CESifo *Working Paper Series*

CONVERTIBLE SECURITIES AND VENTURE CAPITAL FINANCE

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Working Paper No. 217

December 1999

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* I am grateful to Patrick D'Souza, Georg Gebhardt, John Moore, Georg Nöldeke, Hyun Shin, and seminar participants at LSE and in Munich and Ottobeuren for helpful comments and suggestions. Financial support by Deutsche Forschungsgemeinschaft through grant SCHM1196/2-1 is gratefully acknowledged.

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Abstract

This paper offers a new explanation for the prevalent use of convertible securities in venture capital finance. Convertible securities can be used to endogenously allocate cash flow rights as a function of the realized quality of the project. This property can be used to mitigate the double moral hazard problem between the entrepreneur and the venture capitalist. It is shown that an optimally designed convertible security outperforms any mixture of debt and equity and that it can induce both parties to invest efficiently. The result is robust to renegotiation and to changes in the timing of investments and information flows.

Keywords: Convertible securities, venture capital, corporate finance, double moral hazard, incomplete contracts

JEL Classification: D23, G24, G32

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1 Introduction

Venture capital accounts for only a tiny fraction of total corporate investment in the U.S. but it had a dramatic impact on economic growth and the creation of new jobs since the 1970s. Microsoft, Intel, Apple, Federal Express, Cisco Systems, Genentech and many other icons of high technology were all venture-capital backed in their early stages. However, the financing of young entrepreneurial firms is prone to severe incentive problems that make these investments very difficult. In order to deal with these problems venture capital firms have developed sophisticated contracting practices, some of which are unique to the venture capital industry. In particular, the purchase of convertible securities by the venture capitalist is the predominant form of investment.[1] This is surprising because convertible securities are very rarely used by banks or passive outside equity holders who finance the bulk of small (but more established and less risky) companies.

In this paper we offer an explanation for the prevalent use of convertible securities in venture capital finance. The starting point of the analysis is the observation that the ultimate success of high-potential, entrepreneurial firms does not only depend on the quality of the original idea and the abilities of and the effort provided by the entrepreneur, but also on the involvement of the venture capitalist. It is a well documented fact that venture capitalists do not only provide the necessary financial means to develop the project, but that they are also actively engaged in the management of the firm. Venture capitalists are typically well connected in the specific industry, they help to recruit key personnel, they negotiate with suppliers and customers, they advise the entrepreneur on strategic decisions, they play a major role in structuring mergers, acquisitions and initial public offerings, and sometimes they are even engaged in the day to day operations of the firm.[2]

Our model focuses on the incentive properties of convertible securities. We argue that a convertible security is a powerful instrument to mitigate the double moral hazard problem between the entrepreneur and the venture capitalist. It is shown that convertible securities strictly outperform any mixture of debt and equity. Furthermore, we show that under some

[1] See, e.g., Sahlman (1990) and Gompers (1997).

[2] See e.g. Sahlman (1990, p. 508). Gorman and Sahlman (1989) report that on average each venture capitalist is responsible for ten firms, that he visits each firms nineteen times per year and that he spends one hundred hours annually at each firm.

mild conditions on the parameters of the problem an optimally designed convertible security can induce both, the entrepreneur and the venture capitalist, to invest efficiently into the project.

To get some intuition for these result suppose that the ultimate success of the project depends on three factors: the original quality of the project and/or the abilities of the entrepreneur (which we call the realization of the "state of the world"), the effort spent by the entrepreneur, and the effort and financial investment provided by the venture capitalist. At date 0, when the two parties negotiate the terms of the contract, the state of the world is unknown to both parties. They learn the realization of the state of the world only after the initial investment has been sunk. Then the entrepreneur has to spend effort in order to develop the project. Finally the venture capitalist has to decide whether to get further engaged by investing effort and additional money.

Suppose that there are three states of the world. In the bad state, the project is sure to fail and not worth any further investments. In the medium state the project can recover the initial investment if the entrepreneur works sufficiently hard, but it is not profitable enough to warrant the involvement of the venture capitalist. In the good state, the project is highly profitable, but the full involvement of the entrepreneur and the venture capitalist is necessary to develop its full potential. If the initial contract gives the venture capitalist a mixture of standard debt and equity, then it is impossible to induce both parties to invest efficiently in all states of the world. In order to induce the venture capitalist to get engaged in the good state, he has to get some equity in the firm. However, this affects the incentives of the entrepreneur who is then going to spend too little effort.

A properly designed convertible debt contract solves this problem. In the good state the venture capitalist will convert his debt into equity and invest into the project if and only if the company is sufficiently valuable. However, the value of the firm depends on the effort that has been put in by the entrepreneur. By choosing the conversion rate appropriately, the venture capitalist is induced to convert and to invest if and only if the entrepreneur has chosen at least the efficient effort level. This in turn induces the entrepreneur to choose just the right level of effort even though she then loses some fraction of the equity of her firm. In the medium state of the world it does not pay for the venture capitalist to convert his debt and so he will not invest (which is efficient), while the entrepreneur has all the equity and is residual claimant on the margin. Hence, the entrepreneur is induced to choose the efficient effort level. Finally, in the bad state the venture capitalist will not convert and not invest either, the entrepreneur cannot repay her debt, and the firm is liquidated. We also show that this efficiency result is robust to the possibility of renegotiation and to different timings of the information flows.

There are a few other papers that try to explain the prevalent use of convertible securities in venture capital finance. Green (1984) presents a model in which a firm has two investment projects one of which is more risky than the other. If the firm is financed with debt, the entrepreneur has an incentive to engage in too much risk taking. A convertible debt contract limits this incentive because the warrant portion of the debt becomes more valuable as risk increases. However, Green does not allow for the use of equity financing which would easily solve this problem in his model.

Another branch of the literature focuses on conflicts of interests between entrepreneurs, venture capitalists and outside financiers that stem from non-transferable private benefits of control and affect critical decisions such as the liquidation of the venture (Marx, 1998) or the sale to another company or in an IPO (Berglöf, 1994). In these models convertible securities are used to allocate control rights to the right persons in different states of the world. However, Gompers (1997) and Hellmann (1998) argue that the allocation of cash flow rights should be separated from the allocation of control rights by the use of covenants. Gompers and Lerner (1996) document that covenants are indeed frequently used to give the venture capitalist the right to control the board of directors, to approve major expenditures, to liquidate the firm and even to replace the entrepreneur by an outside manager. Typically, the venture capitalist is given these contractual rights independently of the financial structure of the company.

Cornelli and Yosha (1997) focus on the entrepreneurial incentives to engage in "window dressing" in order to induce the venture capitalist to finance the second stage of the project. With a convertible debt contract this signal manipulation is less profitable, because the venture capitalist will convert his debt into equity if the firm looks too good which reduces the entrepreneur's profit.

Repullo and Suarez (1998) consider a double moral hazard problem between the entrepreneur and a wealthy advisor (the venture capitalist). They show that all the returns form the project should be used to improve the effort incentives of the two parties. Hence, outside financiers should not be used. Furthermore, they characterize the second best optimal sharing rule. They show that the venture capitalist should get no compensation for his initial investment in the lower tail and high compensation in the upper tail of the distribution of returns. They demonstrate that this sharing rule can be approximated by the use of warrants.

Our paper is also related to the literature on incomplete contracts and the optimal allocation of ownership rights. Grossman and Hart (1986) argue, that the allocation of ownership rights matters if only incomplete contracts can be written. In their model ownership to an asset is the residual right to control this asset in all contingencies that have not been dealt with in an explicit contract before. Thus, the allocation of ownership rights affects the allocation of bargaining power which in turn affects the investment incentives of the involved parties. In Nöldeke and Schmidt (1998) we show that it may be efficient to use a "conditional ownership structure" in the Grossman-Hart model which can be implemented by using "options on ownership rights" that play a similar role to the convertible securities considered here.[3]

The rest of the paper is organized as follows. In Section 2 the basic model is described. Section 3 establishes our main results. It shows that a mixture of standard debt and/or equity contracts cannot induce the entrepreneur and the venture capitalist to invest efficiently. Furthermore, we demonstrate that the first best can be implemented by using a convertible debt contract. Section 4 extends our main result in several directions. In particular, we show that the result is robust to the possibility of renegotiation, to private benefits of the entrepreneur from running his company, to multi-dimensional investment decisions, and to a different timing of the investments and the information flows. Section 5 concludes.

