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THE FALLACY OF THE FISCAL THEORY OF THE PRICE LEVEL, AGAIN

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Abstract

The Fiscal Theory of the Price Level (FTPL) rejects the fundamental 'Ricardian' proposition, that the government budget constraint must hold identically, that is for all admissible values of the variables entering the budget constraint. Accordingly, if the government is to meet its contractual debt obligations, one of its instruments must be determined residually to ensure the budget constraint is satisfied. If the government overdetermines its fiscal-financial-monetary policy programme, contractual debt obligations will not be met. The FTPL asserts that even when the government overdetermines its policy programme, contractual debt obligations will always be met. The general price level plays the role of a default premium or discount. The paper shows that the FTPL is a fallacy and leads to anomalies and contradictions.

Keywords: Fiscal theory of the price level, Ricardian fiscal rules, government budget constraint, price level indeterminacy

JEL Classification: E31, E41, E51, E52, E62, E63

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I Introduction

The ‘fiscal theory of the price level’ (FTPL) is fatally flawed.[1] The source of the fallacy is an *economic* misspecification. The FTPL denies that the government’s intertemporal budget constraint must hold as an identity. Instead it is required to be satisfied only *in equilibrium*.

Property rights, contract enforcement, budget constraints and voluntary exchange are among the defining features of a market economy with just private agents.[2] When a government is added, a form of ‘involuntary exchange’, is introduced. Unrequited transfers between the private and public sectors, that is, transfers without value-equivalent or utility-equivalent *quid-pro-quo,* are allowed. This reflects the government’s ability to tax, the expression of its monopoly of the legitimate use of force - the power to prescribe and proscribe behaviour.

For a given structure of property rights and contract enforcement, budget constraints define bounds on the uses and sources of funds, and therefore on decision rules, that must always be satisfied by all economic agents. Budget constraints apply equally to private agents (households and firms[3]) and to the government. They apply to agents who are and/or perceive themselves to be small according to some

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[1] The seminal contributors, that is, the original exponents of the fiscal fallacy, are Begg and Haque [1984], followed by Auernheimer and Contreras [1990,. 1991, 1995]. Later contributors tend not to credit (or debit) these earlier authors appropriately. The recent revival includes Leeper [1991], Leeper and Sims [1994], Woodford [1994, 1995, 1996, 1998a,b], Sims [1994, 1997], Cochrane [1996, 1999a,b,c], Dupor [1997], Loyo [1997a,b], Luttmer [1997], Olivei [1997]; critical evaluations include Canzoneri, Cumby and Diba [1998a,b], Buiter [1998], McCallum [1998], Clements, Herrendorf and Valentinyi [1998] and Janssen, Nolan and Thomas [1999].

[2] For there to be a proper market economy, as opposed to, say, bilateral or multilateral bargaining, the terms of trade should be the same for all agents trading a particular commodity for delivery at a specific time, place and state of nature.

appropriate metric. Examples are competitive, price-taking behaviour by households and firms, or household and firms taking tax rates and public spending plans to be exogenous. Budget constraints also apply to agents who are, and perceive themselves to be, large, as in the case of monopolistic or monopsonistic firms and a government, which recognises the impact of its current and future actions on equilibrium prices and quantities. They apply to optimising agents, to satisficing agents, and to agents who follow ad-hoc decision rules. They apply to non-monetary economies and to economies with inside or outside, commodity or fiat money. They apply to economies in which the supply of fiat money is a government monopoly, as is the case in most of the FTPL literature, including this paper.

Specifying the appropriate budget constraints is not always straightforward. Concepts like default, insolvency and bankruptcy are often difficult to formalise in models with uncertainty and incomplete markets. In infinite-horizon models of market economies, the solvency constraint, usually a 'no-Ponzi-finance' condition on the terminal indebtedness of economic agents, is often hard to rationalise in terms of generally acceptable primitive assumptions.[4] The model used in this paper is deterministic and has complete markets. Much of the analysis is done for the finite-horizon case for which the appropriate intertemporal budget constraint is straightforward. All key results are shown to carry over, however, to the infinite-horizon case, with the standard infinite-horizon solvency constraint.

A *fiscal-financial-monetary programme or FFMP* is a complete set of rules specifying public spending, taxes, transfers, money issuance (seigniorage) and bond issuance in each period (and, for stochastic models, in each state of nature).

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[3] In the complete markets models under certainty considered in this paper, the budget constraint of the firm is that (the present discounted value of current and future) profits be non-negative.

The FTPL is based on the distinction between two kinds of FFMPs. Following Woodford I shall refer to these as *Ricardian* and *non-Ricardian* FFMPs. In what follows, the government is to be interpreted as the consolidated general government and central bank. The government spends on goods and services, makes transfer payments and raises taxes, borrows and issues monetary liabilities. When it cannot meet its contractual debt obligations (interest payments and repayment of principal) exactly, it either defaults or has to dispose of its 'supersolvency' surpluses.

Like every agent in a multi-period market economy, the government faces an intertemporal budget constraint or solvency constraint. A *Ricardian* FFMP requires that the government's solvency constraint hold for all admissible sequences of the variables entering into the government's intertemporal budget constraint. That is, the government's intertemporal budget constraint holds identically, not just in equilibrium. With a Ricardian rule, either the government restricts itself to FFMPs that permit it to always meet its contractual debt obligations exactly, or the government will fail to meet its contractual debt obligations. In the latter case, the government's intertemporal budget constraint determines the *effective public debt revaluation factor* (which can e.g. be a default discount factor or a 'supersolvency premium) on the public debt. In other words, the government's intertemporal budget constraint becomes a pricing kernel for the public debt, determining the *effective* value of the public debt and overriding its *notional* or *contractual* value.

A *non-Ricardian* FFMP requires the government's solvency constraint to hold only in equilibrium. It also requires the government to meet its contractual debt obligations exactly.

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[4] See Buiter and Kletzer [1998].

From the Ricardian perspective, non-Ricardian FFMPs are, in general, overdetermined. For instance, a non-Ricardian FFMP would permit the government to fix its sequences of real public spending, real net taxes and real seigniorage exogenously. In general, this will not permit the government to meets its outstanding contractual debt obligations exactly, for all admissible value of the variables entering the government's intertemporal budget constraint, including the initial value of the general price level. However, if, say, the government has a positive initial stock of nominally denominated public debt outstanding, and if the present discounted value of its future primary surpluses plus seigniorage also happens to be positive, there exists a unique value of the initial general price level that will equate the real value of the outstanding stock of contractual debt obligations to the present discounted value of future primary surpluses plus seigniorage. Since non-Ricardian FFMPs only require the government's intertemporal budget constraint to hold *in equilibrium*, the FTPL takes the unique initial price level that satisfies the governments intertemporal budget constraint and allows the government to meet its contractual debt obligations exactly, to be the first element of its equilibrium sequence for the general price level.

The general price level under the (overdetermined) non-Ricardian FFMP therefore plays the role that the endogenous public debt revaluation factor plays under an overdetermined Ricardian FFMP. This paper shows that the attempt to let the general price level mimic the role of the public debt revaluation factor leads to contradictions and anomalies.

Section II presents the model, a finite-horizon model of a deterministic monetary endowment economy, defines the key concepts and characterises the equilibria under Ricardian and non-Ricardian FFMPs. Section III demonstrates the contradictions and anomalies inherent in the FTPL. Section IV confirms that all key results go through for the infinite-horizon case also. Section V concludes.

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II The Model

I use a simple dynamic competitive equilibrium model with a representative private agent and a government sector. There is no uncertainty and markets are complete. It is possible to interpret the sequences of equilibrium prices and allocations as supported by a 'one-shot'-economy in which present value prices for all current and future goods and services are determined in the initial period, and there is no trading in subsequent periods.

Time, indexed by t, is measured in discrete intervals of equal length, normalised to unity. There are N periods indexed by t, $1 \leq t \leq N$. I first consider the finite-horizon case and then the infinite horizon. Initial contractual asset stocks are predetermined, that is, inherited from period *0*.

II.1 Household Behaviour

Households are price takers in all markets in which they transact. They receive an exogenous perishable endowment, $y_t > 0$, each period, consume $c_t \geq 0$ and pay net real lump-sum taxes t_t. They have access to three stores of value: non-interest-bearing fiat money (a liability of the government); a nominal one-period bond with a *notional* or contractual money price $P_t^B \geq 0$ in period t, which entitles the buyer to a single contractual nominal coupon payment worth $\Gamma > 0$ units of money in period $t+1$; and a real or index-linked one-period bond with a *notional* or contractual money price $P_t^b \geq 0$ in period t, which entitles the buyer to a single contractual coupon payment worth $g > 0$ units of real output in period *t+1*. A richer menu of liabilities (longer maturities, contingent coupon payments) could be included, but would not add to the analysis. The quantities of money, nominal bonds and real bonds outstanding at the end of period t (and the beginning of period $t+1$) are denoted M_t, B_t and b_t, respectively. The money price of output in period t is P_t. The government is assumed to have a monopoly of the issuance of base money, so $M_t \geq 0,\ 0 \leq t \leq N$.

Let $i_{t,t+1}$ the one-period risk-free nominal interest rate in period t and $r_{t,t+1}$ the one-period risk-free real interest rate in period t. By arbitrage it follows that

$$1+i_{t,t+1} = \frac{\Gamma}{P_t^B} = \frac{P_{t+1}g}{P_t^b} = (1+r_{t,t+1})\frac{P_{t+1}}{P_t} \qquad (2.1)$$

Notional or contractual bond prices are the prices that prevail if the contractual payments (Γ or g) are know with certainty to be made exactly. The effective bond prices are the prices that actually prevail, if the government does not meet its contractual obligations exactly.

