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# INTERACTIONS BETWEEN <br> INTERNATIONAL MIGRATION AND THE WELFARE STATE 

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# INTERACTIONS BETWEEN INTERNATIONAL migration and the welfare state 


#### Abstract

The intratemporal redistribution feature of the welfare state makes it an attractive destination for immigrants, particularly for low-skill immigrants. George Borjas (1994) reports that foreign-born households in the United States accounted for 10 percent of households receiving public assistance in 1990, and for 13 percent of total cash assisitance distributed, even though they constituted only 8 percent of all households in the United States. In this chapter we explore the implications of various redistribution policies for the attitude of the native-born towards migrants. We analyze the effect of migration on the shape and magnitude of redistribution policies that are determined in a political economy equilibrium; at the same time, we address the question whether the level of migration, when not restricted, is higher or lower in this welfare state than in the laissez-faire (no-redistribution) economy.


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## 1 An Intratemporal Model

There is a continuum of individuals. Each individual is characterized by the innate ability parameter $e$ which is the time cost needed to acquire skill. The c.d.f. of $e$ is given by $G(\cdot)$, so that $G(1)=1$. (The number of individuals is normalized to one). All individuals live for one period. They are born unskilled, each with a unit of labor time and $K$ units of capital. By investing $e$ units of labor time in education, an individual becomes skilled which means that each unit of her remaining labor time (that is, $1-e$ ) is worth one unit of effective labor. If, however, she does not acquire skill (that is, she remains unskilled) her labor time is worth only $q(<1)$ units of effective labor.

The government can only employ an income tax in order to redistribute income. Many studies (for instance, Mirrlees (1971)) suggest that the best egalitarian income tax may be approximated by a linear tax which consists of a flat rate $(\tau)$ and a lump-sum cash demogrant (b). Since all families are of similar size and age structure, the uniform demogrant may capture also free provisions of public services such as health care, education, etc.

In this setup the tax has no effect on the decision to acquire skill. The cutoff ability level $\left(e^{*}\right)$ between acquiring and not acquiring skill is given by the following equation:

$$
\begin{equation*}
e^{*}=1-q \tag{1}
\end{equation*}
$$

Denote the consumption of an e-individual by $c(e)$. It is equal to disposable income: Hence:

$$
c(e)=\left\{\begin{array}{l}
(1-\tau) w(1-e)+[1+(1-\tau) r] K+b \text { for } e 5 e^{*}  \tag{2}\\
(1-\tau) q w+[1+(1-\tau) r] K+b \text { for } e=e^{*}
\end{array}\right.
$$

where $w$ is the wage per unit of effective labor, and $(1+r)$ is the gross rental price of capital. With no loss of generality, it is assumed that capital fully depreciates at the end of the production process; the income tax $(\tau)$ applies to the net rental price of capital $(r)$.

Note that the disposable income (namely, consumption) distribution curve is piecewise linear in the ability parameter $e$. This refers to the native-born population. For individuals who do not acquire skill (i.e. those with an ability parameter $e$ above the cutoff parameter $e^{*}$ ), the ability parameter is irrelevant and they have the same income. Naturally, within the group of individuals who do decide to become skilled (i.e. for $e 5 e^{*}$ ), the more able is the individual (i.e. the lower is $e$ ), then the higher is her disposable income. As can be seen from (4.2), this relationship is linear. The income distribution curve is depicted in Figure 1. Note that the slope of the downward sloping segment is $-(1-\tau) w$. Also, notice that $e^{*}$ is unaffected by the income distribution policy (namely, $\tau$ and $b$ ). We assume that the migrants (whose number is $m$ ) are all unskilled and possess no physical capital. Their disposable income is only $(1-\tau) q w+b$ which is below that of the unskilled native-born individuals.

We assume a standard (concave, constant-returns-to-scale) production function:

$$
\begin{equation*}
Y=F(K, L) \tag{3}
\end{equation*}
$$

where $Y$ is gross output; $K$ is the total stock of capital (recall that each individual possesses $K$ units of capital and the number of individuals is normalized to one), and $L$ is the supply of labor which is given by:

$$
\begin{equation*}
L=\int_{0}^{e^{*}}(1-e) d G+q\left[1-G\left(e^{*}\right)\right]+q m \tag{4}
\end{equation*}
$$

The wage rate and the gross rental price of capital are given in a competitive equilibrium by the marginal productivity conditions:

$$
\begin{equation*}
w=F_{L}(K, L) \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
1+r=F_{K}(K, L) . \tag{6}
\end{equation*}
$$

The income tax parameters $\tau$ and $b$ are related to each other by the government budget constraint:

$$
\begin{equation*}
b(1+m)=\tau(Y-K) \tag{7}
\end{equation*}
$$

Note that the base for the flat income tax rate is net domestic product $(Y-K)$, including labor income of migrants which is subject to the income tax. Also, migrants qualify to the uniform demogrant $b$.

Finally, there are no barriers to migration so that $m$ is determined endogeneously by:

$$
\begin{equation*}
(1-\tau) q w+b=w^{*} \tag{8}
\end{equation*}
$$

where $w^{*}$ is the opportunity income of the migrants in the source countries.
This model is employed in the next sections in order to investigate two issues: (i) How the welfare state attracts migration of various skill levels? (ii) more importantly, what are the effects of migration on the income distribution among the native-born which in turn shape their attitude towards migrants?