[3]The approach taken by Nöldeke and Schmidt differs from the one considered here in several important respects. In particular, there we do not allow for "large" uncertainty that is characteristic for venture capital finance. Furthermore, in the (1998) paper the option to own is an "all or nothing" decision while in this paper the venture capitalist gets the option to convert his debt claim into some fraction $\alpha < 1$ of the equity of the firm. There is no initial investment that has to be financed in Nöldeke and Schmidt. Finally, they focus on the allocation of control rights, while only cash flow rights matter in this paper.

2 The Model

Consider an entrepreneur (E) who has the idea for a potentially profitable project but lacks the funds to finance it. The project requires an initial investment $I > 0$ that has to come from a venture capitalist (VC). Thus, at date 0, E and VC have to negotiate a contract that governs the financing of the project. We assume that there is a perfectly competitive market for venture capital which drives down VC's expected profit from the investment to 0.

The profit that can be generated by the two parties depends on three factors: the quality of the project and/or the abilities of the entrepreneur ("the state of the world"), the effort that is being put in by the entrepreneur, and the effort and further financial investment of the venture capitalist. We assume that the state of the world is unknown to both parties at date 0 and can be observed only after the initial investment I has been sunk.[4] At date 1, E observes the realization of the state of the world and has to make a relationship specific investment, $a \in \mathbb{R}_0^+$, in order to further develop the project. This investment cannot be contracted upon and is best thought of as the effort E puts into the firm. For example, E has to build up her company, she has to engage in additional R&D, she has to market her product, etc. There is no problem to allow for multi-dimensional investments which is briefly discussed in Section 4.3.

The venture capitalist also observes the realization of the state of the world and the effort provided by E before he has to decide at date 2 whether to commit additional effort and capital to the project. We assume that his investment, b, is a binary choice: either VC engages himself at that stage, i.e. $b = \bar{b} > 0$, or he does not invest anymore, i.e. $b = 0$.[5] All investments are measured by their costs.

Finally, at date 3, the surplus $v(a, b, \theta)$ that can be generated by the project is realized

[4]The state of the world could also be realized at some other point in time, e.g. after E chooses her investment. This will be discussed in Section 4.4. However, throughout the paper we maintain the assumption that there is no asymmetry of information between E and VC. Typically, the VC is an expert in the industry, he may be much more experienced than the entrepreneur and he gets closely involved into the project. Therefore, it does not seem to be unreasanoble to assume that both players are symmetrically informed. See Gompers (1993) for a model that uses the capital structure of the firm as a screening device employed by VC in order to get more information on the type of E.

[5]The assumption that VC's involvement for the project is a binary choice is not implausible, in particular when a financial investment or the advice on a key decision of the firm (such as an IPO) is involved. In Section 4.3 we briefly discuss the case where $b \in \mathbb{R}_0^+$.

and split between the two parties according to the initial contract that has been signed at date 0. The time structure of the model is summarized in Figure 1.

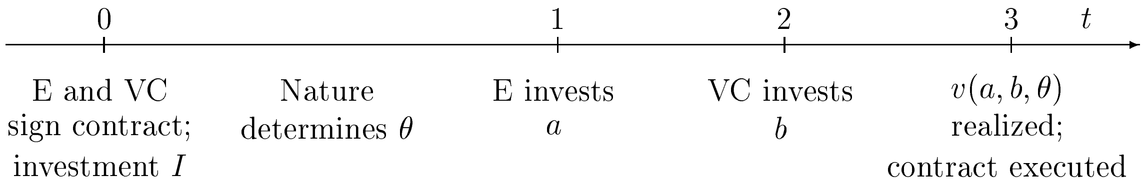

FIGURE 1: Time structure of the model

This model captures some of the specific features of the financing of start-up companies. For a young start-up company entrepreneurial effort is clearly very important. However, for its ultimate success it is also often crucial that the venture capitalist gets actively engaged in the project. For example, the entrepreneur may be a brilliant scientist or engineer with an ingenious idea for a new product, but she may lack the skills to build up a rapidly growing company or to efficiently organize production, controlling, and marketing. On the other hand, experienced venture capitalists tend to have deep industry specific knowledge and they are well connected within the industry. Thus, a VC can give important advise on strategic decisions, he can help to find the right managers to assist the entrepreneur in running the company, and he may even get involved in the day to day operations of the firm. Furthermore, he is often crucial when it comes to organizing an initial public offering. By the nature of VC's services it is natural to assume that his involvement is particularly important in the expansion phase of a successful company, i.e. after E invested a lot of effort already. Because VC's investment is not only a financial commitment but involves the effort he spends for the project, we assume that b cannot be contracted upon either.

Second, these projects are typically very risky. Industry experts estimate that at most 10% of all projects that get venture capital finance are "high flyers", i.e. projects that are going to be very profitable. Venture capitalists are eager to support these firms in every possible respect. Between 20 and 40% of all projects fail completely and are not able to repay the initial investment. They have to be liquidated by VC as soon as possible. The rest are called "living dead". They are moderately successful in that they can repay the initial investment, but they are not worth an additional involvement of the venture capitalist.[6] This risk is reflected by the realization of the state of the world, θ. In order to capture the three potential outcomes of the project we distinguish three different states of the world, $\theta \in \{\theta_h, \theta_m, \theta_l\}$, $\theta_h > \theta_m > \theta_l$, where θ_h represents the possibility that the project is highly profitable, θ_m means that the project is mediocre, while θ_l says that the project fails. The ex ante probabilities of these three states are given exogenously by p, q, and $1-p-q$, respectively, with $0 < p, q, 1-p-q < 1$.[7]

The following assumption characterizes the gross profit function $v(a, b, \theta)$ and the three states of the world:

Assumption 1 *The surplus function $v(a, b, \theta)$ is twice continuously differentiable, strictly increasing and strictly concave in a and strictly increasing in b and θ for all $a \in I\!R_0^+$, $b \in \{0, \overline{b}\}$ and $\theta \in \{\theta_m, \theta_h\}$. Furthermore,*

(a) $v(a, b, \theta) = \underline{v} = 0$ *if either $a = b = 0$ or if $\theta = \theta_l$.*

(b) $v(a, \overline{b}, \theta_m) - \overline{b} < v(a, 0, \theta_m)$ *for all $a \in I\!R_0^+$.*

(c) *There exists an $\underline{a} > 0$ such that $v(a, \overline{b}, \theta_h) - \overline{b} \geq v(a, 0, \theta_h)$ for all $a \geq \underline{a}$.*

(d) $\frac{\partial v(a,b',\theta')}{\partial a} - \frac{\partial v(a,b,\theta)}{\partial a} \geq 0$ *for all $a \in I\!R_0^+$ and $b' \geq b$, $\theta' \geq \theta$, $b', b \in \{0, \overline{b}\}$, $\theta', \theta \in \{\theta_m, \theta_h\}$, with strict inequality if $b' > b$ or $\theta' > \theta$.*

Assumption 1(a) says that the return of the project is a fixed liquidation value $\underline{v}$ if either none of the two parties invests or if the bad state of the world materializes. Without loss of generality the liquidation value can be normalized to zero.[8] Clearly, in the bad state VC should not invest. By Assumption 1(b) it is efficient that VC does not invest ($b = 0$) in the medium state of the world either. However, he should get engaged ($b = \overline{b}$) in the good state if and only if E invested at least some minimum amount $\underline{a}$ (Assumption 1(c)). Assumption 1(d) says that the investments are complements at the margin, i.e. not only the total but also

[6]See, e.g., Sahlman (1990, p. 484).)

[7]Hence, we assume that the probability of the state of the world cannot be affected by the efforts put in by E and VC. These efforts do affect the profit of the firm in each state, but it is not possible to turn a bad idea into a "high flyer" no matter how hard you work on it.

