).

In symbols: $q^{e,i}_{t,t+1}$ is the expectation formed in t by the lender of type $i = 1, 2$ of the price of land in t+1. The framework that we explore in this section is

[6] From a different but equivalent perspective, we can think of $1/\beta$ as the "cost of switching" to a different heuristic.

therefore characterized by *intra-class* heterogeneous expectations *limited to gatherers/lenders*. Agents belonging to this class can be of two types, identified by the predictors they use. As to farmers, in this section we assume they do not form expectations on the future (as in KM). This assumption will be relaxed in section 6.

Given a unit increase in tradable output, the generic farmer decides how much to consume/save comparing the discounted sum of period marginal utilities of the usual strategies. We consider the same KM setting with limited pledgeability of section 3. From Assumptions 1 and 2 and from $R = 1/\gamma'$ follows that the farmer's best strategy consists in consuming only non tradable output ($x_t = ck_{t-1}$) and saving all the tradable output to lever investment. Hence the borrowing constraint is binding.

The borrowing constraint forced on the farmer by the lender of type i in this setting is: $b_t = \frac{1-m}{R} q^{e,i}_{t,t+1} k_t,\ i = 1, 2$. The farmer borrows up to a maximum equal to a fraction $(1 - m)$ of the market value of the collateral *expected by the gatherer/lender* he gets in touch with. Since a fundamentalist gatherer holds expectations on the future value of collateralized land that are in principle different from the expectations of a naif gatherer, the borrowing constraint imposed by the former is in principle different from that imposed by the latter. Thanks to the linearity of the expression above, *on average*, the borrowing constraint becomes

$$b_t = (1-m)\frac{q^e_{t,t+1}}{R}k_t. \tag{13}$$

Equation (13) can be conceived as the borrowing constraint forced on the farmer by the "average gatherer/lender", i.e., by the agent who represents the mean of the naive and fundamentalist gatherers/lenders. The only difference between the (average) borrowing constraint in this setting and the borrowing constraint in a

KM setting with limited pledgeability – i.e., equation (1) – is the fact that in (13) expectations are heterogeneous and formed according to the heuristics switching model.

From (13) follows that, on average, the downpayment is $d_t = q_t - \frac{q^e_{t,t+1}}{R}(1-m)$. From the flow of funds constraint of the farmer $d_t k_t = (a + q_t)k_{t-1} - Rb_{t-1}$ we get the demand for land:

$$k_t = \frac{1}{q_t - \frac{q^e_{t,t+1}}{R}(1-m)}[(a + q_t)k_{t-1} - Rb_{t-1}]. \tag{14}$$

The (average) gatherer's problem consists in maximizing the discounted sum of period utilities subject to a series of period flow of funds constraint over an infinite horizon. The optimization problem requires the formation of expectations on the future values of the variables on interest. In the present setting, expectations are formed using heuristics. We denote with $E^H_t v_{t+1}$ the *average expectation* taken in t of the value of variable v in $t+1$ i.e., the weighted average of the expectations held by fundamentalists and extrapolators, the weights being the fractions of agents who adopt each rule: $E^H_t v_{t+1} = n_{1,t-1}v_{t-1} + n_{2,t-1}v^*$. In symbols:

$$\max_{x'_{t+s}, k'_{t+s}, b_{t+s}} \quad E^H_t \left(\sum_{s=0}^{\infty} \gamma'^s x'_{t+s} \right)$$
$$\text{s.t.} \quad q_{t+s}(k'_{t+s} - k'_{t+s-1}) + b_{t+s} + x'_{t+s} = G(k'_{t+s-1}) + Rb_{t+s-1},$$

Singling out x'_{t+s} from the flow of funds constraint and plugging it into the expected utility function we get:

$$x'_t + \gamma' E^H_t x'_{t+1} + ... = G(k'_{t-1}) + Rb_{t-1} - q_t(k'_t - k'_{t-1}) - b_t +$$
$$+ \gamma'[G(k'_t) + Rb_t - E^H_t q_{t+1}(k'_{t+1} - k'_t) - E^H_t b_{t+1}]$$

From the FOC with respect to b_t we infer that the interest rate is pinned down by the rate of time preference of the lender $R = 1/\gamma'$ as in KM. From the FOC with respect to k_t' we get $q_t = u(k_t) + \frac{E_t^H q_{t+1}}{R}$ where $u(k_t) = \frac{1}{R} G'(\overline{K} - k_t)$ and $E_t^H q_{t+1} \equiv q_{t,t+1}^e = n_{1,t-1} q_{t-1} + n_{2,t-1} q^*$. We can rewrite the FOC above as an asset price equation

$$q_t = u(k_t) + \frac{q_{t,t+1}^e}{R} \tag{15}$$

Note that $d_t = u(k_t) + m \frac{q_{t,t+1}^e}{R}$.

The evolution over time of the state variables of interest is governed by the dynamical system (13) (14) (15). From (13) we get $Rb_t = q_{t-1,t}^e (1-m) k_t$. Plugging this expression in (14), we get:

$$k_t = \frac{a + q_t - (1-m) q_{t-1,t}^e}{q_t - \frac{q_{t,t+1}^e}{R}(1-m)} k_{t-1} \tag{16}$$

We have reduced the dimensionality of the dynamical system to the expectational difference equations (15) (16). The expectations that show up in these equations are given by (11) and (12). Hence:

$$q_{t-1,t}^e = q(q_{t-2}, q_{t-4}, q^*; \beta)$$
$$q_{t,t+1}^e = q(q_{t-1}, q_{t-3}, q^*; \beta)$$

The trajectory of the farmer's land and of the current price of land is therefore affected in a non-linear way by the history of the price of land up to 4 lags. The dynamic system, though simple on surface, is deeply non linear and high-order.

In the steady state $q_t = q_{t-1} = q_{t-2} = q_{t-3} = q_{t-4} = q^*$. It is easy to conclude that

Remark 1 ***Unbiasedness in the steady state***

In the steady state expectations are unbiased because (i) both the fundamentalist *and the naive heuristics lead to the same prediction: the future price of land is expected to be equal to the steady state price of land; (ii) by definition, the future price of land is equal to the steady state price of land. Hence* $\varepsilon_1^* = \varepsilon_2^* = 0$. *Therefore, agents are indifferent between adopting any of the two.*[7] *The choice of the predictor, in this case, can be assimilated to tossing a coin: the probability of choosing predictor 1 (resp. 2) is 1/2. As a consequence* $n_1^* = n_2^* = 1/2$.

The steady state values of the state variables of the dynamical system (13) (14) (15) are identical to those of the RCC model of section 3, namely (7) (8) (9).

5.1 The effects of a productivity shock

In order to study the effects of heterogeneous expectations on the dynamics of state variables, we investigate the consequences of an unexpected temporary shock to the productivity of the farmer's land (e.g., an unexpected good harvest). We suppose that until period t-1 the productivity of the farmer's land were a and the system were in the steady-state. In period t a positive shock occurs so that productivity increases temporarily to $a(1+\Delta)$ where $\Delta > 0$ in period t and $\Delta = 0$ in period t+s for $s \in N, s \geq 1$.

By assumption agents form expectation before the shock. Therefore they are caught by surprise because their expectations are aligned to the steady state $q^e_{t-1,t} = q^e_{t,t+1} = q^*$. As a consequence also expected debt commitments are pinned down to the steady state $Rb^* = q^*(1-m)k^*$. In period t, after the shock, equations

[7] This is due to the fact that, in contrast to the original Brock and Hommes (1997) setting, we have not assumed that agents incur a cost C to employ one of the heuristics. We are implicitly assuming that the fundamentalist predictor does not require more information than the naive one to be used.

(13)(14)(15) become

$$b_t = \frac{q^*}{R}(1-m)k_t \tag{17}$$

$$k_t = \frac{1}{q_t - \frac{q^*}{R}(1-m)}[a(1+\Delta) + q_t)k^* - Rb^*] \tag{18}$$

$$q_t = u(k_t) + \frac{q^*}{R} \tag{19}$$

Substituting (19) into (18) we obtain an expression in k_t from which we can retrieve the first round effect of the shock on the farmer's landholding:

$$k_t \left[u(k_t) + (1-m)\frac{q^*}{R} \right] = [a(1+\Delta) + (q_t - q^*) + mq^*]k^*$$

Substituting this expression for k_t into (19) and (17) yields q_t and b_t.

