CESifo *Working Paper Series*

THE POLITICAL ECONOMY OF SOCIAL SECURITY

Georges Casamatta
Helmuth Cremer
Pierre Pestieau*

Working Paper No. 259

March 2000

CESifo
Poschingerstr. 5
81679 Munich
Germany
Phone: +49 (89) 9224-1410/1425
Fax: +49 (89) 9224-1409
http://www.CESifo.de

* Financial support from the Belgian SSTC and the European Commission is gratefully acknowledged. We thank Philippe De Donder, Louis Gevers, Stéphane Rottier and, particularly, the two referees and the editor for their helpful comments. The third author wrote part of this paper while visiting *CES* at the Univesity of Munich.

Pierre Pestieau was a *CES* visiting scholar in December 1999.

CESifo *Working Paper No. 259*
March 2000

THE POLITICAL ECONOMY OF SOCIAL SECURITY

Abstract

We consider a two-period overlapping generations model in which individual voters differ by age and by productivity. In such a setting, a redistributive Pay-As-You-Go system is politically sustainable, even when the interest rate is larger than the rate of population growth. The workers with medium wages (not those with the lowest wages) and the retirees form a majority which votes for a positive level of social security. This level depends on the difference between population growth and interest rate and on the redistributiveness of the benefit rule.

Keywords: Social security, majority voting

JEL Classification: H55, O41, O9

Georges Casamatta
GREMAQ
Université de Toulouse and
CREPP
Université de Liège

Helmuth Cremer
GREMAQ and IDEI
Université de Toulouse
21, Allee de Brienne
31000 Toulouse
France

Pierre Pestieau
CREPP
Université de Liège,
CORE and Delta
Belgium

1 Introduction

The determination of social security benefits and contribution rates is often studied in majority voting models. This approach was initiated by Browning (1975), who considers a society where people only differ according to age. Within that framework, the decisive voter is the median age individual. The voting equilibrium then leads to an excessive social security budget.

It is more and more recognized that the political forces involved in the debate on social security cannot be represented along the single age axis. There are other dimensions, particularly that of income and, more specifically, income heterogeneity *within* generations. The income dimension is likely to be important when the social security system redistributes not only from younger to older generations, as the pay-as-you-go (PAYG) system often does, but also from high to low wage workers. In other words, introducing differential earning capacities is particularly important when the benefit rule is redistributive rather than actuarially fair.

The voting outcome also depends on the alternatives available for the financing of old age consumption. Consider the hypothetical case where private savings are not available. Then, all individuals vote for a positive tax rate: the old because it determines their benefit level and the young to ensure themselves some consumption during retirement. However, when private savings are available and when their rate of return is higher than that offered by the social security system, individuals will vote against social security. This can occur for at least three reasons. First, if the social security system is of the PAYG type, and the rate of interest is higher than the rate of population growth, private saving is more attractive, at least for a young worker. Second, if the payroll tax implies some deadweight loss, one may get the same bias. Finally, if the social security system is redistributive, those who pay for redistribution, namely workers with earnings above the average level, will prefer private saving. Conversely, workers with lower than average earning capacities benefit from redistribution. Consequently, they may vote in favor of social security, even when there are some distortions and when the rate of interest is higher than the rate of population growth.

In this paper, we adopt a steady state setting with given rates of interest and population growth. We also assume that the benefit rule is given. Some countries, labelled Bismarckian, such as Germany of France, offer replacement ratios that are stable across income levels, whereas others, labelled Beveridgean, such as Canada or the Netherlands, tend to have replacement ratios that fall as income increases. Table 1 illustrates this distinction for 9 countries.

Table1: Size and redistributiveness of social security

	Half of average	Average income	Twice average	Regime*	Spending as % of GDP
Canada	76	44	25	BE	5.4
France	84	84	73	BI	12.5
Germany	76	72	75	BI	12.8
Italy	103 (1/4x)	90	84 (3x)	BI	15.6
Japan	77	56	43	MI	6.6
Netherlands	73	43	25	BE	5.2
New Zealand	75	38	19	BE	5.4
UK	72	50	35	MI	4.4
USA	65	55	32	MI	4.6

*BI: Bismarckian; BE: Beveridgean; MI: mixed.

Source: Johnson (1998)

The assumption that the benefit rule is given implies that we are concentrating on one specific aspect of the problem. In other words, our model is meant to be a building block in a more ambitious setup, encompassing a broader range of decision variables. This is a meaningful approach, for instance, if the decision process is inherently sequential. One may note that while the redistributive character of a system (i.e., its more or less Bismarckian or Beveridgean design) and its size are both essential feature of a retirement system, they are not exactly symmetric. Its redistributive character is, to a large extent, an integral part of the very definition of the system in itself. Bismarckian systems on the one hand and Beveridgean systems on the other hand, imply specific institutional and administrative arrangements which cannot be overturned in the short run. In countries like Germany and France, the Bismarckian system is by now solidly anchored in the traditions. For the UK, on the other hand, the tradition is Beveridgean.

In the concluding section, we briefly discuss the choice of the benefit rule at an earlier, "constitutional", stage. Decisions at this stage can be made either by a welfare maximizing authority or through a voting procedure. In either case, decisions in the first stage will be contingent on the induced outcome in the second stage. Consequently, the characterization of the outcome for any given benefit rule, is a necessary step in the analysis. One of the interesting observations that can be derived from analyzing the constitutional decisions is that, even from a pure Rawlsian viewpoint, it may be optimal to adopt a benefit rule that is not "too redistributive". Interestingly, the less redistributive than otherwise optimal benefit rule is not (or not only) adopted to mitigate labor market distortions but also to induce a majority to opt for generous retirement benefits.

Following Browning (1975), most of the subsequent literature assumes that individuals differ only in age and focuses upon simple majority voting and the median voter outcome. Several variations on the Browning model have been considered; see Myles (1995) for a survey. While these studies have produced different specific results, the fundamental conclusion remains the same: majority voting tends to yield overspending on social security. Among the most representative variations, let us mention Hu (1982) who considers uncertainty of benefits receipts, Boadway and Wildasin (1989) who introduce an explicit capital market, and Veall (1986) who assumes intergenerational altruism.

