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EXCHANGE RATE MISALIGNMENTS AND CRISES

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Abstract

The problem is to evaluate the likelihood that a country will face a currency or balance of payments crisis over a given horizon. When is it rational for market participants to expect a depreciation of the currency? On the basis of considerable empirical studies we know that in both banking and currency crises, there is a multitude of weak and deteriorating economic fundamentals.

Our theme is that there is an economic logic to medium and longer-term movements in exchange rates, within the context of a consistent dynamic stock-flow model. The equilibrium real exchange rate is a trajectory, not a point. We provide objective measures of the real fundamentals that determine the moving equilibrium real exchange rate, and explain the dynamic economic mechanism whereby the actual exchange rate converges to this moving equilibrium exchange rate, called the NATREX. The fundamentals are primarily social consumption/GDP, which is generally driven by fiscal policy, and the productivity of the economy. Trends in social consumption/GDP, and in fiscal policy, reflected political regime changes in France, Germany and Italy.

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Exchange Rate Misalignments and Crises

1. Introduction

There is a need for an empirically implementable theoretical framework that can identify fundamental disequilibria in exchange rates that may lead to crises or to speculative attacks. Such a framework is necessary for the evaluation of policies to be undertaken in order to prevent an attack or to evaluate the responses to attacks on currencies such as the 1992-93 crises in Europe, the Mexican peso in 1994, or the Southeast Asian crises of 1997-98.

During the period 1987 to August 1992, there was relative stability of the exchange rates of the major European currencies against the DMark. Prior to the attacks, the central banks and economists did not know if there were indeed misalignments. The consensus view at the time was that exchange rates were at their "appropriate" or equilibrium levels¹. For example, Bean (1992:33) wrote that: "The fact that there has been no significant realignment since 1987, despite the considerable tensions thrown up by German unification, is a testament to the credibility of the existing parities". Shortly after his article was written, there occurred speculative attacks. In September 1992, the lira and the pound were withdrawn from the ERM, other currencies had come under attack and were devalued. The reason why the attacks were unforeseen is that at the beginning of the 1990s it was clear that virtually every aspect of the theory of international finance was open to question. The models of the 1970s and 1980s lacked explanatory power². The dominant approach has been based upon Purchasing Power Parity (PPP), the quantity theory of money in the medium-longer run and uncovered interest rate parity with rational expectations. Boughton (1997: 792) wrote: "What do we know about the determinants of exchange rates? 'Precious little' would be the common and not unjustified answer...Purchasing power

¹ The European Commission (1995:13) wrote that: "...the international consensus was that after the dollar decline of the previous two years [1985-87] and the January 1987 realignment of the ERM currencies, exchange rates were close to their appropriate levels".

² See Taylor, De Grauwe, Dornbusch and Frankel, Dornbusch 1995, Stein and Paladino, 1997; Stein, Allen 1995, appendix; Boughton, 1997.

parity, almost every one now believes, has only a very long run influence on exchange rates”.

The theoretical/empirical issues remain: What are the determinants and dynamics of equilibrium real exchange rates, how can misalignment be measured, what were their causes, and do the measures of misalignment explain currency crises and whether speculative attacks will be successful?

The consensus³ in 1999 is as follows. (a) The problem is to evaluate the likelihood that a country will face a currency or balance of payments crisis over a given horizon. When is it rational for market participants to expect a depreciation of the currency? (b) We know that the available measures of expectations by market participants display a poor record in anticipating crises. Market expectations of currency crises typically do not occur until very shortly before the crisis. It is important that an early warning system do better than the market itself. (c) In both banking and currency crises, we find a multitude of weak and deteriorating economic fundamentals. This suggests that it would be difficult to characterize them as self-fulfilling crises. (d) Weaknesses in fundamentals are required for a currency attack to persuade the government to abandon the defense of a currency.

In general, the extensive empirical studies⁴ have arrived at the following conclusions. (e) Neither Purchasing Power Parity nor the Monetary view is a good explanation of nominal exchange rates during the recent floating rate period. (f) The Balassa-Samuelson hypothesis per se is an inadequate explanation for movements of real exchange rates. (g) A variety of empirical studies have identified potentially important variables determining real exchange rates in the medium to the longer run. They are: the ratio of social consumption/GDP, fiscal variables, terms of trade, net foreign assets and productivity. (h) The detailed study by Kaminsky et al (1997) evaluates the empirical evidence on currency crises. They studied 76 currency crises

³ See the summary paper by Borensztein et al, “Anticipating Balance of Payments Crises: The Role of Early Warning Systems” (World Bank/IMF/WTO conference on Capital Flows, Financial Crises and Policies:1999); Graciela Kaminsky and Carmen Reinhart, “The Twin Crises: The Causes of Balance of Payments Problems”, Amer. Econ. Rev. 1999. Further references are found in these two papers.

⁴ See Ronald MacDonald and Jerome L. Stein (ed) *Equilibrium Exchange Rates*, Kluwer: 1999. This volume is essentially empirical: what do we know about exchange rates. An evaluation of the theories is in Stein and Paladino (1997).

during the period 1970 - 1995 and compared the predictive power of many sensible early warning indicators. Their main contribution was to evaluate the reliability of the signals. The best indicator of a crisis within the next 24 months is the real exchange rate relative to a trend. This variable gave statistically significant results in 10 out of the 12 studies considered. No economic content was given to the trend: it was just a statistical artifact. There were no other signals that ranked nearly as high on either count.

Our theme is that there is an economic logic to medium and longer-term movements in exchange rates, within the context of a consistent dynamic stock-flow model. The equilibrium real exchange rate is a trajectory, not a point. Deviations from our moving equilibrium are noise. We provide objective measures of the real fundamentals that determine the the moving equilibrium real exchange rate, and explain the economic mechanism whereby the actual exchange rate converges to this moving equilibrium exchange rate. In a regime of adjustable pegs, speculative attacks and crises are some of the ways in which the convergence occurs. Our approach is consistent with the consensus points (a)-(d) and the empirical findings (e)-(h) cited above.

The model that we use has been used to explain the real value of the \$US/G7, the real value of the DMark and other currencies⁵. Here we focus upon misalignments and currency crises. As an example of the explanatory power of our approach relative to some other currently used approaches, we apply our theoretical analysis to explain the movements in the French franc, German Mark and Italian lira over the past twenty years. We identify what are the real fundamentals, why they have changed and explain the transmission process. To explain misalignment, we contrast the French/German with the Italian/German situations prior to the speculative attacks. On the basis of our analysis we show that before the crises, the French franc was not overvalued relative to the DM, but that the Italian lira was overvalued by two standard deviations. We explain the causes of the overvaluation. To what extent did the overvaluation occur because: (1) the actual real exchange rate appreciated, (2) the moving equilibrium real exchange rate depreciated? The PPP hypothesis is that the equilibrium real exchange rate is constant and misalignment occurs because monetary factors produce a deviation from the

⁵ See Stein (1999), the chapters in Stein, Allen et al (1997), Stein and Sauernheimer (1997).

constant equilibrium. Our approach is more general, because we consider (2) as well as (1). We then predict that the French franc would not succumb to any attack. The Italian lira would be devalued because both relative prices were “out of line” and because the equilibrium real exchange declined.

We suggest that our analysis has many useful implications and applications. There is great interest in evaluating the sources of the strengths and weaknesses of currencies such as the Euro/\$US. The PPP approach states that the value of the Euro just depends upon relative prices. PPP ignores the effects of fiscal policy and productivity upon the equilibrium real exchange rate. Another view, starting with Mundell-Fleming, is that an expansionary fiscal policy and a contractionary monetary policy tend to appreciate the exchange rate. Call this “the two instruments approach”. Our analysis implies that “the two instruments approach” is valid only in the shorter run. In the longer run, such a policy will depreciate the equilibrium exchange rate. Another application is that the model explains how to evaluate to what extent overvalued exchange rates contributed to currency crises in Mexico and in South-east Asia.

The paper is organized as follows. Part 2 is a relatively terse summary of the NATREX model for the fundamental determinants of the medium to long run real exchange rate. Part 3 contains the underlying structural equations. Part 4 explains the medium to longer run dynamics of the equilibrium real exchange rate when the fundamentals – productivity and thrift – have changed. Part 5 describes the strategy for the estimation of the NATREX. Part 6 contains the results of alternative econometric estimates. Part 7 is a summary of the previous sections. Part 8 is an application of the analysis to the French-German and Italian-German experiences in the 1990s. The conclusion is in part 9.

2. The NATREX Model⁶

Real Misalignment • $\Phi(t)$ defined in equation (1) is the difference between the actual real exchange rate and our longer term equilibrium real exchange rate. The real exchange rate $R(t) = N(t)p(t)/p'(t)$ can be defined as the nominal exchange rate $N(t)$

⁶ The NATREX is the subject the references in the previous footnote. See the evaluations of the NATREX approach in Boughton (1997) and Goldberg (1996/97).

times the ratio $p(t)/p'(t)$ of domestic/foreign “prices” or “costs”, where each term must be precisely specified⁷. We use primes to denote foreign variables. $N(t)$ is foreign currency/domestic currency, and a rise in $N(t)$ or $R(t)$ is an appreciation. The real exchange rate associated with the internal and external balance in the medium to longer run is referred to as the Natural Real Exchange Rate (NATREX). The equilibrium longer run real exchange rate⁸ denoted $R^*[Z(t)]$ depends upon real fundamentals $Z(t)$ in an explicit dynamic way discussed below.

$$(1) \Phi(t) = N(t)p(t)/p'(t) - R^*[Z(t)].$$

We provide an economic explanation for the empirical observations that: the real exchange rate is often “trend stationary”, and that the most useful early warning signal of a currency crisis is the deviation of the real exchange rate relative to “trend”, cited in point (h) above. The probability of a crisis is related to overvaluation $\Phi(t) > 0$. Misalignment can occur either because the actual real exchange rate $N(t)p(t)/p'(t)$ appreciates due to differential rates of inflation or because the NATREX $R^*[Z(t)]$ depreciates. Our concept of the “trend” in the equilibrium exchange rate is $\Delta R^*[Z(t)] = \nabla R^* \cdot \Delta Z(t)$, which is the change in the NATREX. The gradient of $R^*[Z(t)]$ is vector ∇R^* based upon the model, and $\Delta Z(t)$ is the vector of the changes in the exogenous fundamentals. The real exchange rate converges to the longer run NATREX. The equilibrium real exchange rate NATREX is deterministic, but the convergence process is stochastic. We show that misalignment $\Phi(t)$ is stationary with a zero expectation, for the French/German and Italian/German bilateral real exchange rates.

2.1. The equilibrium real exchange rate

The equilibrium exchange rate depends upon the length of the horizon⁹. The NATREX model focuses upon the medium to the longer run equilibrium real exchange

⁷ This issue of how to measure the real exchange rate is discussed in the empirical section below.

⁸ We use $R^*[Z(t)]$ to denote that the NATREX depends upon the fundamentals $Z(t)$ which vary over time, and the asterisk denotes the longer run NATREX.

⁹ This is true of any relative price. In standard microeconomics, the very short run equilibrium occurs when demand is equal to the current output which is given, the medium run equilibrium occurs when short run marginal cost equals price, and the long

rate, and ignores the short run variations. The real exchange rate which satisfies (A1)-(A3) is the medium run NATREX - denoted $R_m[Z(t)]$ - and the one that satisfies (A1)-(A4) is the longer run NATREX - denoted $R^*[Z(t)]$ - where $Z(t)$ is the vector of real fundamentals affecting social saving, investment and the current account. The longer run equilibrium adds stability condition (A4) which is our intertemporal budget constraint.

Medium run equilibrium (A1)-(A3)

(A1) Internal equilibrium prevails: The deviation $u(t)$ of the rate of capacity utilization from its stationary mean is zero.

(A2) There is external balance: The current account plus investment less saving, evaluated at capacity output, is zero. Since the equilibrium exchange rate must be sustainable, changes in reserves and speculative short term capital flows are equal to zero. Investment less saving, under these conditions, is the ex-ante nonspeculative capital inflow.

(A3) Asset market equilibrium - portfolio balance exists - with Asymptotically Rational Expectations: Real long-term rates of interest adjust such that investors are willing to hold the existing stocks of net foreign assets. The stock equilibrium condition is that the domestic real long-term rate of interest r , plus $E\{\Delta R^*[Z(t)]\} = E\{\nabla R^* \cdot \Delta Z(t)\}$ = the expected appreciation of the domestic currency over the same horizon as the long-term interest rate, is equal to the world rate r' . The expectation is taken over the space of all available information concerning the future course of the exogenous fundamentals $Z(t)$. If the best estimate of a weighted change in the fundamentals $E\{\nabla R^* \cdot \Delta Z(t)\}$ is zero, then real long-term rates of interest converge.

Longer run equilibrium (A1)-(A4)

(A4) The foreign debt intensity, the ratio of the foreign debt / capacity GDP, or foreign debt/effective worker, converges to a constant¹⁰. This implies that the ratio of the trade

run equilibrium: marginal cost equals price occurs when the number and size of firms in the industry adjust.

¹⁰ Insofar as we require that the dynamical system be stable, we must exclude models which contain saddle points. These models are unstable: the slightest perturbation will lead to a growing divergence from the equilibrium. In that case, an equilibrium concept

balance/foreign debt is equal to the interest rate less the growth rate, $(r - n)$. The convergence of the foreign debt/GDP is our intertemporal budget constraint.

To **summarize**: The monetary/PPP approach assumes that the equilibrium real exchange rate is constant. It is independent of the fiscal policies pursued or the growth of productivity in the economy. The NATREX generalizes the PPP to be a function of real and objectively measured fundamentals. Misalignment can occur either because the actual real exchange rate has changed, for example as a result of differential rates of inflation resulting from monetary factors, or because the deterministic equilibrium exchange rate has changed as a result of changes in the real fundamentals. The convergence of misalignment is stochastic stationary process: an $I(0)$ process with a zero expectation.

2.2. Structure of the Model¹¹

NATREX is a generalization of both PPP and the macroeconomic balance models¹² to take into account dynamic stock-flow relations arising from endogenous variations in capital and debt. A crucial distinction is made between social - public plus private - consumption and social investment. Except for the social - public plus private - consumption function, all of the underlying structural equations are widely accepted, and have clear microeconomic foundations. Our discussion can be terse with the appropriate references. Private sector decisions are based upon intertemporal optimization under uncertainty, where each decision is made independently by different agents. The main endogenous variables in the dynamic system are the real exchange rate, current account and debt, and capital. The quantity variables are measured per unit of effective labor.

The uncertainty facing the household or any economic agent concerns the future course of income. The agents know that income will follow a stochastic process. The first part is a trend and the second part is Brownian motion (BM). The BM assumption

is not a useful analytical tool; and it is impossible to predict movements of exchange rates.

¹¹ The basic details of the model and derivation of the underlying equations are in the references above. We shall therefore be as terse as possible and devote our attention to relevant modifications that are necessary to explain the European experience.

¹² See MacDonald and Stein (1999) for a discussion of macroeconomic balance models.

implies that it is impossible to forecast income far into the future. The conditional variance of the stochastic variable increases with the forecast distance; hence knowledge of the mean (expectation) provides information of little reliability for the longer run.

In the standard optimization, an intertemporal consumption plan is formulated on the basis of a budget constraint based upon information and conjectures at initial time. The consumption is the control. The deficiency of this approach is that, due to BM, there are many paths that the system can follow given the controls and initial data. The debt will not evolve in the manner anticipated at the initial date, and it will explode if the initial estimate of the course of income were overly optimistic¹³. There must be a feedback control that corrects errors. In either case, the initial plan will no longer be optimal. The optimization technique used in the NATREX is based upon stochastic optimal control - dynamic programming, with a built in feedback control to correct for errors.

A similar feedback control, derived from dynamic programming determines optimal investment in a growing economy.¹⁴ Infante and Stein (1973) derived optimal investment as a nonlinear function of the difference between the current measurable marginal product of capital and the discount rate. We proved that our suboptimal feedback control (SOFC) converges to the perfect foresight optimal control. Our SOFC, based upon Dynamic Programming, only uses current measurements, does not require perfect foresight, and converges rapidly to the perfect foresight optimum. Equations (2)-(5) in Box 1 describe the structure of the model.

