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Abstract

We construct an asymmetric duopolistic R&D and production behavior model subject to knowledge spillovers. This model is an extension to the symmetric model of d'Aspremont and Jacquemin (A&J (1988)) and aims to determine the cooperative and non-cooperative R&D strategies for two agents of different size. The paper concludes that the introduction of asymmetry into the A&J (1988) model leads to different R&D expenditures and production decisions made by the firms. Simulations show that the bigger agent has larger R&D expenditures and higher output. If firms choose the monopoly collusion or the welfare-maximizing strategy, the optimal solution implies that R&D is conducted asymmetrically by both agents, but that production is conducted only by the largest agent.

Keywords: Innovation, R&D, spillovers, cooperation

JEL Classification: C72, D21, O31

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1. Introduction

The model developed by d'Aspremont and Jacquemin in 1988 has been the subject of several attempts for modification during recent times. The original model (referred to as A&J (1988) below) was built in a symmetric duopolistic setup for two agents making decisions for the research and development (R&D) and production stages in a two-stage process. Agents with equal marginal research and production costs choose whether they are willing to cooperate in one of these stages or in both, or not willing to cooperate at all.

The main goal of this paper is to introduce asymmetry into the A&J (1988) model. It is a rare occasion when two symmetric firms engage in R&D cooperation or competition. Much more often we face 'games' played by asymmetric players of different size and 'power'. It is very interesting to analyze the case of strategic R&D decisions in an asymmetric environment. In our opinion, one of such opportunities can be obtained if we assume that firms in the industry are different in their cost functions.

Certain steps towards the study of asymmetry in the R&D-production strategies have already been made in the literature before. An important finding was presented by Salant and Shaffer (1998) who have shown that a corner (asymmetric) solution provides sub-optimal profits even for the symmetric A&J model with two identical firms. They concentrated their attention on the symmetric A&J (1988) setup, but took into consideration several 'corner' solutions of the problem, where one of the agents has zero R&D expenditures or output. Another typical corner solution when only one firm conducted research in non-cooperative cases was considered in the work by Boivin and Vancachellum (1995) (firms still had identical costs). Our research will also consider corner solutions, but it will be done for the case of two firms of different size. We shall to compare solutions obtained from such asymmetric setup to solutions in the symmetric case.

A very interesting asymmetric model was recently constructed by Petit and Tolwinski (1999). Their model combines the introduction of an asymmetry of agents and a dynamic game approach. For the dynamic setup they introduce a knowledge capital evolution factor, which describes an inter-temporal influence of R&D decisions on the future productivity of dynamic agents. Agents are assumed to have different production cost function parameters and different rates of innovation. In this model the rate of innovation serves as a measure for the R&D capital accumulation in a particular time period. Petit and Tolwinski used their model to derive major implications about the R&D stock for the whole industry and did not consider the R&D expenditures and output of individual firms. In our model we would prefer to keep the static setup, but concentrate more on the individual agent's strategies, still deriving implications on the industry level. From this point of view, the static model makes it easier to reach the conclusions about the firm's individual behavior, which is our main concern in this particular research.

Here we would like to mention other possible developments of the A&J (1988) model investigating different aspects of R&D strategic behavior. De Bondt and Henriques (1995) analyzed the effect of asymmetric spillovers between two agents. In their model agents have a different ability to absorb and accumulate knowledge obtained by their competitors. Rutsaert (1995) has introduced a third party into the R&D production setting, i.e. a foreign firm which competes in R&D and production with two domestic firms in an open economy. Leahy and Neary (1997) considered the case of international knowledge spillovers in another A&J-type model. They studied the feasibility of subsidies towards firms that generate knowledge spillovers in the presence of domestic and international competitors. Suzumura (1992) introduced an output oligopoly into the standard framework. A symmetric oligopolistic extension to the A&J (1988) model was studied by Brod and Shivakumar (1997) and industry-level implications were made. De Bondt (1992) and Kamien (1992) extended the homogeneous symmetric model to the heterogeneous, but still symmetric model with differentiated products under output and price competition.

All these models have one common feature. They still assume that the domestic industry consists of symmetric agents with the same cost structure. In our research we will investigate the strategic R&D behavior of firms that are different in size. We shall study a simple duopolistic structure of the industry in a closed economy. The main attention will be paid to the individual behavior of each agent depending on the level of asymmetry in the industry and the strength of knowledge spillovers.

In this paper we introduce the assumption of two agents being different in size and engaged in the R&D and production processes in a common market. Difference in size between two agents is explicitly formulated by assumptions that one agent has a lower marginal cost of production and a lower per unit marginal cost of research (and which is generally considered to be 'bigger'). These differences are simultaneously introduced into the model. Then we find appropriate solutions for different cooperation-competition strategies and compare obtained implications with results from the previous literature discussed above.

Section 2 of this paper presents an asymmetric R&D and production behavior model subject to knowledge spillovers. Asymmetry in output and research cost functions of the agents is introduced into the model and optimal solutions are found. Section 3 provides a description and results of numeric simulations. Section 4 summarizes the main findings of the research and presents conclusions.

2. The Model

2.1. The Original A&J (1988) Model

The model deals with an industry consisting of two agents operating in a common market. The agents' behavior is described by a two-stage game. In the first stage agents are assumed to make decisions about their R&D expenditures. In the second stage the agents are assumed to build their profit maximizing production plans.

The inverse industry demand function is assumed to be linear:

$$p = a - bQ,$$

where $Q := q_1 + q_2$ is the overall or industry production of two agents with $a, b > 0$.

A particular agent's individual production cost function includes the effect of the research conducted by another agent. The influence of firm *j's* research on firm *i's* marginal cost is multiplied by a constant *b*, which represents the knowledge spillover effect. In the interpretation of Plasmans *et al.* (1999) *b* indicates how intensive firm *i*'s production costs are decreased by the firm *j*'s research efforts without any direct renumeration of firm *i* to firm j. Agent *i*'s production cost function is then:

$$C_i(q_i, x_i, x_j) = (A - x_i - b\,x_j)q_i, \quad i, j = 1, 2, \; i \neq j,$$

where $0 < A < a$ (the intercept of the inverse demand function is higher than the highest value of the marginal cost), $0 < b < 1$ (the interval of the spillover effect levels), $x_i + b\,x_j \leq A$ (the marginal production cost is non-negative), $Q \le \frac{a}{b}$ (means that $p \ge 0$). It is directly visible from this cost function that research reduces the marginal cost of production.

The A&J model uses a quadratic research cost function, which implies that R&D is subject to diminishing returns to research expenditures (assuming that the revenue function is linear in terms of the R&D expenditures). Thus, there are no evident economies to scale for the firm:

$$C_i^r(x_i) = g\,\frac{x_i^2}{2},\ i = 1,2,$$

where $g > 0$ is the per unit marginal cost of research.

2.2. Introduction of Asymmetry into the Model

Extend now the original setup to an asymmetric case. We assume that our agents are different in size and thus have different R&D and production cost functions. A particular agent's R&D cost function is:

$$C_i^r(x_i) = g_i\,\frac{x_i^2}{2},\ i = 1,2,$$

where $g_i > 0$ is agent *i*'s per-unit marginal cost of research.

Agent *i*'s production cost function becomes:

$$C_i(q_i, x_i, x_j) = (A_i - x_i - b\,x_j)q_i,\ i, j = 1,2,\ i \ne j,$$

where A_i is agent *i*'s marginal cost of production when there is no R&D conducted.

Agent 1 is assumed to be larger than agent 2. The difference in size is reflected by having different values of *A*s and *g*s for different agents. According to our assumption we expect $A_1 < A_2$ and $g_1 < g_2$, i.e. we expect that the bigger agent has a lower marginal cost of production and also a lower per unit marginal cost of research. Here we assume that the advantage in size can directly be translated into the advantage in cost.

Similarly to A&J (1988) we will consider three major modes of cooperation between the two agents in the observed industry:

1) competition between the agents in both research and production stages;
2) cooperation between the agents in research, but competition in production;
3) cooperation between the agents in both research and production.

For the purposes of analyzing and comparing the social outcomes of each strategy we will also consider the welfare-maximizing solution of the model.

2.3. The Non-cooperative Mode

Let us assume that the agents decide to compete in both R&D and production. Then the Nash equilibrium for the two-stage game is the optimal equilibrium. We solve the problem backwards starting from the production stage. In this stage each agent chooses his profit-maximizing output quantities depending on the R&D expenditures (conditional on x_i and x_j):

$$\max_{\{q_i\}} p_i = [a - b(q_i + q_j)]q_i - (A_i - x_i - b\, x_j)q_i - g_i \frac{x_i^2}{2}, \; i, j = 1, 2, \; i \neq j,$$

Solution of this problem is derived directly from the first order condition (see Appendix A) and gives the competitive production decision rules for each firm in the second stage:

$$q_i^* = \frac{a - 2A_i + A_j + (2 - b)x_i + (2b - 1)x_j}{3b}, \; i, j = 1, 2, \; i \neq j.$$

so that the firm *i*'s output is positively linearly dependent on its own R&D expenditures, but can positively or negatively depend on (or can be independent of) the firm *j*'s research. If $b > 0.5$, then q_i^* and x_j are positively related, otherwise the relationship is negative for $b < 0.5$ or does not exist for $b = 0.5$.

In the first stage of the game each agent faces another profit maximization problem in order to figure out the optimal amount of R&D expenditures subject to a competitive production decision rule:

$$\max_{\{x_i\}} p_i^* = [a - b(q_i^* + q_j^*)]q_i^* - (A_i - x_i - b\, x_j)q_i^* - g_i \frac{x_i^2}{2}, \; i, j = 1, 2, \; i \neq j.$$

After we substitute the q^*s into the profit function and rearrange the terms we get (see Appendix A.1.):

$$\max_{\{x_i\}} p_i^* = \frac{[a - 2A_i + A_j + (2 - b)x_i + (2b - 1)x_j]^2}{9b} - g_i \frac{x_i^2}{2}, \; i, j = 1, 2, \; i \neq j.$$

The solution of this problem is more complex in terms of an algebraic representation than its symmetric analogue:

$$x_i^* = -2(2 - b)\frac{D_j F_j - CF_i}{D_i D_j - C^2}, \; i, j = 1, 2, \; i \neq j,$$

where:

$$D_i := 2(2 - b)^2 - 9bg_i,$$

$$F_i := a - 2A_j + A_i,$$

$$C := 2(2b - 1)(2 - b),$$

$$i, j = 1, 2,$$

$$i \neq j.$$

Numerical simulations will be run to test the sensitivity of R&D on various parameters.

Additionally we have conducted a test of this solution by plugging into it the parameters corresponding to the symmetric case. We assumed $g_1 = g_2 = g$ and $A_1 = A_2 = A$, which implies having two identical firms in the industry. As is shown in Appendix A.1., our results provide exactly the same non-cooperative solution (x_i^* s and q_i^* s) as the solution given by A&J (1988).

2.4. The R&D Cooperation Mode

In this mode firms choose to cooperate in the research stage and compete in production. The production problem is solved in the same way as the previous case (obtaining the Nash equilibrium). However, the R&D optimization now implies solving for the joint profit maximization:

$$\max_{\{x_1,x_2\}} p = \frac{[a-2A_1+A_2+(2-b)x_1+(2b-1)x_2]^2+[a-2A_2+A_1+(2-b)x_2+(2b-1)x_1]^2}{9b} -$$

$$-g_1\frac{x_1^{\ 2}}{2}-g_2\frac{x_2^{\ 2}}{2}.$$

In this mode we have the following optimal solutions expressed using the same predefined constants D_i, F_i, C (see Appendix A.2.):

$$\hat{q}_i = \frac{a-2A_i+A_j+(2-b)\hat{x}_i+(2b-1)\hat{x}_j}{3b}\text{, with}$$

$$\hat{x}_i = \frac{[(2b-1)^2+\frac{1}{2}D_j][-(2b-1)F_i-(2-b)F_j]+C[(2-b)F_i+(2b-1)F_j]}{[(2b-1)^2+\frac{1}{2}D_i][(2b-1)^2+\frac{1}{2}D_j]-C^2},$$

$i, j=1,2$, $i \neq j$.

The symmetry test of this R&D cooperation mode solution indicates that it is consistent with the corresponding findings given in A&J (1988). Assuming that our firms are identical, we get the same results as the ones obtained by d'Aspremont and Jacquemin (see Appendix A.2.)

