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ON THE INTERGENERATIONAL INCIDENCE OF WAGE AND CONSUMPTION TAXES

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Abstract

This paper analyzes the intergenerational incidence of wage and consumption taxes imposed to finance a given amount of public expenditures. It employs a continuous time overlapping genera-tions framework to demonstrate that it essentially hinges on the relationship between the age-earnings and age-consumption profiles of the households which generations bear the major burden of wage respectively consumption taxes. Furthermore, the paper points to some political economy implications of the incidence of wage and consumption taxes.

Keywords: Intergenerational incidence, wage and consumption taxes, life cycle model

JEL Classification: D91, E62, H22

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I. Introduction

Wage and consumption taxes as alternatives to a general income tax are of classical concern in public economics. A major focus of attention in the literature has been the efficiency and welfare prospects associated with a switch from a general income tax including capital taxation to pure wage or consumption taxes. The ramifications of this research comprise static and dynamic efficiency gains of tax reform, the interplay between taxation, savings and human capital investment, and the effects of taxation on productivity and per capita income growth.[1]

Unlike most of the previous research, the present paper does not address efficiency aspects of taxation but will concentrate on the distributional consequences of alternative tax bases. More precisely, the present paper analyzes the intergenerational incidence of wage and consumption taxes imposed to finance a given amount of public expenditures from the lifetime perspective.[2] As has been pointed out by Summers (1981), wage and consumption taxes lead to quite different intergenerational distributions of the tax burden. Summers and subsequent authors like Auerbach and Kotlikoff (1987, chap. 5) and Fullerton and Rogers (1993) have argued that introducing a consumption tax shifts the major tax burden towards the current old generations and benefits the current young and all future generations, whereas a wage tax shifts the major burden of taxation towards the current young and future generations and benefits the current old.

It is the purpose of this paper to study the exact conditions determining the intergenerational incidence of wage and consumption taxes. In order to introduce and to identify the essentials of the incidence analysis, the paper first considers a simple two-period overlapping generations economy. Subsequently, the paper develops the basic framework for analysis consisting of a continuous time overlapping generations economy in which the household sector is represented by a full-fledged

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[1] This literature includes Summers (1981), Auerbach and Kotlikoff (1983, 1987), Auerbach, Kotlikoff and Skinner (1983), Driffill and Rosen (1983), Chamley (1986), Judd (1987, 1999), Davies and Whalley (1989), Lucas (1990), Gravelle (1991), Jones, Manuelli and Rossi (1993), Pecorino (1993, 1994), Trostel (1993), Devereux and Love (1994), Perroni (1995), Stokey and Rebelo (1995), Krusell, Quadrini and Rios-Rull (1996), and Davies, Milesi-Ferretti and Roubini (1998), Coleman (2000), and Zeng and Zhang (2000).

[2] See Davies, St. Hilaire and Whalley (1984) and Poterba (1989) for discussions on annual versus lifetime incidence of taxation. The intergenerational issue addressed here suggests a lifetime perspective.

life cycle model. The paper demonstrates that it essentially hinges on the relationship between the age-earnings and age-consumption profiles of the households which generations bear the major burden of wage respectively consumption taxes. If age-consumption profiles are steeper than age-earnings profiles, the scenario emphasized by Summers, Auerbach and Kotlikoff, and Fullerton and Rogers obtains, i.e. consumption taxes are less burdensome for the current young and future generations and more burdensome for the current old. However, if age-consumption profiles are flatter than age-earnings profiles, a different scenario obtains. In this case wage taxes are more beneficial not only for the current old but also for the current young and all future generations, and they are more burdensome for the current middle-aged generations.

The basic framework is then extended by considering, in succession, an education decision and a variation in household composition. The education decision has an impact on the shape of the age-earnings profiles and the variation in household composition affects the age-consumption profiles. Both extensions have the consequence of shifting the major burden of consumption taxes towards the current young and all future generations so that again consumption taxes become more burdensome for these generations than wage taxes.

The incidences derived in this paper will be seen to have a straightforward implication with respect to the political economy of taxation. If the conditions hold true under which the current young generations are less burdened by consumption and the current old are less burdened by wage taxes, preferences for the tax regime turn out to be single-peaked with respect to age. Therefore, if the tax regime is determined by majority voting and the median age is rather low, consumption taxes will be established, whereas wage taxes will be established if the median age is rather high. However, if the conditions hold true under which the current young and the current old are less burdened by wage taxes and the current middle-aged are less burdened by consumption taxes, preferences for the tax regime are no longer single-peaked with respect to age. Instead, young and old generations have common interests against the middle-aged.

Clearly, the analysis builds on a variety of simplifying assumptions. The presumably two most significant assumptions are the following. On the macro level the economies considered in this paper are characterized by a constant interest rate and exogenously growing wage rates. On the micro level the paper abstracts from an endogenous labor-leisure choice. The first assumption implies that the economies can either be viewed as small open economies integrated in a perfect capital market or as economies with a linear aggregate technology. The paper, thus, neglects from effects of different tax regimes on general equilibrium factor prices and growth. The second assumption implies that taxes do not distort the labor supply decision. In the present context this assumption is less restrictive as it may seem to be at a first glance. This is because wage and consumption taxes are known to have a similar impact on labor supply so that it cannot be expected that the intergenerational incidence of wage versus consumption taxes is affected by an endogenous labor-leisure choice. There are, however, definite advantages of employing these two assumptions. First, there is a methodological advantage. The procedure allows to isolate intergenerational distribution effects from efficiency considerations that come into play either when taxes are distortionary in a static sense or when the interest rate depends on aggregate savings so that different tax regimes may lead the economy closer to or farther away from the golden rule path. Second, there is a technical advantage. The procedure allows to get clear-cut analytical results and, thus, adds to the literature on lifetime taxation which mainly employs simulation and calibration methods.