Box 1. Summary of the NATREX model

$$(2) S(k,F;Z, u) - I(k,y,R,r;Z, u) = CA(R,y,F,r;Z, u); \quad u = 0$$

$$(3) r + \rho(t) = r'; \quad \rho(t) = E\{\Delta R^*[Z(t)]\} = E\{\nabla R^* \cdot \Delta Z(t)\}$$

$$(4) dF/dt = -A(R,y,F,r;Z, u) = L(R,k,F,r;Z), \quad L=I-S$$

$$(5) dk/dt = I(R,k,r;Z)$$

Endogenous: $\{R, F, r, k\}$. Exogenous: $Z = \{g, g', T, r', \rho, y'\}$;

R = real exchange rate, r = real long-term interest rate, S = saving, I = investment, k = capital, F = foreign debt; $CA = B - r'F$ current account, the trade balance B less $r'F$ the net flow of interest payments to foreigners. y = productivity.

¹³ During the first two oil shocks, the oil producing countries set their consumption patterns on the mistaken basis that the oil crisis is permanent. A debt crisis then developed when the oil prices fell.

¹⁴ When the production function is unknown or subject to considerable errors of estimation, the Maximum Principle of Pontryagin - which requires perfect foresight to travel on the saddle point trajectory - is not useful.

Except for (R,r) variables are measured in terms of effective labor or GDP. Exogenous: u = deviation of rate of capacity utilization from stationary mean; g = fiscal variable = government consumption/GDP; T = terms of trade; r' = foreign real long-term rate of interest; ρ = risk premium = $E\{\Delta R^*[Z(t)]\} = E\{\nabla R^* \cdot \Delta Z(t)\}$; y' = foreign GDP.

Equation (2) is the macroeconomic balance equation. It states that the excess of investment over saving ($I - S$) is equal to the current account deficit ($-CA$), evaluated at internal equilibrium: $u = 0$. Private investment less private saving is the net borrowing of the private sector, and government spending less taxes is the public sector borrowing. Their sum is social investment (I) less social saving (S), and represents foreign borrowing. The equilibrium real exchange rate must adjust the current account deficit to this sum. The functions are evaluated at capacity output $u = 0$. The current account is function $CA(\cdot)$.

Equation (3) is the uncovered interest rate parity theory with Asymptotically Rational Expectations¹⁵. It is the portfolio balance equation for external equilibrium. This equation states that the interest rates at home and abroad adjust so that investors are content to hold the stock of net foreign assets in existence. The “risk premium” $\rho(t) = E\{\Delta R^*[Z(t)]\} = E\{\nabla R^* \cdot \Delta Z(t)\}$ is the rationally expected appreciation of the currency over the same horizon as the real long-term interest rates. The exogenous fundamentals $Z(t)$ have Brownian motion components. The change $\Delta Z(t)$ is iid with a zero expectation. At any time, there will be disturbances; but the portfolio balance equation takes a longer view and is predicated upon the future vector of disturbances. Since the Z 's are Brownian motion terms, the Asymptotically Rational Expectations point of view sets the risk premium $\rho(t) = E[\nabla R^* \cdot \Delta Z(t)] = 0$.

The medium run NATREX is the equilibrium real exchange rate associated with internal equilibrium and external equilibrium - given the stocks of debt and capital. It satisfies equations (2) and (3), which are the medium run equilibrium, conditions (A1) - (A3) above. The equilibrium real exchange rate $R(t)$ and real interest rate $r(t)$ are determined conditional upon the endogenous stock variables: the foreign debt $F(t)$ and capital $k(t)$, and exogenous variables $Z(t)$. This is the macroeconomic balance approach. The NATREX model adds dynamic stock-flow interactions.

¹⁵ See Stein (1997, ch. 2 p.60); Stein (1986, pp. 68-76) for the derivation of Asymptotically Rational Expectations. The second reference concerns anticipations in futures markets.

At the medium run equilibrium there may be a nonzero current account. The current account deficit is the rate of change of the foreign debt¹⁶ $F(t)$. The transition to the longer run equilibrium is obtained by adding the endogenous variations in capital and the foreign debt to conditions (A1) - (A4) above. The dynamics of debt and capital are equations (4) and (5). The change in the foreign debt dF/dt is the current account deficit $(-CA) = -CA(R,k,F,r;Z)$. Equation (2) states that it is equal to investment less saving. Hence the rate of change in the foreign debt is equation (4), where function $L = (I-S)$. Equation (5) describes the growth of capital, the investment equation.

The dynamics of the real exchange rate involve the change in the medium run equilibrium, based upon equations (2)-(3) for external and internal balance, to the longer run equilibrium where the foreign debt and capital intensities stabilize. The model involves both stock and flow equilibrium conditions.

To **summarize**: the endogenous variables are the “equilibrium” real exchange rate $R(t)$, the real rate of interest $r(t)$, the foreign debt $F(t)$ and capital $k(t)$. This is a second order dynamical system. The exogenous variables $Z(t)$ are discussed in the subsequent sections. The medium run equilibrium is the solution of (2)-(3) and the longer run equilibrium is the entire trajectory based upon (2)-(5) including the steady state.

3. Social Consumption, Fiscal Policy and the Current Account

The only structural equation in the model that is not standard is the social consumption function. We first explain how this equation is affected by political regime changes, and hence a crucial exogenous variable in the social consumption equation is the fiscal variable: the ratio of government consumption/GDP. It is important in understanding the fundamental determinants of the real exchange rate.¹⁷ A second issue that needs some justification is what is the appropriate measure of the real exchange rate, and how is it related to internal and external equilibrium.

3.1. Social Consumption and Regime Changes

¹⁶ The foreign debt $F(t)$ is negative if the country is a creditor. The model combines direct and portfolio investment. Hence $dF(t)/dt$ consists of net longer term portfolio and direct investment.

A crucial variable in the model is the fraction of GDP consumed rather than invested. The determinants of the ratio of consumption/GDP in France, Germany and Italy are explained in this section.

A dichotomy must be made between the behavior of the private and the public sectors. The mathematical finance literature, inspired by the work of Robert Merton, discusses the optimal private consumption and portfolio selection when the returns to the risky asset are stochastic¹⁸. The control variables are: the level of household consumption and the fraction of wealth or net worth invested in the risky asset. The object is to maximize the expected discounted value of a HARA utility function over an infinite horizon. Using dynamic programming - stochastic optimal control – this literature proved that the consumption and portfolio decisions could be dichotomized. Optimal private consumption is proportional to net worth $X(t)$. A logarithmic utility function corresponds to the limiting case of a HARA function where relative risk aversion is unity. In that case, the factor of proportionality is the discount rate.

Fleming and Stein (1999) analyze a stochastic optimal control model of international finance and debt, where there are Brownian motion disturbances to the productivity of capital and the world interest rate, and countries can be debtors or creditors. They determine the optimum foreign debt, expected current account and consumption; and we relate optimality to the vulnerability of an economy to the external shocks. They proved that: since optimal consumption and foreign debt are proportional to net worth $X(t)$, equal to capital less foreign debt¹⁹, there will be no bankruptcy or insolvency – regardless of the variance of the disturbances. Consumption as a fixed proportion to net worth is an essential ingredient of an “intertemporal budget constraint”- the avoidance of bankruptcy - and is a stability condition in the NATREX model²⁰.

¹⁷ See point (g) in the Introduction.

¹⁸ Merton (1990: 111, eqn. 4.42); Fleming and Soner (1992); Fleming and Zariphopoulou (1991).

¹⁹ Net foreign asset is a negative foreign debt.

²⁰ Since net worth is capital less debt, a crucial feedback control or stability condition or intertemporal budget constraint is that the ratio of the foreign debt/GDP, variable $F(t)/Y(t)$, should negatively affect social consumption. The weaker is this effect, the slower will be the convergence to a steady state.

Variable $X(t)$ is wealth or net worth, measured as capital $k(t)$ less foreign debt $F(t)$. A negative foreign debt are net foreign assets. Private consumption $C_p(t)$ is proportional to net worth. $C_p(t) = \beta[k(t) - F(t)]$. If the utility function is logarithmic, coefficient β is the discount rate. In the macroeconomy, social consumption $C(t)$ is the sum of private consumption $C_p(t)$ and government consumption $G(t) = g(t)Y(t)$. Variations in fraction $g(t)$ reflect regime changes, described below. The social - public plus private - consumption function is (6a) and social saving $S(t) = Y(t) - C(t)$ is (6b).

$$(6a) C(t) = \beta[k(t) - F(t)] + g(t)Y(t).$$

$$(6b) S(t) = [1 - g(t)] Y(t) - \beta[k(t) - F(t)].$$

We refer to the social consumption ratio $C(t)/Y(t)$ as the social time preference denoted $\delta(t)$, and the ratio $g(t) = G(t)/Y(t)$ as the fiscal variable, whose variations represent regime changes. We now show several very important relationships for France, Germany and Italy.

(1) The variations in the fiscal ratio $g(t)$ of government consumption to GDP are crucial determinants of variations in $\delta(t)$ the ratio of social consumption/GDP. The consumption of the private sectors in the three countries in this study do not cancel the effects of government consumption.

(2) The variations in $g(t)$ represent regime changes. The exogenous fiscal parameter $g(t)$ is a political variable.

(3) The social time preference $\delta(t) = C(t)/Y(t)$ is Granger caused by the fiscal variable.

Table 1 and figures 1-3 describe the relations between social time preference $\delta(t) = C(t)/Y(t)$ and the fiscal variable $g(t) = G(t)/Y(t)$. These two variables are highly correlated. The fiscal variable Granger causes²¹ the social time preference, but not the reverse.

Table 1

Relation between social time preference $\delta(t) = C(t)/Y(t)$ and the Fiscal Variable $g(t) = G(t)/Y(t)$ in France, Germany and Italy, 1973:4 – 1997:1

	Correlation	Granger causation
France	.86	g causes δ

²¹ This was true at lags 2, 4 and 8 quarters.

Germany	.80	g causes δ
Italy	.86	g causes δ

In figures 1 - 3, we plot social time preference δ as a four quarter moving average²² of social private plus government consumption/GNP in France (FRMADIS), Italy (ITMADIS) and Germany (GRMADIS). The fiscal variable $g(t)$ is also a four quarter moving average of the fiscal variable in France (FRGOVYMA), Germany (GRGOVYMA) and Italy (ITGOVYMA). The high correlations between the social time preference and the fiscal variables are seen in table 1 and figures 1-3.

We now explain how the variations in the fiscal variable $g(t) = G(t)/Y(t)$ reflect political regime changes, and that the social time preference variable $\delta(t) = C(t)/Y(t)$ moves along with the fiscal variable. The private sector did not nullify the fiscal variable.

Before reunification, the rise and decline of German fiscal variable $g(t)$ reflected the differences between the Schmidt and Kohl governments. Social time preference $\delta(t)$ then showed the same variations. From 1969-82 the Social Democratic party was in power, led from 1974 by Schmidt. After a period of expansionary fiscal policies in the first half of the seventies, the Schmidt government tried to stabilize the share of government consumption/GDP. At the Bonn summit in mid 1978 Schmidt agreed that Germany should be a locomotive for growth. He put in force government expenditure programs, and government consumption/GNP began to rise again in 1980-82 period. Social time preference $\delta(t) = GRMADIS1$ shows a rapid rise until 1982 for an additional reason. After the election, the Schmidt government promised not to accept any unemployment. This promise made it impossible for trade union leaders to oppose wage demands from the rank and file. As a result, there were large wage increases in 1973-75, especially in the public sector²³, which raised social time preference.

From 1982-90 both the fiscal variable $g(t)$ and social time preference $\delta(t)$ declined drastically. This decline coincided with the switch from a Social Democratic government to a Conservative government led by Kohl. It was part of Kohl's government program to slow down public deficits. This program reduced both fiscal variable $g(t)$ and $\delta(t)$ the social time preference. In addition, wage pressures remained

²² DIS refers to "discount rate", and MA to four quarter moving average.

weak possibly as a result of the severe 1981-82 depression and the substitution of Kohl for Schmidt. During the Kohl period prior to unification 1982-90, there was a decline in both the fiscal variable and in social time preference. The unification reversed the decline: both $g(t)$ and $\delta(t)$ rose from 1990 to 1993, then declined. In 1997 the fiscal variable $g(t)$ was close to its longer run mean, but social time preference $\delta(t)$ was above its longer run mean, due to government transfers to the Eastern part of Germany.

In France there were trends in the fiscal variable $g(t) = \text{FRGOVYMA}$ and in social time preference $\delta = \text{FRMADIS}$. During 1981-82, the newly elected Mitterand Socialist government adopted a budget which explicitly aimed to stimulate the economy by increasing consumption. There were increases in the minimum wage, transfer payments to families and retired people, an enlarged number of government employees, and nationalization of firms. There was a capital flight, current account deficits and a declining value of the French franc. During the period from 1981-85, there was a large rise in both the fiscal variable and the rate of time preference, and a significant decline in the rate of investment/GDP.

A government of the right came to power in 1986. Both the fiscal variable and social time preference declined from 1985 to 1991, and there was privatization. The rate of investment rose from 1986-90. The Socialists returned to power in 1988, both the fiscal variable and social time preference rose, and the rate of investment declined from 1990 to 1997.

In Italy, there was a rising trend in the fiscal variable $g(t) = \text{ITGOVYMA}$ and social time preference $\delta(t) = \text{ITMADIS}$ from 1977 to 1992. Both variables rose drastically from 1979-83. The policy mix increased social consumption to stimulate aggregate demand. The primary deficit/GDP between 1981 and 1987 ranged from 4 to 6.5%. The rate of capacity utilization rose. There was full indexation of wages. These policies produced the inflation of the first half of the 1980s. The rate of monetary expansion was reduced from 1983 to 1988 to offset the inflationary effects of expansionary fiscal policy. The rate of investment declined from 1981 to 1986. The crucial point is that the resulting diversion of resources to consumption and away from capital formation depressed the future growth rate.

²³In 1974, one year after the oil price increase, wage increases of 12% were negotiated.

From 1990 to 1993, the debt became less sustainable²⁴. There was a change in policy in 1992. Both ratios declined after 1992. Technocrats replaced the Craxi government. The full indexation of wages was abolished. There were rising primary surpluses beginning in 1992 reaching to 3.7% in 1995. The change in policies from 1993 diminished the unsustainability, and by 1994-95 the debt dynamics satisfied the sustainability condition.

We **summarize** this section as follows. The ratio of social consumption/GDP primarily reflects changes in fiscal policy, the ratio of government purchases/GDP. These variations result from political regime changes. There is no evidence that the private sector offsets the fiscal variable. Figures 1-3 and table 1 show that the two variables move together when normalized.

3.2 The Measure and Effects of the Real Exchange Rate upon the Current Account and Saving less Investment

Equation (2) states that the equilibrium real exchange rate $R(t)$ equates the current account to saving less investment, evaluated at capacity output. There is an ambiguity in the literature concerning the “appropriate” definition of the real exchange rate. The real exchange rate $R(t) = N(t)p(t)/p'(t)$ can be defined as the nominal exchange rate $N(t)$ times the ratio $p(t)/p'(t)$ of domestic/foreign “prices” or “costs”, where each term must be precisely specified. This issue is pertinent in the empirical section below.

In this theoretical part, we can be general. The current account is the trade balance $B(t)$ less net interest payments to foreigners $r'F(t)$, where r' is the world rate of interest and $F(t)$ is the net foreign debt. The trade balance is exports less imports. The transmission effect of the real exchange rate depends upon how it is defined.

(A) If the real exchange rate is measured in terms of relative prices, then the story is that an appreciation of the real exchange rate leads to a substitution of foreign for domestic goods, and reduces the trade balance. This scenario assumes that there is not perfect substitutability between domestic and foreign goods: the “law of one price” (LOP) is not true for broad based price indexes. If the LOP were valid for broad based

²⁴ The ratio of debt to GDP is $b=B/Y$ where B is the debt, Y is GDP. The rate of change of the debt is: $dB/dt = rB - sB$, where r is the interest rate and s =primary surplus/debt.

indexes, then this concept of the real exchange rate would be constant – a PPP assumption. However, the LOP is a “no-arbitrage” condition in the goods market; and it does not imply that there is external – internal equilibrium in the sense of equations (2) and (3) in Box 1.