When the government does not meet its contractual obligations exactly, its debt should be valued at effective prices, $\tilde{P}_t^B$ and $\tilde{P}_t^b$ respectively. Assume that all debt has equal seniority, that is, any resources available for debt service are pro-rated equally over all outstanding contractual debt. Let $D_{t,t+1}$ denote the fraction of the contractual payments due in period *t+1* that is actually paid. That is, the actual payments in period *t+1* on the two debt-instruments issued in period t are (with certainty, in this simple model),

$$\tilde{\Gamma}_{t+1} = D_{t,t+1}\Gamma \qquad (2.2)$$

$$\tilde{g}_{t+1} = D_{t,t+1}g \qquad (2.3)$$

It follows immediately that

$$\tilde{P}_t^B = D_{t,t+1} P_t^B \tag{2.4}$$

$$\tilde{P}_t^b = D_{t,t+1} P_t^b , \tag{2.5}$$

I shall refer to $D_{t,t+1}$ as the *public debt revaluation factor* for the notional value of the public debt outstanding at the end of period t. When $0 \le D_{t,t+1} < 1$, the debt revaluation factor can be interpreted as a default discount factor.

Note that

$$1+i_{t.t+1} = \frac{\tilde{\Gamma}_{t+1}}{\tilde{P}_t^B} = \frac{P_{t+1}\tilde{g}_{t+1}}{\tilde{P}_t^b} = \left(1+r_{t,t+1}\right)\frac{P_{t+1}}{P_t} \tag{2.6}$$

In principle, households or firms can default as well as the government. However, throughout this literature, households and firms have been assumed to satisfy their intertemporal budget constraints identically. Even though households never default, the household singe-period budget identity and solvency constraint must allow for the possibility that the government does not meet its contractual obligations exactly.

The single-period household budget identity is, for $1 \le t \le N$

$$M_t - M_{t-1} + \tilde{P}_t^B B_t - D_{t-1,t}\Gamma B_{t-1} + \tilde{P}_t^b b_t - D_{t-1,t} g P_t b_{t-1} = P_t(y_t - t_t - c_t) \tag{2.7}$$

The solvency constraint of the household is that at the end of period N, the household cannot have positive debt,

$$\tilde{P}_N^B B_N + \tilde{P}_N^b b_N \ge 0 \tag{2.8}$$

We will only consider equilibria in which money is weakly dominated as a store of value, that is, equilibria supporting a non-negative nominal interest rate sequence. The motive for holding money is that end-of-period real money balances are an argument in the direct utility function. To keep the analysis as transparent as possible, the period felicity function is assumed to be iso-elastic and money is assumed to enter the period felicity function in an additively separable manner. All key propositions in this paper would go through for more general functional forms and for most alternative ways of introducing money into the model including ‘money in the shopping function’ and ‘money in the production function’. For the strict Clower [1967] cash-in-advance models, there exists no finite-horizon equilibrium with a positive price of money unless one introduces another ‘closure rule’ to ensure that money is accepted in exchange for goods and services in the last period of the model.

The representative competitive consumer maximises the utility functional given in equation (2.9) defined over non-negative sequences of consumption and end-of-period real money balances subject to (2.7) and (2.8) and given the initial contractual asset stocks. It takes the tax sequence as given.

$$u_t = \sum_{j=0}^{N-t} \left[\frac{1}{1-h} c_{t+j}^{\;1-h} + f \frac{1}{1-h} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-h} \right] \left(\frac{1}{1+d} \right)^{j} \tag{2.9}$$

$$c_{t+j}, M_{t+j} \geq 0; \;\; h, \; f, \; d > 0$$

Since utility is increasing in consumption and real balances, (2.8) will hold with equality.

The contractual values of the initial financial asset stocks are predetermined, that is,

$$\begin{aligned} B_0 &= \bar{B}_0 \\ b_0 &= \bar{b}_0 \\ M_0 &= \bar{M}_0 > 0 \end{aligned} \tag{2.10}$$

Let R_{t-1t+j} be the nominal discount factor between periods *t-1* and *t+j*, that is,

$$R_{t-1,t+j} \equiv \prod_{k=0}^{j} \frac{1}{1+i_{t-1+k,\,t+k}} \qquad \text{for } j \ge 0$$

$$\equiv 1 \qquad \text{for } j = -1 \tag{2.11}$$

Solving the household budget identity (2.7) forward recursively, using (2.2)-(2.6) yields

$$D_{t-1,t}\left(\Gamma B_{t-1} + P_t g\, b_{t-1}\right) \ge \sum_{j=0}^{N-t} R_{t,t+j}\left(P_{t+j}(c_{t+j} + t_{t+j} - y_{t+j}) + (M_{t+j} - M_{t+j-1})\right)$$
$$+ R_{t,N}(\tilde{P}_N^B B_N + \tilde{P}_N^b b_N) \tag{2.12}$$

Specifically, in the initial period, *t = 1*, we have, imposing the household solvency constraint (2.8),

$$D_{0,1}\left(\Gamma B_0 + P_1 g\, b_0\right) \ge \sum_{j=0}^{N-1} R_{1,1+j}\left(P_{1+j}(c_{1+j} + t_{1+j} - y_{1+j}) + (M_{1+j} - M_{1+j-1})\right) \tag{2.13}$$

The household optimal consumption programme is characterised by

$$\left(\frac{c_{t+1}}{c_t}\right)^h = (1+r_{t,t+1})(1+d)^{-1} \qquad 1 \le t \le N-1 \tag{2.14}$$

$$\frac{M_t}{P_t} = c_t\left[f\left(\frac{1+i_{t,t+1}}{i_{t,t+1}}\right)\right]^{\frac{1}{h}} \qquad 1 \le t \le N-1 \tag{2.15}$$

$$\frac{M_N}{P_N} = c_N f^{\frac{1}{h}} \tag{2.16}$$

Equation (2.15) is the familiar optimality condition relating the optimal money stock in period *t* to optimal consumption in that period. The money-in-the-direct-utility-function approach views money as a consumer durable yielding a flow of unspecified liquidity services each period. In the last period, *N*, money only has value because of the liquidity services it yields that period. Effectively, real money balances in period *N* become a perishable commodity, as shown in equation (2.16), which does not involve any intertemporal relative price.

II.2 The Government

Government decision rules are exogenously given, subject only to a basic feasibility or consistent planning condition, the government's solvency constraint. The government's single-period budget identity for $1 \leq t \leq N$ is given in (2.17), its solvency constraint in (2.18),

$$M_t - M_{t-1} + \tilde{P}_t^B B_t - D_{t-1,t} \Gamma B_{t-1} + \tilde{P}_t^b b_t - P_t D_{t-1,t}\, g\, b_{t-1} \equiv P_t (g_t - t_t) \qquad (2.17)$$

$$\tilde{P}_N^B B_N + \tilde{P}_N^b b_N \leq 0 \qquad (2.18)$$

The government's single-period budget identity and solvency constraint imply that, for $1 \leq t \leq N$

$$D_{t-1,t} \left(\Gamma B_{t-1} + P_t g\, b_{t-1} \right) \leq \sum_{j=0}^{N-t} R_{t,t+j} \left(P_{t+j} (t_{t+j} - g_{t+j}) + (M_{t+j} - M_{t+j-1}) \right) \qquad (2.19)$$

Specifically, in the initial period,

$$D_{0,1} \left(\Gamma B_0 + P_1 g\, b_0 \right) \leq \sum_{j=0}^{N-t} R_{1,1+j} \left(P_{1+j} (t_{1+j} - g_{1+j}) + (M_{1+j} - M_{1+j-1}) \right) \qquad (2.20)$$

For simplicity, assume (2.20) holds with equality (as it will in equilibrium because of the household's intertemporal budget constraint (2.13)). Should one impose the constraint that $0 \leq D_{0,1} \leq 1$? This rules out both $D_{0,1} < 0$ (notional debtors can be effective creditors and vice versa) and $D_{0,1} > 1$ (the default discount factor can be a 'super-solvency premium'). Consider first the case for ruling out $D_{0,1} < 0$ *a-priori*. In that case equation (2.20) applies only if

$$\begin{aligned} &\operatorname{sgn}\left\{ \Gamma B_0 + P_1 g\, b_0 \right\} \\ &= \operatorname{sgn}\left\{ \sum_{j=0}^{N-1} R_{1,1+j} \left(P_{1+j} (t_{1+j} - g_{1+j}) + (M_{1+j} - M_j) \right) \right\} \end{aligned} \qquad (2.21)$$

Consider the case where (2.21) is violated. For instance, let the private sector hold a positive contractual stock of public debt in period *1*, although the government's present discounted value of future primary surpluses plus seigniorage is negative. If one did insist on imposing (2.21) and required $D_{0,1} \geq 0$, it would follow that there exists no feasible FFMP and therefore no equilibrium.

Against this, consider a government that is truly and credibly committed to the spending, tax and monetary issuance sequences on the RHS of (2.20), which incorporates the solvency constraint that there can be no positive debt outstanding at the end of period *N).* If the RHS of (2.20) were to be negative, while the outstanding value of the contractual debt at the beginning of period *1* is positive, this government would have no option but to impose an immediate capital levy on the private sector in period *1*, large enough to create a stock of public sector credit (negative public debt) equal in value to the present discounted value of the excess of current and future public spending over taxes plus seigniorage. Thus a positive notional or contractual value of the public debt would have to be transformed or revalued, in period *1*, into a negative effective value of the public debt. A negative value of $D_{0,1}$, the initial government debt revaluation factor, would, on this interpretation, make perfect economic sense. It is the unavoidable implication of a natural minimal consistent planning requirement on the government's FFMP. I will adopt this second approach and admit negative values of $D_{0,1}$.