From period t+s on, for $s \geq 1$, the dynamic model is as follows:

$$b_{t+s} = \frac{q^e_{t+s,t+s+1}}{R}(1-m)k_{t+s}, \tag{20}$$

$$k_{t+s} = \frac{1}{q_{t+s} - \frac{q^e_{t+s,t+s+1}}{R}(1-m)}[(a + q_{t+s})k_{t+s-1} - Rb_{t+s-1}], \tag{21}$$

$$q_{t+s} = u(k_{t+s}) + \frac{q^e_{t+s,t+s+1}}{R}, \tag{22}$$

We follow KM in linearizing the dynamical system around the steady state. Denoting with hatted variables the percent deviations (of the variables) from the steady state, the dynamic system (17) (18) (19) in period t becomes:

$$\hat{b}_t = \hat{k}_t = \Delta \frac{(1-m)(R-1)}{R-1+m} \tag{23}$$

$$\hat{q}_t = \Delta \frac{(1-m)(R-1)^2}{R\eta} \tag{24}$$

where η is the elasticity of the residual supply of land of the farmer with respect to the opportunity cost at the steady state. On impact, the productivity shock boosts the asset price and the farmer's land; credit increases at the same rate of land.

After linearization, the system (20) (21) (22) in period t+s, $s \geq 1$ becomes:

$$\hat{k}_{t+s} = \frac{R + (R-1)(1-m)}{R-1+m}\hat{k}_{t+s-1} + \frac{1-m}{R-1+m}\hat{q}^e_{t+s,t+s+1} - \frac{R(1-m)}{R-1+m}\hat{b}_{t+s-1} \tag{25}$$

$$\hat{q}_{t+s} = \frac{R-1}{R\eta}\hat{k}_{t+s} + \frac{1}{R}\hat{q}^e_{t+s,t+s+1} \tag{26}$$

$$\hat{b}_{t+s} = \hat{q}^e_{t+s,t+s+1} + \hat{k}_{t+s} \tag{27}$$

This system describes the evolution of state variables after the (temporary) shock has disappeared. Notice that the non-linearity of the original system consisting of (13) (14) (15) has been removed by means of linearization but there is still a hidden non-linearity due to the composition of the population. In fact from Definition 1, using $n_2 = 1 - n_1$ and playing a bit with algebra follows that $q^e_{t+s,t+s+1} - q^* = n_{1,t+s-1}(q_{t+s-1} - q^*)$. Dividing both sides by q^* we get $\hat{q}^e_{t+s,t+s+1} = n_{1,t+s-1}\hat{q}_{t+s-1}$. But $n_{1,t+s-1} = n_1(q_{t+s-1}, q_{t+s-3}, q^*; \beta)$. In the end, therefore

$$\hat{q}^e_{t+s,t+s+1} = n_1(q_{t+s-1}, q_{t+s-3}, q^*; \beta)\hat{q}_{t+s-1} \tag{28}$$

The residual inner non linearity of the linearized system will affect the dynamics remarkably.

In order to track the after-shock dynamics, we calibrate the model as shown in Table 1. For ease of comparison, the numerical values of η, R and Δ are taken from KM, while $(1-m)$ is broadly consistent with the empirical evidence of LTVs in normal times.[8]

[8] During the extraordinary expansion of mortgage lending which led to the subprime crisis LTV increased abnormally reaching and even overcoming unity in the USA.

Table 1. Model calibration

Parameter	Value	Description
η	0.1	Elasticity of the residual supply of land to the farmer
R	1.01	Interest rate
$1-m$	0.8	LTV ratio
Δ	0.01	Productivity shock

We will study the dynamics generated by the linearized system for different values of the lender's intensity of choice. When β takes on extreme values the composition of the population in terms of types is fixed and the deviation from the steady state of the expected price of land formed in t for t+1 is proportional to the actual deviation lagged one period. If $\beta = 0$ then $n_1 = 0.5$ and $\hat{q}^e_{t+s,t+s+1} = 0.5\hat{q}_{t+s-1}$; if $\beta \to \infty$ then $n_1 = 1$ and $\hat{q}^e_{t+s,t+s+1} = \hat{q}_{t+s-1}$.

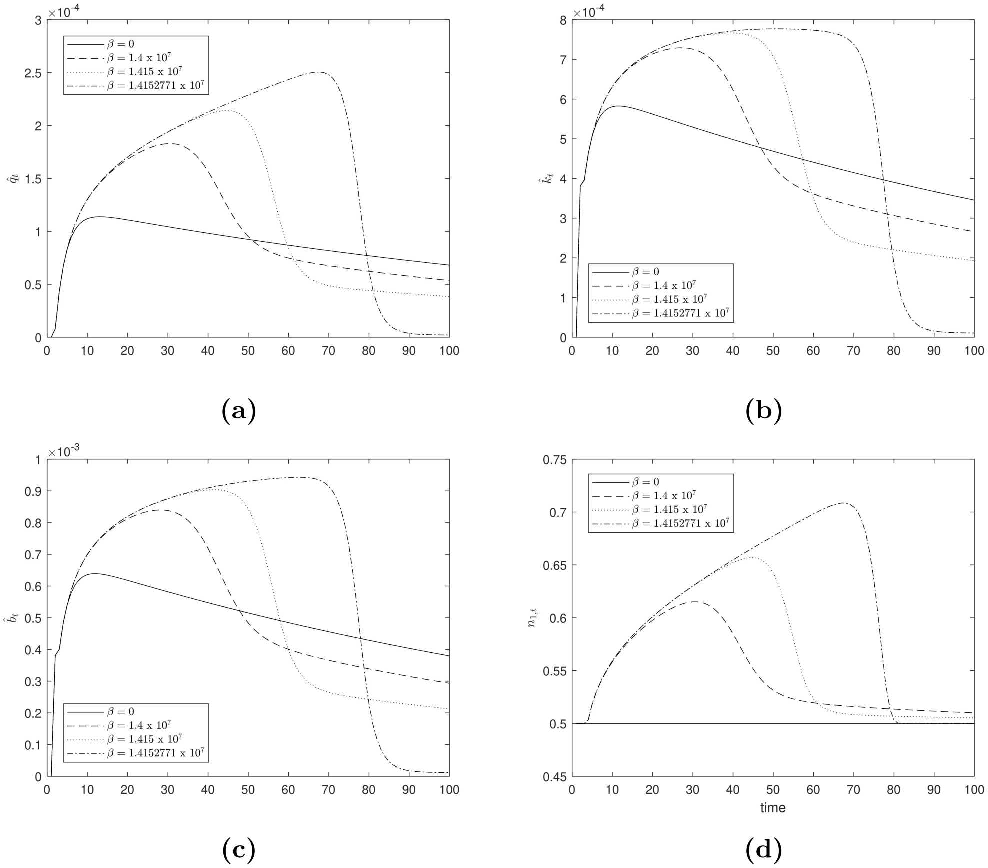


Figure 1. Panels (a), (b), (c) show the effects of a productivity shock $\Delta = 0.01$ on the dynamics of $\hat{q}_t$, $\hat{k}_t$ and $\hat{b}_t$ respectively for different values of β. Panel (d) shows the evolution over time of the fraction of naive agents for different values of β.

The shock generates impulse response functions for $\hat{q}, \hat{k}, \hat{b}$ shown in panels (a),(b),(c) of figure 1 for different values of β. Due to the shock, whatever the value of the intensity of choice, all the variables shoot up and reach a maximum, then decrease and revert to the initial steady state. The deviation from the steady state is positive in any period of the time window considered. The financial accelerator due to the interaction of asset price, collateral constraint and production is at work as in KM, even if the expectation formation mechanism is different.

Let's analyze now the change in the pattern generated by different numerical values of the intensity of choice. Consider first the benchmark case $\beta = 0$. In this case agents are incapable of switching and the population of lenders is split in two: half of the lenders adopt the naive heuristic and the other half the fundamentalist predictor all the time. The magnitude of the positive deviation at the maximum (i.e., the amplitude of the positive fluctuation), however, is "small". A 1% change in productivity generates an increase of the price of land (relative to the steady state) at the maximum of 1/100 of 1%. The maximum deviation of the farmer's landholding is 6/100 of 1% (the dynamic pattern of loans is similar to that of land).

In the benchmark, the composition of the population in terms of heuristics does not change. To explore the effects of heuristic switching we consider positive and increasing values of the intensity of choice.

Positive numerical values of β (see non-solid lines) yield asymmetric hump shaped impulse response functions characterized by the following features:

- As β increases, it takes longer to reach the maximum deviation from the steady state (which measures the amplitude of the fluctuation). The interval of time necessary to reach the maximum increases with β.
- The amplitude of the fluctuation is higher than in the benchmark and is increasing with β.
- Convergence to the steady state after the maximum becomes faster as β increases. The larger the amplitude of the fluctuation, the steeper the decline of the variable after the maximum.