(B) If the real exchange rate is measured as the ratio of “costs”, then it does not matter if goods are perfect substitutes. A rise in the ratio of domestic to foreign costs decreases the profitability-competitiveness of the domestic industries that compete in the world markets. Even if the goods were perfect substitutes and the LOP holds, an appreciation of the cost concept of the real exchange rate diminishes the profitability of the export and import competing industries; and the trade balance would decline.

The level of productivity affects the marginal cost-supply functions in the export and import sectors. Given the prices of output and inputs, a rise in the level of productivity shifts the marginal cost functions downward to the right and increases output and thereby the trade balance.

The main exogenous variable, in exogenous vector $Z(t)$, is the terms of trade, the ratio of export/import prices. Thereby we obtain the current account equation $CA = B - rF = A(R, y, F, r; Z)$ in Box 1.

The cost concept of the real exchange rate also negatively affects the investment function based upon the Keynes-Tobin q -theory of investment. The q -ratio is the ratio of the present value of the expected rents - equal to the capital value of an asset - divided by the supply price of the capital good.

The production function of a typical firm uses labor, capital and imported materials to produce output that it sells at either an exogenously given world price or at a price along a negatively sloped demand curve. It follows that the rent per unit of capital is negatively related to the cost concept of the real effective exchange rate²⁵. An appreciation of the cost concept of the real exchange rate raises unit labor costs. Given the terms of trade, the appreciation reduces the rent per unit of capital and thereby the q -ratio. Investment is less profitable. The net result is that the investment function is equation (5): $I = dk/dt = J(q)$. Since the expected rents also depend upon capital k and the

Then, the proportionate rate of change of the debt/GDP is: $d \ln b / dt = r - (s+n)$, where n is the growth of GDP. A debt is unsustainable if $r > (s+n)$.

²⁵ This is derived via the profit function in Stein (1997: ch. 6, pp. 194-96).

terms of trade T , the investment function is: $I = J(q) = I(k, R, r; T)$, where $I_k < 0$, $I_R < 0$, $I_r < 0$, $I_T > 0$.

4. The Medium Run and Longer Run Equilibrium Real Exchange Rate

The medium run NATREX is the equilibrium real exchange rate associated with both internal equilibrium and external equilibrium: conditions (A1) – (A3) in section 2.1 above - given the stocks of debt and capital. Equation (2) states that the excess of saving over investment ($S - I$) is equal to the current account CA , evaluated at capacity output. This is internal equilibrium. Equation (3) states that the interest rates at home and abroad adjust so that investors are content to hold the stock of net foreign assets in existence. This is an external equilibrium. The medium run NATREX is the real exchange rate $R(t)$ satisfying (2) and (3), given the stocks of debt and capital.

At this equilibrium there may be a nonzero current account. The current account deficit is the rate of change of the foreign debt. Equation (2) states that the current account deficit is equal to investment less saving. Hence the rate of change in the foreign debt is equation (4), where function $L = (I - S)$. The transition to the longer run equilibrium real exchange rate, conditions (A1) - (A4) above is obtained by adding the endogenous variations in capital and the foreign debt. The dynamics of debt and capital are equations (4) and (5) respectively. The dynamics of the real exchange rate involve the change in the medium run equilibrium, based upon equations (2)-(3) for external and internal balance, to the longer run equilibrium using equations (4) and (5) to where the foreign debt and capital intensities stabilize. The model is a generalization of the macroeconomic balance models because it involves both stock and flow equilibrium conditions²⁶.

The model is used to explain how the fundamental determinants $Z(t)$ determine the equilibrium real exchange rate, $R[Z(t)]$. A mathematical analysis and stability conditions are contained in the references to the NATREX model, and a more intuitive graphic presentation of the equations above will suffice for our present purposes.

²⁶ See MacDonald and Stein (1999: 12-17) for a comparison of the macroeconomic balance models and NATREX. See Faruqee, Isard and Masson in MacDonald and Stein,

Figures 4-5 graph the real exchange rate on the ordinate, and saving less investment and the current account (which is the negative of the rate of change of the foreign debt) on the abscissa. Using this figure, based upon the equations above, we describe the dynamic response of the real exchange rate and foreign debt to changes in the real fundamentals. The equilibrium real exchange rate is evaluated at capacity output.

The equation of saving less investment labelled SI is drawn as a positively sloped curve in figures 4-5. The difference between saving less investment is positively related to the real effective exchange rate, because an appreciation of the real exchange rate reduces the rent per unit of capital and decreases the q-ratio which decreases investment, as explained in section (3.2) above. The current account curve, labelled CA, is negatively sloped because an appreciation of the real effective exchange rate reduces competitiveness, as described in section (3.2) above.

In section (4.1) we explain the dynamic effects of changes in the fiscal variable or time preference. In section (4.2) we do the same for changes in productivity. Thereby we explain the economics of the function $R[Z(t)]$, the equilibrium real exchange rate.

4.1. The Dynamics of Changes in Time Preference generated by Changes in the Fiscal Variable

Initially let saving less investment be described by the curve $SI(0)$ and the current account by curve $CA(0)$, in figure 4. The real exchange rate which equilibrates the two is $R(0)$. Let there be a rise in the fiscal variable $g(t)$ of government consumption/GDP. In France, Germany and Italy, private consumption does not decline to nullify this rise and social time preference δ also rises. Evaluated at exchange rate $R(0)$: the curve of social saving less investment shifts to the left to $SI(1)$. A fraction of the rise in social consumption is directed to foreign goods, so the current account function shifts to the left to $CA(1)$. At the initial real exchange rate $R(0)$, saving less investment is less than the current account. This means that: (a) there is an excess demand for goods, or (b) the desired capital inflow I-S generated by the fiscal expansion exceeds the current account deficit. Initially domestic real interest rates rise relative to the foreign rates and stimulate a capital inflow that leads to a restoration of portfolio

(1999: ch. 4) for an analysis of the macroeconomic balance models, particularly those used by the IMF.

balance and appreciates the real exchange rate²⁷. In figure 4, the real value of the domestic currency appreciates to $R(1)$.

The appreciation of the real exchange rate raises unit costs and lowers the q -ratio. Some investment is crowded out²⁸. The appreciated exchange rate $R(1)$ produces a current account deficit. The decline in social saving generated by the fiscal expansion crowds out some investment and increases the current account deficit.

The appreciation of the real exchange rate $R(t) = N(t)p(t)/p'(t)$ could occur via any combination of the appreciation of the nominal rate $N(t)$ or a rise in $p(t)$ the domestic unit costs. In the medium run the appreciation from $R(0)$ to $R(1)$ is consistent with the conventional wisdom: an expansionary fiscal policy appreciates the real exchange rate and decreases the trade balance.

The NATREX model, by considering two dynamic effects, continues where the consensus model stops. The first operates by changing the stock of foreign debt, and the second affects the capital stock and hence productivity. The former is now discussed, and the latter is the subject of section (4.2) below.

The current account deficit $-CA = dF/dt = rF - B$ is the rate of change of the foreign debt dF/dt . It is equal to the interest payments on the debt rF less the trade balance B . The current account deficit raises the foreign debt in the initial period. The resulting rise in interest payments on the debt reduces the current account further. Graphically, the CA curve shifts downwards to the left towards $CA(2)$ as the higher debt requires higher interest payments. The flow of interest payments depreciates the exchange rate below $R(1)$.

If nothing further happened to saving and investment, the $CA(t)$ curve would be shifting downwards to the left along the $SI(1)$ curve, as a result of the rise in the debt. The deficit would continue to grow, which leads to an exponential growth of the foreign debt. The real exchange rate would be expected to depreciate in the south-west direction along curve $SI(1)$. This is an unstable situation. A crisis is inevitable.

²⁷ This is explained in terms of the NATREX model in Stein (1997: figure 2.4, p.62; 1999, figure 4, p. 81).

²⁸ Let saving be relatively insensitive to the real exchange rate.

The dynamics of a stable process require²⁹ that a rise in the debt raises social saving - reduce absorption $C+I+G$. This is the condition that $S_F > 0$ in equation (2) be positive. It should be viewed as the feedback control in the consumption equation (6a) or saving equation (6b). This could come about in several ways.

A rise in the foreign debt $F(t)$ reduces net worth $k(t)-F(t)$ and decreases private consumption and raises private saving. Alternatively, the fiscal variable $g(t)$ is not completely exogenous and is affected by the foreign debt. For example, as the foreign debt and interest payments rise, the incumbent government is voted out of office and the new government reduces its consumption or raises taxes to reduce the government deficit. See the accounts of France, Germany and Italy in section (3.2) above. Insofar as $S_F > 0$, social saving would rise, and the SI curve would shift rightward to $SI(2)$.

The long run equilibrium occurs when the debt/GDP stabilizes. This is our intertemporal budget constraint, derived from the feedback control term in the social consumption function. Abstracting from growth the current account, equal to saving less investment, must be zero³⁰. The higher foreign debt, the summation of current account deficits, requires higher interest payments. Hence the current account curve declines to $CA(2)$ which is below the initial level $CA(0)$. When saving less investment function shifts to $SI(2)$, the real exchange rate $R(2)$ equates the current account $CA(2)$ to $SI(2)$. At a zero current account, the debt is stabilized and the system has converged to a stable equilibrium.

The Mundell-Fleming model implies that an expansionary fiscal policy that stimulates consumption appreciates the exchange rate. The NATREX model shows that the conventional Mundell-Fleming view only describes the short run. The longer run effect of the government budget deficits - an expansionary fiscal policy - is to depreciate the real exchange rate. This is opposite to the conventional wisdom, which just looked at the shorter run appreciation from $R(0)$ to $R(1)$, and ignores the second phase of depreciation from $R(1)$ to $R(2) < R(0)$.

²⁹ The condition that $\delta S/\delta F > 0$ is necessary for stability of the system. See Stein (1997: p. 65, note 24).

³⁰ In general allowing for growth, when the ratio of the debt to GDP stabilizes at F^* , then $dF/dt = rF - B = nF$, where B is the trade balance and n is the growth rate. Hence the current account is $B - rF = -nF$ and $B/F = (r - n) > 0$.

4.2 The Productivity Effect³¹

The second fundamental determinant of the equilibrium exchange rate in the NATREX model is relative productivity of the economy as a whole. The level of productivity affects both the current account function CA and the SI functions in figure 5.

Let the initial current account function be $CA(0)$, the saving less investment function $SI(0)$ and the medium run equilibrium real exchange rate $R(0)$. A rise in productivity lowers marginal costs and increases the supply function of exports and import competing goods, evaluated at any real exchange rate. Thereby, it shifts the current account function from $CA(0)$ to $CA(1)$. The saving function will also be increased because saving is GNP less consumption. The investment function will also increase, because the rise in productivity increases the q-ratio. On balance, the saving less investment function shifts from $SI(0)$ to $SI(1)$. The hypothesis is that a rise in productivity increases the current account function by more than it increases the saving less investment function. Alternatively, the hypothesis in this paper³² is that the rise in productivity $y(t)$ raises the current account plus the desired capital inflow: $d[CA(t) + (I-$

³¹ We do not use the Balassa-Samuelson effect in our analysis because it has been shown to be of negligible importance in explaining exchange rates. The Bundesbank (1995), Clostermann and Friedman (1998), MacDonald and Stein (1999) have shown that the variations in the real exchange rate of Germany – measured in terms of broad based indexes, such as the CPI - are almost completely explained by corresponding variations in the real exchange rate based upon the prices of tradables. In the Balassa-Samuelson hypothesis, the latter reflects the “law of one price” and should be constant. That is clearly not the case. The relative price ratios of non-tradable/tradable in Germany and abroad have negligible explanatory power in explaining the German real exchange rate measured in terms of the CPI or broad based indexes. Other studies by Rogers and Jenkins show that on average 81% of the variance of the real CPI exchange rate is explained by the relative price of traded goods, rather than the relative price of nontraded/traded goods (See the discussion in MacDonald and Stein, 30). These results question the Balassa-Samuelson hypothesis. If alternative measures of the real exchange rate measured as the ratio of non-traded/traded goods at home and abroad are used, such as the ratio of consumer/producer prices, then the results are mixed. See Clark and MacDonald. Mark (1999) and Chinn (1999) show that the productivity of the economy as a whole – much more than relative sector productivity - is an important determinant of the exchange rate.

$S(t)]/dy(t) > 0$. The quantity saving less investment is a desired capital outflow. At initial real exchange rate $R(0)$, the ex-ante current account exceeds the desired capital outflow. The medium run equilibrium real exchange rate appreciates to $R(1)$, where the current account at is again equal to the desired capital outflow.

The positive current account at $R(1)$ lowers the debt. The decline in the debt raises the subsequent current account $CA(t) = B(t) - r'F(t)$. The current account function keeps shifting to the right as long as there are current account surpluses. In addition, the decline in the debt raises net worth capital less debt. Consumption rises and saving decreases. The SI curve shifts to the left as long as the debt is declining. The trajectory will be $R(0)$ - $R(1)$ - $R(2)$. At the new longer run equilibrium $R(2) > R(0)$, the current account is zero³³; and the debt stabilizes.

4. 3 Summary and Reduced Form Equations to be estimated

The dynamics of adjustment in the NATREX model imply reduced form equations (7) - (9) for the equilibrium real exchange rate³⁴. At any time the actual real exchange rate $R(t) = N(t)p(t)/p'(t)$ can be written as the sum of three elements.

$$(7) N(t)p(t)/p'(t) = R[Z(t)] + \{R_m(k(t), F(t);Z(t)) - R[Z(t)]\} + \varepsilon(t)$$

where $\varepsilon(t) = [N(t)p(t)/p'(t) - R_m(k(t), F(t);Z(t))]$

The first term is the longer run equilibrium $R^*(t) = R[Z(t)]$, which corresponds to point $R(2)$ in figures 4-5, when the debt has stabilized. The medium run equilibrium $R_m(t) = R_m(k(t), F(t);Z(t))$, which corresponds to point $R(1)$, results from variations in exogenous fundamental variables $Z(t)$, given the values of the endogenous debt $F(t)$ and capital $k(t)$ or productivity). The endogenous evolution of the debt and capital to the longer run equilibrium is described by trajectory $R(1) - R(2)$; and is the second term $\{R_m(k(t), F(t);Z(t)) - R[Z(t)]\}$. The noise term $\varepsilon(t)$ contains the elements that have been omitted from the NATREX model of the equilibrium exchange rate. They are variations in the rate of capacity utilization, deviations of the interest rate from the foreign rate plus a risk premium, and anticipations unrelated to the fundamentals $Z(t)$.

³² The NATREX model in general allows for several different possibilities, depending upon where the productivity occurs. See Allen (1997, pp. 24-26)..

³³ Again for simplicity of exposition, we are ignoring growth.

³⁴ The technical derivation and stability conditions are in Stein ch. 2 in Stein, Allen et al (1997).

Equation (8) is the VEC econometric counterpart to the discussion above, and the dynamic model implied by Box 1, figures 4-5. The change in the equilibrium real exchange rate $\Delta R(t)$ is the movement along the trajectory $R(0)$ - $R(1)$ - $R(2)$ to the longer run equilibrium $R^*(t) = R[Z(t)]$.

$$(8) \Delta R(t) = -a\{R(t) - R[Z(t)]\}dt$$

$$(9) R^*(t) = R[Z(t)], \quad Z(t) = [g(t), y(t); g'(t), y'(t)]$$

The fundamentals $Z(t)$ that we shall consider are the fiscal variable at home $g(t)$ and abroad (denoted by a prime) and productivity at home $y(t)$ and abroad (denoted by a prime). Alternatively, we use social time preference $\delta(t)$ at home and abroad (denoted by a prime) instead of the fiscal variable. A rise in relative time preference depreciates, and a rise in relative productivity appreciates the equilibrium exchange rate. The fiscal variable or time preference variable is expected to depreciate the longer run equilibrium exchange rate; and the productivity variable is expected to appreciate it. The foreign variables have the opposite effects.

