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OPTIMAL EDUCATION WITH MOBILE CAPITAL AN OLG APPROACH

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Abstract

The paper considers a two-country model of overlapping generations economies with intergenerational transfers carried out in the form of bequest and investment in human capital. We examine in competitive equilibrium the optimal provision of education with and without capital markets integration. First, we explore how regimes of education provision – public, private or mixed – arise and how they affect the dynamics of autarkic economies. Second, we study the transitory and long-run effects of capital markets integration, in equilibrium, on the optimal provision of education and growth. Third, we examine a competition game where countries compete in the provision of public education.

Keywords: Altruism, education, growth, human capital, capital markets integration

JEL Classification: D9, E2, F2, J2

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1 Introduction

Trends manifested during the 1990s suggest a worldwide acceleration in the flows of foreign direct and portfolio investments. International production has become a significant element in the world economy and substantial flows of foreign investments to emerging markets is a recent phenomenon dating only from the beginning of this decade. This would not have been possible if it were not for the ongoing integration of international capital markets (UNCTAD (1997)). The increased mobility of capital coincided with the growing recognition that economies have come to revolve around the production and the use of knowledge. With the continuous upskilling of jobs, investment in education has become a high priority of many developed and developing countries. But does capital mobility explain such a phenomenon or should we look to other causes? This paper seeks to study questions related to human capital aspects of capital markets integration (CMI).

Our main objective is to examine both the transitory and long-term effects of capital markets integration on human capital formation. We consider a two-country model of overlapping generations economies where each country has identical households in each generation and no population growth. Parents care about their offspring's income, hence we observe intergenerational transfers in the form of physical capital (bequest) and investment in education. Due to investments in human capital we obtain endogenous growth. Education to the young generation is provided both publicly and privately. Governments tax income earnings to finance the costs of public education, while parents may use some of their free time to enhance their child's human capital. As capital markets integration affects wages and interest rates differently in different countries, the bequest transfers and the relative sizes of these investments in education are expected to change differently across countries.[1]

[1]This paper integrates few main features in the recent literature on endogenous growth. Investment in human capital is used as an engine for growth (see, e.g., Lucas (1988), Azariadis and Drazen (1990), Lord and Rangazas (1991)). Public education is provided by governments, although private provision of education exists, in order to enhance growth (see, e.g. Glomm and Ravikumar (1992), Eckstein and Zilcha (1994)). Capital markets integration is introduced to question its role in enhancing growth (see, e.g., Barro, Mankiw and Sala-i-Martin (1995), Dellas and De Vries (1995), Leiderman and Razin (1994), Lucas (1990), MacDougall (1960), Mankiw and Sala-i-Martin (1995), Rivera-Batiz and Romer (1991), Ruffin (1985) and Stokey (1996)). Buiter (1981), Ruffin and Yoon (1993) have

We consider first the endogenous growth process in autarkic competitive equilibrium under various educational regimes, and study the effects of capital markets integration on growth and welfare for the capital-exporting country (”domestic”) and the capital-importing country (”foreign”). Later, we examine the impact of CMI on the optimal provision of public education in each country. We find that in our framework, following the introduction of CMI, the allocation of output between the two countries in each date depends upon the relative stock of human capital. Thus governments will enhance the formation of human capital in order to increase their share in the aggregate production. The only tool which can be used in this competition is the level of provision of public education. Various solutions to such a conflict are considered, assuming that CMI takes place and that education is provided only publicly. Under these assumptions we show that the optimal provision of public education is the same whether governments agree on a cooperative solution or the Nash bargaining solution. Moreover, this Pareto optimal provision level is the same as that in autarky case. On the other hand, we indicate that in a Nash equilibrium, between the two governments representing contemporary generations, education provision differs from this Pareto optimal level. We are not aware of any discussion in the literature of such coordination/conflict between governments regarding the provision of public education.

The remainder of the paper is organized as follows. Section 2 presents the OLG model with altruistic representative agents and characterizes the autarky equilibria. It also examines various regimes of education and points out the differences in output growth. Section 3 studies the effects of capital markets integration on the optimal provision of education in a two-country model. It also examines the education provision competition between governments of the integrated economy. We consider the cooperative solution, the Nash bargaining solution and a (stationary) Nash equilibrium. Section 4 concludes the paper. Some of the proofs are relegated to the Appendix to facilitate the reading.

applied the overlapping generations model to study international factor movements.

2 Autarky Equilibrium

2.1 Preferences and Technology

Consider an overlapping generations economy with identical agents in each generation, each economically active during two periods - a working period followed by a retirement period. At the end of the first period, every individual gives birth to one offspring. Denote by G_t the individuals born at the outset of period t and refer to them as generation t. The analysis starts at $t = 0$, where the G_{-1} individuals live during the retirement period, consuming their savings.

In this economy parents derive utility from the future income of their child. This motivates the transfer of wealth to their offspring in the forms of human capital and physical capital. The levels of human and physical capital transfers together with the relevant interest rate and wages determine the offspring's total income. Intergenerational transfers are driven by two main motivations: (a) Improving the earning capability of the offspring via education; (b) the 'joy of giving' and the bequest motive. Denote by b_t the transfer of physical capital to his/her offspring and denote by e_t the effort, measured in time, invested in educating this offspring.

The human capital of the representative individual in G_{t+1}, denoted h_{t+1}, depends upon e_t and the parent's level of human capital h_t. Moreover, we assume that the public sector is a provider of formal education to the young generation and public education expenditure is fully financed by a proportional tax on wage income at each date. Each individual is endowed with two units of time. Labor supply is assumed to be inelastic and equal to one unit of time. The other unit of time is allocated between leisure and private education e_t in such a way that the time each parent devotes to *private education* of his own offspring determines his leisure 1-e_t. Denote by e_t^g the investment (measured in time) in each child provided by the government. Although individuals in each generation G_t are identical[2] we assume that in order to provide public education at a certain level, a fraction of the work force is devoted to this assignment. Let us assume, for example, a continuum of individuals in each G_t, then a proportion (1-e_t^g) of this population is

[2] Aspects of income distribution will therefore be omitted. See e.g. Fernandez and Rogerson (1998) and the references therein.

engaged in production while a proportion e_t^g is engaged in public education (each person works 1 unit of time in each activity).

The mechanism of transfer of human capital to the younger generation and the evolution of this process has attracted a lot of attention in the economic literature during the last decade (see, for example, Lucas (1988), Azariadis and Drazen (1990), Jovanovich and Nyarko (1995), Orazem and Tesfatsion (1997) and many others). It is clear that the human capital level of the younger generation is affected, significantly, by the direct investment in education, by the environment (represented here by the average human capital of the older generation) and by the human capital of the parents. Although this production function is complex, to simplify our analysis we shall take the evolution process of human capital as follows. For some constant $\beta > 1$ we assume that:

$$h_{t+1} = \beta(e_t h_t + e_t^g \hat{h}_t) \qquad t = 0, 1, 2, ... \tag{1}$$

where $\hat{h}_t$ is the average human capital of generation t. β is taken to be constant and it represents the efficiency of the process which generates human capital (β is affected by the schooling system, neighborhood, facilities, etc.)

Lifetime preferences of the individual are assumed to be a Cobb-Douglas utility function:

$$u_t = c_{1t}^{\alpha_1} \quad c_{2t}^{\alpha_2} \quad y_{t+1}^{\alpha_3} \quad [1 - e_t]^{\alpha_4} \tag{2}$$

where α_i are known parameters and $\alpha_i > 0$ for $i = 1, 2, 3, 4$; c_{1t} and c_{2t} denote, respectively, consumption in first and second period of the individual's life; y_{t+1} is the income of the offspring and $(1 - e_t)$ represents leisure. Let b_{t-1} be the intergenerational transfer, r_t and w_t be the interest rate and the wage rate in period t respectively, the lifetime income for G_t is given by:

$$y_t = (1 + r_t)b_{t-1} + (1 - \tau_t)w_t h_t \qquad t = 0, 1, ... \tag{3}$$

where the tax at rate τ_t on wage earning, determined by the government, finances the public education at level e_t^g. However, in our framework, since the human capital of all individuals of generation t is the same, $\hat{h}_t = h_t$. Thus for each t, the government budget constraint is:

$$\tau_t w_t h_t = w_t h_t e_t^g \quad t = 0, 1, ... \tag{4}$$

which implies that $e_t^g = \tau_t$.

Production in this economy is carried out by competitive firms that use labor and capital to produce a single commodity. This commodity serves for consumption as well as an input in the production process. We assume full depreciation of the physical capital. The aggregate level of human capital at each date t (not including the human capital devoted to public education) is an input in the production process. In particular we take the (per-capita) aggregate production function to be:

$$q_t = F(k_t, (1 - e_t^g)h_t) \tag{5}$$

where k_t is the (per-capita) capital stock and $(1 - e_t^g) = \tau_t(1 - \tau_t)h_t$ is the effective human capital used in the production process. $F(\cdot, \cdot)$ is assumed to exhibit constant returns to scale, it is strictly increasing, concave, continuously differentiable and satisfies $F_k(0, (1 - \tau_t)h_t) = \infty$, $F_h(k_t, 0) = \infty$, $F(0, (1 - \tau_t)h_t) = F(k_t, 0) = 0$.

