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Abstract

The interaction of monetary and fiscal policies is a crucial issue in a highly integrated economic area as the European Union. We argue that EMU, which introduced a common monetary policy and restrictions on fiscal policy at the national level, increases the need for macroeconomic policy cooperation. To study the effects of policy cooperation we compare the effects of three alternative policy regimes in a stylized dynamic model of a monetary union: (i) noncooperative monetary and fiscal policies, (ii) partial cooperation, and (iii) full cooperation both in symmetric and asymmetric settings where countries differ in structural characteristics, policy preferences and/or bargaining power. The paper introduces an analysis of coalitional behavior in a dynamic setting into the literature.

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1 Introduction

Two years after the start of the Economic and Monetary Union (EMU), already a considerable amount of experience has accumulated on the functioning of monetary and fiscal policy in this new framework of macroeconomic policy design in the European Union (EU). Monetary policy has been delegated to a supra-national authority, the European Central Bank (ECB), with a complex framework of objectives, policy instruments and decision making procedures. According to the Maastricht Treaty, the ECB should safeguard price stability in the EMU and -subject to the condition that it does not interfere with price stability- promote economic growth in the EMU. Its policies are therefore directed at controlling economic developments of the EMU economy as a whole rather than on individual countries. The design of fiscal policies in the EMU is complicated by the set of constraints on national fiscal policy imposed by the Stability and Growth Pact (SGP). According to the SGP, excessive deficits are to be avoided and subject to sanctions.

In a highly integrated economic area like the EU, policy cooperation is likely to be of crucial importance because of the various interactions, spillovers and externalities from national macroeconomic policies and the complicated design and transmission of the common monetary policy of the ECB. This paper studies alternative regimes of macroeconomic policy cooperation and their effects in a dynamic model of macroeconomic adjustment in the EMU.

Macroeconomic policy cooperation has been one of the central issues in the theory and practice of macroeconomic policy design. This concerns both the coordination of macroeconomic policies within a country and the coordination of macroeconomic policies between different countries in the context of a multi-country setting where macroeconomic policy actions of one country are partly transmitted to the other countries through various channels in goods, labour, money and financial markets. These spillovers create the rationale for policy cooperation. Apart from full cooperation, also settings with partial cooperation, where only a subset of the players cooperates in their policies, have been studied in the literature. In those cases, in particular the effects of international monetary policy cooperation or international fiscal policy cooperation have been analysed, mostly in a static model framework.

The potential gains from cooperation are usually in the middle of the interest in studies of macroeconomic policy cooperation. Much less interest is given to the actual division of gains from policy cooperation. Typically, an egalitarian or a Nash bargaining division is assumed. In a multicountry context in general and the EMU in particular, issues of coalition formation, distributions of (voting) power, the distribution of cooperation gains and the stability of cooperative arrangements over time, however, are likely to be of crucial importance and to have a potential strong effect on policy making. In a dynamic setting these issues are even more important and complicated than in a static setting. This paper therefore analyses extensively these aspects of macroeconomic policy cooperation in the EMU, using a dynamic game approach.

Decision making procedures, coalition formation, voting power and rent sharing inside the EU institutions have been studied in detail. In an influential study Widgrén (1994) analyses voting power and coalition formation in the Council of Ministers and calculates using power indices how the balance of power in the Council changed by the entrance of Austria, Sweden and Finland in 1995. Four regional blocs are distinguished: the Franco-German axis, the Benelux countries, the Mediterranean countries and the Nordic countries. Laruelle and Widgrén (1996) analyse the fairness of voting power in the EU Council. Hosli (1996) also calculates the power distribution in the Council and analyses how it is affected when a qualified majority rule instead of a simple majority rule governs decision making. Bindseil and Hantke (1997) also consider the role of the European Commission and the European Parliament in communal decision making and determine how these affect voting powers and coalition formation. Bindseil (1996) studies coalition formation and power distribution inside the ECB Council. Sutter (1998) analyses voting power and coalition formation in the decision making about the sanctions on excessive deficits according to the Stability and Growth Pact. Levinsky and Silarsky (1998) calculate how the power distribution could be affected by the prospective Eastern Enlargement of the EU. These studies - while enabling us insights into issues of power distribution and coalition formation in communal policy formation -, however, do not consider a next step, namely to analyse the effects of coalition formation and power distribution on economic policies.