The constraint $D_{0,1} \leq 1$ would imply that the public debt could trade at a *discount* on its notional or contractual value if the present value of future primary surpluses plus seigniorage falls short of its contractual value, but not at a *premium* if the opposite applies. If $D_{0,1} > 1$ is ruled out, government bond-holders do not get more than the government is contractually obliged to pay them, if the present discounted value of future primary surpluses and seigniorage exceeds the default-free value of the public debt. This means that equation (2.20) should be replaced by (2.22).

$$D_{0,1} = \min\left\{ 1 \, , \, \frac{\sum_{j=0}^{N-1} R_{1,1+j} \left(P_{1+j} \, (t_{1+j} - g_{1+j}) + (M_{1+j} - M_j) \right)}{\Gamma B_0 + P_1 g \, b_0} \right\} \tag{2.22}$$

If one chose to impose (2.22), one would need a theory for determining how any surplus of the present discounted value of future primary surpluses and seigniorage over the contractual value of the outstanding stocks of public debt, is disbursed or disposed of. If one permits $D_{0,1} > 1$, then, if, say, $\Gamma B_0 + P_1 g \, b_0 > 0$, any surplus resources over and above the contractual value of the outstanding debt are shared out equally among the initial holders of the contractual government debt. I will not restrict the magnitude or sign of $D_{0,1}$. A constraint similar to (2.21) will be relevant when the FTPL is considered below.

The fiscal-financial-monetary programme

I will consider two monetary 'regimes', an exogenous nominal money rule and an exogenous nominal interest rate rule.

The *exogenous nominal money rule* specifies an exogenous positive sequence for the nominal money stock,

$$\begin{aligned} &\left\{ M_t = \bar{M}_t > 0; 0 \le t \le N \right\} \\ &\left\{ \frac{\bar{M}_{t+1}}{\bar{M}_t} \ge \frac{1}{1+d} \right\} \end{aligned} \quad . \tag{2.23}$$

The nominal money stock sequences considered are restricted to those supporting a non-negative nominal interest rate sequence. The second inequality in (2.23) ensures non-negative equilibrium nominal interest rates in our model. The nominal interest rate is endogenous under this rule.

The *exogenous nominal interest rate rule* specifies an exogenous non-negative sequence for the nominal interest rate,

$$\left\{ i_{t-1,t} = \bar{i}_{t-1,t} \geq 0; 1 \leq t \leq N-1 \right\} \qquad (2.24)$$

The nominal money stock is endogenous under this exogenous nominal interest rate rule.

The real government spending sequence is exogenous and, for simplicity, constant.

$$\begin{aligned} & g_t = \bar{g}_t = \bar{g} \qquad\qquad 1 \leq t \leq N \\ & 0 \leq \bar{g} < y_t \end{aligned} \qquad (2.25)$$

Ricardian fiscal-financial-monetary programmes

There are two kinds of Ricardian FFMPs, those which require outstanding contractual debt obligations to be met exactly, that is, those that require the public debt revaluation factor to be identically equal to unity, and those that permit the public debt revaluation factor to be different from unity.

Definition: A Ricardian FFMP with contract fulfilment

A Ricardian FFMP <u>*with contract fulfilment*</u> *is a set of sequences for real public spending,* $\{g_t, t=1,...,N\}$*, net real taxes* $\{t_t, t=1,...,N\}$ *and either a sequence of nominal money stocks,* $\{M_t, t=0,1,...,N\}$ *or a sequence of nominal interest rates,* $\{i_{t,t+1}, t=0,1,...,N\}$ *which identically satisfies the government's* *intertemporal budget constraint (2.20) and ensures that all outstanding contractual debt obligations are met exactly, that is,* $D_{0,1} \equiv 1$.

Given the nominal money stock sequence or given the nominal interest rate sequence, and given the (constant) real public spending sequence, at least one element in the sequence of taxes must become endogenous. Since the model with its representative agent and lump-sum taxes exhibits debt neutrality or Ricardian equivalence, any rule for taxes that permits the government's intertemporal budget constraint (2.20) to be satisfied is appropriate (and equivalent) for our purposes.[5] For concreteness, I shall assume that taxes are set to achieve a zero nominal non-monetary debt from the end of period *1* on, that is,

$$\begin{aligned} t_1 &= \bar{g} - \frac{M_1 - M_0}{P_1} + D_{0,1}\left(\frac{\Gamma B_0}{P_1} + g\, b_0\right) \\ t_t &= \bar{g} - \frac{M_t - M_{t-1}}{P_t} \qquad 2 \le t \le N \end{aligned} \tag{2.26}$$

Equations (2.23), (2.25), (2.26) and $D_{0,1} \equiv 1$ define our Ricardian FFMP with contract fulfilment and an exogenous nominal money rule. Equations (2.24), (2.25), (2.26) and $D_{0,1} \equiv 1$ define our Ricardian FFMP with contract fulfilment and an exogenous nominal interest rate rule.

Many other Ricardian FFMPs with contract fulfilment are possible, including programmes based on ad-hoc feedback rules or optimising rules for the government's instruments.

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[5] Many other Ricardian rules exist. For instance, the net tax sequence could be exogenous and real public spending could adjust endogenously to satisfy the government's intertemporal budget constraint identically according to (2.26). The rule used in the paper simplifies the analysis.

Another Ricardian FFMP with contract fulfilment is the rule studied by Sargent and Wallace [1981] in their *Unpleasant Monetarist Arithmetic* paper. Translating it to the context of the model of this paper, the Unpleasant Monetarist Arithmetic rule specifies exogenous and constant sequences for real public spending and real taxes net of transfers.[6] There are two policy regimes. In regime 1, for $1 \le t < t_1$, the authorities fix the growth rate of the nominal money stock exogenously at $\bar{m}$. Government borrowing is the residual. There is only index-linked or real government debt ($B_t \equiv 0,\ 1 \le t \le N$). Regime 2 starts when, at $t = t_1 \ge 1$, the government stabilises the real stock of public debt, that is, it borrows or lends just enough to keep the real stock of public debt constant until the one-but last period, *N-1*.[7] In the last period, *N*, the government cannot leave any positive debt.[8] In regime 2, the nominal money stock becomes endogenous and adjusts to satisfy the government's single-period budget identity and intertemporal budget constraint. Sargent and Wallace assume that the (endogenous) growth rate of the nominal money stock for $t \ge t_1$ is constant. The Unpleasant Monetarist Arithmetic FFMP is summarised in (2.27).

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[6] The Unpleasant Monetarist Arithmetic model has exogenous growth of the labour force. Real per capita spending and real per capita net taxes are held constant.

[7] In the Sargent and Wallace model, the real *per capita* stock of public debt is stabilised.

[8] The Sargent Wallace model is a 2-period OLG model with an infinite number of generations. In that model, the stock of real per capita public debt is kept constant at its $t = t_1$ value forever after.

$$g_t = \bar{g}$$

$$t_t = \bar{t} \qquad 1 \le t \le N$$

$$M_{t+1} = (1+\bar{m})M_t \qquad 0 \le t \le t_1 - 1$$

$$\frac{M_{t+1}}{M_t} = \frac{M_t}{M_{t-1}} \qquad t_1 \le t \le N-1$$

$$B_t = 0 \qquad 0 \le t \le N$$

$$b_{t+1} = b_t \qquad t_1 \le t \le N-2 \tag{2.27}$$

$$b_N \le 0$$

In the Unpleasant Monetarist Arithmetic model, the government meets its contractual debt obligations exactly, $D_{0,1} \equiv 1$. The Sargent and Wallace FFMP therefore represents a Ricardian FFMP with contract fulfilment and, in regime 1, an exogenous nominal money stock phase, followed by, in regime 2, an endogenous nominal money stock. It is *not* an example of the FTPL fallacy at work.

Definition: A Ricardian FFMP without contract fulfilment

A Ricardian FFMP without contract fulfilment is an overdetermined FFMP which identically satisfies the government's intertemporal budget constraint (2.20), but for which outstanding contractual debt obligations do not have to be met exactly. $D_{0,1}$ *is therefore determined endogenously by the requirement that the government's intertemporal budget constraint be satisfied identically.*

There are many Ricardian FFMPs without contract fulfilment. I will use a very simple rule for spending, net taxes and seigniorage proposed by Woodford [1995] and also used by Cochrane [1999a].

Woodford proposes the following tax rule:

$$t_t = \bar{s}_t - \frac{M_t - M_{t-1}}{P_t} \qquad 1 \le t \le N \qquad (2.28)$$

where $\{\bar{s}_t\}$, $1 \le t \le N$ is an exogenously given real sequence of taxes plus seigniorage.

Equations (2.23), (2.25), and (2.28) define our Ricardian FFMP without contract fulfilment and an exogenous nominal money rule. $D_{0,1}$ is endogenous. Equations (2.24), (2.25), and (2.28) define our Ricardian FFMP without contract fulfilment and an exogenous nominal interest rate rule. Again $D_{0,1}$ is endogenous.

It is clear from equation (2.28) that an exogenous nominal money stock sequence is consistent with the Ricardian fiscal rule without contract fulfilment. With $\{\bar{g}_t, 1 \le t \le N\}$ and $\{\bar{s}_t, 1 \le t \le N\}$ given, the sequence of lump-sum taxes $\{t_t, 1 \le t \le N\}$ can adjust passively to accommodate any exogenous nominal money stock sequence $\{\bar{M}_t, 0 \le t \le N\}$.

Non-Ricardian fiscal-financial-monetary programmes

Definition: A Non-Ricardian FFMP

A Non-Ricardian FFMP is an *<u>overdetermined</u>* *FFMP which satisfies the government's intertemporal budget constraint (2.20)* *<u>in equilibrium only</u>*, *but for which outstanding contractual debt obligations must be met exactly, that is,* $D_{0,1} \equiv 1$.