2.5. The Monopolistic Collusion Mode

In this mode the firms choose to cooperate in both the research and production stages. This mode can also be considered as the case of monopolistic collusion between the two agents. The optimal research expenditures and outputs in this case originate from the solution of the following problem:

$$\max_{\{q_1,q_2,x_1,x_2\}} p = [a-b(q_1+q_2)]q_1-[A_1-x_1-b\,x_2]q_1-g_1\frac{x_1^{\ 2}}{2}+$$

$$+[a-b(q_1+q_2)]q_2-[A_2-x_2-b\,x_1]q_2-g_2\frac{x_2^{\ 2}}{2}.$$

The first order conditions (FOCs) in matrix form are:

$$\begin{pmatrix} -2b & -2b & 1 & b \\ -2b & -2b & b & 1 \\ 1 & b & -g_1 & 0 \\ b & 1 & 0 & -g_2 \end{pmatrix}\begin{pmatrix} q_1^{\ *} \\ q_2^{\ *} \\ x_1^{\ *} \\ x_2^{\ *} \end{pmatrix}=\begin{pmatrix} A_1-a \\ A_2-a \\ 0 \\ 0 \end{pmatrix}.$$

Now we should point out that this FOCs-matrix is only semi-definite, because the second principal minor is singular. This means that for the asymmetric setup monopolistic collusion does not satisfy the second order conditions for maximization, thus does not provide a unique solution for the global maximum given the assumed functional form. One way to resolve this difficulty is to consider the possibility of corner solutions of this cooperative problem. We have to consider all the rationally feasible combinations of corner solutions in the attempt to choose the allocation, which determines the maximum value of joint profit function (presented in Table B.1 of Appendix B).

After numerical simulations and some tests, which are described below, we have concluded that regardless of the level of asymmetry and for any non-zero value of b, one corner solution always yields a higher value of the joint profit function. This is corner solution number 2, which corresponds to the case where both firms conduct research and only firm 1 produces the output (see Table B.1 in Appendix B and Graph C.1 in Appendix C). Then the optimal solution satisfies:

$$\tilde{q}_1 = \frac{g_1 g_2 (A_1 - a)}{b^2 g_1 - 2b g_1 g_2 + g_2},$$

$$\tilde{x}_1 = \frac{g_2 (A_1 - a)}{b^2 g_1 - 2b g_1 g_2 + g_2}, \text{ and } \tilde{x}_2 = \frac{b g_1 (A_1 - a)}{b^2 g_1 - 2b g_1 g_2 + g_2}.$$

This solution represents the optimal strategy for a wide range of asymmetries between the two firms. We tested this fact in the range from a relatively small difference in the marginal cost structure of the firms (see the simulation section below) to a large difference in the marginal cost structure. In this range, given the values of the simulation parameters, the solution presented above is the optimal strategy for monopolistic collusion.

2.6. The Welfare-Maximizing Mode

For reasons of comparison with other strategies we consider the additional problem which maximizes the social welfare in the R&D / production game. The welfare function is obtained by adding the industry's profit and the consumer surplus:

$$\max_{\{q_1,q_2,x_1,x_2\}} p = a(q_1+q_2) - b(q_1+q_2)^2 - [A_1 - x_1 - b\,x_2]q_1 - g_1\frac{x_1^2}{2} - [A_2 - x_2 - b\,x_1]q_2 - g_2\frac{x_2^2}{2}.$$

The corresponding FOCs are:

$$\begin{pmatrix} -b & -b & 1 & b \\ -b & -b & b & 1 \\ 1 & b & -g_1 & 0 \\ b & 1 & 0 & -g_2 \end{pmatrix}\begin{pmatrix} \hat{q}_1 \\ \hat{q}_2 \\ \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ A_2 - a \\ 0 \\ 0 \end{pmatrix}.$$

As it can be seen, the welfare maximization problem has the same drawback as the monopolistic collusion. The FOCs-matrix is not perfectly negatively definite and does not satisfy the second order conditions for the existence of a unique global solution either. In this case we also consider the set of rationally feasible corner solutions in order to derive the welfare maximizing allocation.

Simulation of welfare maximization yields the same strategic implications as in the monopoly case. This is not surprising due to the great similarity of the FOC of these two problems. And similarly to the monopoly case, it is concluded that for the welfare maximizing problem the optimal corner solution is the case where both firms conduct R&D and only firm 1 engages in production (see Table B.1 in Appendix B and Graph C.2 in Appendix C):

$$q_1^{**} = \frac{g_1 g_2 (A_1 - a)}{b^2 g_1 - 2b g_1 g_2 + g_2},$$

$$x_1^{**} = \frac{g_2 (A_1 - a)}{b^2 g_1 - 2b g_1 g_2 + g_2}, \text{ and } x_2^{**} = \frac{b g_1 (A_1 - a)}{b^2 g_1 - 2b g_1 g_2 + g_2}.$$

Similarly to the monopoly case, this solution was also tested in different positions in the range of values for the asymmetry level (see the simulation results below).

3. Simulation

3.1. The Implications from the Second Order Conditions (SOCs) on the Simulation Parameters

Before we could start the simulation of obtained solutions we need to discuss the limitations, which are imposed by the SOCs on the parameters of the model. In particular, there are limitations imposed by the SOCs on the values of parameters b, g_1, and g_2. It will best serve our purposes, if we use in the simulation those values of parameters, which satisfy SOCs for all different problems discussed above. In this case all simulations will describe a situation where each mode has its corresponding solution and thus can be considered as a feasible strategy.

The non-cooperative mode problem always has a unique solution if the following conditions are satisfied (see section A.1.3. in Appendix A):

$$bg_i > \frac{8}{9},\ i=1,2.$$

R&D cooperation requires that (see section A.2.3. in Appendix A):

$$bg_i > \frac{10}{9},\ i=1,2, \text{ and } [10-9bg_1][10-9bg_2] > 64.$$

For the monopolistic collusion mode all SOCs from different corner combinations are summarized in the following set of expressions (see section B.2. in Appendix B):

$$2bg_i > 1,\ 2bg_i - \frac{g_i}{g_2} > 1,\ i=1,2.$$

And correspondingly for the welfare maximization, we have (see section B.2. in Appendix B):

$$bg_i > 1,\ bg_i - \frac{g_i}{g_j} > 0,\ i,j=1,2,\ i \neq j.$$

It is clear that many of these inequalities can be omitted as ones being implicit by other more general conditions. The reduced joint set of parameter restrictions now consists of five inequalities:

$$bg_i > \frac{10}{9},$$

$$bg_i - \frac{g_i}{g_j} > 1,$$

$$[10-9bg_1][10-9bg_2] > 64,$$

$i,j=1,2,\ i \neq j.$

Thus, if we choose values of b, g_1, and g_2 such that all these restrictions will be satisfied, then we will operate in the space, where each mode will provide us with a unique solution. Then we can compare them and derive our implications.

3.2. Calibrating the Model for Simulation

In addition to the limitations on parameters implied by SOCs the model must also be calibrated with respect to the basic rationales. There are two important conditions in the problem setup that must be satisfied. However, the difficulty is that it is impossible to 'tune up' the parameters before solving the problem in order to get these conditions holding. These conditions are: $Q \le \frac{a}{b}$, and $x_i + b x_j \le A_i$. As it can be seen, the conditions use endogenous variables and thus can not be used in a prior calibration of parameters.

We propose the following two-step procedure of calibrating the model for simulation. First, we set the values of the parameters in such a way that SOCs and simpler model conditions are satisfied. After that we run the series of simulations and sensitivity tests to figure out which set of parameters will satisfy the mentioned more complicated conditions.

Because we are interested in the testing of solutions for a wide range of asymmetry levels, we must be sure, that selected parameters also satisfy the above conditions on both upper and lower limits of the selected interval. In this paper we use that parameter set, which gives a valid range of asymmetry: $1 \le A_2 / A_1, g_2 / g_1 \le 5$. We assume a 1.05 difference ratio to represent 'small asymmetry' in marginal costs and a difference ratio of 5 to indicate 'large asymmetry'.

3.3. Simulation 'positions'

We have selected eight different 'asymmetric positions' of interest for simulation. Each position is determined by a pair of asymmetry ratios, indicating the difference in marginal cost of production and per unit marginal cost of research respectively for two agents. These eight positions are divided in two groups: a small asymmetry group (positions 1, 2 and 3 in Figure 1) and a large symmetry group (positions 4, 5, 6, 7, and 8).

Position 1 corresponds to the case where both agents are 'slightly' different in their cost functions ($A_2/A_1 = g_2/g_1 = 1.05$). It is accompanied by two other positions, implying an asymmetry in only one cost component: research costs (position 2, $A_2/A_1 = 1 \neq g_2/g_1 = 1.05$) and production costs (position 3, $A_2/A_1 = 1.05 \neq g_2/g_1 = 1$) respectively.

On the 'large asymmetry' side we have position 4, which assumes that agents are symmetric in marginal production costs and extremely asymmetric in per unit marginal research costs ($A_2/A_1 = 1 \neq g_2/g_1 = 5$). Its companion is position 5 where firms are largely asymmetric in per unit marginal research costs and have only a slight asymmetry in marginal production costs ($A_2/A_1 = 1.05 \neq g_2/g_1 = 5$).

In the opposite 'corner' of the map we have firms being extremely asymmetric in marginal production costs, but symmetric in per unit marginal research costs (position 6, $A_2/A_1 = 5 \neq g_2/g_1 = 1$) or slightly asymmetric in this aspect (position 7, $A_2/A_1 = 5 \neq g_2/g_1 = 1.05$).

Finally, the position of two extremes is position 8, where both firms are largely asymmetric in marginal production and per unit marginal research cost functions ($A_2/A_1 = g_2/g_1 = 5$). We run same simulations in each of the described positions and compare results.

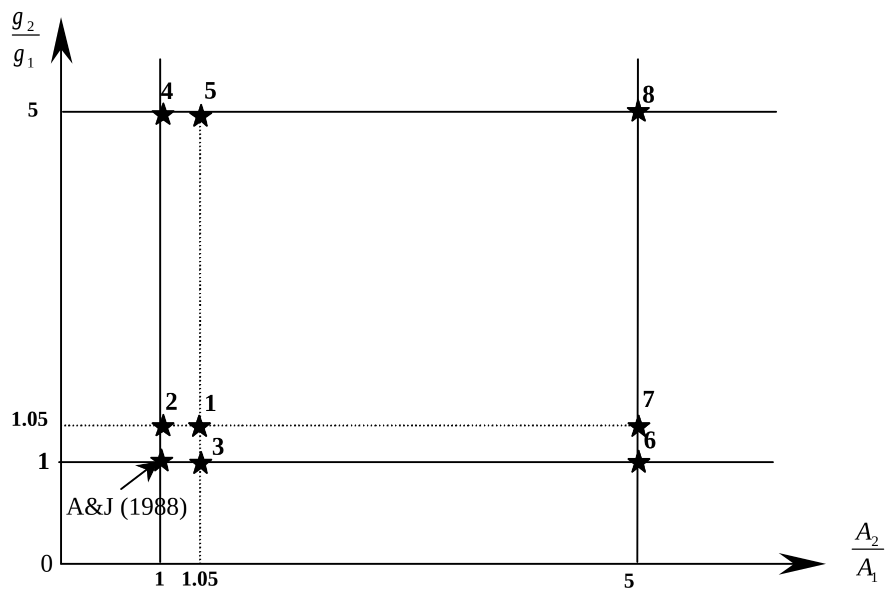


Figure 1. The map of 'asymmetric positions'.

Also we present the table, which contains the values of model parameters corresponding to each (a)symmetric position simulated:

Position	A_1	A_2	g_1	g_2	a	b
1	10	10.5	15	15.75	250	5
2	10	10	15	15.75	250	5
3	10	10.5	15	15	250	5
4	10	10	15	75	250	5
5	10	10.5	15	75	250	5
6	10	50	15	15	250	5
7	10	50	15	15.75	250	5
8	10	50	15	75	250	5

Table 1. Simulation input parameters.

3.4. Simulation results

All relevant graphs illustrating the numerical simulations are presented in Appendix C. The major implications which can be derived from these numerical simulations are grouped in several blocks.

Block 1. Implications common for all 8 positions:

- corner solution 2 yields the highest profits in the monopolistic collusion mode and the highest welfare in the welfare-maximizing mode (see Graphs C.x.1, $x = 1..8$, in Appendix C). Thus, corner solution 2 is considered to be an optimal solution for monopoly and welfare-maximizing problems given the interval of feasible spillover effect's values: $0 \leq b \leq 1$;
- a bigger firm has a higher level of R&D expenditures and a higher output in all modes of cooperation given the interval of feasible spillover effect's values (see Graphs C.x.3, $x = 1..8$, in Appendix C). Even in the position where two firms have identical marginal costs of production, the firm with per unit marginal cost advantages in R&D has higher output. And when two firms have identical per unit marginal costs of research, the firm with marginal cost advantage in production has higher R&D expenditures;
- on the industry level the industry's R&D expenditures are distributed in a way consistent with the implications presented in A&J (1988) (see Graphs C.x.4, $x = 1..8$, in Appendix C):

	Spillover Effect	
	Weak $(b \to 0)$	**Strong** $(b \to 1)$
IMPLICATIONS	$\hat{X} < \tilde{X} < X^* < X^{**}$ $\tilde{Q} < \hat{Q} < Q^* < Q^{**}$ $\tilde{W} < \hat{W} < W^* < W^{**}$	$X^* < \hat{X} < \tilde{X} < X^{**}$ $\tilde{Q} < Q^* < \hat{Q} < Q^{**}$ $\tilde{W} < W^* < \hat{W} < W^{**}$

Table 2. Industry-level implications.

