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TO WHAT EXTENT ARE PUBLIC PENSIONS PARETO-IMPROVING?

ON THE INTERACTION OF MEANS TESTED BASIC INCOME AND PUBLIC PENSIONS

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Abstract

If there is a means tested basic income for old age, households will tend to reduce precautionary savings to an inefficiently low level. This might serve as a justification for a public pension system. In a representative agent framework, indeed, the introduction of a compulsory pension system is shown to be Pareto improving. This analysis is extended to two income types where compulsory savings are found to be Pareto improving only up to a point. Increases in contribution rates beyond that point simply result in increasingly regressive (implicit) taxation, potentially eliminating all redistribution via the means tested basic income. Using these results in a pay-as-you-go framework, we show that an unfunded pensions system (with intragenerational fairness) plays a role similar to compulsory savings in preventing the savings moral hazard and could have the same adverse effects on redistribution if it is too large. If the population is aging, however, an unfunded system with a constant contribution rate is found to become less effective at preventing the savings moral hazard. In this case, the introduction of a funded system of the right size is needed to restore Pareto efficiency.

Keywords: Public pensions, compulsory savings, means tested basic income

JEL Classification: H55, I38

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1. Introduction

As economists we are generally prepared to give consumers the benefit of the doubt that they know their preferred life time consumption pattern best. Public pension systems, however, constrain consumers in their choice of consumption pattern. This leads to a potential loss in welfare. Indeed, from much of the literature on pensions, it would appear that having no public pension system at all would be as good as or in many cases even better than, having a pension system[1].

One important justification for compulsory pension systems is given by Friedrich von Hayek, who should be above any suspicion of having an unfair bias in favor of compulsory schemes[2]:

> "Once it becomes the recognized duty of the public to provide for the extreme needs of old age, unemployment, sickness, etc., irrespective of whether the individuals could and ought to have made provision themselves, and particularly once help is assured to such an extent that it is apt to reduce individuals' efforts, it seems an obvious corollary to compel them to insure (or otherwise provide) against those common hazards of life"

Lindbeck and Weibull [1988] advance a similar argument in a two-stage game with two altruistic agents who consume during two periods and can make transfers to one another after their first period consumption. If agents can, at the second stage of the game, revise their first stage decision about transfers after having observed each other's first period consumption, the subgame perfect Nash equilibrium of this game may be inefficient because of the savings moral hazard. Under certain circumstances agents have an incentive to save too little because they can rely on the transfer of the other agent in their old age. Ex-ante commitment to a certain savings level, for example in the form of compulsory pensions, might solve this problem[3]. In their extensive list of possible explanations for public pensions, Mulligan and Sala-i-Martin [1999] classify this type of consideration as the "Rational Prodigality" argument[4].

The aim of this paper is to explore this argument further for both funded and unfunded pensions systems. In order to reduce complexity, we take the recognized duty of the public to provide for the "extreme needs of old age" in the form of minimum basic income as given.

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[1] Examples here are range from public choice papers like Browning [1975] and Sjoblom [1985] to efficient pension transition literature such as Breyer [1989], Homburg [1990], Feldstein [1995] and Kotlikoff et. al. [1998].

[2] Hayek [1960], p. 286.

[3] In an overlapping generations framework with bi-directional altruism across generations, Laitner [1988] advances in addition the idea of excessive risk taking due to the savings moral hazard.

[4] As opposed to "Myopic Prodigality" used in Feldstein [1985] where agents simply lack foresight.

This is plausible to the extent that minimum basic income for old age is a common feature of the welfare state in all EU member states and very widespread among industrialized countries worldwide.

As a starting point, the second section spells out the rational prodigality argument in a simple public finance setting to explicitly establish the Pareto-improving effect of a funded pension system where there is means tested basic income. In this simple setting, arbitrarily high pension contribution rates will fare just as well as the minimum contribution rate needed to eliminate the savings moral hazard.

The paper then goes on to address both the efficiency and equity effects of a funded pension systems in a model with two income types. Specifically, in the third section, we introduce two income types into this model, rich and poor. We start our analysis from a situation where the pension system is small and both individuals claim means tested basic income in old age. As the size of the pension system is increased, a critical contribution rate is reached where rich ceases to claim minimum income and becomes self-reliant. We show that the tax rate for both individuals can be decreased at this point while keeping government net revenues constant. This implies a clear-cut Pareto improvement. Further increases beyond this critical contribution rate result in a lower effective tax rate for rich and in a higher effective tax rate for poor. Hence, the redistribution from rich to poor via the means tested basic income is reduced, leaving the poor worse off and the rich increasingly better off. This main result is surprising to the extent that equity advocates in practice often fight to keep pension levels high even in countries like Germany where the pension system is intergenerationally fair[5]. Our result establishes that doing this might in fact act against the interest of the poor.[6]

The fourth section translates our result from a funded pensions system into a pay-as-you-go pension system using the equivalence result of Fenge [1995]. Here we show that lowering pension levels in order to stabilize the pay-as-you-go contribution rate with an aging population is highly problematic. In particular, we prove that a pay-as-you-go pension system with a constant contribution rate absorbs aging shocks inefficiently. Adding a funded pension system of the right size will help to restore Pareto efficiency.

In the conclusion we discuss some policy implications of our findings and make some suggestions for further research.

[5] Intergenerational fairness means that within each generation pensions are calculated as a fixed proportion of personal contributions. This implies that the pension system does not redistribute within any given generation, although it might well redistribute across generations.
[6] Or at least against that part of the poor population which is young. Poor old age pensioners will of course generally appreciate any effort directed against lower pensions.