2.2 Competitive Equilibrium

Production at each date t is carried out by competitive firms which borrow capital at date $t-1$ and hire labor services at date t. Thus the factor prices are given, in competitive equilibrium, by the corresponding marginal products. Since the human capital of a worker is observable, the wage payments will depend upon the effective labor supply of the worker, i.e., $w_t h_t$ where $w_t = F_h(k_t, (1 - \tau_t)h_t)$ is the wage rate. The economy starts at period 0 with given capital transfers and human capital endowments, b_{-1} and h_0 respectively.

Let the bequest transfer, b_{t-1}, the stock of human capital, h_t, the effective wage rates w_t, w_{t+1}, the interest rates r_t, r_{t+1} for dates t and $t + 1$ be given. The tax rate at date t, τ_t , is assumed to prevail for the next period as well. An individual chooses the levels of saving, s_t, bequest transfer, b_t, and direct investment in his offspring's education, e_t, so as to maximize:

$$u_t = c_{1t}^{\alpha_1} \quad c_{2t}^{\alpha_2} \quad y_{t+1}^{\alpha_3} \quad [1 - e_t]^{\alpha_4} \tag{6}$$

subject to constraints:

$$c_{1t} = y_t - s_t - b_t \geq 0 \tag{7}$$

$$c_{2t} = (1 + r_{t+1})s_t \tag{8}$$

$$h_{t+1} = \beta(e_t + \tau_t)h_t, \quad e_t \geqq 0 \tag{9}$$

where y_t is defined by (3). Given the initial capital stock k_0, b_{-1} and h_0 at the outset of period 0, a *competitive equilibrium* is a sequence of functions $[c_{1t},\, c_{2t},\, s_t,\, b_t,\, e_t]_{t=0}^{\infty}$, a sequence of prices $(w_t,\, r_t)_{t=0}^{\infty}$ and a sequence of tax rates $(\tau_t)_{t=0}^{\infty}$ such that for $t = 0, 1, 2, ...$

(a) Given the above prices, $[c_{1t}, c_{2t}, s_t, b_t, e_t]_{t=0}^{\infty}$ is the optimum for (6)-(9).
(b) The market clearing conditions hold:

$$w_t = F_h(k_t,\, (1 - \tau_t)h_t) \tag{10}$$

$$1 + r_t = F_k(k_t,\, (1 - \tau_t)h_t) \tag{11}$$

$$k_{t+1} = s_t + b_t \tag{12}$$

Condition (12) is a market clearing condition for the physical capital at the end of period t , equating the aggregate capital stock at date t+1 to the aggregate savings and transfers of physical capital. These conditions, in conjunction with constraints (7) and (8) imply the material balance condition:

$$c_{1t} + c_{2,t-1} + k_{t+1} = F(k_t, (1 - \tau_t)h_t) \quad \text{for } t = 0, 1, ... \tag{13}$$

After substituting the constraints, the first-order conditions that lead to the necessary and sufficient conditions for optimum are:

$$\frac{c_{1t}}{c_{2t}} = \frac{\alpha_1}{\alpha_2(1 + r_{t+1})} \tag{14}$$

$$\frac{c_{1t}}{y_{t+1}} = \frac{\alpha_1}{\alpha_3(1 + r_{t+1})} \tag{15}$$

$$\frac{\alpha_4}{(1 - e_t)} = \frac{\beta\alpha_3(1 - \tau_t)w_{t+1}h_t}{y_{t+1}}, \quad e_t > 0 \tag{16}$$

$$\geqq \text{ if } e_t = 0$$

It is clear from (16) that the optimal amount of time invested in the offspring's education takes into account the gain to his income, due to the choice of the parent's objective function. An increase in either the parents' human capital h_t or the wage at the future date w_{t+1} increases, ceteris paribus, the time spent on education by the parents at the expense of their leisure. Expression (16) establishes also a negative relationship between private and public education: an increase in τ_t decreases the time spent on private education e_t and hence, raises leisure. This substitution among types of provision of education will have a number of implications throughout this paper.

From (8), (14) and (15) we also obtain that:

$$y_{t+1} = \frac{\alpha_3}{\alpha_2}(1 + r_{t+1})s_t \tag{17}$$

Using (3), (7), (8), (14) and (15), we obtain for $e_t > 0$:

$$b_t = \frac{\alpha_3}{\alpha_2}s_t - \frac{(1 - \tau_t)w_{t+1}}{(1 + r_{t+1})}h_{t+1} \tag{18}$$

To simplify the subsequent analysis we assume that the aggregate production function in our economy has the Cobb-Douglas form:

$F(k_t, (1 - \tau_t)h_t) = Ak_t^\theta[(1 - \tau_t)h_t]^{1-\theta}$.

We shall make a technical assumption about the parameters:

Assumption: $\theta(\alpha_2 + \alpha_3) > \alpha_2$.

In equilibrium the following expressions are obtained: $(1+r_t) = \theta A(k_t/(1-\tau_t)h_t)^{\theta-1}$ and $w_t = (1 - \theta)A(k_t/(1 - \tau_t)h_t)^\theta$. Using (12) and (18) we derive:

$$s_t = \frac{\alpha_2}{\theta(\alpha_2 + \alpha_3)}k_{t+1} \tag{19}$$

$$b_t = \frac{(\theta(\alpha_2 + \alpha_3) - \alpha_2)}{\theta(\alpha_2 + \alpha_3)}k_{t+1} \tag{20}$$

Substituting (18) and (19) in (7), while making use of (8) and (14), we obtain an expression for the income at date t :

$$y_t = (1 + \frac{\alpha_1}{\theta(\alpha_2 + \alpha_3)})k_{t+1} \tag{21}$$

However, using (17) and (19) we can also express aggregate income at any date t as a proportion of aggregate output at the same date:

$$y_t = \left(\frac{\alpha_3}{\alpha_2+\alpha_3}\right) q_t \tag{22}$$

This indicates the part of aggregate output which "young" members allocate between current consumption, saving and bequest. By the material balance constraint (13) it is clear that the other part is consumed by the "old" generation.

From (16) and (17) we derive with $e_t \neq 0$ that:

$$\beta e_t h_t = \beta h_t - \frac{\alpha_4}{\alpha_2}\left(\frac{1}{1-\tau_{t+1}}\right)\left(\frac{1+r_{t+1}}{w_{t+1}}\right) s_t$$

Using (1), (4) and (19), the growth factor of human capital is given by:

$$\frac{h_{t+1}}{h_t} = \frac{\beta(1-\theta)(\alpha_2+\alpha_3)(1+\tau_t)}{\alpha_4+(1-\theta)(\alpha_2+\alpha_3)} \equiv \gamma_t, \quad e_t > 0 \tag{23}$$

The growth factor γ_t can be smaller than 1 for β sufficiently close to 1, i.e., for low returns in education and a significant weight α_4 for leisure in the utility. When $e_t = 0$ and leisure equals 1, the growth factor of human capital is readily obtained from (9) :

$$\frac{h_{t+1}}{h_t} = \beta\tau_t \equiv \gamma_t, \quad e_t = 0. \tag{24}$$

The time dependence of γ_t in either (23) or (24) hinges only on the time dependence of the tax rate.

From (21), (22) and the production function we obtain, for e_t >0, that:

$$\frac{k_{t+1}}{k_t} = \frac{\alpha_3}{(\alpha_1+\theta(\alpha_2+\alpha_3))}(1+r_t) = \frac{\alpha_3 A}{(\alpha_2+\alpha_3+\alpha_1/\theta)}\left((1-\tau_t)\frac{h_t}{k_t}\right)^{1-\theta}$$

Dividing by (23):

$$\frac{k_{t+1}}{h_{t+1}} = \frac{\alpha_3 A(\alpha_4+(1-\theta)(\alpha_2+\alpha_3))}{\beta(1-\theta)(\alpha_2+\alpha_3)(\alpha_2+\alpha_3+\alpha_1/\theta)}\frac{(1-\tau_t)^{1-\theta}}{(1+\tau_t)}\left(\frac{k_t}{h_t}\right)^{\theta} \tag{25}$$

This describes the dynamic path of the capital-labor ratio of the economy in autarky.