[8]If $\underline{v} > 0$ the return $v(a, b, \theta)$ should be interpreted as the net return in excess of $\underline{v}$, so $v(a, b, \theta) = \tilde{v}(a, b, \theta) - \underline{v}$, where $\tilde{v}(\cdot)$ is the gross return. See also Section 4.2.

the marginal surplus with respect to a is strictly increasing in b. Furthermore, the marginal return to a is higher in the good than in the medium state.

As a reference point we have to define the first best efficient investment levels of both parties. Let

$$S(a(\theta), b(\theta), \theta) \;=\; v(a(\theta), b(\theta), \theta) - b(\theta) - a(\theta) \tag{1}$$

be the social surplus in state θ if E chooses $a(\theta)$ and VC chooses $b(\theta)$. Suppose that VC chooses the efficient investment level $b^*(\theta)$. Then the first best efficient investment choice of E in state θ is given by

$$a^*(\theta) \;=\; \arg\max_a v(a, b^*(\theta), \theta) - b^*(\theta) - a \tag{2}$$

Clearly, $a^*(\theta_l) = 0$. To make things interesting we assume that there is an interior solution for $a^*(\theta)$ in states θ_m and θ_h. The concavity of $v(\cdot)$ with respect to a implies that $a^*(\theta)$, $\theta \in \{\theta_m, \theta_h\}$, is uniquely characterized by the first order condition

$$S_a(a^*(\theta), b^*(\theta), \theta) \;=\; v_a(a^*(\theta), b^*(\theta), \theta) - 1 \;=\; 0 \tag{3}$$

where subscript a denotes the partial derivative with respect to a, and that $0 < a^*(\theta_m) < a^*(\theta_h)$.

Furthermore, suppose that $a^*(\theta_h) > \underline{a}$. Hence, by Assumption 1 the first best efficient investment choice of VC is $b^*(\theta) = 0$ if $\theta \in \{\theta_l, \theta_m\}$ and $b^*(\theta) = \bar{b}$ if $\theta = \theta_h$.

Finally, we assume that

$$S(a^*(\theta_m), b^*(\theta_m), \theta_m) \;>\; \frac{I}{p+q}\;, \tag{4}$$

i.e. the maximum social surplus in the medium (and thus in the good state, too) is sufficient to cover the (risk adjusted) initial investment cost.

At date 0 the two parties have to agree on a contract that governs their relationship. We argued already that the relationship-specific investments a and b of the two parties cannot be contracted upon.[9] Furthermore, it is impossible to contract on θ, i.e. on the realization of the state of the world. However, the parties can write standard debt and/or equity contracts.

[9] All our results go through if we assume that only some part of VC's investment, e.g. the effort that is being supplied by him, cannot be contracted upon.

For example, the venture capitalist could hold fraction α of the equity of the firm plus a debt claim D. Suppose that VC financed the initial investment I and that the two parties invested a and b at date 1 and 2, respectively. Then, with a debt-equity contract (D, α), the final payoffs of the two parties are given by

$$U^E = \begin{cases} -a & \text{if } v(a,b,\theta) < D \\ (1-\alpha)[v(a,b,\theta) - D] - a & \text{if } v(a,b,\theta) \geq D \end{cases} \tag{5}$$

$$U^{VC} = \begin{cases} v(a,b,\theta) - I - b & \text{if } v(a,b,\theta) < D \\ \alpha[v(a,b,\theta) - D] + D - I - b & \text{if } v(a,b,\theta) \geq D \end{cases} \tag{6}$$

Note that the entrepreneur has no funds on her own, so she is protected by limited liability and cannot be forced to pay more to VC than is available in the firm. Both parties are assumed to be risk neutral.

Furthermore, the parties can issue convertible securities. For example, a convertible debt contract (C, α) says that VC has the option to choose at some date, that is specified in the contract, whether to receive the debt payment C or to convert his debt into fraction α of the equity of the firm.[10] We are going to show that in many interesting cases convertible securities can be used to implement the first best. This is why we do not consider more complicated contractual arrangements here.

3 Convertible Securities vs. Debt-Equity Contracts

In this section we assume for simplicity that the initial contract that has been signed at date 0 cannot be renegotiated if at some point in the relationship there is scope for an efficiency improvement. Renegotiation may be important off the equilibrium path and this may effect investment incentives. We will analyze the renegotiation case in Section 4.1.

Let us start with standard debt-equity contracts. The following proposition shows that these contracts cannot be used to implement efficient investment choices.

Proposition 1 *There exists no debt-equity contract* (D, α) *that induces both parties to invest efficiently.*

[10] In our simple set-up there is no difference between convertible debt and convertible preferred stock.

Proof: Consider any debt-equity contract (D, α) with $\alpha > 0$. E's payoff as a function of a in the good state θ_h is given by

$$U^E(a \mid \theta_h) = (1-\alpha)\max\{0, v(a, b(\theta_h), \theta_h) - D\} - a \tag{7}$$

Taking the derivative with respect to a we get

$$\begin{aligned} \frac{\partial U^E}{\partial a} &\leq (1-\alpha)v_a(a, b(\theta_h), \theta_h) - 1 \\ &< v_a(a, b^*(\theta_h), \theta_h) - 1 \\ &= S_a(a, b^*(\theta_h), \theta_h) \end{aligned} \tag{8}$$

Hence, E's marginal return to her investment is strictly smaller than the social return to her investment. Thus, Assumption 1 implies that she will underinvest. It is easy to see that she will not invest efficiently in state θ_m either.

Consider now a pure debt contract $(D, 0)$. Suppose that E invests efficiently and chooses $a^*(\theta_h)$ in state θ_h. If $D < v(a^*(\theta_h), 0, \theta_h)$ VC will choose $b = 0$ because his return is D which is independent of his investment. VC may only invest if $v(a^*(\theta_h), 0, \theta_h) < D$ and if $\min\{v(a^*(\theta_h), \bar{b}, \theta_h), D\} - \bar{b} > v(a^*(\theta_h), 0, \theta_h)$. In this case VC prefers to spend $\bar{b}$ in order to get a higher debt payment rather than to receive $v(a^*(\theta_h), 0, \theta_h)$. However, in this case E will not invest efficiently in the medium state. To see this note that $D > v(a^*(\theta_h), 0, \theta_h)$ implies $D > v(a^*(\theta_h), 0, \theta_m) > v(a^*(\theta_m), 0, \theta_m)$. Hence, E's marginal return to her investment in the medium state of the world is 0 which induces her to underinvest. *Q.E.D.*

The problem with debt-equity contracts is that they cannot be designed so as to give optimal investment incentives to both parties in all states of the world. In order to induce E to choose a^*, she has to be full residual claimant on profits, i.e. α has to be equal to 0. But if the initial investment is financed with debt only, then either VC is not going to choose $b = \bar{b}$ in the good state of the world or E has no incentive to invest in the medium state. Hence, for any standard debt-equity contract at least one party will not invest efficiently.

Consider now the use of a convertible security, e.g. a convertible debt contract (C, α). Convertible debt implies a contingent allocation of cash flow rights. VC gets the option to decide whether to be repaid C or to convert this debt into fraction α of the equity of the firm. The following proposition shows that a suitably chosen convertible debt contract does implement first best investment decisions of both parties if the investment cost $\bar{b}$ for the venture capitalist is not too large relative to the benefit of this investment.

Proposition 2 *Suppose that*

$$\frac{I}{p+q}\left[\frac{v(a^*(\theta_h),\bar{b},\theta_h) - v(a^*(\theta_h),0,\theta_h)}{v(a^*(\theta_h),0,\theta_h)}\right] \geq \bar{b} \ . \tag{9}$$

Then there exist a convertible debt contract (C, α) which gives VC the option to choose at date 2.5 whether to be repaid C or to get fraction α of the equity of the company that implements first best investment decisions. This convertible debt contract is given by

$$C = \frac{I}{p+q} \tag{10}$$

and

$$\alpha = \frac{\frac{I}{p+q} + \bar{b}}{v(a^*(\theta_h),\bar{b},\theta_h)} < 1. \tag{11}$$

Proof: Note first that the inequality part of (11) is equivalent to $\frac{I}{p+q} < v(a^*(\theta_h),\bar{b},\theta_h) - b$. This inequality is satisfied because $v(a^*(\theta_h),\bar{b},\theta_h) - \bar{b} > v(a^*(\theta_h),0,\theta_h) > v(a^*(\theta_h),0,\theta_m) > v(a^*(\theta_m),0,\theta_m) > \frac{I}{p+q}$ where we used condition (4). Hence, $\alpha < 1$.