2. The Representative Agent Model

The model assumes a representative agent with a utility function $U(C_y, C_o)$ with the usual assumptions of strict quasi-concavity and positive first partial derivatives. C_y and C_o stand for consumption in youth and old age respectively. When he is young the agent supplies his labor inelastically and is paid a total wage of *W*. For notational convenience the price of the consumption good is normalized to unity and the interest rate is set to zero.

The state provides a means tested basic income *m* in old age. If the sum of voluntary savings and compulsory savings due to the pension system of the representative agent at the end of the working period is below *m*, the state provides income support to prop up the total second period consumption to *m*. In order to finance the means tested income and other state expenditure, the state collects income tax at rate *t*.

In addition, the state sets the contribution rate *b* to a funded pension system with individual accounts. This contribution rate *b* can be interpreted as the compulsory savings rate in the context of this model.

Furthermore, the representative agent has access to a perfect credit market. In particular, the agent can sell his pension claims to a bank when he is young. The only restriction on this is imposed by the protected earnings rate (up to which an amount is exempt from seizure) set at the level of the means tested basic income which is a common feature of the welfare state in many countries. Banks are simply not allowed to strip their debtors of the bare essentials they need to live. As a consequence, banks will not accept as collateral that part of compulsory savings which will have to be used to cover the bare essentials in old age. In other words, only pension claims above the basic income can be sold by the agent.

The nature of the budget constraint the representative agent faces is depicted in Figure 1.

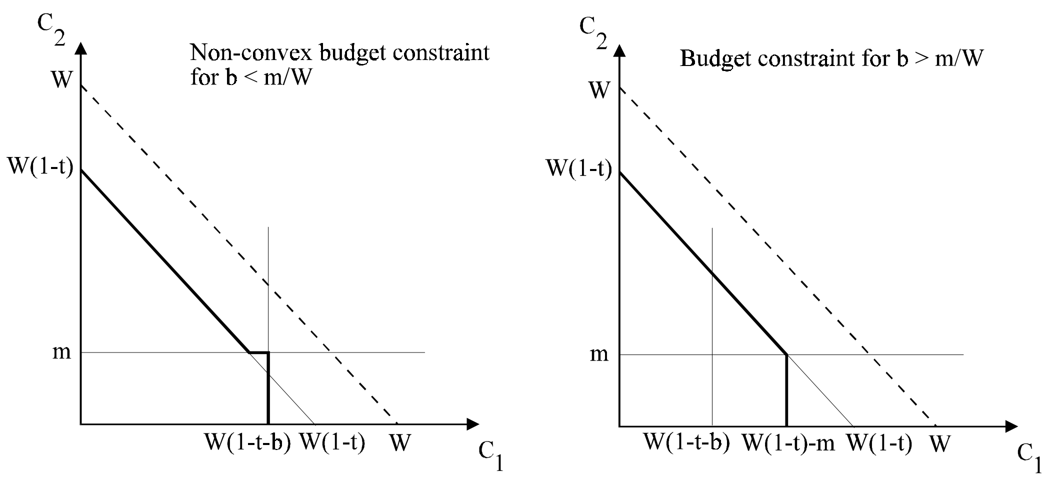

Figure 1: *Budget constraint of the representative agent*

The dotted line in each diagram illustrates the budget constraint with without taxes, basic income or forced savings. The tax t shifts the budget constraint inwards as illustrated. The minimum basic income sets the minimum second period consumption level at m. And the contribution rate b reduces the maximum first period consumption from $W(1-t)$ to $W(1-t-b)$ as illustrated in the left diagram. However, if $b > m/W$, the future pension claim in excess of the basic income m could be used as a collateral and could therefore be consumed already in youth. Therefore, the maximum first period consumption in this case is simply $W(1-t)-m$, as illustrated in the right diagram. As can be seen in the left diagram, the budget set of the representative agent may be non-convex. As a consequence, two separate maximization problems of the representative agent need to be examined:

Case 1: When the agent lives on the basic income in old age, indirect utility is given by

$$\begin{aligned} V_1(W,m,t,b) = \max_{C_y,C_o} & U(C_y,C_o) \\ \text{s.t.} \quad & C_y = W(1-t-b) \\ & C_o = m \end{aligned} \tag{1}$$

It should be noted that, for contribution rates $b \geq m/W$, the representative agent cannot reasonably claim income support, since the compulsory savings are above the level m and negative private savings are restricted by the legal distraint limit in such a way that the sum of private and compulsory savings cannot drop below m. But for the sake of notational completeness which will make the proofs easier, we extend the definition as follows to include all $b \geq m/W$:

$$\begin{aligned} V_1(W,m,t,b) = \max_{C_y,C_o} & U(C_y,C_o) \\ \text{s.t.} \quad & C_y = W[1-t-\min(b,m/W)] \\ & C_o = m \end{aligned} \tag{2}$$

Case 2: When the agent does not live on the basic income in old age, indirect utility is given by

$$V_2(W,m,t,b)=\max_{C_y,C_o} U(C_y,C_o)$$

$$\text{s.t.} \quad C_y + C_o = W(1-t) \quad (3)$$

$$C_y \le W(1-t)-m$$

The indirect utility function of the representative agent can now be defined as

$$V(W,m,t,b)=\max(V_1,V_2)\,. \quad (4)$$

Furthermore, with the population normalized to unity, the government net revenues can be written as

$$\Delta B(t,b)=\begin{cases} Wt & \text{if } V_1 \le V_2 \\ W(t+b)-m & \text{if } V_1 > V_2 \end{cases} \quad (5)$$

Note that we assume that the basic income will not be claimed if the maximum utility when claiming the basic income is equal to the maximum utility when not claiming the basic income.

Having established the basic notation we now look for possible reform policies that involve changes in the tax rate and the contribution rates without making future generations worse off from increased public debt and we find

Proposition 1a: An increase in the contribution rate to the funded pension system is never welfare decreasing.