Three policy regimes can be distinguished for a monetary union as the EMU: (i) non-cooperative monetary and fiscal policies, (ii) partial cooperation, (iii) full cooperation. This paper introduces into the literature an analysis of coalition formation in a dynamic setting. Coalitions between countries and between the ECB and one country are studied in case (ii). In case (iii), full coordination of all macroeconomic policies, i.e. the national fiscal policies and the monetary policy of the ECB are implemented in a cooperative framework. We also consider the effects of asymmetries in players' preferences and structural parameters of the model.

Engwerda *et al.* (1999) have studied the effects of non-cooperative macroeconomic policies in the EMU. They analyse macroeconomic stabilisation among three players (two countries and the ECB) in a dynamic model of the EMU. Cooperation has been analysed in Hughes Hallett and Ma (1996), Acocella and Di Bartolomeo (2000) and Engwerda *et al.* (2001). Hughes Hallett and Ma (1996) find that asymmetries tend to increase the scope for policy cooperation. In their paper the asymmetric cases display for all players larger gains from cooperation than in the symmetric base scenario. This last result is also confirmed in this paper. Acocella and Di Bartolomeo (2000) analyse partial cooperation among a common central bank, trade unions and governments in a monetary union in a static framework. These authors found that, when players' loss functions are distinguished according to different objectives, monetary policy can compensate the governments' actions and neutralise expected coordination benefits of governments, but not those of unions. Engwerda *et al.* (2001) analyse macroeconomic adjustment under non-cooperative and full cooperative fiscal policy design in a monetary union using an open-loop dynamic games approach. They consider how the consequences of fiscal stringency requirements like the Stability and Growth Pact affect fiscal policy design under the EMU and study the introduction of a fiscal transfer mechanism among countries. These authors show that such system deteriorates the internal stability of the economies, but considerably reduces welfare costs.

The analysis is structured as follows: Section 2 proposes a simple dynamic model of the EMU and formulates the dynamic stabilisation game between the monetary and fiscal policy makers in this setup. Sections 3 and 4 study, theoretically, the various equilibria of this dynamic stabilisation game and the resulting design of the common monetary policy and the national fiscal policies. Section 5 studies in detail numerical examples to obtain a deeper insight into the economic properties of the model. The Appendices provide all details about algorithms and calculations in the analytical part of the paper.

2 A dynamic EMU model

To study macroeconomic policy design in the EMU we use the comprehensive model of the EMU used by Engwerda *et al.* (2001). There we considered the scenario where the EMU consists of two symmetric, equally sized countries that share a common central bank, the ECB. In this paper we consider also asymmetric settings and the symmetric Engwerda *et al.* (2001) model is interpreted as a benchmark scenario. The model ignores the external interaction of the EMU countries with the non-EMU countries and also the dynamic implications of government debt and net foreign asset accumulation. It consists of the following equations:

$$\begin{aligned}
y_1(t) &= \delta_1 s(t) - \gamma_1 r_1(t) + \rho_1 y_2(t) + \eta_1 f_1(t) && \text{(1a)}\\
y_2(t) &= -\delta_2 s(t) - \gamma_2 r_2(t) + \rho_2 y_1(t) + \eta_2 f_2(t) && \text{(1b)}\\
s(t) &= p_2(t) - p_1(t) && \text{(2)}\\
r_1(t) &= i_E(t) - \dot{p}_1(t) && \text{(3a)}\\
r_2(t) &= i_E(t) - \dot{p}_2(t) && \text{(3b)}\\
m_1(t) - p_1(t) &= \kappa_1 y_1(t) - \lambda_1 i_E(t) && \text{(4a)}\\
m_2(t) - p_2(t) &= \kappa_2 y_2(t) - \lambda_2 i_E(t) && \text{(4b)}\\
\dot{p}_1(t) &= \xi_1 y_1(t) && \text{(5a)}\\
\dot{p}_2(t) &= \xi_2 y_2(t) && \text{(5b)}
\end{aligned}$$

in which y denotes real output, s competitiveness of country 2 vis-à-vis country 1, r the real interest rate, p the price level, f the real fiscal deficit, i_E the nominal interest rate and m the nominal money balances. All variables are in logarithms, except for the interest rate which is in percentages, and denote deviations from their long term equilibrium (balanced growth path) that has been normalised to zero, for simplicity. A dot above a variable denotes its time derivative.