Consider state θ_l first. In this state E knows that there is no return to her investment and that she will not be able to repay the debt. Hence, she will choose $a^*(\theta_l) = 0$ which is efficient. Clearly VC will not invest and not exercise his option. E cannot repay her debt, so the firm is being liquidated and VC receives the liquidation value $\underline{v} = 0$.

Consider now state θ_m and suppose that a has been chosen such that $v(a,0,\theta_m) < C$. If VC did not invest at date 2, then he should not exercise his conversion option, the firm is liquidated and he receives $v(a,0,\theta_m) - I$. If VC did invest at date 2, then the maximum he can get out of the firm is $v(a,\bar{b},\theta_m) - \bar{b} - I$, but, by Assumption 1(b), this is smaller than $v(a,0,\theta_m) - I$ for all a. Hence, in this case it is optimal for VC not to invest and not to convert.

So suppose that we are in state θ_m and that a has been chosen such that $v(a,0,\theta_m) \geq C$ and $a \leq a^*(\theta_h)$. In this case VC can guarantee himself a payoff of $C - I$ by not investing and not converting his debt. If he does not invest and exercises his option, he gets

$$\begin{aligned}\alpha v(a,0,\theta_m) - I &= \left(\frac{I}{p+q} + \overline{b}\right)\frac{v(a,0,\theta_m)}{v(a^*(\theta_h),\overline{b},\theta_h)} - I \\ &\leq \left(C + C\frac{v(a^*(\theta_h),\overline{b},\theta_h) - v(a^*(\theta_h),0,\theta_h)}{v(a^*(\theta_h),0,\theta_h)}\right)\frac{v(a,0,\theta_m)}{v(a^*(\theta_h),\overline{b},\theta_h)} - I \\ &= C\frac{v(a,0,\theta_m)}{v(a^*(\theta_h),0,\theta_h)} - I \;<\; C - I \qquad (12)\end{aligned}$$

for all $a < a^*(\theta_h)$, where we used (9),(10) and (11). Hence, this cannot be better. Nor can it better to invest and not to exercise the conversion option, which yields $C - \overline{b} - I$. Finally, VC cannot improve his payoff by investing and exercising his conversion option, because

$$\alpha v(a,\overline{b},\theta_m) - \overline{b} - I < \alpha[v(a,\overline{b},\theta_m) - \overline{b}] - I < \alpha v(a,0,\theta_m) - I < C - I \qquad (13)$$

Here the last inequality follows from the argument given in (12). Hence, we have shown that for all $a \leq a^*(\theta_h)$ VC will not invest and not exercise his conversion option in state θ_m. Furthermore, if $a > a^*(\theta_h)$, then VC's payoff must be at least $C - I$ which he can guarantee by not investing and not converting his debt.

Consider now the optimal choice of a for E ist state θ_m. If E chooses a such that $v(a,0,\theta_m) < C$, then her payoff is $-a$ and she should go for $a = 0$. If she chooses a such that $v(a,0,\theta_m) \geq C$ and $a \leq a^*(\theta_h)$, then she can repay her debt and, since VC is not going to exercise his conversion option, she is residual claimant on profits at the margin. Hence, in this range the optimal choice of a is $a^*(\theta_m)$ which maximizes total surplus in state θ_m and gives a strictly positive payoff to E by (4). Finally, it cannot be optimal to choose $a > a^*(\theta_h)$. This reduces social surplus (by the definition of $a^*(\theta_m)$), while VC still gets at least $C - I$. Hence, we have shown that in state θ_m E's payoff is maximized by investing the efficient amount $a^*(\theta_m)$, and VC does not invest and he does not exercise his conversion right in this state in equilibrium.

Finally, consider state θ_h. We want to show that it is optimal for VC to invest and to exercise his option if and only if $a \geq a^*(\theta_h)$. Suppose that E invested at least the efficient amount, i.e. $a \geq a^*(\theta_h)$. Note first that VC can guarantee himself a payoff of $C - I \;=\; \frac{I}{p+q} - I$ by not investing and not converting his debt. Clearly, it cannot be optimal to invest and not to exercise the option because this would yield $C - \overline{b} - I < C - I$. On the other hand, if VC does invest and does exercise his option, then his payoff is given by

$$U^{VC}(a) \;=\; \alpha v(a,\bar b,\theta_h) - \bar b - I \;=\; \frac{\frac{I}{p+q}+\bar b}{v(a^*(\theta_h),\bar b,\theta_h)} v(a,\bar b,\theta_h) - \bar b - I \;. \tag{14}$$

Note that $U^{VC}(a)$ is monotonically increasing with a. VC's payoff if E invested $a^*(\theta_h)$ is

$$U^{VC}(a^*(\theta_h)) \;=\; \frac{\frac{I}{p+q}+\bar b}{v(a^*(\theta_h),\bar b,\theta_h)} v(a^*(\theta_h),\bar b,\theta_h) - \bar b - I \;=\; \frac{I}{p+q} - I \;=\; C - I \;. \tag{15}$$

Hence, VC prefers to invest and to convert his debt rather than not to invest and not to convert if E invested $a \geq a^*(\theta_h)$. On the other hand, if $a < a^*(\theta_h)$, $U^{VC}(a < a^*(\theta_h)) < C - I$, so VC will invest and exercise his option only if $a \geq a^*(\theta_h)$.

To complete this step of the argument we have to show that, given $a \geq a^*(\theta_h)$, VC prefers to invest and to convert his debt rather than not to invest and to convert his debt. Hence, we have to show that

$$\alpha v(a,0,\theta_h) - I \;\leq\; \alpha v(a,\bar b,\theta_h) - \bar b - I \tag{16}$$

for all $a \geq a^*(\theta_h)$. Substituting (11) for α this is equivalent to

$$\frac{\frac{I}{p+q}+\bar b}{v(a^*(\theta_h),\bar b,\theta_h)} \left[v(a,\bar b,\theta_h) - v(a,0,\theta_h)\right] \;\geq\; \bar b \tag{17}$$

Note that by Assumption 1(d) the left hand side of (17) is monotonically increasing with a. Hence, if (17) holds for $a = a^*(\theta_h)$, it also holds for all $a > a^*(\theta_h)$. Substituting $a = a^*(\theta_h)$ and rearranging yields (9). Hence, we have shown that it is optimal for VC to invest and to convert his debt in state θ_h if and only if $a \geq a^*(\theta_h)$.

In the next step we have to show that it is indeed optimal for E to invest $a^*(\theta_h)$. If she chooses $a = a^*(\theta_h)$, VC will invest $\bar b$ and exercise his conversion option, so E's payoff is

$$U^E(a^*(\theta_h),\theta_h) \;=\; (1-\alpha)v(a^*(\theta_h),\bar b,\theta_h) - a^*(\theta_h) \tag{18}$$

Recall that $\alpha v(a^*(\theta_h),\bar b,\theta_h) - \bar b = C = \frac{I}{p+q}$. Substituting this in (18) we get

$$U^E(a^*(\theta_h),\theta_h) \;=\; v(a^*(\theta_h),\bar b,\theta_h) - C - \bar b - a^*(\theta_h) \;. \tag{19}$$

Hence, if E chooses $a = a^*(\theta_h)$ she gets the entire expected social surplus at $a = a^*(\theta_h)$ minus the convertible debt C. Investing more than $a^*(\theta_h)$ reduces the social surplus by the definition of the first best and it increases VC's payoff. Hence, because E's and VC's payoffs must always add up to the social surplus, E's payoff must fall. Hence, E will never invest more than $a^*(\theta_h)$.