Consider for instance the impulse response functions associated to the highest value of the intensity of choice ($\beta = 1.4152771 \times 10^7$). All the state variables (in panels (a),(b),(c)) reach a maximum in about 80 periods, much later than in the benchmark. On the other hand, the amplitude of the fluctuation is much bigger than in the benchmark. A 1% change in productivity generates an increase of the price of land at the maximum of 2.5/100 of 1%. The amplitude of the farmer's land fluctuation is approximately 8/100 of 1%. After the maximum, all the variables decrease steeply so that in 10 periods (around period 90) they are back to the initial steady state while in the benchmark the deviation from the steady state is still sizable even at the end of the time window considered (period 100).

As shown in panel (d) the increase of the intensity of choice enhances the choice of the naive predictor. Around period 80, 70% of the population held naive expectations. The dynamics of the hump-shaped impulse response is clearly driven by lenders' "animal spirits" i.e., by the fraction of lenders who hold naive expectations (according to De Grauwe's definition). As the price of land departs from the steady state due to the shock, the forecasting mistake made by using the fundamental heuristic increases, lenders switch to the extrapolating rule (at a pace governed by the intensity of choice parameter) and the fraction of naive lenders increases. This increase of extrapolators at the beginning of an ascending phase of the asset price captures a wave of optimism that feeds back into further increases of the price of land until the maximum is reached. The turning point is due to the dissipation of the effects of the shock (which is temporary by assumption). As the price of land declines, reverting to the fundamental (steady state) price, the forecasting mistake made by using the fundamental heuristic shrinks and some naive agents turn fundamentalists. A wave of pessimism makes convergence to the steady state faster than in the benchmark. The positive feedback from the expected to the actual price, therefore, enhances the financial accelerator. Figure 2 shows the scatter plot of the fraction of naive lenders and the deviation of the price of land from the steady state. The positive correlation is striking.

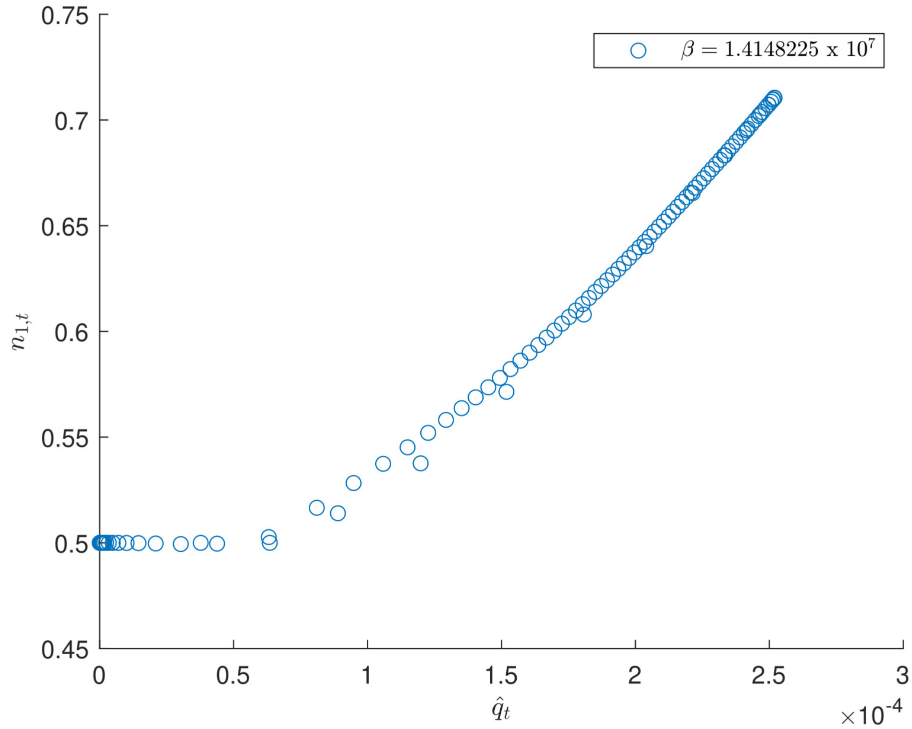


Figure 2. Scatter plot of $\hat{q}$ vs the fraction of extrapolators n_1 (in the population of lenders) in BCC Mark I

While for finite values of β, the trajectories converge to the steady-state. If $\beta \to \infty$ ("neoclassical deterministic choice model") then all the agents immediately switch to the naive predictor ($n_1 = 1$) and $\hat{q}^e_{t+s,t+s+1} = \hat{q}_{t+s-1}$. This case (not shown) is characterized by exponentially increasing deviations of the variables from the steady state.

For the sake of comparison, we have explored the consequences of the same productivity shock in the RCC model (see Figure 3). Consider first RCC in the special case $m = 0$, which coincides with the original KM model (with full pledgeability). A temporary productivity shock in KM generates a much more pronounced but also short lived deviation from the steady state than in BCC Mark I. A 1% change in productivity generates an increase of the price of land at the maximum (in one period) of 10%. The maximum deviation of the farmer's landholding is 100%. After the maximum, all the variables decrease steeply so that in 2 periods they are back to the initial steady state. The magnitude and the persistence (or lack thereof) of the impulse-response functions is clearly unrealistic.

The picture changes dramatically in the case of limited pledgeability. For positive levels of m, the amplitude of the fluctuation decreases and the time to maximum increases. The smallest positive value of m we consider is $m = 0.0001$, which means that 99.99% of the farmer's land is collateralized. In this almost full pledgeability case, a 1% change in productivity generates an increase of the price of land at the maximum of 5%. The maximum deviation of the farmer's landholding is 50%. The amplitude of the fluctuation is cut in half. After the maximum, all the variables decrease but it takes longer to go back to the initial steady state.

For $m = 0.2$ – which is the numerical value used in the simulations of BCC Mark I – a 1% change in productivity generates an increase of the price of land at the maximum of 4.4/1000 of 1%. The amplitude of the farmer's land fluctuation is approximately 4/100 of 1%. After the maximum, all the variables decrease but at an extremely slow pace.

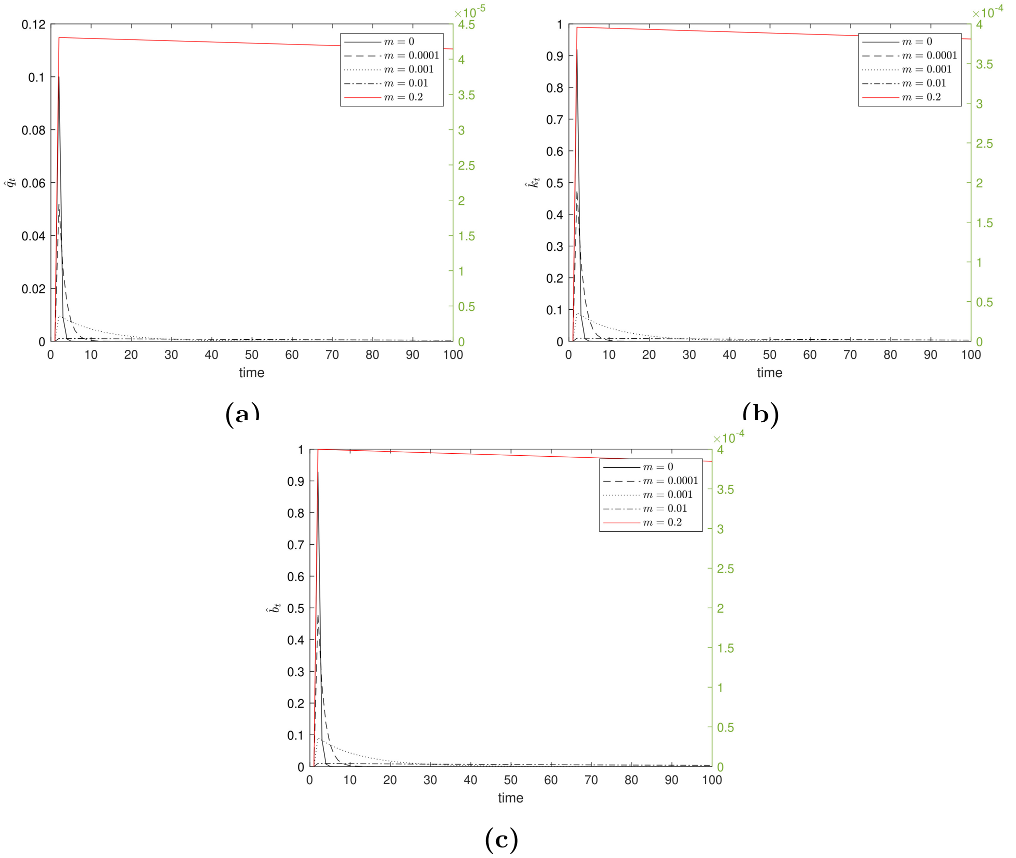


Figure 3. Panels (a), (b) and (c) show the effects of a productivity shock $\Delta = 0.01$ on the dynamics of $\hat{q}_t$, $\hat{k}_t$ and $\hat{b}_t$ respectively in RCC for different values of m. The right y-axis shows the values for $m = 0.2$.