The proof is given in Appendix I. To give an intuition for the result, we examine the three principal cases verbally.

Case 1: The initial contribution rate is already so high that the agent does not claim the basic income in the first place. Further increases in the contribution rate will not change the consumption pattern because the increased compulsory savings can be offset by borrowing, using the increased compulsory savings as collateral. To keep net state revenues constant, the tax rate is left unchanged.

Case 2: The agent claims the basic income at the initial contribution rate, but, as the contribution rate is increased, the agent stops claiming the basic income. Keeping net revenues of the state constant, the tax can now be lowered by the very amount previously used to finance the basic income. The present value of the agent's consumption is thus the same as it was when the basic income was claimed but is no longer conditional on a second period consumption at the level of the basic income *m*. The agent's utility therefore either stays the same or increases.

Case 3: The agent claims the basic income at the initial contribution rate and continues to do so with the increased contribution rate. In this case, the tax rate can be lowered so as to keep the sum of contribution rate and tax rate constant. The utility of the agent is not altered by this type of reform since contributions are perceived as income tax by the agent. And net revenues of the state do not change either, since the increased pensions due to increased contributions make the basic income that much cheaper: the difference between pensions and the basic income that needs to be tax financed diminishes by exactly the right amount.

Having established that it is never welfare decreasing to raise compulsory savings *b*, one important question remains: under what circumstances will an increase in *b* result in a clear-cut Pareto-improvement? This question is answered by

Proposition 1b: If the means tested basic income induces the representative agent to save too little, the first best utility can always be reached by increasing the contribution rate while keeping net revenues of the state constant.

Proof: The first best utility without the basic income while keeping the net revenues of the state constant can be written as $V(W,0,t_0,0)$ where $t_0 = \Delta B(b,t)/W$. "The agent saves too little" implies that the basic income $m < C_2^*$ where C_2^* is the first best second period consumption level. Hence $V(W,m,t,b) = V_1(W,m,t,b) < V(W,0,t_0,0)$. Furthermore, by inspection we have $V(W,0,t_0,0) = V(W,m,t_0,m/W)$. Therefore, if we set $b' = m/W > b$ and $t' = t_0$ it follows that $V(W,m,t,b) < V(W,0,t_0,0) = V(W,m,t',b')$.

With Proposition 1b we conclude that compulsory savings are a complete cure for the savings moral hazard problem if the agent is induced to save too little. Since the basic income is usually set at a level which is unreasonably low for the majority of the population in a first best situation, there is reason to believe that the introduction of a compulsory funded pension system will be welfare improving.

In the case of one representative agent, a more direct solution to the problem would be to do away with the basic income altogether. Indeed, if everybody earns the same wage, redistribution via a means tested basic income makes little sense. Therefore, we extend the model to two types of agents, rich and poor.

3. Extension to Two Types: Rich and Poor

This extension assumes two representative agents, rich and poor, each with the same utility function $U(C_y, C_o)$, only this time homothetic[7]. Both agents supply their labor inelastically. Rich is paid a total wage of W_R. Poor is paid a total wage of $W_P < W_R$. The fraction of rich people in the total population is denoted by α. The fraction of poor people in the population is denoted by $1-\alpha$.

As before, the state provides a means tested basic income *m* in old age. In order to finance the means tested income and other state expenditure, the state collects income tax at rate *t*. In addition, the state sets the uniform contribution rate *b* for a funded pension system with individual accounts. *b* can again be interpreted as the compulsory savings rate.

Both agents have access to a perfect credit market. In particular, the agents can sell their pension claims to the bank when young. Again, the only restriction to this is the protected earnings rate set at the level of means tested basic income.

As illustrated in the preceding section, the maximization problem of each agent generally has a non-convex budget constraint. Using the notation of the last section, the indirect utility of rich is defined as $V(W_R, m, t, b)$, the indirect utility of poor is $V(W_P, m, t, b)$. The net revenues of the state can be written as

$$\Delta B(t,b) = \begin{cases} [\alpha W_R + (1-\alpha)W_P](t+b) - m & \text{if } V_1(W_R) > V_2(W_R) \text{ and } V_1(W_P) > V_2(W_P) \\ \alpha W_R t + (1-\alpha)[W_P(t+b) - m] & \text{if } V_1(W_R) \le V_2(W_R) \text{ and } V_1(W_P) > V_2(W_P) \\ [\alpha W_R + (1-\alpha)W_P]t & \text{if } V_1(W_R) \le V_2(W_R) \text{ and } V_1(W_P) \le V_2(W_P) \end{cases}$$

If both individuals claim the basic income in old age, their contribution *b* is a pure tax because the benefit they receive in retirement is independent of what they contribute. Hence, the net revenues of the state are the contributions to the funded system and the income tax minus the basic income paid to both poor and rich. If only poor claims the basic income and rich relies exclusively on compulsory and voluntary savings, the net state revenues are the contribution payments and the income tax of poor and only the income tax of rich minus the basic income for poor. And if neither poor nor rich claim the basic income, the income tax revenues equal the net revenues of the state.

Lemma A in the Appendix II shows it can never be the case that rich claims the basic income and poor does not. In order to make the point at which rich and poor stop claiming the basic income clearer, we introduce the following two definitions:

Definition 1: The smallest contribution rate for which there exists a tax rate t such that $V_1(W_R, m, t, b) = V_2(W_R, m, t, b)$ and $\Delta B = \Delta B(t, b^*)$ is called b^*.

b^* is the threshold contribution rate for rich. If the tax rate is adjusted reasonably by the state, for any contribution rate $b \geq b^*$ rich will not claim the basic income.

Definition 2: The smallest compulsory savings rate for which there exists a tax rate t such that $V_1(W_P, m, t, b) = V_2(W_P, m, t, b)$ and $\Delta B = \Delta B(t, b^{**})$ is called b^{**}.

