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Thomas D. Jeitschko, Pallavi Pal

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Abstract

In recent years, a significant problem with the carbon credit market has been higher than initially predicted price volatility. It is essential to study the market in a repeated-period dynamic setting to identify the factors enabling high fluctuations in prices. In this paper, we examine the dynamic auction design and propose a method to curb price volatility through a flexible supply cap. The equilibrium analysis shows that modifying the cap on per period supply can decrease price fluctuations. Currently, the government or the auctioneer sets a per-period limit on the supply, which reduces at a fixed rate over time. However, this paper suggests that a flexible cap on the per-period supply would be a better alternative. Specifically, we show that correlating the supply rate with expected future demand results in a more stable price.

JEL-Codes: D430, L110, L420.

Keywords: dynamic mechanism design, auctions, emissions permits, environmental regulation, climate change.

Thomas D. Jeitschko
Department of Economics
Michigan State University
466 W. Circle Dr.
USA – East Lansing, MI 48824
jeitschk@msu.edu

*Pallavi Pal**
Business School
Stevens Institute of Technology
Hoboken / NJ / USA
ppal2@stevens.edu

*corresponding author

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1 Introduction

One of the most popular market-based solutions to limit greenhouse gas (GHG) emissions is to use Cap-and-Trade schemes. Under these schemes, firms use emission credits to pay for GHG emissions. The largest share of GHGs is CO2; thus, we focus on the case of carbon emission allowances, which are called carbon credits.[1]

The carbon credit market has historically exhibited high price fluctuations.[2] Demand shocks are one of the major factors that cause price fluctuations. For example, Figure 2 shows volatility in the California Cap-and-Trade program with a drastic decrease during 2016 and 2017. While price fluctuations do not obstruct the operation of Cap-and-Trade markets and price movements are part of the auction mechanism, large-scale price volatility may impede the reduction of carbon emissions. Sharp price increases can increase the cost of compliance for firms, and sharp price decreases can eliminate the incentive to invest in low-carbon technology.[3] Thus, attenuating large fluctuations in carbon prices in the short run can ensure environmental effectiveness. In this paper, we propose adopting a flexible supply cap that depends on the expected future demand as a possible solution to curb price volatility.

Carbon credit markets have responded to extreme fluctuations by using fixed prices and quantitative caps in the emissions trading system and carbon credit auctions. For example, in the Cap-and-Trade program of California, quarterly carbon credit auctions have a price ceiling and a reserve price, which escalate over time.[4] Similarly, in the European Union Emission Trading System (EU ETS), a "Market Stability Reserve" has been created, which releases permits when there are very few in circulation and withdraws them when there are considerably high amounts in circulation.[5] These fixed cap methods perform well under certainty; however, as noted by Ellerman and Wing (2003), in the presence of uncertainty, flexible caps and fixed caps lead to different results. Our method suggests that in the presence of demand uncertainty, a more flexible limit on

[1]This model is also applicable to other markets for GHG emissions.

[2]For example, Nordhaus (2007) showed that the demand for allowances was likely inelastic in the short run, which caused high price volatility. Dutta (2018), Wang (2017), and Zhang and Sun (2016) also provided evidence of price volatility in carbon credit markets.

[3]For details, see Köppl et al. (2011) and Borenstein et al. (2019)

[4]Additionally, manufactures who are trade-sensitive and energy-intensive have an output-based updating allocation system; under this system free carbon credits are conveyed in proportion to production size in previous periods. For details, refer to Borenstein et al. (2019)

[5]For further details, refer to *https://ec.europa.eu/clima/policies/ets/reform_en*

supply can decrease price volatility. Specifically, the cap on allowances available for sale should be structured to be directly proportional to the intensity of the demand shock.[6]

We consider the primary market where the government or social planner sells carbon credits through an auction. To examine the price volatility, we set up a dynamic model where carbon credits are sequentially sold using a uniform auction design.[7] This is the most common auction format used by markets such as RGGI carbon dioxide (CO2) and the EU ETS (CO2).[8] Additionally, our model attempts to capture essential features of the market, such as stochastic demand and differences in demand urgency across firms. We capture these features within the auction to create a more realistic market design. We allow firms to differ in their urgency of obtaining carbon credits; this is modeled as firms having different deadlines. The firm-specific deadlines for buying carbon credits may be due to the vintage of the carbon credit (i.e., expiry of the credit) or the current portfolio of credits owned by the firm. This feature helps us capture the heterogeneity among firms in terms of their demand duration.

In this setting, we derive the equilibrium bid and show that the bid is truthful and ex post incentive compatible. The optimal bidding strategy indicates that buyers consider their future-period payoffs when deciding the expected value of winning in the current period. Thus, the outside option value is endogenously determined through the expected payoff of auctions in the future.[9]

In a dynamic setting, the introduction of demand uncertainty changes the optimal supply rate. In particular, expectations about future supply and demand impact the equilibrium price and the price fluctuations. Consequently, we show how relating the supply with the expected future demand can decrease the impact of the demand shocks on price.

[6] For literature on the comparison of various price control instruments, see Newell and Pizer (2008), Newell and Pizer (2003), Pizer (2002), Roberts and Spence (1976), Weitzman (1978), Weitzman (1974), Yohe (1978), and Stavins (1996)

[7] We refer to the dynamic setting as the case in which a finite number of goods are sold to buyers that arrive over time. There are two types of dynamic setting in the single-good case. The first type holds the set of buyers fixed and changes their types over time as a function of allocations selected in earlier periods (for example, Athey and Segal (2013), Eso and Szentes (2007)). In the second type, a finite number of goods are sold to buyers that arrive over time. This paper considers the second type of dynamic setting, which we refer to as the changing buyer case. The term "changing buyer type" is taken from the dynamic mechanism design literature review Vohra (2012)

[8] For details, see Lopomo et al. (2011)

[9] Apart from carbon credit auctions, this model can be applied to any market where the bidders differ in their valuation of the object and urgency of acquiring the good, including spectrum auctions (for wireless networks) or electricity markets with different delivery dates.

The comparative statistics section analyzes how demand shocks and the supply rate impact the equilibrium price for carbon credits. The results show that an increase (decrease) in future supply decreases (increases) the current price. The change in future supply affects the current price by increasing the opportunity cost of winning for the firm. A raise in the future supply increases the probability of winning carbon credits in the future, thereby increasing the opportunity cost of winning in the current period. Meanwhile, we find that an increase (decrease) in expected future demand causes an increase (decrease) in the current price. The intuition behind the rise in the current price is that a rightward shift in future demand makes the firm demand more in the current period because the carbon credits can be stored and used in future. The outward shift of the demand curve consequently increases the equilibrium price. Therefore, this paper suggests that the future supply should be a function of the expected future demand. In particular, the government can decrease the supply whenever the expected future demand decreases and vice versa. We show that such a policy can reduce price fluctuations in the market.

This paper is related to the literature on price and supply restrictions imposed in the emissions market.[10] Earlier work such as Weitzman (1974) recognized that the optimal instrument would be a contingency message that provided instructions according to the state of the world.

The current literature has introduced new models that help us understand climate policy responsiveness. They suggest various proposals for the structure of carbon pricing instruments. This literature includes papers on the index regulation that suggests that the emission cap should be proportionate to an index such as the GDP or output. For example, Jotzo and Pezzey (2007) assess how well intensity targets indexed to future realized GDP can handle uncertainties in international GHG emissions trading. Additional papers that consider indexing in the emissions market are Quirion (2005), Sue Wing et al. (2006), Newell and Pizer (2008), Branger and Quirion (2014), and Ellerman and Wing (2003). The literature has investigated flexible caps, and it has considered the net benefit of applying a flexible cap using non-auction models such as cost-benefit analysis and emissions prediction & policy analysis.[11]. This paper extends this literature by analyzing how

[10] The early work on this topic was conducted by Roberts and Spence (1976) and Weitzman (1978). They considered price ceilings and floors under demand and supply uncertainty in a static model.