If E chooses $a < a^*(\theta_h)$ VC will not exercise his conversion option and not invest. Consider a smooth reduction of a starting from $a = a^*(\theta_h)$. There is first a range where E is still able to repay her debt in state θ_h. In this range VC's payoff does not change when a is reduced, so $U^E = S - U^{VC}$ must go down. Finally there is a range where E cannot repay her debt, so her payoff is $-a \leq 0$. Again, she can get a strictly positive payoff by choosing $a = a^*(\theta_h)$, so E has no incentive to choose $a < a^*(\theta_h)$. *Q.E.D.*

The intuition behind this result is as follows. The convertible debt contract endogenously determines the allocation of cash flow rights as a function of the realization of the state of the world and the investment of the entrepreneur.

If the project fails, VC will not invest (which is efficient) and not exercise his conversion option but insist on being paid back his credit. Because E cannot repay, the firm goes bankrupt and is going to be liquidated, so VC gets all of the cash flow.

Consider the medium state of the world. The convertible security has been designed such that in this state VC prefers to get back his credit rather than to convert it into equity. Because he has no equity stake, he has no incentive to get further involved in the firm which is again the efficient thing to do. On the other hand, E becomes full residual claimant on the margin in this state, where only her investment is important. Hence, E invests efficiently.

So far the endogenous allocation of cash flow rights could have been achieved by a debt-equity contract as well. However, a new feature arises in the good state of the world. In this state it is important that VC invests and chooses $b = \bar{b}$. In order to induce this investment, VC has to get some equity stake in the company. However, he will get this equity stake only if he chooses to exercise his conversion option. Note that the value of his conversion option depends on E's investment. The more E invested, the higher is the value of the firm to VC. The convertible security has been designed such that it is worth VC's while to exercise his conversion option and to invest $\bar{b}$ if and only if E invested at least $a^*(\theta_h)$. This property of the convertible security gives rise to a discontinuous jump in E's payoff function. If she invests less than $a^*(\theta_h)$, then it is not worth VC's while to exercise his option and he will not invest.

However, without VC's investment the firm is worth very little to E, so it does not pay for her to reduce a below $a^*(\theta_h)$. If she invests more than $a^*(\theta_h)$, VC will exercise his conversion option and get fraction α of the marginal returns of E's investment. Hence, E has no incentive to invest too much, either. Thus, E is induced to choose exactly the first best investment level $a^*(\theta_h)$. E's payoff as a function of a in state θ_h is depicted in Figure 2.[11]

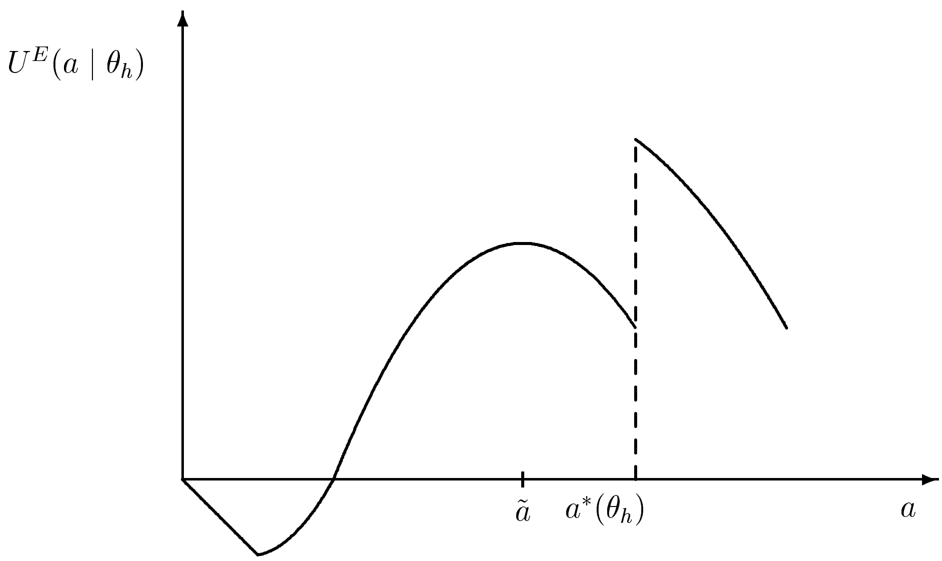

FIGURE 2: E's payoff function with a convertible debt contract in state θ_h

Proposition 2 says that a properly designed convertible debt contract implements the first best if condition (9) holds. It is easy to show that this condition is equivalent to

$$C \;\geq\; \alpha v(a^*(\theta_h), 0, \theta_h) \tag{20}$$

Suppose that (20) does not hold. Then VC prefers to convert his debt and <u>not</u> to invest in the good state even if E has chosen the efficient effort level $a^*(\theta_h)$. Hence, in this case it would not be possible to induce VC to invest efficiently in the good state.

[11]Note that $U^E(a \mid \theta_h)$ must decrease with slope -1 for small levels of a. If a is sufficiently small, E is not able to repay her debt, so her payoff is -a. Her payoff function starts to increase only if she is able to repay C. However, without VC's investment, E's payoff is maximized at $\tilde{a} < a^*(\theta_h)$, where $\tilde{a} = \arg\max_a\{v(a, 0, \theta_h) - a - C\}$. If $a \geq a^*(\theta_h)$, VC will choose $b = \overline{b}$ which boosts E's payoff and gives rise to the discontinuity at $a^*(\theta_h)$. At that point her payoff is $v(a^*(\theta_h), \overline{b}, \theta_h) - \overline{b} - a^*(\theta_h) - C > v(\tilde{a}, 0, \theta_h) - \tilde{a} - C$. For $a > a^*(\theta_h)$ her payoff must fall again.

Condition (9) requires that the value added by VC's investment has to be sufficiently large. This is consistent with the empirical observation that convertible securities are prevalent in venture capital finance, where the effort of the venture capitalist is crucial, while it plays only a minor role in the financing that is provided by banks or passive outside equity holders whose involvement in the firms they finance is less important.

4 Extensions and Robustness

4.1 Renegotiation

We ignored the possibility of renegotiation so far. Let us now assume that whenever there is scope for an efficiency improvement the two parties will renegotiate the initial contract. Recall that E is induced to invest efficiently in state θ_h by the threat that VC is not going to invest if $a < a^*(\theta_h)$. However, if $\underline{a} < a < a^*(\theta_h)$ it is inefficient that VC chooses $b = 0$. Hence, in this case both parties will renegotiate the initial contract and increase the conversion rate so as to make it worth VC's while to exercise the option and to invest efficiently. If E anticipates this, she may have an incentive to invest too little.

To analyze this problem more formally, let us assume that the two parties split the surplus from renegotiation (in excess of the threatpoint utilities given by the original contract) in proportion $(\lambda, 1-\lambda)$, where $\lambda \in (0,1)$ is the fraction that goes to E. Furthermore, define a' by $v(a', 0, \theta_h) = \frac{I}{p+q}$, i.e., a' is the critical investment level below which E is not able to repay $C = \frac{I}{p+q}$ in the good state if VC does not invest. The following proposition shows that the convertible debt contract of Proposition 2 implements the efficient investment choices with renegotiation if one additional condition is met.

Proposition 3 *Under the conditions of Proposition 2 the convertible debt contract (C, α) given by (10) and (11) implements first best investment decisions with renegotiation if either $\underline{a} \geq a'$ or if*

$$\lambda \left[v(a, \overline{b}, \theta_h) - \overline{b} - v(a, 0, \theta_h) \right] \leq a \quad \forall a \in [\underline{a}, a'] \, . \tag{21}$$

Proof: See Appendix.

To see the intuition for this proposition we have to distinguish "small" and "large" deviations from $a^*(\theta_h)$. A small deviation means that given $a < a^*(\theta_h)$ E is still able to repay her debt. Such a deviation cannot be profitable for E. It reduces the social surplus, while VC's payoff does not fall below C. Hence, E's payoff must go down. A "large deviation" means that E is no longer able to repay her debt in the good state.[12] If there are investment levels such that $\underline{a} < a < a'$, i.e., if renegotiation is an issue after such a "large" deviation, then E may have an incentive to deviate: she benefits from the renegotiation in the good state while most of the cost of her deviation are borne by VC who is not being repaid his debt. But, Proposition 3 shows that if E's bargaining power λ is not too large, and/or if the benefit of renegotiation after investment $a \in [\underline{a}, a']$ is not too big, then such a deviation cannot be profitable for E.