II. Intergenerational Incidence in a Simple Two-Period Model[3]

Consider for expository purposes first a two-period overlapping generations model in which at each point in time the young generation works and the old generation consumes. The population grows at the constant rate n, the interest rate is given by r at each time t and the wage rate, given by w_t at time t, grows at the constant rate g_w. Consumption of a representative member of the old generation at time t, c_t, is given by:

$$\frac{1}{1-\tau_c}\, c_t = (1+r)\,(1-\tau_w)\, w_{t-1},$$

where τ_c is the tax rate on consumption and τ_w is the tax rate on wage income. From the budget constraint it can be inferred that a young household is indifferent between a wage and a consumption tax regime if the tax rates under both regimes are equally high. If, on the other hand, the tax rate is lower under one of the two tax regimes, the household's lifetime present value tax burden will also be lower

[3] This section is similar to the exposition of the intergenerational incidence of wage and consumption taxes in Auerbach and Kotlikoff (1987, pp. 58-60) except for the case of a growing economy.

and, henceforth, the household's consumption will be higher.

At time t and all subsequent periods the government either imposes wage or consumption taxes to finance some amount of public expenditures. In order to ensure that the government's share in the economy neither vanishes nor becomes entirely exhaustive in course of time, it is assumed that government expenditures per capita grow at the same rate as labor income. Then, without further loss of generality, the government's budget constraint in case of wage taxation ($\tau_w > 0$ and $\tau_c = 0$) can be written as:

$$\tau_w \, w_t = \bar{G} \, w_t,$$

where $\bar{G} \, w_t$ is the amount of public expenditures per worker with $\bar{G}$ as some positive real number consistent with the requirement that public expenditures do not exceed the tax base. Obviously, one gets:

$$\tau_w = \bar{G}$$

in case of wage taxes. If, in contrast, the government imposes consumption taxes to finance its expenditures, the government's budget constraint becomes:

$$\frac{\tau_c}{1-\tau_c} \frac{1}{1+n} c_t = \bar{G} \, w_t$$

so that in light of the household's budget constraint the consumption tax rate can be written as:

$$\tau_c = \frac{(1+n)(1+g_w)}{1+r} \bar{G}.$$

A comparison of τ_w and τ_c yields $\tau_w > \tau_c$ if $1+r > (1+n)(1+g_w)$. Thus, if the interest rate exceeds the growth rate of the economy, the present value of consumption taxes over the life cycle is lower than the present value of wage taxes.[3] This means that for the current young and all future generations a consumption tax is less burdensome than a wage tax. The economic intuition is as follows.

[3] The cases where $1+r \leq (1+n)(1+g_w)$ are not interesting – not only from an empirical but also from a theoretical point of view. This is because in these cases the budget of an infinitely lived agent like the government is not constrained by a transversality condition requiring that the present value of all future expenditures equals the present value of all future revenues. Without such a constraint, however, the tax problem addressed in this paper is not really meaningful.

Households discount future tax payments at the interest rate. The tax base itself, on the other hand, increases at the growth rate of the economy. Since it is required that the government's budget balances at each point in time, i.e. the government neither incurs debt nor accumulates funds, this means that the government implicitly discounts at the growth rate of the economy.[4] Thus, if the interest rate exceeds the growth rate, households discount at a higher rate than the government. This implies that the government can meet its revenue requirement and can reduce the present value of the households' lifetime tax burden by postponing the extraction of taxes in the households' life cycles. In the simple two-period model households pay wage taxes in their first and consumption taxes in their second period of life. Hence, consumption taxes occur later in life and lead to a lower life cycle tax burden than wage taxes. It should be noted, however, that a lower present value of the life cycle tax burden in case of consumption taxes does not point to a superiority of consumption taxes in efficiency terms. It only shows that wage and consumption taxes exert different intergenerational income effects. This becomes obvious by considering that the old at time t are more heavily burdened under a consumption than under a wage tax because they do not work anymore but only consume. In fact, the present value of the gains that accrue to the current young and all future generations equals the burden imposed on the current old:

$$\sum_{i=0}^{\infty}\left(\frac{1+n}{1+r}\right)^i\left(\tau_w\, w_{t+i}-\frac{\tau_c}{1-\tau_c}\,\frac{1}{1+r}\,c_{t+1+i}\right)=\bar{G}\,w_t=\frac{\tau_c}{1-\tau_c}\,\frac{1}{1+n}\,c_t.$$

In this equation the left hand side is the present value of the sum of the gains accruing to the current young and all future generations when the government imposes consumption rather than wage taxes and the right hand side is the burden born by the current old. Clearly, if the government imposes wage rather than consumption taxes, this will benefit the current old and harm the current young and all future generations. The gains – this time enjoyed by the current old – again equal the present value of the losses born by the current young and all future generations.

Obviously, wage and consumption taxes have very different intergenerational incidences. Given the magnitudes of the interest and the growth rate, the inci-

[4] The balanced budget requirement is essential. If the government were allowed to run deficits or surpluses, a consumption tax could be perfectly mimicked by a wage tax and vice versa. See Bradford (1980) for a detailed discussion of the balanced budget requirement.

dence is determined by the relationship between the age-earnings and the age-consumption profile. In the two-period model with its very simple earnings and consumption patterns consumption taxes benefit the current young and future generations, wage taxes benefit the current old. However, the two-period model only gives a very crude picture of life cycle earnings and life cycle consumption. In order to get more detailed insights into the role of the relationship between the age-earnings and the age-consumption profile for the intergenerational incidence of wage and consumption taxes, the next section introduces a full-fledged life cycle model providing richer patterns of life cycle earnings and consumption.

III. The Basic Framework for Analysis

III.1. *The Macro Economy*

The basic framework consists of a continuous time overlapping generations economy. At each time t the oldest generation dies and a new generation is born. As in the previous section the population grows at the constant rate n, the interest rate on capital r is time-invariant, and wages evolve at the constant rate of labor productivity growth g_w so that the wage rate at time t is given by $w(t) = w_0\, e^{g_w t}$, with w_0 as some initial wage level. In order to make the tax problem interesting and meaningful, it is assumed that the interest rate exceeds the economy's growth rate, i.e. $r > n + g_w$. As has been explained in the previous section, this assumption implies that the present values of the households' life cycle tax burden imposed to finance a given amount of public expenditures are the lower the later the tax extracts revenues in the households' life cycles.

