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POLLUTION AND RESOURCE EXTRACTION: DO THEY MATTER FOR THE DYNAMICS OF GROWTH?

George Economides
Apostolis Philippopoulos*

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CESifo
Poschingerstr. 5
81679 Munich
Germany
Phone: +49 (89) 9224-1410/1425
Fax: +49 (89) 9224-1409
http://www.CESifo.de

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Abstract

This paper shows that whether pollution occurs as a by-product of economic activity (which is supposed to be the case in DCs), or as resource extraction (which is supposed to be the case in LDCs), matters for the dynamics of the optimal growth-environment-policy link. The context is a dynamic general equilibrium model of endogenous growth, in which private agents treat natural resources as a public good and the government chooses second-best environmental policy. We show that resource extraction can lead to indeterminacy, i.e. many different equilibrium transition paths. This can partly explain the observed persistent differences in growth among LDCs with similar fundamentals and endowments.

Keywords: Pollution and resource extraction, growth, dynamics, second-best policy

JEL Classification: Q3, D9, H4

George Economides
Athens University of Economics and Business
76 Patission street
Athens 10434
Greece

Apostolis Philippopoulos
Athens University of Economics and Business
76 Patission street
Athens 10434
Greece
email: aphil@aueb.gr

I. INTRODUCTION

There are two main ways to model the economy-environment interaction.[1] One is to assume that pollution occurs as an inevitable by-product of economic activity; i.e. the environment receives the waste residuals of production and consumption activities. The alternative way is to assume that natural resources are extracted to be used as inputs in production, e.g. land and forestry. Although in both cases the public-good feature of the environment leads to resource misallocation and calls for government intervention, the role of natural resources is rather different. To quote Smulders [1995, p. 327], in the former case “nature acts as a sink”, while in the latter case “nature acts as a source”.

This paper compares the case in which environmental damage takes the form of pollution emissions to the case in which environmental damage takes the form of resource extraction used in production.[2] We show that the role of natural resources matters for the dynamics of the optimal growth-environment-policy link. Our study is motivated not only by academic curiosity, but also by the fact that the former case (i.e. when nature acts mainly as a sink) is believed to be the case of DCs, while the latter case (i.e. when nature acts mainly as a source) is believed to be the case of LDCs.[3]

We model two dynamic general equilibrium economies, which are exactly alike except from the role of natural resources. In the first economy (see section II), pollution is the by-product of final output produced. In the second economy (see section III), recently extracted natural resources are used as an input in private production. In all other respects, the two economies are exactly the same. For instance, in both economies, natural resources are treated as a pure public good by private agents (i.e. firms and households). Moreover, in both economies, the government taxes the polluting firms’ output to finance its clean-up policy. In other words, private activity degrades the environment, but economic policy improves it.

[1] For a survey paper in the context of economic growth, see Smulders [1995]. See also below.

[2] For simplicity, we study separately the two cases. As we explain below, our results for the latter case (i.e. resource extraction) do not change if we also add pollution as a by-product of economic activity.

[3] In DCs, pollution takes the form of carbon dioxide, disposal of garbage, sewage, etc. There is clear evidence that the world’s largest emitters are the rich countries, like the US, Japan and Germany (see e.g. Barrett [1998]). By contrast, in LDCs, the main problem is resource extraction. For instance, Lopez [1997] provides evidence of the quantitative importance of the environment as a factor of production in LDCs. He also argues that natural resources are common property owned by everyone and hence by no one. In other words, they have public good features (see also below).

Furthermore, in both economies, technology - at social level - is linear in capital as in the AK model.[4]

We work as follows (whatever the role of natural resources). We first solve for a competitive decentralized equilibrium, in which private agents take economic policy and natural resources as given.[5] Then, a benevolent government chooses economic policy, and the associated environmental quality, by acting as a Stackelberg leader vis-à-vis private agents. We solve for a long run equilibrium in which the economy can grow at a constant positive rate (this rate is known as Balanced Growth Path, BGP) without damaging the environment.

Our main results are as follows. Under reasonable restrictions on parameter values, there exists a unique BGP, irrespectively of the role of natural resources. However, the transitional dynamics depend crucially on whether natural resources are depleted via pollution or via resource extraction. In particular, when pollution occurs as a by-product of economic activity, the BGP is always locally determinate. That is, there is a unique way to reach the steady state. By contrast, when pollution takes the form of resource extraction that provides a positive externality in private production, the BGP is always locally indeterminate. That is, there can be an infinite number of equilibrium transition paths, each of which is consistent with a given initial condition and with convergence to the same, unique steady state. This is as in Benhabib and Farmer [1994], Benhabib and Perli [1994] and Benhabib and Gali [1995].[6]

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[4] We choose the AK model because it is the simplest model of endogenous growth (see below). Also, its linearity helps us to make our results more focused, i.e. multiplicity can arise even in a "linear" (at social level) model depending on the role of natural resources.

[5] That is, natural resources are treated as a pure public good by private agents. In the case of resource extraction used as an input in private production (see section III), this means that private firms take the economy's natural resources as given. Thus, natural resources increase the productivity of (chosen) private factor inputs. Obviously, this is only one way to model the publicness of natural inputs. An alternative way is to assume that each individual firm chooses its own resource extraction by paying a price and taking the resource extraction of other firms as given; then, in a (Nash) equilibrium, there is too much extraction. However, the important thing is the publicness of natural inputs (due to lack of well-defined property rights) and hence the presence of production externalities. The specific way we model production externalities is not important for what we study here (which is the possibility of indeterminacy, and how this is affected by the menu of externalities present). We therefore choose the simplest possible modeling and assume that recently extracted natural resources enter the private firms' production function as a positive externality.

[6] In the growth literature, there are two types of multiplicity (for a survey on multiplicity in macroeconomics, see Benhabib and Farmer [1998]): First, we can have multiple steady states. In that case, the equilibrium is usually unique, once the initial condition is given. Second, we can have indeterminacy. Here, we focus on indeterminacy and growth and, in particular, endogenous growth (for a survey, see Benhabib and Farmer [1998, section 6]). Note that indeterminacy is no mere intellectual curiosity. Benhabib and Gali [1995] show that the data are not in accord with the dynamics of a growth model with a unique path. Also, indeterminacy can occur for empirically realistic ranges of parameter values.

What causes indeterminacy under resource extraction? Two things: (a) natural inputs offer a positive production externality to individual firms; (b) an individual firm's decision to expand its activity depletes the economy's natural resources, and this provides a negative external effect upon all firms and households. It is the combination of these two public-good external effects (one beneficial and one detrimental) that generates multiplicity, here in the form of dynamic indeterminacy. By contrast, when environmental damage occurs as a by-product of economic activity, only the effect (b) is present. Without externalities in the production process, this is not enough to give indeterminacy. In their seminal work, Baumol and Oates [1988, chapter 8] have also shown that the number of equilibria increases with the number of externality-generating activities.[7]

What does indeterminacy mean? It means that countries with similar fundamentals can grow at completely different rates over time. Any of these different transition paths can be obtained as a self-fulfilling prophecy. Namely, economic agents' actions depend on their initial expectations about the future path of the economy, which in turn depends on economic agents' actions. Note that although the government acts as a Stackelberg leader vis-à-vis the private decentralized economy, it cannot resolve this expectations coordination failure, probably because there are too many externalities relative to policy instruments.