[11] An emissions prediction and policy analysis (EPPA-EU) model is used in Ellerman and Wing (2003) and Sue Wing et al. (2006). The cost-benefit analysis was introduced by Weitzman (1974) and used in Quirion (2005) and Newell and Pizer (2008)

a flexible cap affects the strategic bidding decision of a firm, which affects the final price in the primary sale (during the auction stage).

Apart from indexing, other papers in this literature study how pricing and quantitative instruments can be more responsive to economic fluctuations. Heutel (2012) finds that the optimal policy accommodates the procyclical behavior of carbon emissions. The cap on the emissions trading system in this paper is reduced during recessions and increased during booms. Doda (2016) provides a comprehensive review of the literature comparing fixed and responsive caps in an emissions trading system. Our work extends the above literature by introducing a flexible cap in carbon credit auction design. The previous literature focused on using market equilibrium as the outcome of demand and supply in the emissions trading market. In this study, we focus on examining how a flexible supply rate interacts with Cap-and-Trade auctions. This paper aims to model the responsive supply cap as part of the auction mechanism by including dynamic features in the market, modeling the uncertainty of future demand, and demonstrating how the firm's bid and equilibrium auction price are affected by the flexible supply cap. We consider a dynamic model, which also accounts for heterogeneity in permit lifetime and compliance deadlines.

This paper is also related to the literature on the dynamic mechanism design with multi-dimensional private information (see the survey by Bergemann and Said (2010) for details).[12] It extends this literature to investigate the optimal auction for an auctioneer with an unknown number of buyers and sellers in each period. This paper is also related to the literature on efficient sequential auction with impatient buyers and those on stochastic auctions.[13]

The remainder of the paper is structured as follows: Section 2 and section 3 describe the generalized model setup and derive the equilibrium bid. Section 4 looks at comparative statics

[12] The most relevant paper for our work is Pai and Vohra (2013), who consider the optimal auction for a single seller selling multiple units to stochastically arriving bidders; they also allow the arrival time to be private information. The main difference in our paper is that we consider the auctioneer's problem of identifying an efficient mechanism with stochastically arriving sellers. Furthermore, Pai and Vohra (2013) assumes perfectly patient bidders and focuses on the optimal auction to maximize the seller revenue. Another related paper is Mierendorff (2013), which investigates a revenue-maximizing mechanism for a seller selling a single good in a dynamic environment with buyers having multi-dimensional private information. The main difference in our work is that we consider an efficient mechanism in a dynamic environment with multiple and stochastically arriving sellers.

[13] The most relevant paper in the literature on sequential auctions with impatient buyers is Gershkov and Moldovanu (2010). They examine the allocation of a set of durable goods to a dynamic buyer population. In their setting, objects are durable, whereas in this paper, objects are non-durable, and the total supply in every period is stochastic. For the literature on stochastic auctions, refer to McAfee and McMillan (1987), Mierendorff (2013), Said (2012) and Jeitschko (1999)

focusing on price fluctuations, and section 5 concludes the paper.

2 Model setup

Consider a sequential auction in infinite, discrete-time period model, $t \epsilon \{0, 1, ...\infty\}$. In each period multiple units of a homogeneous good are auctioned. Buyers with single unit demand arrive over time. Buyer i has a valuation v_i, which is an i.i.d. random draw from the distribution $F_v(.)$ on $[\underline{v}, \bar{v}]$. The demand for each buyer lasts for multiple periods. In particular, the buyer has an arrival time a_i and a demand duration k_i. This implies that the unsatisfied demand will last for all $t \epsilon \{a_i, ..., (a_i + k_i)\}$. Thus apart from heterogeneous valuations, buyers also differ in terms of their demand lifetime. The type of a buyer is a triplet consisting of his valuation, arrival time and demand duration, $x = (v_i, a_i, k_i)$; and the type space is given as $X = [\underline{v}, \bar{v}] \times [\underline{a}, \bar{a}] \times [1, \bar{k}]$. A buyer's type is an i.i.d. random draw from a commonly known distribution $F_v \times F_a \times F_k$ over X. We assume that the three components of the type space, i.e. valuation, arrival time and demand duration are independent. For notational ease, we denote the number of active demand periods left for bidder i in period t as $r_{i,t}$. Additionally, for each buyer the probability of his demand surviving in the next period is τ, where $\tau \epsilon [0, 1]$. The parameter τ captures the future demand uncertainty that the buyer might have due to external economic reasons.

In our model, in period t where $r_{i,t} \geq 0$, an agent of type $x_{i,t} = (v_i, r_{i,t})$ who faces a possible payment z_t derives the following instantaneous utility:

$$U(v_i, r_{i,t}) = \begin{cases} (v_i - z_t), & \text{if he wins the auction} \\ 0, & \text{otherwise.} \end{cases}$$

Demand Side The buyer's arrival rate is stochastic: In any period t, n_a new buyers arrive, where n_a is an i.i.d. random draw from the distribution F^n on $\{1, ..., \bar{n}\}$. Additionally, let n_t denote the set of active buyers in period t; n_t is calculated as $n_t = \sum_{x=0}^{\bar{k}} n_{t,x}$, where $n_{t,x}$ denotes the number of bidders with x periods of active demand in period t. The total number of potential buyers is given by $N \epsilon \mathbb{N}$ such that $N \leq \bar{n}\bar{k}$.[14]

[14]Recall that $\bar{n}$ is the upper bound on the new buyers arriving each period and $\bar{k}$ is the upper bound on the number

Supply Side: The supply rate in this model, denoted as m_t, has two main properties. Firstly, in the emission auction markets, the goal is to reduce the supply of emission credits, which reduces the total emission. The reduction in emissions is achieved by having a gradually decreasing supply rate over time. Let the rate at which supply decreases in each period be denoted by $\lambda \ \epsilon \ [0, 1]$. The decreasing rate implies current period supply is λ times the supply in previous period, i.e. $m_t = \lambda m_{t-1}$. Secondly, in this model we allow the supply rate to be a function of demand shocks. Thus, the supply rate λ is a function of the demand uncertainty variable τ, denoted as $\lambda(\tau) = f(\tau)$. Using these two properties, the supply in period t can be written as $m_t = \lambda(\tau) \times m_{t-1}$. The units sold are identical, and the supply decreases at a constant rate.[15] Let the initial period supply be denoted as $m_0 = \bar{m}$, such that $m_t \leq \bar{m} \ \forall \ t > 0$.

Information Structure: The distribution on the buyer's type space, the distribution of buyers' arrival rate, the initial period supply and the decreasing rate of the supply are assumed to be common knowledge. The buyer has private knowledge about his type, composed of his valuation and demand lifetime. He does not have knowledge of the exact number of other buyers, their types, or bid reports. The arrival rate of buyers is stochastic. Thus, the exact per period demand is unknown for future periods.

The timeline of the game is as follows. In each period, t, the active buyers report a bid for a single unit to the auctioneer. The strategy set of the buyers in period t compromises of their bid in period t, i.e. $b_t(v_i, r_{i,t})$, where v_i is the value and $r_{i,t}$ is the number of active periods left at the t^{th} period. The active buyers consist of new buyers arriving in the current period and the existing buyers who are still active and have unfulfilled demand. After receiving bids from the active buyers and an estimate of the number of items to be sold from the social planner, the auctioneer holds an auction to decide the number of items to be traded, denoted as S_t, and the clearing price denoted as z_t.