Equation (1) gives output in the EMU countries as a function of competitiveness in intra-EMU trade, the real interest rate, the foreign output and the domestic fiscal deficit. Competitiveness is defined in (2) as the output price differential. Real interest rates are defined in (3) as the difference between the EMU wide nominal interest rate, i_E, and domestic inflation. Note that (3) implies that, temporarily, real interest rates diverge among countries if inflation rates are different. (4) provides the demand for the common currency where it is assumed that the money market is in equilibrium.

It is assumed that the common interest rate is set by the ECB. Alternatively, we could have assumed - as in Engwerda *et al.* (1999) - that a monetary targeting strategy is implemented by the ECB. In that scenario, the ECB controls the common money supply and the common money market is cleared by the common interest rate. Whereas related, both approaches differ to some degree in their transmission of the ECB's monetary policy. Here the interest rate targeting approach is proposed which seems to be somewhat closer to the policy strategy adopted by the ECB in practice. Domestic output and inflation are related through a Phillips curve type relation in (5).

The model (1-5) can be reduced to two output equations:

$$\begin{aligned}
y_1(t) &= b_1 s(t) - c_1 i_E(t) + a_1 f_1(t) + \frac{\rho_1}{k_1} a_2 f_2(t) && \text{(6a)}\\
y_2(t) &= -b_2 s(t) - c_2 i_E(t) + \frac{\rho_2}{k_2} a_1 f_1(t) + a_2 f_2(t) && \text{(6b)}
\end{aligned}$$

in which $a_1 := \frac{\eta_1 k_2}{k_1 k_2 - \rho_1 \rho_2}$, $a_2 := \frac{\eta_2 k_1}{k_1 k_2 - \rho_1 \rho_2}$, $b_1 := \frac{\delta_1 k_2 - \rho_1 \delta_2}{k_1 k_2 - \rho_1 \rho_2}$, $b_2 := \frac{\delta_2 k_1 - \rho_2 \delta_1}{k_1 k_2 - \rho_1 \rho_2}$, $c_1 := \frac{\gamma_1 k_2 + \rho_1 \gamma_2}{k_1 k_2 - \rho_1 \rho_2}$, $c_2 := \frac{\gamma_2 k_1 + \rho_2 \gamma_1}{k_1 k_2 - \rho_1 \rho_2}$, $k_1 := 1 - \gamma_1 \xi_1$, $k_2 := 1 - \gamma_2 \xi_2$. The dynamics of the model are then represented by the following first-order linear differential equation with competitiveness, $s(t)$, as the scalar state variable, the national fiscal deficits, $f_i(t)$ $i = \{1, 2\}$, and the common interest rate, $i_E(t)$, as control variables:

$$\dot{s}(t) = -\phi_1 f_1(t) + \phi_2 f_2(t) + \phi_3 i_E(t) + \phi_4 s(t) \qquad s(0) =: s_0 \tag{7}$$

in which $\phi_1 := (\xi_1 - \xi_2 \frac{\rho_2}{k_2}) a_1$, $\phi_2 := (\xi_2 - \xi_1 \frac{\rho_1}{k_1}) a_2$, $\phi_3 := \xi_1 c_1 - \xi_2 c_2$ and $\phi_4 := -(\xi_2 b_2 + \xi_1 b_1)$. The initial value of the state variable, s_0, measures any initial disequilibrium in intra-EMU competitiveness. Such an initial disequilibrium in competitiveness could be the result of, e.g., differences in fiscal policies in the past or some initial supply side disturbance in one country.

We assume that the fiscal authorities control their fiscal policy instrument such as to minimise the following quadratic loss function which features the domestic inflation, output and fiscal deficit:

$$\min_{f_1} J^{F_1}(t_0) = \min_{f_1} \frac{1}{2} \int_{t_0}^{\infty} \{\alpha_1 \dot{p}_1^2(t) + \beta_1 y_1^2(t) + \chi_1 f_1^2(t)\} e^{-\theta(t-t_0)} dt \tag{8a}$$

$$\min_{f_2} J^{F_2}(t_0) = \min_{f_2} \frac{1}{2} \int_{t_0}^{\infty} \{\alpha_2 \dot{p}_2^2(t) + \beta_2 y_2^2(t) + \chi_2 f_2^2(t)\} e^{-\theta(t-t_0)} dt \tag{8b}$$

in which θ denotes the rate of time preference and α_i, β_i and χ_i represent preference weights that are attached to the stabilisation of inflation, output and fiscal deficits, respectively. Preference for a low fiscal deficit reflects the costs of excessive deficits such as proposed in the Stability and Growth Pact that sanctions such excessive deficits in the EMU. More in general, costs could also result from undesirable debt accumulation and intergenerational redistribution that high deficits imply and in that interpretation χ_i could also reflect the priority attached to fiscal retrenchment and consolidation.