A more drastic departure from Browning's setting is provided by Tabellini (1990). He assumes that individuals are altruistic (children towards parents and parents towards children) and introduces differences in income as a second source of heterogeneity, along with the traditional age differences. Another specific feature of his model is that there is no commitment to preserve past decisions in the future. The main result is that in such a setting, a coalition of the young poor and the retired may sustain a positive tax rate.

In our paper, there are also two sources of heterogeneity. However, there is no altruism and we return to the conventional commitment assumption. Individual voters differ not only according to age but also according to productivity. In our setting, medium wage workers, rather than the lowest wage ones, join the retirees to form a majority and vote for a positive level of social security. Furthermore, this level is often in excess of the one which maximizes lifetime welfare. The majority equilibrium level is also shown to depend on the difference between population growth and interest rate and on the redistributiveness of the benefit rule.

2 The model

Consider a small open one sector economy with given interest rate, r_t. At each period of time t, two generations overlap: L_t workers and L_{t-1} retirees, with $L_t = L_{t-1}(1 + n_t)$, where n_t is the rate of population growth. Individuals differ in two ways: the generation they belong to and their wage w, a continuous variable with support $[w_-, w_+]$, mean $\overline{w}$ and median $w_m < \overline{w}$. Individual labor supply is given and normalized to 1.

The pension benefits an individual earning w expects to receive is $p_{t+1}(w)$. We assume that $p_{t+1}(w)$ consists of two parts: a contributory part which is directly related to individual earning, w, and a noncontributory part which depends on average earnings, $\overline{w}$. With a Pay-As-You-Go (PAYG) scheme, the average return of the social security system is given by the population growth rate. These properties yield the following expression for $p_{t+1}(w)$:

$$p_{t+1}(w) = (1 + n_{t+1})\,\tau_{t+1}\,(\alpha w + (1-\alpha)\,\overline{w})\,. \tag{1}$$

The parameter α is the Bismarckian factor, that is the fraction of pension benefits that is related to contributions; we assume $0 \leq \alpha \leq 1$. When $\alpha = 1$, the pension scheme is purely Bismarckian or contributory; when $\alpha = 0$, pension benefits are uniform and the scheme is Beveridgean. Finally, throughout the paper, we assume dynamic efficiency: $r_t \geqslant n_t > 0, \forall t$.

We first analyze the optimal saving decision of a working individual born in period t with earning w. He is subject to a payroll tax τ_t and expects the future tax rate to be τ_{t+1} when old. He can then allocate his disposable income between consumption c_t and saving s_t. When he retires his consumption d_{t+1} is equal to the gross return of his saving, $(1 + r_{t+1})\,s_t$, and a pension p_{t+1}. Formally, he solves the following program:

$$\max_{s \geq 0} U_t = u(c_t) + \beta u(d_{t+1}) \tag{2}$$

subject to:

$$w(1-\tau_t) = c_t + s_t \tag{3}$$

and

$$d_{t+1} = (1+r_{t+1})\, s_t + p_{t+1}(w). \tag{4}$$

In (2), $u(.)$ is strictly concave and $\beta \leq 1$ is a factor of time preference. Let σ denote the elasticity of substitution between c_t and d_{t+1}. We assume throughout the paper that σ is constant and that $\sigma < 1$, which means that there is not much substitution in consumption, a widely accepted assumption. The coefficient of relative risk aversion, $R_r(x) = -xu''(x)/u'(x)$ is thus also constant and equal to $\varepsilon = 1/\sigma$. Finally, we restrict savings to be non negative.[1]

The first-order condition associated to an interior solution of s_t is the following:

$$-u'(c_t) + \beta u'(d_{t+1})(1+r_{t+1}) = 0. \tag{5}$$

Denoting $s_t^A \geq 0$ the optimal value of s_t, we can define the (indirect) utility function of an individual with income w as:

$$V_t(\tau_t, \tau_{t+1}, w) = u\left(w(1-\tau_t) - s_t^A\right) + \beta u\left((1+r_{t+1})\, s_t^A + p_{t+1}(w)\right). \tag{6}$$

We now determine the steady state majority voting equilibrium tax rate. We consider α as given; the problem is therefore unidimensional and, under the condition that preferences are single-peaked, the median voter theorem, which ensures the existence of a Condorcet winner, applies. Individuals vote for τ believing that the value of τ chosen by the majority will hold for ever.[2]

[1] Allowing negative savings would generate extreme solutions. Young people in favor of the PAYG pension system would vote for a tax rate of 1 and would rely entirely on borrowing to finance their present consumption.

[2] Or at least for the next two periods; the tax rate in later periods is of no relevance for the individuals.

Formally, they expect that $\tau_{t+1} = \tau_t, \forall t$. We can thus remove all the time subscripts in the following equations.

In the following, we first derive the preferred tax rate of the retirees and that of the workers. Then, to identify the Condorcet winner, we order these preferred alternatives. The Condorcet winner is the tax rate such that half of the population prefer a higher tax rate and half of the population a lower one. For the time being, there is no tax distortion. Next, we derive the comparative statics of the majority voting solution with respect to some parameters of the model, namely α and n. We then compare the PAYG majority voting equilibrium tax rate with a (collective) fully funded (FF) solution, granted that both systems adopt the same benefit rule. Finally, we solve the problem when taxation generates (quadratic) distortions.

3 Preferred tax rates of the different agents

3.1 The retirees

Each retiree has some non negative private savings, s, with return r. These private savings are the result of past decision and the retirees have no control over it. The only variable they can affect is the tax rate which determines their pension level. As there is no altruism, their preferred tax rate, τ^R, is the one which maximizes their consumption:

$$d = (1 + r)\, s + (1 + n)\, \tau\, (\alpha w + (1 - \alpha)\, \overline{w})\,. \tag{7}$$

The solution is given by $\tau^R = 1$ for all retirees.[3] In words, retirees want the tax to be as large as possible. This is because the current tax affects their pension level, but has no impact on their contributions (which have been paid in the previous period).

[3] This result is rather extreme but, introducing tax distortions or some degree of altruism from the old to the young, one would obtain a value of τ^R smaller than 1.