Therefore, our results can partly explain the growth divergence within the group of LDCs with similar endowments. Recall that the data on per capita growth rates reveal, not only that LDCs differ from DCs, but also that it is much harder to demonstrate convergence among countries within the group of LDCs than within the group of DCs.[8] As Benhabib and Farmer [1994], Benhabib and Perli [1994] and Benhabib and Gali [1995] have emphasized, several explanations are possible, and the possibility of indeterminacy in models of endogenous growth is one of them. Here, we have explored the possibility that the role of natural resources can be one factor

[7] Our results are also similar to those in the recent endogenous growth literature. In this literature, the same mechanism (i.e. production externalities) that can generate endogenous growth may also open the door for indeterminacy. In particular, Benhabib and Perli [1994] and Benhabib and Farmer [1994] show that production externalities must be coupled with another condition (on labor elasticity) in order to get indeterminacy.

[8] It is widely accepted that differences in fundamentals (e.g. initial conditions and endowments, preferences, technical progress, demographic factors, social cohesiveness, education, financial markets, shocks, exogenously set government policies, etc) can offer an explanation of why DCs grow at different rates from LDCs; see e.g. Azariadis [1993] and Barro and Sala-i-Martin [1995]. It is persistent differences in growth among LDCs with seemingly similar fundamentals that perplexes researchers.

that can generate indeterminacy and hence contribute to the explanation of the observed patterns of growth.

What is the related environmental economics literature? The link between economic growth, natural resources and environmental policy has been the subject of a rich and still growing literature (for survey papers in the context of growth, see Kolstad and Krautkraemer [1993] and Smulders [1995]). John and Pecchenino [1994], Ligthart and van der Ploeg [1994], Xepapadeas [1997] and others have modeled pollution as a by-product of economic activity. Tahvonen and Kuuluvainen [1993], Lopez [1994, 1997], Bovenberg and Smulders [1995], Nielsen, Pedersen and Sorensen [1995] and others have modeled pollution as an input in production. However, most of these papers do not focus on the joint dynamics of endogenous growth, natural resources and second-best policy. By contrast, here we investigate explicitly the long-run properties and transitional dynamics of the optimal growth-environment-policy link. The focus is on the possibility of multiple steady states and dynamic indeterminacy, and how this possibility depends on the exact role of natural resources.

The rest of the paper is as follows. Section II studies the case in which pollution is a by-product of production. Section III studies the case in which natural resources have also a productive value. Section IV closes the paper. Proofs are gathered in an Appendix.

II. POLLUTION AS A BY-PRODUCT OF OUTPUT

Consider an economy populated by private agents (a representative household and a representative firm) and a government. Households purchase goods, work and save in the form of capital. They get direct utility from consumption and the stock of natural resources. Firms produce output by using a linear, AK-type technology. In doing so, they pollute the environment. That is, pollution increases with final output produced.[9] The government finances its clean-up policy by taxing the polluting firms' output.[10] We assume continuous time, infinite horizons and perfect foresight.

[9] Our main results do not change if pollution is also a by-product of consumption.
[10] Our main results do not change if taxes are imposed on households.

Households

The representative infinite-lived household maximizes intertemporal utility:

$$\int_0^\infty [u(c,N)]e^{-\rho t}\,dt \tag{1a}$$

where c is private consumption, N is the stock of natural resources and the parameter $\rho > 0$ is the rate of time preference. The instantaneous utility function $u(.)$ is increasing and concave in its two arguments, and also satisfies the Inada conditions. For simplicity, we assume that $u(.)$ is additively separable and logarithmic:

$$u(c,N) = \log c + \nu \log N \tag{1b}$$

where the parameter $\nu \geq 0$ is the weight given to environmental quality relative to private consumption.

Households save in the form of capital, k. When they rent out k to firms, they receive a rate of return, r. They also supply inelastically one unit of labor services and get a labor income, w. Further, they receive profits, π. Then, the budget constraint of the representative household is:

$$\dot{k} + c = rk + w + \pi \tag{2a}$$

where a dot over a variable denotes time derivative. The initial stock, k_0, is given.

The household acts competitively by taking prices, policy and natural resources as given.[11] The control variables are c and k, so that the first-order conditions for a maximum are equation (2a) as well as the familiar Euler condition:

$$\dot{c} = (r - \rho)c \tag{2b}$$

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[11] Throughput the paper, we assume that private agents (i.e. households and firms) are quantity-takers, when we consider public goods (e.g. N). This is like assuming a large number of market participants so that no agent feels that he or she can influence the amount of public good that is made available. Alternatively, we could assume that private agents take the quantities chosen by others as given when making their own choices. This would not affect our main results (see also footnote 5 above). See e.g. Oakland [1987] for various categories of public goods.

Firms

The production function takes a linear form, as in the AK model:

$$y = Ak \tag{3}$$

where $A > 0$ is a parameter.[12]

Net profits of the representative firm are:

$$\pi = (1-\theta)Ak - rk - w \tag{4}$$

where $0 \leq \theta < 1$ is a proportional tax rate on firms' output.

The firm acts competitively by taking prices, policy and natural resources as given. This is a static problem. The control variable is k, so that the first-order condition for a maximum is simply:

$$r = (1-\theta)A \tag{5a}$$

which equates the rate of return to the after-tax marginal product of capital.

Using (5a) into (4), and for zero profits, we get:

$$w = 0 \tag{5b}$$

which is a well-known result in the AK model. Namely, in this model, all realized income goes to capital.[13]

The motion of natural resources

The stock of natural resources, N, evolves over time according to:

$$\dot{N} = \delta N - p + g \tag{6}$$

[12] The firm's problem is written in labor intensive form. Then, when the labor market clears, equilibrium employment is one unit of labor services. See Barro and Sala-i-Martin [1995, chapter 4].

[13] That is, if capital is paid its realized marginal product, there is nothing left for labor. See e.g. Barro and Sala-i-Martin [1995, pp. 141-2].

where the parameter $d \geq 0$ is the rate of regeneration of natural resources, p is pollution and g is clean-up policy. The initial stock, N_0, is given.

Government expenditures on clean-up policy, g, are financed by taxes on polluting firms' output, qy.[14] The balanced budget constraint of the government is:

$$g = qy \tag{7}$$

Competitive decentralized equilibrium (given economic policy)

We now characterize a Competitive Decentralized Equilibrium (CDE) for any feasible economic policy.

We start by modeling pollution. In equilibrium, pollution, p, is a by-product of final output produced, y. Specifically, we assume:

$$p = y \tag{8}$$

that is, for simplicity, one unit of output generates one unit of pollution.

We now turn to private agents. Using (5a) and (5b), equations (2b) and (2a) become respectively:

$$\dot{c} = [(1-q)A - r]c \tag{9a}$$

$$\dot{k} = (1-q)Ak - c \tag{9b}$$

which are the private agents' optimal rules for consumption and savings in a CDE.