III.2. *The Household Sector*

Each household's life lasts T time units. During the first R time units of life households inelastically supply one unit of non-leisure time in the labor market. In the last $T - R$ time units of life households are retired and live on the proceeds of their savings. Lifetime utility of a representative member of the generation economically born at time t is given by:

$$U_t = \int_0^T u[c(t,\theta)]\, e^{-\rho\theta} d\theta,$$

where $\rho > 0$ is the subjective discount factor and $c(t,\theta)$ is consumption of a household born at time t at the age of θ. The instantaneous utility function u is assumed to take the form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution in consumption. There are no borrowing constraints so that the household's intertemporal budget constraint reads:

$$\int_0^T \frac{1}{1-\tau_c}\, c(t,\theta)\, e^{-r\theta}\, d\theta = \int_0^R (1-\tau_w)\, w(t+\theta)\, e^{-r\theta}\, d\theta,$$

where τ_c again denotes the tax rate on consumption and τ_w the tax rate on wage earnings. Again, the household is indifferent between a wage and a consumption tax regime if the tax rates under both regimes are equally high, whereas the household's lifetime present value tax payments will be lower and, henceforth, the household's lifetime utility will be higher if the respective tax rate is lower than its other tax regime counterpart.

The household chooses a path of lifetime consumption which maximizes lifetime utility subject to the intertemporal budget constraint. This leads to the following consumption function:

$$c(t,\theta) = c(t,0)\, e^{g_c\theta}, \qquad \theta \in [0,T],$$

where

$$c(t,0) \equiv (1-\tau_c)(1-\tau_w)\, \frac{r-g_c}{1-e^{-(r-g_c)T}}\, \frac{1-e^{-(r-g_w)R}}{r-g_w}\, w(t)$$

is the amount of the household's initial consumption and $g_c \equiv (r-\rho)/\sigma$ is the growth rate of the household's consumption over the life cycle.

III.3. *The Public Sector*

The government again imposes either consumption or wage taxes to finance a given amount of public expenditures. If the government employs wage taxes to finance public expenditures (implying that $\tau_c = 0$), its budget constraint reads:

$$\tau_w \int_0^R w(t)\, e^{-n\theta}\, d\theta = \bar{G}\, w(t),$$

where, now, $\bar{G}\, w(t)$ is the amount of public expenditures per member of the youngest generation at time t. The tax rate on wages is determined by:

$$\tau_w = \frac{n}{1 - e^{-nR}}\, \bar{G}, \tag{1}$$

If, on the other hand, the government meets its revenue requirement by imposing consumption taxes (implying that $\tau_w = 0$), the budget constraint becomes:

$$\frac{\tau_c}{1 - \tau_c} \int_0^T c(t - \theta, \theta)\, e^{-n\theta}\, d\theta = \bar{G}\, w(t) \tag{2}$$

and the tax rate on consumption reads:

$$\tau_c = \frac{r - g_w}{1 - e^{-(r - g_w)R}}\; \frac{n + g_w - g_c}{1 - e^{-(n + g_w - g_c)T}}\; \frac{1 - e^{-(r - g_c)T}}{r - g_c}\, \bar{G}. \tag{3}$$

The next section compares the tax burdens which obtain under the wage and the consumption tax regime. Afterwards, the basic model is extended both with respect to the age-earnings and the age-consumption profile.

IV. Intergenerational Incidence in the Basic Framework

In order to analyze which of the two tax regimes leads to a lower lifetime tax burden and, thus, to a lower utility loss for the current youngest and all future generations, it suffices to compare the tax rates that obtain under the alternative tax regimes. Proposition 1 summarizes the outcome of such a comparison.

Proposition 1.

i) Let $R = T$. Then $\tau_c \gtreqless \tau_w \;\Leftrightarrow\; g_c \lesseqgtr g_w$.

ii) Let $R < T$ and $g_c \geq g_w$. Then $\tau_c < \tau_w$.

iii) Let the length of retirement, $T - R$, be small and $g_c < g_w$. Then $\tau_c > \tau_w$.

Proof: See the Appendix.

Part i) of Proposition 1 says that in the case where households only retire at the end of their life, the tax rate under the consumption tax regime, τ_c, will be higher than (equal to, lower than) the tax rate under the wage tax regime, τ_w, if the households' consumption grows at a lower (equal, higher) rate than the households' labor income. To get an intuition of this result, consider Figure 1 which illustrates a household's age-consumption and age-earnings profile.

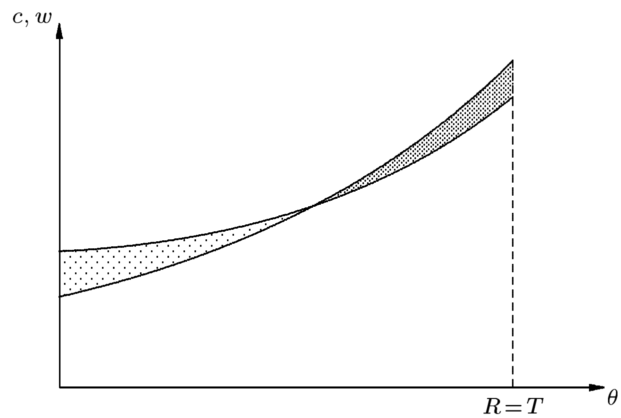

Figure 1. $R\!=\!T$ and $g_c > g_w$

In Figure 1 it is assumed that the growth rate of consumption exceeds the growth rate of labor income. There, and in all further related diagrams the less densely shaded area is associated with periods in the household's life in which a wage tax is more burdensome than a consumption tax, whereas the more densely shaded area is associated with periods in which the opposite holds true. The sizes of the respective areas are associated with the differences in the tax burden due to different tax bases measured in current values. As can be seen from Figure 1, when the household works until the end of life and its consumption grows faster than its labor income, then consumption taxes more heavily burden the household at older ages and wage taxes more heavily burden the household at younger ages. Since for $r > n + g_w$ the present value of taxes imposed to finance a given amount of public expenditures is the lower the later in the life cycle the tax extracts revenue, consumption taxes are less burdensome for the young current and all future generations than wage taxes. The older current generations, on the other hand, are more burdened by consumption taxes. They have already gone through the ages where the burden of wage taxes exceeds the burden of consumption taxes but still will experience the ages where the opposite holds true. Clearly, the opposite intergenerational distribution of wage versus consumption taxes obtains if the households' consumption grows at a lower rate than the households' wage income.