As Lemma B in the Appendix II shows, the threshold contribution rate b^{**} (where poor is indifferent between self-reliance and claiming the basic income in old age) is strictly greater than the threshold b^* for rich. For a given contribution rate, rich is forced to save a higher amount than poor. Therefore, rich will already rely exclusively on his own savings at contribution rates where poor still claims the basic income. Lemma C in the Appendix II shows that, for any contribution rate above the respective threshold values, neither of the agents will start claiming the basic income again. Under the assumption that the basic income is set at such a low level that rich saves too little at b=0 as compared to the first best, we can describe the effects of increases in b as follows: first, rich stops claiming the basic income at $b = b^*$ and then poor stops claiming the basic income at $b = b^{**}$.

Furthermore, given any compulsory savings rate b ($0<b<1$) and tax rate t where $V(W_R, m, t, b)$ and $V(W_P, m, t, b)$ are well defined we deduce

[7] More generally, the following propositions hold if the first best income expansion path is either a straight line (homothetic utility) or bent towards second period consumption. This is the plausible assumption that rich

Proposition 2: For reform policies that increase the contribution rate *b* and adjust the tax rate *t* in order to keep net revenues constant, the following statements hold:

(i) Below the critical contribution rate b^*, contribution rate increases allow the state to reduce the tax rate one to one. Such changes are Pareto indifferent, since the utility of neither rich nor poor changes.

(ii) An increase of the contribution rate to the critical rate b^* allows the state to reduce the tax by more than the contribution rate was increased. Such changes are strictly Pareto improving since the utility of both rich and poor increases.

(iii) Further increases from *b** to a contribution rate between b^* and b^{**} allow the state to reduce the tax rate by less than the increase in the contribution rate. Such changes are Pareto incomparable, since the utility of rich increases, whereas the utility of poor decreases. They simply reduce the redistribution from rich to poor via means tested basic income.

(iv) Increases of the contribution rate to and beyond b^{**} eliminate all redistribution via means tested basic income.

Proof: Differentiating the state budget constraints yields

$$\frac{dt}{db} = -1 \qquad \text{for } b \in \left[0, b^*\right[\tag{6}$$

$$\frac{dt}{db} = -\frac{(1-\alpha)W_P}{\alpha W_R + (1-\alpha)W_P} > -1 \qquad \text{for } b \in \left]b^*, b^{**}\right[\tag{7}$$

$$\frac{dt}{db} = 0 \qquad \text{for } b \in \left]b^{**}, 1\right] \tag{8}$$

Propostion 2(i) follows from (6). Proposition 2(iii) follows from (7). Proposition 2(iv) follows from (8). To prove Proposition 2(ii), we investigate how *t* behaves at the critical value b^*.

For $b = b^*$, there exists a tax rate $\underline{t}$ such that $V_1(W_R, m, \underline{t}, b^*) = V_2(W_R, m, \underline{t}, b^*)$. To reach the required level of government net revenues ΔB,

$$\underline{t} = \frac{\Delta B + (1-\alpha)m}{\alpha W_R + (1-\alpha)W_P} - b^* \frac{(1-\alpha)W_P}{\alpha W_R + (1-\alpha)W_P}$$

attaches, in relative terms, more importance to future consumption than poor.

At this borderline case, if rich decided to continue claiming the basic income, $\bar{t} = \frac{\Delta B + m}{\alpha W_R + (1-\alpha) W_P} - b^*$. Since $b^* < \frac{m}{W_R}$ by assumption as rich saves too little compared to first best at $b = 0$, it follows that $\bar{t} > \underline{t}$, so the tax indeed drops vertically at b^*. Hence Proposition 2(ii) holds. €

Closer investigation of the proof shows that there are small ranges of *b* above both *b**and *b*** where there are two possible tax rates that satisfy the state budget contraint.[8] The higher tax rate is inefficient in the Laffer sense. The proof of Proposition 2 describes the nature of the trade-off between the compulsory savings rate b and the tax rate t exhaustively, as depicted in Figure 2.

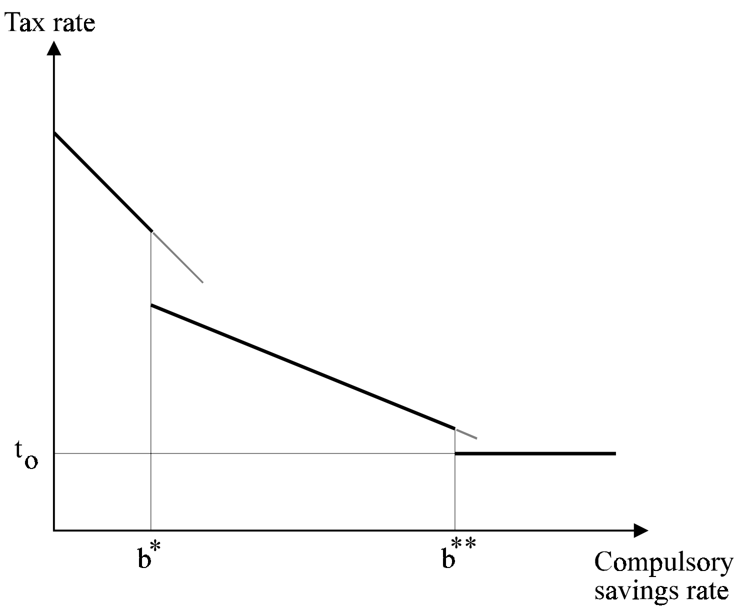

Figure 2: *The trade-off between tax rate and compulsory savings rate*

For contribution rates ranging from 0 to b^*, the tax rate *t* which satisfies the state budget restriction falls one to one as *b* increases: any increase in b helps to finance the basic income. At b^* the tax rate drops vertically, since rich no longer claims the basic income, thus reducing government expenditure. Between b^* and b^{**} the tax rate decreases with a slope of absolute value smaller than 1, since only the increased contributions by poor help to finance the basic income. Increased contributions by rich no longer have this effect, since rich no longer claims

[8] $\bar{t} > \underline{t}$ in the above proof implies that somewhat above the critical contribution rates the agents can still be made to claim the basic income with an inappropriately high tax rate.

the basic income in the first place. From b^{**} onwards the tax rate no longer varies with the contribution rate since higher contributions have no impact on state expenditure.