In BCC Mark I the amplitude of the fluctuation generated by the shock (when $m = 0.2$) is much higher and more persistent than in the corresponding RCC. Lower numerical values of m (i.e., a higher fraction of collateralized land) yield more pronounced fluctuations in the RCC setting. In order to account for the empirically observable persistence of a shock, in models with rational expectations it is frequently assumed that the shock has an auto-regressive component. In our setting the persistence-enhancing mechanism is the co-existence of heterogeneous expectations that slows down the pace at which variables converge to the steady-state.

6 Behavioural Credit Cycles (Mark II)

In this section we modify the framework in order to take into account the possibility of default. Default, in fact, dramatically modifies the shape of behavioural credit cycles by introducing a crucial type of non-linearity. To do so, first of all we assume that not only gatherers but also farmers form expectations. This is a necessary condition to provide a rationale for the voluntary default decision on the part of borrowers.

Assumption 4 ***Heuristic switching among lenders and borrowers***
Both gatherers and farmers form expectations on the future price of land by means of simple heuristics. Expectations can be either naive (predictor of type 1) or fundamentalist (type 2).

The framework that we explore in this section is therefore characterized by *intra-class* heterogeneous expectations *both for lenders and for borrowers.*[9] In the following we will "lump together" agents of the same class adopting different heuristics

[9] The population of this economy – consisting of a continuum of unit mass of (impatient) farmers/borrowers and a continuum of unit mass of (patient) gatherers/lenders – can be partitioned in 4 sets, characterized by agents of class/type F1 (naive farmers), F2 (fundamentalist farmers), G1 (naive gatherers), G2 (fundamentalist gatherers). A borrowing/lending relationship is a link between an agent of class F and one of class G. Therefore a naive farmer can be matched to a naive gatherer – creating a connection between F1 and G1 – or a fundamentalist gatherer (F1,G2). Analogously, a fundamentalist farmer can borrow from a naive gatherer (F2,G1) or from a fundamentalist gatherer (F2,G2). Expectations are uniform across classes of agents in connections (F1,G1) and (F2,G2), expectations are different across classes in connections (F1,G2) and (F2,G1). We will abstract from the intricacies of the matching process which would require an analysis of the effects of expectational heterogeneity for each of the 4 types of connections (hence a more granular networked heterogeneous agents model). We will instead imagine that an average farmer gets in touch with an average gatherer.

by imagining that an *average* farmer (i.e., a fictitious borrowing agent in which naive and fundamentalist beliefs coexist) gets in touch with an *average* gatherer (a fictitious lending agent that consists of a naive and a fundamentalist fractions). In order to allow for heterogeneity of expectations between farmers and gatherers we make the following

Assumption 5 ***Intensity of choice heterogeneity***
The gatherer's intensity of choice is different from the farmer's: $\beta^F \neq \beta^G$.

Thanks to this assumption, we can write the fractions of types in each class as follows

$$n^F_{i,t-1} = \frac{\exp[\beta^F(U_{i,t-1})]}{\exp[\beta^F(U_{1,t-1})] + \exp[\beta^F(U_{2,t-1})]}$$

$$n^G_{i,t-1} = \frac{\exp[\beta^G(U_{i,t-1})]}{\exp[\beta^G(U_{1,t-1})] + \exp[\beta^G(U_{2,t-1})]}$$

where $i = 1, 2$. The measure of fitness of predictor $U_{i,t-1}$ is negatively correlated with the forecasting error made using that predictor, that in turn depends exclusively on the actual and predicted price. The latter is equal to q_{t-1} for the naive agent and q^* for the fundamentalist agent, *independently of the class* the agent belongs to. In other words a naive (fundamentalist) farmer has the same expected price as a naive (fundamentalist) gatherer. Therefore, by construction, Assumption 5 is key in assuring *interclass* expectations heterogeneity. In fact, thanks to this assumption $n^F_{i,t-1} \neq n^G_{i,t-1}$ and therefore $q^F_{t,t+1} \neq q^G_{t,t+1}$. We don't have a prior on the relative size of the intensity of choice of the farmer and the gatherer. In simulations we assume that the latter is more capable than the former to switch to the best heuristic: $\beta^F < \beta^G$.

Taking into account the definitions of the forecasting errors, in the end, each fraction turns out to be a non linear function of the price of land in t-1 and in t-3 and of the fundamental value, given the intensity of choice:

$$n_{1,t-1}^{F} = n_1(q_{t-1}, q_{t-3}, q^*; \beta^F) := \frac{\exp[-\beta^F(q_{t-1} - q_{t-3})^2]}{\exp[-\beta^F(q_{t-1} - q_{t-3})^2] + \exp[-\beta^F(q_{t-1} - q^*)^2]}$$

$$n_{2,t-1}^{F} = n_2(q_{t-1}, q_{t-3}, q^*; \beta^F) := \frac{\exp[-\beta^F(q_{t-1} - q^*)^2]}{\exp[-\beta^F(q_{t-1} - q_{t-3})^2] + \exp[-\beta^F(q_{t-1} - q^*)^2]}$$

Substituting these expressions in (10) we end up with:

$$q_{t,t+1}^{F} = q(q_{t-1}, q_{t-3}, q^*; \beta^F) := n_1(q_{t-1}, q_{t-3}, q^*; \beta^F)q_{t-1} + n_2(q_{t-1}, q_{t-3}, q^*; \beta^F)q^* \tag{29}$$

Applying the same line of reasoning to the gatherer we can write:

$$q_{t,t+1}^{G} = q(q_{t-1}, q_{t-3}, q^*; \beta^G) := n_1(q_{t-1}, q_{t-3}, q^*; \beta^G)q_{t-1} + n_2(q_{t-1}, q_{t-3}, q^*; \beta^G)q^* \tag{30}$$

6.1 Debt: Validation vs. repudiation

Interclass expectations heterogeneity allows to explore a new scenario in the borrower-lender relationship – characterized by the farmer's default – that was, by construction, not conceivable both in the original KM model and in the previous setting characterized by intra-class heterogeneity among lenders only. Inter-class heterogeneity may lead to a misalignment between the loan the farmer expects to receive and the loan the gatherer actually extends – grounded, in the end, in a misalignment of expectations on the future price of land – that generates uncertainty on the outcome of the borrower/lender relationship and possibly default.

To explore these issues we assume that the preferences of the average farmer are represented by the usual discounted sum of future consumption flows (on an infinite horizon) expected in t. From Assumptions 1 and 2 follows $x_t = ck_{t-1}$. Given the timing, at the moment the farmer decides how much land to buy, he must form an expectation on the size of the loan he will get. He knows that the lender will limit the size of the loan to the future value of land. Hence he forms an expectation on the future price of land in order to assess the (maximum) level of credit she could possibly get. In symbols:

$$b_t^F = \frac{q_{t,t+1}^F}{R}(1-m)k_t \tag{31}$$

As a consequence the farmer's demand for land is

$$k_t = \frac{1}{q_t - \frac{q_{t,t+1}^F}{R}(1-m)}\left[(a+q_t)k_{t-1} - Rb_{t-1}\right]. \tag{32}$$

Notice, however, that the actual size of the loan is determined by the lender's expectation:

$$b_t = \frac{q_{t,t+1}^G}{R}(1-m)k_t \tag{33}$$

Having received the loan, the farmer decides whether to actually purchase k_t or to "take the money and run", i.e., default on debt commitments. In the former case we are in a regime of Normal times; if the farmer repudiates debt, period t is characterized by default.

According to KM "creditors must never allow a farmer's debt obligations to rise above the value of his land; otherwise he will simply abscond, leaving the land behind but taking all the fruit with him."(KM 1997, p. 218)[10] In the present setting the size of the loan the farmer actually receives b_t can indeed be greater than the credit expected by the farmer b_t^F. This is due to the fact that in the

[10]We interpret the expression "taking all the fruit with him" as the appropriation of the loan in real terms to consume it.

present context the farmer is active in evaluating collateralizable wealth while in KM and in the previous setting (section 5) she is not. Therefore there can be a misalignment of expectations on the future price of land between borrower and lender which is a necessary condition for default. As in Assenza & Berardi (2009) we put forward the following default condition.