2.3 Education Regimes

Given the above framework let us consider the optimal level of public provision of education in autarky. This is the level e_t^g that maximizes welfare of G_t. Hence,

Consider the utility function (2). Let us substitute for y_{t+1} in (2) and in (6). Assuming that $e_t>0$ first we make use of (23) to obtain an expression for leisure:

$$(1-e_t)=\frac{\alpha_4}{\beta(1-\theta)(\alpha_2+\alpha_3)}\gamma_t, \quad e_t>0. \tag{26}$$

The substitution of (23) and (26) in (2) leads to an expression for the lifetime utility of individuals in the tth generation:

$$u_t=\Omega_m(1+\tau_t)^{\alpha_4+(1-\theta)(\alpha_2+\alpha_3)}(1-\tau_t)^{(1-\theta)[\alpha_1+\alpha_2+\alpha_3+\theta(\alpha_2+\alpha_3)]} \tag{27}$$

where Ω_m groups parameters and variables that are predetermined at the outset of period t. It is assumed that any chosen τ_t will stay in place in the next period. Maximizing (27) with respect to τ_t, we derive the optimal level of public education under the "mixed" regime:

$$\tau_m=\frac{\alpha_4-(1-\theta)[\alpha_1+\theta(\alpha_2+\alpha_3)]}{\alpha_4+(1-\theta)[\alpha_1+(\alpha_2+\alpha_3)(2+\theta)]} \tag{28}$$

which establishes our claim. Note also that τ_m is independent of the size of private provision of education e_t. We are now left with the determination of the optimal τ_t when $e_t=0$. To that end let us consider the following optimization problem of the t-th generation:

$$Max_{s_t,b_t} \quad u_t=c_{1t}^{\alpha_1} \quad c_{2t}^{\alpha_2} \quad y_{t+1}^{\alpha_3}$$

subject to constraints (7), (8) and $h_{t+1}=\beta\tau_t h_t$. Using the first order conditions, and repeating the same steps as above, we obtain:

$$u_t=\Omega_p\tau_t^{(1-\theta)(\alpha_2+\alpha_3)}(1-\tau_t)^{(1-\theta)[\alpha_1+\alpha_2+\alpha_3+\theta(\alpha_2+\alpha_3)]} \tag{29}$$

The maximization of (29) with respect to τ_t leads to the optimal public provision of education:

$$\tau_p=\frac{\alpha_2+\alpha_3}{\alpha_1+(\alpha_2+\alpha_3)(2+\theta)} \tag{30}$$

with $\tau_p > \tau_m$.

Regardless of initial conditions, the optimal provision rate of public education depends on the parameters of the utility and production functions only. It is independent of time and of the efficiency of the education process β. It is also important to note that τ_m in (28) is decreasing in α_1, α_2 and α_3. However, it is increasing in α_4, that is, the extent to which public education is supplied increases in the weight to leisure in the utility function. As education regimes are substitutes the next proposition states the condition for their coexistence in terms of α_4 while fixing the other parameters of the model.[3]

Proposition 1 *Regardless of initial conditions, the weight to leisure in utility determines the optimal regime of education. There exists an interval* (α_4^m, α_4^p) *for values of* α_4 *which sustain a regime of both public and private education. For* $\alpha_4 \geq \alpha_4^p$*,it is optimal to provide only public education. For* $\alpha_4 \leq \alpha_4^m$*,it is optimal to provide only private education.*

Proof. There exists a value α_4^m for which τ_m in (28) is equal to zero:

$$\alpha_4^m = (1-\theta)[\alpha_1 + \theta(\alpha_2 + \alpha_3)]$$

For $\alpha_4 \leq \alpha_4^m$, only private education exists. For $\alpha_4 > \alpha_4^m$, it is optimal to provide public education. Substituting (23) in (26), making use of (28), one obtains:

$$(1-e_t) = \frac{2\alpha_4}{\alpha_4 + (1-\theta)[\alpha_1 + (\alpha_2 + \alpha_3)(2+\theta)]}$$

which gives the extent of leisure (and of private education) in a mixed regime of education. As $\partial(1-e_t)/\partial\alpha_4 > 0$, $e_t = 0$ at:

$$\alpha_4^p = (1-\theta)[\alpha_1 + (\alpha_2 + \alpha_3)(2+\theta)]$$

with $\alpha_4^p > \alpha_4^m$. For $\alpha_4 \geq \alpha_4^p$, private education ceases. □

The above proposition is illustrated in Figure 1. It depicts the optimal provision of public education τ for combinations of α_3 and α_4 while assuming

[3]The next result can be generalized to the case where both α_3 and α_4 vary. Intervals will be substituted by planes in this case.

fixed values for the other parameters of the model. The emphasis is on α_3 and α_4 because of their offsetting effects on leisure (see (16)) and hence, on utility. Figure 1 is divided into three planes, each representing a regime of education. For any given value of α_3, there is a first range of values of α_4 starting from 0 which gives rise to private education only ($\tau = 0$); for a second range of intermediate values of α_4, it is optimal to have both public and private education; finally, a third range of values of α_4 justifies public education only (τ taking values around 0.35).

[**Insert Figure 1**]

The preceding results demonstrate that the optimal rate of financing of public education is independent of a country's initial levels of physical and human capital. An important implication is that any heterogeneity observed across education systems is reflected by a nation's preferences rather than the nation's wealth. In other words, there should not be a discernible relationship between how rich a country is in terms of national income per-capita and the proportion of this income that it allocates to education.

2.4 Long-run Growth

Our main purpose now is to compare the equilibrium paths of a single economy in autarky under the three regimes of education discussed in Proposition 1. Consider the competitive equilibria from given initial conditions and compare the long-run properties of this economy under each regime. Note first that the time independence of the tax rate in the previous section implies time independence of γ in (23) and (24) as well.

Let us apply a one-period lead to (22) and then, divide by (21) to obtain the expression for output and income growth (for any τ) :

$$\frac{q_{t+1}}{q_t} = \frac{y_{t+1}}{y_t} = \frac{\alpha_3 A}{[(\alpha_2 + \alpha_3) + \alpha_1/\theta]} \left[\frac{k_{t+1}}{(1-\tau)h_{t+1}}\right]^{\theta-1} \tag{31}$$

In the long-run $k_{t+1}/h_{t+1} = k_t/h_t = k/h$. From (25), we obtain the long-run capital-labor ratio:

$$\frac{k}{h} = \left[\frac{\alpha_3 A}{\gamma(\alpha_2 + \alpha_3 + \alpha_1/\theta)}\right]^{\frac{1}{1-\theta}} (1-\tau) \tag{32}$$

Substituting this in (31) gives:

$$\frac{q_{t+1}}{q_t} = \gamma$$

This relationship holds whatever regime of education, that is for any τ, which establishes our claim. The long-run economic growth in autarky coincides with the human capital growth factor γ, regardless of initial conditions and the education regime.

Now let us use some notation to differentiate education regimes. Let τ_i $(i = a, m, p)$ denote the optimal provision rate of formal education under, respectively, private education ($\tau_a = 0$), mixed provision (τ_m given in (28)) and public provision (τ_p given in (30)). Likewise, using (23) and (24), let γ_i $(i = a, m, p)$ denote the corresponding growth factors. Now we prove the following relationship regarding the long-run rates of growth of each regime.

Proposition 2 *Regardless of initial conditions, growth factors across education regimes rank as follows:* $\gamma_a > \gamma_m > \gamma_p$.

The proof is relegated to the Appendix. An implication of Proposition 2 is that for any two economies which differ, let us say only in preferences for leisure, their long-run endogenous growth rates will differ. This results from the stronger impact that private education has on growth as the weight of leisure in the utility function decreases. In contrast, any two economies which differ only in the initial conditions will grow in the long-run at the same endogenous growth rate[4]

[4] It is worth noting that the proof of Proposition 2 assumes similar efficiency in the education process regardless of the education regime (namely, the same β). Though there are reasons to believe that efficiency of public education is higher than that of private education, the ratio γ_a/γ_p indicates by how much the former has to exceed the latter to achieve a similar long-run growth rate. Straightforward numerical calculations (using $\alpha_1 = \alpha_2 = 1$, $\alpha_3 = 2$, $\theta = 0.5$, $A = 4$) indicate that public education (with $\alpha_4 = 2.75$) has to be 2.4 times more efficient than private education (when characterized by $\alpha_4 = 0.25$) to achieve the same growth rate. However, even if growth rates are similar, output levels under public education are lower, in our framework, since part of the human capital resources are diverted from production activities to education (while raising the leisure).

3 Capital Markets Integration

Consider two economies in autarky: the *domestic* economy and the *foreign* economy, whose variables are marked with "*". They are assumed to differ *only* in the initial physical capital transfers and human capital. At date $t = 0$, the following variables are given: b_{-1}, h_0 for the domestic economy; b^*_{-1}, h^*_0 for the foreign economy. Denote the domestic equilibrium by $\{(c_{1t}, c_{2t}, s_t, b_t, e_t), (w_t, r_t)\}$. The equilibrium abroad is denoted by $\{(c^*_{1t}, c^*_{2t}, s^*_t, b^*_t, e^*_t), (w^*_t, r^*_t)\}$. As the utility and production functions are assumed to be similar, both countries provide public education at the same rate, i.e., $\tau = \tau^*$. In this case, as (25) holds for both economies, if $(k_0/h_0) > (k^*_0/h^*_0)$ then $(k_t/h_t) > (k^*_t/h^*_t)$ for all t. This implies $(1 + r_t) < (1 + r^*_t)$ and $w_t > w^*_t$ for all t.

Assume that at date $t = 0$ the domestic and foreign economies are integrated to form a single commodity market and a single capital market (while labor remains internationally immobile). Upon the integration of capital markets, physical capital will flow from the low return to the high return country until interest rates are equalized in the integrated economy. The type of international capital movement we consider, involves a change in the location but not the ownership of physical capital. In the sequel, we use capitals to distinguish post-integration variables from their autarky counterparts. Hence, $(1 + R_t)$ stands for the post-integration interest rate, W_t and W^*_t the wage rates, Υ_t and Υ^*_t the provision rate of public education, etc.