As discussed above, for certain values of *b*, there are two possible tax rates as indicated by the gray segments, in addition to the black line, that guarantee government net revenues ΔB. Moving from the higher to the lower tax rate is Pareto improving just as moving from one side of a Laffer curve to the other is Pareto improving. However, if the government were to mistakenly equate its efforts to achieve social justice with social justice itself, there would be a real danger of choosing the inefficient rather than the efficient tax rate for a given compulsory savings rate *b* because the "social" expenditure for the basic income is bigger for the inefficiently high tax rate.

For any $b < b^{*}$, both representative agents view their compulsory savings as an additional implicit tax, since additional compulsory savings do not result in additional consumption in old age. For any *b* between b^{*} and b^{**}, rich does not view the compulsory savings as an implicit tax, since additional compulsory savings will result in additional old age income. Poor, however, still views compulsory savings as a tax, since additional compulsory still will not result in higher old age income. For $b > b^{**}$, neither of the agents views *b* as an implicit tax. Taking these considerations into account, Figure 3 shows the variations of the effective marginal tax rates for rich and poor as *b* is increased. In addition, the effective level of redistribution from rich to poor via means tested basic income as *b* is increased is depicted.

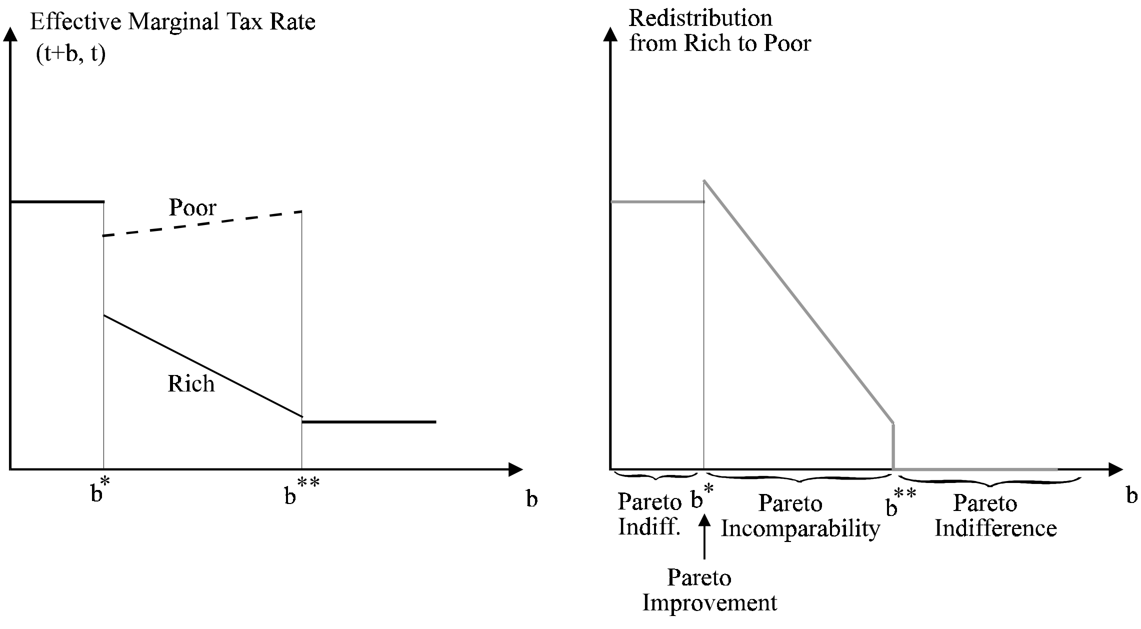

Figure 3: *Variation of the effective marginal tax rate and the redistribution with an increasing compulsory savings rate b.*

Raising the contribution rate from any $b < b^*$ to $b = b^*$ results in a Pareto improvement. The redistribution[9] is actually increased. Increasing the contribution rate from b^* any further is generally not Pareto improving. Rich becomes increasingly better off whereas poor loses out. If *b* is increased to b^{**}, any redistribution from rich to poor is eliminated. Further increases of *b* have no real effect. Figure 4 illustrates the regressive effects of contribution rate increases above b^*:

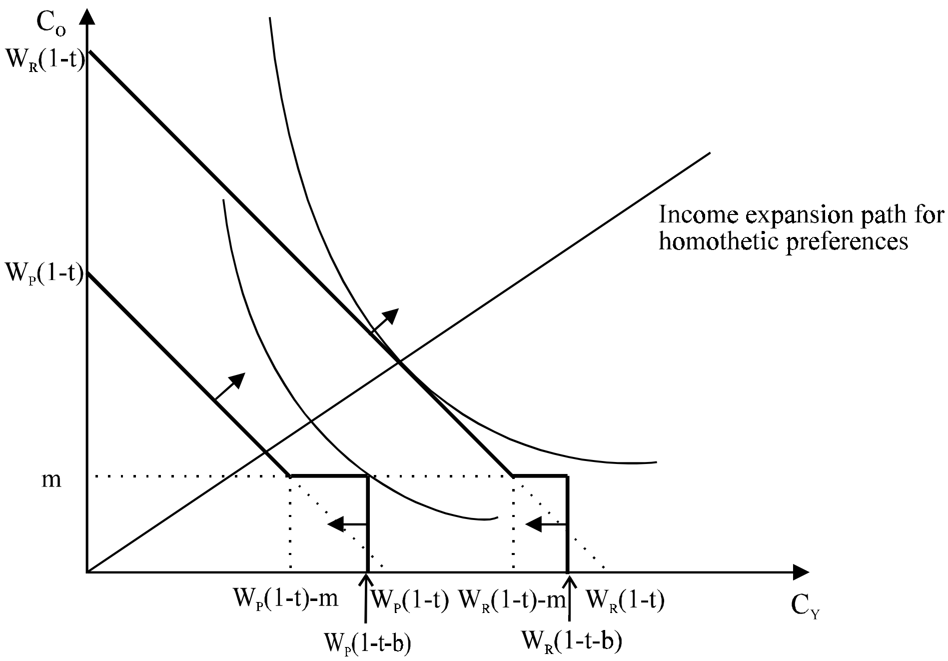

Figure 4: *If contribution rate b is increased from b* on the utility of the rich is increased and the utility of the poor is decreased.*

The bold lines show the budget sets of rich and poor if *b* is slightly above the critical value b^*. Rich lives in old age from his savings, so his indifference curve is tangent to the upper left segment of his budget constraint. Poor consumes the basic income in old age. His indifference curve touches the corner of his budget set. The bold arrows indicate how the budget sets shift if *b* is increased further. Rich continues not to claim the basic income and receives his savings in old age. Poor continues to rely on the basic income and loses his

[9] We measure redistribution by the percentage of gross wage income consumed by poor compared to the percentage of gross income consumed by rich: $\frac{(C_y^P + C_o^P)/W_P}{(C_y^R + C_o^R)/W_R} - 1$, normalizing redistribution to zero for proportional taxation in the absence of publicly provided goods. The increase in redistribution at b^* is due to the fact that rich all of a sudden no longer gets the basic income and this increases redistribution. The resulting tax cut in turn benefits both rich and poor by the same proportion, so redistribution is altogether increased.

increased contribution payments. In order to keep the state budget constant, the tax rate will be decreased by less than the contribution rate is increased. Thus, rich benefits from an increase in net income due to a declining tax rate whereas poor suffers from an increase in his effective overall tax rate which is the sum of the contribution rate b and the implicit tax rate t.

In this section we have shown how crucial the contribution rate b really is. If b is smaller than b^*, then the whole system is inefficient. If b is larger than b^*, then further increases in b will result in redistribution from poor to rich. This result is counter intuitive to the extent that, traditionally, the efficiency advocates have called for a small compulsory pensions system hoping for small government, whereas the equity advocates have called for large compulsory pension system in the name of social justice. Our results challenge this view. Efficiency advocates not interested in social justice should be quite happy to call for high contribution rates even above b^{**} whereas equity advocates should promote a small contribution rate b^*

The next section explains why these results are particularly relevant in a pay-as-you-go pension system faced with an aging crisis.

4. Pay-as-you-go pension system

In a model with labor-leisure distortion, Fenge (1995) has shown that the transition from a pay-as-you-go (PAYG) pension system with intragenerational fairness to a fully funded system cannot be Pareto-improving. However, in our present model with a savings distortion due to means tested basic income things are somewhat different. Even though an intragenerationally fair PAYG pension system with a sufficiently high effective contribution rate is, in principle, able to eliminate the moral hazard savings problem, in times when the population in most countries with a PAYG system is aging, the effective implied savings rate may become too low to do so.

To derive this result, we write the individual maximization problem in an economy with a PAYG pension system and a means tested basic income as:

$$\begin{aligned} &Max\, U(C_Y, C_o) \\ &s.t. \qquad C_y + s = W(1-\delta-\theta) \\ &\qquad\qquad C_o = s + p \end{aligned} \tag{9}$$

where δ is the contribution rate to the PAYG system, θ is the explicit income tax rate, and *s* denotes the voluntary savings. The pension *p* in the PAYG pension system with intragenerational fairness is given by:

$$p = \Omega \cdot \delta \cdot W \tag{10}$$

where Ω denotes one plus the implicit rate of return to the PAYG which is equal to the growth rate of wage income[10]. The contribution to the PAYG system may be split into a part that yields the market interest and a part that will be seen as an implicit tax by the individual[11]:

$$\delta W = \underbrace{\Omega\delta}_{b} \, W + \underbrace{(1-\Omega)\delta}_{\tau} \, W \,. \tag{11}$$

The first term on the right side gives the part that pays the capital market interest. The effective contribution rate is denoted by *b*. The second term on the right side is the part that is seen as an implicit tax where the implicit tax rate is denoted by τ[12]. The lifetime budget restriction results from equation (9), (10) and (11):

$$C_y + C_o = W(1-\delta-\theta) + \Omega\delta W = W(1-\theta) - \underbrace{(1-\Omega)\delta}_{\tau} W = W(1-\theta-\tau)$$

By denoting the sum of the implicit tax rate τ and the explicit tax rate θ as *t*, we re-establish the notation of preceding sections. When the individual decides to live on the basic income in old age, the indirect utility is given by:

$$\begin{aligned} V_1(W,m,t,b) = \max_{C_y,C_o} & \, U(C_y,C_o) \\ \text{s.t.} \quad & C_y = W(1-t-b) \\ & C_o = m \end{aligned} \tag{12}$$

[10] If we were to introduce a time subscript in our notation the growth factor of wage income would be written as $\Omega_{t+1} = \frac{N_{t+1} \cdot W_{t+1}}{N_t \cdot W_t}$, where N_t is the working population in period t and W_t their wage.