As stipulated in the Maastricht Treaty, the ECB directs the common monetary policy at stabilising inflation and, as long as not in contradiction to inflation stabilisation, stabilising output in the aggregate EMU economy. Moreover, we will assume that the active use of monetary policy implies costs for the monetary policymaker: other things equal it would like to keep its policy instrument constant, avoiding large swings. Consequently, we assume that the ECB is confronted with the following optimization problem:

$$\min_{i_E} J^{ECB,A}(t_0) = \min_{i_E} \frac{1}{2} \int_{t_0}^{\infty} \{(\alpha_{1E} \dot{p}_1(t) + \alpha_{2E} \dot{p}_2(t))^2 + (\beta_{1E} y_1(t) + \beta_{2E} y_2(t))^2 + \chi_E i_E^2(t)\} e^{-\theta(t-t_0)} dt \tag{9a}$$

Alternatively, we could consider a case where the ECB is governed by national interests rather than EMU-wide objectives.[1] In that scenario, the ECB would be rather a coalition

[1]See also Gros and Hefeker (2000). These authors compare, in a static framework, a similar specification of the ECB's cost function with the standard one (9a) (e.g. Cukierman (1992)) and discuss the welfare implications of the two specifications. Also van Aarle *et al.* (1997) and De Grauwe (2000) compare outcomes under an ECB's objective function based on national and aggregate variables, respectively.

of the (former) national central banks that decide cooperatively on the common monetary policy but based on individual, national interests rather than EMU-wide objectives. In this scenario the monetary policy of the ECB will typically be more sensitive to individual country variables. In that case, the ECB seeks to minimize a loss function, which is quadratic in the individual countries' inflation rates and outputs -rather than in EMU-aggregate inflation and output as in (9a)- and the common interest rate,

$$\min_{i_E} J^{ECB,N}(t_0) = \min_{i_E} \frac{1}{2} \int_{t_0}^{\infty} \{\alpha_{1E}\dot{p}_1^2(t) + \alpha_{2E}\dot{p}_2^2(t) + \beta_{1E}y_1^2(t) + \beta_{2E}y_2^2(t) + \chi_E i_E^2(t)\} e^{-\theta(t-t_0)} dt \tag{9b}$$

Disregarding the monetary instrument, which in this paper is the interest rate rather than the money supply, equation (9a) can be seen as a generalisation of the loss function used in Engwerda *et al.* (1999), equation (14), p.262,

$$\min_{i_E} J^{ECB,N}(t_0) = \frac{1}{2} \int_{t_0}^{\infty} \{\alpha_E \dot{p}_E^2(t) + \beta_E y_E^2(t) + \chi_E i_E^2(t)\} e^{-\theta(t-t_0)} dt \tag{9c}$$

with $\alpha_{1E} := \omega\sqrt{\alpha_E}$ and $\alpha_{2E} := (1-\omega)\sqrt{\alpha_E}$, $\beta_{1E} := \omega\sqrt{\beta_E}$ and $\beta_{2E} := (1-\omega)\sqrt{\beta_E}^2$, where ω equals the weight defined in the average inflation rate and output: $\dot{p}_E := \omega\dot{p}_1 + (1-\omega)\dot{p}_2$ and $y_E := \omega y_1 + (1-\omega)y_2$. The loss function in (9b) is equivalent to a loss function in which the ECB is a coalition of national central bankers which all have a share in the decision making proportional to the size of their economies defined by ω, here. Hence, (9b) is equivalent to the minimisation of the following expression,

$$\min_{i_E} J^{ECB}(t_0) = \frac{1}{2} \int_{t_0}^{\infty} \{\alpha_E \left(\omega\dot{p}_1(t)^2 + (1-\omega)\,\dot{p}_2(t)^2\right) + \beta_E \left(\omega y_1(t)^2 + (1-\omega)\,y_2(t)^2\right) + \chi_E i_E^2(t)\} e^{-\theta(t-t_0)} d \tag{9d}$$

if: $\alpha_{1E} := \omega\alpha_E$ and $\alpha_{2E} := (1-\omega)\alpha_E$, $\beta_{1E} := \omega\beta_E$ and $\beta_{2E} := (1-\omega)\beta_E$.