## 2 The Attractiveness of the Welfare State to Migrants.

Within this framework we address the first issue of whether the welfare state indeed attracts migrants. More generally, is it true that more taxes and more transfers attract more migrants in the context of our stylized model? Specifically, we study the sign of $d m / d \tau$.

To simplify the analysis we assume a uniform distribution of the ability parameter $e$ over the interval $[0,1]$. This assumption yields a simple labor supply function as follows:

$$
L=\frac{1}{2}(1-q)^{2}+q(1+m)
$$

where use is made of (1).
Substituting (3), (4'), (5) (6) and (8) into (7) and rearranging terms yields:

$$
\begin{align*}
& \left\{w^{*}-(1-\tau) q F_{L}\left[K, \frac{1}{2}(1-q)^{2}+q(1+m)\right]\right\}(1+m)  \tag{9}\\
= & \tau\left\{F\left[K, \frac{1}{2}(1-q)^{2}+q(1+m)\right]-K\right\} .
\end{align*}
$$

Total differentiation of the latter equation with respect to $\tau$ yields:

$$
\begin{equation*}
\left[w^{*}-q F_{L}-(1+m)(1-\tau) q^{2} F_{L L}\right] \frac{d m}{d \tau}=F-K-(1+m) q F_{L} \tag{10}
\end{equation*}
$$

By substituting (5), (8), $F=(1+r) K+w L$ (Euler's equation) and (4') into (10) we conclude that:

$$
\begin{equation*}
\left[b-q \tau w-(1+m)(1-\tau) q^{2} F_{L L}\right] \frac{d m}{d \tau}=r K+\frac{1}{2}(1-q)^{2} w . \tag{11}
\end{equation*}
$$

It is straightforward to see from the government budget constraint (namely, equation (7)) that the tax on labor income paid by an unskilled individual (namely, $\tau q w$ ) must fall
short of her demogrant (namely, $b$ ), that is $b>\tau q w .{ }^{6}$ Since $F_{L L}<0$, it follows from (11) that:

$$
\begin{equation*}
\frac{d m}{d \tau}>0 \tag{12}
\end{equation*}
$$

Thus, more taxes and transfers attract more unskilled migrants.
This unambiguous conclusion that the more intensive is the welfare state, the more attractive it becomes to migrants is restricted naturally to the case of low-skill migration. If we allow for high-skill migrants as well, we can see in a natural extension of our stylized model that the welfare state attracts more low-skill migrants but fewer high-skill migrants, as long as "supply-side economics" does not prevail (that is, as long as raising taxes does not yield less revenues). This is shown in Appendix 1. As we have seen, migration changes the income distribution among the native-born and the attitude of the native-born towards migrants is shaped accordingly.

## 3 The Attitude of Native-Born Towards Migration

Migration changes the income distribution among the native-born, and the attitude of nativeborn towards migrants is shaped accordingly.

### 3.1 A Benchmark Case: No Redistribution Policies

Let us start with a benchmark case where the government does not engage in redistributing income. This benchmark case highlights the gains from trade effect of labor mobility. In this case we set the tax-transfer parameters at zero (i.e., $\tau=b=0$ ) and drop out the government budget constraint (7).

Suppose initially that there is no migration, so that $m$ is set equal to zero and the migration equilibrium condition (4.8) is dropped out. The resulting income distribution among the native-born is depicted by the curve ABC in Figure 4.2 , which is based on numerical simulations. Assuming that $e$ is uniformly distributed, the area under the income distribution curve is equal to net output (i.e., $Y-K$ ), less payments to migrants (i.e., $w^{*} m$ ) which is initially zero.

Now we allow free migration. That is, we reinstate the migration equilibrium condition (4.8) and reintroduce $m$ as an endogenous variable. The ensuing income distribution among the native-born is described by the curve DEF in Figure 4.2. As expected, the gains from trade effect is impeccable in the absence of any costly redistribution: total income of the native-born (i.e., the area under the income distribution curve) rises as a result of the influx of migrants.

The determination of the free migration number of immigrants is neatly described in Figure 3. The aggregate labor supply of the native-born is perfectly inelastic. (Capital is also fixed.) Thus, the labor supply of migrants changes the total domestic labor supply one-to-one. The downward-sloping curve describes the marginal product of low-skilled migrants (namely, $q w$ ) as a function of the number of migrants. The equilibrium level of $m$ occurs
at point $A$, where $q w$ is euated to $w^{*}$. The standard gains from trade (to the native born) is measured by the triangle-like area $A B C$, which consists of the total output produced by the migrants $(O C A m)$, less the amount of wages paid to them $(O B A m)$.

However, the distributional effects of migration are in general not clear: Some must always gain, but others may lose. In our particular model and for our specific parameter values, it so happens that some individuals (those with an ability parameter above $\bar{e}$; see Figure 2) gain, but other individuals (those with $e<\bar{e}$ ) lose. Nevertheless, with an active redistribution policy all may lose as we shall see below.