We now give a detailed description of the auction mechanism. The auctioneer holds a series of static uniform price auctions. Bidders simultaneously submit sealed bids for the item. In each period t, the auctioneer calculates the total number of items to be traded, denoted by S_t, by

of periods a demand can last for a buyer

[15] To focus on the effect of supply and demand dynamics, we will be abstracting away from the multi-unit demand nature of this market

equating the demand (denoted by n_t) and supply (denoted by m_t). The buyers are ranked in ascending order of their bids, and S_t highest bids are accepted to allocate the item. [16] The total traded items S_t is a function of demand n_t and supply m_t, i.e. $S_t(n_t, m_t)$; for convenience we will suppress the dependence and use notation S_t. We will reintroduce the dependence in the notation for the comparative statics section as it would be relevant in that section. Each winning bidder gets one unit of the item and pays the price equal to the highest losing bid.

To formally determine the clearing price, we denote the order statistic for bid as $b^{(l)}$, which represents the l^{th} highest order statistic of the bidding values. Using the ordered bids, the auctioneer selects the S_t highest bids in period t to trade. So the bid of a buyer is accepted for trade if $b_t(v_i, r_{i,t})$ ϵ $\{b^{(1)}, b^{(2)}............b^{(S_t)}\}$. Every winning bidder pays the bid of the highest losing bidder, i.e. $z_t = b^{(S_t+1)}$. The price paid by the winning buyers, in any period t, is equal to the $(S_t + 1)^{th}$ highest order statistic of the bids. All buyers who bid above z_t win the item at a price z_t and each accepted sale request yields a payment equal to z_t for social planner.

We solve the repeated period problem using Subgame Perfect Bayesian Nash equilibrium as the solution concept. In each period, the buyers report a bid for a single unit of the item to the auctioneer. The equilibrium defines a set of strategies and beliefs, such that given the opponents' strategies, the expected payoff of every buyer is maximized in each period. Table **??** summarizes all the notations used in this paper.

Observation 1. *Notice that, although the arrival time a_i is private knowledge, the buyer does not have any incentive to lie about the arrival period. This is due to the independence of the buyer's payoff function and the arrival time a_i. Thus the relevant private information of buyer i is (v_i, k_i).*

3 Equilibrium analysis

Bidder i's bidding strategy is defined as $b^i = \{b_1(v_i, r_{i,1}), b_2(v_i, r_{i,2}), ...b_\infty(v_i, r_{i,\infty})\}$, where $b_t(v_i, r_{i,t})$ denotes the bid in the t^{th} period given that the bidder's value is v_i and demand will last for $r_{i,t}$ periods after period t.[17] As this is a dynamic setting, we start by defining the state variables. Let

[16] If the number of items demanded is equal to the number of available units, i.e. $m_t = n_t$, the auctioneer sells all the available supply in period t, i.e. $S_t = m_t = n_t$. On the other hand, if the number of items demanded is less than available units i.e. $n_t < m_t$, the auctioneer sells $S_t = n_t$ number of carbon credits.

[17] All this assumes, of course, that the particular bidder has not already won an object so is still active in period t.

$\sigma_t = \{m_t, n_t\}$ denote the auction "state" in period t. Recall, n_t denotes the set of active buyers in period t and m_t represents the number of sale units available in period t.

Let us now consider, in period t the maximizing objective function for the buyer, which is the total lifetime payoff after realization of demand and supply, denoted by $V(v_i, r_{i,t}|\sigma_t)$.

$$V(v_i, r_{i,t}|\sigma_t) \equiv \Bigg\{ Pr\Bigg(b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg|\sigma_t\Bigg)\mathbb{E}\Bigg[v_i - b^{(S_t+1)}(v_i, r_{i,t})\Bigg|b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg] \\ + \Bigg[1 - Pr\Bigg(b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg|\sigma_t\Bigg)\Bigg]\tau \int_{n_{t+1}} V(v_i, r_{i,t} - 1|\sigma_{t+1})\Bigg\} \tag{1}$$

The first term represents the expected payoff from the auction in the first period and the second term is the future integrated over possible arrival rates σ_{t+1}. The future payoff is equal to the total payoff for value v_i and $(r_{i,t} - 1)$ periods left. Let $W(v_i, r_{i,t})$ denote the total payoff ex-ante, i.e., before the realization of the state variable 'σ_t',evaluated as follows

$$W(v_i, r_{i,t}) \equiv \int_{r_{i,t}} \tau\Bigg\{ Pr\Bigg(b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg|\sigma_{t_{r_{i,t}}}\Bigg)\mathbb{E}\Bigg[v_i - b^{(S_t+1)}(v_i, r_{i,t})\Bigg|b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg] \tag{2} \\ + \Bigg[1 - Pr\Bigg(b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg|\sigma_t\Bigg)\Bigg]W(v_i, r_{i,t} - 1)\Bigg\}$$

Using the total payoff ex-ante, we can rewrite the ex-post per period payoff in Equation 1 as

$$V(v_i, r_{i,t}|\sigma_{t_{r_{i,t}}}) \equiv \Bigg\{ Pr\Bigg(b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg|\sigma_{t_{r_{i,t}}}\Bigg)\mathbb{E}\Bigg[v_i - W(v_i, r_{i,t} - 1) - b^{(S_t+1)}(v_i, r_{i,t})\Bigg|b(v_i, r_{i,t}) > b_{j\neq i}^{(S_t)}\Bigg] \\ + W(v_i, r_{i,t} - 1)\Bigg\} \tag{3}$$

Pseudo type for each period:

Observe that we have the probability of winning in each period defined in terms of the bid, in order to define that in term of the bidder's type, we will define a per period pseudo type for each period t and bidder i as $\eta_{i,t}$. Let $G_t(.)$ be the distribution for $\eta_{t,i}$. The pseudo type is defined as follows:

$$\eta_{i,t} = \begin{cases} v_i - \sum_{l=t+1}^{k_i} \int_{n_l} \tau^{l-t} G_l^{(S_l)}(\eta_{i,l}|\sigma_l)\Big(\eta_{i,l} - \mathbb{E}[(\eta_l^{S_l})|\eta_{i,l} > \eta_l^{S_l}]\Big), & \text{if } t \geq a_i \text{ or } t \leq k_i \\ 0, & \text{otherwise.} \end{cases}$$

The pseudo type helps simplify the payoff function as well as the bid. The following lemma shows the relation between the pseudo type and the payoff function

Lemma 1. *The equilibrium bid is increasing in pseudo type and the total payoff in period t can be rewritten as follows :*

$$W(\eta_{i,t}) = \int_{n_t} \tau G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\bigg(\eta_{i,t} - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\bigg) + W(\eta_{i,t+1}) \quad (4)$$

$$V(\eta_{i,t}) = G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\bigg(\eta_{i,t} - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\bigg) + W(\eta_{i,t+1}) \quad (5)$$

We can use this equation in order to determine equilibrium bid functions, as demonstrated in the following result. We focus on the symmetric Bayesian Nash equilibrium.

Theorem 1. *The equilibrium per period bidding strategy in the dynamic auction game is given as:*

$$b(\eta_{i,t}) = v_i - W(\eta_{i,t+1})$$

$$or$$

$$b(\eta_{i,t}) = \eta_{i,t}$$

4 Analysis of flexible cap and its effect on price fluctuation

This section evaluates how the supply rate can be strategically used to stabilize price fluctuation arising from demand shocks. Demand shocks can occur from weather conditions, changes in government policies regarding environmental issues, and unpredictable events such as the onset of Covid-19. Section 4.1 looks at the advantage of using flexible cap for curbing price volatility. Additionally, section 4.2 gives preliminary data evidence to motivate the need for a flexible cap.