A *non-Ricardian FFMP with an exogenous nominal money rule* will be defined by an exogenous sequence of real public spending, g_t, an exogenous sequence of real net taxes plus real seigniorage, $\overline{s}_t$, and an exogenous strictly positive sequence of nominal money stocks, that is, by equations (2.23), (2.25), (2.28), and $D_{0,1} \equiv 1$.

A *non-Ricardian FFMP with an exogenous nominal interest rate rule* will be defined by an exogenous sequence of real public spending, g_t, an exogenous sequence of real net taxes plus seigniorage, $\overline{s}_t$ and an exogenous non-negative sequence of nominal interest rates, that is, by equations (2.24), (2.25), (2.28) and $D_{0,1} \equiv 1$.

II.3 Market Clearing

The goods market clears each period, that is,

$$y_t = c_t + g_t \qquad 1 \leq t \leq N \qquad (2.29)$$

For simplicity, I assume in what follows that the real fundamentals are constant, that is,

$$\begin{aligned} y_t &= \overline{y} \\ g_t &= \overline{g} \qquad 1 \leq t \leq N \end{aligned}$$

Only non-negative equilibrium price sequences are permissible.

II.4 Equilibrium under the Ricardian FFMP with Contract Fulfilment and an Exogenous Nominal Money Rule

The equilibrium is characterised by equation (2.23) and (2.30) to (2.36). Note that $D_{0,1} \equiv 1$, since contract fulfilment is imposed.

$$c_t = c = \overline{y} - \overline{g} \qquad 1 \leq t \leq N \qquad (2.30)$$

$$r_{t,t+1} = d \qquad 1 \le t \le N-1 \qquad (2.31)$$

$$1 + i_{t,t+1} = (1+d)\frac{P_{t+1}}{P_t} \qquad 1 \le t \le N-1 \qquad (2.32)$$

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+d)\frac{P_{t+1}}{P_t}}{(1+d)\frac{P_{t+1}}{P_t} - 1}\right]^{\frac{1}{h}} \qquad 1 \le t \le N-1 \qquad (2.33a)$$

$$\frac{M_N}{P_N} = (\bar{y} - \bar{g})f^{\frac{1}{h}} \qquad (2.33b)$$

$$\frac{\Gamma B_0}{P_1} + g\, b_0 \equiv \sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j \left[t_{1+j} + \left(\frac{M_{1+j} - M_j}{P_{1+j}}\right) - \bar{g}\right] \qquad (2.34)$$

$$\begin{aligned} t_1 &= \bar{g} - \frac{M_1 - M_0}{P_1} + \frac{\Gamma B_0}{P_1} + g\, b_0 \\ t_t &= \bar{g} - \frac{M_t - M_{t-1}}{P_t} \qquad 1 < t \le N \end{aligned} \qquad (2.35)$$

$$\begin{aligned} M_0 &= \bar{M}_0 > 0 \\ B_0 &= \bar{B}_0 \\ b_0 &= \bar{b}_0 \end{aligned} \qquad (2.36)$$

This economy has multiple equilibria for the general price level sequence.[9] One equilibrium has an infinite price level (a zero price of money) in each period. A second has an infinite price level only in the last period, *N*. Since money is worthless in period *N*, the demand for money in period *N-1* takes the same form as the demand for money in the terminal period, given in equation (2.33b). One can work backwards from this to an initial value for price level and the real money stock. Indeed, for every

[9] There are therefore also multiple equilibria for the real money stock sequence and, if non-zero contractual nominal debt is outstanding, for the real debt sequence.

period t, $N \geq t > 1$, there exists an equilibrium in which money is valueless for all periods s, $N \geq s \geq t$, but valued up to that time.[10]

There is also a unique equilibrium in which money has positive value in each period. The monetary equilibrium conditions (2.33a,b) provide N equations that uniquely determine the N (finite) equilibrium prices P_t, $t = 1, \ldots, N$. Equation (2.33b) determines P_N as a function of the nominal stock of money in the last period, $\bar{M}_N$. The remaining $N-1$ monetary equilibrium conditions given by (2.33a) determine P_{N-1} down to P_1, given the solution for the price level in period N and the exogenous values of the nominal money stocks in periods 1 to $N-1$. The tax rule given in (2.35) then determines the N values of the lump-sum tax sequence. Given that tax rule, the government's solvency constraint holds identically.

Another way of putting this is that the government's solvency constraint (and the assumed exogeneity of the real public spending sequence and the nominal money stock sequence) forces the tax sequence to become endogenous, if all contractual debt obligations are to be met exactly.

The equilibrium real and nominal interest rate sequences and the equilibrium consumption sequence are always uniquely determined.

Under this Ricardian FFMP with contract fulfilment and an exogenous nominal money rule, money is *conditionally neutral* (see Buiter [1998]). Holding constant the initial stock of nominal non-monetary debt, B_0, equal proportional changes in the sequence of nominal money stocks (including the initial nominal stock

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[10] I am indebted to Chris Sims for pointing out, in private correspondence, the existence of more than one equilibrium in which money is valueless in some period(s). He asserted that this only was the case in the absence of nominal bonds. This is incorrect. Any change in the real value of the nominal stock of bonds associated with a change in the general price level will, because of our assumption of

of money), $\{\bar{M}_t\},\ 0 \le t \le N$, and in the sequences of all endogenous nominal prices $\{P_t, P_t^B, P_t^b\}, 1 \le t \le N$ leave the real equilibrium unchanged. If the initial stock of non-monetary nominal debt is non-zero, the sequence of (endogenous) real lump-sum taxes will change (according to (2.35)), because the real value of the initial stock of nominal non-monetary government debt, $\frac{B_0}{P_1}$, changes when the initial price level changes.

Under the Ricardian FFMP with contract fulfilment and an exogenous nominal money rule, money and the initial stock of nominal non-monetary debt are *jointly unconditionally neutral* (see Buiter [1998]). Equal proportional changes in the sequence of nominal money stocks (including the initial nominal stock of money), $\{\bar{M}_t\},\ 0 \le t \le N$, in the initial stock of nominal non-monetary debt, B_0, and in the sequences of all endogenous nominal prices $\{P_t, P_t^B, P_t^b\}, 1 \le t \le N$ leave the real equilibrium unchanged. The (endogenous) sequence of real lump-sum taxes will not need to change (again according to (2.35)).

I summarise this as Proposition 1.

Proposition 1.

Under the Ricardian FFMP with contract fulfilment and an exogenous nominal money rule, money is neutral in equilibria in which money has value in each period.

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a Ricardian FFMP with contract fulfilment, be offset by a matching change in real lump-sum taxes, according to (2.35).

II.5 Equilibrium under the Ricardian FFMP with Contract Fulfilment and an Exogenous Nominal Interest Rate Rule

With an exogenous non-negative nominal interest rate sequence (and endogenous nominal money stocks), the equilibrium under a Ricardian FFMP with contract fulfilment is characterised by equations (2.24), and (2.30), to (2.36). It is helpful to rewrite the two monetary equilibrium conditions as follows:

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+\bar{i}_{t,t+1})}{\bar{i}_{t,t+1}}\right]^{\frac{1}{h}} \qquad 1 \leq t \leq N-1 \qquad (2.37a)$$

$$\frac{M_N}{P_N} = (\bar{y} - \bar{g}) f^{\frac{1}{h}} \qquad (2.37b)$$

The monetary equilibrium conditions (2.37a,b) provide *N* equations that uniquely determine the *N* equilibrium real money stocks, M_t/P_t , $t = 1, \ldots, N$. The endogenous equilibrium nominal money stock sequence $\{M_t\}$, *1 £ t £ N* and the equilibrium price sequence $\{P_t\}$, *1 £ t £ N* are indeterminate. The tax rule given in (2.35) then determines the *N* values of the lump-sum tax sequence. If the initial stock of nominal non-monetary debt, B_0, is non-zero, the first term in the equilibrium real tax sequence, t_1, which depends on $\frac{B_0}{P_1}$, is also indeterminate. However, it continues to be the case that, given that tax rule in (2.35), the government's intertemporal budget constraint holds identically. Whatever the general price level turns happens to be, the period *1* lump-sum tax will assume the value required to satisfy the first equation in (2.35). The equilibrium real interest rate sequence, the equilibrium inflation rate sequence and the equilibrium consumption sequence are also uniquely determined.

Price level indeterminacy under a Ricardian nominal interest rate rule is a familiar result. It is not paradoxical or surprising, let alone anomalous. In a frictionless economy, with flexible (that is, non-predetermined), market-clearing nominal prices, an exogenous nominal interest rate sequence does not provide a nominal anchor for the system. The reason is that, despite its name, the short *nominal* interest rate is a *real* variable, the real pecuniary opportunity cost of holding money balances.

I summarise this as Proposition 2.

Proposition 2.

Under the Ricardian FFMP with contract fulfilment and an exogenous nominal interest rate rule, all nominal equilibrium values are indeterminate.

Price level indeterminacy under a Ricardian FFMP with contract fulfilment and with an exogenous nominal interest rate rule is a feature of the class of flexible price level, general equilibrium models considered in this paper rather than a problem for monetary policy in the real world. More policy-relevant models would view the price level (and/or the money wage) in any given period as predetermined. With such 'Keynesian' money wage or price rigidities, nominal indeterminacy is eliminated under a Ricardian FFMP with contract fulfilment and an exogenous nominal interest rate rule (see Buiter [1999]).