Our basic hypothesis can be stated in several ways. (a) The NATREX is an attractor. The latter is a deterministic equilibrium. (b) The actual real exchange rate $N(t)p(t)/p'(t)$ converges to the NATREX according to stochastic differential equation (10). The first term in (10) is the convergence of the medium term equilibrium to the longer run equilibrium. The term $\sigma d\eta$ is the Brownian motion term which produces the variations of the actual real exchange rate $N(t)p(t)/p'(t)$ from the NATREX. It is the third term in equation (7) above. (c) Misalignment $\Phi(t) = \{N(t)p(t)/p'(t) - R[Z(t)]\}$ is stationary with a zero expectation. It converges to a distribution (10a).

$$(10) d\Phi(t) = -c\Phi(t)dt + \sigma d\eta \quad d\eta = \varepsilon(t) \sqrt{dt} \quad \varepsilon(t) \sim N(0,1)$$

$$(10a) \lim \Phi(t) = N(0, \sigma^2/2c)$$

5. Estimation Strategy for the Equilibrium Real Exchange Rate

5.1 Measurement of Real Exchange Rate

The equilibrium real exchange rate is the endogenous variable that equilibrates the current account to saving less investment and also affects the investment demand function in the manner described in section (3.2). The adjustment can occur either

through changes in the nominal rate $N(t)$ or the ratio of domestic/foreign prices or costs $p(t)/p'(t)$.

As indicated in section (3.2) above, there are several definitions of the real exchange rate $R(t) = N(t)p(t)/p'(t)$, depending upon the deflators $p(t)/p'(t)$. The International Monetary Fund- International Financial Statistics uses unit labor costs in manufacturing as the deflators to derive the measure of the real effective exchange rate, denoted REU. In that case, the bilateral real exchange rate of France/Germany or Italy/Germany would be the ratio of the multilateral French/ multilateral German, or multilateral Italian/ multilateral German REU's.

Economists at the Deutsche Bundesbank³⁵ pointed out that the REU measure of the real exchange rate gives a very different picture of the competitiveness of the German economy than do the broader based indexes. The competitiveness of the economy also depends upon the labor costs included in intermediate products purchased from other domestic and foreign sectors. They constructed and used measures of the real exchange rate based upon: unit labor costs in the economy, deflators of total expenditures and the CPI's. The latter three measures of the real exchange rate moved together, and differently from the REU measure.

The question is which measure of the real exchange rate affects exports and imports? The Bundesbank economists used the following alternative deflators $p(t)/p'(t)$. Unit labor costs in manufacturing (the REU index), price or cost deflator of total sales, consumer prices, producer prices of industrial products, terms of trade. The basic estimation results in the export and import functions were similar for each measure. Their preferred measures are the price deflators for total expenditures: GDP plus imports.

Figures 6 and 7 contain the basic data used in our study for France/Germany and Italy/Germany. The series FRGRREU, ITGRREU are the ratios of the French/German and Italian/German real effective exchange rates. A rise in either is an appreciation of the French or Italian real exchange relative to the German. The series FRGREX, ITGREX are the corresponding French/German and Italian/German real exchange rates

³⁵ See Clostermann and Friedmann (1998), Deutsche Bundesbank, Monthly Report, 50, No. 11 (1998). We refer to these two articles as the Bundesbank studies, simply because Clostermann and Friedmann are at that institution.

using the CPI index as deflators. It is seen from the graphs that the two measures produce different trend movements for each country. The correlation coefficient for France relative to Germany is $r(\text{FRGRREU}, \text{FRGREX}) = 0.187$ and for Italy relative to Germany it is $r(\text{ITGRREU}, \text{ITGREX}) = -0.174$.

5.2 Measurement of the Exogenous Fundamental affecting the Consumption Ratio

One of the two basic fundamentals is the ratio of social consumption/GDP. Define $\delta(t) = [C(t) + G(t)]/Y(t)$ as social time preference, where $C(t)$ is private consumption and $G(t)$ is government consumption. Section (4.1) explained how variations in social consumption lead to medium to long run real equilibrium exchange rate movements.

It was shown (figures 1-3) that variations in the fiscal variable, the ratio of government consumption/GDP, have been the major determinants of variations in social time preference in France, Germany and Italy. Private consumption has not offset the variations in government consumption.

We use two measures for relative time preference in France/Germany and Italy/Germany, graphed in figures 6 and 7. The reason for using two alternative measures is that, using national income accounting, $\delta(t) = [C(t) + G(t)]/Y(t) = 1 - I(t)/Y(t) - B(t)/Y(t)$, where $I(t)$ is investment and $B(t)$ is the trade balance. It may be the case that what we attribute to the exogenous time preference variable $\delta(t) = [C(t) + G(t)]/Y(t)$ is just $[-B(t)/Y(t)]$, the effect of the trade balance. Since we have shown that variations in $[C(t) + G(t)]/Y(t)$ are produced by variations in the fiscal variable $g(t) = G(t)/Y(t)$, we use two alternative measures of the relative exogenous time preference: $\delta(t)/\delta'(t)$, and $g(t)/g'(t)$. Germany is viewed as the foreign country, denoted by a prime.

For France/Germany: DISCFRA is $\delta(t)/\delta'(t)$, the ratio of social consumption/GDP in France relative to Germany; FRGRGOV is $g(t)/g'(t)$, the ratio of a 4-quarter moving average of French government consumption/GDP divided by the corresponding measure for Germany. For Italy/Germany: DISCITA is $\delta(t)/\delta'(t)$; and ITGRGOV is $g(t)/g'(t)$.

In each country, the two measures of this fundamental $\delta(t)/\delta'(t)$ and $g(t)/g'(t)$ are closely correlated. For the period 1975:4 – 1993:3 the correlations are: $r(\delta(t)/\delta'(t), g(t)/g'(t)) = r(\text{FRGRGOV}, \text{DISCFRA}) = 0.70$,
 $r(\delta(t)/\delta'(t), g(t)/g'(t)) = r(\text{ITGRGOV}, \text{DISCIT}) = 0.84$.

5.3 Productivity

Many empirical studies of the determinants of the longer run real exchange rate have found that the ratio of productivity - GDP/worker - in the two countries is a highly significant variable³⁶. This is not the Balassa-Samuelson effect³⁷, but the total productivity of the economy: $y(t) = \text{GDP}/\text{employment}$.

In section (4.2) we explained the dynamics of productivity upon the trajectory of the equilibrium real exchange rate. Here we measure the productivity effect³⁸ $y(t)/y'(t)$ as $\text{PRODFRA} = \text{French/German productivity}$ and $\text{PRODIT} = \text{Italian/German productivity}$. The scenario was that productivity affects the current account function and the investment function. The longer run equilibrium real exchange rate appreciates along trajectory $R(0)$ - $R(1)$ - $R(2)$ in figure 5.

5.4 Variables not used in the estimation of the Longer Run NATREX

The medium run NATREX, based upon equations (2)-(3), is $R_m(t) = R_m(k(t), F(t); Z(t))$, where $F(t)$ is the net foreign debt – the negative of net foreign assets – and exogenous vector $Z(t) = [\delta(t)/\delta'(t), y(t)/y'(t)]$ or $Z(t) = [g(t)/g'(t), y(t)/y'(t)]$. Conditions (A1)-(A3) in section 2.1 prevail: internal balance, portfolio balance – uncovered interest rate parity/asymptotically rational expectations, and the rate of capacity utilization is at

³⁶ Clostermann and Friedmann (1998), Mark (1999), Clark and MacDonald (1999), MacDonald (1999), Chinn (1999) all show the importance of relative productivity in affecting the longer run real exchange rate. To put it another way, the PPP hypothesis fails to take into account productivity.

³⁷ The Balassa Samuelson effect was shown to be feeble. See the MacDonald and Stein, chapter 1, pp. 11-12 and figure 2 for evaluation of the empirical work.

³⁸ An alternative concept of the productivity effect is an exogenous disturbance to the investment function. The q-ratio is raised which stimulates investment and growth. The growth variable is then used to empirically examine this effect. This is done for the \$US/G7 real exchange rate in Stein (1999).

its long-term stationary mean. The net foreign debt³⁹ depreciates the medium run real exchange rate $\delta R_m(t)/\delta F(t) < 0$, in the manner described in figures 4 and 5.

Many empirical studies have examined the effect of the net foreign debt, or net foreign assets, upon the real exchange rate. For example, Clark and MacDonald (1999) have found that a rise in net foreign assets ($-F(t)$) appreciates the exchange rate for the US, Germany and Japan.⁴⁰ We rely upon these studies for evidence of the foreign debt effect $\delta R_m(t)/\delta F(t) < 0$.

In the long run, the foreign debt or net foreign assets, is a state variable: Box 1 equation (4). The trajectories of both the real exchange rate $R(t)$ and net foreign debt $F(t)$ are endogenous variables, determined by the exogenous variables $Z(t)$. The long run equilibrium values are: $R(t) = R[Z(t)]$ and $F(t) = F[Z(t)]$, when $dF(t)/dt$ and $dk(t)/dt$ are zero. For this reason, we do not use $F(t)$ as a regressor in our longer run relation $R(t) = R[Z(t)]$ of the equilibrium real exchange rate.

An important variable that affects the actual exchange rate or balance of payments is the rate of capacity utilization. In the standard open economy macro-economic models, the exchange rate and rate of capacity utilization⁴¹ are simultaneously determined. In the countries examined: France, Germany and Italy, the rates of capacity utilization are highly correlated⁴². The NATREX model is concerned with the “equilibrium” exchange rate, when the rate of capacity utilization is at its stationary

³⁹ Our model makes no distinction between indirect and direct investment. Variable $F(t)$ is the sum of current account deficits, and $r'(t)F(t)$ is the net flow of interest and dividend payments abroad plus reinvested foreign earnings.

⁴⁰ Net foreign assets have also been found to be a significant variable in the studies by: Mark (1999), MacDonald (1999), Chinn (1999). On the other hand, this effect was not found to be significant for Germany in the study by Clostermann and Friedmann (1998), a result quite opposite to that found by Clark and MacDonald. This is another example that the estimate of the effect of a variable x upon the real exchange rate depends upon the other variables z included in the regression.

⁴¹ This is the ratio of actual/ capacity output. Hence it is the endogenous output rate in the Keynesian type models.

⁴² The correlations of the rate of capacity utilization for France (F), Germany (G) and Italy (I) are as follows. $r(F,G) = .81$, $r(F,I) = .79$, $r(G,I) = .88$.

mean.⁴³ Consequently the rate of capacity utilization only affects misalignment: the deviation of the actual exchange rate from its equilibrium - the NATREX⁴⁴.

Many empirical studies of the exchange rate show that the interest rate differentials are important determinants of the actual exchange rate. In some, it is true in the shorter run, and in others the real long-term interest rate differential affects the real exchange rate in the long run⁴⁵. The NATREX model allows for real interest rate differentials in the shorter run⁴⁶. The longer run convergence of real long-term interest rates depends upon the risk premium term in equation (2) above $\rho = E\{\Delta R^*[Z(t)]\} = E\{\nabla R^* \cdot \Delta Z(t)\}$, the asymptotic rational expectation of a weighted sum of changes in the fundamentals $Z(t)$. In the longer run we expect $\rho = 0$, because the ΔZ 's are $I(0)$ variables with zero expectations. In estimating the longer run NATREX, we do not include the real long-term interest rate differential as a regressor and just consider the relation of the interest rate differential to misalignment⁴⁷.

6. Econometric Estimation of the Equilibrium Real Rate

6.1 Research Design

The NATREX model implies equations (8)-(9) above for the evolution of the equilibrium real exchange rate. For the actual estimation, we use equation (8a) that contains a lag structure. As indicated in section (5.2), we use two different estimates of "time preference" $\delta(t)/\delta'(t)$ and $g(t)/g'(t)$ in (9a) and (9b). Moreover, we consider alternative measures of the real exchange rate as indicated in section (5.1). The hypotheses are that:

(A) A rise in time preference relative to Germany depreciates the longer run value of the currency relative to Germany: $b_1 < 0$.

⁴³ Stein (1999: figure 3, p. 79) explains the relation of the medium run NATREX and the Keynesian models where actual /capacity output is endogenous.

⁴⁴ In the study of the \$US/G7 exchange rate, Stein (1999: tables A2, A4) showed how the rate of capacity utilization affects the misalignment.

⁴⁵ See Clark and MacDonald (1999).

⁴⁶ See Stein (1999: pp. 80-81).

⁴⁷ In an application of the NATREX model to the real exchange rate of the \$US/G7, Stein (1999: tables A2 – A4) there is an estimate of both the shorter run effects of variations in the rates of capacity utilization and interest rate differentials, and the longer run effects of $Z(t)$ productivity and thrift, upon the real exchange rate.

(B) A rise in y productivity relative to Germany appreciates the longer run value of the currency relative to Germany: $b_2 > 0$.

(C) The actual real exchange rate converges to the NATREX. Specifically, misalignment $\Phi(t) = N(t)p(t)/p'(t) - R^*[Z(t)]$ converges to a distribution with a mean of zero and variance of $\sigma^2/2c$. The NATREX is an attractor.

$$(8a) \Delta R(t) = \alpha[R(t-1) - BZ(t-1)] dt + \Sigma B'\Delta Z(t-h) + \varepsilon(t) \quad \alpha < 0$$

$$(9a) R^*(t) = BZ(t) = b_1 \delta(t)/\delta'(t) + b_2 y(t)/y'(t)$$

$$(9b) R^*(t) = BZ(t) = b_1 g(t)/g'(t) + b_2 y(t)/y'(t) \quad B = (b_1 < 0, b_2 > 0)$$

$$(10a) \lim \Phi(t) = N(t)p(t)/p'(t) - R^*[Z(t)] \Rightarrow (0, \sigma^2/2c)$$

Our estimates of the coefficients of vector Z involve several steps. First, we examine whether there is a unique cointegrating vector (R, Z) relating the real exchange rate R to the fundamentals Z . This is a VEC estimate of equation (8a). Both the likelihood ratio and trace tests are used. We call this the unconstrained VEC estimates. Second, we examine whether the Z 's are weakly exogenous, such that the only variable that is adjusting to the gap $[R(t) - BZ(t)]$ is the real exchange rate. That is the α in (8a) is negative but in the other equations such as $\Delta Z(i) = a(i)[R(t) - BZ(t)]$ the coefficients $a(i)$ are not significantly different from zero. Third, having ascertained weak exogeneity of the Z 's, we estimate the cointegrating equation using the constraint that all coefficients $a(i)$ are zero. We call this estimation the VEC-constrained estimates. Steps one through three are explicitly presented in the appendix tables. Fourth, once we ascertain weak exogeneity and one cointegrating equation, we are able to perform an OLS estimate of equation (9).

We consider the various measures of the real exchange rate and measures of time preference, for both France/Germany and Italy/Germany.

6.2 Results when (a) the real effective exchange rate is based upon the IMF definition, and (b) time preference is social consumption/GDP

Table 2 summarizes the econometric results, and draws upon the appendix tables A1 – A4. The top half of the table refers to the real value of the French franc relative to the Dmark, (FRGRREU). The bottom half refers to the real value of the Italian lira relative to the Dmark (ITGRREU). The real bilateral exchange rate is the ratio of the

real multilateral exchange rate of France or Italy, relative to the real multilateral exchange rate of Germany. Since each multilateral equilibrium real rate produces equilibrium between the country and the rest of the world, the equilibrium bilateral rate is the ratio of two equilibrium rates. A rise is an appreciation of the currency relative to the DM. The first column refers to the relative discount rates, or rates of time preference, between the country and Germany. The second column is the ratio of productivity, GNP/employee, in the country relative to Germany.

The results are the same for both pairs of countries, regardless of the method of estimation: VEC Johansen, constrained or unconstrained, and OLS.

- (1) There is only one cointegrating equation: $R - BZ = e$, where e is stationary. This is true for lags of 4, 8 quarters.
- (2) The Z 's are weakly exogenous. Only the real exchange rate is affected by the deviation $[R(t-1) - BZ(t-1)]$. Only α in equation (8a) is significant. The $a(i)$'s in $\Delta Z_i(t) = a(i) [R(t-1) - BZ(t-1)]$ are not significantly different from zero.
- (3) In all three methods of estimation: time preference depreciates, and productivity appreciates, the longer run real exchange rate. All three methods of estimation of equation (9a) produce similar coefficient estimates.