3.1 Two-Country Equilibrium

Distinguish between the capital stock used in the production in the home country, K_t, and the stock of physical capital, located at home and abroad, *owned* by domestic residents, T_t. Hence (12) becomes:

$$T_{t+1} = S_t + B_t \tag{33}$$

Similarly we define T^*_t for the foreign country. Any difference $(T_t - K_t)$ corresponds to a net outflow of domestic capital abroad. At any date t, the following international identity must hold:

$$T_t + T^*_t = K_t + K^*_t. \tag{34}$$

Hence, the above difference corresponds also to a foreign inflow of capital, $(K^*_t - T^*_t)$. After substituting (32) and making use of (33), the first-order conditions for both countries under integration lead to:

$$S_t + S_t^* = \frac{\alpha_2}{\theta(\alpha_2+\alpha_3)}(K_{t+1} + K_{t+1}^*) \tag{35}$$

$$B_t + B_t^* = \frac{[\theta(\alpha_2+\alpha_3)-\alpha_2]}{\theta(\alpha_2+\alpha_3)}(K_{t+1} + K_{t+1}^*) \tag{36}$$

These two equations are the analogues of (19) and (20) for the integrated economy. As for the autarky equilibrium, we obtain:

$$\frac{[H_{t+1}(1-\Upsilon_{t+1}) + (1-\Upsilon_{t+1}^*)H_{t+1}^*]}{[H_t(1-\Upsilon_t) + (1-\Upsilon_t^*)H_t^*]} = \Gamma_t \tag{37}$$

$$\frac{K_{t+1} + K_{t+1}^*}{K_t + K_t^*} = \frac{\alpha_3}{(\alpha_1 + \theta(\alpha_2+\alpha_3))}(1+R_t) \tag{38}$$

$$Y_t + Y_t^* = \left[1 + \frac{\alpha_1}{\theta(\alpha_2+\alpha_3)}\right](K_{t+1} + K_{t+1}^*) \tag{39}$$

$$Y_t + Y_t^* = (\frac{\alpha_3}{\alpha_2+\alpha_3})(Q_t + Q_t^*) \tag{40}$$

It is worth noting that equations (35) to (40) that describe the dynamic path of the integrated economy are similar to those obtained for the autarky case.

A central issue that can be analyzed within this framework is to know the extent countries are expected to modify their formal education policy as capital markets become more integrated. Before doing so, it is important to raise two questions, namely: What are the benefits, if any of CMI, to the integrated economy? What can be said about the division of the gains between the capital-exporting and the capital-importing countries? These questions, which have been largely untouched in the literature, will be taken up in the rest of this section.

Proposition 3 *Regardless of the education regime, total output and total capital stock of the integrated economy increase at all dates compared to the autarkic case.*

The proof is relegated to the Appendix. This proposition confirms the robustness of the traditional static gains from international capital mobility and extends the result to a dynamic framework with public education. As there are dynamic gains in terms of income, even though the provision rate stays at the autarky level, each country's expenditures on public education will nevertheless be affected by CMI. It stands to reason that, as a result of the latter, wages in the country which is endowed with higher initial capital stock increase and decrease in the other country. Therefore, the relative expenses on public education as a percentage of a country's national income vary differently across countries.

Numerical Simulations

Our main purpose now is to numerically compute the dynamic paths of the domestic and foreign economies for the cases of autarky and capital markets integration. This allows the computation and comparison of the benefits of capital integration across education regimes. Initial values were taken to be: $\alpha_1 = \alpha_2 = 1$, $\alpha_3 = 2$, $\theta = 0.5$, $A = 4.0$, $k_0 = T_0 = 2$, $k_0^* = T_0^* = 1$, $h_0 = H_0 = h_0^* = H_0^* = 1$. The weight to leisure α_4 takes three values: $\alpha_4(a) = 0.5$, $\alpha_4(m) = 1.5$ and $\alpha_4(p) = 5.0$. Each value of α_4 is chosen such that, according to Proposition 2, each education regime is represented (”a” stands for private, ”m” for mixed and ”p” for public). This implies that $\tau_m = 0.044$ and $\tau_p = 0.353$. The efficiency of educational regimes is chosen to be $\beta(a) = 1.44$, $\beta(m) = 2.07$ and $\beta(p) = 3.06$ such that the growth rate of output is the same, that is $\gamma = 1.08$.

Given this information, a representative agent in each economy, with perfect foresight, attains his/her optimal consumption-bequest in two steps. First, the paths of w_t, r_t, h_t, k_t and y_t are simultaneously solved for. Second, given these parameters, the paths of c_{1t}, c_{2t}, s_t, b_t and u_t can be computed. Simulations under capital markets integration must satisfy (33) and the adding-up constraint for capital balances (34).

[Insert Figure 2]

Figure 2 plots the change in income following capital markets integration as a percentage of their autarky values. The three panels of Figure 2 represent the three education regimes namely, private education in panel (a), mixed in panel (b) and public in panel (c). The broken line gives the time pattern of the gains for the integrated economy resulting from capital markets integration. Gains in income of the order of 1.5-2 percent are observed in the short-run but fade away with time whatever the education regime. Note that the gains in utility turn out to be much larger (not shown).

Proposition 3 demonstrated the existence of gains from capital markets integration for the integrated economy as a whole. It is now important to determine the partition of these gains between the capital importing country and the capital exporting country. It will become apparent that there exists a source of conflict among countries which justifies strategic behavior in the provision of public education.

Following capital market integration, equal returns to physical capital implies:

$$\frac{K_t}{(1-\Upsilon_t)H_t} = \frac{K_t^*}{(1-\Upsilon_t^*)H_t^*} =$$

$$= \frac{K_t + K_t^*}{(1-\Upsilon_t)H_t + (1-\Upsilon_t^*)H_t^*} \quad t = 0, 1, 2, .. \tag{41}$$

From the properties of the production functions:

$$\frac{Q_t}{K_t} = \frac{Q_t^*}{K_t^*} = \frac{Q_t + Q_t^*}{K_t + K_t^*} \quad t = 0, 1, 2, ..$$

Combining these two expressions:

$$\frac{Q_t}{Q_t + Q_t^*} = \frac{K_t}{K_t + K_t^*} = \frac{(1-\Upsilon_t)H_t}{(1-\Upsilon_t)H_t + (1-\Upsilon_t^*)H_t^*} \quad t = 0, 1, 2, .. \tag{42}$$

Following capital markets integration, each country's share in total output and its share in the stock of physical capital of the integrated economy is equal to its share in the stock of human capital.

Reconsidering the above numerical example, we note that the use of (42) enables us to compute numerically the individual gains of participating countries as displayed in Figure 2. We observe from Figure 2 that the first generation of both countries G_0 and G_0^* are better off after CMI whatever education regime is considered. In the case of public education foreign individuals are worse off in all generations t, $t > 0$, following the CMI. The same applies to the domestic individuals in the private provision case in all generations t, $t > 2$. In the latter case (and in the mixed regime), the domestic wages decrease after integration which lowers the return to private education and, hence, the stock of domestic human capital compared to its autarky level (and vice versa for the foreign country). In the public education regime, as the path of human capital is unaffected by integration and the foreign country is (by assumption) poorer at the outset, the chain of effects is triggered by the decrease in interest rates abroad compared to autarky.[5]

To simplify matters, consider the regime of public education only. As the two countries have similar representative agent's utility and same aggregate production function, then $\Upsilon_t = \Upsilon_t^* = \tau_p$. Making use of (24):

$$H_t = \beta\tau_p H_{t-1} = (\beta\tau_p)^t H_0$$

$$H_t^* = \beta\tau_p H_{t-1}^* = (\beta\tau_p)^t H_0^*$$

Substitution in (42) gives:

$$\frac{Q_t}{Q_t + Q_t^*} = \frac{K_t}{K_t + K_t^*} = \frac{H_0}{H_0 + H_0^*} \quad t = 0, 1, 2, ..$$

It is clear from this last expression that the partition of output in the integrated economy at any date t is determined by the initial levels of human capital. Thus, with a cooperative symmetric solution for the provision rates, none of the two countries can improve its relative position.

3.2 Nash Equilibrium Under Symmetry

Consider now the case where both countries are identical at $t = 0$ in *all parameters*. Introducing capital markets integration will result in variations in Υ even though we have a symmetric case. We can find the reaction curves in terms of Υ and Υ^* using the usual Cournot-Nash behavior: the domestic government chooses Υ in a way that maximizes the utility of its representative consumer, while Υ^* is assumed to be given, and vice versa. We demonstrate now that for Nash equilibria the provision of public education differs from the cooperative, or autarkic, level even in the symmetric case:

Proposition 4 *When we consider Nash equilibrium for this symmetric game the optimal provision Υ of public education is different from τ_p in each date.*

[5] It is important to stress that the simulation results in Figure 2 can be generalized to a very broad range of parameter values. Whereas the pattern of country responses to CMI is very robust with respect to changes in A, β, and α's, it is sensitive to the choice of capital share θ. For example, for $\theta = 0.35$ the changes in foreign national income in Figure 2(c) are all negative. However, independent of parameter values gains from CMI are always attained.

The proof is to be found in the Appendix. The extent by which the Nash equilibrium (NE) differs from the cooperative one is shown in the proof as well. Numerically it can be added that, when countries are fully symmetric, the cooperative solution τ_p is 0.353 and the NE solution at date $t = 0$ is 0.468 for the usual parameter values.