II.6 Equilibrium under the Ricardian FFMP without Contract Fulfilment and an Exogenous Nominal Money Rule

Define the *effective* real value of the initial net public debt, $\tilde{L}_0$, as follows:

$$\tilde{L}_0 \equiv D_{0,1}\left(\frac{\Gamma\bar{B}_0}{P_1}+g\,\bar{b}_0\right) \qquad (2.38)$$

Let $L_0 \equiv \frac{\Gamma B_0}{P_1} + g\, b_0$ be the contractual or notional value of the government's initial contractual debt obligations, so $\tilde{L}_0 = D_{0,1} L_0$. We now substitute the rule given by (2.28), that real tax revenue plus the real value of seigniorage is exogenously given, into the government's intertemporal budget constraint (2.20), but without imposing the constraint that all contractual debt obligations are met exactly. We can then represent the key equilibrium conditions determining $D_{0,1}$ and the equilibrium price sequence by (2.39), and (2.33a,b).

$$\tilde{L}_0 \equiv D_{0,1} L_0 \equiv D_{0,1} \left(\frac{\Gamma \bar{B}_0}{P_1} + g\, \bar{b}_0 \right) \equiv \sum_{j=0}^{N-1} \left(\frac{1}{1+d} \right)^j \left[\bar{s}_{1+j} - \bar{g} \right] \tag{2.39}$$

The remaining equilibrium conditions are given by (2.30), (2.31), (2.32) and (2.36).

For reasons of space I will concentrate exclusively on the unique equilibrium in which money has positive value in each period.

The right-hand side of (2.39) is exogenous. Everything on the left-hand-side of (2.39), except for P_1 and $D_{0,1}$, is exogenous or predetermined. Assume again that the exogenous and strictly positive nominal money stock sequence satisfies $\frac{\bar{M}_{t+1}}{\bar{M}_t} \geq \frac{1}{1+d}$. The monetary equilibrium conditions (2.33a,b) still provide N equations that uniquely determine the N equilibrium prices P_t , $t = 1, \ldots, N$.

Given the value of the initial price level, determined by the monetary equilibrium conditions and the exogenous nominal money stock sequence, the government's intertemporal budget constraint determines the government debt revaluation factor, $D_{0,1}$. Except for a set of parameter configurations of measure zero, this endogenous value of $D_{0,1}$ will be different from *1*.

For a Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule to always support an equilibrium, it must be possible to turn positive (negative) contractual net debt into negative (positive) effective net debt ($D_{0,1} < 0$), and to permit contractual debt not only to be effectively discounted ($D_{0,1} < 1$) but also to be effectively priced at a premium ($D_{0,1} > 1$).

If one does not, on a-priori grounds, accept values of $D_{0,1}$ that are greater than *1* or negative, one would have to conclude that no equilibrium exists under a Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, if equation (2.39) were to yield a value for $D_{0,1}$ that was negative or greater than *1*. My interpretation of the government's intertemporal budget constraint as a consistency requirement imposed on all FFMPs permits values for $D_{0,1}$ that are negative or greater than *1*. The critique of the FTPL does not depend on whether one excepts Ricardian equilibria with $D_{0,1} < 0$ or $D_{0,1} > 1$.

The government's intertemporal budget constraint can therefore be viewed as an effective public debt pricing kernel, that is, an equation determining the effective real value of the net public debt or the public debt revaluation factor. The present discounted value of future primary surpluses and seigniorage equals ('determines', if the real seigniorage and real primary surplus sequences are taken as given) the effective real value of the initial net government debt. If the notional or contractual value of the initial debt differs from its effective value, the government solvency 'overwrites' the contractual value.

I summarise this discussion as Proposition 3.

Proposition 3.

Under a Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, the government's intertemporal budget constraint and the overdetermined FFMP determine the effective real value of the net public debt. This will in general be different from the notional or contractual value of the government's outstanding debt obligations.

The remaining properties of the equilibrium are familiar:

Proposition 4.

Under the Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, money is neutral in equilibria in which it has value in each period.

II.7 Equilibrium under the Ricardian FFMP without Contract Fulfilment and an Exogenous Nominal Interest Rate Rule

As under the Ricardian FFMP with contract fulfilment and an exogenous non-negative nominal interest rate rule (and endogenous nominal money stocks), the monetary equilibrium conditions (2.37a,b) determine the real money stock sequence under the Ricardian FFMP without contract fulfilment and with an exogenous nominal interest rate rule. The remaining equilibrium conditions are (2.28), (2.30), (2.31), (2.32), (2.36) and (2.39).

The government's intertemporal budget constraint (2.39), with its overdetermined FFMP, now determines the effective real value of the initial net public debt, $\tilde{L}_0$. The endogenous nominal money stock sequence and the price level sequence are indeterminate. Note that, if the initial stock of contractual nominal debt,

B_0, is non-zero, both the initial price level, P_1, and the government debt revaluation factor, $D_{0,1}$ are indeterminate. The effective real value of the initial net public debt, $\tilde{L}_0$, however, is well-determined. All other real equilibrium values, including the inflation rate, are well-determined.

II.8 Equilibrium under the Non-Ricardian FFMP with an Exogenous Nominal Money Rule

The equilibrium conditions are the same as for the Ricardian FFMP without contract fulfilment and with an exogenous nominal money rule, except for the imposition of the additional constraint that contractual government debt obligations are met exactly, that is, $D_{0,1} \equiv 1$. The equilibrium price sequence is determined by the government's intertemporal budget constraint with $D_{0,1} = 1$, reproduced below as (2.40), and the monetary equilibrium conditions, (2.33a,b) with the exogenous nominal money stock sequence imposed.

$$\frac{\Gamma \overline{B}_0}{P_1} + g\,\overline{b}_0 \equiv \sum_{j=0}^{N-1} \left(\frac{1}{1+d} \right)^j \left[\overline{s}_{1+j} - \overline{g} \right] \qquad (2.40)$$

The remaining equilibrium conditions are given by (2.28), (2.30), (2.31), (2.32) and (2.36).

Restricting consideration again to equilibria with a positive value for money in each period, it is clear that the system (2.40) and (2.33a,b) is overdetermined. The initial price level is determined twice, once from the monetary equilibrium conditions and once from the government's intertemporal budget constraint. Except through a fluke, these two values of the initial price level will not be the same. This should not be surprising. The equilibrium under the Ricardian FFMP with an exogenous nominal money rule (with or without contract fulfilment) was exactly determined. The non-Ricardian FFMP with an exogenous nominal money rule has a further restriction imposed on it.

Proposition 5.

Under the non-Ricardian FFMP with an exogenous nominal money rule, the price level is overdetermined.

II.9 Equilibrium Under the non-Ricardian FFMP with an Exogenous Nominal Interest Rate Rule: could this be the FTPL?

Under a non-Ricardian FFMP with an exogenous nominal interest rate rule, the equilibrium conditions are the same as under the Ricardian FFMP without contract fulfilment and with an exogenous nominal interest rate rule, except for the addition of the constraint $D_{0,1} \equiv 0$. Outstanding contractual debt obligations have to be met exactly, despite the overdetermined FFMP.

It may seem that the price level indeterminacy characteristic of the Ricardian FFMPs with a fixed nominal interest rate rule can now be resolved. The monetary equilibrium conditions (2.33a,b) determined the equilibrium real money balances for each period. The government's intertemporal budget constraint (2.40) (which has $D_{0,1} \equiv 1$ imposed) determines the initial price level, P_1, and equation (2.41) permits all subsequent price levels to be determined.

$$1+\bar{i}_{t,t+1} = (1+d)\frac{P_{t+1}}{P_t} \qquad 1 \le t \le N-1 \qquad (2.41)$$

The FTPL, with its overdetermined non-Ricardian FFMP, lets the initial price level do the work done by the government debt revaluation factor in the overdetermined Ricardian FFMP. The general price level revalues the outstanding stock of contractual nominal government debt to make it consistent with the overdetermined real spending, tax and seigniorage sequences. The effective real value of the initial public debt adjusts to satisfy the government's intertemporal budget constraint in equilibrium, *and* remains equal to the notional or contractual real value of the initial public debt. Could this be the FTPL?

Three questions arise. First, when is this fiscal theory of the price level mathematically consistent, within the confines of the specific model of this paper? Second, what else does the FTPL imply, and do these other implications make sense? Third, how robust is the FPTL?

III Implications of the Fiscal Theory of the Price Level: contradictions and anomalies

III.1 An arbitrarily restricted domain of existence

Unlike the government debt revaluation factor, $D_{0,1}$, the general price level, P_1, cannot be negative. A necessary condition for the government's intertemporal budget constraint under the non-Ricardian FFMP to support an equilibrium is therefore that condition (3.1) be satisfied. Note that (3.1) is similar to condition (2.21), which ensures a non-negative value of the public debt revaluation factor. It is the same as (2.21) if all government debt is nominally denominated.

$$\operatorname{sgn}\left\{\bar{B}_0\right\} = \operatorname{sgn}\left\{\sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j\left[\bar{s}_{1+j}-\bar{g}\right]-g\,\bar{b}_0\right\} \qquad (3.1)$$

Condition (3.1) says that the initial stock of non-monetary nominal public debt must be positive (negative) if the excess of the present discounted value of future real primary government surpluses plus future real seigniorage revenues over the value of the initial stock of index-linked government debt is positive (negative).

Everything on either side of equation (3.1) is exogenous or predetermined. There is no reason why arbitrary configurations of $B_0, b_0, d, \bar{g}$ and $\{\bar{s}_t\}, 1 \le t \le N$ would always satisfy (3.1), although they may do so. If there is only index-linked public debt, there can be no FTPL. If, in an open economy extension of this model, all public debt is foreign-currency-denominated, there likewise is no FTPL. Arbitrary restrictions on the predetermined and exogenous variables in the government solvency constraint are required to support a non-negative equilibrium price level sequence.