- the analysis of industry's profit functions shows the following relationship of profits in different modes: $p^{**} < p^{*} \leq \hat{p} < \tilde{p}$ (see Graphs C.x.2, $x = 1..8$, in Appendix C), i.e. R&D cooperation is in general (except for one point) preferred to non-cooperative behavior.

Block 2. Implications specific to positions 4 and 5 (firm 1 has large cost advantage in R&D, but not in production, see Graphs C.x.2, $x = 1..8$, in Appendix C)*:*

- in the 'production competitive' (non-cooperative and R&D cooperation) modes firm 1 prefers R&D cooperation to non-cooperative strategy (except one point, where it is indifferent), because its profits in R&D cooperation mode are higher or equal to those in the non-cooperative mode;
- in the production competitive modes firm 2 prefers non-cooperative strategy to R&D cooperation (except in one point where it is indifferent).

Block 3. Implications specific to positions 6 and 7 (firm 1 has large cost advantage in production, but not in R&D, see Graphs C.x.2, $x = 1..8$, in Appendix C).

The results in these positions are directly opposite to the results in the positions of Block 2: firm 1 prefers non-cooperative behavior to R&D cooperation in production competitive modes, but firm 2 prefers R&D cooperation to competition.

4. Conclusions

Research and simulations conducted in this paper have shown that the introduction of asymmetry into the A&J (1988) model does imply a substantial difference into the behavior of incumbent firms and the industry as a whole. The existence of asymmetry and also its degree have an influence on the optimal R&D and production strategies of the firms. We can summarize the major findings in the following statements.

- Asymmetric firms make different expenditures and production decisions. The firm, which has a marginal cost advantage over another, has higher R&D expenditures and output. Furthermore, the cases of the monopolistic collusion and welfare maximization do not have a unique non-zero solution for both firms, and thus corner solutions are considered. The optimal profit-maximizing corner solution of the monopoly and welfare maximizing cases implies that the research is conducted asymmetrically by two firms, and only one firm (with the lower marginal cost of production) should produce the output.
- The analysis of simulated profit functions provided several very interesting results.
  - (i) From the profit-maximizing point of view the monopolistic collusion is the most desirable on the industry level. But asymmetric individual firms can have certain considerations about that. If the asymmetry is 'not balanced', i.e. one firm has large advantage only in one aspect (either marginal cost of production or per unit marginal cost of R&D), the firms prefer different strategies in the same situations.
  - (ii) The firm, which has large advantage in R&D costs, but small or no advantage in production, generally chooses to compete and not to engage in R&D cooperation with another agent. The bigger firm has a higher level of its own 'cheaper' R&D in the non-cooperative mode. In the case of R&D cooperation its research expenditures are lower and thus there are smaller positive benefits for its production costs. But the smaller agent prefers the R&D cooperation, to be able to benefit from a cheaper research of his bigger partner.
  - (iii) When one agent has a large cost advantage in production and not a similar advantage in research, the situation becomes quite opposite. The bigger agent prefers to cooperate in R&D, in order to benefit more from another agent's research at similar cost. But the smaller firm prefers not to cooperate, because overall benefits of joint research are unable to offset the initial disadvantage in production costs, which the smaller firm exhibits in this case.
- Finally we conclude that the major industry-level implications about the total R&D expenditures, output, and total welfare are very similar to the findings from the original A&J (1988) model. Thus the presence of asymmetric agents in the industry does not alter the main features of industry's R&D and production behavior.

All these findings now indicate that the asymmetric agents' behavior in the R&D-intensive industries depends very much on their relative size and the size of knowledge spillovers. Here we should realize the importance of a proper empirical investigation of asymmetry in different sectors. It is interesting to find out what is the major 'asymmetry range' in which firms operate. The spillover effect is an another important object for future study. What could be the most appropriate measure for the knowledge spillover? On which factors does it depend? All these questions open a wide field for a future inquiry.

APPENDIX A

The Model Solutions for the Non-cooperative and R&D Cooperation Modes

A.1. The Non-cooperative Mode

A.1.1. First Order Conditions (FOCs)

The FOCs in the non-cooperative mode problem are represented by the two systems of equations corresponding to each step of a non-cooperative game. We use matrix notation to represent them. In the *production step* solutions of the problem:

$$\max_{\{q_i\}} p_i = [a - b(q_i + q_j)]q_i - (A_i - x_i - b\,x_j)q_i - g_i \frac{x_i^2}{2},\ i, j = 1,2,\ i \neq j.$$

must satisfy the following system of equations, which was built using the FOC from the firms' maximization problems:

$$\begin{pmatrix} -2b & -b \\ -b & -2b \end{pmatrix}\begin{pmatrix} q_1^* \\ q_2^* \end{pmatrix} = \begin{pmatrix} A_1 - a - x_1 - b\,x_2 \\ A_2 - a - x_2 - b\,x_1 \end{pmatrix}.$$

so that the optimal solution is obtained by applying Cramer's Rule:

$$q_i^* = \frac{a - 2A_i + A_j + (2-b)x_i + (2b-1)x_j}{3b},\ i, j = 1,2,\ i \neq j.$$

The *research step* problem requires certain preliminary transformations before we proceed to solving it. The initial problem is:

$$\max_{\{x_i\}} p_i^* = [a - b(q_i^* + q_j^*)]q_i^* - (A_i - x_i - b\,x_j)q_i^* - g_i \frac{x_i^2}{2},\ i, j = 1,2,\ i \neq j.$$

We note that:

$$Q^* := q_1^* + q_2^* = \frac{2a - A_1 - A_2 + (1+b)x_1 + (1+b)x_2}{3b}.$$

Substituting Q^* into p_i^* we get:

$$p_i^* = (a - b\frac{2a - A_i - A_j + (1+b)x_i + (1+b)x_j}{3b} - A_i + x_i + b\,x_j)q_i^* - g_i \frac{x_i^2}{2} =$$

$$= \frac{a - 2A_i + A_j + (2-b)x_i + (2b-1)x_j}{3} q_i^* - g_i \frac{x_i^2}{2} =$$

$$= \frac{[a - 2A_i + A_j + (2-b)x_i + (2b-1)x_j]^2}{9b} - g_i \frac{x_i^2}{2}.$$

Thus the initial (2nd stage) problem is equivalent to:

$$\max_{\{x_i\}} p_i^* = \frac{[a - 2A_i + A_j + (2-b)x_i + (2b-1)x_j]^2}{9b} - g_i \frac{x_i^2}{2},\ i, j = 1,2,\ i \neq j,$$

and we can find solutions by uniting the FOCs of the individual problems into the system:

$$\begin{pmatrix} 2(2-b)^2-9bg_1 & 2(2b-1)(2-b) \\ 2(2b-1)(2-b) & 2(2-b)^2-9bg_2 \end{pmatrix}\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix}=\begin{pmatrix} -2(2-b)(a-2A_1+A_2) \\ -2(2-b)(a-2A_2+A_1) \end{pmatrix}.$$

After we obtain x_i^* we substitute them into q_i^* and get the solution explicitly written out in terms of the exogenous variables.

$$x_i^* = -2(2-b)\frac{D_jF_j - CF_i}{D_iD_j - C^2},\ i, j = 1, 2,\ i \neq j,$$

where:

$$\begin{aligned} &D_i := 2(2-b)^2 - 9bg_i, \\ &F_i := a - 2A_j + A_i, \\ &C := 2(2b-1)(2-b), \\ &i, j = 1, 2, \\ &i \neq j. \end{aligned}$$

We substitute x_i^* s into the expressions of q_i^* s to obtain the finalized optimal outputs.

A.1.2. The Symmetry Check

Here we would like to make one additional check of the solution, which we called a ‘symmetry check’. We will set $g_1 = g_2 =: g$ and $A_1 = A_2 =: A$ (which will give us two identical firms), substitute them into our solution and see if it will differ from the corresponding solution of the A&J (1988) problem.

First we find the values of x_i^* s. Considering two identical firms we get:

$$\begin{aligned} &D_1 = D_2 = D = 2(2-b)^2 - 9bg, \\ &F_1 = F_2 = F = a - A, \\ &C := 2(2b-1)(2-b). \end{aligned}$$

Then:

$$\begin{aligned} &x_1^* = x_2^* = x^* = -2(2-b)F\frac{D-C}{D^2-C^2} = -\frac{2(2-b)F}{D+C}, \\ &x^* = -\frac{2(a-A)(2-b)}{2(2-b)^2 - 9bg + 2(2b-1)(2-b)} = \\ &\quad = \frac{(a-A)(2-b)}{4.5bg - (2-b)(2-b+2b-1)} = \\ &\quad = \frac{(a-A)(2-b)}{4.5bg - (2-b)(1+b)} \end{aligned}$$

As we can see, this result is the same as the result obtained in A&J (1988, p.1134). Considering q_i^* s, we get:

$$q_1^* = q_2^* = \frac{a - A + (1 + b)x}{3b} =$$

$$= \frac{a - A}{3b} + \frac{1 + b}{3b}\left[\frac{(a - A)(2 - b)}{4.5bg - (2 - b)(1 + b)}\right].$$

Thus for two identical firms in the non-cooperative mode our solution provides the same results as the non-cooperative solution of the A&J (1988) model.

A.1.3. The Second Order Condition (SOC) Implications

In the non-cooperative mode each stage of the game provides its own SOCs set. We should analyze them in order to figure out the limitations imposed on the simulation parameters.

In the second stage firms solve individual profit maximization problems for optimal outputs in terms of the R&D expenditures (see section A.1.1.) SOCs that must be satisfied for obtaining the unique profit-maximizing solution of these problems are expressed as:

$$\frac{\partial^2 p_i}{\partial q_i^2} = -2b\,.$$

This derivative will always be negative, because it is assumed that *b>0*. In the first stage firms maximize their profits in terms of the R&D expenditures. Their SOCs (for profit-maximization) are:

$$\frac{\partial^2 p_i^*}{\partial x_i^2} = \frac{2}{9b}(2 - b)^2 - g_1 < 0$$

This implies:

$$bg_i > \frac{2(2 - b)^2}{9}$$

As it was mentioned in the body of the paper, our goal is to select such simulation parameters, which will satisfy all restrictions necessary for having the solutions of every single R&D behavior mode. Therefore we want our SOCs to be satisfied also for any given value of ***b***. It can be shown that:

$$1 \le (2 - b)^2 \le 4\,.$$

Thus successful maximization of firms' profits in the first-stage non-cooperative problem for any value of b implies that:

$$bg_i > \frac{8}{9}\,.$$

We use a similar procedure in the analysis of SOCs for other modes.

A.2. R&D Cooperation Mode

A.2.1. First Order Conditions (FOCs)

In this mode the production step problem is solved in the same way as it was done for the non-cooperative mode leading to the same structural form of optimal outputs. But in the research stage we solve the following problem:

$$\max_{\{x_1,x_2\}} p = \frac{1}{9b}[(a-2A_1+A_2+(2-b)x_1+(2b-1)x_2)^2+(a-2A_2+A_1+(2-b)x_2+(2b-1)x_1)^2]$$
$$-g_1\frac{x_1^2}{2}-g_2\frac{x_2^2}{2}.$$

In matrix notation FOCs are presented by:

$$\begin{pmatrix} 2(2b-1)^2+D_1 & 2C \\ 2C & 2(2b-1)^2+D_2 \end{pmatrix}\begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix}=\begin{pmatrix} -2(2-b)F_2-2(2b-1)F_1 \\ -2(2b-1)F_2-2(2-b)F_1 \end{pmatrix},$$

where

$$D_i := 2(2-b)^2-9bg_i,$$
$$F_i := a-2A_j+A_i,$$
$$C := 2(2b-1)(2-b),$$
$$i, j = 1, 2,$$
$$i \neq j.$$

The expressions of $\hat{x}_i$ s are obtained by solving this system and are used to get $\hat{q}_i$ s:

$$\hat{q}_i = \frac{a-2A_i+A_j+(2-b)\hat{x}_i+(2b-1)\hat{x}_j}{3b}, \text{ with}$$

$$\hat{x}_i = \frac{[(2b-1)^2+\frac{1}{2}D_j][-(2b-1)F_i-(2-b)F_j]+C[(2-b)F_i+(2b-1)F_j]}{[(2b-1)^2+\frac{1}{2}D_i][(2b-1)^2+\frac{1}{2}D_j]-C^2}, \qquad i, j = 1, 2,$$

$$i \neq j.$$

A.2.2. The Symmetry Check

Similarly to the non-cooperative mode solution, we will apply a symmetry check now. It is visible, that it is enough to prove that our $\hat{x}_i$ s are the same with the corresponding solutions obtained from A&J (1988) (see the symmetry check for the non-cooperative mode). From the assumption that $g_1 = g_2 = g$ and $A_1 = A_2 = A$ follows that:

$$\hat{x}_1 = \hat{x}_2 = \hat{x} = \frac{-[(2b-1)^2 + \frac{1}{2}D](1+b)F + (1+b)CF}{[(2b-1)^2 + \frac{1}{2}D]^2 - C^2} =$$

$$= \frac{-(1+b)F[(2b-1)^2 + \frac{1}{2}D - C]}{[(2b-1)^2 + \frac{1}{2}D]^2 - C^2} =$$

$$= \frac{-(1+b)F}{(2b-1)^2 + \frac{1}{2}D + C},$$

$$\hat{x} = \frac{-(1+b)(a-A)}{(2b-1)^2 + (2-b)^2 - 4.5bg + 2(2b-1)(2-b)} =$$

$$= \frac{(a-A)(1+b)}{4.5bg - [(2-b)+(2b-1)]^2} =$$

$$= \frac{(a-A)(1+b)}{4.5bg - (1+b)^2}$$

So we conclude that if we substitute the symmetric parameters into the solution of our R&D cooperation model, it will produce results consistent with the results of A&J (1988). Such fact serves as evidence that the R&D cooperation problem in the A&J (1988) model can be considered as a particular case of the R&D cooperation problem of our asymmetric model.