Thus if governments choose at date 0 the NE Υ_0 and Υ_0^* , assuming that this can be repeated in each date, we find deviations from the level τ_p, which basically mean that we are not in a Pareto optimal situation.

3.3 Education Policy: Cooperative Solution

Public education policy is called 'optimal' when at each date t both countries (governments) decide jointly upon the public education provision rate in a way that maximizes some *weighted sum* of the tth generations' utilities.

Proposition 5 *Following capital market integration, and coordination of education policy, the optimal provision rate of public education is the same as that under autarky. It is independent of time, initial levels of transfers and human capital across countries.*

See the Appendix for the proof. Proposition 5 implies that $\Upsilon_t = \Upsilon_{t+1} = \Upsilon_t^* = \Upsilon_{t+1}^* = \tau$. Given this result, Propositions 2, 3 and 4 hold for the integrated economy as a whole. In particular, $\Gamma_t = \gamma$ for all t implies that the integrated economy will grow in the long-run as in the autarky case. A feature of this cooperative solution is that capital market integration affects neither the optimal provision rate of formal education nor the long-run growth rate when compared to autarky.

3.4 Public Education Policy: The Nash Bargaining Solution

In considering non-cooperative solutions we shall take the following approach. Public education policy cannot be varied too often by governments, hence only stationary behavior will be considered here. Since the share in production of each country will depend upon its share in the stock of human capital in the integrated economy, education policy in the foreign country will affect the production volume in the domestic country and vice-versa. Thus the two governments in negotiating the CMI will also have to deal with public education provision level. In our framework it is unreasonable to expect an agreement where the public education levels differ, since it implies that in the long run only one country will produce. We assume now that the resolution of this conflict concerning public education provision is implemented via the Nash bargaining solution. The "disagreement point" will be given by the autarkic utility levels, i.e., when $\tau = \tau_p$ and no transfer of capital between the two countries.

We shall consider the Nash bargaining solution for this conflict assuming that it takes place at the outset of each period to determine the provision of public education in each country. Given the initial capital holdings of residents and the human capital levels at the beginning of date t, to define the set of feasible utilities for the Nash bargaining problem let us derive the set of all utilities "attainable" at period t (under the given parameters at the outset of this period). In this case we must consider also *partial* capital transfers between the two markets (so far we looked at competitive equilibrium with unrestricted flow of capital which is a "Pareto optimal" case).

Let λ be a parameter in $[0, 1]$. We say that the two economies, or capital markets, are λ-*integrated*, $0 \leqq \lambda \leqq 1$, if only a proportion λ of the unrestricted competitive equilibrium (CE) flow of capital is allowed to move between the domestic and the foreign capital markets. Let us elaborate on this type of arrangement. Given the ownership pattern of capital by each country's residents at the outset of date t and the initial distribution of human capital, if the flow of capital between the two countries for this period under unrestricted CE is Δ_t, then we impose the restriction: only $\lambda\Delta_t$ is allowed to move from one country to the other, and under this restricted transfer of capital we consider the CE in each economy. For $\lambda = 1$ it is the CMI case we have considered before while $\lambda = 0$ is the autarkic case. Clearly, such equilibria will also depend upon the identical Υ, $0 \leqq \Upsilon \leqq 1$, chosen by the two governments. For each given values of (λ_t, Υ_t) at the outset of date t the competitive equilibrium, after capital transfers at level $\lambda\Delta_t$ take place, is called: $(\lambda_t, \Upsilon_t) - CE$. Denote the corresponding utilities of the representative consumers in each country by $U_t(\lambda_t, \Upsilon_t)$ and $U_t^*(\lambda_t, \Upsilon_t)$ where $(\lambda_t, \Upsilon_t) \in [0, 1]^2$.

To formulate our Nash bargaining problem at the outset of date t, $\langle S_t, d_t \rangle$, given the initial conditions of the two economies, we define:

$$S_t = \{(u, u^*) \in R^2 \quad | \text{ For some } (\lambda, \Upsilon) \in [0,1]^2$$
$$0 \leqq u \leqq U_t(\lambda, \Upsilon) \text{ and } 0 \leqq u^* \leqq U_t^*(\lambda, \Upsilon)\}$$

The "disagreement point" d_t is given by the autarkic utility levels, i.e., when capital is not allowed to move between the two countries,

$$d_t = \langle U_t(0, \tau_p), \quad U_t^*(0, \tau_p) \rangle$$

Before we look for the Nash bargaining solution let us note the following property for the "full CMI" case (i.e., when $\lambda = 1$) :

Claim For any Υ, $0 \leqq \Upsilon \leqq 1$ **,** $U_t(1, \Upsilon)/U_t^*(1, \Upsilon) = \xi_t$ *where* ξ_t *is a constant which depends only on the initial conditions of the two economies at date t.*

The proof is relegated to the Appendix. From the proof we see that we can write:

$U_t(1, \Upsilon) = B_t \Upsilon^{(1-\theta)(\alpha_2+\alpha_3)}(1-\Upsilon)^{(1-\theta)[\alpha_1+(\alpha_2+\alpha_3)(1+\theta)]}$

and

$U_t^*(1, \Upsilon) = B_t^* \Upsilon^{(1-\theta)(\alpha_2+\alpha_3)}(1-\Upsilon)^{(1-\theta)[\alpha_1+(\alpha_2+\alpha_3)(1+\theta)]}$

where B_t and B_t^* are given by the initial parameters at the outset of period t. Making use of this claim and the expressions for the utility levels we prove:

Proposition 6 *The Nash bargaining solution yields a provision level of public education as in the cooperative solution case; namely,* $\overline{\Upsilon} = \tau_p$.

Proof. To solve for the Nash bargaining solution we should note first that any $\langle U_t(\lambda, \Upsilon), U_t^*(\lambda, \Upsilon) \rangle$ is not Pareto optimal if $\lambda < 1$. This follows from our earlier analysis which shows that for $\lambda = 1$ we have positive gains to aggregate outputs, compared to $\lambda = 0$, but this can be generalized to any $\lambda < 1$. We shall allow asymmetry in the bargaining power of the two countries (see Zhou (1996)); thus for some positive constants μ and ν the Nash solution will be attained by maximizing $(u - d_t)^\mu (u^* - d_t^*)^\nu$ over the set

$$\hat{S}_t = \{(U_t(1, \Upsilon), U_t^*(1, \Upsilon)) \mid \quad 0 \leqq \Upsilon \leqq 1\}$$

rather than over the whole feasible set S_t. Using the above expressions for the utility levels under full CMI and noting that

$$\max_{0\leq\Upsilon\leq1}[(U_t(1,\Upsilon)-d_t)^{\mu}(U_t^*(1,\Upsilon)-d_t^*)^{\nu}]$$

cannot be obtained for $\Upsilon=0$ or $\Upsilon=1$, we derive from the optimum condition:

$$(1-\theta)(\alpha_2+\alpha_3)\Upsilon^{(1-\theta)(\alpha_2+\alpha_3)-1}(1-\Upsilon)^{[\alpha_1+(1+\theta)(\alpha_2+\alpha_3)]}$$

$$-\Upsilon^{(1-\theta)(\alpha_2+\alpha_3)}(1-\Upsilon)^{(1-\theta)[\alpha_1+(1+\theta)(\alpha_2+\alpha_3)]-1}(1-\theta)[\alpha_1+(1+\theta)(\alpha_2+\alpha_3)]=0$$

which yields that

$$\overline{\Upsilon}=\frac{\alpha_2+\alpha_3}{\alpha_1+(2+\theta)(\alpha_2+\alpha_3)}=\tau_p$$

□

[Insert Figure 3]

Namely, the Nash bargaining solution chooses the same provision level of public education, and hence allocation of production between the two countries, as the cooperative solution and hence as the autarkic case. To understand this result let us refer to Figure 3 which displays the feasible set together with a map of iso-indifference curves for the integrated economy. It is important to note that $\hat{S}_t$ lies on a straight line via (d_t, d_t^*) with a slope B_t^*/B_t. Thus the Nash bargaining solution $\overline{\Upsilon}$ must coincide with τ_p which maximizes the *weighted sum* of utilities since both $U_t(1,\Upsilon)$ and $U_t^*(1,\Upsilon)$ are either increasing in Υ or decreasing in Υ simultanously. Moreover, this argument also demonstrates that the Kalai-Smorodinsky solution to this bargaining problem is also obtained at $\Upsilon=\tau_p$, since $(U_t(1,\tau_p), U_t^*(1,\tau_p))$ is the "ideal point" and it is in S_t.

4 Discussion

The objective of this paper is to examine in a dynamic framework the optimal provision of education in equilibrium with and without capital markets integration. Endogenous growth in this economy is attained via investment in education. The evolution of human capital, and its effects on economic growth, has attracted tremendous attention of economists since the mid-80s, where provision of education has been introduced explicitly in economic models. There is strong evidence that parents' human capital and parents' investment in the upbringing of their child play an important role. Thus the factors we chose in the evolution of human capital are: (1) public education (financed by taxing labor earnings), (2) the parents' human capital and (3) 'private education', which is, in our case, the time spent by parents to enhance their children's human capital.