### 3.2 Redistribution Policy

Now, consider a typical welfare state which redistributes income from the rich to the poor. That is, it levies a positive flat $\operatorname{tax}(\tau>0)$ on income (labor and capital) and uses the proceeds to finance a positive demogrant $(b>0)$. The immigrants are typically not only subject to the income tax, but also eligible for the benefits of the welfare state, in contrast to guest workers.

We perform the following exercise. Suppose first that there is no migration. The closed economy equations described above (that is, (1), (3)-(7)), allow the government one degree of freedom in designing its redistribution policy (that is, the $\tau$ and $b$ parameters). Thus, for each $\tau$ there is a corresponding equilibrium $b$. Consider a certain configuration of the equilibrium pair $(\tau, b)$. For this pair we find the income distribution curve given by (2). We then allow free migration, that is, we endogenize $m$ and reinstate the free migration equilibrium equation (8). We next redesign the tax-transfer pair $(\tau, b)$ in such a way so as
to maintain the income of the native-born unskilled individuals at its pre-migration level; and ask what happens to the income of the skilled individuals. The above exercise is carried out for various (pre-migration) tax-transfer configurations, starting from a very low level of redistribution up to a very high level.

Notice that in the absence of migration, the redistribution is not distortionary: In the absence of a pecuniary cost of acquiring education, the redistribution policy affects neither the individual decision whether to become skilled or remain unskilled (that is, the determination of $e^{*}$ ), nor the supply of labor and capital. A dollar taxed away from some individuals ends up entirely, with no deadweight loss whatsoever, at the hands of some other or the same individuals. With migration, there is still no deadweight loss in the common use of this term: It is still the case that a dollar taxed away from some individuals ends up entirely at the hands of some other or the same individuals. But there is a loss from the point of view of the native-born individuals because the low-skilled migrants are typically net beneficiaries of the welfare state in the sense that their tax payments (namely, $\tau q w m$ ) fall short of their gross benefits (namely, $b m$ ); thus, a dollar of revenues collected from the native-born does not end up entirely at the hands of the native-born, as a portion of it "leaks" to the migrants.

Furthermore, note that with a redistribution policy the gains from trade (to the native-born) may disappear altogether: Total income of the native-born may actually decline as a result of migration. To see this, refer again to Figure3. The migrants who are low-skilled and do not own any capital are net beneficiaries of the welfare state. That is, $\tau q w<b$ which means that their net income (namely, $(1-\tau) q w+b)$, is above their net marginal product
(namely, $q w$ ). Since their net income is equal to their reservation income $w^{*}$, it follows that free migration occurs at a point such as $D$, where $q w<w^{*}$. In this case the net gain to the native-born is measured by the area ABC, less the triangular-like area AED. This "gain" from trade could well become negative when $\tau$ (and $b$ ) are sufficiently high. When this happens, it may also be the case that all (skilled and unskilled) native-born individuals lose from free migration.

Our simulations show (see Table 1) that when the flat tax rate $(\tau)$ in the absence of migration is between $35 \%$ to $55 \%$ (and the corresponding demogrant (b) is between $17.7 \%$ to $25.7 \%$ of GDP), indeed the skilled individuals all strictly lose from migration, if the redistribution policy is adjusted in order to maintain the disposable income of low-skill native-born at its pre-migration level. The aggregate gains (losses) to the skilled individuals are presented in the last column of Table 1. These gains (losses) to the skilled individuals are also the aggregate gains (losses) to the entire native-born population, as the redistribution policy is geared at leaving the unskilled individuals intact. Thus, migration cannot be a Pareto-improving shock for the native-born population, when $\tau$ originally (before any migration takes place) exceeds $35 \%$.

As was already mentioned, when the income distribution policy is geared to maintaining the income of the native-born unskilled individuals intact, then the net gain (or loss) to the native-born skilled individuals measures the standard gain (or loss) from trade to the native-born population. For instance, when pre-migration $\tau$ is between $35 \%$ to $55 \%$ (and the corresponding $b$ is between $17.7 \%$ to $25.7 \%$ of GDP), then the curves describing the disposable income distribution among the native-born look like the curve ABC in Fig-
ure 4.4. Now, if we allow free migration and adjust the tax-transfer parameters so as to maintain the disposable income of the native-born unskilled intact, then the new disposable income distribution curves look like the curve DBC. (Note that among the native-born the triangle-like area ADB in Figure. 4 measures the total net loss to the native-born and is therefore equal to the area AED, less the area ABC in Figure 3.)

## 4 Political-Economy Effects on the Host Country

The preceding section analyzed the attitude of the native-born towards migration. We examined the effects of migration on the aggregate income of the native-born people and its distribution among them. The scope of the welfare state itself was not the focus of analysis as the tax-transfer parameters were assumed exogenous (though, of course, constrained by the government budget constraint).

In this section we examine how the redistribution policy is determined in a political economy equilibrium. We then address the following issues in this setup: Does migration necessarily tilt the political power balance in favor of heavier taxation and more intensive redistribution? Relatedly, how does migration affect income ineqality among the nativeborn?