The basic data for France/Germany is in figure 6, and for Italy/Germany in figure 7. Figure 8 compares the actual real effective exchange rate FRGRREU with the two NATREX estimates in table 2: FRGRJ based upon the Johansen method and FRGROLS based upon the OLS method. Table 9 compares the Italian real effective exchange rate ITGRREU with the Johansen estimate ITGRJ and OLS estimate ITGROLS in table 2. The variables have been normalized to facilitate orders of magnitude. The two estimates are practically identical and explain the trend movements in the real exchange rate. A rise signifies an appreciation of the French franc or Italian Lira relative to the German Mark.

Table 2

Summary of Estimates of the Long run Cointegrating equation for the real effective exchange rates of France (FRGRREU) and Italy (ITGRREU), relative to Germany 1974:3 1996:3

	Discount rates, relative to Germany DISCFRA, DISCITA	Productivity, relative to Germany PRODFR, PRODIT	constant	adj. R-SQ
	FRGRREU	FRGRREU	FRGRREU	FRGRREU
France- Germany	coeff (se) [t-stat]	Coeff (se) [t-stat]		
VEC Cointegration Lags = 4,6,8	-5.88 (0.71) [-8.23]	4.15 (0.46) [8.96]	4.1	
VECJ- constrained a(i) = 0; EC α	-6.48 (0.91) [-7.2] -0.169 (0.029) [-5.5]	4.29 (0.58) [7.89]	4.6	
OLS stationary residuals	-5.68 (0.51) [-11.0]	3.53 (0.31) [11.1]	4.4	0.69
	ITGRREU	ITGRREU	ITGRREU	ITGRREU
Italy- Germany	coeff (se) [t-stat]	coeff (se) [t-stat]		
VEC Cointegrat ion Lags = 4,6,8	-1.94 (0.57) [-3.4]	3.37 (0.34) [9.7]	1.3	
VECJ- constrained a(i) = 0 EC α	-1.95 (0.57) [3.38] -0.27 (0.05) [-5.4]	3.37 (0.34) [9.79]	- 0.68	

OLS					
stationary		-1.36	2.28	-	0.47
residuals	(0.53)		(0.27)[8.4]	0.08	
	[-2.6]				

Notes: French (Italian) relative to German Real Effective Exchange Rate FRGRREU (ITGRREU). This is the ratio of the French (Italian) multilateral to the German multilateral real effective exchange rates. The relative French/German (Italian/German) discount rates, time preference, are DISCFRA (DISCITA) and relative French/German (Italian/German) productivity, GDP/employed worker are PRODFR (PRODITA). All variables are I(1). See appendix tables A1 and A2 for the cointegration and weak exogeneity tests. Tables A3, A4 are the OLS estimates.

6.3 Results when (a) the real effective exchange rate is based upon the IMF definition, and (b) time preference is the fiscal variable government consumption/GDP

An alternative measure of time preference is the fiscal variable: the ratio (g/g') of government consumption/GDP in France and Italy, relative to Germany. Table 1 and figures 1-3 above show that the social consumption ratio and the fiscal variable are highly correlated, and that the causation runs from the fiscal variable to social consumption.

Tables 3a-3b summarize the mixed results in the estimation of equation (9b). (a) For France/Germany, there is a cointegrating equation for lags 4 and 8 quarters. (b) Using the Johansen VEC method, neither the fiscal nor the productivity variable is significant. (b) Using OLS, the fiscal variable significantly depreciates, and the productivity variable significantly appreciates, the real exchange rate.

For Italy/Germany, there is no cointegration at lag 4, but there is cointegration at lag 8 quarters. (a) Using the Johansen VEC, the fiscal variable depreciates, and the productivity variable appreciates, the real exchange rate. (b) Using OLS, the fiscal variable depreciates the real exchange rate. The productivity variable is not significant.

Table 3a

France/Germany Real Exchange rate, rise is an appreciation of the French franc

	Fiscal g/g'	productivity y/y'
VEC-Johansen	ns	ns
OLS	depreciate	appreciate

Cointegration: 1 equation at either lag 4,8 quarters; ns= not significant

Table 3b

Italy/Germany Real Exchange Rate, rise is an appreciation of Italian lira

	Fiscal variable g/g'	productivity y/y'
VEC-Johansen	depreciate	appreciate
OLS	depreciate	ns
Cointegration: none at lag 4; 1 equation at lag 8; ns = not significant		

The differences between the results of tables 2 and 3a and 3b may arise because although social consumption $\delta(t)$ is dominated by the fiscal variable $g(t)$ -government consumption/GDP - there are other factors affecting movements in social consumption. For example, variations in the tax system may affect social consumption, but only the expenditure component of fiscal policy is measured by the variable $g(t)$.

6.4 The Real Exchange Rate based upon CPI Weights

The different concepts of the real exchange rate between France-Germany and Italy-Germany were discussed in section 5.1 above. Figures 6 and 7 show that the two concepts are quite different⁴⁸. The bilateral real exchange rate based upon unit labor costs in manufacturing is the ratio of the French or Italian to the German multilateral real effective exchange rates (suffix REU in the International Monetary Fund International Financial Statistics). A rise is an appreciation of the French franc or Italian lira relative to the DMark. Neither the French/German (FRGRREU) nor the Italian/German (ITGRREU) real exchange rate is stationary⁴⁹. There is no PPP relationship for the REU concept of the real exchange rate.

The real bilateral exchange rate based upon the CPI indexes is denoted by FRGREX for France relative to Germany, and ITGREX for Italy relative to Germany. See figures 6 and 7. The variable FRGREX for France is stationary, but ITGREX for Italy is not stationary. For France/Germany, one can claim that there is a PPP relationship. Such a claim cannot be made for Italy; there is no central tendency.

We estimated equations (9a) and (9b) for France/Germany and Italy/Germany for the CPI weighted exchange rates, denoted by the suffix REX. We found that neither measure of time preference depreciated the real exchange rate; but that relative productivity appreciated the real exchange rate.

⁴⁸ For the US relative to the G7, when relative GDP deflators are used for $p(t)/p'(t)$, the REX and REU concepts of the real exchange rate are practically identical,

⁴⁹ A histogram shows that there is no central tendency, but discrete distributions.

These results were not too surprising. First, the two concepts of the real exchange rate were quite different. Second, the study by Friedmann and Clostermann cited above used the REX definition – the CPI weighted bilateral exchange rate. They considered the following regressors: relative productivity, relative real interest rates and net foreign assets. They obtained the following results for the real exchange rate of Germany. (a) Net foreign assets had no significant influence on the REX real exchange rate. (b) The relative productivity was significant. (c) The real interest rate differential was the dominant explanatory variable in both the long run and the short run.

For the nominal exchange rate, they concluded the following. In the long run: (d) the price differential between foreign countries and Germany is the major determinant of the nominal exchange rate – the PPP hypothesis, (e) internationally divergent productivity trends, and (c) real interest rate differentials are basic determinants of the REX exchange rate.

Several questions can be raised. First, they used the uncovered interest rate parity rational expectations (UIRP-RE) hypothesis that the change in the real exchange rate $d(\log R(t))$ is equal to the appropriate foreign less the domestic real interest rate $(r'(t) - r(t))$. However, they found that the interest rate differential was $I(1)$ but that the change in the exchange rate was $I(0)$. This result questions the validity of the UIRP-RE hypothesis.

Second: If relative CPI prices are constant, is it really true that fiscal policy – budget deficits – has no significant effect upon the equilibrium real exchange rate? What is the theoretical justification for ignoring fiscal policy?

Third: what is the direction of causation? Suppose that the German-French goods are close substitutes in demand and that transport costs between the two countries are low. Then, the “law of one price” would be relatively valid. That means that the $R(t) = N(t)p(t)/p'(t) = \text{REX}$ concept of the real exchange rate is always relatively constant, regardless of the fiscal policy followed. It follows that the REX measure of the real exchange rate cannot equate the current account to the capital outflow: saving less investment, evaluated at internal equilibrium.

The stationarity of the REX concept is a “no arbitrage” concept but not an equilibrium concept for internal-external balance. Budget deficits and current account deficits would have no influence upon the value of the real exchange rate $N(t)p(t)/p'(t)$.

Fourth: they found that relative prices were the main determinant of the nominal exchange rate. Are relative prices exogenous? They did not consider the crucial hypothesis that relative prices are determined by relative money/output. Studies by Husted and MacDonald have examined the PPP-Monetary hypothesis that: the nominal exchange rate is determined by relative prices, and that the nominal exchange rate is determined by relative money/output. The former hypothesis fares relatively well. However, the latter fares badly⁵⁰. It would have been more informative if they used relative money/GDP instead of relative CPI in their regressions of the nominal D-Mark.

The Bundesbank economists have rightly questioned the use of narrow based indexes such as the REU, and have advocated the use of broader based indexes. However, the broader the index the closer it is to the CPI. The real exchange rate based upon the CPI deflators may be more a reflection of a “law of one price”, which varies within the range of transport costs. It may not be the variable that adjusts the current account to saving less investment at capacity output and portfolio balance.

The Bundesbank economists have convinced us to have doubts concerning the appropriate empirical counterpart to $R(t)$ in the model. The results reported in this and the earlier parts of section 6 lead us to the view that the more appropriate real exchange rate corresponding to the equilibrium value in the model in box 1 may be the one labeled REU, based upon unit labor costs in manufacturing. This is determined by productivity and thrift as shown in table 2. However the real exchange rate measure REX in terms of relative CPI may be closer to a “law of one price – no arbitrage” equation in the goods market.

7. Summary

The PPP hypothesis implies that a reliable early warning signal of a currency crisis is the deviation of the real exchange rate $N(t)p(t)/p'(t)$ from a stationary mean: this constant is usually measured as an average denoted $av [R]$. Define the PPP measure of misalignment as $\Phi_1(t) = N(t)p(t)/p'(t) - av [R]$. However there is no theory as to what

⁵⁰ Husted and MacDonald (1999: tables 2,5), see also the summary evaluation of their work in MacDonald and Stein (1999: 8, table 1), where the nominal exchange rates for France, Germany, Italy, UK and Japan are relative to the \$US.

this constant should be, what are its economic determinants and what should be the sample period for finding ΔR .

The main empirical finding by Kaminsky et al concerning early warning signals of currency crises is that the best indicator is the real exchange rate relative to a non-constant trend⁵¹. The Kaminsky et al measure of misalignment is $\Phi_2(t) = \{N(t)p(t)/p'(t) - [R(0) + a(t)t]\}$ where the trend value is $R(0) + a(t)t$. One can call this a “trend PPP” measure. This measure does not explain what determines $a(t)$, the time varying trend.

The NATREX is a dynamic stock-flow model that generalizes the macroeconomic balance and the “trend PPP” approaches. It emphasizes the role of “supply” or real variables in determining the equilibrium trajectory of the exchange rate. The fundamental determinants are social time preference and productivity, vector⁵² $Z(t)$. Thus the “trend” value $R(0) + a(t)t$ corresponds to $R[Z(t)]$, the NATREX.

Social time preference $\delta(t)$ is the sum of private and government consumption/GDP. A driving force behind time preference movements is the fiscal variable: $g(t) =$ government consumption/GDP. The trends in $g(t)$ reflect political regime changes, and trends in social time preference $\delta(t)$ follow those in $g(t)$.

Productivity $y(t) =$ GDP/worker is a highly complex variable that reflects past investment/GDP, the efficiency of the allocation of resources and the diffusion of technology. We make no attempt to explain what the literature has not succeeded in doing: the determinants of the underlying determinants of productivity. We just take it as a smoothly varying exogenous parameter related to the rate of capital formation in equation (5).

The NATREX model then explains the transmission of variations in each component of $Z(t)$ upon the equilibrium trajectory $R[Z(t)]$, graphically described in figures 4 and 5 above. Our measure of **misalignment** is equation (1) repeated here: $\Phi(t) = N(t)p(t)/p'(t) - R[Z(t)]$. Comparing $\Phi(t)$ with $\Phi_1(t)$ and $\Phi_2(t)$, one sees how NATREX generalizes PPP and gives economic content to “trend”.

An important implication of the NATREX model is the qualification of the “two instruments approach” to exchange rates. A traditional view is that an expansionary

⁵¹ See point (h) in the introduction above.

⁵² Other fundamentals in the NATREX model are: the terms of trade and the world real long-term rate of interest.

fiscal policy and/or a contractionary monetary policy raise interest rates, attract an inflow of capital and appreciate the exchange rate. The expansionary fiscal, and contractionary monetary, policy can keep the price level constant and yet appreciate the exchange rate. Hence, the two instruments approach to nominal exchange rates is different from PPP.

While the two instrument approach is true in the short run, the NATREX model shows that the opposite is true in the long run. An expansionary fiscal policy leads to a rise in the foreign debt – a decline in net foreign assets – that depreciates the exchange rate along trajectory R(0)-R(1)-R(2) in figure 5. The contractionary monetary policy crowds out investment and leads to a decline in the growth of productivity. The latter affects the trajectory of the real exchange rate in the manner described in section 4.2, and leads to a depreciated real exchange rate – the reverse of the short run effect.

Misalignment $\Phi(t)$ can occur in several ways. The traditional approach concentrates upon the actual exchange rate $N(t)p(t)/p'(t)$, and attributes misalignment and crises to rises in relative prices $p(t)/p'(t)$ or to overvalued nominal exchange rate $N(t)$. The rises in relative wages and prices are generally linked to excessive money growth. The NATREX model does not deny the importance of the traditional approach. We supplement it by also considering variations in the equilibrium real exchange rate $R[Z(t)]$, the NATREX. Rises in relative time preference and/or declines in relative productivity depreciate the real equilibrium exchange rate. In this manner we explain the sources of misalignment, and the mechanism of adjustment. The hypothesis is that our measure of misalignment $\Phi(t) = N(t)p(t)/p'(t) - R[Z(t)]$ is stationary: it converges to a zero mean, but with a positive variance.

In section 6 we derived estimates of the NATREX, using several different methods of estimation. The three econometric methods yielded the same basic results and are consistent with the hypotheses in the model. The qualifications concerning the appropriate measurements of variables were stated in sections 5 and 6.

8. Application to the 1992-93 Crises in Italy/Germany, France/Germany

The model and econometric approach are quite general, and can be applied to all currency crises. We now show how the model and estimates can be applied to the 1992-93 crises in Europe. We pose and answer the following questions for Italy/Germany and

France/Germany.

1. Were the attacks due to fundamental disequilibria, defined as misalignment $\Phi(t)$, or to attacks against otherwise sustainable exchange rates?
2. Were the crises produced by appreciations of the real exchange rate relative to PPP?
3. Was the misalignment $\Phi(t)$ due to an appreciation of the of the actual exchange rate or to a depreciation of the NATREX?
4. Why did the NATREX depreciate relative to Germany?
5. Why did the crises occur in 1992 and not earlier?

8.1. Italy/Germany

There are several candidates for an early warning signal: the misalignment $\Phi_1(t)$ based upon the PPP concept using relative CPI, and $\Phi(t)$ the NATREX concept of “misalignment”. Our estimate of the NATREX labeled ITGRJ, based upon the estimate using the Johansen method in table 2 above, is compared in figure 9 with the actual value ITGRREU.

A necessary condition⁵³ for the PPP hypothesis to be valid is that ITGREX should be mean reverting. Its normalized value should be $I(0)$, with a zero mean⁵⁴. If the NATREX hypothesis is valid, then misalignment $ITGRMISJ = \Phi(t) = N(t)p(t)/p'(t) - R[Z(t)] = ITGRREU - ITGRJ$ should revert to a zero mean. That is, $\Phi(t)$ should be $I(0)$. Figure 10 graphs the normalized PPP misalignment $\Phi_1(t)$ labeled ITGREX and normalized NATREX misalignment $\Phi(t)$ labeled ITGRMISJ.

Figure 10 shows that the PPP measure has been appreciating from the mid-1970s to 1992. The PPP measure of the real exchange rate did not revert to a constant mean during the sample period⁵⁵. It has exceeded its mean since the early 1980s. Therefore it was not a good early warning signal. Table 4 confirms the impression derived from figure 10 that the PPP hypothesis is not valid: $ITGREX = N(t)p(t)/p'(t)$ is not $I(0)$.

Table 4 confirms the impression from figure 10 that the NATREX measure of misalignment $ITGRMISJ$ was stationary = $\Phi(t)$ is $I(0)$. The Italian/German real exchange rate reverted to a time varying mean $R[Z(t)]$.