Part ii) of Proposition 1 says that in case of a retirement phase of life the consumption tax rate will be lower than the wage tax rate if the households' growth rate of consumption is at least as high as the households' growth rate of wage income. Figure 2 illustrates this result.

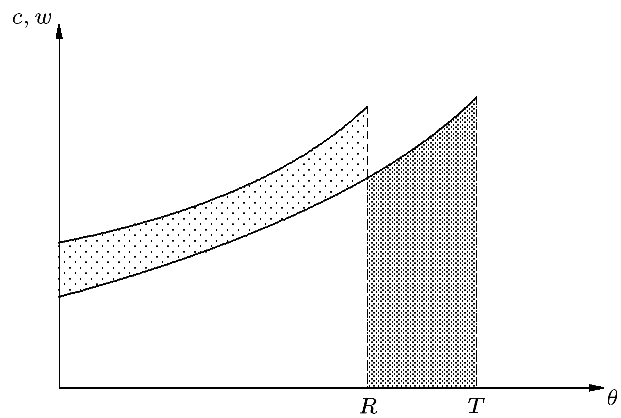

Figure 2. $R < T$ and $g_c \geq g_w$

Obviously, the major burden of consumption taxes occurs later in the life cycle than the one of wage taxes. As a consequence, the young current and all future generations are less burdened by a consumption than by a wage tax. The older current generations, on the other hand, are less burdened by a wage than by a consumption tax as they have already gone through some or all of their working life but are still subject to consumption taxes.

Part iii) of Proposition 1 says that the consumption tax rate exceeds the wage tax rate if consumption grows at a lower rate over the life cycle than wage income and the phase of retirement is sufficiently short. This result is illustrated in Figure 3, where it is assumed that the growth rate of consumption equals zero whereas the growth rate of labor income is positive. Clearly, if wage income grows at a higher rate than consumption and the phase of retirement is short, consumption of young households will exceed their wage income. As a consequence, consumption taxes are more burdensome than wage taxes in early stages of life resulting in a higher present value of the life cycle tax burden.

If the conditions stated in part i) or part ii) of Proposition 1 hold true, a

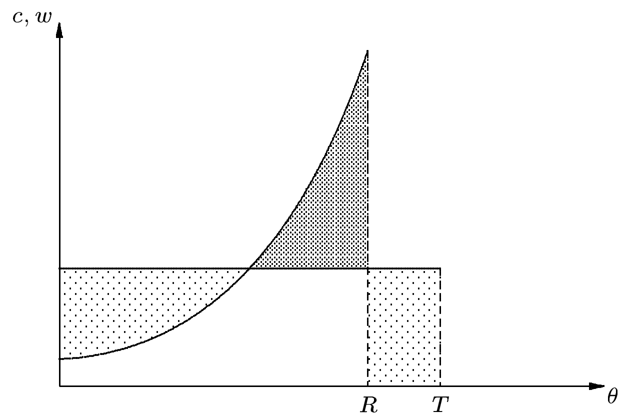

Figure 3. $R < T$ and $g_c = 0 < g_w$

very simple political economy conclusion can be drawn. Younger living households prefer consumption taxes (wage taxes if $R = T$ and $g_c < g_w$) and older living households prefer wage taxes (consumption taxes if $R = T$ and $g_c < g_w$). In fact, preferences for wage versus consumption taxes are single-peaked with respect to age. As age is the only dimension in which households differ, the tax regime preferred by the household with median age will be established if the tax regime is determined by majority voting. Thus, if the median age is rather low, consumption taxes are imposed (wage taxes if $R = T$ and $g_c < g_w$), whereas the opposite holds true if the median age is rather high. However, such a simple policy conclusion cannot be drawn if the conditions stated in part iii) of Proposition 1 hold true. In this case young households and old households prefer consumption taxes, whereas middle-aged households prefer wage taxes (Figure 3 reveals that middle-aged households have gone through some of the ages where consumption taxes are very burdensome but still will experience the ages where wage taxes are very burdensome). Thus, preferences for the tax regime are not single-peaked with respect to age and a stable majority for one of the two tax regimes cannot be expected. Interestingly, in the case described by part iii) of Proposition 1 the young and the old have common interests which are in opposition to the interests of the middle-aged.

V. Extensions of the Basic Framework

In the basic framework the age-consumption and the age-earnings profile are fully determined by the growth rates of consumption and wage income and the ages of retirement and death. This section extends the basic model both with respect to the age-earnings and the age consumption profile. A richer pattern of the age-earnings profile obtains by considering full-time education prior to work. The age-consumption profile, on the other hand, takes a more complex form when a variation in household composition is considered. The basic framework is first extended by considering education. Subsequently, a variation in household composition is introduced into the basic framework. Both extensions will be seen to have a strong impact on the intergenerational incidence of taxation.

V.1. *Education*

In order to introduce an education decision, it is assumed that the household's instantaneous labor income depends not only on the wage rate but also on the household's endowment with human capital. Households accumulate human capital during a phase of full-time education preceding the working life.[5] Human capital is labor-augmenting. Thus, if a household has spent S units of time for full-time education, it is endowed with $h(S)$ units of labor. The function h is strictly increasing in S and satisfies $h(0) = 1$, implying that a household without any full-time education is endowed with one unit of labor during the working life.

Instantaneous labor earnings y of a household born at time t at the age of $\theta \geq S$ are given by:

$$y(t+\theta) = w(t+\theta)\, h(S). \tag{4}$$

The function h is specified by employing the human capital earnings function approach developed by Mincer (1974).[6] This function has a semi-logarithmic form as follows:

$$\ln y = \text{constant} + \delta\, S. \tag{5}$$

[5] Undertaking a full-time education at the very beginning of the economic life is not only an empirical regularity but also optimal as has been shown by Ishikawa (1975).