When each economy is considered in isolation, the *optimal provision* of public education is shown to depend on the parameters of household's utility function and of the economy's aggregate production function. In particular, the weight to leisure in the utility function may explain how 'optimal regime' of education (public, private or "mixed") is chosen. A striking feature of our model is the irrelevance of the specific levels of physical and human capital in each country in determining the optimal share in total output allocated to public education. We believe that the specific choices of preferences and production function are the main reason for this outcome.

That capital markets integration matters for the international allocation of productive resources has been a theme of numerous theoretical and empirical studies. Less known, however, is the fact that as a consequence of integration of capital markets the allocation of the total capital between the two countries depends on the share of each country in the total human capital of the integrated economy. Thus, it opens the possibility for governmental intervention in provision of public education in order to attract a larger share of the available stock of physical capital. In this paper we point out one policy variable which has received less attention in the literature: attracting physical capital, and hence local production, by increasing the relative share in the stock of human capital. The tools for competition in this case are the investments in public education. The analysis rests on the reasoning that an increase in a country's investment in education raises this country's return to physical capital, and thus acts as an incentive for capital inflows. This gives rise to a strategic behavior by governments due to the competition in attracting investors. Some interesting results demonstrated in our framework are: (a) The optimal provision of public education in the autarkic case is the same as that of the cooperative solution and the Nash bargaining solution; (b) Nash equilibrium to this conflict produces efficiency losses compared to the cooperative solution (or the Nash bargaining solution).

5 Appendix

Proof of Proposition 2. Let $\alpha_4(i)$, for $i = a, m, p$, denote the weight to leisure in utility which corresponds to each education regime. Let us use (23) twice to obtain the growth factors γ_a and γ_m. Setting $\tau_t = 0$ in (23):

$$\gamma_a = \frac{\beta(1-\theta)(\alpha_2+\alpha_3)}{\alpha_4(a)+(1-\theta)(\alpha_2+\alpha_3)} \tag{A.1}$$

Substituting (28) in (23):

$$\gamma_m = \frac{2\beta(1-\theta)(\alpha_2+\alpha_3)}{\alpha_4(m)+(1-\theta)[\alpha_1+(\alpha_2+\alpha_3)(2+\theta)]} \tag{A.2}$$

Substituting (30) in (24):

$$\gamma_p = \frac{\beta(\alpha_2+\alpha_3)}{\alpha_1+(\alpha_2+\alpha_3)(2+\theta)} \tag{A.3}$$

Hence $\gamma_m > \gamma_p$ if:

$$(1-\theta)[\alpha_1+(\alpha_2+\alpha_3)(2+\theta)] > \alpha_4(m)$$

which, by Proposition 2, is always satisfied. Likewise, $\gamma_a > \gamma_m$ if:

$$(1-\theta)[\alpha_1+\theta(\alpha_2+\alpha_3)] > 2\alpha_4(a)-\alpha_4(m)$$

which, by Proposition 2, is always satisfied. Otherwise there would not be a mixed regime in the first place. □

Proof of Proposition 3. At date $t = 0$, we have $k_0 + k_0^* = K_0 + K_0^*$. With capital markets integration, we have:

$$\frac{H_0(1-\tau)}{K_0} = \frac{H_0^*(1-\tau^*)}{K_0^*} = \frac{H_0(1-\tau)+H_0^*(1-\tau^*)}{K_0+K_0^*}$$

Denote:

$$\lambda_t = \frac{k_t}{k_t+k_t^*} \qquad \text{for} t = 0,1,2,...$$

Since, at date $t = 0$:

$$(1-\tau)h_0 = (1-\tau)H_0 \text{ and } (1-\tau^*)h_0^* = (1-\tau^*)H_0^*$$

are given, we can write:

$$\frac{H_0(1-\tau)}{K_0} = \lambda_0 \frac{h_0(1-\tau)}{k_0} + (1-\lambda_0)\frac{h_0^*(1-\tau^*)}{k_0^*}$$

Therefore, by the concavity of the production function:

$$\frac{q_0+q_0^*}{k_0+k_0^*} = \lambda_0 F\left(1, \frac{h_0(1-\tau)}{k_0}\right) + (1-\lambda_0)F\left(1, \frac{h_0^*(1-\tau^*)}{k_0^*}\right)$$

$$< F\left(1, \quad \lambda_0 \frac{h_0(1-\tau)}{k_0} + (1-\lambda_0)\frac{h_0^*(1-\tau^*)}{k_0^*}\right) = F(1, \frac{H_0(1-\tau)}{K_0})$$

$$= F\left(1, \frac{H_0^*(1-\tau^*)}{K_0^*}\right)$$

Thus:

$$\frac{q_0+q_0^*}{k_0+k_0^*} < \frac{Q_0}{K_0} = \frac{Q_0^*}{K_0^*} = \frac{Q_0+Q_0^*}{K_0+K_0^*}$$

However, since $k_0 + k_0^* = K_0 + K_0^*$ then:

$$q_0 + q_0^* < Q_0 + Q_0^*$$

This implies that:

$$y_0 + y_0^* < Y_0 + Y_0^*$$

Therefore:

$$k_1 + k_1^* < K_1 + K_1^*$$

As $\tau = \tau^*$ in autarky and with integration, aggregate labor supply is unaffected by integration and, hence:

$$\frac{q_1+q_1^*}{k_1+k_1^*} = \lambda_1 F\left(1, \frac{h_1(1-\tau)}{k_1}\right) + (1-\lambda_1)F\left(1, \frac{h_1^*(1-\tau^*)}{k_1^*}\right)$$

$$< F\left(1, \lambda_1 \frac{h_1(1-\tau)}{k_1} + (1-\lambda_1)\frac{h_1^*(1-\tau^*)}{k_1^*}\right) =$$

$$= F\left(1, \frac{h_1(1-\tau) + h_1^*(1-\tau^*)}{k_1+k_1^*}\right)$$

Rewriting this expression:

$$q_1 + q_1^* < F(k_1 + k_1^*, h_1(1-\tau) + h_1(1-\tau^*))$$
$$< F(K_1 + K_1^*, H_1(1-\tau) + H_1^*(1-\tau^*))$$

since $h_1 + h_1^* = H_1 + H_1^*$. Dividing both sides by $(K_1 + K_1^*)$:

$$\frac{q_1 + q_1^*}{k_1 + k_1^*} < F\left(1, \frac{H_1(1-\tau) + H_1^*(1-\tau^*)}{K_1 + K_1^*}\right) = F\left(1, \frac{H_1(1-\tau)}{K_1}\right)$$
$$= F\left(1, \frac{H_1^*(1-\tau^*)}{K_1^*}\right) = \frac{Q_1}{K_1} = \frac{Q_1^*}{K_1^*} = \frac{Q_1 + Q_1^*}{K_1 + K_1^*}$$

Hence, $q_1 + q_1^* < Q_1 + Q_1^*$, which implies that $k_2 + k_2^* < K_2 + K_2^*$. This process continues for all $t = 2, 3, 4, ...$ proving our claim that $q_t + q_t^* < Q_t + Q_t^*$.□

Proof of Proposition 5. We will demonstrate the result for public education only. The proof for the mixed regime can be obtained by analogy. Authorities of both countries that participate to capital markets integration choose a single $\Upsilon_t (= \Upsilon_t^*)$ such as to maximize the following weighted sum of domestic and foreign utilities:

$$\max \left\{ a_t C_{1t}^{\alpha_1} \quad C_{2t}^{\alpha_2} \quad Y_{t+1}^{\alpha 3} + a_t^* C_{1t}^{*\alpha_1} \quad C_{2t}^{*\alpha_2} \quad Y_{t+1}^{*\alpha 3} \right\} \tag{A.4}$$

where the weights a_t and a_t^* are independent of Υ_t. Authorities also expect futures rates to be equal to the current one, that is $\Upsilon_t = \Upsilon_{t+1} = \Upsilon_t^* = \Upsilon_{t+1}^*$. Making use of the first-order conditions, the optimization problem (A.4) becomes:

$$\max_{\Upsilon_t} \left\{ \left(\frac{\alpha_1}{\alpha_3}\right)^{\alpha_1} \left(\frac{\alpha_2}{\alpha_3}\right)^{\alpha 2} (1 + R_{t+1})^{-\alpha_1} \left(a_t Y_{t+1}^{a_1 + \alpha_2 + \alpha_3} + a_t^* Y_{t+1}^{*a_1 + \alpha_2 + \alpha_3} \right) \right\}$$

Making use of (3), (7), (9) and (17), one obtains:

$$Y_{t+1} = \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \left[Y_t (1 + R_{t+1}) + (1 - \Upsilon_{t+1}) W_{t+1} H_t \Upsilon_t \beta \right] \tag{A.5}$$

Likewise, one obtains Y_{t+1}^*. Under capital integration:

$$\frac{K_t}{(1 - \Upsilon_t) H_t} = \frac{K_t^*}{(1 - \Upsilon_t^*) H_t^*} = \frac{K_t + K_t^*}{(1 - \Upsilon_t) H_t + (1 - \Upsilon_t^*) H_t^*} \tag{A.6}$$