4.1 Derivation of the optimal flexible cap

In this section we analyze the impact of demand shock on price fluctuations; specifically looking at whether flexible cap can curb demand induced price fluctuations. We compare the flexible cap case to the non-flexible cap case, in order to see the relative benefit of switching to flexible cap. Note that the current model accommodates these two cases, i.e. whether supply rate changes with demand shock, as special cases. Hence we can use the above model to derive comparative analysis in both of these two cases.

$$\text{Non-Flexible cap: } \frac{dm_t}{d\tau} = \lambda'_{NF}(\tau) \times m_{t-1} = 0 \tag{6}$$

$$\text{Flexible cap 1: } \frac{dm_t}{d\tau} = \lambda'_{F_1}(\tau) \times m_{t-1} \geq 0 \tag{7}$$

$$\text{Flexible cap 2 : } \frac{dm_t}{d\tau} = \lambda'_{F_2}(\tau) \times m_{t-1} \leq 0 \tag{8}$$

In the flexible cap cases, the supply rate changes in the event of a demand shock. In this model, demand shocks are interpreted as a change in the demand survival rate, denoted by τ. Note that a negative (positive) demand shock decreases (increases) the survival rate τ.

Proposition 1 states the main result of this section. It shows that the proposed flexible cap 1 decreases price fluctuation and can be used to stabilize the price. From Corollary 1 and 2 we see that λ and τ have an opposite effect on the price. Thus, an optimal strategy will negatively correlate the future supply rate and expected demand. The rest of the section explains the intuition and steps behind the result.

Proposition 1. *In the event of change in demand survival rate (or a demand shock), the change in price would be lower in case of the new supply rate.*

$$\frac{d\mathbb{E}(P_t|\lambda_{F_1})}{d(\tau)} < \frac{d\mathbb{E}(P_t|\lambda_{NF})}{d(\tau)} < \frac{d\mathbb{E}(P_t|\lambda_{F_2})}{d(\tau)}$$

where λ_{F1}, λ_{F_2}, λ_{NF} are defined using equations 7, 8, and 6 correspondingly. To see how the supply rate can stabilize the effect of demand uncertainty on the price, we need first to look at the basic non-flexible case, where we individually analyze the impact of supply rate and the impact of future demand uncertainty on the equilibrium price.

The results from Proposition 2 and Corollary 1 show that a positive demand shock, i.e. an increase in demand survival rate, increases the bidder's total payoff and decreases the current period bid and price. The intuition is straight forward as we would expect that an increase in future demand would make bidders bid more aggressively, which leads to increase in current period bid and price.

Proposition 2. *The bidder's expected payoff is decreases when the demand survival rate decreases.*

$$\frac{d(V(\eta_{i,t}|\sigma_t))}{d(\tau)} \geq 0 \tag{9}$$

Proposition 2 shows that the payoff is positively affected when there is a positive demand shock, i.e. future demand survival rate increases.

Corollary 1. *The current bid and price increase with increase in demand survival rate.*

$$\frac{d(b(\eta_{i,t}))}{d(\tau)} \geq 0$$
$$\frac{d\mathbb{E}(P_t)}{d(\tau)} \geq 0$$

This shows that an increase (decrease) in demand survival rate increases (decreases) the price.

The effect of non-flexible supply rate on market outcomes: As expected, an increase in the supply rate will positively affect the bidder's total payoff and negatively impact the current bid and price. Increasing future supply rate, would lead to a higher supply in the which would increase the opportunity cost of winning the auction in the current period. As we show in the equilibrium section the current period bid is negatively impacted by opportunity cost. Thus, increase in future supply rate decreases the current bid and final price. We provide the formal derivation in Proposition 3 and Corollary 2 to show that the general results of a rightward shift in supply still hold even after the introduction of stochastic demand and multi-dimensional bidder type in the model.

Proposition 3. *The bidder's expected payoff is increasing with the increase in the future supply rate.*

$$\frac{d(V(\eta_{i,t}|\sigma_t))}{d(\lambda)} \geq 0 \tag{10}$$

The above proposition shows that the expected payoff in each period t is positively correlated with the future periods' supply rates. The intuition behind the result is that as the supply increases, the chances of acquiring credits in the future increase, thereby increasing the total expected payoff of the buyer.

Corollary 2. *The bid and price are decreasing with the increase in the future supply rate.*

$$\frac{d(b(\eta_{i,t}))}{d(\lambda)} \leq 0$$
$$\frac{d\mathbb{E}(P_t)}{d(\lambda)} \leq 0$$

The price and the bid in the current period are negatively affected by an increase in the supply rate. Thus, even though the total payoff increases for the bidder, they will still decrease the current period's bid. The intuition behind the increase in the current period bid is that an increase in the supply rate increases the payoff in the future, which increases the opportunity cost of winning in the current period (t^{th} period). Thus, the buyer's bid and the auction price are negatively correlated to the supply rate.

Now that we have established the individual effects of the supply rate and demand shocks on price, we look at how to utilize it to stabilize the price path.

Using proposition 3 and 2 we have analyzed the impact on price in the absence of the adjustment in the supply rate. In this case, impact of a demand shock on expected price is given by $\frac{d\mathbb{E}(P_t|\lambda)}{\delta(\tau)}$. This is true because only τ changes due to a demand shock. On the other side, in the flexible supply rate case, a demand shock will impact the expected price in two ways. First, it will change due to the change in τ; additionally, it will change due to change in supply. Mathematically this would look like

$$\text{Case1: fixed rate} \rightarrow \frac{d\mathbb{E}(P_t|\lambda)}{\delta(\tau)} \tag{11}$$

$$\text{Case2: proposed flexible rate} \rightarrow \frac{\delta\mathbb{E}(P_t|\lambda)}{\delta(\lambda)}\frac{\delta(\lambda)}{\delta\tau} + \frac{\delta\mathbb{E}(P_t|\lambda)}{\delta(\tau)} \tag{12}$$

Note that from Proposition 1 we know that the expected price and demand survival rate are positively correlated. Additionally, from the above two equations, it is evident that the effect in the two cases differ due to the first term in Equation 12, which is negative due to Proposition 2 and Equation 10. Thus, the change in price due to demand shock is lower in the case of the proposed supply rate. □

4.2 Discussion:

This section provides data evidence that confirms the equilibrium bidding behavior derived in our model. Additionally, we look at price movements in the emission markets with fixed caps and identify potential residual fluctuations in the price due to short-term shocks. The evidence of price fluctuation in the markets with fixed caps further motivates an examination of flexible caps.

Recall that the optimal bidding strategy indicates that the buyers consider their future payoffs when deciding the value of winning in the current period. This consideration would result in a correlation between the current price and the advance price of carbon credits. The dependence of current prices on the expected outcome of future auctions is evident if we compare the current and future prices in the California Cap-and-Trade market. Figure(2) plots the prices for current and advance prices, which appear highly correlated. Apart from the auctions held in the early periods, the current price closely follows the future price, which indicates that the current bid also accommodates the effect of future prices.

As noted by Nordhaus (2015) ,Dutta (2018), and Zhang and Sun (2016), high volatility in price is a problem in the carbon credit market. Even after the price caps, there can be a high variance in price within the bounded prices. Let us use the example of the RGGI carbon credit market. Figure(1) shows the price path with significant events during the timeline. Here, we observe that the price fluctuates with changes in demand. For example, 2016 saw a substantial fall in price when the demand uncertainty increased because the supreme court halted the Environmental Protection Agency's Clean Power Plan.[18] This example shows that demand shocks can cause sharp price increases. In our model, the proposed flexible supply rate would have been temporarily adjusted, thereby reducing the intensity of price drops from the temporary demand shock.