There is another potential problem with renegotiation. The convertible debt contract of Propositions 2 and 3 gives VC the right to exercise his option at date 2.5, i.e. after both parties made their investment decisions. Suppose that the exercise date of the conversion right is set before date 2, say at date 1.5, i.e. after VC observed E's investment and the realization of the state of the world, but before VC has made his own investment. Consider the case where we are in the good state of the world and E invested $a^*(\theta_h)$. In this case VC has an incentive not to exercise his option but to claim the debt payment C. To see this note that if he does not own fraction α of the firm, then he has no incentive to invest. But without VC's investment the firm is worth very little to E. Hence, E wants to renegotiate the ownership structure and to sell some fraction of her firm to VC in order to make it worth VC's while to invest. However, if VC gets fraction $1 - \lambda > 0$ of the surplus from renegotiation, then E's payoff from investing $a^*(\theta_h)$ is reduced which may give her an incentive to invest less than $a^*(\theta_h)$.[13] The cause of the problem is that by insisting on being paid back his credit at date 1.5, VC can credibly threaten not to invest. However, if the exercise date of the conversion option is set after VC had to decide on his investment, or if VC gets the right to exercise his conversion option at any point up to a date that is sufficiently far in the future, then such a

[12]The idea is that in this case VC cannot simply take over the company and complete the project on his own. He still needs E, even though E's investment decision has been taken already. If VC could complete the project on his own, then it would never pay for E to choose an $a \in (\underline{a}, a')$.

[13]See Nöldeke and Schmidt (1998) for a more detailed discussion of this problem.

threat is not credible. When it comes to date 2, it is optimal for VC to invest and, given that he invested, he cannot benefit at date 2.5 from not exercising his conversion option. Hence the timing of the conversion option is important.

There is some empirical evidence that the conversion date of convertible securities is indeed put at the end of the relationship between E and VC. Gompers (1997, p. 16) reports that in his sample of 50 convertible preferred equity venture investments 92% had mandatory conversion that occurs at the time of the IPO.[14]

Finally, the reader may wonder whether Proposition 1 still holds if we allow for renegotiation. Is it possible to implement the first best with a sequence of simple debt-equity contracts? For example, VC could give a credit to E that finances I. At date 1.5, i.e. after VC observed E's investment and the realization of the state of the world, he could buy an additional equity stake from E that induces him to invest $\bar{b}$ in the good state. However, it is easy to see that any such sequence of contracts can implement the first best only if E has all the bargaining power at the renegotiation stage. Otherwise, some fraction of the surplus in the good state will accrue to VC. If E anticipates this she will underinvest.

Hence, a sequence of simple debt-equity contracts may be efficient only if there is enough competition at date 1.5 between several VCs or other financial intermediaries. If, however, there is a close relationship between E and VC that puts VC at an advantage when it comes to the renegotiation stage, then a sequence of debt-equity contracts is inefficient.

4.2 Private Benefits of the Entrepreneur and the Liquidation Decision

Entrepreneurs are not only interested in their monetary payoffs. It is often argued that they get a non-transferable private benefit from running their company. This can be a problem if the project turns out to be a failure. In this case the venture capitalist would like to liquidate the firm as soon as possible in order to get at least the liquidation value while the entrepreneur has an incentive to continue the project in order to receive her private benefits. The optimal contract between E and VC has to make sure that VC has the legal right and the appropriate

[14] Furthermore, a "reasonable estimate of the number of contracts that are converted is between twenty and thirty percent, the fraction of venture-backed projects that eventually go public. A small number are likely converted when a venture-financed company is acquired by an already public firm." This suggests that the conversion option is exercised for the very successful ventures only which is consistent with our model.

incentives to intervene in the bad state and to liquidate the firm.

It is straightforward to extend the model in this direction and to show that a convertible debt contract is well suited to deal with the liquidation problem. In order to do so we cannot normalize the liquidation value to zero anymore but have to assume that $\frac{I}{p+q} > \underline{v} > 0$. Suppose that in order to liquidate the firm, VC has to become active and to spend some effort cost $\underline{b} > 0$. Furthermore, suppose that E gets a private benefit z from running her company. We assume that E's private benefit is smaller than the liquidation value of the firm minus VC's effort cost $\underline{b}$, i.e. $z < \underline{v} - \underline{b}$. If the firm is kept in operation then the liquidation value does not materialize and the value of the firm is zero. Hence, it is efficient to liquidate the firm in the bad state.

This modification has two effects in our model. The obvious effect is that a convertible debt contract is well suited to achieve efficiency in the bad state. In this state VC will never use his conversion option but call his debt. Because the firm cannot pay, VC gets control of the firm and he has the right incentives to liquidate. The somewhat less obvious effect is that renegotiation is now less of a problem in the good state of the world. If E deviates and chooses an effort level with $\underline{a} < a < a'$, then she is not able to repay her debt. Renegotiation makes sure that the firm is not being liquidated and that VC chooses $b = \overline{b}$ in this case. However, E's threatpoint utility in the renegotiation game is now reduced, because she is being deprived of her private benefits if the firm would be liquidated. This relaxes condition (21) to

$$\lambda \left[v(a, \overline{b}, \theta_h) - \overline{b} - v(a, 0, \theta_h) \right] - z \;\leq\; a \quad \forall a \in [\underline{a}, a'] \tag{22}$$

in Proposition 3.

4.3 Multi-dimensional Investments

So far we assumed that E's investment is one-dimensional, $a \in \mathbb{R}_0^+$. Suppose now that E has to choose a multi-dimensional investment vector, $a \in \mathcal{A} \subset \mathbb{R}^N$. For example, E may have to invest in R&D, she may have to spend effort in order to organize her firm and to hire the right employees, she may have to invest in marketing, in supplier and customer relations, etc. Assume that given VC's optimal investment choice $b^*(\theta)$ there is a well defined ex post optimal investment vector $a^*(\theta)$. Then the convertible debt contract of Proposition 2 still implements the first best investments of both parties. In the bad state of the world, E is not going to invest at all. In the medium state she is full residual claimant on profits, so multi-dimensional investments are no problem. In the good state, she wants to choose a so as to make it worth VC's while to exercise his conversion option. Recall that if VC exercises his option, then E gets the entire social surplus minus a constant. Hence, private and social incentives to invest coincide and E is going to invest efficiently.

Consider now VC's investment. So far we assumed that VC's investment choice is binary. Hence, it does not matter whether b is one- or multi-dimensional. Suppose that VC's investment is a continuous choice variable, $b \in \mathbb{R}_0^+$. In order to induce VC to invest efficiently, he has to become full residual claimant on the margin after E took her investment decision. This can be achieved by setting $\alpha = 1$, i.e., VC can convert his debt into 100% of the equity of the firm. Such a contract is feasible, if VC can pay a lumpsum upfront to the E that exceeds the initial investment cost I. This drives up the amount of convertible debt C and the conversion rate α. If VC becomes full residual claimant on the margin, then there is no problem with multi-dimensional investments for him either.

However, such arrangements where $\alpha = 1$ and E gets a lumpsum ex ante that exceeds the amount that is necessary for the first round of investment are not observed in the venture capital industry. There are several obvious additional problems that would arise from this type of contract. In particular, if VC takes over all of the equity in the good state, then there is no incentive for E to stay with the firm after date 2 and to help to further develop it. Furthermore, the entrepreneur could simply take the lumpsum and run at date 0 already.

4.4 The Timing of Investments and Information Flows

The assumption that E and VC invest sequentially is clearly important for our results to hold. However, in the context of venture capital finance, this assumption is not unrealistic. The entrepreneur probably has the strongest impact on his company at the beginning of the project. On the other hand, by the nature of his services, the venture capitalist is more helpful for the development and marketing of a well defined product in the expansion stage than, say, for the construction of a first prototype. Hence, this sequential order is quite natural.