The extent of taxation and redistribution policy in our analytical framework is determined by a direct democracy voting. The political economy equilibrium is then determined by a balance between those who gain and those who lose from a more extensive tax-transfer policy. The model captures two conflicting effects of migration on taxation and redistribution. On the one hand, the low-skill, low-income migrants who are net beneficiaries from
the tax-transfer system will join forces with the native-born low-income voters in favor of higher taxes and transfers. On the other hand, redistribution becomes more costly to the native-born population, as the migrants share some of the benefits at their expense.

## 5 Redistribution Policy in a Direct Democracy

We continue to employ the basic intratemporal model of an economy with migration and redistribution which is described in section 1 with two modifications. As explained in that section, the tax-transfer policy is not distortionary in the absence of migration. With no migration, there is also no "leakage" of tax revenues to migrants (through the demogrant) and, as a result, there need not be an interior solution for the equilibrium tax rate: It may go all the way either to zero or to $100 \%$. We therefore introduce a positive pecuniary cost of acquiring skill which is not tax deductible; thus, $e^{*}$ is now by:

$$
(1-\tau) w\left(1-e^{*}\right)-\gamma=(1-\tau) w q
$$

and, by rearranging terms:

$$
\begin{equation*}
e^{*}=1-q-\frac{\gamma}{(1-\tau) w} \tag{13}
\end{equation*}
$$

The second modification is done for the sake of simplicity: We consider the case
where migration is restricted by quotas. Formally, it means that $m$ is exogenously given, so that equation (8) which specifies the equilibrium level of free migration is dropped out. It turns out that in this case of exogenous $m$, one can analytically derive the results when factor prices are not variable. Thus, for analytical tractability in this chapter we assume a linear production function:

$$
\begin{equation*}
Y=w L+(1+r) K \tag{14}
\end{equation*}
$$

where the marginal productivity conditions for setting up factor prices (namely, equations (5)-(6)) were already substituted into the production function. We continue to assume that $e$ is distributed uniformly over $[0,1]$, so that the labor supply equation (4) becomes:

$$
\begin{equation*}
L=e^{*}-\frac{1}{2}\left(e^{*}\right)^{2}+\left(1-e^{*}+m\right) q . \tag{15}
\end{equation*}
$$

Finally, the government's budget constraint (7) implies that:

$$
\begin{equation*}
b=\frac{\tau(w L+r K)}{1+m} . \tag{16}
\end{equation*}
$$

For any tax rate $\tau$, and exogenously given migration quota $m$, equations (13), (15) and (16) determine $e^{*}, L$ and $b$ as functions of $\tau$ and $m: e^{*}=e^{*}(\tau, m), L=L(\tau, m)$ and
$b=b(\tau, m)$. The number of migrants $(m)$ is exogenous, but we nevertheless write $e^{*}, L$ and $b$ as functions also of $m$, because we wish to explore in this chapter the effect of $m$ on these variables. Recall that consumption is a strictly decreasing function of the innate ability parameter ( $e$ ) for the native-born skilled; then constant for the native-born unskilled. It is also constant for the migrants, but at a lower level than for the native-born unskilled since the migrants do not own any capital. This function is given by:

$$
c(e, \tau, m)= \begin{cases}(1-\tau) w(1-e)-\gamma+[1+(1-\tau) r] K+b(\tau, m) & \text { for } 05 e 5 e^{*}(\tau, m)  \tag{17}\\ (1-\tau) w q+[1+(1-\tau) r] K+b(\tau, m) & \text { for } e=e^{*}(\tau, m) \\ (1-\tau) w q+b(\tau, m) & \text { for } 15 e 51+m\end{cases}
$$

where for ease of exposition we artificially attribute a parameter $e$ between 1 and $1+m$ to the migrants, simply in order to indicate that their consumption is below that of native-born unskilled. For a given tax rate $\left(\tau_{0}\right)$, consumption as a function of $e$ is depicted in Figure 5 . by the curve $A B C D E F$ ( $m$ is supressed).

The political economy $\tau$ is then determined by majority voting. By twice differentiating $c(e, \tau, m)$ with respect to $e$ and to $\tau$ we find that:

$$
\frac{\partial^{2} c(e, \tau, m)}{\partial e \partial \tau}= \begin{cases}w & \text { for } 0 \leq e<e^{*}(\tau)  \tag{18}\\ 0 & \text { for } e^{*}(\tau)<e<1 \\ 0 & \text { for } 1+m=e>1\end{cases}
$$

Thus, $\partial^{2} c / \partial e \partial \tau=0$. Therefore, if $\partial c / \partial \tau>0$ for some $e_{o}$, then $\partial c / \partial \tau>0$ for all $e \geq e_{o}$. Similarly, if $\partial c / \partial \tau<0$ for some $e_{o}$, then $\partial c / \partial \tau<0$ for all $e \leq e_{o}$. This implies that if an increase in the income tax rate $(\tau)$ benefits a certain individual (because the higher tax rate can support a higher transfer $b$ ), then all individuals who are less able (that is, those who have a higher innate ability parameter $e$ ), including the migrants, must also gain from this tax increase. Similarly, if an income tax increase hurts a certain individual (because the increased transfer does not fully compensate her for the tax hike), then it must also hurt all individuals who are more able (that is, those who have a lower innate ability parameter $e)$. These considerations imply that the median voter is a pivot in determining the outcome of majority voting. That is, the political equilibrium tax rate maximizes the consumption of the median voter.