Below, we will (for notational convenience) only elaborate problem (9a). In a similar way formulae can be obtained for the other performance criterion.

Using (5a) and (5b) we can rewrite (8a), (8b) and (9a) as follows,

$$J^{F_1} = \frac{1}{2} d_1 \int_{t_0}^{\infty} \{y_1^2(t) + \frac{\chi_1}{d_1} f_1^2(t)\} e^{-\theta(t-t_0)} dt, \tag{10a}$$

$$J^{F_2} = \frac{1}{2} d_2 \int_{t_0}^{\infty} \{y_2^2(t) + \frac{\chi_2}{d_2} f_2^2(t)\} e^{-\theta(t-t_0)} dt, \tag{10b}$$

$$J^{ECB} = \frac{1}{2} \int_{t_0}^{\infty} \{d_{1E}y_1^2(t) + d_{2E}y_2^2(t) + 2d_{3E}y_1(t)y_2(t) + \chi_E i_E^2(t)\} e^{-\theta(t-t_0)} dt \tag{10c}$$

where $d_i := \alpha_i\xi_i^2 + \beta_i$, $d_{iE} := \alpha_{iE}\xi_i^2 + \beta_{2E}$ with $i = \{1,2\}$ and $d_{3E} := \alpha_{1E}\alpha_{2E}\xi_1\xi_2 + \beta_{1E}\beta_{2E}$.

[2] α_E and β_E are the preference parameters w.r.t. inflation and output respectively of equation (14) in Engwerda *et al.* (1999), where $\alpha_E := 1$ because of normalisation.

Defining $x^T(t) := (s(t),\ f_1(t),\ f_2(t),\ i_E(t))$, we can rewrite (6a) and (6b) as,

$$y_1(t) = (b_1, a_1, \frac{\rho_1}{k_1}a_2, -c_1)x(t) =: m_1 x(t) \tag{11a}$$

$$y_2(t) = (-b_2, \frac{\rho_2}{k_2}a_1, a_2, -c_2)x(t) =: m_2 x(t) \tag{11b}$$

The policy makers' loss functions (10) can then be rewritten as:

$$J^{F_1} = \frac{1}{2}d_1 \int_{t_0}^{\infty} \{x^T(t)(m_1^T m_1 + \frac{\chi_1}{d_1}e_2^T e_2)x(t)\} e^{-\theta(t-t_0)} dt \tag{12a}$$

$$J^{F_2} = \frac{1}{2}d_2 \int_{t_0}^{\infty} \{x^T(t)(m_2^T m_2 + \frac{\chi_2}{d_2}e_3^T e_3)x(t)\} e^{-\theta(t-t_0)} dt \tag{12b}$$

$$J^{ECB} = \frac{1}{2} \int_{t_0}^{\infty} \{x^T(t)(d_{1E}m_1^T m_1 + d_{2E}m_2^T m_2 + 2d_{3E}m_1^T m_2 + \chi_E e_4^T e_4)x(t)\} e^{-\theta(t-t_0)} dt \tag{12c}$$

where e_j is the j^{th} vector of the basis of $\mathbb{R}^4$ (i.e. $e_1 := (1\ 0\ 0\ 0)^T$, *etc.*). Then considering:

$$M_{F_1} := \begin{pmatrix} b_1^2 & a_1 b_1 & a_2 b_1 \frac{\rho_1}{k_1} & -b_1 c_1 \\ a_1 b_1 & a_1^2 + \frac{\chi_1}{d_1} & a_1 a_2 \frac{\rho_1}{k_1} & -a_1 c_1 \\ a_2 b_1 \frac{\rho_1}{k_1} & a_1 a_2 \frac{\rho_1}{k_1} & a_2^2 \frac{\rho_1^2}{k_1^2} & -a_2 c_1 \frac{\rho_1}{k_1} \\ -b_1 c_1 & -a_1 c_1 & -a_2 c_1 \frac{\rho_1}{k_1} & c_1^2 \end{pmatrix}$$