On the other hand, it can be argued that the information on the realization of the state of the world is not revealed before E has to decide on her investment. If E does not know the realization of θ before she invests, she cannot choose the ex post efficient effort level $a^*(\theta)$. Instead, she would have to choose an ex ante optimal effort level a^* that maximizes the *expected* social surplus. We are now going to show that a convertible debt contract can implement the first best under this information structure as well.[15]

The expected social surplus (over θ) if E chooses a and VC chooses $b(\theta)$ is given by

$$S^e(a, b(\theta)) \;=\; p\cdot[v(a, b(\theta_h), \theta_h) - b(\theta_h)] + q\cdot[v(a, b(\theta_m), \theta_m) - b(\theta_m)] + (1-p-q)[0 - b(\theta_l)] - a\ . \tag{23}$$

Thus, the first best efficient investment choice of E is given by

$$a^* \;=\; \arg\max_a p\cdot[v(a, \bar{b}, \theta_h) - \bar{b}] + q\cdot v(a, 0, \theta_m) - a\ . \tag{24}$$

We assume that there is an interior solution with $a^* > \underline{a}$. Hence, the first best efficient investment choice of VC is $b^*(\theta) = 0$ if $\theta \in \{\theta_l, \theta_m\}$ and $b^*(\theta) = \bar{b}$ if $\theta = \theta_h$. Furthermore, we assume that first best expected surplus of the project is sufficient to cover the initial investment, I, i.e. $S^e(a^*, b^*(\theta)) > I$. Finally, we assume

$$v(a^*, 0, \theta_m) \;>\; \frac{I}{p+q}\ , \tag{25}$$

and

$$p\left[v(a, 0, \theta_h) - \frac{I}{p+q}\right] - a \;\leq\; 0 \qquad \forall a \in \mathbb{R}_0^+. \tag{26}$$

Condition (25) requires that the return generated in the medium state of the world (given that both players invested efficiently) is sufficient to pay for the (risk adjusted) initial investment cost. Condition (26) says that if VC does not invest, then the expected return in the good state minus $\frac{I}{p+q}$ is not sufficient to cover E's investment cost a. Both conditions simplify the exposition but they are not necessary for our results.[16]

The following proposition shows that efficient investments $(a^*, b^*(\theta))$ can be implemented by a convertible debt contract under this information structure, too.

[15] If θ is revealed only after VC has taken his investment decision, then uncertainty would not matter at all, because both parties are risk neutral and care only about expected values.

[16] See Footnotes (18) and (19) in the Appendix.

Proposition 4 *Suppose that*

$$\frac{I}{p+q}\left[\frac{v(a^*,\overline{b},\theta_h)-v(a^*,0,\theta_h)}{v(a^*,0,\theta_h)}\right] \;\geq\; \overline{b}\;. \tag{27}$$

Then there exists a convertible debt contract $(C,\hat{\alpha})$ *which gives VC the option to choose at date 2.5 whether to be repaid* C *or to get fraction* $\hat{\alpha}$ *of the equity of the company that implements first best investment decisions. This convertible debt contract is given by*

$$C \;=\; \frac{I}{p+q} \tag{28}$$

and

$$\hat{\alpha} \;=\; \frac{\frac{I}{p+q}+\overline{b}}{v(a^*,\overline{b},\theta_h)} \;<\; 1\;. \tag{29}$$

Proof: See Appendix.

The proof is similar to the proof of Proposition 2 and relegated to the Appendix. In one respect the proof is simpler, because we do not have to make sure that E chooses the ex post efficient investment level $a^*(\theta)$ in each of the different states of the world. Rather, we have to show that E is induced to choose the investment level a^* that is optimal on average. Again, this can be achieved by using a convertible security. The conversion rate has been chosen such that VC is going to exercise his conversion and to choose $\overline{b}$ if and only if the good state materialized and E invested at least a^*. E will never invest more than a^* because fraction $\hat{\alpha}$ of the benefits of this investment are going to VC. Nor will E underinvest. In this case VC would not exercise his conversion option and not invest in the good state which considerably reduces the value of the firm to E. Thus, the intuition is very similar to the intuition for Proposition 2.

On the other hand, the argument is slightly more complicated because off the equilibrium path it is now possible that E chooses an $a < a^*$ such that she is able to repay C in the good state but not in the medium state of the world. Hence, we do need different case distinctions which somewhat complicate the argument.

The next proposition extends the analysis of renegotiation to the case where E invests before she observes θ.

Proposition 5 *Under the conditions of Proposition 4 the convertible debt contract $(C, \hat{\alpha})$ given by (28) and (29) implements first best investment decisions with renegotiation if either $\underline{a} \geq a''$ or if*

$$\lambda p \left[v(a, \bar{b}, \theta_m) - \bar{b} - v(a, 0, \theta_h)\right] + p \max\{v(a, 0, \theta_h) - C, 0\} \leq a \quad \forall a \in [\underline{a}, a''] \quad (30)$$

where a'' is defined by $v(a'', 0, \theta_m) = \frac{I}{p+q}$.

The proof is a straightforward adaptation of the proof of Proposition 3 and left to the reader. Note that we now have to deal with the possibility that E can repay her debt in the good but not in the medium state which explains the use of a'' instead of a'.

As a final extension of the model, suppose that there is some uncertainty in addition to the realization of θ. Any uncertainty that resolves after VC invested and exercised his conversion right is no problem. We can simply interpret $v(a, b, \theta)$ as an expected value. Given that both parties are risk neutral, this is all they care about.

If there is additional uncertainty that resolves after the initial investment I has been sunk but before E invests, the parties could renegotiate the initial contract and adapt the conversion rate without distorting the investment incentives. Given that both parties are symmetrically informed they will always reach an efficient agreement.

Finally, there could be some additional uncertainty that resolves after the realization of θ and after E invested but before VC took his investment decision. Again, if the parties are free to renegotiate, they will always adapt the conversion rate such that VC is induced to invest efficiently. Note that with renegotiation and with additional uncertainty the discontinuity of E's payoff function in Figure 2 is smoothed out. However, it can be shown that if the uncertainty is not too large, then the maximum remains at a^* and E is still going to invest efficiently.[17]

5 Conclusions

Convertible securities endogenously allocate cash flow rights as a function of the realization of the state of the world and of the effort provided by the entrepreneur. We have shown that

[17]See Nöldeke and Schmidt (1998, Section 5) for a formal analysis of this case.

this is a powerful instrument to solve the double moral hazard problem between the venture capitalist and the entrepreneur. Our results offer an explanation for the prevalent use of convertible securities in venture capital finance.

The paper also shows why convertible securities are not being used by banks or outside equity holders who do not get involved in the management of the firms they finance. Propositions 2 and 3 show that a convertible security implements the first best only if the investment of VC is sufficiently important to increase the value of the firm. If this is not the case, a convertible security only dilutes the equity stake of the entrepreneur without providing additional incentives.

Appendix

Proof of Proposition 3: Note first that there is scope for renegotiation only in the good state in order to ensure that VC is going to invest $\overline{b}$ if this is efficient but not privately optimal for VC given the initial contract. Consider first a "small" deviation from $a^*(\theta_h)$ such that $v(a, 0, \theta_h) \geq C = \frac{I}{p+q}$. In this case VC can still guarantee himself a return of C. Clearly, VC will not agree to a new contract that gives him less than C in this state. Hence, E's payoff in state θ_h after renegotiation is given by

$$\begin{aligned} U^E(a \mid \theta_h) &= v(a, 0, \theta_h) - C - a + \lambda \left[v(a, \overline{b}, \theta_h \right] - \overline{b} - v(a, 0, \theta_h)) \\ &< v(a, \overline{b}, \theta_h) - \overline{b} - a - C \\ &< v(a^*, \overline{b}, \theta_h) - \overline{b} - a^* - C \;=\; U^E(a^*) \;. \end{aligned} \tag{31}$$

The last inequality follows from the definition of the first best. The intuition is simply that if E invests (slightly) less than $a^*(\theta_h)$, then the total surplus becomes smaller while the return to VC does not decrease. Hence, such a deviation cannot pay for her. Note that renegotiation is only an issue if $a \geq \underline{a}$. Hence, if E can repay her debt in state θ_h for all $a \in [\underline{a}, a^*]$, i.e., if $\underline{a} > a'$, then renegotiation is not a problem.

