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THE OPTIMAL PORTFOLIO OF START-UP FIRMS IN VENTURE CAPITAL FINANCE

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Abstract

A venture capitalist faces a trade-off between the extent of managerial advice allocated to each start-up and the total number of firms advised. Diminishing returns to advice per firm call for a larger portfolio. As advice gets diluted, further expansion of the portfolio eventually becomes unprofitable.

Keywords: Venture capital finance, double-sided moral hazard, company portfolio

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1 Introduction

It is a stylized fact that the typical portfolio of a venture capitalist (VC) includes several firms, although a limited number of them (see Gorman and Sahlman (1989), Sahlman (1990), Norton and Tenenbaum (1993) and Reid, Terry and Smith (1997). Gompers and Lerner (1999) summarize recent research on venture capital). How many start-ups should a VC include in her portfolio? We argue that the optimal company portfolio results from a trade-off between the number of firms included and the advisory effort allocated to each one. With diminishing returns on advice in each firm, it is better to expand the number of companies rather than concentrating advice on a single project. On the other hand, the VC's effort cost increases progressively as more firms are included. Advice is easily stretched too thin, thereby reducing the survival chances of all firms in the portfolio. When projects become riskier, the VC must cede a higher profit share to entrepreneurs to secure their effort which is critical for survival. With her own profit share eroded, the VC eventually finds it unattractive to expand the portfolio further. In developing a simple model of VC activity with double-sided moral hazard, our analysis draws on Repullo and Suarez (1999), Casamatta (1999) and Strobel (2000). These authors assume, like all other contributers in this field, that a VC finances only one entrepreneur.

2 A Simple Model

Venture Capital Activity: We assume that each project yields $R > 0$ if successful and zero if it fails. An entrepreneur pursues exactly one project, all being identical. In exchange for a profit share $1 - s_i$, the VC must finance the entire start-up cost I of $i = 1, \ldots, n$ symmetric projects since entrepreneurs are assumed to have no resources of their own. Apart from supplying funds, the VC provides managerial assistance a_i. The entrepreneur's contribution $e_i \in \{0, 1\}$ is deemed critical. Her shirking will certainly result in business failure. We assume a survival probability $P(e_i, a_i) = e_i p(a_i)$, satisfying $p'(a_i) > 0 > p''(a_i)$ and $p(a_i) < 1$ over the relevant range. Active managerial consulting thus adds value and enhances the probability of success. Such services are increasingly costly, however. In supporting a portfolio of n companies, the VC's total managerial input amounts to $A = \sum_{i=1}^{n} a_i = an$, where $a_i = a$ by symmetry. The VC's effort cost $c(A)$ is increasing and convex, the entrepreneur's is discrete, $l(e_i) \in \{0, \beta\}$. We work with isoelastic functions

$$c(A) = \gamma \frac{A^{1+\varepsilon}}{1+\varepsilon}, \qquad p(a) = \frac{a^{1-\theta}}{1-\theta}, \qquad \varepsilon > 0, \qquad 0 < \theta < 1. \tag{1}$$

Neither the effort of each entrepreneur nor the extent of managerial advice are verifiable and contractable. The informational asymmetry is reflected in the following sequence of decisions. First, the VC chooses a number n of start-up firms, offering an equity share s_i to each entrepreneur. Next, given n and s_i, efforts are chosen. Finally, risk is resolved and payments distributed. The model is solved by backward induction. Both parties are risk neutral. The VC's problem is

$$\begin{aligned}
\max \quad \pi \quad &= \quad \sum_{i=1}^{n} \left[e_i p(a_i)(1-s_i) R - I \right] - c(A) \qquad s.t. && (2)\\
PC^E \quad &: \quad \pi_i^E = e_i p(a_i) s_i R - l(e_i) \geq 0, \qquad i = 1, ..., n, && \text{(i)}\\
IC^E \quad &: \quad p(a_i) s_i R - \beta \geq 0, \qquad i = 1, ..., n, && \text{(ii)}\\
IC^F \quad &: \quad \{a_i\} = \arg\max \left\{ \left[\textstyle\sum_i e_i p(a_i)(1-s_i) R \right] - c(A) \right\}. && \text{(iii)}
\end{aligned}$$

Condition (i) is the participation constraint of entrepreneurs. To attract them, the contract must at least yield an expected income equal to the alternative income, normalized to zero. Conditions (ii) and (iii) reflect the *ex post* incentive constraints. Given that contracts are already fixed and investments are sunk, agents choose effort to maximize the remaining income that is still at their discretion. Emphasizing the critical nature of the entrepreneurs' effort, their choice is restricted to two alternatives, high effort and shirking.