Also, using (3), (7) and (8) into (6), we get:

$$\dot{N} = dN - (1-q)Ak \tag{9c}$$

which is the motion of natural resources in a CDE.

[14] For simplicity, we assume that one unit of resources (i.e. tax revenues) used for clean-up policy can improve environmental quality by one unit.

To sum up, equations (9a), (9b) and (9c) summarize a Competitive Decentralized Equilibrium (CDE). In this equilibrium: (i) households maximize utility and firms maximize profits; (ii) all constraints are satisfied and all markets clear. In this equilibrium, individual firms have failed to internalize the adverse external effect of their output decisions on the economy's natural resources (and that this constitutes a disutility to consumers). Also, this equilibrium holds for given initial conditions and any feasible economic policy, where the latter is summarized by the tax rate on polluting firms' output, q. We now move on to endogenize the choice of q. By choosing q, the government will attempt to control externalities and also raise funds to finance clean-up policy.[15]

Optimal policy and growth in general equilibrium

We solve for optimal tax policy, q. We endogenize q by assuming that the government is benevolent and plays Stackelberg *vis-a-vis* private agents. Thus, the government chooses the paths of q, c, k, N to maximize (1a)-(1b) subject to the Competitive Decentralized Equilibrium (9a), (9b) and (9c). The current-value Hamiltonian, H , of this problem is:[16]

$$H \equiv \log c + n \log N + lc[(1-q)A - r] + g[(1-q)Ak - c] + m[dN - (1-q)Ak] \qquad (10)$$

where l, g and m are the multipliers associated with (9a), (9b) and (9c) respectively. That is, l is the social value of private marginal utility of assets, g is the social price of capital and m is the social price of natural resources.

The first-order conditions with respect to q, c, l, k, g, N, m are respectively:

$$lc + gk = mk \qquad (11a)$$

$$\dot{l} = rl - \frac{1}{c} - l[(1-q)A - r] + g \qquad (11b)$$

$$\dot{c} = c[(1-q)A - r] \qquad (11c)$$

$$\dot{g} = rg - (1-q)Ag + (1-q)Am \qquad (11d)$$

[15] In addition to market failures associated with externalities, tax policy is distortionary.

$$\dot{k} = (1-q)Ak - c \tag{11e}$$

$$\dot{m} = rm - \frac{n}{N} - dm \tag{11f}$$

$$\dot{N} = dN - (1-q)Ak \tag{11g}$$

These necessary conditions are completed with the addition of a transversality condition that guarantees utility is bounded. A sufficient condition for this to hold is:

$$[(1-q)A - r] + d < r \tag{11h}$$

so that the growth rate of consumption, $[(1-q)A - r]$, plus the rate of regeneration of natural resources, d, is less than the rate of time preference, r.[17]

Following usual practice, we transform the variables to facilitate analytical tractability. Let define $z \equiv \frac{c}{k}$, $y \equiv mk$ and $f \equiv mN$. Then, Appendix A shows that the dynamics of (11a)-(11g) are equivalent to the dynamics of (12a)-(12d) below:

$$\dot{z} = (z - r)z \tag{12a}$$

$$\dot{y} = \left[(1-q)A - z + r - \frac{n}{f} - d\right]y \tag{12b}$$

$$\dot{f} = \left[r - \frac{n}{f} - \frac{(1-q)Ay}{f}\right]f \tag{12c}$$

$$\left[z + \frac{n}{f} + d\right]y = 1 \tag{12d}$$

where (12a)-(12d) constitute a system in z, y, f, q. Since (12d) is static, the dynamics of q follow from the dynamics of z, y, f (see also below).

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[16] We assume commitment technologies on behalf of the government so that policies are chosen once-and-for-all. Thus, we do not study time-consistency issues.

[17] If $l \geq 0, g \geq 0, m \geq 0$, and since the objective function and the constraints in (10) are concave in q, c, k, N, the necessary conditions are also sufficient for optimality.

Long-run equilibrium

We now study the long-run properties of (12a)-(12d). The steady state is characterized by $\dot{z} = \dot{y} = \dot{f} = 0$. In this steady state, consumption, capital and natural resources can grow at the same constant positive rate.[18] This is typical of AK-type endogenous growth models, in which all the per capita variables grow at the same rate.[19] Hence, this is a Balanced Growth Path (BGP). It is also a sustainable BGP.[20]

Let us denote the steady state values of (z, y, f, q) by $(\tilde{z}, \tilde{y}, \tilde{f}, \tilde{q})$. The rest of this subsection solves for $(\tilde{z}, \tilde{y}, \tilde{f}, \tilde{q})$ and discusses long-run equilibrium properties. In particular, Appendix B shows:

PROPOSITION 1: If the parameter values satisfy the following restrictions:

$$A > r + d \qquad (13a)$$

$$r > 2d \qquad (13b)$$

$$2nd > r - d \qquad (13c)$$

there exists a unique well-defined long-run pollution tax rate, $\tilde{q}$*, which lies in the region* $0 < 1 - \frac{r+d}{A} < \tilde{q} < 1 - \frac{r}{A} < 1$ *and is a solution to:*

$$n[r + A(1-\tilde{q})][r + d - A(1-\tilde{q})] = A(1-\tilde{q})[A(1-\tilde{q}) - d] \qquad (13d)$$

This tax rate supports a unique well-defined steady state in which consumption, capital and natural resources can grow at the same constant positive rate. Hence, the steady state is a sustainable Balanced Growth Path (BGP).

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[18] Since $z \equiv \frac{c}{k}$, $\dot{z} = 0$ implies that c and k grow at the same rate by following (11c) and (11e) respectively, i.e. $\frac{\dot{c}}{c} = \frac{\dot{k}}{k}$. Also, since $y \equiv mk$ and $f \equiv mN$, $\dot{y} = \dot{f} = 0$ implies $\frac{\dot{k}}{k} = \frac{\dot{N}}{N}$. Therefore, c, k and N grow at the same rate.

[19] See also e.g. Barro and Sala-i-Martin [1995, p. 40]. One could argue that there are natural resources that cannot "grow" (e.g. minerals and fuel). However, there are also natural resources that are renewable, in the sense that they are capable of growth, especially if one takes into account human intervention and maintenance policy. Popular examples include agricultural land, fishery and forestry resources (see e.g. Munro and Scott [1985]). Also, we have used the AK model for algebraic simplicity and we do not think that our main results depend on this particular model specification.

[20] In particular, at this BGP: (i) the economy can grow without damaging the environment; (ii) natural resources, N, are valued positively, i.e. $m > 0$; (iii) the transversality condition (11h) holds.

Conditions (13a)-(13c) are jointly sufficient for a well-defined and unique long-run equilibrium to exist. The algebra is in Appendix B. Here, we just discuss the results. Condition (13a) requires the productivity of private capital, A, to be higher than the rate of time preference, r, plus the regeneration rate of natural resources, d. Although conditions of this type are familiar in endogenous growth models, here we require a stronger condition than usually (see e.g. Barro and Sala-i-Martin [1995, p. 142]) because our economy must also devote resources to clean-up policy. Condition (13b) guarantees that the transversality condition (11h) holds and so utility is bounded. It means that if r is not high enough, households are over-saving and the transversality condition is violated; utility would increase if current consumption were higher (see e.g. Barro and Sala-i-Martin [1995, chapters 2 and 4]). Finally, as Appendix B shows, condition (13c) is required for existence. It states that existence obtains more easily when the rate of regeneration of natural resources is high (i.e. a high d helps existence), we care about the environment/public good (i.e. a high n helps existence) and we care about the future (i.e. a low r helps existence).