3.2 The workers

A worker with earning w chooses $\tau^A(w)$ which maximizes:

$$v(\tau, w) = u\left(w(1-\tau) - s^A\right) + \beta u\left((1+r)s^A + (1+n)\tau(\alpha w + (1-\alpha)\overline{w})\right) \tag{8}$$

where $s^A \geq 0$ is the optimal level of private saving and $v(\tau, w) = V(\tau, \tau, w)$.

Note that a worker will always be in favor of a zero tax if:

$$1 + r > (\alpha + (1-\alpha)\overline{w}/w)(1+n). \tag{9}$$

This means that the return from private saving is higher than the return from PAYG social security. Without tax distortions, these returns are independent of the tax rate. Consequently, a given individual will either prefer a zero tax rate and positive private saving if (9) holds or a strictly positive tax rate and no private saving in the opposite case.

In other words, an individual prefers private saving if his wage is higher than $\hat{w}$ defined as:

$$\hat{w} = \frac{1-\alpha}{\frac{1+r}{1+n} - \alpha}\overline{w} \leq \overline{w}. \tag{10}$$

One easily checks that $\hat{w} = \overline{w}$ if $n = r$, $\partial\hat{w}/\partial n > 0$ and $\partial\hat{w}/\partial\alpha < 0$.

The solution to the worker's problem is characterized in Proposition 1 which is proved in the Appendix.

Proposition 1 *The preferred tax rates of the workers have the following properties:*

(i) $\tau^A(w) = 0$ *if* $w > \hat{w}$ *and* $\tau^A(w) > 0$ *if* $w \leqslant \hat{w}$;
(ii) $\dfrac{\partial \tau^A(w)}{\partial w} > 0$ *if* $w < \hat{w}$;
(iii) $Max\ \tau^A(w) = \tau^A(\hat{w}) < \tau^R = 1.$

The first point of the proposition recalls that only workers with a sufficiently low wage level want a positive tax rate. The second point says that preferred tax rates are increasing with income. This result arises because the intertemporal elasticity of substitution, σ, is smaller than one. Intuitively, the relationship between the low intertemporal substitution and the property that the preferred tax rate increases with income is easily understood. Consider, for example, the extreme case where there is no substitution at all: individuals want to equalize their two periods consumptions. Because the rate of return of PAYG social security is decreasing with income, the high wage workers would like to transfer a greater proportion of their income from the first period to the second period than the low wage one.[4] Finally, point (*iii*) states that the maximal preferred tax rate of the workers is lower than 1, the preferred tax rate of the retirees. This simply comes from the fact that individuals want to consume when young.[5]

4 The majority voting solution

Because their utility is increasing with the value of the tax rate, preferences of the retirees are obviously single-peaked. The objective function of the workers can easily be shown to be strictly concave, which implies that it is single-peaked; see the proof of Proposition 1. The workers are divided into two classes, those preferring a zero tax rate and positive savings and those preferring a positive tax rate and no savings. The utility of individuals in the first group decreases with the tax rate. For the second group, the preferred value of the tax rate is given by an interior solution.

[4] The property that preferred tax rates are increasing with income may seem surprising. Whithin our framework, it results from the (realistic) assumption that the intertemporal elasticity of substitution is low. However, the assumption of a strictly proportional tax is crucial. If instead we had assumed that the poorest individuals are exempted from taxation, they would have a high preferred tax rate, thereby joining the retirees to sustain a generous social security system.

[5] Only for a linear utility function (which corresponds to an infinite intertemporal elasticity of substitution), one would obtain a corner solution with $\tau^A = 1$.

Summing up, a fraction $1/(2+n)$ of citizens, the retirees, is in favor of $\tau^R = 1$. Further from Proposition 1, all workers with earnings above $\hat{w}$ are in favor of a zero tax. The preferred tax rate for the workers with earnings below $\hat{w}$ increases with w. The profile of preferred tax rates is represented in Figures 1 and 2 for the cases $r = n$ and $r > n$ respectively. We can now determine the decisive voter and the majority voting equilibrium tax rate is reported in the following proposition.

Proposition 2 *If $\int_{w_-}^{\hat{w}} f(w)\,dw < n/2(1+n)$, the majority voting equilibrium tax rate, τ^* , is 0. If $\int_{w_-}^{\hat{w}} f(w)\,dw \geq n/2(1+n)$, the majority voting equilibrium tax rate is the rate preferred by the workers with earning $\widetilde{w}$ defined as follows:*

$$\int_{\widetilde{w}}^{\hat{w}} f(w)\,dw = \frac{n}{2(1+n)} \,. \tag{11}$$

Proof.

The Condorcet winner is the tax rate such that one half of the total population prefer a higher tax rate and the other half a lower tax rate. The total number of individuals preferring a tax rate higher than 0 is $L + L(1+n)\int_{w_-}^{\hat{w}} f(w)\,dw$, where L is the number of old individuals. The total population is $L + L(1+n) = L(2+n)$. Consequently, if $L + L(1+n)\int_{w_-}^{\hat{w}} f(w)\,dw < L(2+n)/2 \Leftrightarrow \int_{w_-}^{\hat{w}} f(w)\,dw < n/2(1+n)$, less than one half of the total population prefer a strictly positive tax rate and the Condorcet winner is 0. On the other hand, when $\int_{w_-}^{\hat{w}} f(w)\,dw \geq n/2(1+n)$, the individuals preferring a strictly positive tax rate constitute more than one half of the total population and the Condorcet winner is strictly positive. It is the tax rate preferred by the individual with income $\tilde{w}$ such that the population who prefer a higher tax rate constitutes half of the total population:

$$L + L(1+n)\int_{\tilde{w}}^{\hat{w}} f(w)\,dw = \frac{L(2+n)}{2}. \qquad (12)$$

Straightforward manipulations lead to (11). ■

The proposition first states that the majority voting equilibrium tax rate is zero when the number of working individuals in favor of a positive tax rate is too low. This may occur in particular when r is large relatively to n. In this case, the redistributive effect of PAYG social security is dominated by the high return of private saving and even the poorest of the workers may prefer to save privately. The second part of the proposition states that when the Condorcet winner is strictly positive, the majority coalition consists of the retirees and of the workers with *medium* wages. This result is reminiscent of Epple and Romano's (1996) "ends against the middle" equilibrium in which there is a coalition made of the tails of the income distribution; see also Casamatta *et al.* (1999). This property clearly hinges upon our assumption on the elasticity of substitution. With $\sigma > 1$, preferred tax rates would be decreasing with income and the majority coalition would be composed by the retirees and the poorest working individuals.