Effort and Advice: Optimal managerial advice in (2iii) must satisfy

$$e_i p'(a_i)(1-s_i) R = c'(A), \qquad i = 1, \ldots, n. \tag{3}$$

Anticipating *the ex* post incentive constraints, the VC will always offer a profit share sufficient to satisfy (2ii), eliciting high effort $e_i^* = 1$. Otherwise, she would earn no revenue. What then is the minimum profit share s_i to retain the entrepreneurs' incentive? Taking logarithmic differentials of (2ii) and (3) at the symmetric equilibrium solution and using (1), we obtain[1]

$$IC^E: \quad \hat{s} = -\hat{R} - (1-\theta)\,\hat{a}, \qquad IC^F: \quad (\theta+\varepsilon)\,\hat{a} = \hat{R} - \frac{s}{1-s}\hat{s} - \varepsilon\hat{n}. \tag{4}$$

A higher return R and a larger profit share $1-s$ for the VC boost the marginal benefits of advice while a larger portfolio raises the marginal cost of advice. The entrepreneur's profit share may be reduced if her incentives are strengthened by a higher project value or a higher success probability $\hat{p} = (1-\theta)\,\hat{a}$. Using IC^E to replace $\hat{s}$ in (4) gives

$$\hat{a} = \frac{1}{\Psi}\left[\frac{1}{1-s}\hat{R} - \varepsilon\hat{n}\right], \qquad \Psi \equiv \theta + \varepsilon - \frac{s\,(1-\theta)}{1-s} > 0. \tag{5}$$

When the VC increases advice because of a higher project value R, she boosts the firm's survival chance. A smaller profit share then suffices to retain the entrepreneur's incentive. With her own profit share larger, she advises even more intensively. When this cycle converges, the total effect is positive, $\Psi > 0$, and exceeds the direct effect.

Optimal Contract: Substituting (5) back into IC^E in (4) gives

$$\hat{s} = \frac{1}{\Psi}\left[(1-\theta)\,\varepsilon\hat{n} - (1+\varepsilon)\,\hat{R}\right]. \tag{6}$$

When more firms call for support, the VC advises each one less. As the success rate falls, she must offer higher shares to her entrepreneurial partners to enlist their effort.

Optimal Company Portfolio: With an optimal number of firms, the contribution of the marginal start-up to profits is zero. Differentiating (2) and imposing symmetry yields $\pi_n \equiv \frac{d\pi}{dn} = [p\,(1-s)\,R - I] - ac' - npR\frac{\partial s}{\partial n}$. Although a larger portfolio dilutes advice in

[1] The hat notation indicates a logarithmic differential, $\hat{a} \equiv d\ln(a) = da/a$ etc. Because of (1), $\hat{p} = (1-\theta)\,\hat{a}$, $\hat{p}' = -\theta\hat{a}$, $\hat{c} = (1+\varepsilon)\,\hat{A}$, and $\hat{c}' = \varepsilon\hat{A}$.

(5), the marginal effect on profits is zero by the *envelope theorem* applied to (2iii). The square bracket indicates the marginal contribution of an extra firm to VC profits. The second term reflects the additional effort cost from extending managerial support to the marginal firm. The last term captures a profit destruction effect. Having more firms leads the VC to advise each one less, which erodes survival chances. To preserve incentives in face of higher risk, the VC must cede a higher profit share to entrepreneurs. Insert (6) together with (2ii), replace c' by (3) and use $ap' = (1-\theta)\,p$ from (1) to get

$$\pi_n \equiv \frac{d\pi}{dn} = \left[\theta p\,(a)\,(1-s)\,R - I\right] - \beta\frac{(1-\theta)\,\varepsilon}{\Psi\,(s)} = 0, \qquad \pi_{nn} < 0. \tag{7}$$

The number of firms is determined by (7). By (5) and (6), n reduces a but raises s. Since $\Psi'\,(s) = -\frac{1-\theta}{(1-s)^2} < 0$, all terms in π_n decline with n, thereby fulfilling the sufficient condition. As more firms are financed, the profit destruction effect becomes more severe. With small n, on the other hand, the VC advises rather intensively and can appropriate a large profit share without losing the entrepreneur's effort. Marginal benefits (net of effort cost) of expanding the portfolio are then relatively high. A separate appendix proves

Proposition 1 *A unique optimal number of portfolio companies exists,* $0 < n^* < \infty$.