III.2 The FTPL and the price of phlogiston: the price of money in an economy without money

If, in section III.1, the FTPL is seen to do too little, the point of section III.2 is that the FTPL does too much. Taken at face value, the FTPL can determine the price of money (the reciprocal of P) when (3.1) is satisfied, even in a world in which there is no demand for money. In our model this will be the case when $f = 0$. As there is no private demand for money balances, money does not enter into any budget constraint. There may be a physical or virtual substance called 'money', the private sector may hold these worthless objects and the government may issue or retire them at will, no-one's choices or budget constraints are affected in any way by its existence. Think of this as a world with non-interest-bearing government fiat money, or cash, in which cash has become redundant as a medium of exchange and means of payment, and in which cash is dominated as a store of value by money-denominated securities with a positive nominal interest rate.[11]

There are interesting and important issues that arise when an economy gradually demonetises over time, say in response to technological and regulatory

[11] Money could be held is a zero nominal interest rate world. Since it is not germane to the issues under consideration, I ignore this case in what follows.

developments that permit households to economise on money to an ever-increasing degree and that may, ultimately, make money completely redundant. The FTPL sheds no light on these issues. It does, however, permit the price of money to be determined in a barter economy, in which no one now demands and holds money or ever has. In this world, money may not exist at all, either as a physical object or as a financial claim. Something called ‘money’ could of course be the numéraire, unit of account and invoicing unit, and private or public agents could denominate securities in terms of this pure numéraire ‘money’.

According to the FTPL, the price of ‘money’ in this world without money can be determined uniquely from the government’s intertemporal budget constraint, under a non-Ricardian FFMP with an exogenous nominal interest rate rule, if three conditions are satisfied. First, ‘money’ exists as a word, or a name. It can be thought of as an imaginary substance, like *phlogiston*, the imaginary element formerly believed to cause combustion. Second, an interest-bearing financial claim denominated in terms of this 'money' is issued by the government. This means, in our example, that the purchaser of the security in period t is entitled to a payment worth Γ units of money in period $t+1$. The payment cannot, of course, be made in money, that is, by the transfer of monetary claims, since no one is willing to hold these or accept them in payment. Third, (3.1) is satisfied.

Under these conditions, the government’s intertemporal budget constraint (2.40) alone can, under the non-Ricardian exogenous nominal interest rate rule, determine the initial general price level, P_1. From (2.41) all future price levels can then be determined. Equations (2.37a,b) play no longer any role, with $f = 0$. A theory capable of pricing phlogiston, something that does not exist, except as a name, is an intellectual bridge too far.

The anomaly is eliminated if we replace the non-Ricardian intertemporal budget constraint (2.40) by the Ricardian intertemporal budget constraint without contract fulfilment (2.39).

From the Ricardian perspective, the only thing determined by the government's intertemporal budget constraint when the FFMP is overdetermined, as it is in (2.39), is the effective real value of the net public debt in the initial period, $\tilde{L}_0$. If $B_0 = 0$, the public debt revaluation factor $D_{0,1}$ is determined uniquely. The initial price level, P_1, is indeterminate. In the case most commonly considered in the FTPL literature, which excludes index-linked contractual debt obligations, that is, $b_0 = 0$, the public debt revaluation factor and the general price level are not determined individually. Only their ratio, $\frac{D_{0,1}}{P_1}$, is determined (and so is $\tilde{L}_0$).

It really does not matter what the contractual debt is denominated in, be it 'money', commodities or phlogiston. The government's intertemporal budget constraint determines the effective real value of the initial net public debt regardless of the denomination of the contractual debt obligations. Arbitrage ensures that, from period *1* on, households will receive the same real rate of return, d, on all their securities, including those denominated in 'money' or phlogiston.

Now consider the Ricardian FFMP with contract fulfilment ($D_{0,1} \equiv 1$), with an exogenous nominal interest rate rule and without money ($f = 0$ and therefore $\frac{M}{P} \equiv 0$). The natural interpretation is that the nominal stock of money is zero[12] and that the price level is indeterminate. The Ricardian tax rule (2.35) ensures that the government's intertemporal budget constraint (2.34) holds identically. If there is a non-zero outstanding initial contractual stock of government debt, $B_0 \neq 0$, both P_1 and t_1 are indeterminate, although they are constrained to jointly satisfy (2.35).

Therefore, for both Ricardian FFMPs, since households will only hold nominally denominated government debt from period *1* on if it yields the equilibrium real rate of interest, an inflation rate for 'money' is implied in this model without 'money'.

When the government calls out a non-negative sequence of nominal interest rates, $\{\bar{i}_{t,t+1}\},\ 0 \leq t \leq N-1$, and when $B_0 \neq 0$, the private sector and the government must jointly agree and believe, that the price of real goods in terms of 'money' must rise at a proportional rate $\frac{1+\bar{i}_{t,t+1}}{1+r} - 1,\ 0 \leq t \leq N-1$. If this condition were not satisfied, there would be arbitrage opportunities between nominal debt and real debt.

Thus, to keep households indifferent between 'money'-denominated debt and real debt, given the equilibrium real interest rate sequence determined by the model, a sequence of inflation rates $\frac{P_{t+1} - P_t}{P_t} = \frac{1+\bar{i}_{t,t+1}}{1+r} - 1,\ 0 \leq t \leq N-1$, must be implied. The only equilibrium relationship involving this nominal interest sequence and this inflation rate sequence, is the no-arbitrage condition between nominal and real debt, (2.41). Nothing else either influences this rate of inflation (or this nominal interest rate) or is influenced by it. Indeed there is no way of verifying from subsequent observation of the rates of exchange between money and goods, whether the inflation rate defined by (2.41) has indeed materialised. The interpretation of (2.41) in a world without money, is therefore that the parties engaged in the sale and purchase of nominal debt must agree on both the nominal interest rate sequence and on the

[12] This would certainly be the only possible interpretation if 'money' were phlogiston.

inflation rate sequence that will define such debt contracts. Unless the contracting parties agree on sequences of nominal interest rates and inflation rates that satisfy (2.41), the private demand for nominal debt with either be infinitely negative (if $1+\overline{i}_{t,t+1} < (1+d)\frac{P_{t+1}}{P_t}$) or infinitely positive (if $1+\overline{i}_{t,t+1} > (1+d)\frac{P_{t+1}}{P_t}$). Any sequences of nominal rates and inflation rates that satisfy (2.41) are equivalent. 'Money' is, not surprisingly, superneutral in this economy without money. Thus the existence of nominal debt in a world without money gives rise to a form of contractual verbal 'shadow boxing'. This is not surprising, since we know that, in a barter economy, only relative prices and real quantities are determined. The general price level, unlike the rate of inflation, is not involved in a complete characterisation of the nominal debt contracts and is therefore indeterminate.

III.3 The FTPL and the HTPL

Substituting the household first-order conditions (2.14), (2.15) and (2.16) into the household intertemporal budget constraint (2.13) (assumed to hold with equality) yields equation (3.2).

$$\begin{aligned}\frac{M_0+\Gamma B_0}{P_1}+g\,b_0 \equiv \sum_{j=0}^{N-1} R_{1,1+j}(t_{1+j}-y_{1+j}) \\ +\overline{c}_1\left[\sum_{j=0}^{N-2}\left(\left(\frac{1}{1+d}\right)^j\left[1+f^{\frac{1}{h}}\left(\frac{1+i_{1+j,2+j}}{i_{1+j,2+j}}\right)^{\frac{1-h}{h}}\right]\right)+\left(\frac{1}{1+d}\right)^{N-1}(1+f^{\frac{1}{h}})\right]\end{aligned} \tag{3.2}$$

When the household intertemporal budget constraint is viewed as a constraint that must be satisfied not only in equilibrium, but for all admissible values of the economy-wide endogenous variables, (3.2) represents a constraint on c_1, that is, it determines the optimal value of c_1 as a function of the variables the household takes to be exogenous or predetermined.

Applying the logic of the FTPL to the household sector, however, one can overdetermine the household's optimal consumption and portfolio allocation programme and fix c_1 at some arbitrary positive level, $c_1 = \overline{c}_1$, say. The household solvency constraint then determines the period *1* price level. This gives us the 'household intertemporal budget constraint theory of the price level' or HTPL. For the initial price level to be non-negative, it would of course have to be the case that a condition analogous to (3.1) is satisfied.

This HTPL would be recognised immediately as a complete economic non-sense. Household decision rules, be they derived from optimising, satisficing or other behavioural principles, must satisfy the household intertemporal budget constraint identically. That is correct, but the same holds for all agents, private or public, in a market economy.

The economy-wide equilibrium implications of the HTPL would be rather disconcerting. An exogenous private consumption level would violate the goods market equilibrium condition, $\overline{y}_1 = \overline{c}_1 + g_1$, unless government spending passively accommodated the private consumption programme. Of course, if the government did adopt a Ricardian FTPL (with contract fulfilment, say), with the real net tax sequence exogenous and real public spending determined residually to satisfy the government's intertemporal budget constraint (reversing the roles of t and g in (2.35), we would have a HTPL that formally exactly mirrors the HTPL.[13]

The HTPL is no stranger than the FTPL. The household could be 'large' rather than competitive and 'tax-taking', leading to first-order conditions for the household optimal consumption programme that are different from the competitive

[13] We would have to also impose the constraint $0 < \overline{c}_1 \leq \overline{y}_1$

ones used in (2.14), (2.15) and (2.16). Indeed the household sector could have its behaviour specified in a completely ad-hoc manner (like the government in this paper), subject only to non-negativity constraints on consumption and money holdings. The constraint $M_t \geq 0$ affects both households and the government.

The source of the fallacy in the HTPL is the same as that in the FTPL: the failure to recognise that, in a market economy, intertemporal budget constraints have to hold identically, that is, both in and out of equilibrium.