So suppose that $a' > \underline{a}$ and consider a "large" deviation, i.e. $\underline{a} \leq a < a'$. In this case E cannot repay her debt. Thus, E's expected payoff is given by

$$U(\underline{a} \leq a < a') \;=\; \lambda \left[v(a, \overline{b}, \theta_h) - \overline{b} - v(a, 0, \theta_h) \right] \Big] - a \tag{32}$$

If E chooses $a = a^*(\theta_h)$, her payoff is strictly positive. Thus, a sufficient condition that guarantees that E has no incentive to deviate is that the right hand side of (32) is less than or equal to 0, which is condition (21) of the proposition. *Q.E.D.*

Proof of Proposition 4: Note first that $v(a^*, \overline{b}, \theta_h) - \overline{b} > v(a^*, 0, \theta_h) > v(a^*, 0, \theta_m) > \frac{I}{p+q}$ (by condition (25). Hence, $\hat{\alpha} < 1$.

Suppose that E chose a^* at date 1. In state θ_l VC will clearly not invest and not exercise his option. E cannot repay her debt, so the firm is being liquidated and VC receives the liquidation value $\underline{v} = 0$.

Consider now state θ_m. If VC chooses not to invest and not to exercise his conversion option, then, by condition (25), E will be able to repay her debt and VC can guarantee himself a payoff of $C - I = \frac{I}{p+q} - I$.[18] Clearly, it cannot be better to invest and not to exercise the option, because in this case he would get $C - \overline{b} - I$. Nor is it better not to invest and to exercise the option, because

$$\begin{aligned}
\hat{\alpha} v(a^*, 0, \theta_m) - I &= \left(\frac{I}{p+q} + \overline{b}\right) \frac{v(a^*, 0, \theta_m)}{v(a^*, \overline{b}, \theta_h)} - I \\
&\leq \left(C + C\frac{v(a^*, \overline{b}, \theta_h) - v(a^*, 0, \theta_h)}{v(a^*, 0, \theta_h)}\right) \frac{v(a^*, 0, \theta_m)}{v(a^*, \overline{b}, \theta_h)} - I \\
&= C\frac{v(a^*, 0, \theta_m)}{v(a^*, 0, \theta_h)} - I \ < \ C - I \qquad (33)
\end{aligned}$$

where we used (27), (28) and (29). Finally, it is not better to invest and to exercise the conversion option, because

$$\hat{\alpha} v(a^*, \overline{b}, \theta_m) - \overline{b} - I < \hat{\alpha}[v(a^*, \overline{b}, \theta_m) - \overline{b}] - I < \hat{\alpha} v(a^*, 0, \theta_m) - I < C - I \qquad (34)$$

Here the last inequality follows from the argument given in (33). Hence, we have shown that given a^* VC will not invest and not exercise his conversion option in state θ_m.

Finally, consider state θ_h. We want to show that it is optimal for VC to invest and to exercise his option if and only if $a \geq a^*$. Suppose that E invested at least the efficient amount a^*. Note first that VC can guarantee himself a payoff of $C - I = \frac{I}{p+q} - I$ by not investing and not converting his debt. Clearly, it cannot be optimal to invest and not to exercise the option because this would yield $C - \overline{b} - I < C - I$. On the other hand, if VC does invest and does exercise his option, than his payoff is given by

$$U^{VC}(a) \ = \ \hat{\alpha} v(a, \overline{b}, \theta_h) - \overline{b} - I \ = \ \frac{\frac{I}{p+q} + \overline{b}}{v(a^*, \overline{b}, \theta_h)} v(a, \overline{b}, \theta_h) - \overline{b} - I \ . \qquad (35)$$

Note that $U^{VC}(a)$ is monotonically increasing with a. Substituting $\hat{\alpha}$ by (29), VC's payoff if E invested a^* is

$$U^{VC}(a^*) \ = \ \frac{\frac{I}{p+q} + \overline{b}}{v(a^*, \overline{b}, \theta_h)} v(a^*, \overline{b}, \theta_h) - \overline{b} - I \ = \ \frac{I}{p+q} - I \ = \ C - I \ . \qquad (36)$$

[18] If (25) does not hold, C would have to be chosen such that $I = pC + qv(a^*, 0, \theta_m)$ which complicates the exposition but does not change our main result.

Hence, VC prefers to invest and to convert his debt rather than not to invest and not to convert if E invested $a \geq a^*$. On the other hand, if $a < a^*$, $U^{VC}(a < a^*) < C - I$, so VC will invest and exercise his option only if $a \geq a^*$.

It remains to be shown that, given $a \geq a^*$, VC prefers to invest and to convert his debt rather than not to invest and to convert his debt. Hence, we have to show that

$$\hat{\alpha} v(a, 0, \theta_h) - I \;\leq\; \hat{\alpha} v(a, \bar{b}, \theta_h) - \bar{b} - I \tag{37}$$

for all $a \geq a^*$. Substituting (29) for $\hat{\alpha}$ this is equivalent to

$$\frac{\frac{I}{p+q} + \bar{b}}{v(a^*, \bar{b}, \theta_h)} \left[v(a, \bar{b}, \theta_h) - v(a, 0, \theta_h) \right] \;\geq\; \bar{b} \tag{38}$$

Note that by Assumption 1(d) the left hand side of (38) is monotonically increasing with a. Hence, if (38) holds for $a = a^*$, it also holds for all $a > a^*$. Substituting $a = a^*$ and rearranging yields (27). Hence, we have shown that it is optimal for VC to invest and to convert his debt in state θ_h if and only if $a \geq a^*$.

In the next step we have to show that it is indeed optimal for E to invest a^*. If she chooses $a = a^*$, VC will invest $\bar{b}$ and exercise his conversion option, so E's expected payoff is

$$U^E(a^*) \;=\; p(1 - \hat{\alpha}) v(a^*, \bar{b}, \theta_h) + q[v(a^*, 0, \theta_m) - C] - a^* \tag{39}$$

Recall that $\hat{\alpha} v(a^*, \bar{b}, \theta_h) - \bar{b} = C = \frac{I}{p+q}$. Substituting this in (39) we get

$$\begin{aligned} U^E(a^*) &= p[v(a^*, \bar{b}, \theta_h) - C - \bar{b}] + q[v(a^*, 0, \theta_m) - C] - a^* \\ &= p[v(a^*, \bar{b}, \theta_h) - \bar{b}] + q v(a^*, 0, \theta_m) - a^* - I \;=\; S(a^*, b^*(\theta)) - I \,. \end{aligned} \tag{40}$$

Hence, if E chooses $a = a^*$ she gets the entire expected social surplus at $a = a^*$ minus the initial investment cost I. Investing more than a^* reduces the social surplus by the definition of the first best and it increases VC's payoff. Thus, because E's and VC's payoffs must always add up to the social surplus, E's payoff must fall. Hence, E will never invest more than a^*.

If E chooses $a < a^*$ VC will not exercise his conversion option and not invest. Consider a smooth reduction of a starting from $a = a^*$. There is first a range where E is still able to repay her debt in states θ_h and θ_m. In this range VC's payoff does not change when a is reduced, so $U^E = S - U^{VC}$ must go down. Then there comes a range of a where E is able to repay her debt in the good but not in the medium state. In this range E's payoff is given by $p[v(a,0,\theta_h) - C] - a = p[v(a,0,\theta_h) - \frac{I}{p+q}] - a \leq 0$ by condition (26), while E would get $S(a^*, b^*(\theta)) > 0$ if she chooses $a = a^*$.[19] Finally there is range where E cannot repay her debt in any state of the world, so her payoff is $-a \leq 0$. Again, she can get a strictly positive payoff by choosing $a = a^*$, so E has no incentive to choose $a < a^*$. *Q.E.D.*

[19] Condition (26) has been imposed to guarantee that E has no incentive to reduce her effort level in such a way that the cost of this reduction are mainly born by VC (who receives a lower repayment in state θ_m) while the benefits (in terms of lower investment costs a) accrue to E. However, it is obvious that condition (26) can be relaxed as long as $\max_a p[v(a,0,\theta_m) - I/(p+q)] - a < S(a^*, b^*(\theta))$.

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