The interest rate $(1+R_{t+1})$ and wage W_{t+1} can be expressed in terms of (A.6) at period $t+1$. Equations (37) and (38) give the dynamic path of the human capital and of the stock of the physical capital of the integrated economy respectively. Hence, all period $t+1$ variables can be expressed in terms of period t variables which cannot be influenced by the choice of Υ_t. Hence,

$$(1+R_{t+1}) = \Upsilon_t^{1-\theta}(1-\Upsilon_t)^{(1-\theta)\theta} \quad cst1$$

$$Y_{t+1} = \Upsilon_t^{1-\theta}(1-\Upsilon_t)^{(1-\theta^2)} \quad cst2$$

where cst1 and cst2 stand for groups of parameters and of predetermined variables. Y^*_{t+1} can be obtained by analogy. Hence,

$$a_t U_t + a_t^* U_t^* = \Upsilon_t^{(1-\theta)(\alpha_2+\alpha_3)}(1-\Upsilon_t)^{(1-\theta)[\alpha_1+\alpha_2+\alpha_3+\theta(\alpha_2+\alpha_3)]} \quad cst3 \tag{A.7}$$

where cst3 is another constant. Maximization of (A.7) with respect to Υ_t leads to:

$$\Upsilon_p = \frac{\alpha_2+\alpha_3}{\alpha_1+(\alpha_2+\alpha_3)(2+\theta)} \tag{A.8}$$

which is similar to τ_p in (30).□

Proof of Proposition 4. The proof is for the regime of public education only. The maximization problem of the domestic country's authority is to choose Υ_t given Υ_t^* such as to maximize U_t :

$$Max_{\Upsilon_t} \quad U_t = C_{1t}^{\alpha_1} \quad C_{2t}^{\alpha_2} \quad Y_{t+1}^{\alpha_3} \quad \text{given } \Upsilon_t^* \tag{A.9}$$

By analogy, the foreign country's maximization problem is:

$$Max_{\Upsilon_t^*} \quad U_t^* = C_{1t}^{*\alpha_1} \quad C_{2t}^{*\alpha_2} \quad Y_{t+1}^{*\alpha_3} \quad \text{given } \Upsilon_t \tag{A.10}$$

Making use of the first-order conditions, the maximization problem of the domestic authority is:

$$Max_{\Upsilon_t} \left(\frac{\alpha_1}{\alpha_3}\right)^{\alpha_1}\left(\frac{\alpha_2}{\alpha_3}\right)^{\alpha_2}(1+R_{t+1})^{-\alpha_1}Y_{t+1}^{\alpha_1+\alpha_2+\alpha_3} \tag{A.11}$$

given Υ_t^*. The expression for Y_{t+1} is given by (5), that of $(1+R_{t+1})$ depends on (A.6). Like Proposition (5), (37) and (38) can be used to express $t+1$ variables in terms of period t variables. After substitution, the final expression for U_t is:

$$U_t = \Omega_3[b_t + \Omega_2\Omega_{1t}(1-\Upsilon_t)\Upsilon_t]^{\alpha_1+\alpha_2+\alpha_3}\Omega_{1t}^{(\theta-1)(\alpha_2+\alpha_3)} \tag{A.12}$$

where Ω_2 and Ω_3 are constant grouping parameters of the model and:

$$\Omega_{1t} = [(1-\Upsilon_t) + (1-\Upsilon_t^*)]^{1-\theta} / [(1-\Upsilon_t)\Upsilon_t + (1-\Upsilon_t^*)\Upsilon_t^*] \qquad \text{(A.13)}$$

Maximization of U_t in (A.12) with respect to Υ_t for a given Υ_t^* leads to the following FOC:

$$(\alpha_1+\alpha_2+\alpha_3)(1-2\Upsilon)\Omega_{1t} - (1-\theta)(\alpha_2+\alpha_3)[(1-\Upsilon_t)\Upsilon_t$$

$$+b_t/\Omega_4]\frac{\partial \Omega_{1t}}{\partial \Upsilon_t} + (\alpha_1+\alpha_2+\alpha_3)(1-\Upsilon_t)\Upsilon_t \frac{\partial \Omega_{1t}}{\partial \Upsilon_t} = 0 \qquad \text{(A.14)}$$

where $\Omega_4 = A(1-\theta)\alpha_3 K_t^\theta H_t^{1-\theta}/[\alpha_1+\theta(\alpha_2+\alpha_3)]$. By imposing symmetry, i.e. $\Upsilon_t = \Upsilon_t^*$, (A.14) simplifies to:

$$(\alpha_1+\alpha_2+\alpha_3)(1-2\Upsilon) = -\Delta(1-\Upsilon(1-\theta)) \qquad \text{(A.15)}$$

where $\Delta = -(1-\theta)(\alpha_2+\alpha_3)(2-2\Upsilon)^{\theta-1} b_t/\Omega_4 + (\alpha_1+\theta(\alpha_2+\alpha_3))/2$.

From (A.15), it is clear that the symmetric Nash equilibrium (NE) is $\Upsilon_t = \Upsilon_t^* = 0.5$ when $\Delta = 0$ (because let us say $b_t = 0$). If one assumes $\Delta < 0$ instead, then assuming [1- $\Upsilon_t(1+\theta)] > 0$ leads to a symmetric NE solution $\Upsilon_t = \Upsilon_t^* < \frac{1}{2}$. Taking $[1-\Upsilon_t(1+\theta)] < 0$ the NE solution is then $\Upsilon_t = \Upsilon_t^* > \frac{1}{1+\theta} > \frac{1}{2}$. Hence, the NE solution at date $t = 0$ is different from the cooperative solution $\tau_p < \frac{1}{2}$ except by a fluke. □

Proof of the Claim. Let Υ be in [0,1], the common tax rate for both countries, i.e. $\Upsilon_t = \Upsilon_t^* = \Upsilon$. Making use of the first-order conditions, the expression for domestic utility is:

$$U_t(1,\Upsilon_t) = \left(\frac{\alpha_1}{\alpha_3}\right)^{\alpha_1} \left(\frac{\alpha_2}{\alpha_3}\right)^{\alpha_2} (1+R_{t+1})^{-\alpha_1} Y_{t+1}^{\alpha_1+\alpha_2+\alpha_3}$$

Repeating the same steps as in the proof of Proposition 5 (which makes use of equations (37) and (38)), all period $t+1$ variables can be expressed in terms of period t variables which cannot be influenced by the choice of Υ_t. Denote by B_t the constant which depends upon parameters fixed at date t such that:

$$U_t(1,\Upsilon_t) = B_t \Upsilon_t^{(1-\theta)(\alpha_2+\alpha_3)} (1-\Upsilon_t)^{(1-\theta)[\alpha_1+(\alpha_2+\alpha_3)(1+\theta)]}$$

Similarly, we obtain that:

$$U_t^*(1, \Upsilon_t) = B_t^* \Upsilon_t^{(1-\theta)(\alpha_2+\alpha_3)}(1-\Upsilon_t)^{(1-\theta)[\alpha_1+(\alpha_2+\alpha_3)(1+\theta)]}$$

The ratio of the last two expressions

$$\frac{U_t(1, \Upsilon_t)}{U_t^*(1, \Upsilon_t)} = \frac{B_t}{B_t^*} = \xi_t$$

is a constant which depends upon parameters which cannot be influenced by the choice of Υ_t. □

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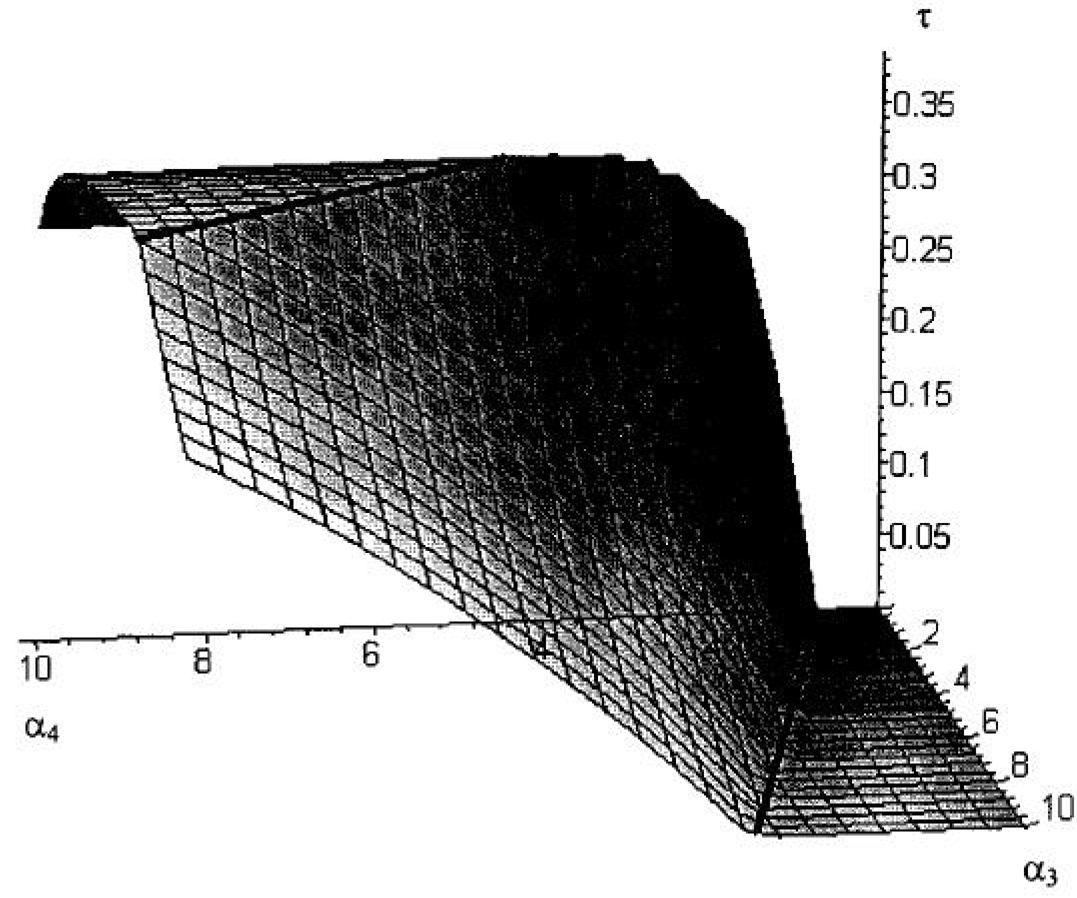
Figure 1