Denote the innate ability parameter of the median voter by $e_{M}$. Assuming that migrants are allowed to vote, then:

$$
\begin{equation*}
e_{M}(m)=(1+m) / 2 . \tag{19}
\end{equation*}
$$

(Recall that the size of the native-born population was normalized to one and the ability parameter is uniformly distributed.) Diagramatically, suppose that $\tau_{o}$ in Figure 5 is a political equilibrium tax rate. Suppose further for the sake of concreteness that the median voter is skilled, that is $(1+m) / 2<e^{*}\left(\tau_{o}\right)$. An increase of $\Delta \tau>0$ in the tax rate must tilt the income distribution curve from $A B C D E F$ to $A^{\prime} B C^{\prime} D^{\prime} E^{\prime} F^{\prime}$, so that all individuals who are more able than the median voter lose and all the rest gain. Similarly, if
the tax rate is lowered to $\tau_{o}-\Delta \tau$, then the income distribution curve tilts from $A B C D E F$ to $A^{\prime \prime} B C^{\prime \prime} D^{\prime \prime} E^{\prime \prime} F^{\prime \prime}$, so that all individuals who are more able than the median voter gain and all the rest lose.

As noted, the political equilibrium $\tau$ (denoted by $\tau_{o}(m)$ ) maximizes the consumption of the median voter, that is:

$$
\begin{equation*}
\tau_{o}(m)=\arg \max _{\{\tau\}} c\left(e_{M}(m), \tau, m\right) \tag{20}
\end{equation*}
$$

Therefore, $\tau_{o}(m)$ is implicitly defined by:

$$
\begin{equation*}
\frac{\partial c\left(e_{M}(m), \tau, m\right)}{\partial \tau} \equiv B(\tau, m)=0 \tag{21}
\end{equation*}
$$

where, by (5):

$$
B(\tau, m)= \begin{cases}-w(1-m) / 2-r K+b_{\tau}(\tau, m) & \text { if } 0<e_{M}(m)<e^{*}(\tau, m)  \tag{22}\\ -w q-r K+b_{\tau}(\tau, m) & \text { if } e^{*}(\tau, m)<e_{M}(m)<1 \\ -w q+b_{\tau}(\tau, m) & \text { if } e_{M}(m)>1\end{cases}
$$

As a second-order condition for maximization we have:

$$
\begin{equation*}
\frac{\partial^{2} c\left(e_{M}(m), \tau_{o}(m), m\right)}{\partial \tau^{2}}=B_{\tau}\left(\tau_{o}(m), m\right) 50 \tag{23}
\end{equation*}
$$

where subscripts stand for partial derivatives.
Note that the equation $B(m, \tau)=0$ which determines the political equilibrium tax rate $\left(\tau_{o}(m)\right)$ depends, among other things, on the median income versus the average income. For instance, consider the case where the median voter is an unskilled native-born person, that is: $e^{*}(\tau, m)<e_{M}(m)<1$. Since equation (16) implies that $b$ is equal to $(w L+r K) /(1+$ $m$ ), it follows that the equation $B(\tau, m)=0$ implies that:

$$
I_{M}=\frac{\partial(\tau \bar{I})}{\partial \tau}
$$

where $I_{M}=w q+r K$ is pre-tax median income (net of depreciation) and $\bar{I}=(w L+r K) /(1+$ $m)$ is pre-tax mean income.

### 5.1 The Effects of Migration on Redistribution

Having described the political economy equilibrium, we now turn to the question of how this equilibrium is affected by migration.

Total differentiation of (21) with respect to $m$ implies that:

$$
\begin{equation*}
\frac{d \tau_{o}(m)}{d m}=\frac{B_{m}\left(\tau_{o}(m), m\right)}{B_{\tau}\left(\tau_{o}(m), m\right)} \tag{24}
\end{equation*}
$$

Since $B_{\tau} 50$ (see (23), it follows that the direction of the effect of migration $(m)$ on the equilibrium tax rate $\left(\tau_{o}\right)$ is determined by the sign of $B_{m}\left(\tau_{o}(m), m\right)$.

By differentiating equation (22) with respect to $m$ and evaluating it at $\tau=\tau_{o}(m)$, we conclude that:

$$
B_{m}\left(\tau_{o}(m), m\right)= \begin{cases}\frac{w(q+m)}{1+m}-\frac{r K}{1+m} & \text { if } e_{M}<e^{*}  \tag{25}\\ -\frac{r K}{1+m} & \text { if } e^{*}<e_{M}<1 \\ 0 & \text { if } e_{M}>1\end{cases}
$$

See Appendix 2 for the derivation of the latter equation.
As noted, if the sign of $B_{m}\left(\tau_{o}(m), m\right)$ is negative, then an increase in the number of migrants lowers the political equilibrium tax rate $\left(\tau_{o}\right)$ and, consequently, the demogrant (b). Whether this is what actually happens depends on whether the median voter is skilled or unskilled. Consider first, the case where the median voter is skilled, that is, $e_{M}>e^{*}$. As can be seen from equation (25), the sign of $B_{m}$ is a priori not determined. In this case, an increase in the number of migrants can either raise or lower the political equilibrium tax rate and demogrant. Consider next the case where the median voter is a native-born unskilled individual, that is $e^{*}<e_{M}<1$. In this case, an increase in the number of migrants unambiguously lowers the political equilibrium tax rate and demogrant. In the extreme case
where the median voter is an (unskilled) migrant, an increase in the number of migrants has no effect on the tax rate and the demogrant.