A.2.3. The Second Order Condition (SOC) Implications

The second-stage problem in the R&D cooperation mode is identical to the one in the non-cooperative mode and leads to the same conclusion that its SOCs are automatically satisfied. The first-stage problem is more complicated. We maximize profits as a function of two variables x_1 and x_2. FOCs are expressed as a 2x2 matrix equation (see section A.2.1.) Maximization requires the first principal minor of the coefficient matrix (Hessian matrix) to be negative, and the second principal minor to be positive. The first principal minor is:

$$H_1 = 2(2b-1)^2 + 2(2-b)^2 - 9bg_1 < 0,$$

which implies:

$$bg_1 > \frac{2}{9b}[(2b-1)^2 + (2-b)^2].$$

Showing that:

$$2 \le (2b-1)^2 + (2-b)^2 \le 5,$$

we have:

$$bg_1 > \frac{10}{9}. \qquad (1)$$

The second principal minor:

$$H_2 = [2(2b-1)^2 + 2(2-b)^2 - 9bg_1][2(2b-1)^2 + 2(2-b)^2 - 9bg_2] - 16(2b-1)^2(2-b)^2 > 0$$

b-interval. When =*0* we get:

$$[10-9bg_1][10-9bg_2]-64>0\ . \qquad (2)$$

And when b =

$$[4-9bg_1][4-9bg_2]-16>0\ . \qquad (3)$$

It can be shown that when (1) is satisfied, condition (2) automatically implies (3). The first term of the left-hand side of (3) is bigger than the corresponding term in (2), and the second term is smaller, thus condition (3) is the relaxed version of (2) and can be omitted.

APPENDIX B

The Corner Solutions of the Monopolistic Collusion and Welfare Maximization Modes

B.1. The First Order Conditions (FOCs)

Since the FOCs for the monopoly case and the welfare maximization case are very much similar, we will discuss their corner solutions together. There are 15 possible different zero / non-zero combinations of the four-variable solutions. From these we choose 11 which are rationally feasible. We leave out such solutions like:

- the solution where all the variables are non-zero (non-corner solution) is not rationally feasible, because it corresponds to the not well defined initial problem (semi-definite FOCs matrix);
- three solutions with zero outputs for two firms, which are not rationally feasible under the assumption of firm's profit maximizing behavior.

Table B.1 shows the list of rationally feasible solutions. Each solution was obtained by solving the system of FOCs where each condition corresponding to the zero-value variable is relaxed in such a way that allows it to have any non-positive value. Solutions for the monopolistic collusion and the welfare maximizing modes are given in the same table, because they are very similar.

The difference between the monopoly and welfare maximizing solution is captured in a constant h (with $h{:=}2b$ in the monopoly problem and $h{:=}b$ in the welfare maximizing one). The FOCs for these two problems can be presented as:

$$\begin{pmatrix} -h & -h & 1 & b \\ -h & -h & b & 1 \\ 1 & b & -g_1 & 0 \\ b & 1 & 0 & -g_2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ A_2 - a \\ 0 \\ 0 \end{pmatrix}.$$

The symmetry check is not applicable for the monopolistic and welfare-maximizing modes, because for these models solutions are obtained using rationales being different from those in the A&J (1988) setup.

Table B.1. The list of feasible corner solutions for the monopolistic collusion problem (*h=2b*) and welfare maximization (*h=b*).

No	q_1	q_2	x_1	x_2	Description
1.	0	$\frac{g_1 g_2 (A_2 - a)}{g_1 - h g_1 g_2 + b^2 g_2}$	$\frac{b g_2 (A_2 - a)}{g_1 - h g_1 g_2 + b^2 g_2}$	$\frac{g_1 (A_2 - a)}{g_1 - h g_1 g_2 + b^2 g_2}$	Firms 1 & 2 research, firm 2 produces
2.	$\frac{g_1 g_2 (A_1 - a)}{b^2 g_1 - h g_1 g_2 + g_2}$	0	$\frac{g_2 (A_1 - a)}{b^2 g_1 - h g_1 g_2 + g_2}$	$\frac{b g_1 (A_1 - a)}{b^2 g_1 - h g_1 g_2 + g_2}$	Firms 1 & 2 research, firm 1 produces
3.	no solution	no solution	0	no solution	Firm 2 researches, firms 1 & 2 produce (The problem is not well defined)
4.	no solution	no solution	no solution	0	Firm 1 researches, firms 1 & 2 produce (The problem is not well defined)
5.	no solution	no solution	0	0	No research, firms 1 & 2 produce (The problem is not well defined)
6.	0	$\frac{g_2 (a - A_2)}{h g_2 - 1}$	0	$\frac{a - A_2}{h g_2 - 1}$	Firm 2 researches, firm 2 produces
7.	$\frac{g_1 (a - A_1)}{h g_1 - 1}$	0	$\frac{a - A_1}{h g_1 - 1}$	0	Firm 1 researches, firm 1 produces

Table B.1. The list of feasible corner solutions for the monopolistic collusion problem (*h*=2*b*) and welfare maximization (*h*=*b*). (Continued)

No	q_1	q_2	x_1	x_2	Description
8.	0	$\frac{g_1(a-A_2)}{hg_1-b^2}$	$\frac{b(a-A_2)}{hg_1-b^2}$	0	Firm 1 researches, firm 2 produces
9.	$\frac{g_2(a-A_1)}{hg_2-b^2}$	0	0	$\frac{b(a-A_1)}{hg_2-b^2}$	Firm 2 researches, firm 1 produces
10.	$\frac{a-A_1}{h}$	0	0	0	No research, firm 1 produces
11.	0	$\frac{a-A_2}{h}$	0	0	No research, firm 2 produces

B.2. The Second Order Condition (SOC) Implications.

We will consider all rationally feasible corner solutions one by one providing their FOCs, SOCs and the implications for the simulation input parameters. In the same way as it was done above we use a constant h to distinguish between the FOCs belonging to the monopolistic collusion problem ($h:=2b > 0$) and the FOCs for the welfare-maximizing problem ($h:=b > 0$).

Solution 1: firms 1 and 2 research, and firm 2 produces.
FOCs are:

$$\begin{pmatrix} -h & b & 1 \\ b & -g_1 & 0 \\ 1 & 0 & -g_2 \end{pmatrix}\begin{pmatrix} q_2 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_2 - a \\ 0 \\ 0 \end{pmatrix}$$

Corresponding SOCs are:

$$H_1 = -h < 0,$$
$$H_2 = hg_1 - b^2 > 0,$$
$$H_3 = g_1 - hg_1g_2 + b^2g_2 < 0.$$

The SOCs imply that (using similar steps as in A.1.3. and A.2.3.):

$$hg_1 > 1,$$
$$hg_1 - \frac{g_1}{g_2} > 1.$$

Solution 2: firms 1 and 2 research, and firm 1 produces.
FOCs are:

$$\begin{pmatrix} -h & 1 & b \\ 1 & -g_1 & 0 \\ b & 0 & -g_2 \end{pmatrix}\begin{pmatrix} q_1 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ 0 \\ 0 \end{pmatrix}$$

Corresponding SOCs are:

$$H_1 = -h < 0,$$
$$H_2 = hg_1 - 1 > 0,$$
$$H_3 = b^2g_1 - hg_1g_2 + g_2 < 0.$$

The SOCs imply that:

$$hg_1 > 1,$$
$$hg_2 - \frac{g_2}{g_1} > 1.$$

Solution 3: firm 2 researches, firms 1 and 2 produce.
FOCs are:

$$\begin{pmatrix} -h & -h & b \\ -h & -h & 1 \\ b & 1 & -g_2 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ A_2 - a \\ 0 \end{pmatrix}$$

This problem has a semi-definite Hessian. Thus there is no unique solution satisfying this case's assumption.

Solution 4: firm 1 researches, firms 1 and 2 produce.
FOCs are:

$$\begin{pmatrix} -h & -h & 1 \\ -h & -h & b \\ 1 & b & -g_1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ A_2 - a \\ 0 \end{pmatrix}$$

This problem has a semi-definite Hessian. Thus there is no unique solution satisfying this case's assumption.
Solution 5: no research, firms 1 and 2 produce.
FOCs are:

$$\begin{pmatrix} -h & -h \\ -h & -h \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ A_2 - a \end{pmatrix}$$

Obviously, this problem does not have a unique solution, because of the singular Hessian.
Solution 6: firm 2 researches, firm 2 produces.
FOCs are:

$$\begin{pmatrix} -h & 1 \\ 1 & -g_2 \end{pmatrix} \begin{pmatrix} q_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_2 - a \\ 0 \end{pmatrix}$$

Corresponding SOCs are:

$$H_1 = -h < 0,$$
$$H_2 = hg_2 - 1 > 0.$$

The SOCs imply that:

$$hg_2 > 1.$$

Solution 7: firm 1 researches, firm 1 produces.
FOCs are:

$$\begin{pmatrix} -h & 1 \\ 1 & -g_1 \end{pmatrix} \begin{pmatrix} q_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ 0 \end{pmatrix}$$

Corresponding SOCs are:

$$H_1 = -h < 0,$$
$$H_2 = hg_1 - 1 > 0.$$

The SOCs imply that:

$$hg_1 > 1.$$

Solution 8: firm 1 researches, firm 2 produces.
FOCs are:

$$\begin{pmatrix} -h & b \\ b & -g_1 \end{pmatrix} \begin{pmatrix} q_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} A_2 - a \\ 0 \end{pmatrix}$$

Corresponding SOCs are:

$$H_1 = -h < 0,$$
$$H_2 = hg_1 - 1 > 0.$$

The SOCs imply that (using similar steps as in A.1. and A.2.):

$$hg_1 > 1.$$

Solution 9: firm 2 researches, firm 1 produces.

FOCs are:

$$\begin{pmatrix} -h & b \\ b & -g_2 \end{pmatrix} \begin{pmatrix} q_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_1 - a \\ 0 \end{pmatrix}$$

Corresponding SOCs are:

$$H_1 = -h < 0,$$

$$H_2 = hg_2 - 1 > 0.$$

The SOCs imply that (using similar steps as in A.1. and A.2.):

$$hg_2 > h.$$

Solution 10: no research, firm 1 produces.

FOC is:

$$-hq_1 = A_1 - a\,.$$

SOC is $-h < 0$, which is automatically satisfied.

Solution 11: no research, firm 2 produces.

FOC is:

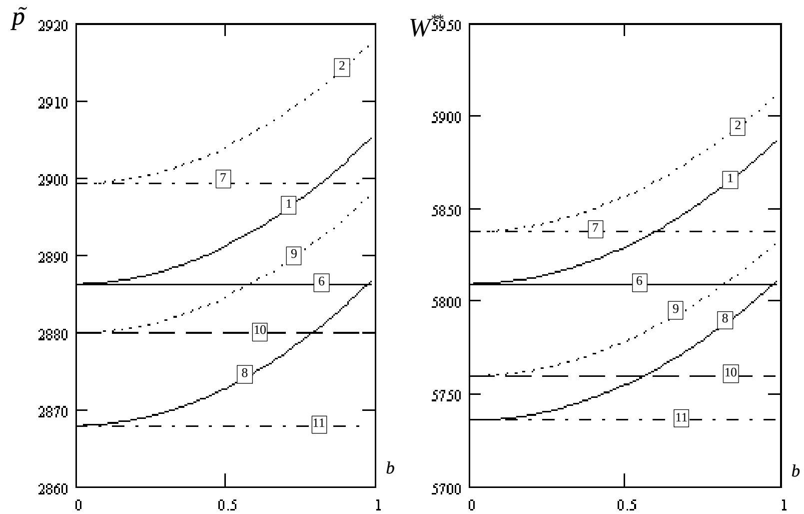
$$-hq_2 = A_2 - a\,.$$

SOC is $-h < 0$, which is automatically satisfied.