Assumption 6 ***Default condition***

The farmer defaults on his payment if actual debt turns out to be greater than the farmer's expected debt: $b_t > b_t^F$*. From* (31) *and* (33) *follows that this inequality is satisfied if*

$$q_{t+1}^G > q_{t+1}^F, \tag{34}$$

where q_{t+1}^G and q_{t+1}^F are given by (30) and (29). If inequality (34) is satisfied, at the beginning of period t the farmer repudiates his debt.

1. In period t, the (average) farmer chooses k_t (the farmer's demand for land or desired landholding) given his expectation $q_{t,t+1}^F$ about the future price of land. She therefore will form an expectation on the size of the loan she will receive: $b_t^F = \dfrac{q_{t,t+1}^F}{R}(1-m)k_t$;

2. The gatherer decides the amount of credit to extend given k_t and his expectation $q_{t,t+1}^G$ about the future price of land. The actual size of the loan is determined by the lender's expectation: $b_t = \dfrac{q_{t,t+1}^G}{R}(1-m)k_t$;

3. After receiving the loan, the farmer decides whether to actually purchase k_t or to "take the money and run" defaulting on debt commitments. In the former case we are in a regime of normal times, in the latter one we are in default. By assumption 6 the farmer repudiates debt if the loan he expected turns out to be smaller than the loan the gatherer is willing to extend.

- In case of default, the farmer will not purchase land in t (the market for land will be put on hold) so that the farmer's landholding is the same as in $t-1$: $k_t = k_{t-1}$.

4. In period $t+1$, if the farmer has not defaulted he will be able to reimburse debt $Rb_t = q^G_{t,t+1}(1-m)k_t$. If, on the contrary, the farmer has repudiated debt:

   - He will consume b_t;
   - The gatherer will terminate the debt contract, repossess $(1-m)k_t = (1-m)k_{t-1}$ and sell it at the price q_t (realized price). We assume that the defaulting farmer cannot access credit because the gatherer is not willing to lend (the credit market is put on hold in $t+1$ in case of deafult in t). However, the farmer still holds mk_{t-1} of (uncollateralized) land and can use this land to consume and to generate savings. The farmer therefore can in principle increase landholding by using internal funds.
   - To simplify matters to the greatest possible extent, we assume that lenders have a very short memory so that the farmer can re-access the credit market the period after $t+1$.[11]. In other words, we assume that the credit market is put on hold only for one period. After $t+1$, therefore, we are back to step 1.

[11] This is the main novelty with respect to Assenza and Berardi (2009), where, after the first bankruptcy, "the relationship borrower/lender comes to an end [...] and farmers disappear from the economy."

6.2 Debt commitments are validated

Suppose that the default condition 6 is not satisfied. The gatherer's optimization problem is the same as in section 5. We rewrite it here for the reader's convenience:

$$\max_{x'_{t+s},k'_{t+s},b_{t+s}} \quad E_t^H \left(\sum_{s=0}^{\infty} \gamma'^s x'_{t+s} \right)$$
$$\text{s.t.} \quad q_{t+s}(k'_{t+s} - k'_{t+s-1}) + b_{t+s} + x'_{t+s} \leq G(k'_{t+s-1}) + Rb_{t+s-1},$$

Following the usual procedure and recalling (33), in the optimum:

$$q_t - \frac{q^G_{t,t+1}}{R} = u(k_t). \tag{35}$$

where $q^G_{t,t+1} = q(q_{t-1}, q_{t-3}, q^*; \beta^G)$. In normal times the dynamics of the model with heterogeneous expectations are determined by equations (32) and (35). This dynamical system is essentially the same as in Mark I, the only difference being that in the present setting the intensity of choice is not uniform across classes so that the expectations of the farmer and of the gatherer can be different. Also in Mark II, therefore, in normal times the trajectory of the farmer's land and of the current price of land is affected in a non-linear way by the price of land in the past. In the steady state $q_t = q_{t-1} = q_{t-2} = q_{t-3} = q_{t-4} = q^*$ and expectations are unbiased for the reasons discussed above. Normal times, however, can be disrupted. We discuss the consequences of default in the following section.

6.3 Default

Suppose now that in t the default condition 6 is satisfied: the farmer repudiates debt.

At the beginning of period t – before default – the farmer got b_t. Due to default, he will consume it in $t+1$ instead of using it to purchase land in t as in Normal times.

The flow of funds constraint of the farmer in $t+1$ – after the default occurred in t – is $q_{t+1}(k_{t+1} - mk_t) + x_{t+1} \leq (a+c)mk_t + b_t$. Notice that $k_t = k_{t-1}$ in case of default. The farmer consumes $cmk_t + b_t$ and employs tradable output amk_t to purchase land at the price q_{t+1}. From the equation above we get the farmer's demand for land:

$$k_{t+1} = \frac{1}{q_{+1}}(a + q_{t+1})mk_t. \tag{36}$$

Let us now consider the behaviour of the gatherer. At the beginning of period t she has extended a loan b_t to the farmer. At the end of the period, since the farmer has defaulted, the gatherer terminates the debt contract, repossesses $(1-m)k_{t-1}$ and sells it at the price q_t.

After default, in t+1 the flow of funds constraint will be

$$q_{t+1}(k'_{t+1} - k'_t) + x'_{t+1} \leq G(k'_t) + K^r_t,$$

where $K^r_t = q_t(1-m)k_t$ is the market value of the land the farmer collateralized and the gatherer has repossessed and sold because of default. Hence the FOC of the optimization problem of the gatherer after the farmer's default with respect to k'_t is exactly the same as in normal times, i.e., equation (35).

6.4 Dynamics

As said above, for simplicity we assume that the farmer can re-access the credit market the period after $t+1$. Therefore, the credit market is de-activated only in period $t+1$.

Under this assumption, we can write the model as follows:

$$k_t = \begin{cases} \frac{1}{q_t - \frac{q^F_{t+1}}{R}(1-m)}[(a+q_t)k_{t-1} - Rb_{t-1}] & \text{if } q^G_{t+1} \le q^F_{t+1} \wedge (q^G_t \le q^F_t \vee q^G_{t-1} > q^F_{t-1}) \\ k_{t-1} & \text{if } q^G_{t+1} > q^F_{t+1} \wedge q^G_t \le q^F_t \\ \frac{1}{q_t}(a+q_t)mk_{t-1} & \text{if } q^G_t > q^F_t \wedge q^G_{t-1} \le q^F_{t-1} \end{cases} \tag{37}$$

$$q_t = u(k_t) + \frac{1}{R}q^G_{t+1} \tag{38}$$

$$b_t = \begin{cases} 0 & \text{if } q^G_t > q^F_t \wedge q^G_{t-1} \le q^F_{t-1} \\ \frac{q^G_{t+1}}{R}(1-m)k_t & \text{otherwise} \end{cases}. \tag{39}$$

In words: in period t, if the default condition is not satisfied the system proceeds along the trajectory of Normal times. This goes on until the default condition is met. When default happens the credit market and the debt contract are still a crucial part of the story since the gatherer has indeed lent b_t and repossessed collateralized land as guaranteed by the debt contract. Since the farmer has defaulted, however, in the next period the credit market is put on hold.

Linearizing around the steady-state we get

$$\hat{k}_t = \begin{cases} \frac{R+(R-1)(1-m)}{R-1+m}\hat{k}_{t-1} + \frac{1-m}{R-1+m}\hat{q}^F_{t+1} - \frac{R(1-m)}{R-1+m}\hat{b}_{t-1} & \text{if } \hat{q}^G_{t+1} \le \hat{q}^F_{t+1} \wedge (\hat{q}^G_t \le \hat{q}^F_t \vee \hat{q}^G_{t-1} > \hat{q}^F_{t-1}) \\ \hat{k}_{t-1} & \text{if } \hat{q}^G_{t+1} > \hat{q}^F_{t+1} \wedge \hat{q}^G_t \le \hat{q}^F_t \\ \frac{m[\eta(R-1)(1-m)+R\eta]-R\eta}{R\eta+R-1} + m\frac{\eta(R-1)(1-m)+R\eta+R-1}{R\eta+R-1}\hat{k}_{t-1} + \frac{\eta(m-1)}{R\eta+R-1}\hat{q}^G_{t+1} & \text{if } \hat{q}^G_t > \hat{q}^F_t \wedge \hat{q}^G_{t-1} \le \hat{q}^F_{t-1} \end{cases} \tag{40}$$

$$\hat{q}_t = \frac{R-1}{R\eta}\hat{k}_t + \frac{1}{R}\hat{q}^G_{t+1} \tag{41}$$

$$\hat{b}_t = \begin{cases} -1 & \text{if } \hat{q}^G_t > \hat{q}^F_t \wedge \hat{q}^G_{t-1} \le \hat{q}^F_{t-1} \\ \hat{q}^G_{t+1} + \hat{k}_t & \text{otherwise} \end{cases}, \tag{42}$$

with $\hat{q}^F_{t+1} = n^F_{1,t-1}\hat{q}_{t-1}$ and $\hat{q}^G_{t+1} = n^G_{1,t-1}\hat{q}_{t-1}$.