The rationale for this result is as follows. It is most instructive to begin with the case where the median voter is a native-born unskilled individual (that is, $e^{*}<e_{M}<1$ ). In this case, the majority of the voters are unskilled and they are certainly pro-tax. This majority has already pushed upward the tax rate to the limit (constrained by the efficiency loss of taxation). A further increase in the number of migrants who join the pro-tax group does not change the political power balance which is already dominated by the pro-tax group. However, the median voter who is a native-born member of this group (and, in fact, all the unskilled native-born individuals) would now lose from the "last" (marginal) percentage point of the tax rate because a larger share of the revenues generated by it would "leak" to the migrants whose number has increased. (Recall that before more migrants arrived, this median voter was indifferent with respect to the marginal percentage point of the tax rate.) Therefore, the median voter and all unskilled native-born individuals support now a lower tax rate. Indeed, $B_{m}$ which is equal to $-r K /(1+m)$ in this case reflects the marginal increase in tax revenues that are collected from the median voters (but not the migrants who own no capital) and "leak" to the migrants. This is also why $B_{m}=0$ in the case in which the median voter is an unskilled migrant (that is, $e_{M}>1$ ) because the "leakage" element does not exist. In this case, an increase in the number of migrants does not change the political equilibrium tax rate and demogrant.

Turn now to the case where the median voter is a native-born skilled individual. The "leakage" elements, as in the case where the median voter was a native-born unskilled
individual, works for lowering the tax rate when $m$ increases. However, now an increase in $m$ tilts the political power balance towards a median-voter who is less able and has a lower income; she benefits more from a tax hike than the original median voter. Thus, an increase in $m$ generates two conflicting effects on the political equilibrium tax rate. Therefore, one cannot unambiguously determine the effect of $m$ on $\tau$ and $b$.

A further insight into these conflicting effects can be gained when the second effect (that is, the shift in the political power balance) is elminated by assuming that migrants are not entitled (or choose not) to vote. In this case (see Appendix 2) one can show that:

$$
B_{m}\left(\tau_{o}(m), m\right)= \begin{cases}\frac{w}{1+m}\left(-\frac{1}{2}+q\right)-\frac{r K}{1+m} & \text { if } e_{M}<e^{*} \\ -\frac{r K}{1+m} & \text { if } e^{*}<e_{M}<1 \\ 0 & \text { if } e_{M}>1\end{cases}
$$

As noted before, when the median voter is either a native-born unskilled individual or an unskilled migrant, then even if the migrants were to exercise their voting rights, they do not effectively tilt the political balance power; and indeed equations (25) and (25') are identical when $e_{M}>e^{*}$. However, when the median voter is a native-born skilled individual, it does matter whether the migrants do or do not vote. If they do not vote, then $B_{m}$ is unambiguously negative (see Appendix 2 for the proof). When migrants do not vote, the tilting power-balance effect vanishes and only the "leakage" effect is at play and an increase in $m$ lowers $\tau$ and $b$.

The effect of $m$ on $\tau$ and $b$ has an interesting implication for the income distribution
among the native-born. Recall that we showed that more migration leads or can lead to lower taxation and redistribution. For instance, this is always the case when migrants do not participate in the political process (namely, they do not vote), or when the median voter is an unskilled native-born individual. Then more migration which leads the native-born to vote for a lower tax rate and a lower demogrant has the unintended consequence of a greater inequality of the income distribution among the native-born.

## Appendix 1: The Welfare State and the Skill Mix of Migration

Let us allow for high-skill migrants as well as low-skill migrants. Denote the number of low-skill migrants and high-skill migrants by $m_{\ell}$ and $m_{h}$, respectively. Suppose that their reservation wages in their home countries are $w_{\ell}^{*}$ and $w_{h}^{*}$, respectively. Then equation (8) is replaced by two equations, one for each skill type:

$$
\begin{equation*}
(1-\tau) q w+b=w_{\ell}^{*} \tag{A1.8a}
\end{equation*}
$$

and

$$
\begin{equation*}
(1-\tau) w+b=w_{h}^{*} . \tag{A1.8b}
\end{equation*}
$$

The labor supply equation (4') becomes now

$$
L=\frac{1}{2}(1-q)^{2}+q\left(1+m_{\ell}\right)+m_{h}=\frac{1}{2}(1-q)^{2}+q+m_{1},
$$

where $m_{1} \equiv q m_{\ell}+m_{h}$ is the labor supply of the migrants in efficiency units. The government's budget constraint (namely, equation (7)) becomes now:

$$
\begin{equation*}
b\left(1+m_{2}\right)=\tau(Y-K) \tag{A1.7}
\end{equation*}
$$

where $m_{2} \equiv m_{\ell}+m_{h}$ is the total number of low and high skill migrants. Finally, the other equations of the model, namely (1), (3), (5) and (6), remain intact.