APPENDIX C.

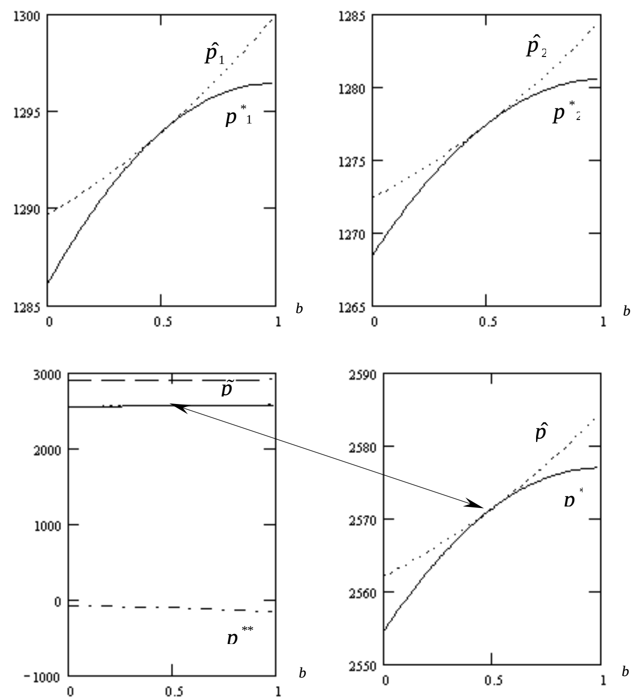
Simulation Results

Position 1: $A_2 / A_1 = g_2 / g_1 = 1.05$



Graph C.1.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions.

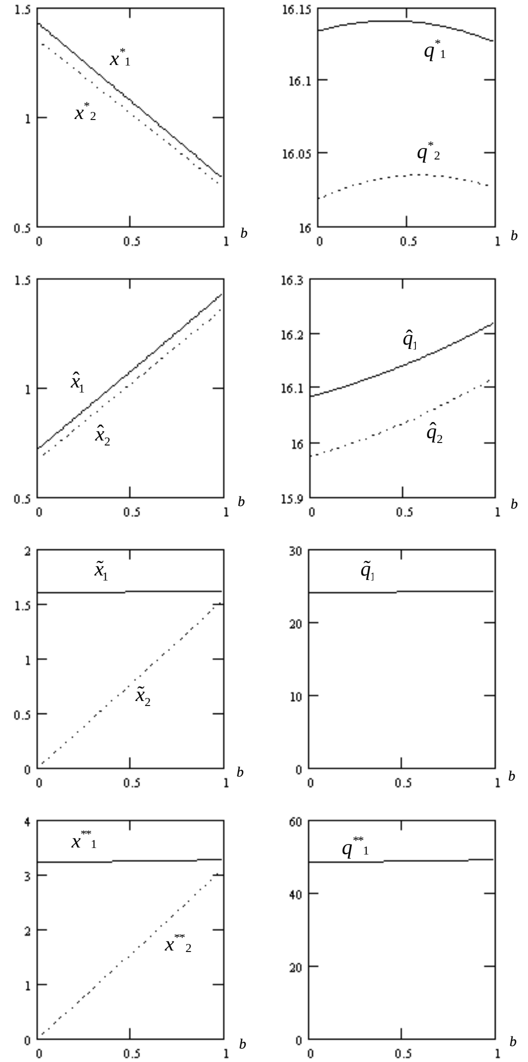
Position 1: $A_2 / A_1 = g_2 / g_1 = 1.05$



Graph C.1.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

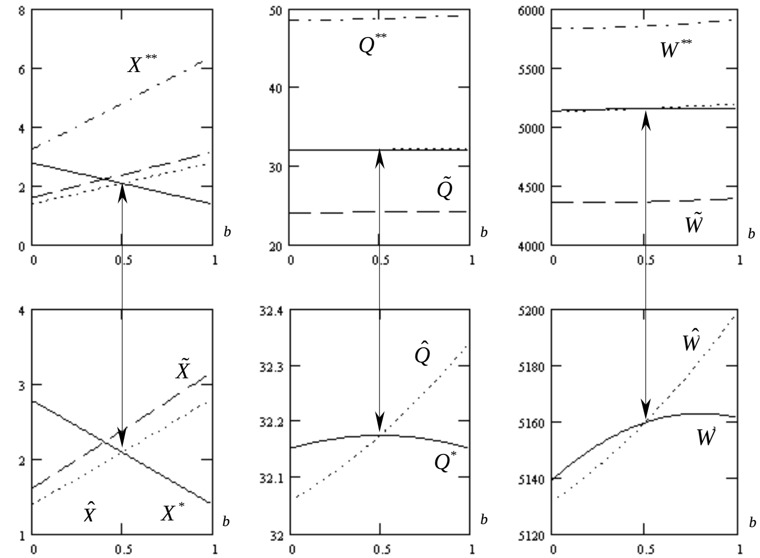
Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode, $z = x, q, p, W, X, Q.$

Position 1: $A_2 / A_1 = g_2 / g_1 = 1.05$



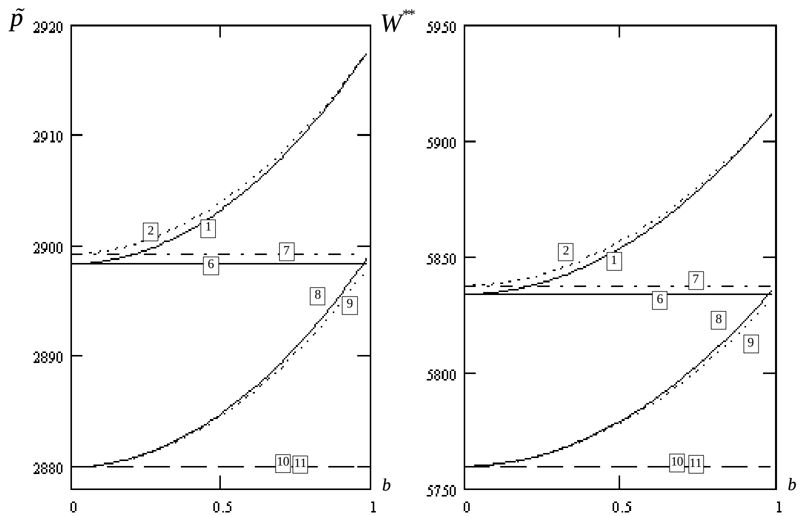
Graph C.1.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 1: $A_2 / A_1 = g_2 / g_1 = 1.05$



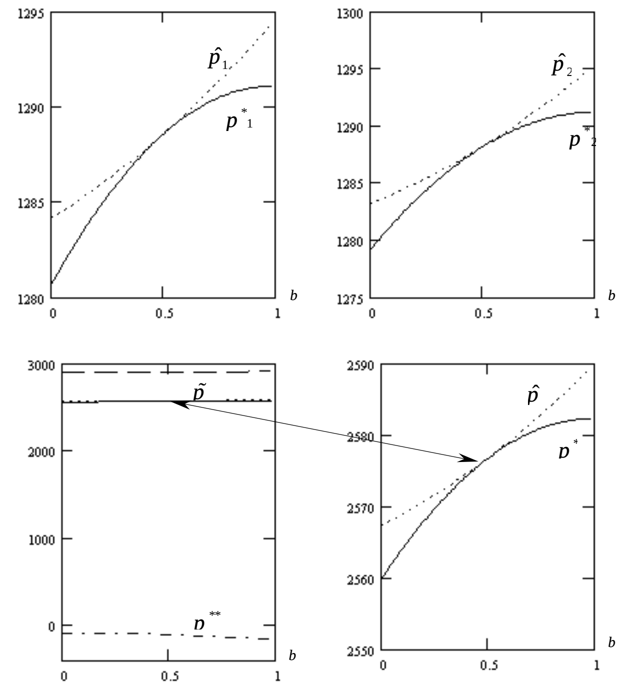
Graph C.1.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

Position 2: $A_2 / A_1 = 1 \neq g_2 / g_1 = 1.05$



Graph C.2.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions.

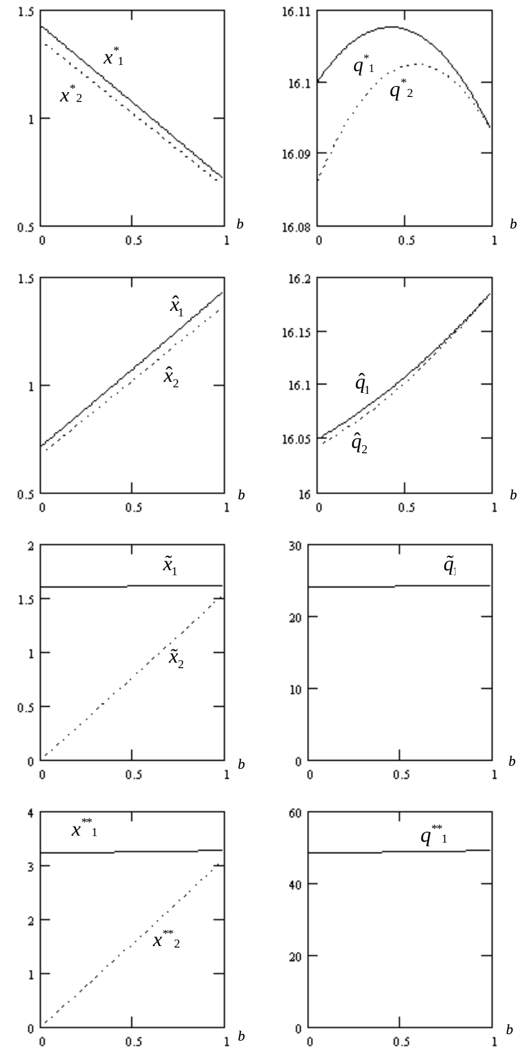
Position 2: $A_2 / A_1 = 1 \neq g_2 / g_1 = 1.05$



Graph C.2.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

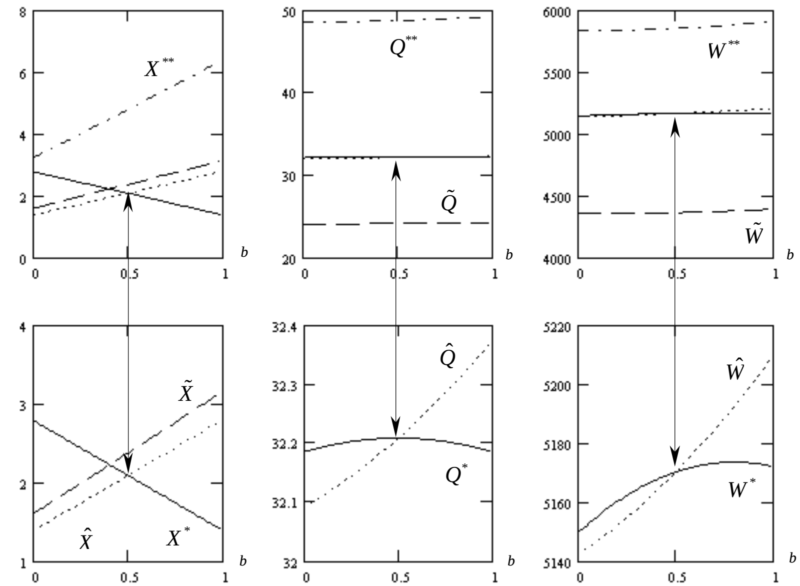
Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode,
$z = x, q, p, W, X, Q.$