Additionally, in the European Union Emissions Trading System (EU ETS), shocks such as technological progress, weather conditions, and prices in related industries have caused a high degree of price fluctuation in the market. For example, EU allowances saw a drastic increase in prices in January 2005. According to the Carbon Market Monitor 2005 Review at PointCarbon, the sharp price change was due to high gas and oil prices (specifically observed in the UK), low

[18] For details, see "Opinion: Supreme Court puts the brakes on the EPA's Clean Power Plan" - February 9 2016 Washington Post.

coal prices, and the onset of cold weather.[19] More recently, the economic downturn caused by COVID-19 has caused a drop in carbon credit prices. For example, the EU ETS allowance prices decreased in the first quarter of 2020 to €17/tCO2e (US$19/tCO2e) compared to approximately €25/tCO2e (US$27/tCO2e) over 2019.[20]

Thus, past data shows the necessity of introducing responsive quantity caps that responds to uncertainty in the market. As discussed in the literature, multiple papers have suggested such a mechanism; our paper contributes by suggesting a demand-dependent cap at the auction stage. This cap may also have economic appeal. Adjusting the cap according to demand shocks can decrease the expected costs incurred for reaching a particular environmental target. This work shows that linking the supply rate to changing market factors will stabilize the price. Further, empirical analysis is required to estimate the exact functional form of the flexible cap.

5 Conclusion

In this paper, we analyzed a dynamic auction setting with the stochastic arrival of bidders and multi-dimensional bidder's type. We derived the BNE bid in the repeated auctions setting. The setup was used to understand the price volatility in the Cap-and-Trade scheme auction of carbon credits. Price uncertainty is a significant concern in the Cap-and-Trade market because firms need a more stable short-term supply of carbon credits to change to more renewable energy. The paper identifies two factors that affect the price fluctuation: the rate of supply and uncertainty in future demand. The uncertainty in future demand is not in the control of the auctioneer (or the government in this case). However, the rate of supply is decided by the government. Thus, the suggested policy in this paper is that the government should correlate the supply rate with the uncertainty in the market. Specifically, they should decrease the future supply rate when the future demand uncertainty in the market increases. In other words, increase the supply rate as the expected future demand increases. The results show that this policy will result in a more stable price over time. The paper also analyzed the general model setup, which can be applied to other

[19] For details, see Carbon Market Monitor 2005 Review, Jan. 2006, available at: http://www.pointcarbon.com/research/carbonmarketresearch/monitor/ and Mason (2009)

[20] For details, refer to https://openknowledge.worldbank.org/handle/10986/33809

markets. One key feature of the paper is that we looked at the multi-dimensional type of bidders. Thus, this model can fit any market where the bidders differ by more than only the object's value. Future works can extend this setting to multi-unit demand and examine double auction settings, which can further generalize the auction design.

This work shows that linking the supply rate to demand shocks can stabilize the price. Furthermore, empirical analysis is required to understand the implementation of such policy. Borenstein et al. (2019) conducted an extensive study on how different factors affected the price fluctuation and provided simulations. Since they looked at the secondary trading market, similar studies are required to analyze the price fluctuation in the primary market of selling carbon credits through auction.

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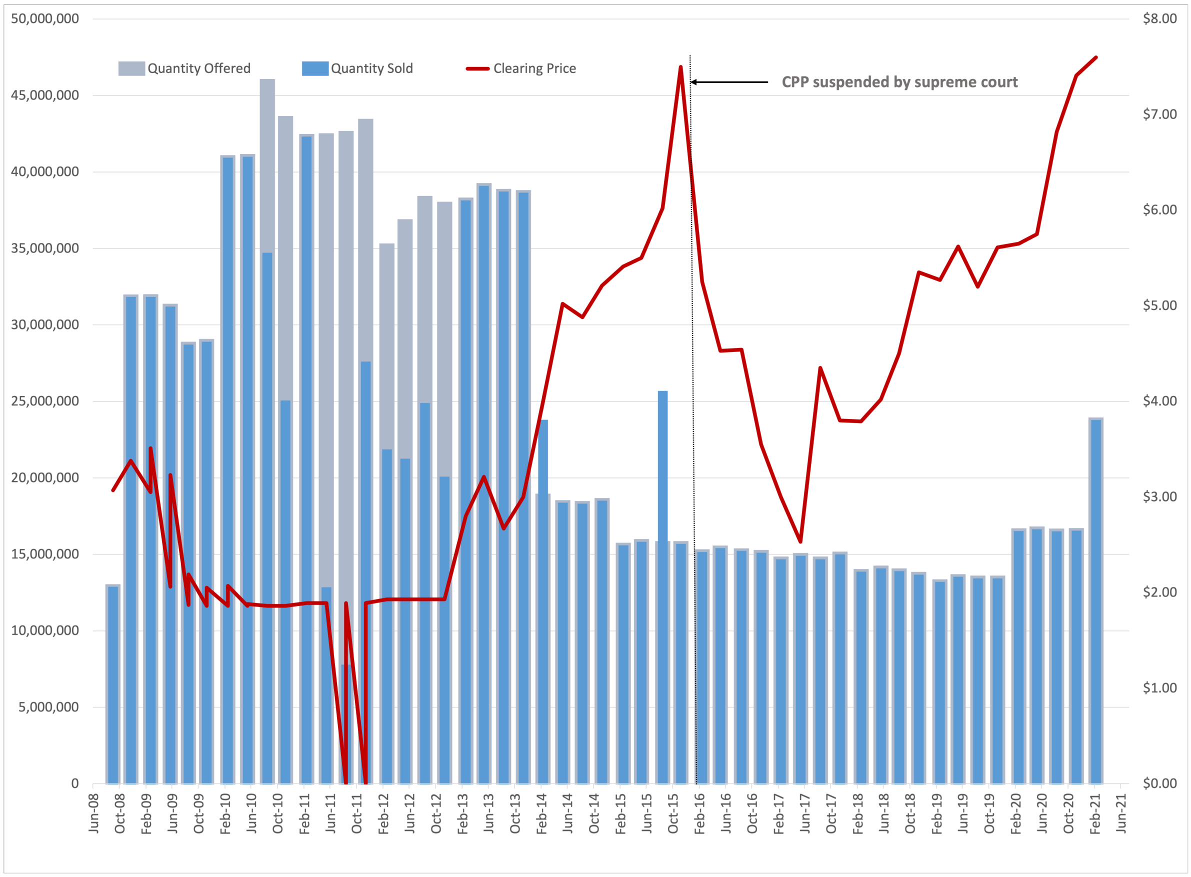
6 Appendix