⁵³ If the PPP concept results from a law of one price – no arbitrage – relation, then the stationarity of the REX concept is not sufficient for internal-external balance.

⁵⁴ The actual value would be $I(0)$ with a constant mean.

To be sure, in 1990 – 91, both measures showed large deviations from the means. But the PPP index was signaling a crisis almost all of the time since the 1980s. The NATREX measure was only signaling a crisis from 1990.

⁵⁵ The PPP hypothesis is that ITGREX reverts to a constant mean C , or that $\Phi_1(t) = (ITGREX - C)$ reverts to a constant mean of zero.

Table 4 Italy/Germany

Tests of the hypothesis that the PPP measure is mean reverting and that Misalignment $\Phi(t)$ is mean reverting. The test is the null that each is I(1).

ADF	ITGREX = $N(t)p(t)/p'(t)$	ITGRMISJ = $\Phi(t)$
UROOT(C,4)	-1.97	-3.43*
UROOT(C,8)	-1.85	-3.4*

5% critical value is -2.9; * reject hypothesis of I(1).

A more general test of the two hypotheses can be made. If the PPP hypothesis were valid, then the ITGREX measure “Granger Causes” changes in the nominal exchange rate $\Delta[N(t)] = \Delta(DMLIRA)$. Current and lagged values of the ITGREX⁵⁶ are the hypothesized predictors of changes in the nominal value of the Italian lira relative to the German Mark. If the NATREX hypothesis is valid, then misalignment $\Phi(t) = ITGRMISJ$ should “Granger Cause” changes in the nominal value of the Italian lira. In both cases we use lags of 4,6,8 quarters.

The results are in tables 5a and 5b. The null is that one variable does not Granger Cause the other variable. We present the F statistics and the asterisk denotes the rejection of the null hypothesis.

Table 5a

Tests of Granger Causality between the Change in the Nominal exchange rate, $\Delta(DM/Lira)$ and ITGRMISJ the NATREX measure of Misalignment

Lags	$\Delta(DMLIRA)$ does not cause	$\Phi(t) = ITGRMISJ$ does not cause
4	F = 2.4	F = 13.2*
6	F = 1.4	F = 9.4*
8	F = 2.0	F = 3.8*

$$ITGRMISJ = \Phi(t) = N(t)p(t)/p'(t) - R[Z(t)] = ITGRREU - ITGRJ$$

⁵⁶ The appropriate variable is ITGREX relative to the constant mean. Since the mean is hypothesized to be constant, the test in table 4a can ignore the constant mean.

Table 5b

Tests of Granger Causality between the Change in the Nominal exchange rate, $\Delta(\text{DM/Lira})$ and ITGREX the PPP measure of Misalignment

Lags	$\Delta(\text{DMLIRA})$ does not cause	$\Phi_1(t) = \text{ITGREX}$ does not cause
4	F = 0.58	F = 0.89
6	F = 0.48	F = 0.55
8	F = 1.4	F = 2.3*

$$\text{ITGREX} = N(t)p(t)/p'(t); \Phi_1(t) = \text{ITGREX} - \text{constant}$$

The **conclusions** from tables 4 – 5 are that: (a) the PPP concept of the real exchange rate is not mean reverting, whereas the NATREX measure of misalignment is mean reverting, and (b) changes in the nominal exchange rate are Granger caused by the NATREX measure of misalignment rather than by the PPP. (c) From 1987 to 1991, the PPP measure of the real exchange rate (ITGREX) rose and in 1991 it was about 1.5 standard deviations above the entire sample mean. Since this variable is not mean reverting, it provides an unreliable early warning signals: there are too many false signals. The actual real effective exchange rate ITGRREU relative to Germany rose from 1988 to 1991, but was at its mean at the later date. The NATREX hypothesis uses misalignment $\Phi(t) = (\text{ITGRREU} - \text{ITGRJ}) = \text{ITGRMISJ}$ as an early warning signal. Variable $\Phi(t)$ is mean reverting. Figure 10 shows that ITGRMISJ rose from 1987 to 1991 and was about 2.5 standard deviations above the entire sample mean. This is a serious early warning signal, based upon tables 4 and 5 above.

We now come to the reasons for crises 1992-93. The Lira was indeed overvalued relative to the DMark before the crisis because the NATREX fell drastically relative to the real exchange rate. The NATREX depreciated because the relative time preference – generated by the fiscal variable – rose and the relative productivity declined because of the policy mix

The misalignment ITGRMISJ rose because the NATREX was depreciating, rather than that the actual real exchange rate was appreciating. The NATREX of Italy relative to Germany has been depreciating, $\Delta R[Z(t)] = R'[Z(t)] \Delta Z < 0$, since 1987 for several reasons. See figure 7 for normalized variables $Z(t)$, relative time preference and relative productivity.

(1) Time preference had been rising since 1978. The relative time preference Italy/Germany denoted DISCITA rose from -2 standard deviations in 1977 to +2 standard deviations in 1990. (2) The fiscal variable Italy/Germany, denoted ITGRGOV, was the fuel for the rise in time preference. There was an upward trend since 1977, and in 1990 it was 2 standard deviations above the sample mean. (3) The ratio of Italian/German productivity PRODIT declined drastically from 1988 to 1991.

The traditional Mundell-Fleming view that the inflationary effects of an expansionary fiscal policy can be offset by a contractionary monetary policy led the Italian authorities to adopt a tight money policy to offset the expansionary fiscal policy. The NATREX model claims that this policy mix may appreciate the exchange rate in the short run, but the exact opposite occurs in the longer run: This policy mix leads to a longer run depreciation. As a result of the expansionary fiscal and contractionary monetary policy, a crowding out of investment occurred. The rate of capital formation/GDP fell from 1981 to 1993, which decreased the growth of productivity. Thereby, the Italian economy lost competitiveness. The relative productivity PRODIT declined from 1989 to 1992. In addition the rise in time preference led to a decreased current account. The current account deficits increased from 1987 to 1992. The rise in the foreign debt exerted further downward pressure on the lira. While the NATREX declined from 1987 to 1992, the actual real exchange rate - the ratio of domestic to foreign unit labor costs measured in a common currency - appreciated.

We have shown that the Misalignment $\Phi(t)$ based upon the NATREX was an early warning signal, and we have explained the causes of the misalignment. One must ask: why did the crisis occur in 1992 and not earlier? The resulting misalignment increased drastically from 1989 to 1992. When capital controls were abolished in 1991 the pressures for devaluation were no longer restrained.

8.2 France/Germany

Some have claimed that the German unification produced the overvaluations of the other European currencies⁵⁷ and engendered the speculative attacks. We do not find this to be the case. The experiences of France and Italy, relative to Germany, were quite different. The speculative attacks against the French franc in 1992 and 1993 were not

successful - the nominal value of the French franc relative to the Dmark, (DMFF) did not depreciate. It has been stable since 1987. The attack on the Lira was successful and it did depreciate after the attack.

The two measures of the real exchange rate of the French franc relative to the DMark graphed in figure 6 are quite different. The PPP measure $FRGREX = N(t)p(t)/p'(t)$ using the CPI weights (a rise is an appreciation of the French franc) is $I(0)$ stationary or mean reverting, as shown in table 6. The real effective exchange rate, based upon the ratio of the IMF measure of the French to the German real multilateral effective exchange rates $FRGRREU$, is not $I(0)$ stationary.

Figure 8 plots the real effective exchange rate $FRGRREU$ and two estimates of the NATREX: the Johansen estimate $FRGRJ$ and the OLS estimate $FRGROLS$ based upon table 2. Both estimates show that there is a downward trend in the equilibrium value: the NATREX. The NATREX of France relative to Germany had a downward trend because of the trends in the two fundamentals relative to Germany: time preference ($DISCFRA$) and productivity ($PRODFR$). See figure 6. The time preference variable tends to depreciate the franc. The productivity variable $PRODFR$ tends to appreciate the French franc.

From 1977 to 1988, the two fundamentals had opposite effects⁵⁸. The fiscal variable $FRGRGOV$ of government consumption/GDP in France relative to Germany had a strong upward trend, figure 6. Social time preference is correlated with the fiscal variable (see table 1). The relative rate of social time preference $DISCFRA$ rose from -2 standard deviations in 1978 to more than 1 standard deviation in 1990: a 3 standard deviation rise. From 1980 – 89, the relative productivity in France $PRODFR$ rose by 1 standard deviation and was a force for appreciation. The net effect was a downward non-monotonic trend in the NATREX for France from 1978 to 1990. From 1990, the time preference variable stabilized and declined towards its long- term mean, even though the fiscal variable continued to rise. On the other hand, the productivity variable deteriorated.

⁵⁷ Mundell (1995:469)

⁵⁸ Just as Crouhy-Veyrac and Saint Marc found (1995, ch. 4), these forces operated in opposite directions.

Which measure can provide a reliable early warning signal of a change in the nominal exchange rate? We have two estimates of “misalignment”, depending upon which measure of the real exchange rate we select. These measures are graphed in figure 11. The curve labeled FRGREX is the PPP measure of the real exchange rate based upon relative CPI. Table 6 shows that it is stationary, so that the normalized variable is a measure of “Misalignment”. Call the PPP measure of misalignment $\Phi_1(t)$. The curve labeled FRGRMISJ is the deviation of the real effective exchange rate from the NATREX, where the NATREX is the Johansen estimate in table 2. Misalignment $\Phi(t) = \text{FRGRMISJ} = \text{FRGRREU} - \text{FRGRJ}$ in figure 11 is stationary with a mean of zero. As shown in table 6, it is I(0) mean reverting.

Table 6 France/Germany

Test of the hypothesis that the PPP measure is mean reverting and that Misalignment $\Phi(t)$ is mean reverting. The test is that each is I(1)

Lag	$\Phi_1(t) = \text{FRGREX}$	$\Phi(t) = \text{FRGRMISJ}$
4	-3.37*	-3.5*
8	-3.8*	-3.0*

*Reject the null hypothesis of I(1)

The conclusion from table 6 and figure 11 is that both measures of misalignment are mean reverting. The PPP-CPI measure reverts to a constant mean. Similarly, the bilateral real effective exchange rate reverts to its NATREX.

The next question is which measure of “misalignment”, the PPP-CPI measure $\Phi_1(t)$ or the NATREX measure $\Phi(t)$, Granger causes changes in the DM/FF nominal exchange rate, where the appreciation or depreciation is denoted $\Delta(\text{DMFF})$. Tables 7a and 7b indicate that both measures Granger cause changes in the nominal exchange rate.

Figure 11 shows that neither one of the two measures indicates an overvaluation of the French franc relative to the German mark in the period 1990 to just prior to the attacks. The French/German results are quite different from the Italian/German situation described in figure 10.

Table 7a

Tests of Granger Causality between the Change in the Nominal exchange rate, $\Delta(\text{DMFF})$ and FRGREX, the PPP measure $\Phi_1(t)$ of Misalignment

Lags	$\Delta(\text{DMFF})$ does not cause	$\Phi_1(t) = \text{FRGREX}$ does not cause
4	F = 1.65	F = 4.68*
8	F = 0.65	F = 4.22*

*Reject the null hypothesis

Table 7b

Tests of Granger Causality between the Change in the Nominal exchange rate, $\Delta(\text{DMFF})$ and FRGRMISJ, the NATREX measure $\Phi(t)$ of Misalignment

Lags	$\Delta(\text{DMFF})$ does not cause	$\Phi(t) = \text{FRGRMISJ}$ does not cause
4	F = 1.3	F = 6.2*
8	F = 0.68	F = 2.7*

*Reject the null hypothesis

9. Conclusion

The problem is to evaluate the likelihood that a country will face a currency or balance of payments crisis over a given horizon. When is it rational for market participants to expect a depreciation of the currency? On the basis of considerable empirical studies we know that in both banking and currency crises, there is a multitude of weak and deteriorating economic fundamentals. Many empirical studies have identified potentially important variables determining real exchange rates in the medium to the longer run. They are: the ratio of social consumption/GDP, fiscal variables, terms of trade, net foreign assets and productivity. The best indicator of a crisis within the next 24 months is the real exchange rate relative to a trend. No economic content was given to the trend: it was just a statistical artifact.

Our theme is that there is an economic logic to medium and longer-term movements in exchange rates, within the context of a consistent dynamic stock-flow model. The equilibrium real exchange rate is a trajectory, not a point. We provide objective measures of the real fundamentals that determine the moving equilibrium real

exchange rate, and explain the dynamic economic mechanism whereby the actual exchange rate converges to this moving equilibrium exchange rate, called the NATREX. The fundamentals are primarily social consumption/GDP, which is generally driven by fiscal policy, and the productivity of the economy. Trends in social consumption/GDP, and in fiscal policy, reflected political regime changes in France, Germany and Italy.

The “two instruments” approach claims that an expansionary fiscal policy and a contractionary monetary policy will appreciate the exchange rate and can preserve price stability. Our analysis explains that this may be true in the shorter run, but that the opposite is true in the longer run. The foreign debt will rise and the growth of productivity will decline. Each leads to a depreciation of the real value of the currency.

Deviations from our moving equilibrium are defined as misalignment. Misalignment Granger causes changes in nominal exchange rates. Misalignment can occur either because the actual real exchange rate has appreciated, due to rises in relative nominal magnitudes, or because the moving equilibrium has declined. In either case, the source of the misalignment is identified. Misalignment is stationary with a mean of zero. In a regime of adjustable pegs, speculative attacks and crises are some of the ways in which the convergence occurs. Our measure of misalignment was a reliable “early warning signal” of problems. On an intuitive level, one should pay close attention to three variables: the fiscal policy, the growth of productivity and the relative nominal costs. The first two affect the NATREX and can produce misalignment even though relative prices have not diverged. The NATREX is a dynamic generalization of the PPP and macroeconomic balance approaches to take into account how variations in fiscal policy and productivity affect the equilibrium real exchange rate.

There are several problems, which are not entirely resolved. First: what is the appropriate measure of the real exchange rate? Is it the ratio of CPI indexes adjusted for the nominal exchange rate? If the “law of one price” were relatively valid, then a PPP measure of the exchange rate deviation from a stationary mean would lead to arbitrage in the goods market. It is a necessary condition for equilibrium, but it is not sufficient. It would not reflect whether there was internal and external equilibrium in the economy, and says nothing about the effects of fiscal policy, the terms of trade and productivity upon internal-external equilibrium. The measure of the real bilateral exchange rate as the ratio of the real effective multilateral exchange rates in the two countries is generally

based upon unit labor costs in manufacturing. Thereby, it omits more general concepts of costs. Before the 1992-93 crises we found that: in Italy/Germany both measures of misalignment were present. However, the PPP-CPI measure was sending signals of depreciation steadily for over a decade. It was not a reliable signal, as was the NATREX measure of misalignment.

In France/Germany before the crisis, neither measure indicated misalignment. We think that the stationarity of the PPP-CPI measure of the real exchange rate between France and Germany mainly reflects a “law of one price” in the goods market. This measure is silent concerning the effects of fiscal policy and productivity upon internal-external equilibrium.

Second is the question of stability, outside the sample period, of the coefficients of the fundamentals acting upon the equilibrium real exchange rate. It seems that they are not sufficiently stable as to give us much confidence in precise numbers. The estimates cannot justify exchange rate management or a regime of stabilized nominal exchange rates. The best that one can do is to consider the trend movements in our fundamentals of time preference – fiscal policy and productivity in the two countries. If both point to depreciation or appreciation in the NATREX relative to the actual real exchange rate, then this is a serious early warning signal. If they point in opposite directions, then the expected change in the equilibrium should be based upon confidence limits of the parameters.

Appendix A1 France-Germany

Cointegration analysis is performed among: bilateral real effective exchange rate France-Germany, a rise is an appreciation of the French franc, ratio of French/German social discount (time preference) rates, and French/German productivity.