Position 2: $A_2 / A_1 = 1 \neq g_2 / g_1 = 1.05$



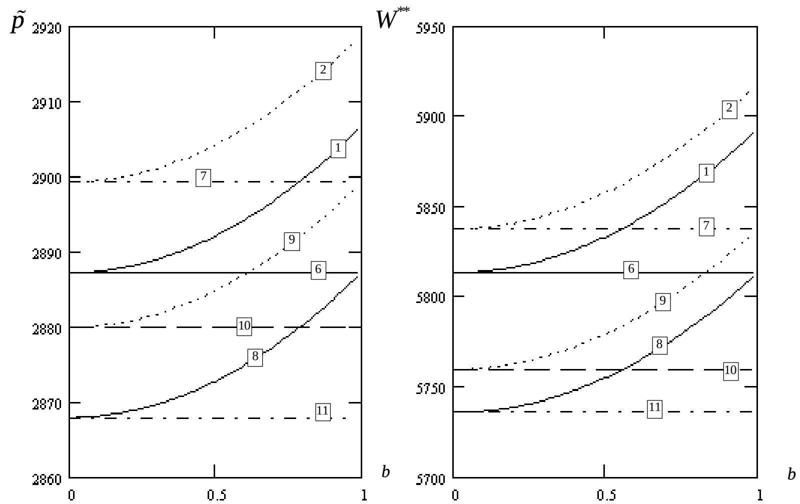
Graph C.2.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 2: $A_2 / A_1 = 1 \neq g_2 / g_1 = 1.05$



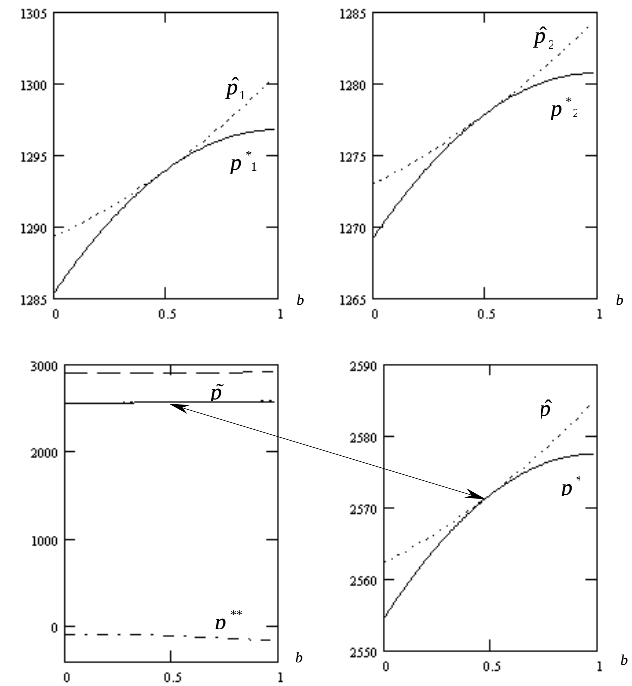
Graph C.2.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

Position 3: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 1$



Graph C.3.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions.

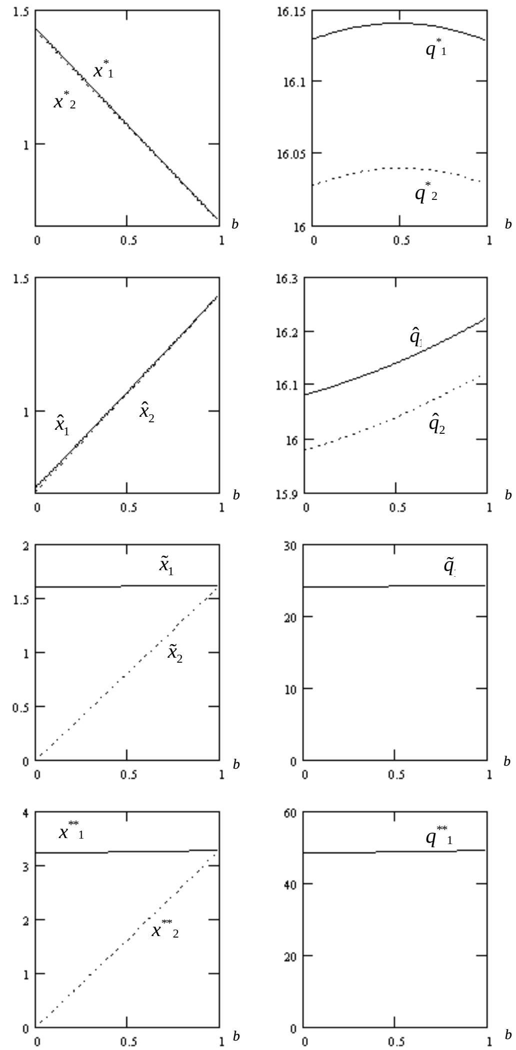
Position 3: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 1$



Graph C.3.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

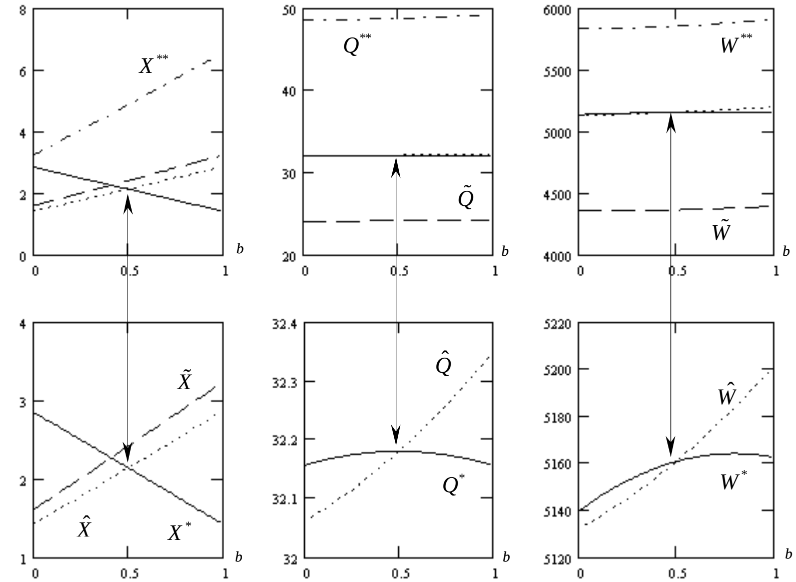
Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode, $z = x, q, p, W, X, Q.$

Position 3: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 1$



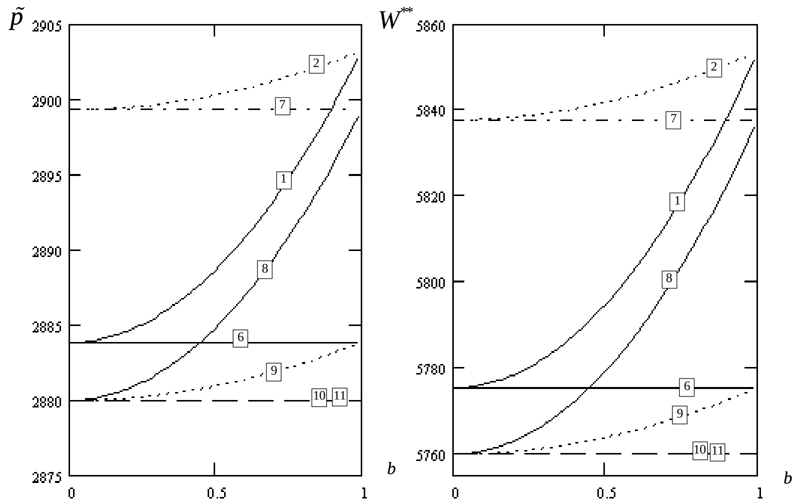
Graph C.3.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 3: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 1$



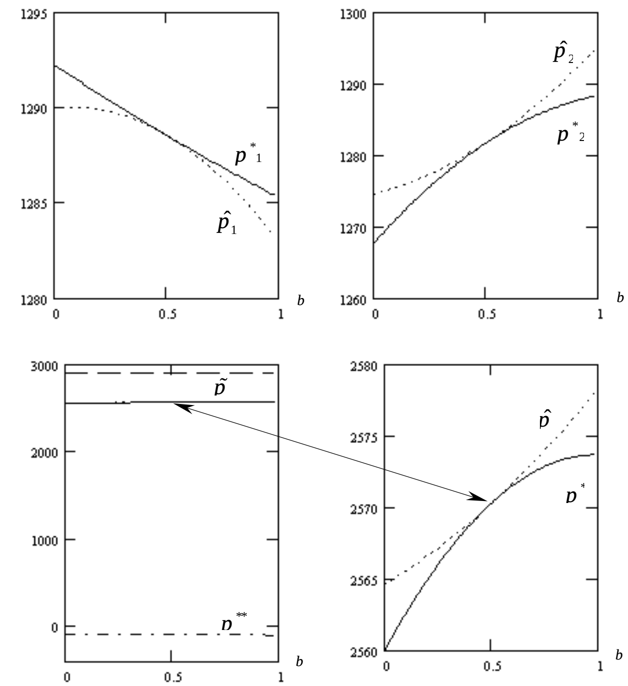
Graph C.3.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

Position 4: $A_2 / A_1 = 1 \neq g_2 / g_1 = 5$



Graph C.4.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions.

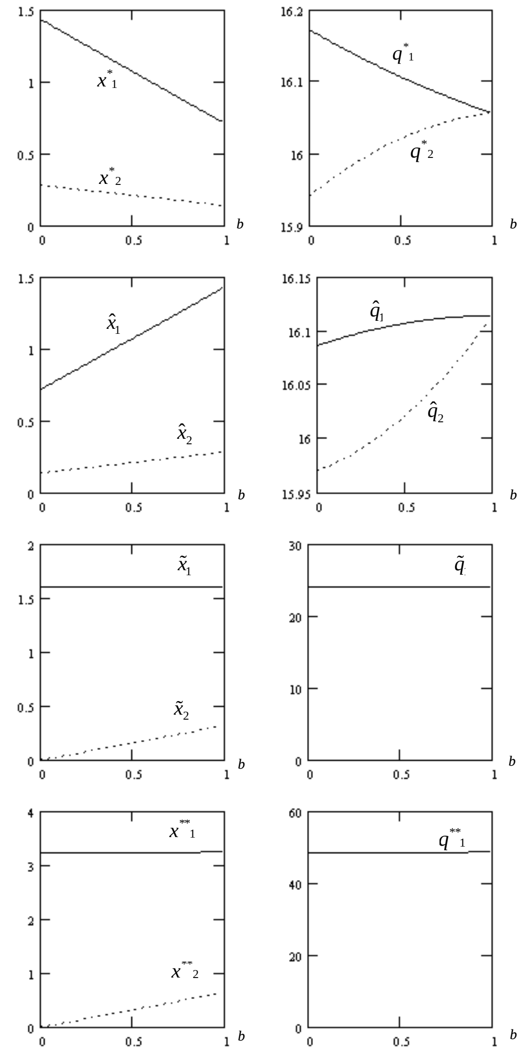
Position 4: $A_2 / A_1 = 1 \neq g_2 / g_1 = 5$



Graph C.4.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

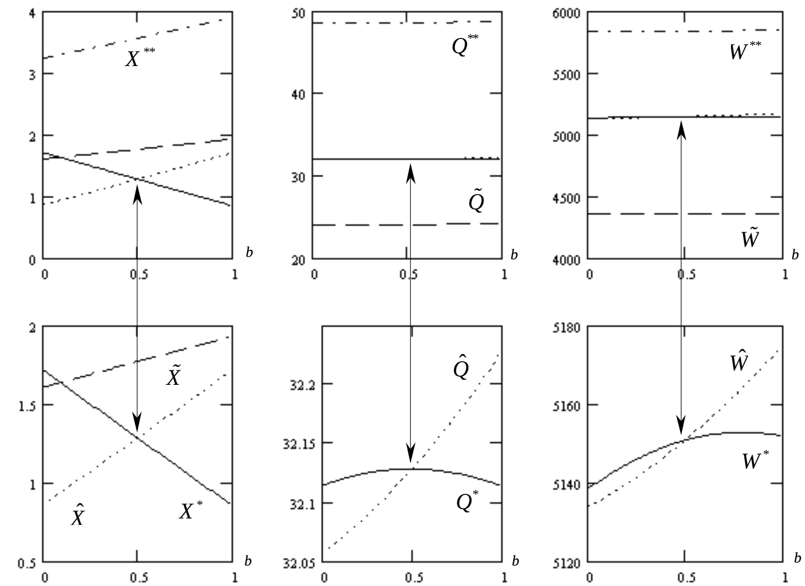
Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode, $z = x, q, p, W, X, Q.$