[6] See also Kalemli-Ozcan et al. (2000) for an incorporation of the human capital earnings function into a theoretical framework.

The parameter δ can be interpreted as the return on education. Furthermore, in the present context, the constant can be interpreted as the log of instantaneous labor earnings of a household without any full-time education. Equations (4) and (5) together imply that the function h takes the form $h(S) = e^{\delta S}$.

Households choose educational time S so as to maximize the present value of net lifetime wage income. More precisely, a household born at time t solves:

$$\max_{0 \le S \le R} \int_S^R (1-\tau_w)\, w(t+\theta)\, e^{\delta S}\, e^{-\theta r}\, d\theta.$$

This problem has an interior solution if:

$$\delta > \underline{\delta} \equiv \frac{r-g_w}{1-e^{-(r-g_w)R}}.$$

In this case the optimal length of full-time education is determined by:

$$S^* = R + \frac{1}{r-g_w} \ln \left[1 - \frac{r-g_w}{\delta}\right].$$

Note that S^* is neither distorted by wage nor by consumption taxes. As in the basic model, both types of taxation only exert income effects.

If the household spends S^* units of time in the educational system before participating in the labor market, this does not affect the growth rate of consumption over the household's life cycle. However, the level of initial consumption will generally be affected. Initial consumption of a household born at time t becomes:

$$\begin{aligned} c(t,0) \equiv & (1-\tau_c)(1-\tau_w)\, \frac{r-g_c}{1-e^{-(r-g_c)T}} \\ & \times \frac{1-e^{-(r-g_w)(R-S^*)}}{r-g_w}\, e^{-(r-g_w)S^*}\, e^{\delta S^*}\, w(t). \end{aligned}$$

The government again either employs wage or consumption taxes to meet its revenue requirement. In case of wage taxes the government budget constraint reads:

$$\tau_w \int_{S^*}^R w(t)\, e^{\delta S^*}\, e^{-n\theta}\, d\theta = \bar{G}\, w(t)$$

which leads to the following wage tax rate:

$$\tau_w = \frac{n}{1 - e^{-(r-g_w)(R-S^*)}}\, e^{(n-\delta)S^*}\, \bar{G}. \tag{6}$$

If, on the other hand, the government meets its revenue requirement by imposing consumption taxes, the budget constraint becomes:

$$\frac{\tau_c}{1-\tau_c} \int_0^T c(t-\theta,\theta)\, e^{-n\theta}\, d\theta = \bar{G}\, w(t)$$

and the consumption tax rate is determined by:

$$\tau_c = \frac{r - g_w}{1 - e^{-(r-g_w)(R-S^*)}}\, \frac{n + g_w - g_c}{1 - e^{-(n+g_w-g_c)T}}\, \frac{1 - e^{-(r-g_c)T}}{r - g_c}\, e^{(r-g_w-\delta)S^*}\, \bar{G}. \tag{7}$$

A comparison of equations (6) and (7) leads to the following result.

Proposition 2. Let the length of retirement, $T - R$, be small and $g_c \leq g_w$. Then $\tau_c > \tau_w$ for all $\delta > \underline{\delta}$.

Proof: See the Appendix

Thus, when the phase of retirement is sufficiently short and the growth rate of the households' consumption does not exceed the growth rate of their labor income, consumption taxes create a larger present value of the lifetime tax burden than wage taxes if the households undertake a full-time education prior to work (recall that $S^* > 0$ if $\delta > \underline{\delta}$). Figure 4 illustrates this result.

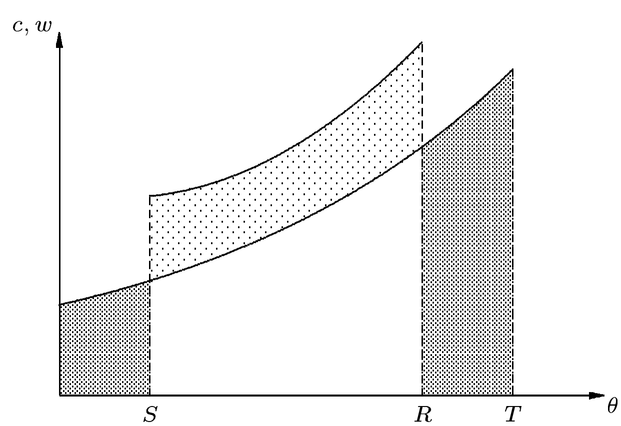

Figure 4. Education

The phase of full-time education at the beginning of life postpones wage taxes so that the households are only subject to consumption taxes during the first S time units of their life. If the retirement phase is short and consumption does not grow faster than wage income, the advantage of consumption taxes over wage taxes disappears. As a consequence, wage rather than consumption taxes benefit the current young, the current old and all future generations, whereas the current middle-aged generations are more heavily burdened by wage taxes.

Again, preferences concerning the tax regime of living generations are not single-peaked with respect to age. The young and the old generations prefer wage to consumption taxes whereas the middle-aged generations prefer consumption to wage taxes.

V.2. *Household Composition*

This section extends the basic model by assuming that the composition of the household varies over its life cycle. Assume that a household economically born at time t sets up a family at the age of $F \geq 0$ by bringing up children until the latter become economically independent at the age of E. Figure 5 illustrates the extended life cycle model. From the age of 0 until the age of F the household is a single, from the age of F until the age of $F+E$ the household is a family, and from the age of $F+E$ until death the household is again a single. In Figure 5 it is assumed that the children leave home before the head of the household retires.

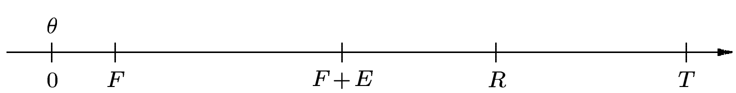

Figure 5. Life-Cycle Model with Variation in Household Composition

In order to capture the variation in the household's composition it is assumed that lifetime utility takes the form:

$$U_t = \int_0^F u\, e^{-\rho\theta} d\theta + \int_F^{F+E} (1+\alpha)\, u\, e^{-\rho\theta} d\theta + \int_{F+E}^T u\, e^{-\rho\theta} d\theta,$$

where the argument of the instantaneous utility function u has been suppressed for notational simplicity. The parameter $\alpha > 0$ measures to what extent the household's utility increases because of the fact that the household is a family rather than a single. This form of household utility can be motivated by the observation that to some extent consumption within households has public good character, either because of parental altruism towards their children or because of non-rivalry of goods like, e.g., TV sets.