Total differentiation of (13d) implies the comparative static results:[21]

$$\tilde{q} = q\left(\overset{-}{d}, \overset{-}{n}, \overset{-}{r}, \overset{+}{A}\right) \tag{14}$$

That is, (i) when natural resources regenerate themselves (i.e. d is high), the need for pollution taxes is smaller; (ii) when private agents themselves value environmental standards (i.e. n is high), the need for environmental policy is less acute; (iii) the more we care about the future (i.e. the lower is r), the higher the chosen pollution tax rate; (iv) when private capital is productive (i.e. A is high), we can afford higher pollution taxes. These are intuitive results for the long-run pollution tax rate, $\tilde{q}$.[22]

In turn, the properties of the BGP (i.e. the common rate at which consumption, capital and natural resources can grow in the long-run) follow directly from the properties of the tax rate, $\tilde{q}$.[23] Specifically, the properties of the BGP are symmetrically opposite from those of $\tilde{q}$. That is, a lower (resp. higher) tax rate leads

[21] Signs above parameters give equilibrium properties.
[22] Also, the comparative static properties are in logical accordance with the results for existence above.
[23] See footnote 18 and equation (11c) above.

to higher (resp. lower) economic growth and improving (resp. deteriorating) environmental quality. Intuitively, lower tax rates lead to higher capital accumulation, higher economic growth and therefore larger tax bases, which lead to a greater ability to engage in clean-up policy. This improves environmental quality, despite the adverse effect of higher economic growth and pollution.[24] These results are consistent with the main result of the literature: economies that achieve a sustained growth path will ultimately be characterized by improving environmental quality (see John and Pecchenino [1994] and Economides and Philippopoulos [1999]).

Transitional dynamics

We now study the transitional dynamics of (12a)-(12c). We study stability properties around steady state. Linearizing (12a)-(12c) around the steady state solution $\left(\tilde{z}, \tilde{y}, \tilde{f}, \tilde{q}\right)$ given in Proposition 1 above implies that the local dynamics are approximated by the linear system:

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} r & 0 & 0 \\ -\tilde{y} & 0 & \dfrac{n\tilde{y}}{\tilde{f}^{2}} \\ 0 & -(1-\tilde{q})A & r \end{bmatrix} \begin{bmatrix} z \\ y \\ f \end{bmatrix} \qquad (15)$$

where the elements of the Jacobian matrix have been evaluated at the steady state.

The determinant of the Jacobian matrix in (15) is $\det(J) = r(1-\tilde{q})A\dfrac{n\tilde{y}}{\tilde{f}^{2}}$. This is positive. Hence, if there are three roots, there are two possibilities: either there are three positive roots, or one positive and two negative roots. Since all three variables (z, y, f) are jump variables, the former possibility (i.e. three positive roots) implies local determinacy, while the latter possibility (i.e. one positive and two negative roots) implies local indeterminacy. As we show in Appendix C by applying

[24] In a more general model than the one we use here, we would also have a short-run effect that works in an opposite direction from our long-run effect on growth. Specifically, in the short-run, capital tax bases are inelastic so that a lower tax rate would push tax revenues down. If the short-run effect dominates, then lower tax rates can lead to less clean-up policy and worse environmental quality. For details, see Economides and Philippopoulos [1999].

Descartes' Theorem, the characteristic equation of the Jacobian matrix excludes the latter possibility. Hence, there is local determinacy.

What does it mean? Without predetermined variables, determinacy means that the jump variables jump immediately, and in a unique way, to take their long-run values and stay there (until the system is disturbed in some way). There are no transitional dynamics and the saddle-path solution is equivalent to the steady state, as in the basic AK model (see e.g. Barro and Sala-i-Martin [1995, chapter 4]).

These results are summarized by the following proposition:

PROPOSITION 2: Under the conditions in Proposition 1, the unique long-run pollution tax rate and the associated BGP are locally determinate. That is, when pollution occurs as a by-product of final output produced, the economy jumps immediately and in a unique way to its steady state.

III. RESOURCE EXTRACTION AS AN INPUT OF PRODUCTION

In this section, the environment has also a productive value. In particular, recently extracted natural resources are used as an input in private production. Also, given the public-good feature of the environment, we assume that economy-wide extracted natural resources enter the private firms' production function as a positive externality (see footnote 5 above). Everything else is the same as in the previous section.

Equations (1)-(2) describing the household's behavior do not change. However, the production technology and the firm's optimization problem do change.

Firms

The firm's production function changes from (3) to:

$$y = Ap^{1-a} k^{a} \tag{16}$$

where p denotes now resource extraction and $0 < a \leq 1$ is a parameter. Extraction of natural resources, p, is complementary to private inputs, k, in the sense that an increase in p raises the marginal product of k. Note that for fixed p, the firm faces diminishing returns to k. Also, note that if a is one, we go back to the model of the previous section.

Net profits of the representative firm change from (4) to:

$$p = (1-q)Ap^{1-a}k^{a} - rk - w \tag{17}$$

The firm acts competitively by taking prices, policy and natural resources as given. The control variable is k, so that the first-order conditions, which also imply zero profits, change from (5a) and (5b) to:

$$r = a(1-q)Ap^{1-a}k^{a-1} \tag{18a}$$

$$w = (1-a)(1-q)Ap^{1-a}k^{a} \tag{18b}$$

which equate rates of return to after-tax marginal products.

Competitive decentralized equilibrium (given economic policy)

We now characterize a Competitive Decentralized Equilibrium (CDE) for any feasible economic policy.

Equations (6)-(8) still hold. However, although the algebraic forms of (6) and (8) are as in the previous section, the working of the model is different because now pollution plays a dual role. Namely, on the one hand, private production uses natural resources, and this deteriorates the environment; on the other hand, resource extraction provides productive services. Thus, equation (8) now reads that, in equilibrium, resource extraction is proportional to output produced. Specifically, the assumption is that one unit of output produced requires one unit of natural resources. Note that this modeling makes our results directly comparable to those in the previous section. It is also consistent with modeling in the relevant endogenous growth theory.[25]

Using (8) into (16), we have for output, y, in a CDE:

[25] In the theory of endogenous growth with production externalities, the externality is usually provided by an aggregate variable (e.g. capital or output). Here, it is provided by resource extraction, which - in equilibrium - is proportional to final output produced.

$$p = y = \hat{A}k \tag{19a}$$

where $\hat{A} \equiv A^{\frac{1}{a}}$. Thus, economy-wide output is linear in capital. In other words, while there are decreasing returns to scale at the firm's level (see (16)), there are constant returns to scale at social level (see (19a)).[26]

In turn, using (19a) into (18a) and (18b), we get factor returns in a CDE:

$$r = a(1-q)\hat{A} \tag{19b}$$

$$w = (1-a)(1-q)\hat{A}k \tag{19c}$$

Then, using (19b) and (19c), equations (2b) and (2a) become respectively:

$$\dot{c} = [a(1-q)\hat{A} - r]c \tag{20a}$$

$$\dot{k} = (1-q)\hat{A}k - c \tag{20b}$$

which are the private agents' optimal rules for consumption and savings in a CDE. Observe that in (20a), the decentralized net rate of capital return, which is what drives capital accumulation in equilibrium, is $a(1-q)\hat{A}$. By contrast, (19a) implies that the social net rate of capital return is $(1-q)\hat{A}$, which is higher than $a(1-q)\hat{A}$ because $0 < a < 1$. That is, in the presence of production externalities that are not internalized by private agents, the economy's growth rate is inefficiently low.