5 Comparative statics

The illustrative figures presented in Table 1 suggest that the size and the redistributive character of a system are inversely related. Put differently, the most generous systems appear to be also those which redistribute the least. To see whether this stylized fact is consistent with our model let us now study the impact of the contributive part on the equilibrium tax rate. Consider the case where the Condorcet winner is strictly positive.

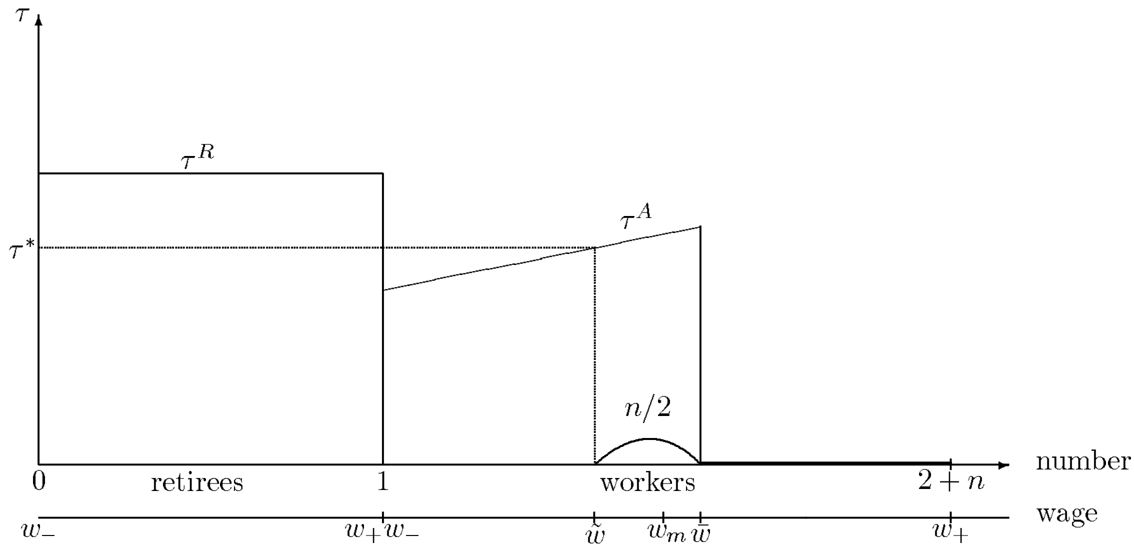

Figure 1: preferred tax rates when $r = n$

Differentiating (11) with respect to α yields:

$$\frac{\partial \tilde{w}}{\partial \alpha} = \frac{\partial \hat{w}}{\partial \alpha} \frac{f(\hat{w})}{f(\tilde{w})} \leq 0. \tag{13}$$

Furthermore, the first-order condition for τ^A, implies:

$$\frac{\partial \tau^A}{\partial \alpha} \left(\tilde{w}\left(\alpha\right), \alpha\right) = \frac{\beta\left(1+n\right)\left(\tilde{w}-\overline{w}\right) u'\left(d\right)\left(1-R_r\left(d\right)\right)}{-D_\tau}. \tag{14}$$

Keeping in mind that $\tilde{w} < \overline{w}$ and that $R_r\left(.\right) > 1$, this expression is positive. Finally, recalling that $\tau^*\left(\alpha\right) = \tau^A\left(\tilde{w}\left(\alpha\right), \alpha\right)$ yields:

$$\underset{?}{\frac{\partial \tau^*}{\partial \alpha}} = \underset{+}{\frac{\partial \tau^A}{\partial \alpha}} \left(\tilde{w}\left(\alpha\right), \alpha\right) + \underset{+}{\frac{\partial \tau^A}{\partial w}} \left(\tilde{w}\left(\alpha\right), \alpha\right) \underset{-}{\frac{\partial \tilde{w}}{\partial \alpha}}. \tag{15}$$

Clearly the sign of $\partial \tau^* / \partial \alpha$ is positive when $r = n$; in that case the identity of the decisive voter does not depend on α and the second term vanishes. The first, positive, term then determines the sign of the expression

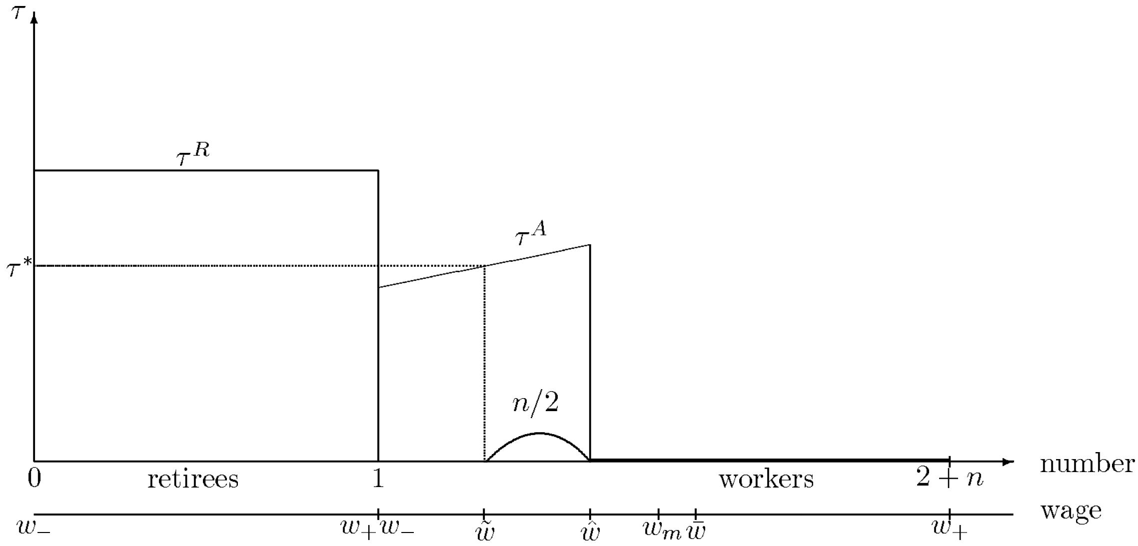

Figure 2: preferred tax rates when $r > n$

and we get the expected positive relation between α and τ^*. When $r < n$, however, the sign of (14) is ambiguous. There are now two opposite effects. On the one hand, when α rises, the preferred tax rate of the decisive voter rises. However, on the other hand, when α changes, the identity of the decisive voter changes as well. When α rises, PAYG public pension becomes less attractive to low wage workers (who support this system). Some of them (the more productive) "switch" to the private sector and the new decisive voter is poorer than before. As preferred tax rates are increasing with income, the tax rate chosen by this new decisive voter is smaller than before.