6.1 Figures and Tables

Table 1: Notation

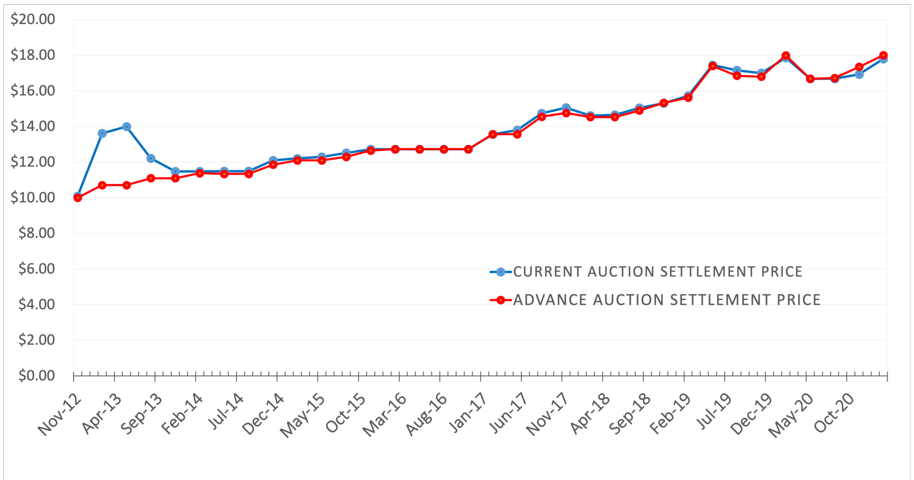
List of Symbols	
v_i	Buyer i's value
a_i	arrival time of buyers
k_i	number of periods buyer's demand is active
$F_v \times F_a \times F_k$	buyers type distrubution for value, arrival time and demadn duration.
$r_{i,t}$	The number of active demand periods left for bidder i in period t
τ	the future demand uncertainty for the buyer
n_a	New buyers in period t drawn from F^n
λ	rate of reducing supply each period
m_t	Supply in period t
n_t	Total demand in period t
S_t	Total items traded in period t
b_t	Bid in period t
$b^{(l)}$	The l^{th} highest order statistic of the bids
$\sigma_t = \{m_t, n_t\}$	the auction "state" (demand and supply) in period t
$\eta_{i,t}$	per period pseudo type for each period t and bidder i
$G_l(.)$	distribution of pseudo type in period l
$V(\eta_{i,t}\|\sigma_t)$	The total lifetime payoff after realization of demand and supply
$W(\eta_{i,t})$	denote the total payoff ex-ante

Figure 1: California Quarterly Auction Revenue Since 2018



Source: The Regional Greenhouse Gas Initiative (RGGI) — www.rggi.org

Figure 2: Cap-and-Trade auction current and advance prices in California



Source: The California Air Resources Board (CARB)

6.2 Proof

Proof of Lemma 1 Recall the definition of η is as follows:

$$\eta_{i,t} = \begin{cases} v_i - \sum_{l=t+1}^{k_i} \int_{n_l} \tau^{l-t} G_l^{(S_l)}(\eta_{i,l}|\sigma_l)\Big(\eta_{i,l} - \mathbb{E}[(\eta_{j,l}^{(S_l)})_{j\neq i}|\eta_{i,l} > \eta_{j,l}^{(S_t)}]\Big), & \text{if } t \geq a_i \text{ or } t \leq k_i \\ 0, & \text{otherwise} \end{cases}$$

Through recursive addition and subtraction, it is easy to see that the above is equivalent to the following :

Replacing probability of bid with probability of pseudo type we can rewrite the payoff functions as

$$W(\eta_{i,t}) = \int_{n_t} \tau G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\Big) + (1 - G_t^{(S_t)}(\eta_{i,t}|\sigma_t))W(\eta_{i,t+1}) \tag{13}$$

$$V(\eta_{i,t}) = G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\Big) + (1 - G_t^{(S_t)}(\eta_{i,t}|\sigma_t))W(\eta_{i,t+1}) \tag{14}$$

This can be rewritten as

$$W(\eta_{i,t}) = \int_{n_t} \tau G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - W(\eta_{i,t+1}) - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\Big) + W(\eta_{i,t+1}) \tag{15}$$

$$V(\eta_{i,t}) = G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\Big(v_i - W(\eta_{i,t+1}) - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\Big) + W(\eta_{i,t+1}) \tag{16}$$

Now, we rewrite the pseudo type in terms of the payoff function. Using addition and subtraction and using the definition of η we can rewrite the definition of η as :

$$\begin{aligned} \eta_{i,t} = v_i &- \int_{n_{t+1}} \tau G_t^{(S_t)}(\eta_{i,t}|\sigma_l)\Big(v_i - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\Big) \\ &+ \sum_{l=t+2}^{k_i} \int_{n_l} \Big(\prod_{q=1}^{l-(t+1)} (1 - \int_{n_q} G_q^{S_q}(\eta_{i,q}))\Big) \tau^{l-t} G_l^{(S_l)}(\eta_{i,l}|\sigma_l)\Big(v_i - \mathbb{E}[(\eta_l^{S_l})|\eta_{i,l} > \eta_l^{S_l}]\Big), \quad \text{if } t \geq a_i \text{ or } t \leq k_i \end{aligned}$$

Note that the second term in the equation above is equal to $W(\eta_{i,t+1})$. Thus we have

$$\eta_{i,t} = \begin{cases} v_i - W(\eta_{i,t+1}), & \text{if } t \geq a_i \text{ or } t \leq k_i \\ 0, & \text{otherwise} \end{cases}$$

Thus, using the above equation we can rewrite Equation 15 and Equation 16 as

$$W(\eta_{i,t}) = \int_{n_t} \tau G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\Big(\eta_{i,t} - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\Big) + W(\eta_{i,t+1}) \tag{17}$$

$$V(\eta_{i,t}) = G_t^{(S_t)}(\eta_{i,t}|\sigma_t)\Big(\eta_{i,t} - \mathbb{E}[(\eta_t^{S_t})|\eta_{i,t} > \eta_t^{S_t}]\Big) + W(\eta_{i,t+1}) \tag{18}$$

□

Proof of Theorem 1

The symmetric Bayesian Nash equilibrium bid in period t maximizes the following payoff of bidder t:

$$V(\eta_{i,t}) = \left\{ G_l^{(S_t)}(\eta_{i,t}|\sigma_l)\mathbb{E}\left[n_{i,t} - b^{(S_t)}(n_{i,t})\middle| b(n_{i,t}) > b_{j\neq i}^{(S_t)}\right] + W(\eta_{i,t+1})\right\}$$

Notice that $W(\eta_{i,t})$ in the above expression is merely an additive constant.

We will use the above equation and backward induction to solve for the equilibrium bidding function.

First from the structure of $V(\eta_{i,t}|\sigma_t)$, it is clear that after the last active period k_i, the buyer's equilibrium bid will be equal to zero, i.e. $b_t^i = 0 \ \forall \ t > k_i$. This is because the buyer is only active till period k_i and would earn a negative profit from winning if he is active after the actual deadline. Thus we can rewrite the equilibrium bidding strategy as a set of finite bids that corresponds to bids in the periods with active demand. WLOG let time period 1 be the start of active demand for bidder i, this implies $a_i = 1$. The biding strategy can be rewritten as follows

$b^i = \{b(\eta_{i,t_1}), b(\eta_{i,2}), ...b(\eta_{i,k_i})\}$

Now we will first show that in the last active round of bidder i's lifetime i.e. k_i^{th} period, bidder bids their pseudo valuation, so $b(\eta_{i,k_i}) = \eta_{i,k_i}$. Note that in the last period pseudo type is equal to value of the bidder, i.e. $\eta_{i,k_i} = v_i$

- If $b' < v_i$.

  In cases where the price for the object is in-between $b(v_i, k_i)$ and v_i, i.e. $b(v_i, k_i) < z_{t_{k_i}} < v_i$, the current period discounted utility from winning is positive i.e $(v_i - z_{t_{k_i}}) > 0$ but the buyer does not win. Thus this is not optimal

- If $b' > v_i$.

  In cases where the price for the object is in-between $b(v_i, k_i)$ and v_i, i.e. $b(v_i, k_i) > z_{t_{k_i}} > v_i$, the current period discounted utility from winning is negative i.e $(v_i - z_{t_{k_i}}) < 0$. Thus this is not optimal.

From above we get that any other bid than $b(\eta_{i,k_i}) = v_i = \eta_{i,k_i}$ would decreases buyers payoff. Thus $b(v_i, k_i) = \eta_{i,k_i}$ is an optimal bid in the last active period (k_i^{th} period) for bidder i.