The consumption function that results from utility maximization in the presence of a variation in the household composition reads:[7]

$$c(t,\theta) = c(t,0)\, j(\theta)\, e^{g_c\theta}, \qquad \theta \in [0,T], \tag{8}$$

where

$$c(t,0) = (1-\tau_c)(1-\tau_w)\, \frac{1-e^{-(r-g_w)R}}{r-g_w}\, \phi(r-g_c)\, w(t),$$

with

$$\phi(x) \equiv \frac{x}{1-e^{-xT}+[(1+\alpha)^{1/\sigma}-1](e^{-xF}-e^{-x(F+E)})}$$

is the household's initial consumption and

$$j(\theta) = \begin{cases} (1+\alpha)^{1/\sigma}, & \text{if } \theta \in [F, F+E] \\ 1, & \text{otherwise.} \end{cases}$$

The function j gives rise to an upward jump in consumption when the household settles down a family and a downward jump when the children leave home and the household becomes a single again. Figure 6 plots the household's age-consumption profile of the extended life cycle model. Until the age of F the household is a single and its periodical consumption continuously increases at rate g_c. At the age of F the household becomes a family, i.e. the number of consumers in the household increases which leads to a discontinuous upward jump in the household's consumption. As long as the household is a family, periodical consumption again grows continuously at the rate g_c. When the household reaches the age of $F+E$, the children become economically independent and leave home. This means

[7] In addition to the standard Euler equation, the optimal consumption path satisfies $\lim_{\theta\to F^-} u'[c(t,\theta)] = (1+\alpha) \lim_{\theta\to F^+} u'[c(t,\theta)]$ and $(1+\alpha) \lim_{\theta\to F+E^-} u'[c(t,\theta)] = \lim_{\theta\to F+E^+} u'[c(t,\theta)]$. This implies that although there are discontinuous jumps in consumption at the ages of F and $F+E$, marginal instantaneous utilities directly before and after the ages of F and $F+E$ are equalized.

that the number of consumers in the household decreases and the consumption discontinuously jumps down and then again grows continuously at the rate g_c until death.

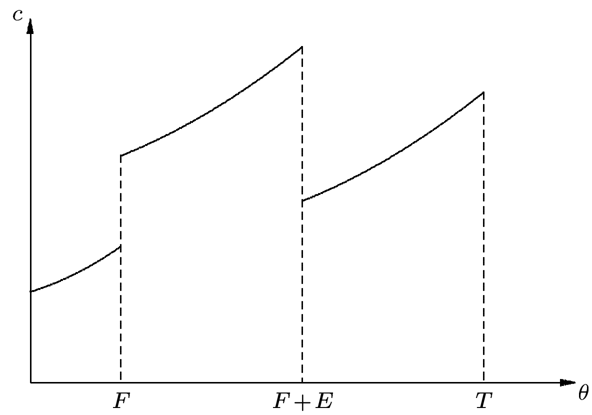

Figure 6. Age-Consumption-Profile for $\alpha > 0$

The government again finances its expenditures either by wage or consumption taxes. Since the variation in household composition does not affect the working pattern of the household, the wage tax rate τ_w is determined by (1) as in the basic framework. In order to determine the consumption tax rate τ_c in the presence of a variation in household composition, substitute (8) into (2) to find that:

$$\tau_c = \frac{r - g_w}{1 - e^{-(r-g_w)R}} \, \frac{\phi(n + g_w - g_c)}{\phi(r - g_c)} \, \bar{G}. \tag{9}$$

A comparison of equations (1) and (9) leads to the following proposition.

Proposition 3. Let $F < F + E < R < T$, F small, and $g_c \leq g_w$. Then there exists some $\bar{\alpha} > 0$ so that $\tau_c > \tau_w$ for all $\alpha > \bar{\alpha}$.

Proof: See the Appendix.

Thus, on the conditions that the household becomes a family at rather young age, children leave home before the household retires, and instantaneous consumption growth does not exceed instantaneous labor income growth, the present value of tax payments is higher under consumption than under wage taxes if the impact of household composition on household consumption is sufficiently strong. In order to interpret this result, consider Figure 7.

Figure 7 reveals that the variation in household composition and the asso-

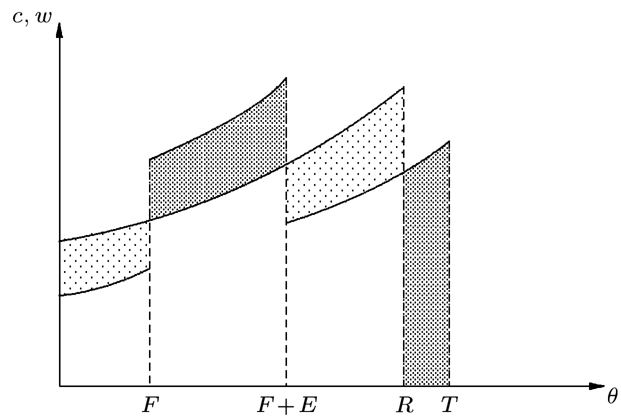

Figure 7. Household Composition

ciated jump in consumption shifts a major part of the household's lifetime consumption towards younger ages in the life cycle. Therefore, the extraction of consumption taxes is forwarded and, consequently, the present value of tax payments under consumption taxes increases and eventually exceeds the payments under wage taxes.

As in the case of high income relative to consumption growth and full-time education prior to work, the consideration of a variation in household composition suggests that wage taxes create a lower life cycle tax burden than consumption taxes and, therefore, benefit not only the current old but also the current young and all future generations. However, the middle-aged working generations whose children are going to leave or have already left home are more heavily burdened by a wage tax. Thus, again the living young and the living old prefer wage taxes and the living middle-aged prefer consumption taxes.