We should however note that, when $r > n > 0$, the majority voting equilibrium tax rate jumps discontinuously to 0 for α high enough. This comes from the fact that, when α is sufficiently close to 1, the PAYG system operates so little redistribution that even the poorest of the workers does not find it attractive anymore and vote for the abandonment of the system.

Following the same approach, one can also study the comparative statics of $\partial\tau^*/\partial n$. The result is also ambiguous: the negative "direct" effect induced by the decrease in the rate of return of the PAYG system has to be balanced against an ambiguous "indirect" effect which arises because the identity of the decisive voter changes. Consequently, a decline in fertility *may* indeed result in an increase in the majority voting equilibrium tax.[6]

6 Pay-as-you-go versus fully funded

So far, we have concentrated on voting over PAYG systems. Let us now compare the majority voting equilibrium under PAYG and the equilibrium that arises with an "equivalent" fully funded (FF) scheme. We are not concerned by the transition from one to the other. We just assume that vote takes place in two alternative steady states. To make the comparison fair we assume $r = n > 0$, so that the average return is the same under both systems. Furthermore, we consider identical benefit rules to ensure that the two schemes operate the same degree of redistribution. In other words, we do *not* compare a redistributive PAYG system to a totally individualized FF system, as it is often done in the literature on the privatization of social security.

The major difference between the two systems is that under a FF scheme, the retirees have no stake in the vote. All the decisions which are relevant for them have been taken in the past: private saving, s, and collective saving

[6] For a "large" decline in fertility, however, the tax rate falls to zero, as long as the condition $w_- > \overline{w}/(1+r)$ is satisfied. The explanation is simple: for n close to 0, $\hat{w}$ is close to $(1-\alpha)\overline{w}/(1+r-\alpha)$. Moreover we know that $\hat{w}$ declines with α. Hence the maximal value of $\hat{w}$ when n tends to 0 is $\overline{w}/(1+r)$. Therefore, if $w_- > \overline{w}/(1+r)$, we are sure that the poorest individual does not support the PAYG pension system so that a majority votes for the abandonment of the system.

through pension funds, if any. Recall that the interest rate, r, and the Bismarckian factor, α, are also given.

Consequently, only the active population matters for the determination of the voting equilibrium. Further, preferred tax rates of workers continue to be characterized by Proposition 1 and the decisive voter has earnings $\breve{w}$ such that:

$$\int_{\breve{w}}^{\overline{w}} f(w)\, dw = 1/2. \tag{16}$$

In other words, the young population is divided into two groups: those with earnings between $\breve{w}$ and $\overline{w}$ and who want a tax rate higher than $\tau^A(\breve{w})$, and those with earnings below $\breve{w}$ or above $\overline{w}$ who want a lower tax rate. Note that $w_m < \overline{w}$ ensures that majority voting yields a strictly positive tax rate.

Comparing (16) and (11) while recalling that $r = n$ implies $\hat{w} = \overline{w}$, it follows that $\breve{w} < \tilde{w}$. Consequently, the decisive voter under the FF scheme has a lower wage than the decisive voter under the PAYG system. This leads to Proposition 3.

Proposition 3 *The majority voting equilibrium size of social security is larger under a PAYG than under a FF scheme.*

This result can be explained as follows. With a PAYG system, the retirees favor a payroll tax rate which is as large as possible. They do not have the majority and the median voter is a worker who backs social security because of its redistributiveness. Nevertheless, it is clear that the vote of the retirees tends to increase the size of the social security system. With a FF system, on the other hand, the retirees do not have any leverage on the workers' tax payments.

In a sense, the voting under PAYG amounts to give the older generation a larger weight than the younger one. The current young will be old in the next period and take this into account for their vote. Consequently, they put some weight on old age consumption, as do of course the currently old. First period consumption, on the other hand, affects only the currently young.

What about social welfare? In the setting with heterogenous individuals, the comparison is not straightforward and depends on the welfare function which is used. With a Rawlsian criterion FF always dominates PAYG, as it gives the decisive vote to workers with lower earnings (recall that, $\breve{w} < \tilde{w}$).[7] With a general utilitarian criterion, the comparison is ambiguous.

7 Extension: distortionary payroll taxation

Up to now the tax system was assumed to imply no efficiency loss. Let us now briefly examine the implications of distortionary taxation. For simplicity, we use a quadratic loss function so that the steady state relationship between contributions and pensions is now given by:

$$p(w) = (1+n)\,\tau\left(\alpha w + (1-\gamma\tau)(1-\alpha)\,\overline{w}\right), \tag{17}$$

where $\gamma > 0$ is the distortion factor. Note that the distortion applies only to the noncontributory part of social security. In other words, voters see through the budgetary veil and realize that the fraction α of their tax payment is given back to them with a return of n.[8]

[7] Welfare depends on (ex ante) lifetime utilities.

[8] This specification is a reduced form of a more general (and complicated) model where labour supply is endogenous. Indeed the FOC for an interior value of τ would be:

$$-yu'(c) + \beta(1+n)(\alpha y + (1-\alpha)\overline{y})\,u'(d) +$$
$$\beta\left(\tau(1+n)(1-\alpha)\int_{w_-}^{w_+} w\partial l^A/\partial\tau f(w)\,dw\right)u'(d) = 0,$$

where $\overline{y} = \int_{w_-}^{w_+} yf(w)\,dw$, $y = wl^A(w)$ and $l^A(w)$ is the optimal labour supply of an individual with productivity w. It appears clearly that tax distorsions are associated to the third term of this FOC.