Next we prove reporting bid equal to $b(\eta_{i,t})$ is optimal in an arbitrary t during the active demand period, i.e., $a_i \leq t < k_i$, assuming it is optimal in all period after t. Recall that equilibrium bid maximizes $V(\eta_{i,t})$

$$V(\eta_{i,t}) = \left\{ G_l^{(S_t)}(\eta_{i,t}|\sigma_l)\mathbb{E}\left[n_{i,t} - b^{(S_t)}(n_{i,t})\middle| b(n_{i,t}) > b_{j\neq i}^{(S_t)}\right] + W(\eta_{i,t+1}) \right\}$$

Here the first term represents the expected current period discounted utility and the second term represents the expected utility from the future if he loses the current period auction . Notice that the second term is independent of the bid in period t_r. Thus, this is equivalent to the bid maximizing the first term.

Note that $\eta_{i,t} = v_i - W(\eta_{i,t+1})$ represents the adjusted value for bidder i in period t.We will now show $b(\eta_{i,t}) = \eta_{i,t}$ maximizes eqn(6). Consider any arbitrary $b' \neq b(\eta_{i,t})$.

- If $b' < b(\eta_{i,t})$

  In cases where the price for the object is in-between b' and $b(\eta_{i,t})$, i.e. $b' < z_t < b(\eta_{i,t})$, the current period discounted utility from winning is positive i.e $v_i - W(\eta_{i,t+1}) - z_{t_r} > 0$ but the buyer does not win. Thus this is not optimal.

- If $b' > b(\eta_{i,t})$

  In cases where the price for the object is in-between b' and $b(\eta_{i,t})$, i.e. $b' > z_t > b(\eta_{i,t})$, the current period discounted utility from winning is negative i.e $v_i - W(\eta_{i,t}) - z_t < 0$. Thus this is not optimal.

Which gives the optimal bidding strategy as, $b(\eta_{i,t}) = v_i - W(\eta_{i,t+1})$. $\square$

Proof for Proposition 3 First we look at how the rate of future supply effects the equilibrium payoff of the bidder . The period t payoff for bidder i after realization of state σ_t is

$$V(\eta_{i,t}|\sigma_t)) = G^{S_t}(\eta_{j,t})\mathbb{E}\Big[v_i - b^{(S_t)}(\eta_{j,t})_{j\neq i}\Big| b(\eta_{j,t}) > b^{(S_t)}(\eta_{j,t})\Big] + (1 - G^{S_t}(\eta_{j,t}))W(\eta_{i,t+1}|\lambda) \quad (19)$$

Note that the bid is made after the supply and demand realization, so λ only affects future payoff. Also, $W(\eta_{i,t+1})$ is dependent on the number of traded items, i.e. S_t which is a function of m_t and thereby λ. For ease of representation we usually suppress the notation for the dependence of $S_{t+1}(\lambda)$ and $W(.|\lambda)$ on λ. However, we reintroduce it here as it is critical. Using integral-form envelopee theorem we get :

$$\frac{\delta(V(\eta_{i,t}|\sigma_t))}{\delta(\lambda)} = \left(1 - G_t^{(S_t)}(\eta_{i,t})\right)\frac{\delta(W(\eta_{i,t+1}|\lambda))}{\delta(\lambda)} \quad (20)$$

Now to show $\frac{\delta(W(\eta_{i,t+1}|\lambda))}{\delta(\lambda)} \geq 0$, we will use Lemma 1 and proof by induction starting from the last active period.

Let us prove this for non-active demand period, i.e., the case where $r_{i,t} = 0$. From the proof of Theorem 1, we know that the equilibrium bid for non-active demand period will be $b_i(\eta(v_i, 0)) = 0$, implying $W(\eta_i(v_i, 0)) = 0$. The derivative is shown below

$$\frac{\delta(W(\eta_i(v_i, 0)))}{\delta(\lambda)} = 0 \geq 0$$

Now let us assume that the proposition is true for any arbitrary $a_i \leq t' + 1 \leq k_i$, i.e. assume

$\frac{\delta(W(\eta_{i,t'+1}))}{\delta(\lambda)} \geq 0$, we show that this hold in t' period too. Rewriting Equation 4 for $t'+1$

$$\frac{d(W(\eta_{i,t'}))}{d(\lambda)} = \frac{d\int_{n_{i,t'}} \tau G^{S_{t'}(\lambda)}(\eta_{i,t'})\left(\eta_{i,t'} - \mathbb{E}[\eta_{t'}^{S_{t'}(\lambda)}|\eta_{i,t'} > \eta_{t'}^{S_{t'}(\lambda)}]\right) + W(\eta_{i,t'+1})}{d\lambda}$$

Using envelope theorem we get

$$= \int_{n_{t'+1}} \tau \frac{\delta G^{S_{t'+1}}(\eta_{i,t'+1})}{\delta\lambda}\left(\eta_{i,t'+1} - \mathbb{E}[\eta_{t'+1}^{S_{t'+1}}|\eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]\right)$$

$$- G^{S_{t'+1}}(\eta_{i,t'+1})\frac{\delta\mathbb{E}[\eta_{t'+1}^{S_{t'+1}}|\eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]}{\delta\lambda} + \frac{\delta W(\eta_{i,t'+1})}{\delta\lambda}$$

using $\frac{\delta G^{S_{t'+1}}(\eta_{i,t'+1})}{\delta\lambda} \geq 0$ and the assumption $\frac{\delta(W(\eta_{i,t'+1}))}{\delta(\lambda)} > 0$, we get

$\frac{d(W(\eta_{i,t'}))}{d(\lambda)} \geq 0$ as long as $\frac{\delta\mathbb{E}[\eta_{t'+1}^{S_{t'+1}}|\eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]}{\delta\lambda} \leq 0$

Thus, it is sufficient to prove $\frac{\delta\mathbb{E}[\eta_{t'+1}^{S_{t'+1}}|\eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]}{\delta\lambda} \leq 0$. Expanding the above term while suppressing the subscript $t'+1$ and reintroducing the subscript for number of demanded credits, i.e., $n_{t'+1}$ we get:

$$\begin{aligned}\mathbb{E}[(\eta^{S:n}|\eta_i > \eta^{S:n}] &= \mathbb{E}[(\eta^{1:n-S}|\eta_i \geq \eta^{1:n-S}] \\ &= \int_0^{\eta_i} x\frac{(n-S)F^{n-S-1}(x)f^n(x)}{F^{n-S}(x)}dx\end{aligned}$$

The differentiation of the above term gives us

$$\frac{\delta\left(\mathbb{E}[(\eta^{S:n}|\eta_i > \eta^{S:n}]\right)}{\delta\lambda} = \int_0^{\eta_i} x\frac{-F^{n-S-1}(x) - (n-S)(n-S-1)F^{n-S-2}(x) + (n-S)F^{n-S-1}(x)}{F^{2(n-S)}(x)}\frac{dS}{d\lambda}f^n(x)dx$$

$$\to \frac{\delta\left(\mathbb{E}[(\eta^{S:n}|\eta_i > \eta^{S:n}]\right)}{\delta\lambda} \leq 0 \text{ as } (n-S)(n-S-1)F^{n-S-2}(x) \geq (n-S)F^{n-S-1}(x)$$

hence proved. □

Proof for Corollary 2 first we show bid is decreasing in supply rate:

$$\frac{\delta(b(\eta_{i,t})}{\delta(\lambda)} = \frac{\delta(v_i - W(\eta_{i,t+1}|\lambda))}{\delta\lambda} \tag{21}$$

$$- \frac{\delta(W(\eta_{i,t+1}|\lambda))}{\delta\lambda} \tag{22}$$

$$\leq \text{as a result of proposition(3)} \tag{23}$$

Notice that this also implies $\frac{\delta(\eta_{i,t})}{\delta(\lambda)} \leq 0$, thus we have

$$\mathbb{E}(P_t) = \mathbb{E}\left(\eta_t^{(\lambda * m_t)}\right)$$

$$\rightarrow \frac{\delta\mathbb{E}(P_t)}{\delta(\lambda)} = \frac{\delta(\mathbb{E}[\eta_t^{(\lambda * m_t)}]}{\delta(\lambda * m_t)} m_t + \frac{\delta(\mathbb{E}[\eta_t^{(\lambda * m_t)}]}{\delta(\eta_t)} \frac{\delta(\eta_t)}{\delta(\lambda)} \leq 0$$