We can solve equations (A1.8a) and (A1.8b) for $b$ and $w:^{1}$

$$
\begin{equation*}
b=\frac{w_{\ell}^{*}-q w_{h}^{*}}{1-q} \tag{A1.1}
\end{equation*}
$$

and

$$
\begin{equation*}
w=\frac{w_{h}^{*}-w_{\ell}^{*}}{(1-\tau)(1-q)} . \tag{A1.2}
\end{equation*}
$$

Substituting (A4.4 ) and (A4.1) into (A4.7) we get:

$$
\begin{align*}
& \left(\frac{w_{\ell}^{*}-q w_{h}^{*}}{1-q}\right)\left(1+m_{2}\right)  \tag{A1.3}\\
= & \tau\left\{F\left[K, \frac{1}{2}(1-q)^{2}+q+m_{1}\right]-K\right\} \equiv R\left(\tau, m_{1}\right),
\end{align*}
$$

where $R\left(\tau, m_{1}\right)$ is tax revenues. Substituting (A1.2) and (A1.4') into (1.5) yields:

$$
\begin{equation*}
w_{h}^{*}-w_{\ell}^{*}=(1-\tau)(1-q) F_{L}\left[K, \frac{1}{2}(1-q)^{2}+q+m_{1}\right] \tag{A1.5}
\end{equation*}
$$

The latter two equations (namely, (A1.3) and (A1.5)) can be solved for the labor supply $\left(m_{1}\right)$ and the number $\left(m_{2}\right)$ of the migrants as functions of the tax rate $(\tau)$. Total differentiation of (A1.5) with respect to $\tau$ yields:

$$
\frac{d m_{1}}{d \tau}=F_{L}\left[(1-\tau) F_{L L}\right]^{-1}<0
$$

because we assume that the marginal product of labor is diminishing (that is, $F$ is concave). Upon inspection of (A1.3) we can see that:

$$
\operatorname{sign}\left(\frac{d m_{2}}{d \tau}\right)=\operatorname{sign}\left(\frac{d R}{d \tau}\right)
$$

where $d R / d \tau=\partial R / \partial \tau+\left(\partial R / \partial m_{1}\right)\left(d m_{1} / d \tau\right)$. Suppose that "supply-side economics" does not prevail, that is $d R / d \tau>0$. (This is always true for small $\tau^{\prime} s$.) Then, $d m_{2} / d \tau>0$.

Thus, we have established that the labor supply of the migrants $\left(m_{1}\right)$ falls while their number $\left(m_{2}\right)$ rises, when the tax rate $(\tau)$ is raised. That is:

$$
\frac{d m_{1}}{d \tau} \equiv q \frac{d m_{\ell}}{d \tau}+\frac{d m_{h}}{d \tau}<0
$$

while

$$
\frac{d m_{2}}{d \tau} \equiv \frac{d m_{\ell}}{d \tau}+\frac{d m_{h}}{d \tau}>0
$$

This can happen, if, and only if, $d m_{\ell} / d \tau>0$ and $d m_{h} / d \tau<0$. Thus, more taxes and transfers attract more low-skill migrants but fewer high-skill migrants.

## Appendix 2: Migrant Vote

In this appendix we prove equation (25) and $\left(25^{\prime}\right)$.
Differentiating equation (22) with respect to $m$ implies that:

$$
B_{m}(\tau, m)= \begin{cases}\frac{w}{2}+b_{\tau m}(\tau, m) & \text { if } e_{M}<e^{*}  \tag{A2.1}\\ b_{\tau m}(\tau, m) & \text { if } e^{*}<e_{M}<1 \\ b_{\tau m}(\tau, m) & \text { if } e_{M}>1\end{cases}
$$

Using equation (16), we conclude that:

$$
\begin{equation*}
b_{\tau}(\tau, m)=\frac{w L+r K}{1+m}+\frac{\tau w}{1+m} \frac{\partial L}{\partial \tau} . \tag{A2.2}
\end{equation*}
$$

Differentiating equation (15) with respect to $\tau$ implies that:

$$
\begin{equation*}
\frac{\partial L}{\partial \tau}=\left(1-e^{*}-q\right) \frac{\partial e^{*}}{\partial \tau} \tag{A2.3}
\end{equation*}
$$

where $\frac{\partial e^{*}}{\partial \tau}$ is derived from equation (5.1).
Substituting equation (A2.3) into equation (A2.2) yields:

$$
\begin{equation*}
b_{\tau}(\tau, m)=\frac{w L+r K}{1+m}-\frac{\gamma \tau\left(1-e^{*}-q\right)}{(1+m)(1-\tau)^{2}} \tag{A2.4}
\end{equation*}
$$

Differentiate $b_{\tau}$ in equation (A2.4) with respect to $m$ to obtain:

$$
\begin{equation*}
b_{\tau m}(\tau, m)=-\frac{b_{\tau}(\tau, m)}{1+m}+\frac{w q}{1+m} \tag{A2.5}
\end{equation*}
$$

where use is made of equation (15) in order to obtain $\partial L / \partial m=q$.
Since $B\left(\tau_{o}(m), m\right)=0$, we conclude from equation (22) that:

$$
b_{\tau}\left(\tau_{o}(m), m\right)= \begin{cases}\frac{w(1-m)}{2}+r K & \text { if } e_{M}<e^{*}  \tag{A2.6}\\ w q+r K & \text { if } e^{*}<e_{M}<1 \\ w q & \text { if } e_{M}>1 .\end{cases}
$$