VI. Conclusion

This paper has identified the conditions which determine the intergenerational lifetime incidence of wage and consumption taxes. Generally, if the households' age-consumption profiles are steeper than their age-earnings profiles, wage taxes shift the major burden of taxation towards the current young and future generations, whereas consumption taxes shift the major burden towards the current old. In contrast, if age-consumption profiles are flatter than age-earnings profiles, wage taxes shift the major burden towards current middle-aged generations, whereas consumption taxes shift the major burden towards the current young and old and all future generations.

These results have been derived in a framework with exogenously evolving factor prices. Thus, the paper has abstracted from dynamic efficiency issues associated with the impact of aggregate savings and investment on the interest rate. Clearly, the consideration of an interest rate determined by aggregate (domestic) savings will generally have an effect on the incidence of wage versus consumption taxes. Consumption taxes favor aggregate savings relative to wage taxes so that they will move the economy closer to the golden rule path – given, of course, that there is underaccumulation under both tax regimes relative to the golden rule. This will benefit future generations and will possibly offset the negative impact of consumption taxes on young and future generations in case of flat age-consumption profiles. However, this effect becomes an issue mainly in the closed economy context.

The paper has also abstracted from intragenerational incidence issues by assuming homogeneous households in each generation. Besides methodological arguments this can be justified by the fact that the paper has focused on the incidence of wage versus consumption taxes. It has not been concerned with the differential incidence of not directly taxing capital income which – possibly – affects high income and low income households differently.[8] Deemphasizing questions of intragenerational incidence can also be justified by the observation that most households are wandering between different income groups during their life cycle [see, e.g., Poterba (1989)]. As households – according to the life cycle hypothesis – smooth intertemporal consumption streams, it can be expected that the intragenerational incidence component of a consumption tax becomes less significant. However, the intragenerational incidence of wage versus consumption taxes is affected if households differ in annual wage income streams even if they are identical with respect to lifetime income. This is because the timing of wage income streams is most essential for the actual burden of wage taxes as the present analysis has revealed. Furthermore, the intragenerational incidence of wage and consumption taxes becomes an issue in the context of a variation in household composition. The present paper has demonstrated that increasing the number of household members by rearing children affects the lifetime incidence of wage versus consumption taxes.

[8] See, e.g., Feenberg, Mitrusi and Poterba (1997) and Gentry and Hubbard (1997) for treatments of the distributional implications of not directly taxing capital income.

Therefore, considering that there are intragenerational differences with respect to the number of children, wage and consumption taxes can be expected to differ systematically with respect to their intragenerational incidence. This, however, is a subject for future research.

Although the paper has only been concerned with the intergenerational incidence of different tax bases, it also has implications in a broader public economics context. In the policy debate on social security reform it has been proposed to finance social security benefits by a consumption rather than a payroll tax in order to shift part of the increasing burden of social security financing towards the elderly. The results derived in this paper suggest that such a reform strategy may also lead to an increased burden of social security financing for the current young and future generations and, thus, increases the fiscal strain of those generations which are expected to bear an increased financial burden anyway.

Appendix

In order to prove the propositions stated in the text, the following lemma will be established.

Lemma 1. Define the function $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = x/(1 - e^{-x})$. Then

i) $f(x) > 0$ and $f'(x) > 0$ for all $x \in \mathbf{R}$,

ii) $f(a)\, f(d) < f(b)\, f(c)$ for all $\{a, b, c, d\} \subset \mathbf{R}$ constrained by $a < b \leq c < d$ and $a + d = b + c$.

Proof of i): Since f is continuously differentiable and $\lim_{x \to -\infty} f(x) = 0$, it suffices to show that

$$f'(x) = \frac{1 - (1 + x)\, e^{-x}}{(1 - e^{-x})^2} > 0$$

for all $x \in \mathbf{R}$. Making use of L'Hospital's Rule, it follows that $f'(0) = 1/2 > 0$. Thus, let $x \neq 0$ and suppose, on the contrary to Lemma 1.i), that there exists some $x_0 \neq 0$ so that $f'(x_0) \leq 0$. It then follows that $e^{x_0} \leq 1 + x_0$ which only holds true for $x_0 = 0$ – a contradiction.

Proof of ii): It will be shown that the function $g : \mathbf{R}^2 \to \mathbf{R}$ defined by $g(x, y) = f(x)\, f(y)$ is strictly quasi-concave and that the solution to the problem $\max_{\{x,y\}} g(x, y)$ subject to $x + y = c$ for an arbitrarily chosen $c \in \mathbf{R}$ is given by $x = y = c/2$. Standard methods show that f is C^2 in $\mathbf{R}$ so that g is C^2 in $\mathbf{R}^2$. Therefore, g is strictly quasi-concave if $g_1^2 > 0$ and $2\, g_1\, g_2 - g_2^2\, g_{11} - g_1^2\, g_{22} > 0$, where g_i is the i-th partial derivative of g and so forth. Considering the definition of g, one finds that $g_1^2 = f_x'^2\, f_y^2$, where $f_x = f(x)$ and so forth. Thus, in light of Lemma 1.i) it follows that $g_1^2 > 0$. It remains to show that $2\, g_1\, g_2 - g_2^2\, g_{11} - g_1^2\, g_{22} > 0$ which is equivalent to:

$$f_x\, f_y\, [f_y'^2\, (f_x'^2 - f_x\, f_x'') + f_x'^2\, (f_y'^2 - f_y\, f_y'')] > 0,$$

where $f_x = f(x)$ and so forth. Since $f > 0$ and $f'^2 > 0$, it suffices to show that $f'^2 - f\, f'' > 0$. Considering the definition of f, one gets:

$$f'(x)^2 - f(x)\, f''(x) = \frac{1}{(1 - e^{-x})^2} - \frac{x^2\, e^{-x}}{(1 - e^{-x})^4}.$$