Also, using (7), (8) and (19a) into (6), we get:

$$\dot{N} = dN - (1-q)\hat{A}k \tag{20c}$$

which is the motion of natural resources in a CDE.[27]

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[26] This is a usual result in endogenous growth models with production externalities (see e.g. Barro and Sala-i-Martin [1995, chapter 4] and Jones and Manuelli [1997]).

[27] Recall that, in this section, pollution takes the form of resource extraction. We could easily have pollution both as a by-product of final output produced, denoted by p_1, and as resource extraction, denoted by p_2. In that case, both activities diminish the economy's natural resources, i.e.

To sum up, equations (20a), (20b) and (20c) summarize a Competitive Decentralized Equilibrium (CDE). In this equilibrium: (i) households maximize utility and firms maximize profits; (ii) all constraints are satisfied and all markets clear. In this equilibrium, individual firms have failed to internalize the adverse external effect of their output decisions on the economy's natural resources, as well as the positive external effect of the economy's natural resources on their own production activity. Also, this equilibrium holds for given initial conditions and any feasible economic policy, where the latter is summarized by the tax rate, q. As in the previous section, we now endogenize the choice of q.

Optimal policy and growth in general equilibrium

We solve for optimal tax policy, q. We again endogenize q by assuming that the government is benevolent and plays Stackelberg *vis-a-vis* private agents. Thus, the government chooses the paths of q, c, k, N to maximize (1a)-(1b) subject to the Competitive Decentralized Equilibrium (20a), (20b) and (20c). The current-value Hamiltonian, H, of this problem is:

$$H \equiv \log c + n \log N + lc\left[a(1-q)\hat{A} - r\right] + g\left[(1-q)\hat{A}k - c\right] + m\left[dN - (1-q)\hat{A}k\right] \qquad (21)$$

where l, g and m are new multipliers associated with (20a), (20b) and (20c) respectively.

The first-order conditions with respect to q, c, l, k, g, N, m are respectively:

$$alc + gk = mk \qquad (22a)$$

$$\dot{l} = rl - \frac{1}{c} - l\left[a(1-q)\hat{A} - r\right] + g \qquad (22b)$$

$$\dot{c} = c\left[a(1-q)\hat{A} - r\right] \qquad (22c)$$

$$\dot{g} = rg - (1-q)\hat{A}g + (1-q)\hat{A}m \qquad (22d)$$

$\dot{N} = dN - p_1 - p_2 + g$. Then, our structure (specifically, that one unit of output generates one unit of pollution, and that one unit of output requires one unit of natural resources) implies that in equilibrium $p_1 = p_2 = y$, so that - instead of (20c) - we would have $\dot{N} = dN - 2(1-q)\hat{A}k$. This does not change the qualitative results of this section.

$$\dot{k} = (1-q)\hat{A}k - c \tag{22e}$$

$$\dot{m} = rm - \frac{n}{N} - dm \tag{22f}$$

$$\dot{N} = dN - (1-q)\hat{A}k \tag{22g}$$

Also, the transversality condition holds if:

$$a\left[(1-q)\hat{A} - r\right] + d < r \tag{22h}$$

Again, we transform the variables to facilitate analytical tractability. Let define $z \equiv \frac{c}{k}$, $y \equiv mk$, $f \equiv mN$ and $c \equiv gk$. Appendix D shows that the dynamics of (22a)-(22g) are equivalent to the dynamics of (23a)-(23e) below:

$$\dot{z} = [z - r - (1-a)(1-q)\hat{A}]z \tag{23a}$$

$$\dot{y} = \left[(1-q)\hat{A} - z + r - \frac{n}{f} - d\right]y \tag{23b}$$

$$\dot{f} = \left[r - \frac{n}{f} - \frac{(1-q)\hat{A}y}{f}\right]f \tag{23c}$$

$$\dot{c} = \left[r - z + \frac{(1-q)\hat{A}y}{c}\right]c \tag{23d}$$

$$\left[z + \frac{n}{f} + d\right]y = a + (1-a)zc \tag{23e}$$

where (23a)-(23e) constitute a system in z, y, f, c, q. Since (23e) is static, the dynamics of q follow from the dynamics of z, y, f, c (see also below). Comparison of (12a)-(12d) with (23a)-(23e) reveals that the dimensionality of the dynamic system increases by one when pollution plays the role of an input of production.

Long-run equilibrium

We now investigate the long-run properties of (23a)-(23e). The steady state is characterized by $\dot{z} = \dot{y} = \dot{f} = \dot{c} = 0$. As in the previous section, in this steady state, cosumption, capital and natural resources can grow at the same constant positive rate. Thus, it is again a sustainable Balanced Growth Path (BGP).

Let us denote the new steady state values of (z, y, f, c, q) by $(\tilde{z}, \tilde{y}, \tilde{f}, \tilde{c}, \tilde{q})$. The rest of this subsection solves for $(\tilde{z}, \tilde{y}, \tilde{f}, \tilde{c}, \tilde{q})$. Appendix E shows:

PROPOSITION 3: If the parameter values satisfy the following restrictions:

$$a\hat{A} > 2d \qquad (24a)$$

$$\frac{3d}{2} < r < 2d \qquad (24b)$$

$$n(2d - r)d > r(r - d) \qquad (24c)$$

there exists a unique well-defined long-run pollution tax rate, $\tilde{q}$*, which lies in the region* $0 < 1 - \frac{2d}{a\hat{A}} < \tilde{q} < 1 - \frac{r}{a\hat{A}} < 1$ *and is a solution to:*

$$n[2d - a(1-\tilde{q})\hat{A}][r + d - a(1-\tilde{q})\hat{A}] = a(1-\tilde{q})\hat{A}[a(1-\tilde{q})\hat{A} - d] \qquad (24d)$$

This tax rate supports a unique well-defined steady state in which consumption, capital and natural resources can grow at the same constant positive rate. Hence, the steady state is a sustainable Balanced Growth Path (BGP).

Conditions (24a)-(24c) are jointly sufficient for a well-defined and unique long-run equilibrium to exist. Condition (24a) is similar to (13a); it requires the "effective" productivity of private capital, $a\hat{A}$, to be high enough. Condition (24b) differs from (13b), since now there is also an upper boundary on r. This is because, when resource extraction provides positive production externalities to private firms, there must be a balance between a too low r (that typically leads to over-saving and violates the transversality condition) and a too high r (that leads to over-consumption, over-production and over-use of natural resources in the short-run). Condition (24c) is required for the existence of a long-run equilibrium and has the same qualitative properties with (13c) above.