Notice that, albeit linearized around the steady state, the system above is non-linear not only because the fractions of agents adopting one predictor or the other are non linear functions of the price of land (as in Mark I) but also because default generates a regime switch.

We perform a numerical analysis of the model in order to investigate its evolution after an unexpected and temporary shock to productivity at time t:

$$a_{t+s} = \begin{cases} a(1+\Delta) & \text{if } s = 0 \\ a & \text{if } s \geq 1 \end{cases}$$

Given (40) (41) and (42) and the calibration in Table 2, we explore the trajectories of $\hat{q}_t$, $\hat{k}_t$ and $\hat{b}_t$ for different positive values of β^G. As said above, we assume that the farmer is less capable than the gatherer to switch to the best heuristics. To simplify matters we set $\beta^F = 0$.[12] This means that the population of farmers is equally split between naive and fundamentalist agents and the composition is constant over time: $n_1^F = n_2^F = 1/2$.

Table 2. Model calibration

Parameter	Value	Description
β^F	0	Farmer's intensity of choice
η	0.1	Elasticity of the residual supply of land to the farmer
R	1.01	Interest rate
$(1-m)$	0.8	LTV ratio
Δ	0.01	Productivity shock

[12] The dynamics as both β^G and β^F vary are shown in Appendix B.

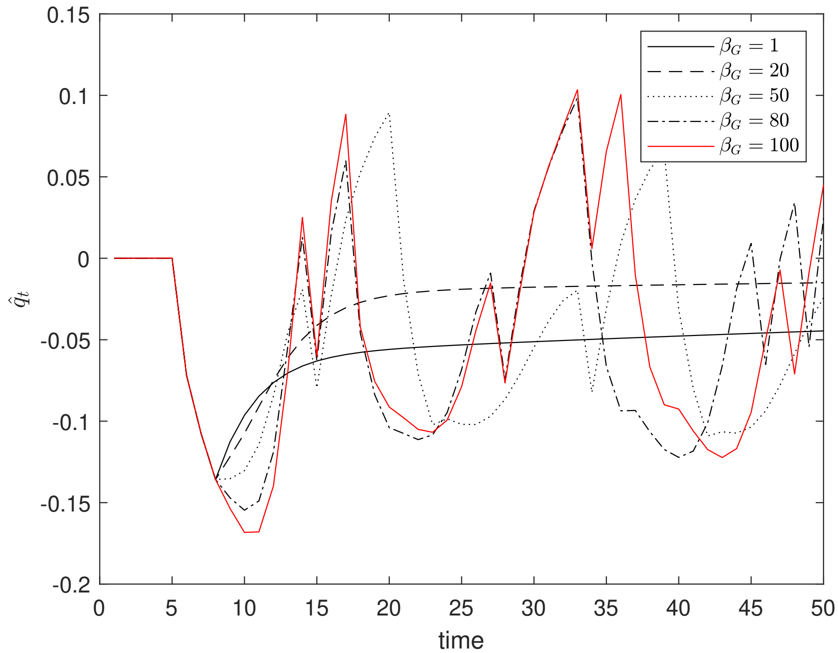
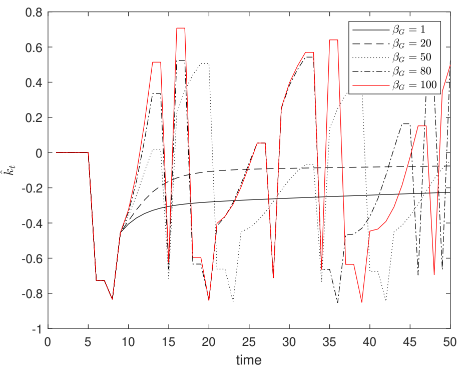
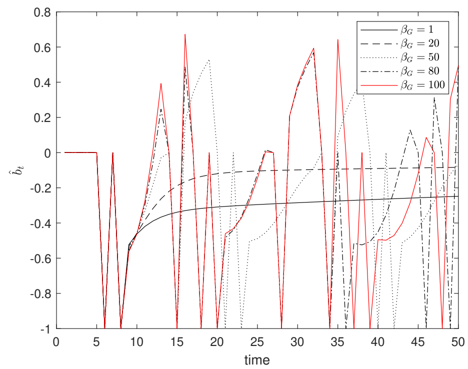


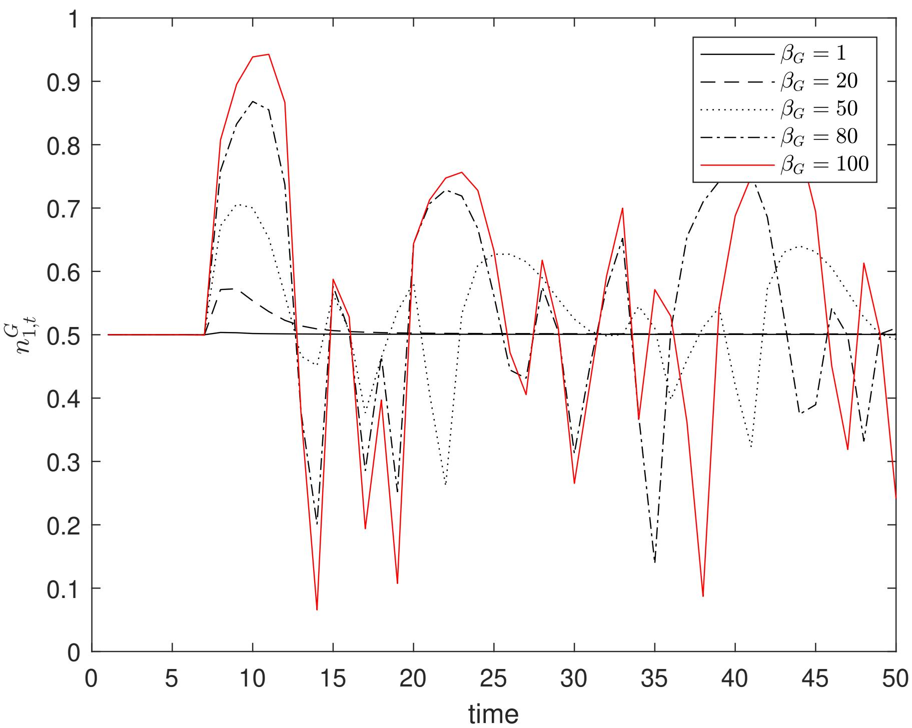
Figure 4. Dynamics of $\hat{q}_t$ after a temporary productivity shock $\Delta = 0.01$ for different values of β^G.



(a)



(b)



(c)

Figure 5. Panels (a) and (b) show the dynamics of $\hat{k}_t$ and $\hat{b}_t$ after a temporary productivity shock $\Delta = 0.01$ for different values of β^G. Panel (c) shows the evolution over time of the fraction of naive gatherers for different values of β^G.

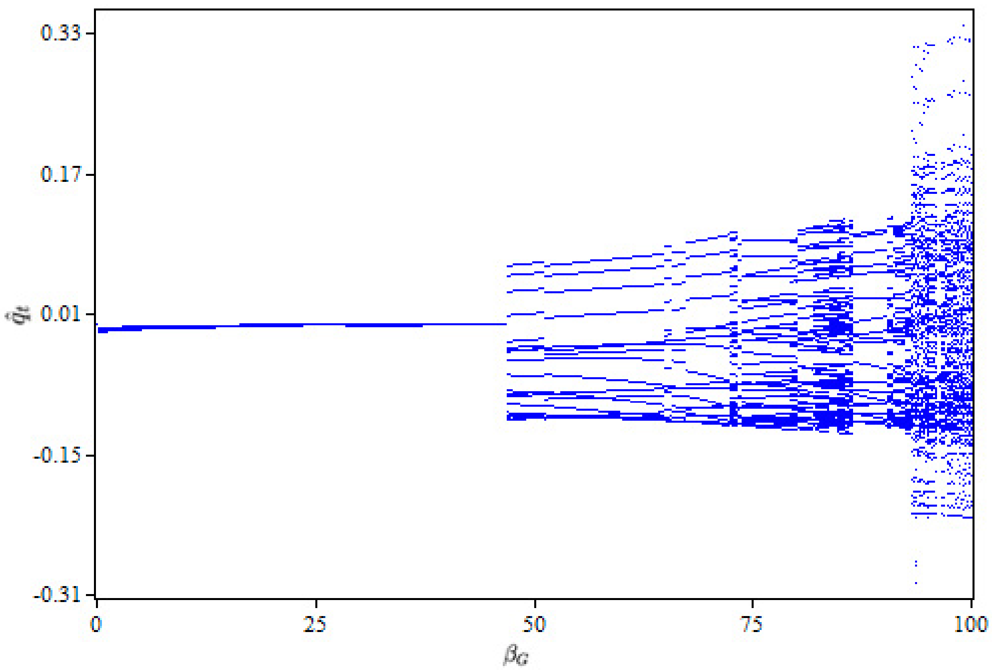


Figure 6. Bifurcation diagram for the price of land with respect to the intensity of choice β^G.