1975:4 - 1996:3				
eigenvalue	log likelihood		rank	
	1319.8		0	
0.327217	1336.44		1	
0.103182	1341.02		2	
0.0278199	1342.2		3	
Ho: rank p	LR	95%	trace	95%
p = 0	33.29	22.0	44.81	34.9
p ≤ 1	9.14	15.7	11.52	20.0
p ≤ 2	2.37	9.2	2.37	9.2

We find one cointegrating equation. We then examine the VEC equations to test for weak exogeneity of the Z 's, as specified above for Italy/Germany. Wald test $a(i) = 0$, $i > 1$, for dependent variable other than the real exchange rate (variable 1) yields a Chi-square = 2.1021, probability of an equal or larger statistic 0.3496

Appendix A2: Italy/Germany

Cointegration analysis is performed among: bilateral real effective exchange rate Italy-Germany, a rise is an appreciation of the Lira, ratio of Italian/German social discount (time preference) rates, and Italian/German productivity.

1975:4 - 1996:3				
eigenvalue	log likelihood		rank	
	1253.38		0	
0.291914	1267.87		1	
0.125431	1273.50		2	
0.03484	1274.99		3	
Ho: rank p	LR	95%	trace	95%
p = 0	29.00	22	43.23	34.9
p ≤ 1	11.26	15.7	14.24	20.0
p ≤ 2	2.97	9.2	2.97	9.2

Hence we find one cointegrating equation. The VEC equations are $\Delta R = \alpha [R(t-1) - BZ(t-1)] + B'\Delta Z(t-1)$, and $\Delta X_i(t) = a(i) [R(t-1) - BZ(t-1)] + B'\Delta Z(t-1)$, where $X_1 = R$ and the $i > 1$ refer to the Z 's. We test if only $\alpha = a(1)$ is significant. This is the weak exogeneity test. The Wald test restriction $a(i) = 0$, for all $i > 1$, that is for variables other than the real exchange rate, yields a Chi-square (2) = 0.5575, probability of an equal or larger value 0.7567. Hence we accept weak exogeneity. Only α is significant in the error correction.

Appendix A3 OLS Estimates France/Germany

LS // Dependent Variable is FRGRREU; Sample: 1975:4 1996:3

Included observations: 84

Variable	Coefficient	Std. Error	T-Statistic	Prob.
DISCFRA	-5.302412	0.419700	-12.63381	0.0000
PRODFR	3.499072	0.254322	13.75841	0.0000
C	4.022544	0.423750	9.492720	0.0000

R-squared	0.774685	Mean dependent var	1.082154
Adjusted R-squared	0.769122	S.D. dependent var	0.162583
S.E. of regression	0.078121	Akaike info criterion	-5.063934
Sum squared resid	0.494333	Schwartz criterion	-4.977119
Log likelihood	96.49439	F-statistic	139.2487
Durbin-Watson stat	0.218031	Prob(F-statistic)	0.000000

In the OLS regression (8a), the ADF test of the residuals indicates that they are stationary for various lags.

APPENDIX A4 OLS Estimates Italy/Germany

LS // Dependent Variable is ITGRREU

Date: 08/23/99 Time: 11:14; Sample: 1975:4 1996:3;

Included observations: 84

Variable	Coefficient	Std. Error	T-Statistic	Prob.
DISCITA	-1.141550	0.484143	-2.357881	0.0208
PRODIT	2.184997	0.245918	8.885080	0.0000
C	-0.226978	0.612792	-0.370400	0.7121

R-squared	0.547301	Mean dependent var	0.970840
Adjusted R-squared	0.536123	S.D. dependent var	0.166119
S.E. of regression	0.113141	Akaike info criterion	-4.323173
Sum squared resid	1.036879	Schwartz criterion	-4.236358
Log likelihood	65.38244	F-statistic	48.96337

Appendix A5 OLS Estimates France/Germany

LS // Dependent Variable is FRGRREU

Sample: 1975:4 1993:3

Included observations: 72 after adjusting endpoints

Variable	Coefficient	Std. Error	T-Statistic	Prob.
FRGRGOV	-1.277042	0.138698	-9.207351	0.0000
PRODFR	1.065809	0.385624	2.763858	0.0073
C	1.594035	0.310749	5.129650	0.0000
R-squared	0.573596	Mean dependent var		1.157882
Adjusted R-squared	0.561236			
S.D. dependent var		0.141908; S.E. of regression		0.093999
Akaike info criterion	-4.688171; Sum squared resid			0.609670
Schwartz criterion	-4.593310			
Log likelihood	69.61058	F-statistic	46.40914	
Durbin-Watson stat	0.131139			
Prob(F-statistic)		0.000000		

Appendix A6 OLS Estimates Italy/Germany

LS // Dependent Variable is ITGRREU

Date: 08/18/99; Sample: 1975:4 1993:3; Included observations: 72 after adjusting endpoints

Variable	Coefficient	Std. Error	T-Statistic	Prob.
ITGRGOV	-0.677086	0.155599	-4.351470	0.0000
PRODIT	-0.355429	0.339629	-1.046521	0.2990
C	1.994424	0.465433	4.285091	0.0001
R-squared	0.255600	Mean dependent var	1.041321	
Adjusted R-squared	0.234023; S.D. dependent var			0.112246
S.E. of regression	0.098238; Akaike info criterion			-4.599951
Sum squared resid	0.665899; Schwartz criterion			-4.505089
Log likelihood	66.43465	F-statistic	11.84605	
Durbin-Watson stat	0.368392	Prob(F-statistic)		0.000038

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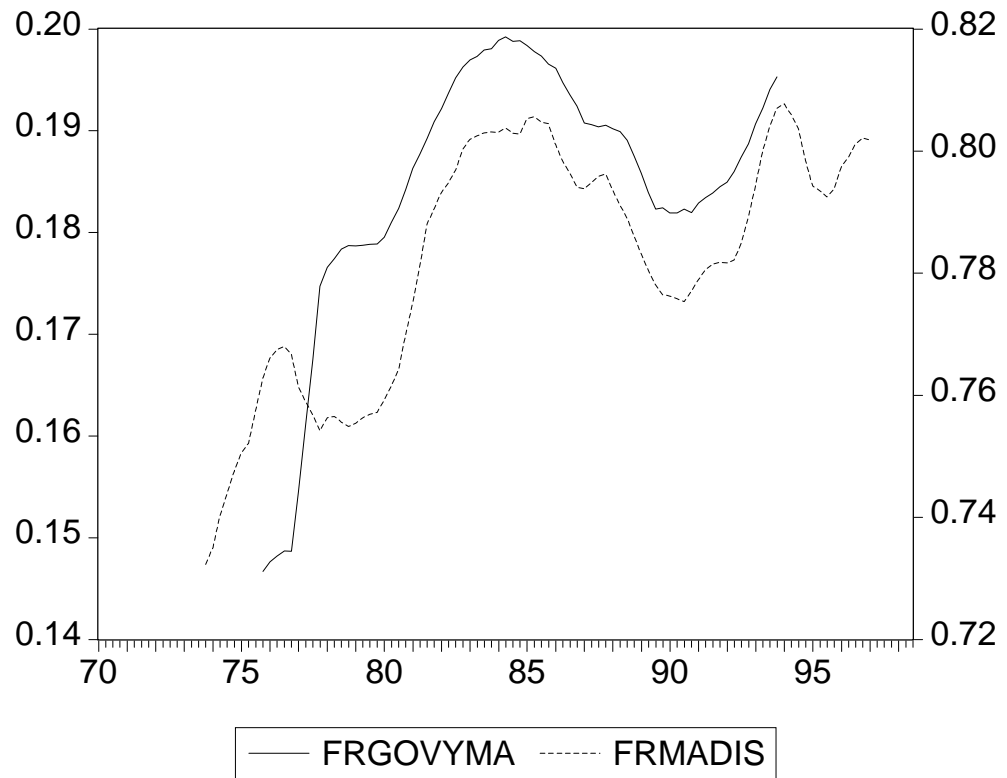
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FRGOVYMA = French social, private + public, consumption/GDP
FRMADIS = French government consumption/GDP

4 quarter moving average

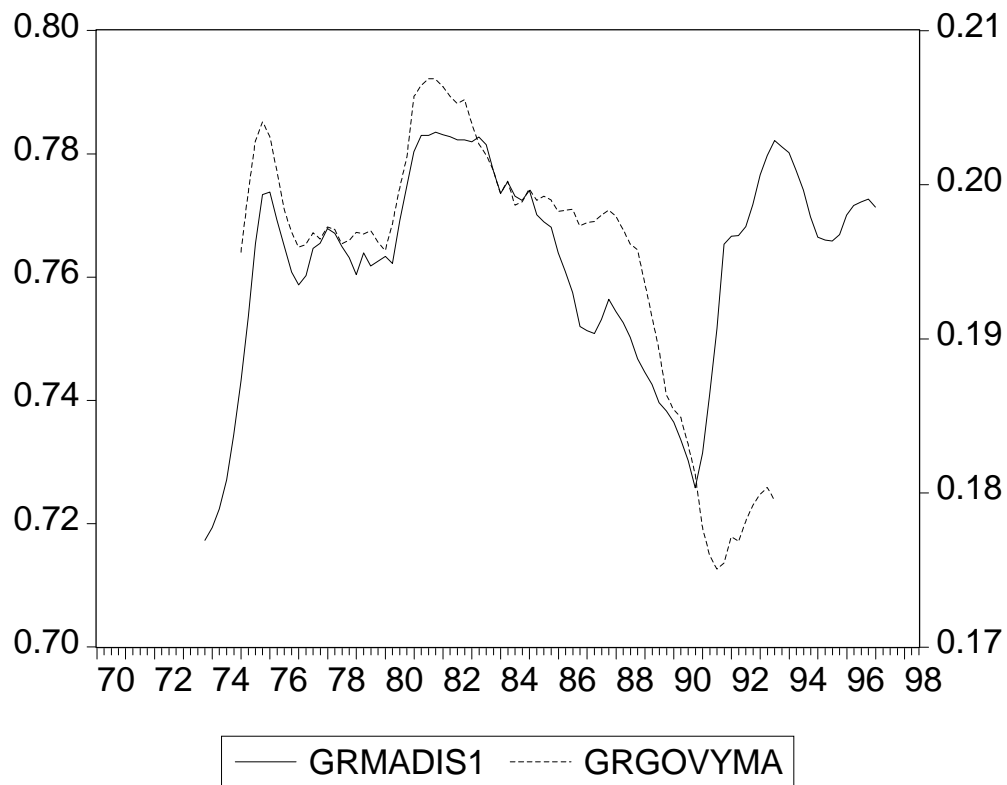
Figure 1



GRMADIS1 German social, private + public, consumption/GNP
GRGOVYMA German government consumption/GNP

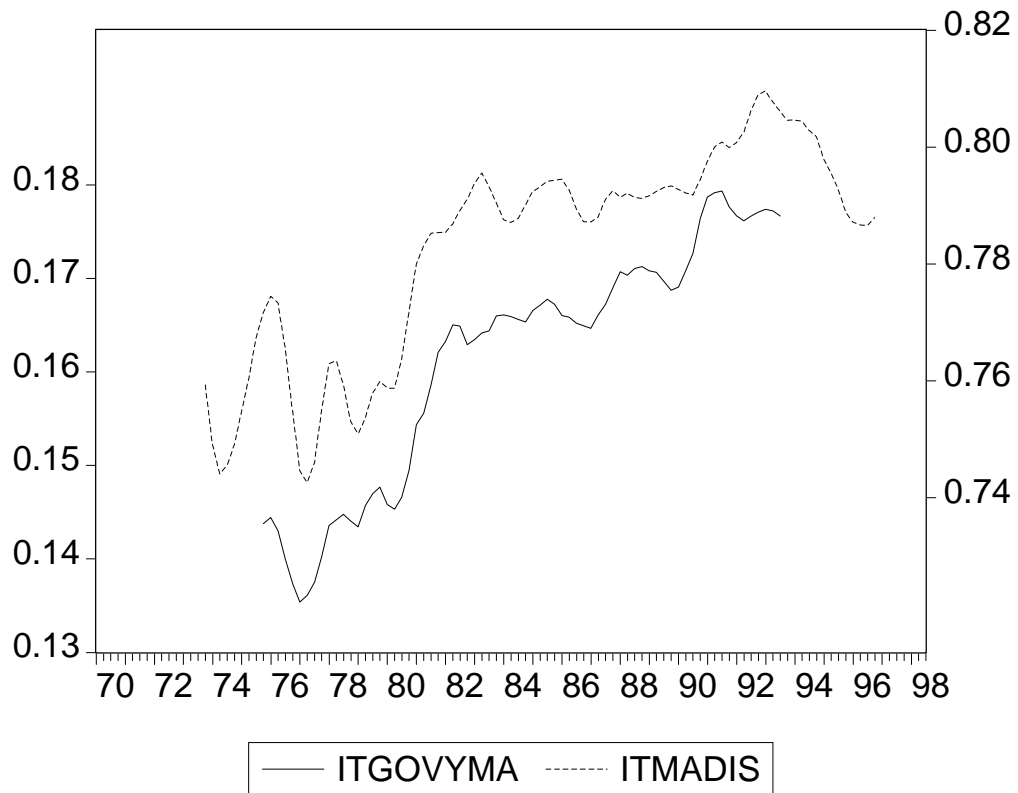
4 quarter moving average

Figure 2



ITGOVYMA = Italian government consumption/GDP
ITMADIS = Italian social, private + public, consumption/GDP
4 quarter moving average

Figure 3



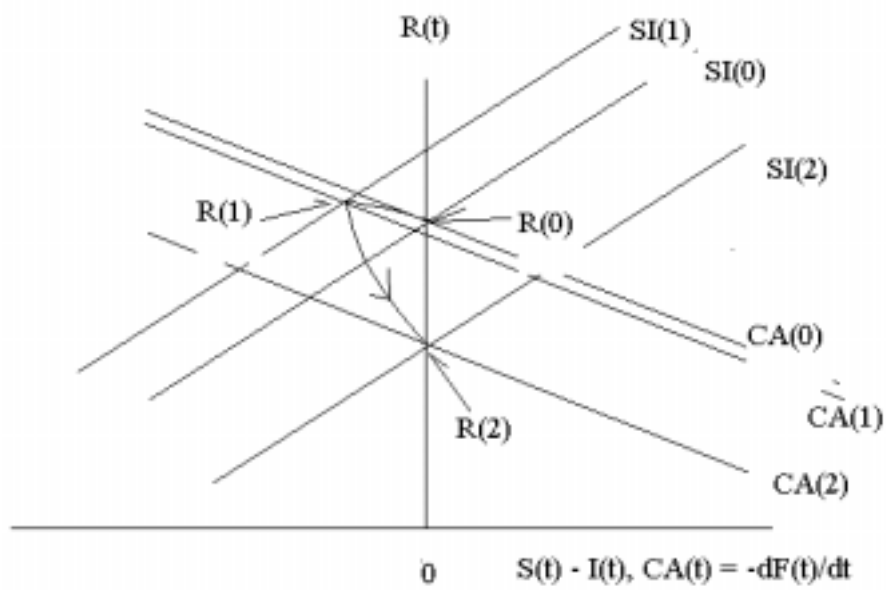


Figure 4 Rise in time preference. Initial $R(0)$ - $R(1)$ Final depreciation $R(1)$ - $R(2) < R(0)$