In case of $x = 0$ one finds by L'Hospital's Rule that $f'(0)^2 - f(0)\,f''(0) = 1/12 > 0$. In case of $x \neq 0$ one has to show that $(1 - e^{-x})^2 > x^2\,e^{-x}$. This can be done by using the fact that $\sinh(x) \gtreqless x \Leftrightarrow x \gtreqless 0$ (sinh = hyperbolic sine) which implies:

$$\sinh^2\left(\frac{x}{2}\right) > \left(\frac{x}{2}\right)^2.$$

Considering that $\sinh(x) = (e^x - e^{-x})/2$, it follows that:

$$\left[\frac{1}{2}\left(e^{x/2} - e^{-x/2}\right)\right]^2 > \left(\frac{x}{2}\right)^2,$$

which is easily shown to be equivalent to $(1 - e^{-x})^2 > x^2\,e^{-x}$. Thus, g is strictly quasi-concave. The first-order condition of the maximization problem defined above reads $f'(x)\,f(x-c) = f(x)\,f'(x-c)$ implying that $x = y = c/2$. *Q.E.D.*

Proof of Proposition 1

Considering equations (1) and (3), it follows that:

$$\tau_c \gtreqless \tau_c \;\Leftrightarrow\; f[(r-g_w)R]\,f[(n+g_w-g_c)T] \gtreqless f(nR)\,f[(r-g_c)T] \tag{A1}$$

since $f > 0$ from Lemma 1.i).

Proof of i): Let $R = T$. Obviously, for $g_c = g_w$ it follows that $\tau_c = \tau_w$. For $g_c < g_w$ one has $n < \min\{r-g_w, n+g_w-g_c\}$ and $r-g_c > \max\{r-g_w, n+g_w-g_c\}$. Since $(r-g_w)+(n+g_w-g_c) = n+r-g_c$, it follows from Lemma 1.ii) that $\tau_c > \tau_w$. For $g_c > g_w$ one has $n+g_w-g_c < \min\{n, r-g_c\}$ and $r-g_w > \max\{n, r-g_c\}$ and, by the same reasoning, it follows that $\tau_c < \tau_w$.

Proof of ii): Let $R < T$ and $g_c \geq g_w$. Two cases need to be distinguished: $nR \geq (r-g_c)T$ and $nR < (r-g_c)T$. Consider first the case $nR \geq (r-g_c)T$. It then follows that: $f[(r-g_w)R]\,f[(n+g_w-g_c)T] < f[(r-g_w)R+(r-n-g_w)(T-R)]\,f[(n+g_w-g_c)T] < f(nR)\,f[(r-g_c)T]$. The first equality follows from Lemma 1.i), i.e. the fact that $f' > 0$. The second inequality follows from Lemma 1.ii) since $(r-g_w)R + (r-n-g_w)(T-R) = nR + (r-n-g_w)T > nR \geq (r-g_c)T > (n+g_w-g_c)T$ and $nR + (r-n-g_w)T + (n+g_w-g_c)T = nR + (r-g_c)T$. Thus, considering equation (A1), it follows that $\tau_c < \tau_w$ for $nR \geq (r-g_c)T$. Next consider the case $nR < (r-g_c)T$. By the same reasoning it then follows that $f[(r-g_w)R]\,f[(n+g_w-g_c)T] < f(nR)\,f[(r-g_c)T-(r-n-g_c)R] < f(nR)\,f[(r-g_c)T]$ which again implies $\tau_c < \tau_w$.

Proof of iii): Follows from i) and the continuity of τ_c and τ_w in R and T. *Q.E.D.*

Proof of Proposition 2

Let $g_c \leq g_w$. It will be shown that Proposition 2 holds true for $R = T$ Then, by continuity, the same holds true if $T - R$ is small. Thus, let $R = T$. Considering equations (6) and (7), it follows that $\tau_c > \tau_w$ is equivalent to:

$$e^{(r-g_w-n)S^*} f[(r-g_w)(R-S^*)]\,f[(n+g_w-g_c)R] > f[n(R-S^*)]\,f[(r-g_c)R].$$

Considering that $e^{-x} f(x) = f(-x)$, straightforward algebra reveals that the expression above is equivalent to:

$$f[-(r-g_w)(R-S^*)]\,f[-(n+g_w-g_c)R] > f[-n(R-S^*)]\,f[-(r-g_c)R].$$

For $\delta > \underline{\delta}$ it follows that $S^* > 0$ and therefore: $f[-n(R-S^*)]\,f[-(r-g_c)R] < f[-n(R-S^*)+(r-n-g_w)S^*]\,f[-(r-g_c)R] < f[-(r-g_w)(R-S^*)]\,f[-(n+g_w-g_c)R]$. The first inequality follows from Lemma 1.i) and the second inequality follows from Lemma 1.ii) and $g_c \leq g_w$. Thus, $\tau_c > \tau_w$. *Q.E.D.*

Proof of Proposition 3

Let $F < F+E < R < T$ and $g_c \leq g_w$. It will be shown that $\tau_c > \tau_w$ for $F = 0$ and $\alpha \to \infty$. Then, Proposition 3 follows by continuity. Thus, let $F = 0$ and observe that:

$$\lim_{\alpha\to\infty} \frac{\phi(n+g_w-g_c)}{\phi(r-g_c)} = \frac{f[(n+g_w-g_c)E]}{f[(r-g_c)E]}.$$

Considering equations (1) and (9) and $F = 0$, it follows that $\tau_c > \tau_w$ if:

$$f[(n+g_w-g_c)E]\,f[(r-g_w)R] > f[(r-g_c)E]\,f(nR). \qquad \text{(A2)}$$

Since $f(xE)/f(xR) = E(1-e^{-xR})/[R(1-e^{-xE})]$ strictly decreases in x for $E < R$, it follows from (A2) that $\tau_c > \tau_w$ if:

$$f[(n+g_w-g_c)R]\,f[(r-g_w)R] > f[(r-g_c)R]\,f(nR).$$

In light of Lemma 1.ii) this holds true for $g_c \leq g_w$ since $(n+g_w-g_c)R+(r-g_w)R=(r-g_c)R+nR$ as well as $r-g_c > \max\{n+g_w-g_c, r-g_w\}$ and $n < \min\{n+g_w-g_c, r-g_w\}$. *Q.E.D.*

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