7.1 The retirees

It is straightforward to show that the preferred tax rate of a retiree with (past) wage of w is given by:

$$\tau^R(w) = \min\left\{\frac{1}{2\gamma}(1 + \frac{\alpha}{1-\alpha}\frac{w}{\overline{w}}), 1\right\}. \tag{18}$$

The retiree choose the tax rate that maximizes his pension under the constraint that $\tau \leq 1$. Note that $\tau^R(w)$ is increasing in w. This is because tax distortions are related to the Beveridgean part of pensions. Consequently, poor retirees suffer more from a tax increase than the rich.

7.2 The workers

The problem of the workers is drastically modified when tax distortions are introduced. The rate of return of the PAYG system now depends on the value of the tax rate. Consequently, it is now possible that a given individual prefers the PAYG system to private saving for some values of τ and that his preferences change in favor of private saving for some other values. This individual will then choose a positive tax rate, such that the rates of return of public pension and private saving are equalized, and will supplement these mandatory public savings with private savings. Consequently, the possibility of a mixed choice, public pension supplemented with private savings, cannot be ruled out anymore.

Formally, the preferred tax rate, $\tau^A(w)$, and the savings, $s^A(w)$, of a worker are the solutions to the following problem:

$$\underset{\tau,s}{Max}\, U = u(c) + \beta u(d) \tag{19}$$

subject to:

$$w(1-\tau) = c + s$$

$$d = (1 + r)\, s + (1 + n)\, \tau\, (\alpha w + (1 - \alpha)\,(1 - \gamma\tau)\, \overline{w}) \tag{20}$$

and

$$\tau \geq 0, s \geq 0. \tag{21}$$

The next proposition, proved in the appendix, states that low wage individuals want a positive tax rate but no private savings, middle wage individuals will make a mixed choice, with both PAYG social security and private saving, and the higher wage individuals will prefer to rely only on private saving.

Proposition 4 *(i)* $\max \tau^A(w) < \min \tau^R(w)$.

Moreover, there exists a value of w, $w' < \hat{w}$ (defined in (10)), such that:

(ii) if $w \leq w'$, $\tau^A(w) > 0$, $s^A(w) = 0$,

(iii) if $w' < w < \hat{w}$, $\tau^A(w) > 0$ and $s^A(w) > 0$,

(iv) if $w \geq \hat{w}$, $\tau^A(w) = 0$ and $s^A(w) > 0$,

(v) if $w < w'$, $\partial\tau^A(w)/\partial w > 0$ and if $w' < w < \hat{w}$, $\partial\tau^A(w)/\partial w < 0$.

The first point in Proposition 4 says that the maximal preferred tax rate of the workers is lower than the minimal preferred tax rate of the retirees. The underlying idea is the same as in the case without distortions: workers do not want the tax to be too high because it decreases their first period consumption. The other points imply that $\tau^A(w)$ is first increasing and then decreasing with w; it is equal to zero when $w = \hat{w}$. For workers with $w < \hat{w}$ social security is attractive up to a certain point; its relative return is now equal to $(\alpha + (1-\alpha)(1-\gamma\tau)\overline{w}/w)$, which can be lower than $1 + r$ for some τ. For workers with wage close to w_- , there is no saving and the preferred tax rate increases with w. Figure 3 presents these results for the case where $r = n$, so that $\hat{w} = \overline{w}$.

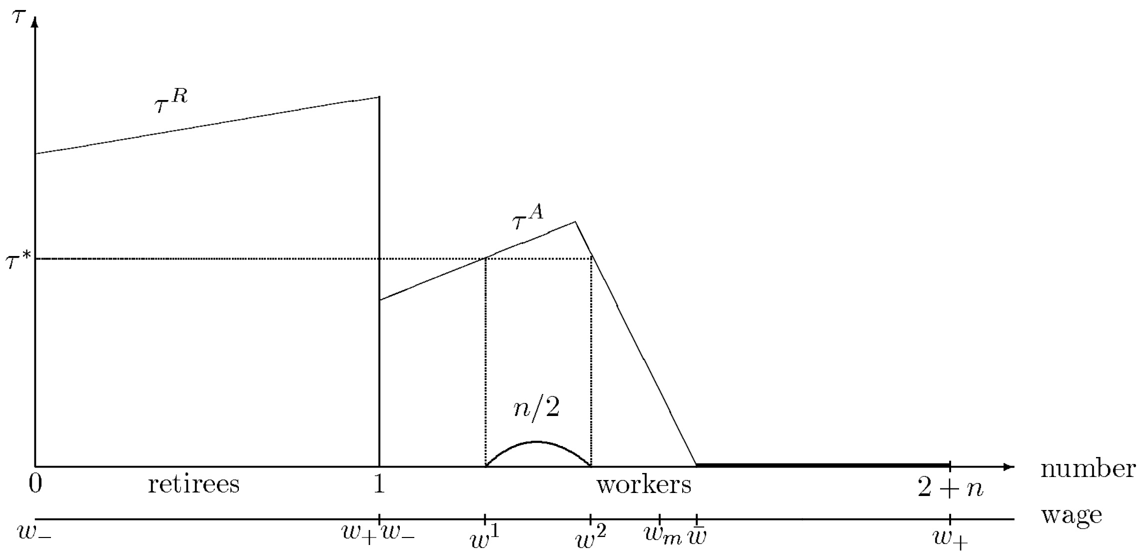

Figure 3: preferred tax rates with quadratic tax distortions when $r = n$

7.3 The majority voting solution

One can easily show that the objective function of each worker is concave relatively to its arguments, τ and s. Therefore, preferences over τ are single-peaked and the median voter theorem applies. In the next proposition, we characterize the Condorcet winner. For this purpose, we define w^* as the income level satisfying $\tau^A(w_-) = \tau^A(w^*)$.