Having set $\beta^F = 0$, in the simulations there is no heuristic switching among farmers. Hence the price of land expected by the (average) farmer is: $q^F_{t,t+1} = 0.5q_{t-1} + 0.5q^*$. From this equation and (30) we infer that the farmer defaults (i.e., condition (6) is satisfied) if: (i) animal spirits prevail among lenders ($n^G_{1,t} := n_1(q_{t-1}, q_{t-3}, q^*; \beta^G) > 0.5$) and the price of land is higher than the fundamental price $\hat{q}_{t-1} > q^*$ or (ii) animal spirits prevail among farmers ($n^G_1 < 0.5$) and the price of land is lower than the fundamental price $\hat{q}_{t-1} < q^*$. Case (i) occurs during a positive deviation of the asset price from the steady state (a "boom") while case (ii) occurs during a negative deviation of the asset price from the steady state (a "bust").

In figure 4 we plot the time series of the deviation of the price of land from the steady state following a temporary positive productivity shock in BCC Mark II.[13] Two striking facts emerge from this simulation.

First, after the departure from the steady state the price of land returns monotonically back to the steady state (as in the usual impulse-response functions) only for relatively "low" numerical values of the intensity of choice. For high values of the intensity of choice ($\beta_G > 50$ in the figure) the shock triggers ample and repeated oscillations *on both sides of the steady state* (positive and negative deviations). From the bifurcation diagram 6 in fact we infer that the steady state is unique for $\beta_G < 47$. Higher values of the intensity of choice yield persistent fluctuations of variable periodicity (more on this momentarily). This feature is not present in BCC Mark I where deviations form the steady state occur always in the usual impulse-response fashion (deviations are always non-negative).

Second, contrary to BCC Mark I, in BCC Mark II, a *positive* productivity shock generates (from period 6 onward) a *negative* deviation of the price of land from the steady state (almost) from the beginning of the time window considered. This surprising feature of BCC Mark II is due exactly to the possibility of default. From the artificial time series generated by the simulation we observe that the positive productivity shock yields, in fact, a positive deviation of the price of land from the steady state (as expected) on impact. Even if – at the beginning of the time window considered – this increase is extremely small (not visible in the figure), it is sufficient for lenders to switch to the extrapolating heuristic, which yields a forecasting error smaller than the fundamental rule of thumb. The fraction of lenders who hold naive expectations shoots up (see figure 5, panel (c)). Therefore, very soon (period 6) the default condition is satisfied: $q_G > q_F$. This leads to the farmer's default which in turn triggers the fall in the price of land. The higher the intensity of choice, the deeper the price of land goes and the larger in size the

[13]For the sake of clarity, the model has been simulated for 50 periods.

negative deviation from the steady state. With $\beta_G = 100$ for instance the negative deviation of the price of land reaches the trough at -17% in period 10. During the bust, the fraction of naive gatherers increases up to 90%.

The change of dynamic pattern after the trough leads to smaller and smaller negative deviations from the steady state until a positive deviation occurs (see period 15). In the upswing, the fraction of naive lenders goes down (see again figure 5). Thereafter there is a series of short run fluctuations whose motivation and development are similar to the initial one discussed above. The other state variables follow trajectories similar to that of the price of land.

Figure 7 shows the scatter plot of the fraction of naive lenders and the deviation of the price of land for high intensity of choice in BCC Mark II. The relationship is clearly non monotonic. The fraction of naive lenders is high not only for positive high deviations from the steady state but also for sizable absolute values of negative deviations.

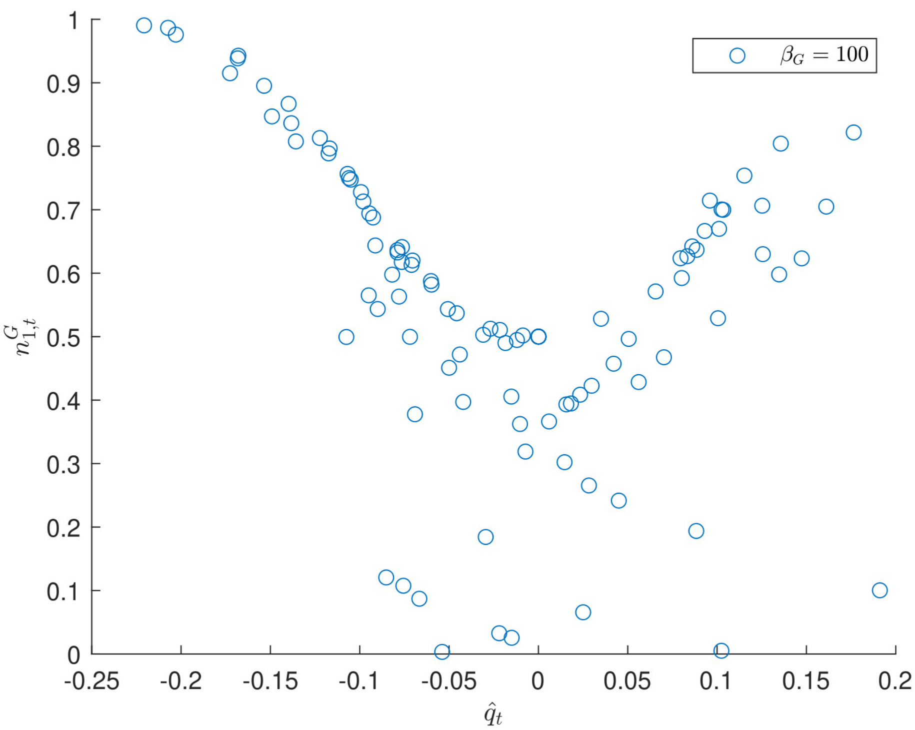


Figure 7. Scatter plot of $\hat{q}$ vs the fraction of extrapolators n_1^G in the population of lenders in BCC Mark II

In the end, therefore, expectations misalignments between farmers and gatherers may generate defaults that trigger an alternation of booms and busts.[14] Figure 6 shows that for high values of the intensity of choice the price of land either converges to n-cycles or displays chaotic dynamics. This means that, regardless of how good the gatherer's understanding of the economy is, she will still not be able to prevent bankruptcies.

[14]One may argue that such dynamics are not robust to the introduction of hedging markets against external shocks, as Krishnamurthy (2003) has shown in the KM model. However, the emergence of these additional frictions only requires the system not to be at the steady-state and it does not depend on the size of the initial departure. The dual role of land is certainly necessary, but it is not the financial accelerator per se to generate the persistent boom-bust cycle observed here.

7 Conclusions

In this paper we have explored the intertwined dynamics of asset prices and the macroeconomy in an environment characterized by a credit friction *à* la Kiyotaki and Moore augmented with heterogeneous expectations and heuristic switching *à* la Brock and Hommes. This behavioural twist allows to better understand and replicate the effects of shocks.

Even in the absence of actual defaults, following a positive productivity shock, our behavioural model of the credit cycle (BCC Mark I) generates hump-shaped impulse-response functions that are more realistic than the impulse response functions generated by the same shock in a corresponding model with rational expectations (RCC).

When heterogeneous expectations characterize both gatherers and farmers, the bahavioural model can take into account also defaults and the reopening of the credit market (BCC Mark II). This framework generates ample and persistent fluctuations of the state variables involved (if the intensity of choice of the lender is sufficiently high), a feature that is absent in BCC Mark I (and of course in RCC).

In a richer setting, where more economic agents are involved (e.g., a New-Keynesian framework), it would be interesting to derive policy suggestions taking into account the expectations-driven frictions we study here. Such analysis should relax some strict assumptions we adopt to keep the model simple and more manageable, for instance the presence of a single asset, the simple exit-entry mechanism and the absence of direct one-to-one borrowing-lending relationships between farmers and gatherers holding different heuristics.