□

Proof for Proposition 2

First we look at how the future demand survival rate effects the equilibrium payoff of the bidder . The period t payoff for bidder i after realization of state σ_t is

$$V(\eta_{i,t}|\sigma_t)) = G^{S_t}(\eta_{j,t})\mathbb{E}\left[v_i - b^{(S_t)}(\eta_{j,t})_{j \neq i} \middle| b(\eta_{j,t}) > b^{(S_t)}(\eta_{j,t})\right] + (1 - G^{S_t}(\eta_{j,t}))W(\eta_{i,t+1}|\tau) \tag{24}$$

Note that the bid is made after the supply and demand realization, so τ only affects future payoff. Also, $W(\eta_{i,t+1})$ is dependent on the number of traded items, i.e. S_t which is a function of m_t and thereby τ. For ease of representation we usually suppress the notation for the dependence of $S_{t+1}(\tau)$ and $W(.|\tau)$ on τ. However, we reintroduce it here as it is critical. Using integral-form envelopee theorem we get :

$$\frac{\delta(V(\eta_{i,t}|\sigma_t))}{\delta(\tau)} = \left(1 - G_t^{(S_t)}(\eta_{i,t})\right) \frac{\delta(W(\eta_{i,t+1}|\tau))}{\delta(\tau)} \tag{25}$$

Now to show $\frac{\delta(W(\eta_{i,t+1}|\tau))}{\delta(\tau)} \geq 0$, we will use Lemma 1 and proof by induction starting from the last active period.

Let us prove this for non-active demand period, i.e., the case where $r_{i,t} = 0$. From the proof of Theorem 1, we know that the equilibrium bid for non-active demand period will be $b_i(\eta(v_i, 0)) = 0$, implying $W(\eta_i(v_i, 0)) = 0$. The derivative is shown below

$$\frac{\delta(W(\eta_i(v_i, 0)))}{\delta(\tau)} = 0 \geq 0$$

Now let us assume that the proposition is true for any arbitrary $a_i \leq t' + 1 \leq k_i$, i.e. assume $\frac{\delta(W(\eta_{i,t'+1}))}{\delta(\tau)} \geq 0$, we show that this hold in t' period too. Rewriting Equation 4 for $t' + 1$

$$\frac{d(W(\eta_{i,t'}))}{d(\tau)} = \frac{d \int_{n_{i,t'}} \tau G^{S_{t'}(\tau)}(\eta_{i,t'}) \Big(\eta_{i,t'} - \mathbb{E}[\eta_{t'}^{S_{t'}(\tau)} | \eta_{i,t'} > \eta_{t'}^{S_{t'}(\tau)}]\Big) + W(\eta_{i,t'+1})}{d\tau}$$

Using envelope theorem we get

$$\begin{aligned}
&= \int_{n_{t'+1}} G^{S_{t'+1}}(\eta_{i,t'+1}) \Big(\eta_{i,t'+1} - \mathbb{E}[\eta_{t'+1}^{S_{t'+1}} | \eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]\Big) \\
&+ \int_{n_{t'+1}} \tau \frac{\delta G^{S_{t'+1}}(\eta_{i,t'+1})}{\delta\tau} \Big(\eta_{i,t'+1} - \mathbb{E}[\eta_{t'+1}^{S_{t'+1}} | \eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]\Big) \\
&- G^{S_{t'+1}}(\eta_{i,t'+1}) \frac{\delta \mathbb{E}[\eta_{t'+1}^{S_{t'+1}} | \eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]}{\delta\tau} + \frac{\delta W(\eta_{i,t'+1})}{\delta\tau}
\end{aligned}$$

using $\frac{\delta G^{S_{t'+1}}(\eta_{i,t'+1})}{\delta\tau} \geq 0$ and the assumption $\frac{\delta(W(\eta_{i,t'+1}))}{\delta(\tau)} > 0$, we get

$\frac{d(W(\eta_{i,t'}))}{d(\tau)} \geq 0$ as long as $\frac{\delta \mathbb{E}[\eta_{t'+1}^{S_{t'+1}} | \eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]}{\delta\tau} \leq 0$

Thus, it is sufficient to prove $\frac{\delta \mathbb{E}[\eta_{t'+1}^{S_{t'+1}} | \eta_{i,t'+1} > \eta_{t'+1}^{S_{t'+1}}]}{\delta\tau} \leq 0$. Expanding the above term while suppressing the subscript $t' + 1$ and reintroducing the subscript for number of demanded credits, i.e., $n_{t'+1}$ we get:

$$\begin{aligned}
\mathbb{E}[(\eta^{S:n} | \eta_i > \eta^{S:n}] &= \mathbb{E}[(\eta^{1:n-S} | \eta_i \geq \eta^{1:n-S}] \\
&= \int_0^{\eta_i} x \frac{(n-S)F^{n-S-1}(x) f^n(x)}{F^{n-S}(x)} dx
\end{aligned}$$

The differentiation of the above term gives us

$$\frac{\delta\Bigg(\mathbb{E}[(\eta^{S:n}|\eta_i > \eta^{S:n}]\Bigg)}{\delta\tau} = \int_0^{\eta_i} x \frac{-F^{n-S-1}(x) - (n-S)(n-S-1)F^{n-S-2}(x) + (n-S)F^{n-S-1}(x)}{F^{2(n-S)}(x)} \frac{dS}{d\tau} f^n(x)dx$$

$$\rightarrow \frac{\delta\Bigg(\mathbb{E}[(\eta^{S:n}|\eta_i > \eta^{S:n}]\Bigg)}{\delta\tau} \leq 0 \text{ as } (n-S)(n-S-1)F^{n-S-2}(x) \geq (n-S)F^{n-S-1}(x)$$

hence proved. □

Proof of Corollary 1 first we show bid is decreasing in uncertainty:

$$\frac{\delta(b(\eta_{i,t})}{\delta(\tau)} = \frac{\delta(\tau v_i - W(\eta_{i,t+1}|\tau))}{\delta\tau} \tag{26}$$

$$v_i - \frac{\delta(W(\eta_{i,t+1}|\tau))}{\delta\tau} \tag{27}$$

$$\geq 0 \tag{28}$$

Notice that this also implies $\frac{\delta(\eta_{i,t})}{\delta(\tau)} \geq 0$, thus we have

$$\mathbb{E}(P_t) = \mathbb{E}\Bigg(\eta_t^{(S_t)}\Bigg)$$

$$\rightarrow \frac{\delta\mathbb{E}(P_t)}{\delta(\tau)} = \frac{\delta(\mathbb{E}[\eta_t^{(m_t)}]}{\delta(\eta_t)} \frac{\delta(\eta_t)}{\delta(\tau)} \geq 0$$

□

Proof of Proposition 1 Proof in the text □