If effort cost were linear ($\varepsilon = 0$), advice and profit share in (5) and (6) would be independent of n. The profit destruction effect would disappear, making marginal benefits a constant $\pi_n = \theta p\,(a)\,(1-s)\,R - I \gtrless 0$ and leaving the individual portfolio problem indeterminate. If, on the other hand, the survival probability were linear ($\theta = 0$), a would fall and s would increase in n as before. In this case, however, the benefit of an extra firm (net of marginal effort) would be unambiguously negative, making $\pi_n < 0$ in (7). The optimal number of firms would be driven to one, if that was still profitable.

Proposition 2 *The number of start-up firms in the VC's portfolio increases with R.*

For proof, apply the implicit function theorem to (7), giving $\frac{dn}{dR} = -\frac{\pi_{nR}}{\pi_{nn}}$. This result follows from $\pi_{nn} < 0$ and $\pi_{nR} = \theta p\,(1-s) + \theta\,(1-s)\,Rp'a_R - \left[\theta pR + \frac{\beta\varepsilon(1-\theta)^2}{(1-s)^2\Psi^2}\right] s_R > 0$.

Depending on her profit share, a larger return R directly raises the VC's income by $\theta p(1-s)$. The profit from an extra firm is also strengthened by the fact that a higher project value R encourages the VC to advise more intensively, making the firm more likely to survive, $\theta(1-s)Rp'a_R > 0$. By reducing risk, the VC may increase her own profit share without provoking the entrepreneur to shirk, which raises profits by $-\theta pRs_R > 0$. Finally, funding an additional start-up dilutes advice over more firms, leaving less support and a higher risk for each individually. To preserve incentives, entrepreneurs must be compensated with a higher profit share, giving rise to the profit destruction effect in (7). When projects become more valuable, however, the VC starts to advise more intensively and cuts the entrepreneurs' profit share on account of lower risk. In obtaining a larger share for herself, the VC is able to alleviate the profit destruction effect by $-(\frac{1-\theta}{(1-s)\Psi})^2\varepsilon\beta c^E \cdot s_R > 0$, which boosts the incentive to expand portfolio size.

3 Final Remarks

In real life, a VC finances a small number of start-up enterprises, typically fewer than ten. This note has developed a model which rationalizes this fact. It is shown that both diminishing returns to advice and convex effort cost are necessary to determine the optimal number of firms in a VC's portfolio.

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Appendix: Proof of Proposition 1

To prove existence and uniqueness, we must first show how a and s in (7) depend on n.

Advice and Profit Share: The solution for a and s follows from (2ii) and (3). Since we must impose $p \leq 1$, the form of $p(a)$ in (1) implies an upper limit for advice of $\bar{a}$. By the same argument, (2ii) implies a minimum profit share $\underline{s}$,

$$\bar{a} \equiv (1-\theta)^{1/(1-\theta)}, \qquad \underline{s} = \beta/R. \tag{A.1}$$

Figure 1 now plots the IC^Ecurve given in (2ii) in s, a-space:

$$IC^E: \qquad a = E(s) = \bar{a} \cdot (\underline{s}/s)^{1/(1-\theta)}. \tag{A.2}$$

This curve hits the upper limit at $\underline{s}$, i.e. $\bar{a} = E(\underline{s})$. Since the profit share cannot exceed one, it is bounded below by $\underline{a} = E(1) = \bar{a}\underline{s}^{1/(1-\theta)}$. It is falling and convex, $E' < 0 < E''$. The financier's incentive constraint (3) is

$$IC^F: \qquad a = F(s) = \left[\frac{R}{\gamma n^{\varepsilon}}(1-s)\right]^{1/(\theta+\varepsilon)}. \tag{A.3}$$

This curve is negatively sloped, $F'(s) < 0$. Since $F''(s) = \frac{-F'(s)(1-\theta-\varepsilon)}{(\theta+\varepsilon)(1-s)} \gtrless 0$, it is concave for $1-\theta < \varepsilon$ and convex otherwise. It satisfies $F(1) = 0$. To have an interior solution with IC^E binding, we must impose

$$F(\underline{s}) \leq \bar{a} \qquad \Leftrightarrow \qquad (1-\theta)^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta} \geq (R-\beta)^{1-\theta}. \tag{A.4}$$