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A The farmer's optimal consumption/saving choice

Consider the farmer's marginal unit of tradable fruit at time t. The farmer considers three heuristics ("strategies" in KM): (1) save tradable output and reinvest it to maximize the levered increase of landholding; (2) save tradable output in t and use the return (maturing in t+1) to start investing from t+1 onward; (3) consume tradable output in t.

If he follows strategy (1), the amount of land the farmer can obtain by investing the marginal unit of tradable fruit in land is $1/d_t$. The additional land will bear additional fruit equal to $(a+c)\dfrac{1}{d_t}$ in $t+1$; the farmer will consume $\dfrac{c}{d_t}$ in $t+1$ and use the additional tradable output $\dfrac{a}{d_t}$ as downpayment to obtain additional land in $t+1$ equal to $\dfrac{a}{d_t}\dfrac{1}{d_{t+1}}$. The additional land will bear additional fruit equal to $(a+c)\dfrac{a}{d_t}\dfrac{1}{d_{t+1}}$ in $t+2$; the farmer will consume $c\dfrac{a}{d_t}\dfrac{1}{d_{t+1}}$ in $t+2$ and so on. In the steady state the farmer will consume $\dfrac{c}{d^*}$ in t+1, $c\dfrac{a}{d^{*2}}$ in t+2 and so on, where, as shown above, $d^* = a\dfrac{R-1+m}{(R-1)(1-m)}$. In the absence of shocks and in the neighbourhood of the steady-state, the discounted sum of marginal utilities (that coincide with consumption flows) with strategy (1) is

$$\sum_{s=0}^{\infty}\gamma^s x_{(1),t+s} = 0+\gamma\frac{c}{d^*}+\gamma^2\frac{a}{d^*}\frac{c}{d^*}+\gamma^3\frac{a}{d^*}\frac{a}{d^*}\frac{c}{d^*}+... = \frac{c}{a}\times\frac{\gamma\frac{a}{d^*}}{1-\gamma\frac{a}{d^*}} \tag{43}$$

If he follows strategy (2), in t the farmer saves one unit of tradable output (so that he does not consume) and gets R in t+1. In t+1, he invests R and gets R/d_{t+1} in land. Hence hhe does not consume also in t+1. The additional land will bear additional fruit equal to $(a+c)\dfrac{R}{d_{t+1}}$ in $t+2$; the farmer will consume $c\dfrac{R}{d_{t+1}}$ and use the additional tradable output $a\dfrac{R}{d_{t+1}}$ as downpayment to obtain additional land in $t+2$ equal to $a\dfrac{R}{d_{t+1}}\dfrac{1}{d_{t+2}}$. The additional land will bear additional fruit equal to $(a+c)\,a\dfrac{R}{d_{t+1}}\dfrac{1}{d_{t+2}}$ in $t+3$; the farmer will consume $ca\dfrac{R}{d_{t+1}}\dfrac{1}{d_{t+3}}$ in $t+3$ and so on. In the absence of shocks and in the neighbourhood of the steady-state, the discounted sum of marginal utilities in the case of strategy (2) is

$$\sum_{s=0}^{\infty}\gamma^s x_{(2),t+s} = 0+0+\gamma^2\frac{R}{d^*}c+\gamma^3\frac{Ra}{d^*}\frac{c}{d^*}+... = \gamma R\frac{c}{a}\times\frac{\gamma\dfrac{a}{d^*}}{1-\gamma\frac{a}{d^*}} \tag{44}$$

Finally, strategy (3) yields $x_{(3),t}=1; x_{(3),t+1}=x_{(3),t+2}=...=0$ so that $\sum_{s=0}^{\infty}\gamma^s x_{(3),t+s} = 1$.

Proposition 1 *Strategy (1) dominates strategy (2).*

Proof: By assumption $\gamma<\gamma'$. Moreover in the optimum $\gamma' R=1$. Hence $\gamma R<1$ and $\sum_{s=0}^{\infty}\gamma^s x_{(1),t+s} > \sum_{s=0}^{\infty}\gamma^s x_{(2),t+s}$ under perfect foresight and in the steady state.

Proposition 2 *If $c>a\left(\dfrac{R-1+m}{\gamma(R-1)(1-m)}-1\right)$ then strategy (1) dominates strategy (3).*

Proof: Strategy (1) dominates strategy (3) if $x_{(1),t+s}=\dfrac{c}{a}\times\dfrac{\gamma\dfrac{a}{d^*}}{1-\gamma\dfrac{a}{d^*}}>1$, i.e., if

$$c>a\left(\frac{1}{\gamma\frac{a}{d^*}}-1\right). \tag{45}$$

From the equation of the downpayment in the steady state follows $\dfrac{a}{d^*}=\dfrac{(R-1)(1-m)}{R-1+m}$. Substituting this expression into (45) we get the condition that must be satisfied for strategy (1) to dominate strategy (3). If this condition is satisfied the farmer will always choose to postpone consumption and will borrow as much as possible to buy land.

B Further analysis on β^F and β^G

So far, we have seen how the model behaves when β^F is given and β^G varies. One may ask what happens when they both vary and the best way to show it is by running Monte Carlo simulations. The idea is to investigate the dynamics of the main variables, the fraction of naive gatherers and the fraction of naive farmers as β^G and β^F are randomly selected.

The following figures show the values across 100 Monte Carlo simulations with different random seeds for each value of parameters β^G ranging from 20 and 100 and β^F ranging from 0 to 25.[15]

[15]Again, the values of the intensity of choice follow the order of magnitude of the performance measures. In particular, the intervals of β^G and β^F have been selected to reproduce a situation in which it may happen that a fraction of borrowers has a better understanding of the evolution of the price than lenders.

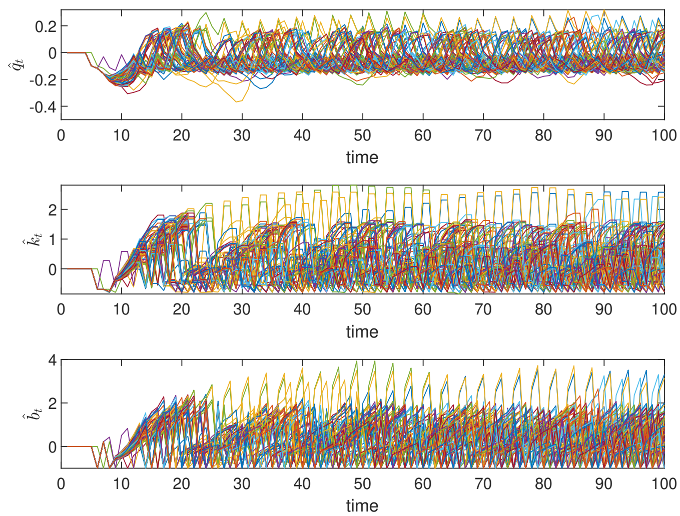

Figure 8. $\hat{q}_t$, $\hat{k}_t$ and $\hat{b}_t$ after a productivity shock $\Delta = 0.01$. Values across 100 Monte Carlo simulations.

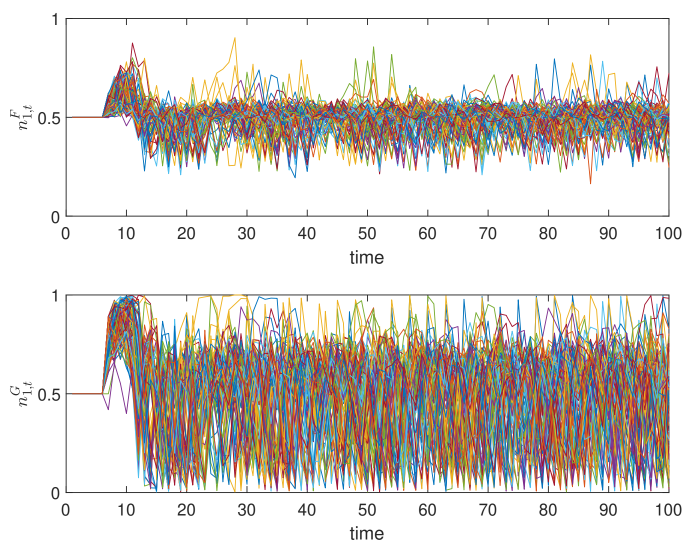

Figure 9. Fractions of naive farmers and naive gatherers. Values across 100 Monte Carlo simulations.

Even if it may be the case of some economies with wider price variations, the endogenous "boom and bust" nature of the model is preserved and there are no significant qualitative differences with respect to the analysis in section 6.4.