III.4 The FTPL when the price-level is predetermined

From now on, I only consider the case where $f > 0$, for which there always exists an equilibrium with a positive price of money in each period. If one departs from the assumption made thus far in this paper, that the general price level is non-predetermined, or flexible, a further anomaly is associated with the FPTL. If the price level is predetermined, that is, the price level in period *t* depends on the price level in one or more periods before *t*, the FTPL leads to an overdetermined price level even when the authorities adopt an exogenous nominal interest rate rule. In Buiter [1999], I consider a simple 'Keynesian' example of such an economy, in which output is demand-determined and the price level and the rate of inflation are predetermined through a simple augmented Phillips curve.

With the price level in period *1* predetermined, it cannot do the job of mimicking a revaluation factor on the public debt in the government's intertemporal budget constraint. Under Ricardian FFMPs, the model continues to be exactly determined.

I can think of no good reason why the decision as to whether to treat the government's intertemporal budget constraint as an equilibrium condition or an identity should depend on whether the price level is non-predetermined or predetermined. This need to switch the FTPL 'on' or 'off' for no good economic reason in order to avoid contradictions or anomalies extends to the details of the monetary regime as well. There is no economic argument for treating the government's intertemporal budget constraint as an identity when the nominal money stock is exogenous, but as an equilibrium conditions when the nominal money stock is endogenous.

IV The Infinite Horizon Case

To obtain the infinite horizon case, we let $N \rightarrow ¥$ in (2.9) and follow the standard procedure of replacing the household solvency constraint (2.8) by the 'no Ponzi-finance' condition:[14]

$$\lim_{N\rightarrow\infty} R_{t,N} \left(\tilde{P}_N^B B_N + \tilde{P}_N^b b_N \right) \geq 0 \qquad (4.1)$$

This produces the following intertemporal budget constraint for the household

$$D_{t-1,t} \left(\Gamma B_{t-1} + P_t g\, b_{t-1} \right) \geq \lim_{N\rightarrow\infty} \sum_{j=0}^{N-t} R_{t,t+j} \left(P_{t+j} (c_{t+j} + t_{t+j} - y_{t+j}) + (M_{t+j} - M_{t+j-1}) \right) \qquad (4.2)$$

The household optimal consumption and money holdings programme is characterised by equations (2.14) and (2.15) for all $t \geq 1$. While equation (2.16), which characterises monetary equilibrium in the finite terminal period, N, applies when N is very large but finite, it cannot hold if the economy truly has no terminal period.[15]

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[14] We now also have to impose $d > 0$.

[15] Taking the limit as $N \rightarrow \infty$ of (2.16) and of (10), we get an inconsistency unless $\lim_{t\rightarrow\infty} \left\{ \frac{P_{t+1}}{P_t} \right\} = \pm\infty$, which would not make economic sense.

The government's solvency constraint is (again following standard practice) also given by a 'no-Ponzi finance' condition:

$$\lim_{N\to\infty} R_{t,N}\left(\tilde{P}_N^B B_N + \tilde{P}_N^b b_N\right) \le 0 \tag{4.3}$$

This produces the government intertemporal budget constraint (4.4)

$$D_t\left(\Gamma B_{t-1} + P_t g\, b_{t-1}\right) \le \lim_{N\to\infty} \sum_{j=0}^{N-t} R_{t,t+j}\left(P_{t+j}(t_{t+j} - g_{t+j}) + (M_{t+j} - M_{t+j-1})\right) \tag{4.4}$$

In the infinite-horizon case, equilibrium is characterised by (2.36) and the following conditions.

$$1 + i_{t,t+1} = (1+d)\frac{P_{t+1}}{P_t} \qquad 1 \le t \tag{4.5}$$

$$\frac{M_t}{P_t} = (\bar{y} - \bar{g})\left[\frac{f(1+d)\frac{P_{t+1}}{P_t}}{(1+d)\frac{P_{t+1}}{P_t} - 1}\right]^{\frac{1}{h}} \qquad 1 \le t \tag{4.6}$$

$$D_{0,1}\left(\frac{\Gamma\bar{B}_0}{P_1} + g\,\bar{b}_0\right) \equiv \lim_{N\to\infty}\left(\sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j\left[t_{1+j} + \left(\frac{M_{1+j} - M_j}{P_{1+j}}\right) - \bar{g}\right]\right) \tag{4.7}$$

Under the Ricardian FFMP with contract fulfilment we have in addition

$$D_{0,1} \equiv 1 \tag{4.8}$$

and

$$\begin{aligned} t_1 &= \bar{g} - \frac{M_1 - M_0}{P_1} + D_{0,1}\left(\frac{\Gamma\bar{B}_0}{P_1} + g\,\bar{b}_0\right) \\ t_t &= \bar{g} - \frac{M_t - M_{t-1}}{P_t} \qquad 1 < t \end{aligned} \tag{4.9}$$

Under the Ricardian FFMP without contract fulfilment we have (4.10) instead of (4.9), and $D_{0,1}$ is endogenous.

$$t_t + \frac{M_t - M_{t-1}}{P_t} = \bar{s}_t \qquad 1 \le t \tag{4.10}$$

Under the Non-Ricardian FFMP, we have (4.10) and $D_{0,1} \equiv 1$.

Under an exogenous rule for the nominal money stock we have

$$\left\{ M_t = \bar{M}_t > 0;\ 0 \le t \right\}$$
$$\left\{ \frac{\bar{M}_{t+1}}{\bar{M}_t} \ge \frac{1}{1+d};\ t \ge 0 \right\} \tag{4.11}$$

Under an exogenous rule for the nominal interest rate rule we have

$$\left\{ i_{t-1,t} = \bar{i}_{t-1,t} \ge 0; 1 \le t \right\} \tag{4.12}$$

Again, only non-negative equilibrium price level sequences are admissible. All results, inconsistencies and anomalies of the finite-horizon case carry over to the infinite-horizon case with one exception. Proposition 5, that under a non-Ricardian FFMP with an exogenous rule for the nominal money stock, the general price level is overdetermined, now only applies when the velocity of circulation of money does not depend on the nominal interest rate and, through that, on expected future price levels.

Consider the simple cash-in-advance-model due to Helpman [1981] and Lucas [1982]. Each unit period is subdivided into two sub-periods. In the first sub-period, the market for consumer goods is closed, but financial markets are open. The asset menu is the same as before. During this first sub-period, households collect interest and principal from last period's bond purchases, receive their endowments and pay taxes or receive transfers. They allocate these resources among money and new bond purchases. In the second sub-period financial markets are closed, but the market for consumer goods is open. Consumption must be paid for with money: $\bar{M}_t \geq P_t c_t$.

Assume that the equilibrium short nominal interest rate sequence is strictly positive. The cash-in-advance constraint for consumption purchases will be binding. Monetary equilibrium is characterised by[16]

$$\bar{M}_t = P_t c_t \qquad t \geq 1 \qquad (4.13)$$

Using the same endowment economy used before, we have

$$c_t = \bar{y} - \bar{g}$$

Therefore,

$$P_t = \frac{\bar{M}_t}{\bar{y} - \bar{g}} \qquad t \geq 1 \qquad (4.14)$$

This applies in all periods, including the initial period, $t = 1$. However, according to the FTPL, the period *1* price level is also determined by the government solvency constraint under the non-Ricardian FFMP (given by (2.40), since $D_{0,1} \equiv 1$, provided (3.1) is satisfied) (see also Buiter [1998]). The price level is overdetermined under an exogenous rule for the nominal money stock.

When the demand for money depends on the nominal interest rate, even with the price level in period *t,* determined by the government's intertemporal budget constraint for period *t,* monetary equilibrium in period *t* can be achieved through an appropriate value of the (expected) price level in period *t+1.* In Buiter [1999], I show that even though there is no overdermination of the price level when velocity is

[16] In the Helpman [1981]-Lucas [1982] variant of the cash-in-advance model, consumers can use cash acquired at the beginning of period t for consumption later in that period. The producers who receive the cash must hold it between periods. An alternative model constrains the household to use only money acquired in period *t-1* for consumption purchases in period *t*. The cash-in-advance constraint for that variant

endogenous and forward-looking, other anomalies arise. For the logarithmic special case of the household period felicity function ($h = 1$), the monetary equilibrium condition (4.6) implies the following first-order difference equation for the general price level.

$$P_{t+1} = \frac{M_t P_t}{(1+d)[M_t - f(\bar{y} - \bar{g})P_t]} \qquad t \geq 1 \tag{4.15}$$

For the equilibrium to be well-defined, we require $\frac{M_t}{P_t} > f(\bar{y} - \bar{g})$.

Under a Ricardian FFMP, P_1 is a 'free' variable and there is a continuum of equilibrium price level sequences. Consider the case where the nominal money stock is kept constant, $M_t = \bar{M} > 0$. The unique steady-state price level, $\bar{P}$, for which money has a bounded positive value, is given by

$$\bar{P} = \left(\frac{d}{1+d}\right)\left(\frac{\bar{M}}{f(\bar{y} - \bar{g})}\right)$$

Equation (4.15) can be rewritten as

$$P_{t+1} = \left(\frac{1}{1 + \frac{d}{\bar{P}}(\bar{P} - P_t)}\right) P_t$$

Thus, when $P_t > \bar{P}$, the price level will be rising further away from its steady state value. When $P_t < \bar{P}$, the price level will be falling further away from its steady state value. The 'well-behaved' steady state is unstable. According to the FTPL, P_1 is determined from (2.40), provided condition (4.16), the infinite horizon analogue of (3.1), is satisfied.

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would be $M_{t-1} \geq P_t c_t$. Overdeterminacy under a monetary rule characterises this variant also.

$$\text{sgn}\left\{\bar{B}_0\right\} = \text{sgn}\left\{\lim_{N\to\infty}\left\{\sum_{j=0}^{N-1}\left(\frac{1}{1+d}\right)^j\left[\bar{s}_{1+j} - \bar{g}\right]\right\} - g\,\bar{b}_0\right\} \tag{4.16}$$

Under these conditions, there is a unique equilibrium price level sequence. Not surprisingly, the equilibrium price sequence determined in this manner can behave anomalously.