[11] See Homburg/Richter (1989)

[12] Strictly speaking, in an efficient world with zero interest rate, $\Omega < 1$ and $\tau < 0$ which would leave us in an Aaron [1966] world. However, we need not worry since zero interest was only assumed for simplification. All results we rely on in this section carry through for a strictly positive interest rate as well.

When the individual decides not to live on the basic income, the indirect utility is given by:

$$
\begin{aligned}
V_2(W,m,t,b) = \max U(C_y,C_o) \\
s.t. \quad C_y + C_o = W(1-t) \\
C_y \leq W(1-t) - m
\end{aligned}
\tag{13}
$$

The formal equivalence of the funded and PAYG pension system shows that PAYG pensions systems can, in principle, fight the savings moral hazard just as well as a funded system. In particular, in an economy where the wage *W* and the basic income *m* stay constant over time and the population has constant average age or rejuvenates (which corresponds to a constant or accelerating population growth rate), the effective contribution rate *b* will also stay constant or will increase. If *b* is sufficiently high to begin with, the whole economy will therefore be in an eternally efficient state. A Pareto improving transition to a funded system would not be possible in this case.

Next we consider an economy which faces a demographic crisis due to an aging population. If the reaction to this crisis is to keep PAYG contribution rate constant, we can write:

$$\delta = \overline{\delta} \tag{14}$$

Assuming the wage *W* to be constant across generations, an aging population implies a decrease in Ω. In order to avoid intergenerational redistribution, we keep the net revenues of the state constant. As a consequence we obtain

Proposition 3a: If population ages, the policy of a fixed contribution rate in a PAYG-system tends to induce contributors to save too little and to claim the means tested basic income.

Proof:

If the population ages (Ω decreases), the effective contribution rate *b* decreases to b'. This can be seen from equation (11). If we write $b' = b - \Delta b$, the implicit tax rate *t* increases to $t' = t + \Delta t$, where:

$$\Delta b + \Delta t = 0\,. \tag{15}$$

Consider now an effective contribution rate $b<m/W$. We want to prove:

$$V_1(W,m,t,b) \geq V_2(W,m,t,b) \Rightarrow V_1(W,m,t',b') > V_2(W,m,t',b') \tag{16}$$

Since $t + b = t' + b'$:

$$V_1(W,m,t',b') = V_1(W,m,t,b). \tag{17}$$

Since $V_2(\cdot)$ decreases with an increasing tax rate t and does not depend on the compulsory savings rate b we deduce that

$$V_2(W,m,t,b) > V_2(W,m,t',b') \tag{18}$$

Equation (17) and (18) yield the implication in (16). In particular we have shown that:

$$V_1(W,m,t,b) = V_2(W,m,t,b) \Rightarrow V_1(W,m,t',b') > V_2(W,m,t',b') \tag{19}$$

Hence, if the nominal contribution rate δ is fixed, a decreasing population tends to induce a higher demand for means tested basic income in old age.

If the population ages and consequently the implicit tax τ in the PAYG system increases by too much, individuals will refuse to save out of their diminishing net income and decide to live on the means tested basic income in old age. For a country with a constant contribution rate, an aging population poses a real danger to the efficiency of the welfare state since the pension system may no longer be effective in fighting the savings moral hazard.

Proposition 3b: Adding a funded pension system of the right size to the PAYG in times of aging population will restore Pareto efficiency. If the funded system exceeds a critical size, the initial Pareto improvement turns into a pure redistribution from poor to rich.

Proof: For one representative agent, Proposition 1 shows that the moral hazard savings problem can be eliminated by the introduction of a fully funded pension. Even increases above the threshold rate will have the desired effect. According to Proposition 2 (ii), the inefficiency of an aging shock in the described pension system will disappear if an additional fully funded pension system is introduced, so that the total effective contribution rate equals b^*. Proposition 2 (iii) now tells us, that, if the contribution rate of the funded system is increased from b^* to even higher effective contribution rates, pure redistribution from poor to rich will result.

Thus, a government set upon a constant contribution rate the the PAYG system and faced with an aging population can restore efficiency by introducing a supplementary funded system. However, the government should be careful to choose the right size for the supplementary funded system. If it is too small, the savings moral hazard of the rich will not be cured. If it is too big, it will simply have an regressive effect by reducing redistribution from rich to poor via the means tested basic income.

Those results can readily applied to current policy debate about pension reform in Germany. The Wissenschaftliche Beirat [1998], for example, recommends a policy of keeping the sum of the contribution rates to the PAYG system and to the additional funded system constant. According to our analysis it would be preferable to keep the effective contribution rate constant instead.

5. Conclusion

If a government employs the means tested basic income as a redistribution device, then it is generally Pareto-improving to additionally install a compulsory funded pension system. Having established this result as a starting point, we find that the size of this compulsory funded system as given by its contribution rate is important. It should be sufficiently high to insure that average citizens will not start claiming the basic income in old age. On the other hand it should be sufficiently small so that the means tested basic income still results in redistribution from rich to poor. To some extent this challenges conventional wisdom that equity advocates should argue for bigger pension systems whereas efficiency advocates should argue for smaller pension systems.

By translating these results into the framework of pay-as-you-go pension systems, we are able to draw important conclusions for the current pension debate in Europe. Europe is faced with a serious aging crisis and most European countries have pay-as-you-go pension systems that run into severe financial difficulties as a consequence. One popular belief is that, as a reaction to this crisis, old age pensions should be decreased so as to ensure that contributions for the working population stay constant over time. We show that this policy option is highly problematic. When the general pension level falls below the level of the basic income, there is a real danger of savings moral hazard causing serious inefficiencies. Under these circumstances, the introduction of an additional funded system of the right size is needed in order to restore Pareto efficiency.