Proposition 5 *When the noncontributory part of social security implies quadratic distortions, the majority voting equilibrium tax rate is characterized as follows:*

(i) If $\int_{w_-}^{\hat{w}} f(w)\,dw < n/2(1+n)$, *the majority voting equilibrium tax rate,* τ^*, *is 0.*

(ii) If $\int_{w_-}^{w^*} f(w)\,dw \leq n/2(1+n) \leq \int_{w_-}^{\hat{w}} f(w)\,dw$, *the majority voting*

equilibrium tax rate is the rate preferred by the workers with earning w^p *defined as follows:*

$$\int_{w_-}^{w^p} f(w)\,dw = \frac{n}{2(1+n)} \ . \tag{22}$$

(iii) If $\int_{w_-}^{w^*} f(w)\,dw > n/2(1+n)$, *the majority voting equilibrium tax rate is the rate preferred by the workers with earnings* w^1 *or* w^2 *satisfying:*

$$\int_{w^1}^{w^2} f(w)\,dw = \frac{n}{2(1+n)} \text{ and } \tau^A\left(w^1\right) = \tau^A\left(w^2\right). \tag{23}$$

The proof goes along the same lines as the one of proposition 2, so that we do not develop the formal argument here. The idea is that one must find a sufficient number of individuals in the working population to form a majority coalition with the retirees and sustain a positive equilibrium tax rate. Under the condition in (i), the number of young favoring a positive tax rate is not sufficient to form a majority coalition with the old and the PAYG pension system is not sustainable in the steady state. In (ii), the number of young individuals favoring the PAYG scheme is sufficient but the young with income between w^1 and w^2 are not enough to form a majority coalition with the old. Therefore, some young people with income $w > w^2$ belong also to that majority. Finally, in (iii), there are enough people in the interval $\left[w^1, w^2\right]$ and workers with income $w > w^2$ belong to the minority in favor of a lower tax rate than $\tau^A\left(w^1\right) = \tau^A\left(w^2\right)$. It is then clear that the set of workers who join the retirees to form a majority in favor of a positive tax rate is different from what it was without distortion. In particular, when $\int_{w_-}^{\hat{w}} f(w)\,dw$ is not close to $n/2(1+n)$, those with earnings equal or just below $\hat{w}$ do not belong to that majority.

8 Concluding comments

Throughout the paper, the more or less Bismarckian feature of the social security system was considered as given. A natural extension of our analysis would be to study the determination of α. A possible approach is to assume that the benefit formula is determined by a welfare maximizing authority at a first, constitutional stage. This approach is in line with the sequential nature of the decision process alluded to in the introduction. It has been adopted by Casamatta *et al.* (1998) but within a different context, namely social insurance. In that paper the choice of the Bismarckian parameter, α, is made at the constitutional level upon the expectation that the payroll tax is determined later through majority voting. One of the main results is that a positive Bismarckian parameter can be desirable, even though with full control of α and τ, the constitutional planner would set α equal to zero. This result arises because the redistributive character of the system has an impact on its political support. Specifically, when α is positive, the decisive voter chooses a tax rate that fits better the preferences of the low wage individuals. A Rawlsian constitutionalist can then favor such a positive α

Alternatively, one could consider a setting where both α and τ are chosen by majority voting. In this two-dimensional collective choice problem, a Condorcet winner does not generally exist. Therefore, one must give more structure to the political process. Some examples, again for the context of social insurance, can be found in Casamatta (1999), who studies sequential voting procedures, or in De Donder and Hendricks (1998), who study (two parties) electoral competition.

Another extension is to study the properties of our setting out of the steady state. In particular, it is interesting to study the transition following the occurrence of a shock like, for example, a sudden drop in fertility or in productivity. In such a setting, the issue of voting is more complex and depends very much on how expectations are formed. Also, the degree of redistributiveness can be very important when looking at social security reforms. It is often observed that the resistance to change by vested interests is related to entitlements founded on the contributory part of social security. In other words, the choice of α has an effect on political support in the steady state but also on political resistance out of steady state when reforms are contemplated. But this is clearly another story.

References

[1] Boadway, R. and D. Wildasin, (1989), A median voter model of social security, *International Economic Review*, 30, 307-28.

[2] Browning, E. K., (1975), Why the social insurance budget is too large in a democracy, *Economic Enquiry*, 13, 373-388.

[3] Casamatta, G., (1999), The political economy of social protection and redistribution, PhD dissertation.

[4] Casamatta, G., H. Cremer and P. Pestieau, (1999), Political sustainability and the design of social insurance, *Journal of Public Economics*, forthcoming.

[5] Conde Ruiz, J.I. and V. Galasso, (1999), "Positive arithmetic of the welfare state", *mimeo*, Universidad Carlos III de Madrid.

[6] De Donder, Ph. and J. Hindricks, (1998), The political economy of targeting, *Public Choice*, 95, 177-200.

[7] Epple, D. and R. Romano, (1996), Public provision of private goods, *Journal of Political Economy*, 104, 57-84.

[8] Hu, S. C., (1982), Social security, majority voting and dynamic efficiency, *International Economic Review*, 23, 269-287.

[9] Johnson, P., (1998), Pension provision and pensioner's incomes in 10 OECD countries, ISF, unpublished.

[10] Myles, G., (1995), *Public Economics*, Cambridge University Press.

[11] Tabellini, G., (1990), A positive theory of social security, NBER Working Paper #3272.

[12] Veall, M. R., (1986), Public pensions as optimal social contracts, *Journal of Public Economics*, 31, 237-51.

Appendix

A Appendix 1: proof of proposition 1

First observe that some straighforward algebra is sufficient to show that $v(\tau, w)$ is a concave function of τ. To prove (i), one differentiates $v(\tau, w)$ at $\tau = 0$:

$$\left.\frac{\partial v}{\partial \tau}\right|_{\tau=0} = -wu'(c) + \beta(1+n)(\alpha w + (1-\alpha)\overline{w})u'(d). \qquad (24)$$

At $\tau = 0$, everyone saves. Hence, substituting (5) in the previous expression, we obtain:

$$\left.\frac{\partial v}{\partial \tau}\right|_{\tau=0} = u'(c)\left(\frac{(1+n)}{(1+r)}(\alpha w + (1-\alpha)\overline{w}) - w\right). \qquad (25)$$

This expression is greater than 0 iff $w \leq \hat{w}$. Therefore, only these individuals will have a strictly positive preferred tax rate.