With a logarithmic period felicity function, the equilibrium behaviour of the price level under the non-Ricardian monetary rule will be explosive (or implosive) unless the value of the initial price level determined from (2.40) happens to be the steady state price level, that is, unless $P_1 = \bar{P}$. If, with a constant nominal money stock, the price level were to start off below its steady state value, it would decline towards zero. The stock of real money balances would go to infinity.

In many monetary models, infinite real money balances would cause private consumption to become unbounded, which would violate the economy's real resource constraint. In the simple 'money in the direct utility function' model of this paper, unbounded real money balances do not violate the equilibrium conditions, because the nominal interest rate would go to zero, creating an unbounded equilibrium demand for real money balances without consumption becoming unbounded.[17] Obstfeld and Rogoff [1983, 1986, 1996] discuss plausible restrictions on the utility function that would rule out such deflationary bubbles.[18] [19]

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[17] The resource constraint and the money demand function imply that

$$\bar{y} - \bar{g} = \frac{1}{f}\left(\frac{i_{t,t+11}}{1+i_{t,t+1}}\right)\frac{M_t}{P_t}.$$

This condition can be satisfied even if $\frac{M_t}{P_t} \to \infty$, as long as $i_{t,t+1} \to 0$ at the appropriate rate.

[18] They conclude that there are no plausible a-priori restrictions on the utility function that would rule out hyperinflationary bubbles.

V Conclusion

Ricardian FFMPs require the government's intertemporal budget constraint to hold identically. All government decision rules, optimising, satisficing or ad-hoc, are constrained to satisfy the government's intertemporal budget constraint for all admissible values of the variables entering the budget constraint. If the government is to meet its outstanding contractual debt obligations exactly, the Ricardian view implies that, when the government has k instruments to which it can assign values in each of N periods, at most $Nk - 1$ instrument values can be assigned independently. If the government assigns Nk instrument values independently, its FFMP is overdetermined, and, in general, the government's outstanding contractual debt obligations cannot be met exactly. According to the Ricardian view, when the FFMP is overdetermined, the government's intertemporal budget constraint becomes an effective public sector debt pricing kernel. It determines the effective real value of the initial net public debt as that value that permits the government's intertemporal budget constraint to be satisfied identically given its overdetermined FFMP.

[19] Infinite real money balances would violate the economy-wide real resource constraint in the 'unpleasant monetarist arithmetic' model of Sargent and Wallace [1981] (see also Buiter [1987]). When the period felicity function of the 'poor' households who hold all the money (and for whom money is the only store of value) in this model generates a demand for real money balances that is sensitive to the (expected) rate of inflation, the adoption of a non-Ricardian FFMP can force the economy to follow a solution trajectory along which real money balances increase without bound, even though real endowments are finite.

In the 'money in the shopping function' model used in Buiter [1998], a similar real resource constraint violation problem would arise if the price level were to *rise* without bound with a constant nominal money stock. Such behaviour could be implied in that model by the adoption of a non-Ricardian FFMP. As the stock of real money balances goes to zero, the real resources used up in the shopping function technology would go to infinity, violating the economy's real resource constraint.

Equivalently, when the FFMP is overdetermined, the government's intertemporal budget constraint determines the public debt revaluation factor, that is, the ratio of the effective real value of the initial net public debt to its contractual value.

The Ricardian approach respects the key property of any model of a market economy, that all economic agents' decision rules must satisfy their budget constraints identically. The government may have instruments that are different from those available to private agents. Taxes are one example. Money issuance, based on a monopoly of the issuance of a financial instrument that is useful to private agents even though it is rate of return-dominated as a store of value because it bears a zero nominal interest rate, is another. Whatever the government's instruments, the government's intertemporal budget constraint must be satisfied for all admissible values of the variables entering the budget constraint.

The Ricardian approach is general. It applies when the government is, or perceives itself to be, small and if the government is, and perceives itself to be, large in the markets in which it operates. It applies regardless of how the government's debt instruments are denominated, that is, it holds even when there is no nominally denominated government debt. It applies when the government specifies an exogenous sequence for the nominal money stock, when the government specifies an exogenous sequence for the nominal interest rate, when the government adopts a rule relating the nominal interest rate to the (real or nominal) stock of money, or for any other monetary or interest rate rule. It applies when money plays an essential role in the economy. It also applies, and makes sense, in a barter economy, that is, in an economy without money as a transactions medium, means of payment and store of value. It applies regardless of how the general price level is determined. Specifically, it applies both when the general price level is non-predetermined, as in many New

Classical models, and when the general price level is predetermined or ‘sticky’, as in many New and Old Keynesian models with nominal price or wage rigidities.

The FTPL rejects the fundamental property of any market economy, that for each agent, budget constraints must be satisfied identically. Instead, the FTPL asserts that, subject to a long list of conditions, the government’s intertemporal budget constraint need only hold in equilibrium. Since the FTPL, with its Non-Ricardian FFMP, assumes that fiscal-financial-monetary programmes are overdetermined, it can only determine the equilibrium price level in situations where a Ricardian FFMP would yield price level (or nominal) indeterminacy. This requires a monetary regime under which the nominal money stock is endogenous. The example in this paper considers an exogenous nominal interest rate rule with an endogenous nominal money stock. Under these conditions, the general price level under the FTPL can, sometimes, mimic the role played by the public debt revaluation factor under a Ricardian FFMP. Even then, anomalies abound.

The FTPL cannot explain why the government’s intertemporal budget constraint should hold as an identity when the nominal money stock is exogenous, but not when it is endogenous. From a Ricardian perspective, the budget constraint holds identically regardless of how the government chooses its fiscal, financial and monetary instruments. It must hold identically when the nominal interest rate is exogenous, when the nominal money stock is exogenous, when the nominal interest rate is specified as a function of the nominal or real money stock, and indeed for any optimising, satisficing or ad-hoc behavioural rule for the nominal money stock or the nominal interest rate.

The FTPL has to impose arbitrary restrictions on the composition of the financial liabilities and assets of the government. In the model considered in this paper, there is no FTPL if there is only index-linked government debt or if all government debt is foreign-currency denominated. Even if there is a non-zero stock of nominal government debt outstanding, further arbitrary restrictions are required to ensure that the price level implied by the government's intertemporal budget constraint is positive.

Non-Ricardian FFMPs are not permitted in barter economies, unless the government, for some reason, chooses to denominate one of its debt instruments in terms of something which may be called 'money', but ought to be called 'phlogiston' - a substance without physical presence, which does not yield utility, directly or indirectly, is not used as a productive input, and is not used as a transactions medium, means of payment or store of value. In that case, the FTPL determines the price of this pure numéraire 'money', provided the conditions are satisfied for the overdetermined non-Ricardian FFMP to yield a positive value for the price of this numéraire.

Non-Ricardian FFMPs also are not permitted when the general price level is predetermined. The FTPL cannot explain why the appropriate specification of the government's intertemporal budget constraint should depend on the way the general price level is determined in general equilibrium.

The fiscal theory of the price level starts with an untenable economic assumption. Its denial of a fundamental property of any model of a market economy, that all agents' decision rules are subject to budget constraints that must hold identically, makes it a non-starter for positive and normative models of monetary and budgetary policy in a market economy. The unfortunate starting point is compounded by defective analysis, which fails to uncover the many contradictions and anomalies, outlined in this paper, that follow from the misspecification of the conditions under which the government's intertemporal budget constraint must hold.

Adoption of a non-Ricardian FFMP could have painful consequences when the Ricardian reality dawns. Consider the case where the original public spending, tax and seigniorage plans imply a discount on the contractual value of the public debt. Something will have to give. If government default on its contractual debt obligations is not permitted to occur and if real primary surpluses are not boosted sharply, the two familiar *Ricardian* mechanisms linking public debt and inflation the anticipated and unanticipated inflation taxes, will come into play.

The effect of public debt on the 'anticipated inflation tax' is familiar from the 'unpleasant monetarist arithmetic' model of Sargent and Wallace [1981]. A higher (constant) ratio of public debt to GDP will, holding constant the sequence of real primary surpluses as a fraction of GDP, require a higher real seigniorage sequence if the government's solvency constraint is to be satisfied.[20] If the economy is operating on the upward-sloping segment of the seigniorage Laffer curve, the need for higher real seigniorage implies a higher rate of nominal money growth and a higher rate of inflation.[21]

If there is nominally denominated fixed rate public debt outstanding, an unexpected increase in future short nominal interest rates will, by unexpectedly reducing the price of fixed-rate nominal debt, reduce the real value of the outstanding stock of such debt. This effect will be stronger the longer the maturity of the outstanding fixed rate nominal debt. An unanticipated increase in the inflation rate is

[20] The analysis assumes that the interest rate exceeds the growth rate.
[21] For reasons made familiar by Oliveira and Tanzi [1978], anticipated inflation may affect both the spending and revenue sides of the primary government budget.

one way for the policy maker to engineer an unanticipated increase in expected future short nominal interest rates. This is the 'unanticipated inflation tax'.

Of course, even maximal use of the anticipated and unanticipated inflation taxes (with the government moving to the top of the seigniorage Laffer curve and the real value of the outstanding stock of nominal contractual debt reduced to zero) may not be enough to ensure consistency of the fiscal-financial-monetary programme. In that case default on the government's index-linked or foreign-currency-denominated debt will occur, or the spending and tax programmes will have to be revised.

The FTPL emperor wears no clothes. It is time to return to serious general equilibrium theory and monetary economics.

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