More generally, the number of old age pensioners claiming the basic income could be used as an important indicator of whether the pension system has had the right size in the past. In Germany, for example, currently less than 4 percent of the population aged 60 and over claim basic income support. This might indicate that the German pension system has been somewhat oversized during the past decades at the expense of redistribution. Therefore, there might be some room for moderate pension cuts. However, the German government has to be careful not to cut pensions by too much: if it does, Germany might end up in a situation where the resulting decrease in pension contribution rates is more than offset by the increasing financing needs for the means tested basic income.

For future research we suggest introducing the labor-leisure distortion and two working periods into our model. First, this would make it possible to analyze the trade-off between Beveridge and Bismarck pensions[13] when fighting the savings moral hazard and labor-leisure distortions at the same time. Second, new light could be shed on the problem of selling future pension claims. Although even very high compulsory savings might not adversely affect total working life consumption, they distort the consumption during working life.

[13] Cremer and Pestieau [1998] coined these terms to denote flat and contribution related benefits respectively.

Appendix I

Proof for Proposition 1a: Three situations need to be treated separately where $b' > b$ is the increased contribution rate to be examined.

A. We examine the situation where, even prior to reform, the representative agent did not rely on state support: $V_1(W,m,t,b) \leq V_2(W,m,t,b)$. It follows by inspection that, for any contribution rate $b' > b$ and an unaltered tax rate $t' = t$, the indirect utility doesn't change $V(W,m,t,b) = V(W,m,t',b')$ and the net revenues of the state do not change $\Delta B(t,b) = \Delta B(t',b')$.

B. Alternatively, we might be in the situation where, prior to reform, the representative agent did rely on state support

$$V_1(W,m,t,b) > V_2(W,m,t,b) \tag{20}$$

From the comments about equation (1) it is clear that $b \leq m/W$.

B1. If we are in a situation where

$$V_1(W,m,t+b-m/W,b') \leq V_2(W,m,t+b-m/W,b'), \tag{21}$$

we chose the tax rate $t' = t + b - m/W$. Because $b < b' \leq m/W$ we deduce $t' \leq t$ and $t' + b' \leq t + b$. Therefore $V_1(W,m,t',b') \geq V_1(W,m,t,b)$. Using (20) and (21) we deduce $V(W,m,t,b) \leq V(W,m,t',b')$. It also follows that $\Delta B(t',b') = t'W = (t+b)W - m = \Delta B(t,b)$.

B2. For all the remaining cases (20) and the following inequality holds:

$$V_1(W,m,t+b-m/W,b') > V_2(W,m,t+b-m/W,b'). \tag{22}$$

We chose a tax rate $t' = t - b' + b$. By inspection it follows that $V_1(W,m,t,b) = V_1(W,m,t',b')$. If $V_1(W,m,t',b') > V_2(W,m,t',b')$ it follows directly that $V(W,m,t,b) = V(W,m,t',b')$ and that $\Delta B(t',b') = (t - b' + b)W + b'W - m = tW + bW - m = \Delta B(t,b)$. If $V_1(W,m,t',b') \leq V_2(W,m,t',b')$, it follows that $V(W,m,t,b) \leq V(W,m,t',b')$ and $\Delta B(t',b') = (t - b' + b)W = (t+b)W - b'W \geq (t+b)W - m = \Delta B(t,b)$ since $b' \leq m/W$ because of (22).

Appendix II

Lemma A: If rich claims the basic income, poor will as well. Equivalently, if poor does not claim the basic income, neither will rich.

Proof: If rich claims the basic income, $V_1(W_R, m, t, b) > V_2(W_R, m, t, b)$. Because of homothetic utility it follows that $V_1(W_P, \frac{W_P}{W_R} m, t, b) > V_2(W_P, \frac{W_P}{W_R} m, t, b)$. But V_1 strictly increases whereas V_2 weakly decreases as m increases. Hence $V_1(W_P, m, t, b) > V_1(W_P, \frac{W_P}{W_R} m, t, b) > V_2(W_P, \frac{W_P}{W_R} m, t, b) \geq V_2(W_P, m, t, b)$. Poor will therefore claim the basic income as well.

Lemma B: $b^* < b^{**}$

Proof: From Lemma A it is clear that $b^* \leq b^{**}$. Assuming $b = b^*$, by definition $V_1(W_R, m,t,b) = V_2(W_R, m,t,b)$. Using homothetic utility, it follows that $V_1(W_P, \frac{W_P}{W_R} m,t,b) = V_2(W_P, \frac{W_P}{W_R} m,t,b)$. But V_1 strictly increases with *m* whereas V_2 weakly decreases. Hence $V_1(W_P, m,t,b) > V_1(W_P, \frac{W_P}{W_R} m,t,b) = V_2(W_P, \frac{W_P}{W_R} m,t,b) \geq V_2(W_P, m,t,b)$. This contradicts $b = b^{**}$, where $V_1(W_P, m,t,b) = V_2(W_P, m,t,b)$ by definition. Therefore $b^* \neq b^{**}$.

Lemma C: If an agent does not claim the basic income for *b*, he will not claim the basic income for any $b' > b$.

Proof: If $V_2(W, m,t,b) \geq V_1(W, m,t,b)$ and the government budget is kept at ΔB, for any $b' > b$ the corresponding $t' \leq t$. Hence, by inspection, $V_2(W, m,t,b) \leq V_2(W, m,t',b')$ and $V_1(W, m,t',b') < V_1(W, m,t,b)$. Therefore it follows that $V_2(W, m,t',b') \geq V_1(W, m,t',b')$.

The above Lemmas establish that, if for $b = 0$ both agent claim the basic income and rich saves too little, when *b* is increased, first rich stops claiming the basic income at $b = b^*$ and then poor stops claiming the basic income at $b = b^{**}$.

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