Total differentiation of (24d) implies:

$$\tilde{q} = q\left(\overset{-}{d}, \overset{-}{n}, \overset{-}{r}, \overset{+}{A}\right) \tag{25}$$

so that the signs are as in (14) above. In turn, the properties of the long-run growth rate, *BGP*, are also as in the previous section.[28] Therefore, the qualitative properties of the long run equilibrium (i.e. conditions for existence and comparative static results) are the same irrespectively of the role of natural resources.

Transitional dynamics

We now study the transitional dynamics of (23a)-(23d). As above, we study stability properties around steady state. Linearizing (23a)-(23d) around the steady state solution for $\left(\tilde{z}, \tilde{y}, \tilde{f}, \tilde{c}, \tilde{q}\right)$ given in Appendix E implies that the local dynamics are approximated by the linear system:

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{j} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} z & 0 & 0 & 0 \\ -\tilde{y} & 0 & \dfrac{n\tilde{y}}{\tilde{f}^2} & 0 \\ 0 & -(1-\tilde{q})\hat{A} & r & 0 \\ -\tilde{c} & (1-\tilde{q})\hat{A} & 0 & -(\tilde{z}-r) \end{bmatrix} \begin{bmatrix} z \\ y \\ j \\ c \end{bmatrix} \tag{26}$$

where the elements of the Jacobian matrix have been evaluated at the steady state.

The determinant of the Jacobian matrix in (26) is $\det(J) = -\tilde{z}(1-a)[(1-\tilde{q})\hat{A}]^2 \dfrac{n\tilde{y}}{\tilde{f}^2}$. This is negative. Hence, if there are four roots, there are two possibilities: Either there are three positive and one negative root. Or there is one positive and three negative roots. In either case, since all four variables (z, y, f, c) are jump variables, the number of stable roots is higher than the number of predetermined variables (i.e. zero) so that there is local indeterminacy.

These results are summarized by the following proposition:

PROPOSITION 4: Under the conditions in Proposition 3, the unique long-run pollution tax rate and the associated BGP are locally indeterminate. That is, when

[28] That is, as equation (22c) shows, the *BGP* is decreasing in $\tilde{q}$.

pollution takes the form of resource extraction that provides production externalities to private firms, there exists an infinite number of equilibrium trajectories, each of which is consistent with a given initial condition and with convergence to the unique steady state.

As argued in the Introduction, it is the combination of the two public-good externalities (one positive and one negative) that generates indeterminacy. Indeterminacy can partly explain why economies with similar fundamentals can grow at completely different rates and enjoy completely different levels of welfare over time. Only in the long run, will these economies converge to the same growth rate of consumption, output and natural resources (although not to the same level).

As e.g. Benhabib and Farmer [1994], Benhabib and Perli [1994] and Benhabib and Gali [1995] explain, indeterminacy implies that there can be many (infinite) pairs of expectations/outcomes over time, each of which is consistent with optimizing behavior on behalf of private agents and the government, market clearing, perfect foresight, given initial conditions and convergence to a single steady state. Any of these different equilibrium transition paths can be obtained as a self-fulfilling prophecy. Namely, economic agents' actions depend on their initial expectations about the future path of the economy, which in turn depends on economic agents' actions. Thus, here it is not the initial conditions that dictate the long-run outcome. Rather it is the initial choice of the jump variables (z, y, f, c), which determines which of the transition paths the economy will follow.

Therefore, there is an expectations coordination failure associated with multiplicity of the equilibrium transition path. It is important to note that, although the government acts as a Stackelberg leader, it cannot coordinate expectations and move the economy into a good equilibrium. We think that this happens because, in our decentralized economy, there are more externalities than policy instruments. Specifically, in the model developed in section III, there are two externalities (one beneficial and one detrimental) and only one policy instrument, which is also distortionary (i.e. the income tax rate). Rodrik [1996] also gets multiplicity in a model of specialization patterns. When he discusses mechanisms that may help the government to select a good equilibrium, he basically seems to presuppose the availability of sufficient policy instruments.

VI. CONCLUSIONS AND EXTENSIONS

We have investigated how the two main ways of modeling natural resources (i.e. pollution as a by-product of economic activity and as resource extraction) affect the long-run properties and transitional dynamics of endogenous growth, natural resources and second-best policy. This is when private agents treat the environment as a public good. Our focus has been on the possibility of multiple steady states and dynamic indeterminacy, and how this possibility is affected by the role of natural resources.

We close with two possible extensions. First, we can study the strategic interaction of two economies, in one of which pollution occurs as a by-product of economic activity, while in the other environmental damage takes the form of resource extraction used in production. This can be thought of as being a game between a DC and a LDC respectively, when environmental quality is a global, public good. A second extension could be to consider different policy instruments. Here, we have studied (second-best) taxes on polluting firms' output. Quantitative controls (e.g. pollution targets and tradable pollution permits) are alternative policy instruments, which seem to be particularly popular nowadays. We leave these extensions for future work.

APPENDICES

APPENDIX A: From equations (11a)-(11g) to equations (12a)-(12d)

Taking logs on both sides of (11a) and differentiating with respect to time, we get:

$$\frac{\dot{l}c + l\dot{c} + \dot{g}k + g\dot{k}}{lc + gk} = \frac{1}{mk}\left(\dot{m}k + m\dot{k}\right) \tag{A.1}$$

Substituting (11b), (11c), (11d), (11e) and (11f) for the rates of growth of l, c, g, k, m respectively into (A.1), we obtain:

$$1 = mc + dmk + \frac{nk}{N} \tag{A.2}$$

If $z \equiv \frac{c}{k}$, (11c) and (11e) give (12a) in the text. Also, if $y \equiv mk$ and $f \equiv mN$, (11e)-(11g) give (12b) and (12c) in the text. Finally, (A.2) is (12d) in the text.

APPENDIX B: Proof of Proposition 1

Setting (12a) equal to zero, the solution for $\tilde{z}$ is:

$$\tilde{z} = r \tag{B.1}$$

Setting (12b) equal to zero and using (B.1), we get for $\tilde{f}$:

$$\tilde{f} = \frac{n}{A(1-\tilde{q}) - d} \tag{B.2}$$

Setting (12c) equal to zero and using (B.1)-(B.2), we get for $\tilde{y}$:

$$\tilde{y} = \frac{n[r + d - A(1-\tilde{q})]}{(1-\tilde{q})A[A(1-\tilde{q}) - d]} \tag{B.3}$$

Then, using (B.1)-(B.3) into (12d), we get:

$$n[r + A(1-\tilde{q})][r + d - A(1-\tilde{q})] = A(1-\tilde{q})[A(1-\tilde{q}) - d] \tag{B.4}$$

which is (13d) in the text. This is a quadratic equation in $\tilde{q}$ only. Once we solve (B.4) for $\tilde{q}$, (B.2) and (B.3) will give $\tilde{f}$ and $\tilde{y}$. So the main task is to solve (B.4) for $0 \le \tilde{q} < 1$ and check whether the solution is well-defined.