To prove (ii), let's write the first-order condition on τ for an individual with income $w < \hat{w}$:

$$-wu'(c) + \beta(1+n)(\alpha w + (1-\alpha)\overline{w})u'(d) = 0. \qquad (26)$$

Differentiating this expression with respect to w, we obtain:

$$\frac{\partial \tau^A}{\partial w} = \frac{\beta(1+n)(1-\alpha)\frac{\overline{w}}{w}u'(d)(1-\varepsilon)}{D_\tau}, \qquad (27)$$

where $D_\tau < 0$ is the second-order derivative of $v(\tau, w)$ with respect to τ. Clearly, under our assumption that $\sigma < 1$, the preferred tax rate of individuals with income below $\hat{w}$ is increasing with w.

To prove (iii), we note that the FOC $\partial v/\partial\tau = 0$ for $w \leq \hat{w}$ implies that workers with less than break-even level of earnings oppose too high a tax rate because it decreases their first period consumption. Indeed, when $\tau \to 1$, $u'(c) \to +\infty$ whereas the limit of $u'(d)$ is finite, which implies that the FOC cannot be satisfied.

B Appendix 2: proof of proposition 4

The first-order conditions associated to the program of a working individual are:

$$-wu'(c) + \beta(1+n)(\alpha w + (1-\alpha)\overline{w}(1-2\gamma\tau))u'(d) + \lambda_\tau = 0 \tag{28}$$

and

$$-u'(c) + \beta(1+r)u'(d) + \lambda_s = 0 \tag{29}$$

where λ_τ and λ_s are the Lagrange multipliers respectively associated to the non-negativity constraints on τ and s.

To solve the program, we have to consider different cases, depending on which constraint binds.

Case 1: $\lambda_\tau = \lambda_s = 0$

In this first case, none of the nonnegativity constraints bind, which means that we have an interior solution for both τ and s. Substituting (29) into (28), we obtain:

$$\tau^A(w) = \frac{(1+n)(\alpha w + (1-\alpha)\overline{w}) - w(1+r)}{2\gamma(1-\alpha)\overline{w}(1+n)}. \tag{30}$$

A necessary and sufficient condition for $\tau^A(w)$ to be positive is that $w < \hat{w} = (1+n)(1-\alpha)\overline{w}/((1+r) - \alpha(1+n))$. All the people with earnings above this threshold will have a preferred tax rate of 0 and will choose to rely exclusively on private savings. For people with wage lower than $\hat{w}$ who save privately, we have:

$$\frac{\partial \tau^A}{\partial w} = \frac{\alpha(1+n) - (1+r)}{2\gamma(1-\alpha)\overline{w}(1+n)} < 0. \tag{31}$$

Consequently, preferred tax rates are decreasing with income.

Using (29), we obtain:

$$\frac{\partial s^A(w)}{\partial w} = -\frac{\left(-\left(1-\tau^A\right) + w\frac{\partial \tau^A}{\partial w}\right)u''(c)}{u''(c) + (\beta(1+r))^2 u''(d)} -$$

$$\frac{\beta(1+r)\left(\frac{\partial\tau^A}{\partial w}(1+n)(\alpha w+(1-\alpha)\overline{w})+\alpha(1+n)\tau^A\right)u''(d)}{u''(c)+(\beta(1+r))^2u''(d)} = \frac{\alpha((1+n)(\alpha w+(1-\alpha)\overline{w})-w(1+r))}{\gamma(1-\alpha)\overline{w}}. \tag{32}$$

This expression is positive for individuals with wage below $\hat{w}$. The function $s^A(w)$ is thus increasing. Moreover, from (29), it is negative for w sufficiently close to 0 and positive for w sufficiently close to $\hat{w}$. Therefore, there exists a value of w, w', such that each individual with income above this value has a strictly positive saving and for people with income below w', the constraint $s \geq 0$ is binding.

To sum up, we have found that all the people with earnings between w' and $\hat{w}$ have an interior solution for both τ and s. These solutions are respectively given by (30) and (29). The other individuals have either a positive preferred tax rate with no savings or positive savings and a zero preferred tax rate. We will analyze these two cases one after the other.

Case 2: $\lambda_\tau = 0, \lambda_s > 0$

As shown before, individuals with earnings less than w' will choose a strictly positive tax rate and no savings. The value of the optimal tax rate is given by the condition (28).

Case 3: $\lambda_\tau > 0, \lambda_s = 0$

The richer individuals (those with earnings above $\hat{w}$) will rely exclusively on savings which value is given by (29).

In order to determine the majority voting solution, we must know how the preferred tax rates vary with income for individuals below w'. We obtain from (28) that:

$$\frac{\partial\tau^A}{\partial w} = \frac{(1-\varepsilon)(\beta\alpha(1+n)u'(d)-u'(c))-\alpha(1-\alpha)\beta\gamma\tau^2(1+n)^2\overline{w}u''(d)}{-D_\tau}. \tag{33}$$

For these people who do not save privately, we know from (29) that $u'(c) > \beta(1+r)u'(d)$ which implies that $u'(c) > \beta\alpha(1+n)u'(d)$. Hence, when $\varepsilon > 1$, this expression is positive, so that preferred tax rates are increasing with income.

Finally we show that the maximal preferred tax rate of the workers is lower than the minimal preferred tax rate of the retirees. The maximal preferred tax rate of the workers is:

$$\tau^A\left(w^{'}\right) = \frac{(1+n)\left(\alpha w^{'} + (1-\alpha)\overline{w}\right) - w^{'}(1+r)}{2\gamma(1-\alpha)\overline{w}(1+n)}, \tag{34}$$

and, assuming an interior solution, the minimal preferred tax rate of the retirees is:

$$\tau^R(w_-) = \frac{1}{2\gamma}\left(1 + \frac{\alpha}{1-\alpha}\frac{w_-}{\bar{w}}\right). \tag{35}$$

Therefore, the condition for the maximal preferred tax rate of the workers being higher than the minimal preferred tax rate of the retirees is $\tau^A\left(w^{'}\right) > \tau^R(w_-) \Leftrightarrow w^{'}(\alpha(1+n)-(1+r)) - \alpha w_-(1+n) > 0$. Knowing that $\alpha(1+n) < 1+r$, this is impossible. Hence the result.