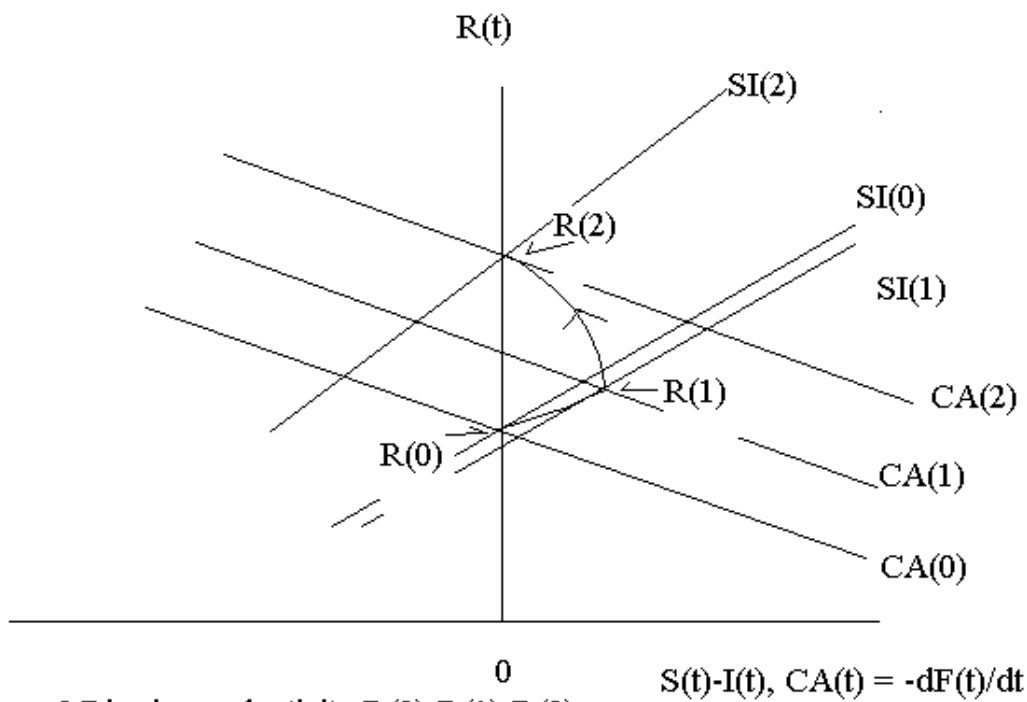
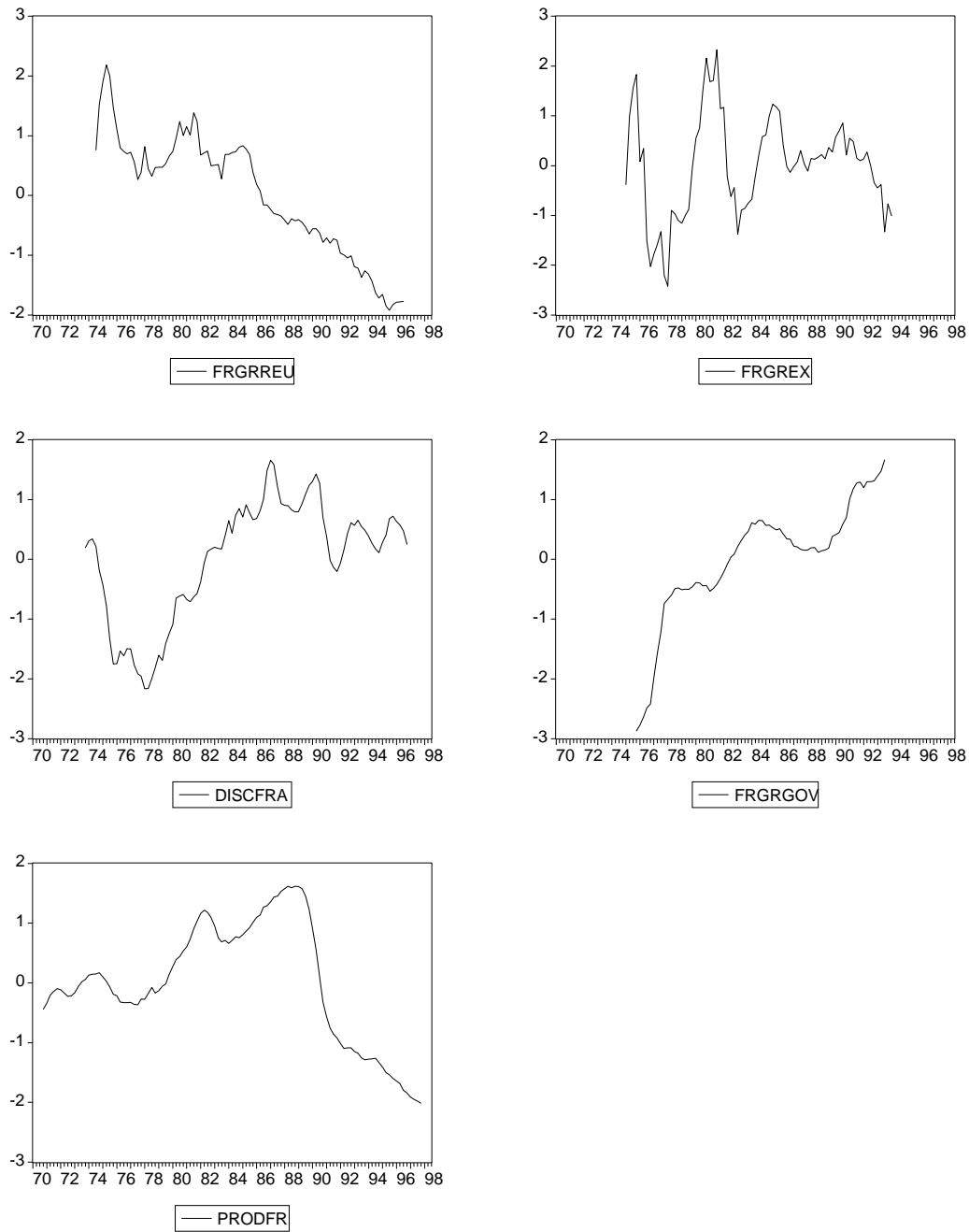


Figure 5 Rise in productivity $R(0)$ - $R(1)$ - $R(2)$

FRGRREU real effective exchange rate France/Germany, unit labor costs, IFS
 FRGREX bilateral French German real exchange rate CPI weights
 DISCFRA social consumption/GDP ratios France/Germany
 FRGRGOV government consumption/GDP France/Germany
 PRODFRA productivity France/Germany

normalized variables, rise in real exchange rate is appreciation for France

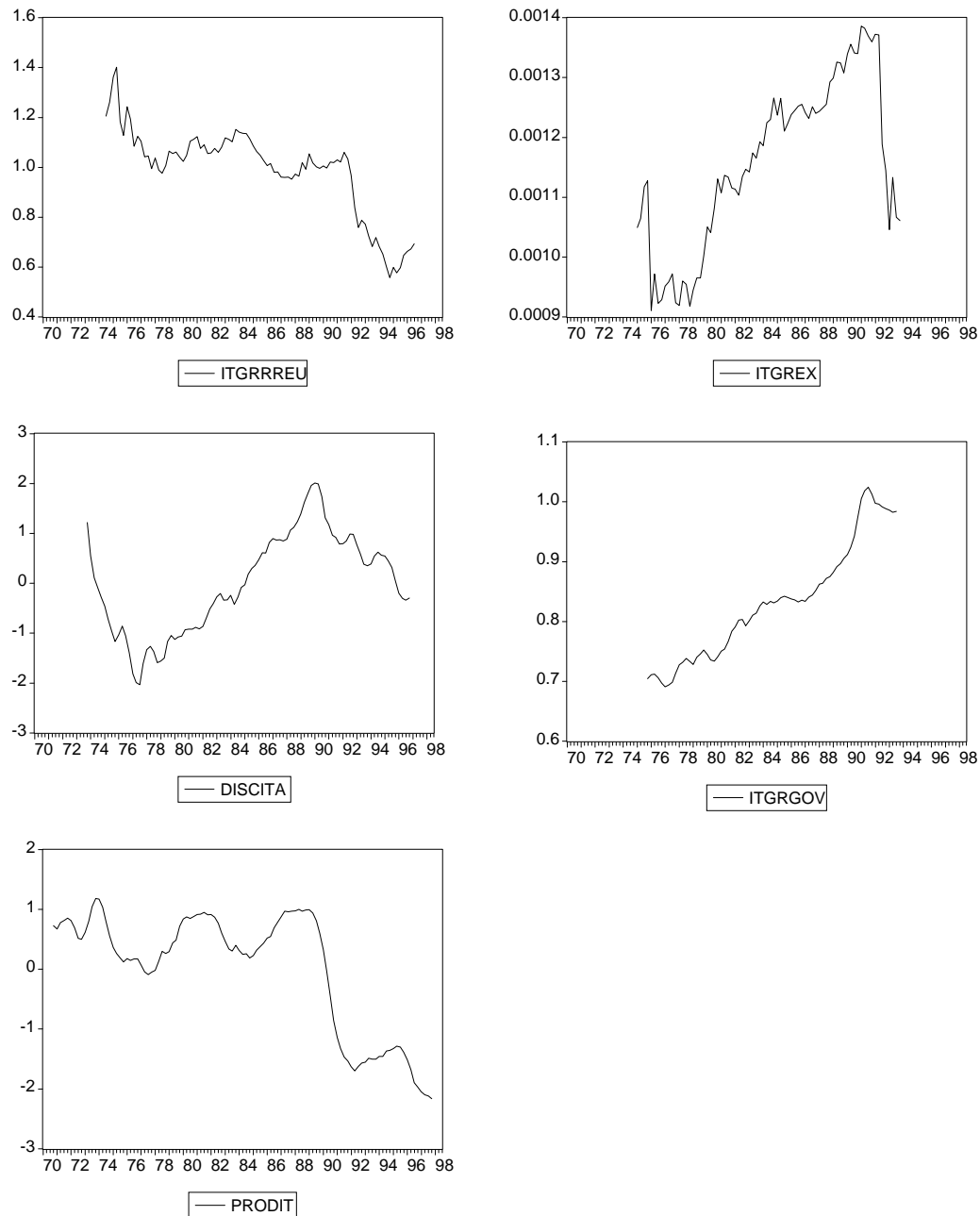
Figure 6



ITGRREU = Italian/German real effective exchange rate, unit labor costs, IFS
 ITGREX = Italian/German real exchange rate CPI weights
 DISCITA = Italian/German social consumption/GDP
 PRODIT = Italian/German productivity
 ITGRGOV = Italian/German government consumption/GDP

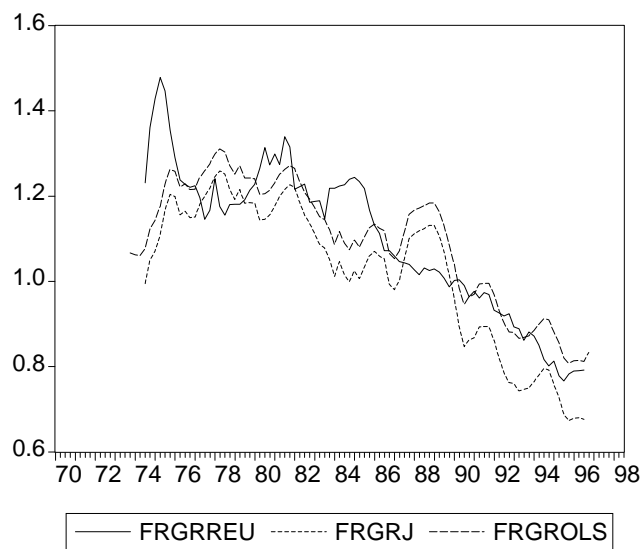
normalized variables

Figure 7



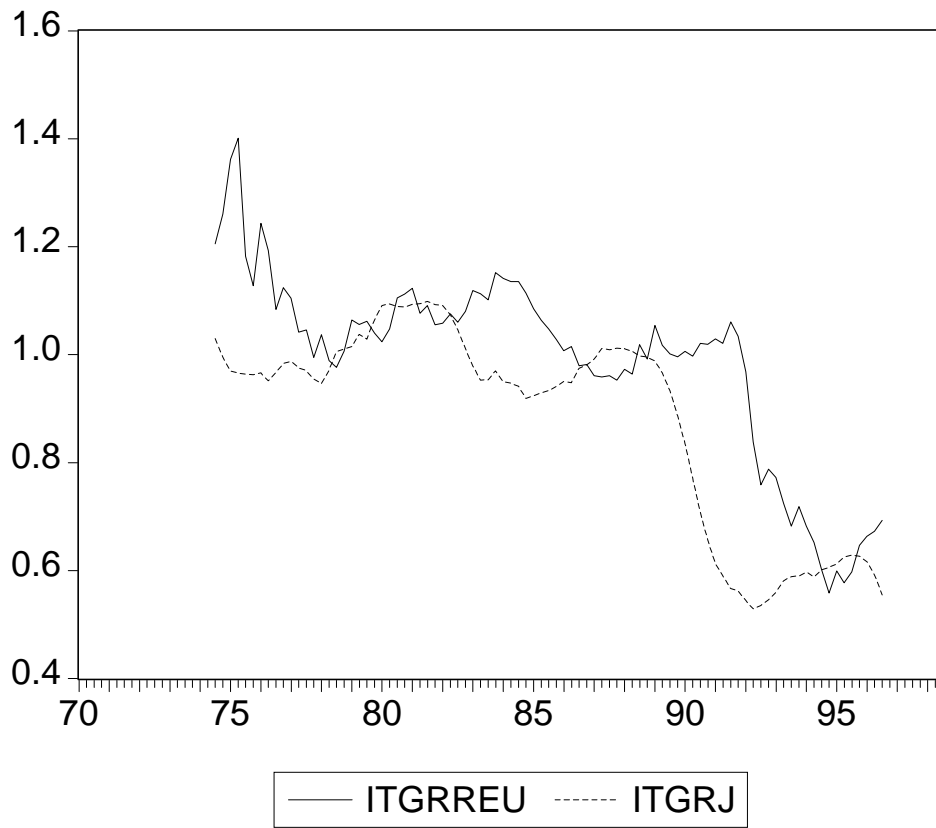
FRGRREU = French/German real effective exchange rate, rise is an appreciation of French franc
FRGRJ = NATREX Johansen estimate
FRGROLS = NATREX OLS estimate

Figure 8



ITGRREU = Italian/German real effective exchange rate IFS
ITGRJ = NATREX Johansen estimate

Figure 9



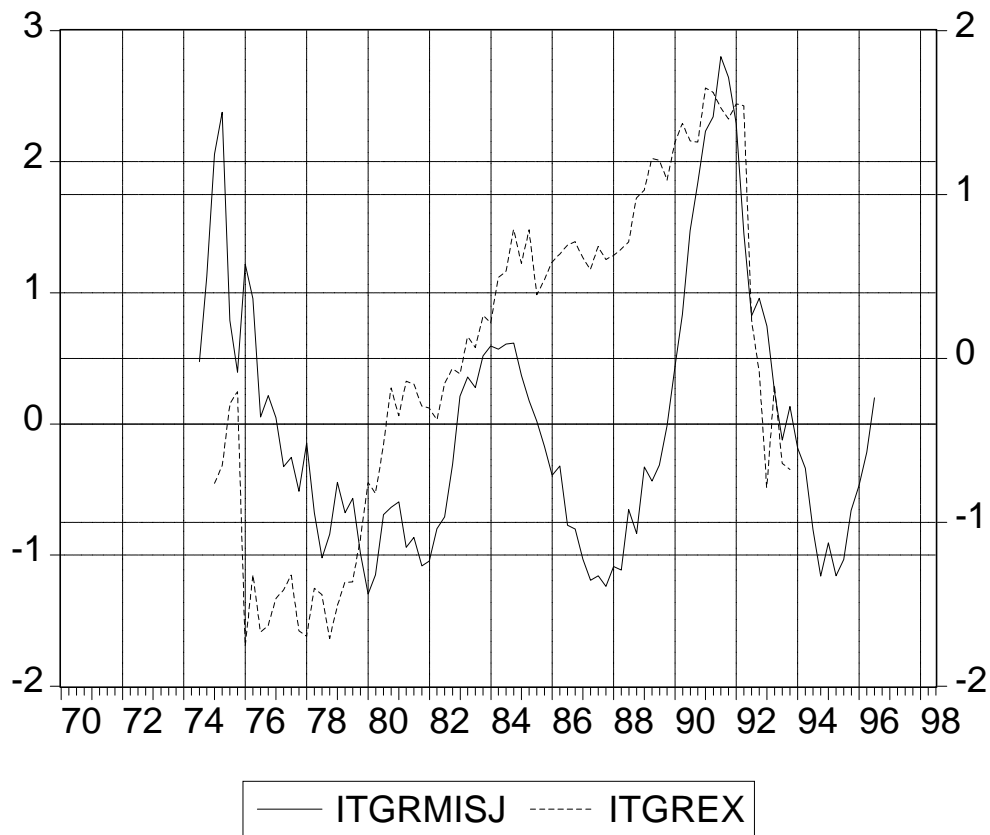
Italy-Germany

$ITGRMISJ = ITGREU - NATREX = \text{Italian Misalignment}$

$ITGREX = \text{Italian Misalignment using CPI-PPP}$

normalized variables

Figure 10



France-Germany

FRGREX= Normalized misalignment using PPP-CPI

FRGRMISJ = FRGREU - FRGRJ = Normalized misalignment using NATREX

Figure 11