We work in steps. In the first step, we specify the region in which a well-defined solution (if any) for $\tilde{q}$ should lie. A well-defined solution requires: (i)

$(1-\tilde{q})A-r>0$, i.e. $\tilde{q}<1-\frac{r}{A}$. This is required for long-term growth. (ii) $(1-\tilde{q})A+d<2r$, i.e. $1-\frac{2r-d}{A}<\tilde{q}$. This is required for the transversality condition (11h) to hold. (iii) $r+d-(1-\tilde{q})A>0$, i.e. $1-\frac{r+d}{A}<\tilde{q}$. This follows from inspection of (B.2)-(B.4). (iv) $(1-\tilde{q})A-d>0$, i.e. $\tilde{q}<1-\frac{d}{A}$. Again this follows from inspection of (B.2)-(B.4). (iv) $2(1-\tilde{q})A-d>0$, i.e. $\tilde{q}<1-\frac{d}{2A}$. This is required for the left-hand side of (B.4) to be monotonically increasing in $\tilde{q}$ (see below why we need this). Now, if we combine (i)-(iv), and given the parameter restrictions in (13a) and (13b) in Proposition 1, it follows that the "binding" lower boundary for $\tilde{q}$ is $0<1-\frac{r+d}{A}$,[29] while the "binding" upper boundary for $\tilde{q}$ is $1-\frac{r}{A}<1$.[30] Thus, $0<1-\frac{r+d}{A}<\tilde{q}<1-\frac{r}{A}<1$, which is the region in which a well-defined solution (if any) for $\tilde{q}$ should lie.

Consider now the second step. We study whether such a solution for $\tilde{q}$ actually exists and is unique. Define the left-hand side of (B.4) by $L(\tilde{q})$ and the right-hand side by $R(\tilde{q})$. Then, $L_q(\tilde{q})>0$ (see condition (iv) above) and $R_q(\tilde{q})<0$. Concerning the lower boundary, i.e. $1-\frac{r+d}{A}$, we have $L\left(1-\frac{r+d}{A}\right)=0$ which is always smaller than $R\left(1-\frac{r+d}{A}\right)>0$. Concerning the upper boundary, i.e. $1-\frac{r}{A}$, we

[29] In particular, if $r>2d$ [which is (13b)], $1-\frac{2r-d}{A}<1-\frac{r+d}{A}<\tilde{q}$. That is, when $1-\frac{r+d}{A}<\tilde{q}$, we also have $1-\frac{2r-d}{A}<\tilde{q}$ so that the transversality condition is always satisfied. We also assume $A>r+d$ [which is (13a)] so that $1-\frac{r+d}{A}>0$. Thus, the binding lower boundary for $\tilde{q}$ is $1-\frac{r+d}{A}$ which is positive.

[30] In particular, $r>2d$ implies $1-\frac{r}{A}<1-\frac{d}{A}<1-\frac{d}{2A}$. Thus, the binding upper boundary for $\tilde{q}$ is $1-\frac{r}{A}$.

have $L\left(1-\frac{r}{A}\right) > R\left(1-\frac{r}{A}\right) > 0$, if the parameter values satisfy (13c) in the text. Since $L_q(\tilde{q}) > 0$ and $R_q(\tilde{q}) < 0$ monotonically, these values of $L(\tilde{q})$ and $R(\tilde{q})$ at the lower and upper boundaries mean that $L(\tilde{q})$ and $R(\tilde{q})$ intersect once, as it is shown in Figure 1 below.

Figure 1 here

Therefore, a unique, well-defined solution for $\tilde{q}$ exists. This in turn supports - via (B.2) and (B.3) - a unique solution for $\tilde{f}$ and $\tilde{y}$.

APPENDIX C: Transitional Dynamics of (15)

The characteristic equation of the Jacobian evaluated at the steady state is:

$$e^3 - 2re^2 + \left[r^2 + \frac{A(1-\tilde{q})n\tilde{y}}{\tilde{f}^2}\right]e - \frac{rA(1-\tilde{q})n\tilde{y}}{\tilde{f}^2} = 0 \qquad \text{(C.1)}$$

where e is an eigenvalue. The coefficient on e^2 is negative, the coefficient on e is positive, and the constant term is negative (i.e. $-\frac{rA(1-\tilde{q})n\tilde{y}}{\tilde{f}^2} < 0$). That is, there are three sign alterations in (C.1). Now, we use Descartes' Theorem (see e.g. Azariadis [1993]) which states that the number of positive roots cannot be higher than the number of sign alterations. Hence, we cannot have more than three positive roots. Next define $e' \equiv -e$. In this case, there are no sign alterations in (C.1). Hence, we cannot have a negative root. Therefore, there are three positive roots.

APPENDIX D: From equations (22a)-(22g) to equations (23a)-(23e)

Taking logs on both sides of (22a) and differentiating with respect to time, we get:

$$\frac{a(\dot{l}c + l\dot{c}) + \dot{g}k + g\dot{k}}{alc + gk} = \frac{1}{mk}\left(\dot{m}k + m\dot{k}\right) \qquad \text{(D.1)}$$

Substituting (22b), (22c), (22d), (22e) and (22f) for the rates of growth of l, c, g, k, m respectively into (D.1), we obtain:

$$a + (1-a)cg = cm + dmk + \frac{nk}{N} \qquad \text{(D.2)}$$

With $z \equiv \frac{c}{k}$, (22c) and (22e) give (23a) in the text. Also, if $y \equiv mk$, $f \equiv mN$ and $c \equiv gk$, (22d)-(22g) give (23b), (23c) and (23d) in the text. Finally, (D.2) is (23e) in the text.

APPENDIX E: Proof of Proposition 3

Setting (23a) equal to zero, we get:

$$\tilde{z} - r = (1-a)(1-\tilde{q})\hat{A} \tag{E.1}$$

Setting (23b) equal to zero and using (E.1), we get for $\tilde{f}$:

$$\tilde{f} = \frac{n}{a(1-\tilde{q})\hat{A} - d} \tag{E.2}$$

Setting (23c) equal to zero and using (E.1)-(E.2), we get for $\tilde{y}$:

$$\tilde{y} = \frac{n[r + d - a(1-\tilde{q})\hat{A}]}{(1-\tilde{q})\hat{A}[a(1-\tilde{q})\hat{A} - d]} \tag{E.3}$$

Setting (23d) equal to zero and using (E.1), we get:

$$\frac{\tilde{y}}{\tilde{c}} = 1 - a \tag{E.4}$$

Then, using (E.2)-(E.4) into (23e), we get:

$$n[2d - a(1-\tilde{q})\hat{A}][r + d - a(1-\tilde{q})\hat{A}] = a(1-\tilde{q})\hat{A}[a(1-\tilde{q})\hat{A} - d] \tag{E.5}$$

which is (24d) in the text. This is a quadratic equation in $\tilde{q}$ only. Once we solve (E.5) for $\tilde{q}$, (E.1)-(E.4) can give $\tilde{z}$, $\tilde{f}$, $\tilde{y}$ and $\tilde{c}$. So the main task is to solve (E.5) for $0 \leq \tilde{q} < 1$ and check whether the solution is well-defined.