Substituting equation (A2.6) into equation (A2.5) yields:

$$
b_{\tau m}\left(\tau_{o}(m), m\right)= \begin{cases}\frac{w}{1+m}\left(-\frac{1-m}{2}+q\right)-\frac{r K}{1+m} & \text { if } e_{m}<e^{*}  \tag{A2.7}\\ -\frac{r K}{1+m} & \text { if } e^{*}<e_{M}<1 \\ 0 & \text { if } e_{M}>1\end{cases}
$$

Finally, combining equation (A2.7) with equation (A2.1), we conclude that:

$$
B_{m}\left(\tau_{o}(m), m\right)= \begin{cases}\frac{w(q+m)}{1+m}-\frac{r K}{1+m} & \text { if } e_{M}<e^{*}  \tag{A2.8}\\ -\frac{r K}{1+m} & \text { if } e^{*}<e_{M}<1 \\ 0 & \text { if } e_{M}>1\end{cases}
$$

This completes the derivation of (25).

Consider now the case where migrants are not entitled (or choose not) to vote. Then the ability index of the median voter is $e_{M}=\frac{1}{2}$, independently of $m$. In this case, a straightforward application of the same procedure yields:

$$
B_{m}\left(\tau_{o}(m), m\right)= \begin{cases}\frac{w}{1+m}\left(-\frac{1}{2}+q\right)-\frac{r K}{1+m} & \text { if } e_{M}<e^{*}  \tag{A2.9}\\ -\frac{r K}{1+m} & \text { if } e^{*}<e_{M}<1 \\ 0 & \text { if } e_{M}>1\end{cases}
$$

This completes the derivation of $\left(25^{\prime}\right)$.
Note also that when $e_{M}=\frac{1}{2}<e^{*}$, then $q<\frac{1}{2}$ (see equation (13)), which implies that $B_{m}<0$ in this case.

## FOOTNOTES

${ }^{1}$ Note from equation (A1.7) that positive $b$ and $\tau$ are possible in this case of migration of both low and high skill migrants only when the wage differential at the source country (that is, $\left.w_{h}^{*} / w_{\ell}^{*}\right)$ is lower than the wage differential at the destination country which is $q$.

Table 1. Free Migration and Income Distribution Policy: Taxes, Transfers and the Gains from Trade

| Pre-migration(1) |  | Post-Migration(2) |  | Gains from Trade |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\tau$ | $b / Y$ | $\tau$ | $b / Y$ |  |  |
| 0.35 | 0.2434 | 0.4024 | 0.1687 | 0.8748 | $(0.0646)$ |
| 0.40 | 0.2782 | 0.3921 | 0.1648 | 0.8771 | $(0.0478)$ |
| 0.45 | 0.3130 | 0.3902 | 0.1628 | 0.9009 | $(0.0341)$ |
| 0.50 | 0.3478 | 0.3798 | 0.1587 | 0.9062 | $(0.0167)$ |
| 0.55 | 0.3825 | 0.3737 | 0.1552 | 0.9261 | $(0.0011)$ |
| 0.60 | 0.4173 | 0.3737 | 0.1539 | 0.9517 | $(0.0116)$ |

$\tau=$ tax rate
$\mathrm{b}=$ demogrant
$\mathrm{m}=$ ratio of migrants to native-born individuals
$\mathrm{Y}=\mathrm{GDP}$
(1) exogenously given tax rate
(2) endogenous tax rate: tax rate is determined so as to restore post-migration disposable income of low-skilled individuals to its pre-migration level, for each tax rate shown in the pre-migration cell. For example, $\tau=0.4024$ is the endogenously determined tax rate corresponding to a post-migration disposable income of low skilled, which is equal to its pre-migration level at a pre-migration tax rate of 0.35 .


Figure 1: The Income Distribution Curve


Figure 2: The Effect of Migration on the Income Distribution among the Native -Born (With No Income Redistribution Policy) Notes: The parameter values are: $\mathrm{q}=0.5 ; \mathrm{K}=1$; $\mathrm{w}^{*}=0.95 \mathrm{qw}$ where w is the wage rate in the no-tax-transfer, no migration case; e is uniformly distributed over [ 0,1$]$; the production function is a Cobb-Douglas $\mathrm{F}(\mathrm{K}, \mathrm{L})=\mathrm{AK} \mathrm{L}^{\alpha} \mathrm{L}^{1-\alpha}$, with $\alpha=0.33$ and $\mathrm{A}=4.5$.


Figure 3: Free Migration: The Income Gain to the Native-Born


Figure 4: The Effect of Migration on the Income Distribution among the Native -Born (with an Income Distribution Policy)


Figure 5: Income Distribution and a Political Economy Equilibrium


[^0]:    * This paper draws on Chapters 4 and 5 in A. Razin and E. Sadka, Cross-Border Flows:

    Labor, Capital, Finance, Cambridge University Press, forthcoming.