For a solution to exist, the incentive constraints in Figure 1 must intersect. By equating $F(s) = E(s)$, we get $H(s) \equiv (1-s)^{1-\theta} s^{\theta+\varepsilon} = [(1-\theta)\beta]^{\theta+\varepsilon} (\gamma n^{\varepsilon})^{1-\theta} / R^{1+\varepsilon} \equiv X$. The H-schedule satisfies $H(0) = H(1) = 0$ and attains a maximum at $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon} < 1$ which follows from $H'(s) = s^{\varepsilon}\left[(\theta+\varepsilon)(\frac{1-s}{s})^{1-\theta} - (1-\theta)(\frac{s}{1-s})^{\theta}\right] = 0$. Evaluating $H(s)$ at its maximum gives the condition $H(\bar{s}) > X$ for the existence of a solution,

$$H(\bar{s}) = \frac{(1-\theta)^{1-\theta}(\theta+\varepsilon)^{\theta+\varepsilon}}{(1+\varepsilon)^{1+\varepsilon}} > \frac{[(1-\theta)\beta]^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta}}{R^{1+\varepsilon}}. \quad \text{(A.5)}$$

Under this condition, $H(s) = X$ has two solutions for s, meaning that the incentive constraints in Figure 1 intersect twice. For the solutions to be in the relevant range, (A.5) and (A.4) must be satisfied simultaneously. Multiplying (A.5) by $R^{1+\varepsilon}/\beta^{\theta+\varepsilon}$ and comparing with (A.4) gives the condition

$$(1-\theta)^{1-\theta}\left(\frac{\theta+\varepsilon}{\beta}\right)^{\theta+\varepsilon}\left(\frac{R}{1+\varepsilon}\right)^{1+\varepsilon} > (1-\theta)^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta} \geq (R-\beta)^{1-\theta}. \quad \text{(A.6)}$$

Choosing R large and β small opens a wide wedge, allowing placement of the middle term to this interval by choice of appropriate values for γ and n.

Of the two intersection points in Figure 1, A is the solution. To see this, note that all combinations to the north east of the E-schedule are admissible choices for the VC. For any given s, the F-curve gives the VC's optimal advice according to (3). Applying the envelope theorem to (2), the VC maximizes profit by increasing her own profit share, i.e. by reducing s. She moves along the F-curve to the north west until the entrepreneur's incentive constraint binds at A. Equation (4) linearizes the two constraints at solution A. The condition $\Psi > 0$ in (5) reflects the fact that IC^E is steeper than IC^F at A.[2]

The comparative statics in n is also illustrated in Figure 1. A larger number of firms n leaves the E-schedule unaffected but shifts down the F-schedule, moving solution A to the south east. Advice per firm is reduced, and the entrepreneur's equity share must be increased on account of higher risk.

[2] Note that $E'(s) = \frac{-a}{(1-\theta)s} < 0$ and $F'(s) = \frac{-a}{(\theta+\varepsilon)(1-s)} < 0$, whence $E'(s) < F'(s) \Leftrightarrow \Psi(s) > 0$.

Optimal Number of Firms: To prove proposition 1 claiming the existence and uniqueness of n^*, we use $p(a) = \underline{s}/s$ from IC^E. Write (7) as $\pi_n = z_1(s) - z_2(s)$. Note that n enters the condition only via its effect on $s(n)$ which is the intersection of (A.2) and (A.3) with the lowest share s. The profit creation and destruction effects, z_1 and z_2, are

$$z_1(s) \equiv \theta R \underline{s} \frac{1-s}{s} - I, \qquad z_2(s) \equiv \frac{(1-\theta)\varepsilon\beta}{\Psi(s)}. \tag{A.7}$$

Evaluating these terms at the lowest admissible equity share (see Figure 1), we get

$$z_1(\underline{s}) \equiv \theta(R-\beta) - I, \qquad z_2(\underline{s}) \equiv \frac{(1-\theta)\varepsilon\beta}{\Psi(\underline{s})}. \tag{A.8}$$

Since $\Psi' < 0$, Ψ gets larger for low values of s, making $z_2(\underline{s})$ comparatively small. In raising R relative to β [see also the discussion of (A.6)], we make $z_1(\underline{s})$ arbitrarily large, both directly and indirectly on account of a smaller $\underline{s}$. The effect on $\underline{s}$ also squeezes the profit destruction effect at the lower boundary of s. With R appropriately set, we have $z_1(\underline{s}) > z_2(\underline{s}) > 0$ in Figure 2.