We work as in Appendix B above. In the first step, we specify the region in which a well-defined solution (if any) for $\tilde{q}$ should lie. A well-defined solution requires: (i) $a(1-\tilde{q})\hat{A} - r > 0$, i.e. $\tilde{q} < 1 - \frac{r}{a\hat{A}}$. This is required for long-term growth. (ii) $a(1-\tilde{q})\hat{A} + d < 2r$, i.e. $1 - \frac{2r-d}{a\hat{A}} < \tilde{q}$. This is required for the transversality condition (22h) to hold. (iii) $d < a(1-\tilde{q})\hat{A} < 2d$, i.e. $1 - \frac{2d}{a\hat{A}} < \tilde{q} < 1 - \frac{d}{a\hat{A}}$. (iv) $r + d > a(1-\tilde{q})\hat{A}$, i.e. $\tilde{q} < 1 - \frac{r+d}{a\hat{A}}$. Note that conditions (iii) and (iv) follow from inspection of (E.2)-(E.5); they also imply that both sides of

(E.5) are positive. Now, if we combine (i)-(iv), and given the parameter restrictions in (24a) and (24b) in Proposition 3, it follows that the "binding" lower boundary for $\tilde{q}$ is $0 < 1 - \frac{2d}{a\hat{A}}$,[31] while the "binding" upper boundary for $\tilde{q}$ is $1 - \frac{r}{a\hat{A}} < 1$.[32] Thus, $0 < 1 - \frac{2d}{a\hat{A}} < \tilde{q} < 1 - \frac{r}{a\hat{A}} < 1$, which is the region in which a well-defined solution (if any) for $\tilde{q}$ should lie.

Consider now the second step. We study whether such a solution for $\tilde{q}$ actually exists and is unique. Define the left-hand side of (E.5) by $L(\tilde{q})$ and the right-hand side by $R(\tilde{q})$. Then, from conditions (iii) and (iv) above, $L_q(\tilde{q}) > 0$ and $R_q(\tilde{q}) < 0$. Concerning the lower boundary, i.e. $1 - \frac{2d}{a\hat{A}}$, we have $L\left(1 - \frac{2d}{a\hat{A}}\right) = 0$ which is always smaller than $R\left(1 - \frac{2d}{\hat{a}A}\right) > 0$. Concerning the upper boundary, i.e. $1 - \frac{r}{a\hat{A}}$, we have $L\left(1 - \frac{r}{a\hat{A}}\right) > R\left(1 - \frac{r}{a\hat{A}}\right) > 0$, if the parameter values satisfy (24c) in the text. Since $L_q(\tilde{q}) > 0$ and $R_q(\tilde{q}) < 0$ monotonically, these values of $L(\tilde{q})$ and $R(\tilde{q})$ at the lower and upper boundaries imply that $L(\tilde{q})$ and $R(\tilde{q})$ intersect once, as it is shown in Figure 2 below.

Figure 2 here

Therefore, a unique, well-defined solution for $\tilde{q}$ exists. This in turn supports - via (E.1)-(E.4) - a unique solution for $\tilde{z}$, $\tilde{y}$, $\tilde{f}$, and $\tilde{c}$.

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[31] In particular, if $\frac{3d}{2} < r < 2d$ [which is (24b)], $1 - \frac{r+d}{a\hat{A}} < 1 - \frac{2d}{a\hat{A}}$ and $1 - \frac{2r-d}{a\hat{A}} < 1 - \frac{2d}{a\hat{A}}$. That is, when $1 - \frac{2d}{a\hat{A}} < \tilde{q}$, we also have $1 - \frac{2r-d}{a\hat{A}} < \tilde{q}$ so that the transversality condition is always satisfied. We also assume $a\hat{A} > 2d$ [which is (24a)] so that $1 - \frac{2d}{a\hat{A}} > 0$. Thus, the binding lower boundary for $\tilde{q}$ is $1 - \frac{2d}{a\hat{A}}$ which is positive.

[32] In particular, (24b) implies $1 - \frac{r}{a\hat{A}} < 1 - \frac{d}{a\hat{A}}$. Thus, the binding upper boundary for $\tilde{q}$ is $1 - \frac{r}{a\hat{A}}$.

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FIGURE 1

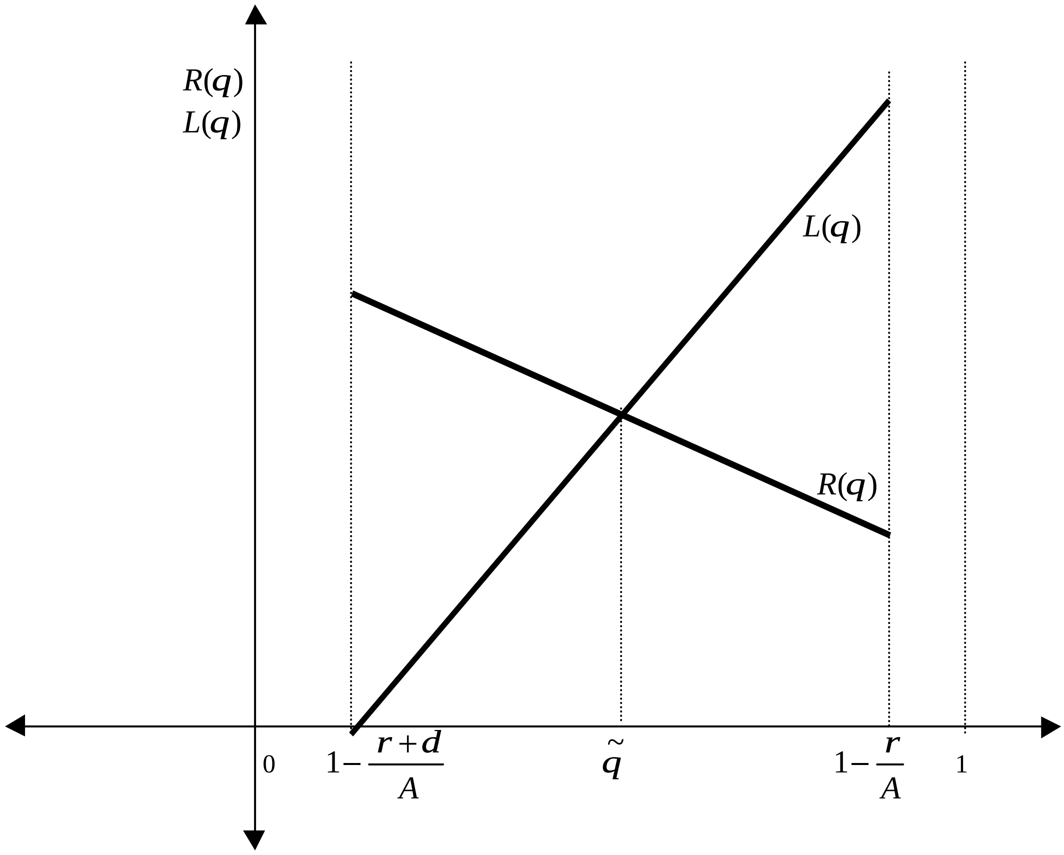

FIGURE 2