Optimal Public Education in Autarky*

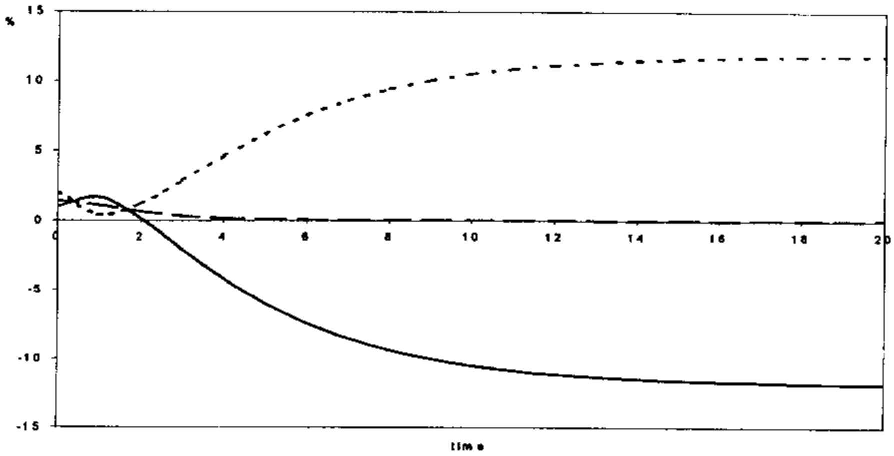


*Note. Parameter values: $\alpha_1 = \alpha_2 = 1$, $\Theta = 0.5$, $\beta = 1.33$, $A = 4.0$.

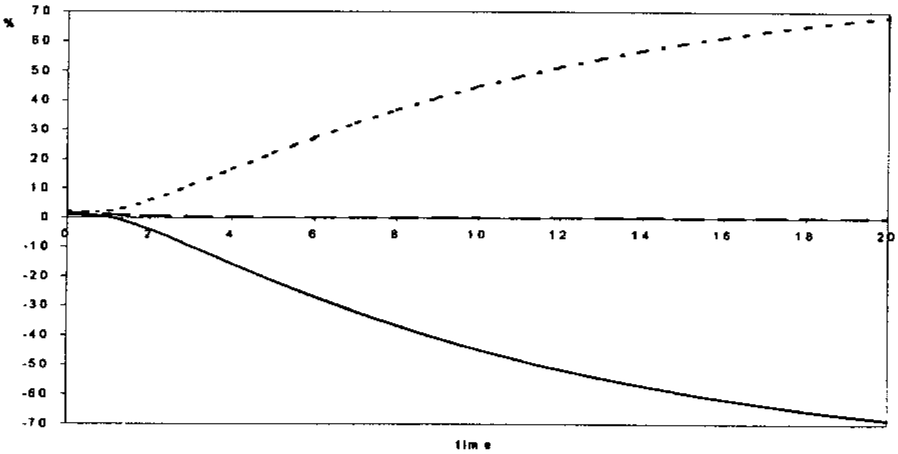
Figure 2

Gains from Capital Markets Integration*

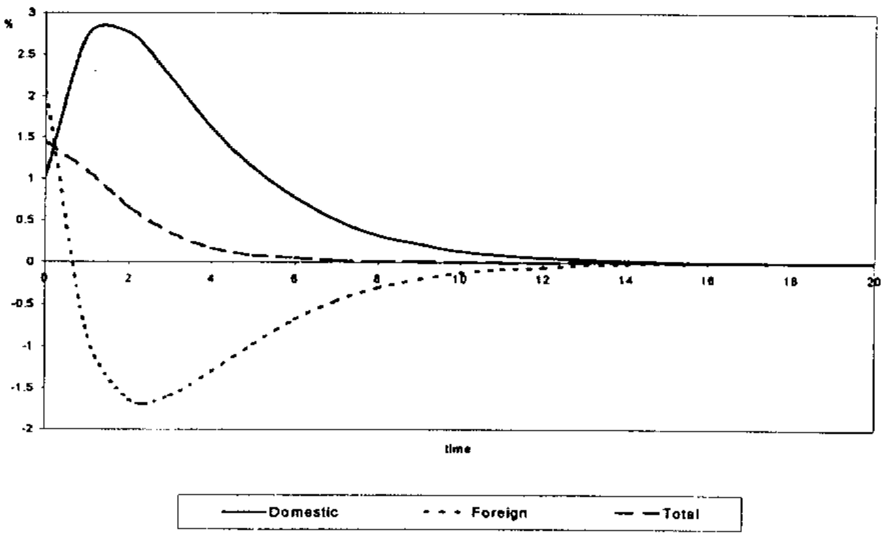
(a) Private



(b) Mixed

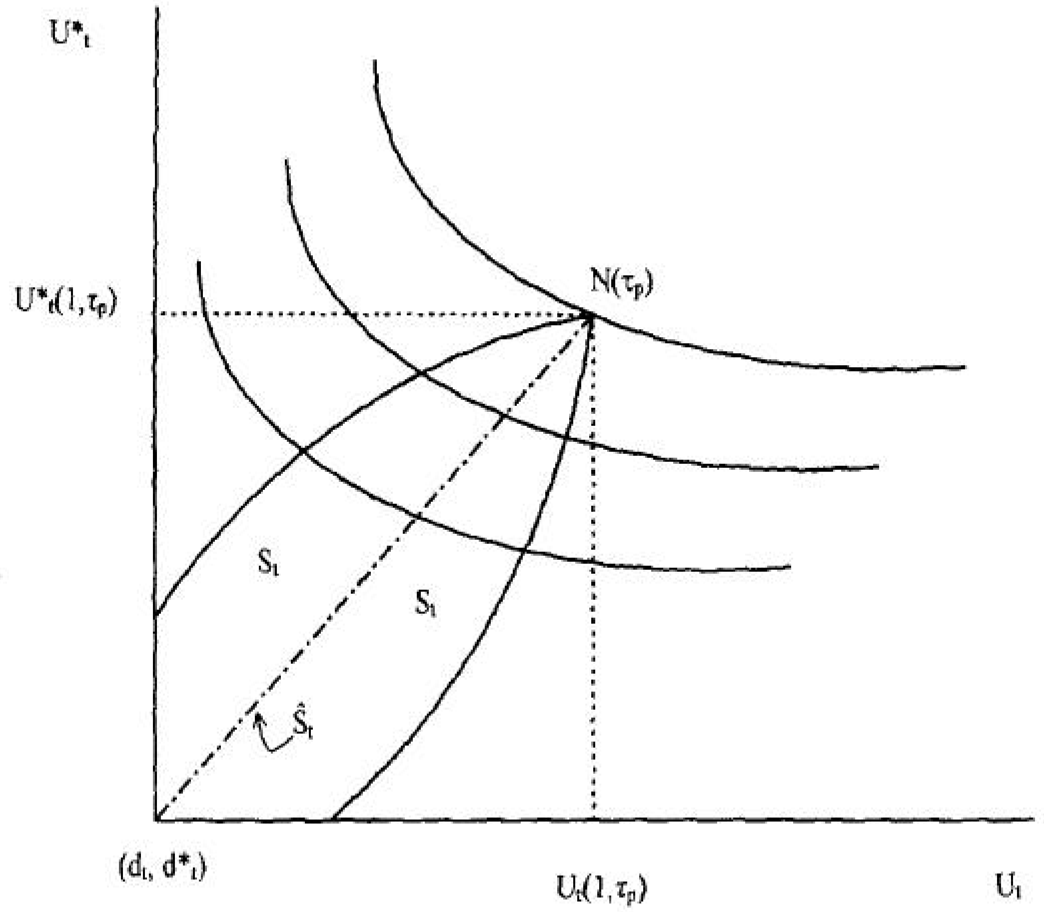


(c) Public



*Note. Change in income of individual countries (as a percentage of autarky values)

Figure 3
Education Game: the Nash Bargaining Solution*



*Note. (d_t, d^*_t) are disagreement points (autarkic utility levels); $N(\tau_p)$ is the Nash Bargaining Solution; $U_t(1,\tau_p)$ and $U^*_t(1, \tau_p)$ are Kalai-Smorodinsky's ideal values.