Position 4: $A_2 / A_1 = 1 \neq g_2 / g_1 = 5$



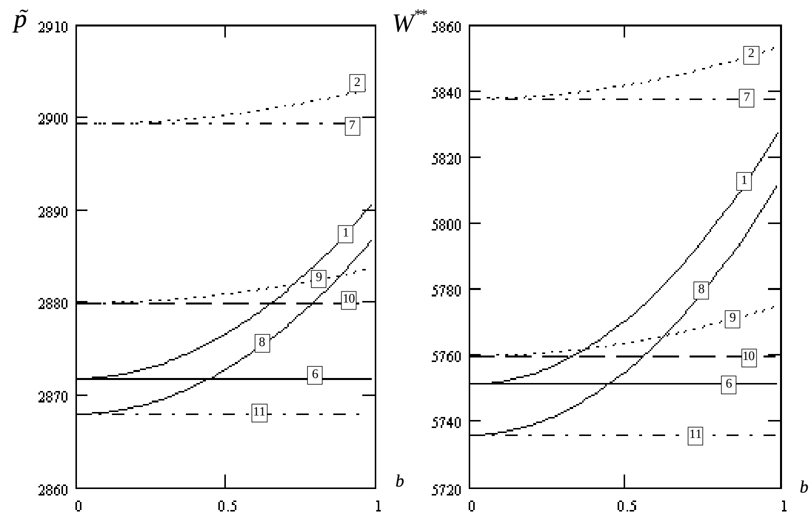
Graph C.4.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 4: $A_2 / A_1 = 1 \neq g_2 / g_1 = 5$



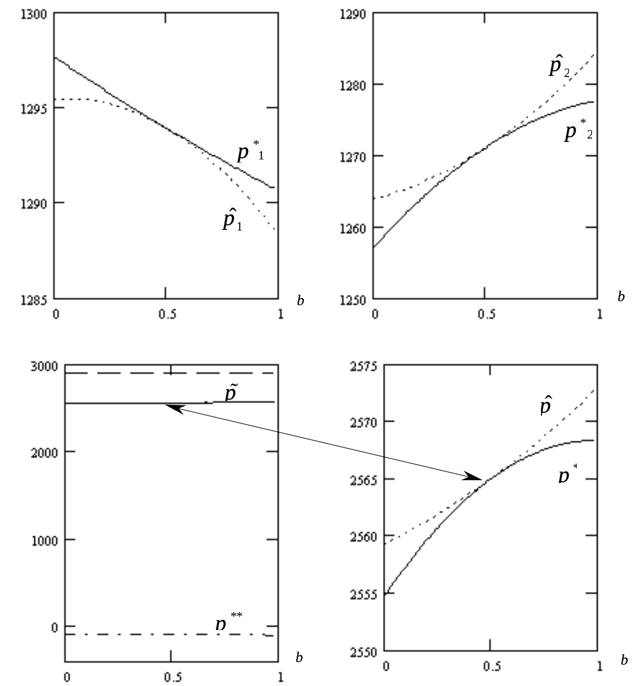
Graph C.4.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

Position 5: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 5$



Graph C.5.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions.

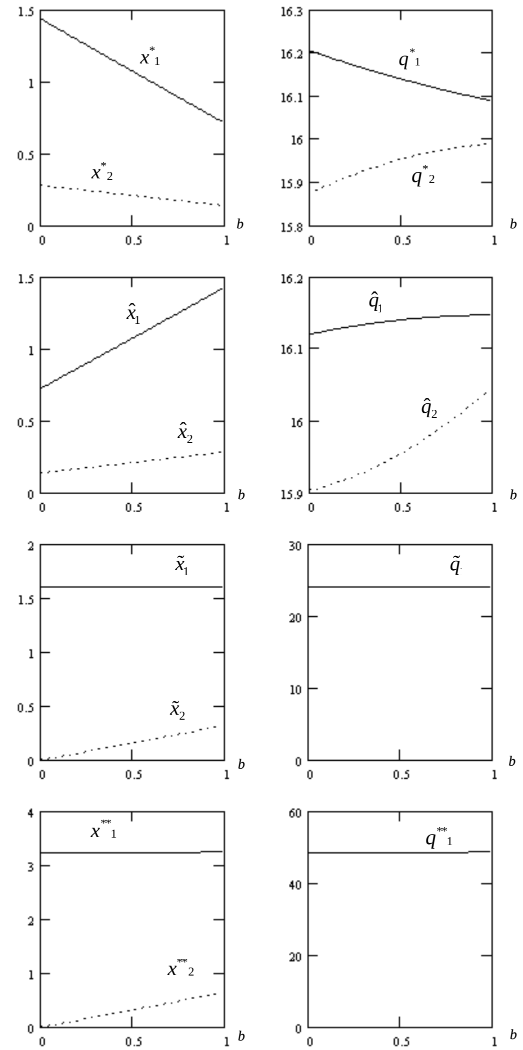
Position 5: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 5$



Graph C.5.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

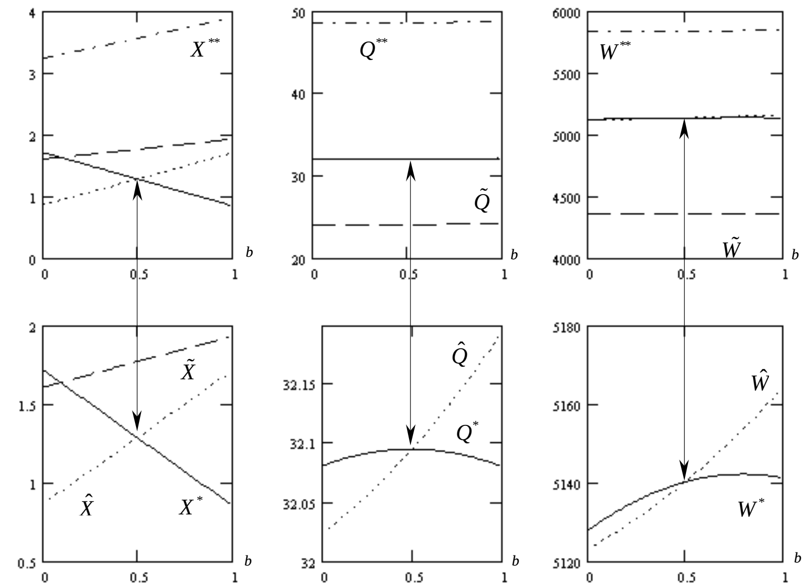
Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode, $z = x, q, p, W, X, Q.$

Position 5: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 5$



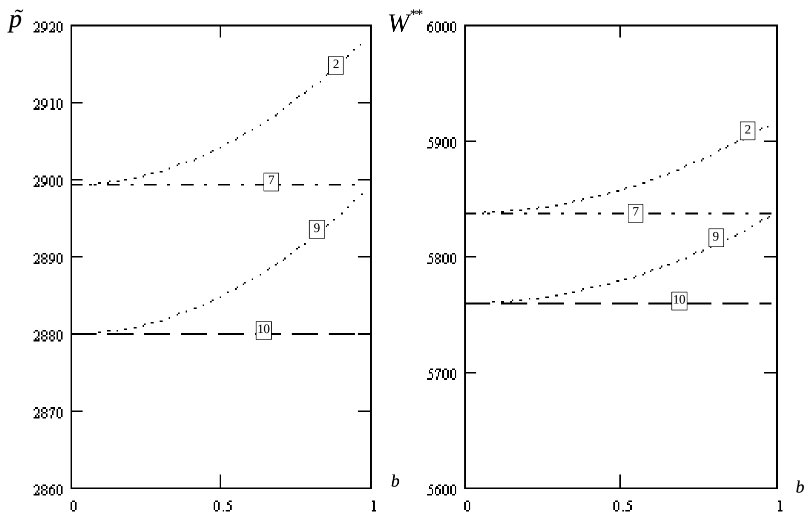
Graph C.5.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 5: $A_2 / A_1 = 1.05 \neq g_2 / g_1 = 5$



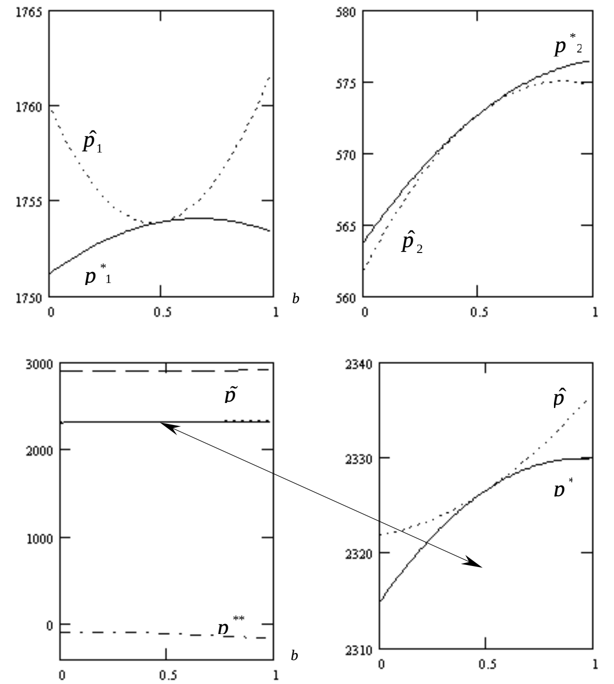
Graph C.5.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

Position 6: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1$



Graph C.6.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions (only four 'higher' curves due to the scaling difficulties).

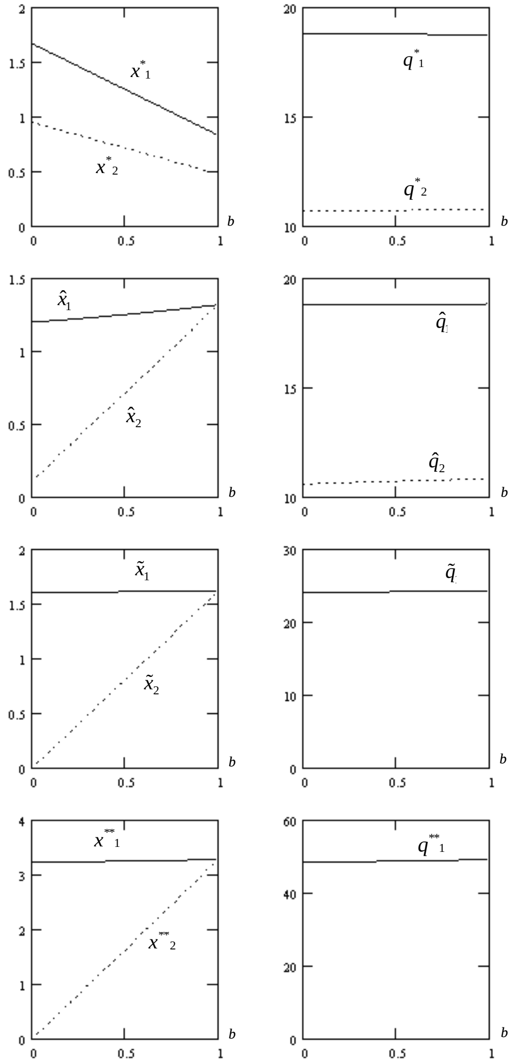
Position 6: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1$



Graph C.6.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

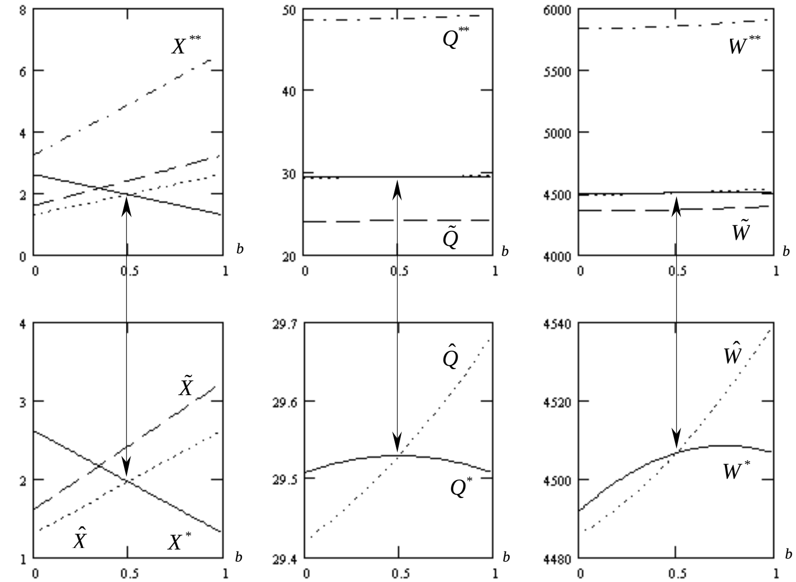
Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode,
$z = x, q, p, W, X, Q.$

Position 6: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1$



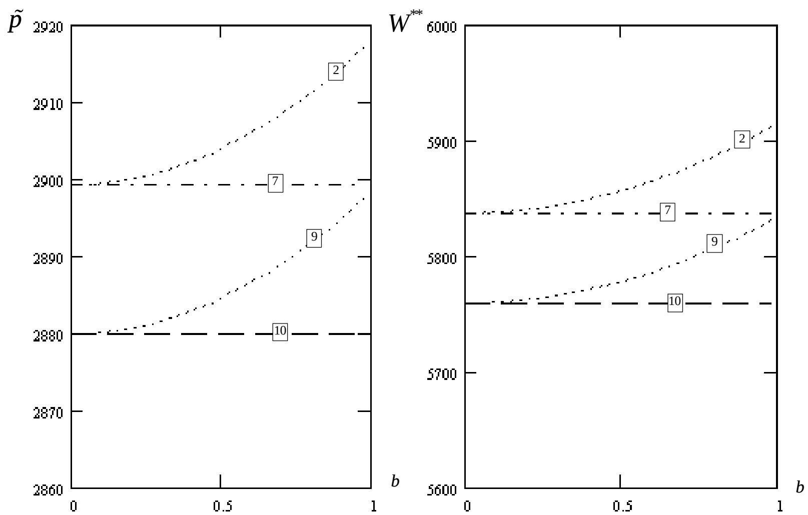
Graph C.6.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 6: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1$