Expanding portfolio size n raises the equity share s on account of the "dilution of advice" effect, see Figure 1. Since $z_1'(s) < 0$ and $z_2'(s) > 0$, the profit creation effect melts down while the profit destruction effect becomes ever larger. In particular, $\Psi(\bar{s}) = 0$ for $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon} < 1$, which makes $z_2(s) \to \infty$ for $s \to \bar{s}$.[3] Since both schedules are monotonic, a unique solution n^* exists in the interval $[\underline{n}, \bar{n}]$ which corresponds to the interval $[\underline{s}, \bar{s}]$.

[3]Evaluating at $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon}$ gives $E'(\bar{s}) = \frac{-E(\bar{s})(1+\varepsilon)}{(1-\theta)(\theta+\varepsilon)}$ and $F'(\bar{s}) = \frac{-F(\bar{s})(1+\varepsilon)}{(1-\theta)(\theta+\varepsilon)}$. By (A.2) and (A.3), there is one $\bar{n}$ such that $E(\bar{s}) = F(\bar{s})$, implying a tangency solution $E'(\bar{s}) = F'(\bar{s})$ and $\Psi(\bar{s}) = 0$ in Figure 1. The value of $\bar{s}$ corresponds to the maximum of $H(s)$ as noted prior to (A.5).

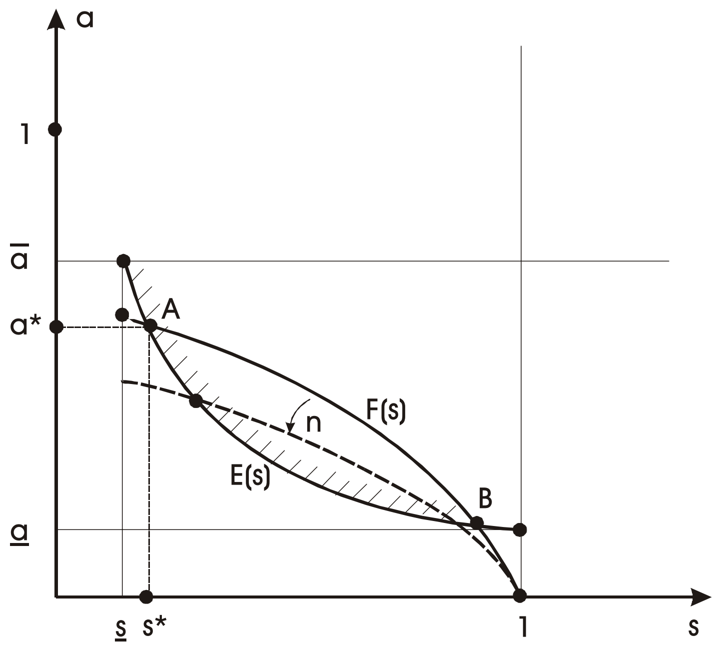

Figure 1: Optimal Advice and Profit Share

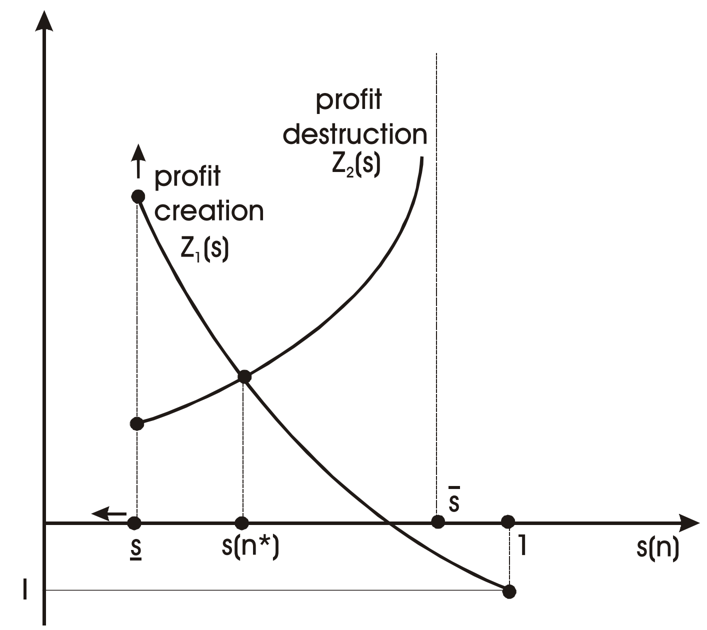

Figure 2: Optimal Number of Firms