Graph C.6.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

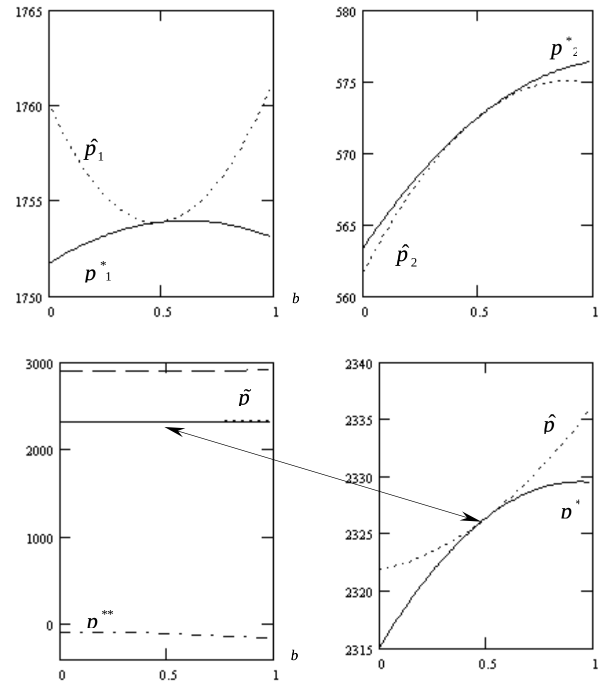
Position 7: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1.05$



Graph C.7.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions (only four 'higher' curves due to the scaling difficulties).

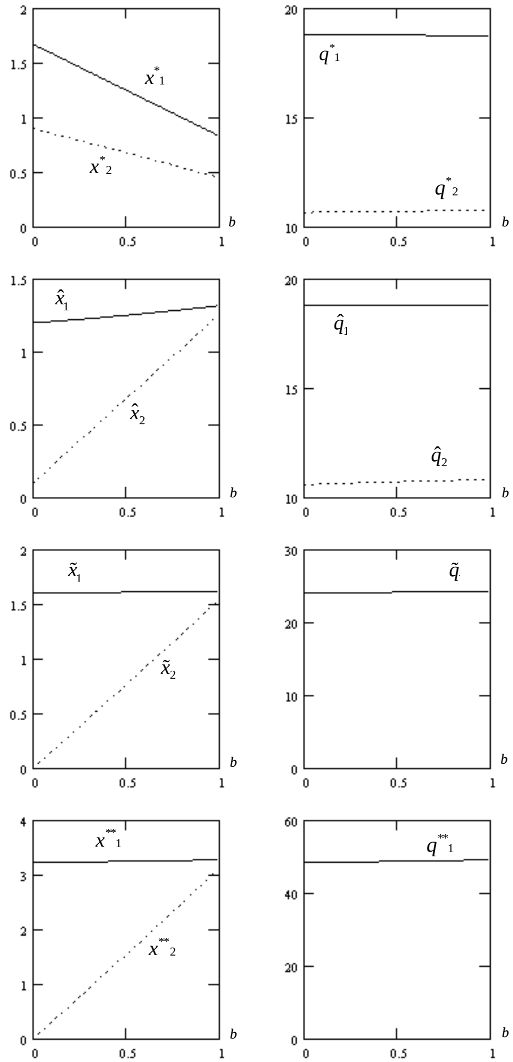
Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode, $z = x, q, p, W, X, Q$.

Position 7: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1.05$



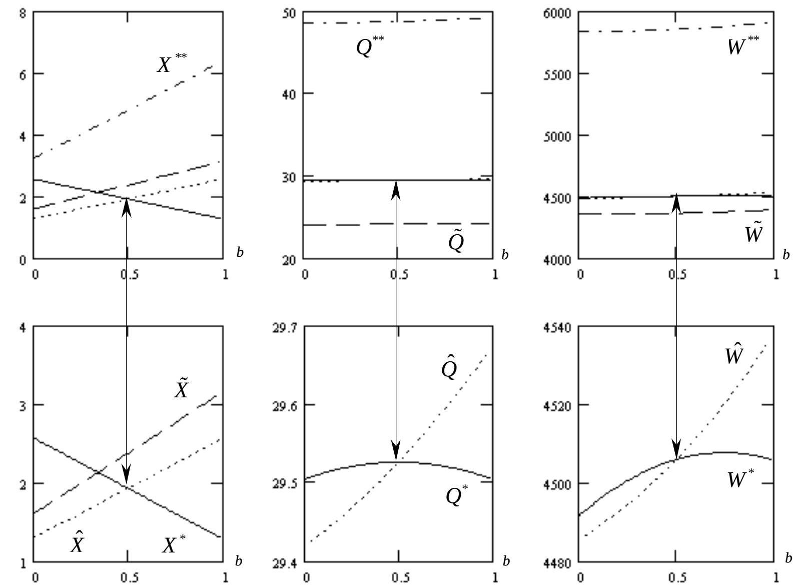
Graph C.7.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

Position 7: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1.05$



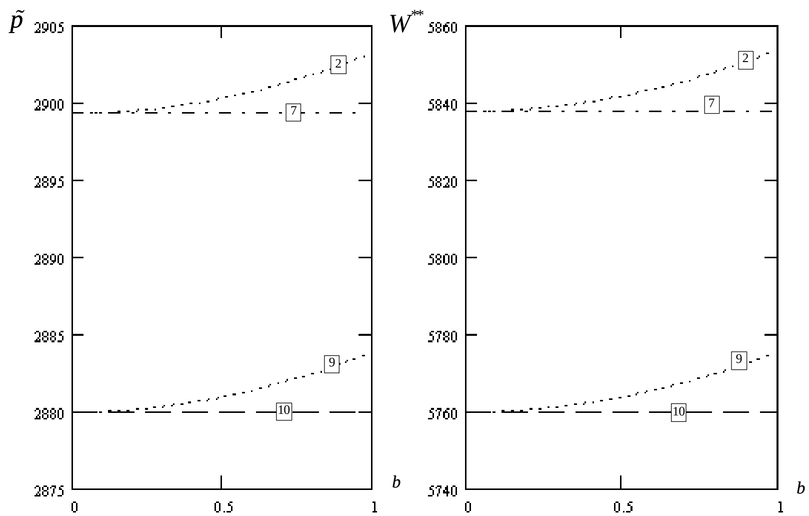
Graph C.7.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 7: $A_2 / A_1 = 5 \neq g_2 / g_1 = 1.05$



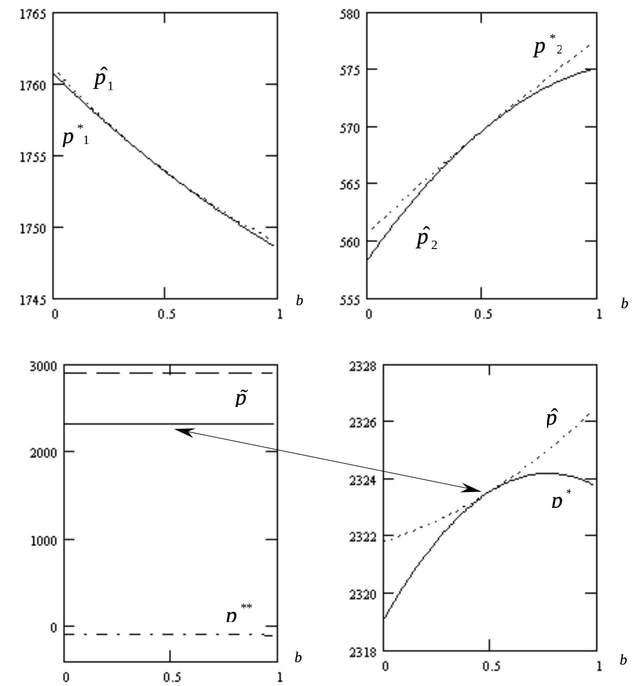
Graph C.7.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

Position 8: $A_2 / A_1 = g_2 / g_1 = 5$



Graph C.8.1. Profit ($\tilde{p}$, for the monopoly mode) and welfare (W^{**}, for the welfare maximizing mode) functions corresponding to different corner solutions (only four 'higher' curves due to the scaling difficulties).

Position 8: $A_2 / A_1 = g_2 / g_1 = 5$

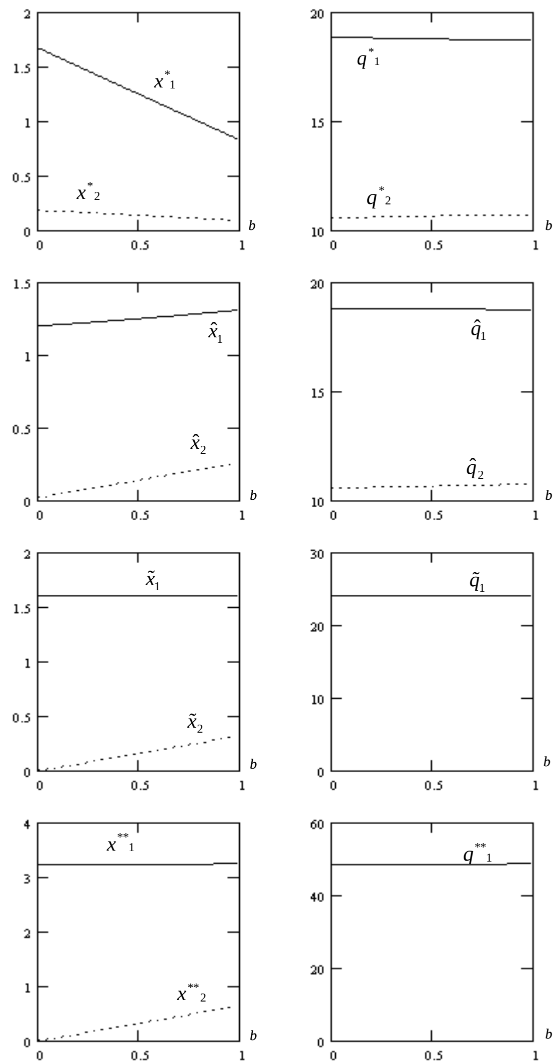


Graph C.8.2. Individual firms' profits in production-competitive modes (1st row) and industry-level profit functions in all modes (2nd row).

Notation agreements: z^* - non-cooperative mode, $\hat{z}$ - R&D cooperation mode, $\tilde{z}$ - monopoly mode, z^{**} - welfare-maximizing mode,

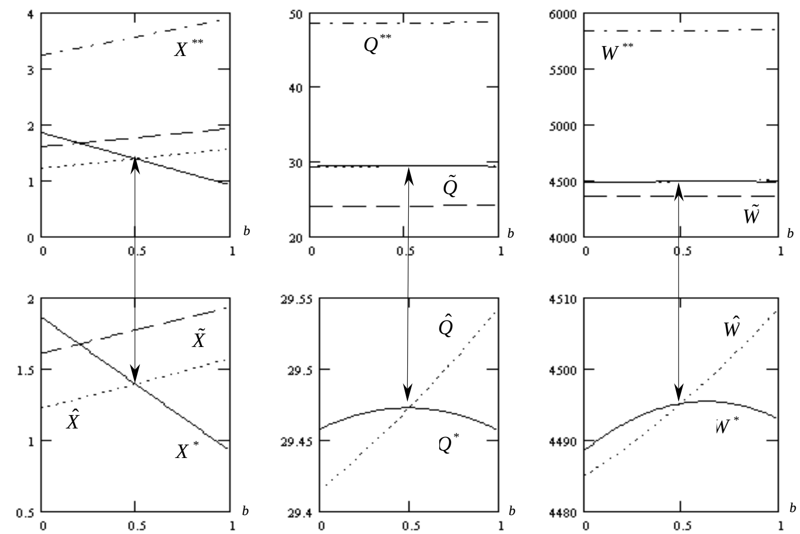
$z = x, q, p, W, X, Q.$

Position 8: $A_2 / A_1 = g_2 / g_1 = 5$



Graph C.8.3. Simulated optimal R&D expenditures and outputs as functions of *b*.

Position 8: $A_2 / A_1 = g_2 / g_1 = 5$



Graph C.8.4. Industry-level solutions for different modes of firms' behavior as functions of *b*.

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