$$M_{F_2} := \begin{pmatrix} b_2^2 & -a_1 b_2 \frac{\rho_2}{k_2} & -a_2 b_2 & b_2 c_2 \\ -a_1 b_2 \frac{\rho_2}{k_2} & a_1^2 \frac{\rho_2^2}{k_2^2} & a_1 a_2 \frac{\rho_2}{k_2} & -a_1 c_2 \frac{\rho_2}{k_2} \\ -a_2 b_2 & a_1 a_2 \frac{\rho_2}{k_2} & a_2^2 + \frac{\chi_2}{d_2} & -a_2 c_2 \\ b_2 c_2 & -a_1 c_2 \frac{\rho_2}{k_2} & -a_2 c_2 & c_2^2 \end{pmatrix}$$

and

$$M_E \ : \ = \begin{pmatrix} b_1^2 d_{1E} + b_2^2 d_{2E} - 2b_1 b_2 d_{3E} & a_1\left[b_1 d_{1E} - b_2 \frac{\rho_2}{k_2} d_{2E} + \left(b_1 \frac{\rho_2}{k_2} - b_2\right) d_{3E}\right] & a_2\left[b_1 \frac{\rho_1}{k_1} d_{1E} - b_2 d_{2E} + \left(b_1 - b_2 \frac{\rho_1}{k_1}\right) d_{3E}\right] & b_2 c_2 d_{2E} - b_1 c_1 d_{1E} - (b_1 c_2 - b_2 c_1) d_{3E} \\ a_1\left[b_1 d_{1E} - b_2 \frac{\rho_2}{k_2} d_{2E} - \left(b_2 - \frac{\rho_2}{k_2} b_1\right) d_{3E}\right] & a_1^2\left[d_{1E} + \frac{\rho_2^2}{k_2^2} d_{2E} + 2\frac{\rho_2}{k_2} d_{3E}\right] & a_1 a_2\left[\frac{\rho_1}{k_1} d_{1E} + \frac{\rho_2}{k_2} d_{2E} + \left(1 + \frac{\rho_1 \rho_2}{k_1 k_2}\right) d_{3E}\right] & -a_1\left[c_1 d_{1E} + c_2 \frac{\rho_2}{k_2} d_{2E} + \left(c_1 \frac{\rho_2}{k_2} + c_2\right) d_{3E}\right] \\ a_2\left[b_1 \frac{\rho_1}{k_1} d_{1E} - b_2 d_{2E} + \left(b_1 - b_2 \frac{\rho_1}{k_1}\right) d_{3E}\right] & a_1 a_2\left[\frac{\rho_1}{k_1} d_{1E} + \frac{\rho_2}{k_2} d_{2E} + \left(1 + \frac{\rho_1 \rho_2}{k_1 k_2}\right) d_{3E}\right] & a_2^2\left(\frac{\rho_1^2}{k_1^2} d_{1E} + d_{2E} + 2\frac{\rho_1}{k_1} d_{3E}\right) & -a_2\left[c_1 \frac{\rho_1}{k_1} d_{1E} + c_2 d_{2E} + \left(c1 + c2\frac{\rho_1}{k_1}\right) d_{3E}\right] \\ -b_1 c_1 d_{1E} + b_2 c_2 d_{2E} + (b_2 c_1 - b_1 c_2) d_{3E} & -a_1\left[c_1 d_{1E} + c_2 \frac{\rho_2}{k_2} d_{2E} + \left(c_1 \frac{\rho_2}{k_2} + c_2\right) d_{3E}\right] & -a_2\left[c_1 \frac{\rho_1}{k_1} d_{1E} + c_2 d_{2E} + \left(c_1 + c_2 \frac{\rho_1}{k_1}\right) d_{3E}\right] & c_1^2 d_{1E} + c_2^2 d_{2E} + 2c_1 c_2 d_{3E} + \chi_E \end{pmatrix}$$

the loss functions can also be written as:

$$J^{F_1} = \frac{1}{2}d_1 \int_{t_0}^{\infty} \{x^T(t) M_{F_1} x(t)\} e^{-\theta(t-t_0)} dt \quad (13a)$$

$$J^{F_2} = \frac{1}{2}d_2 \int_{t_0}^{\infty} \{x^T(t) M_{F_2} x(t)\} e^{-\theta(t-t_0)} dt \quad (13b)$$

$$J^{ECB} = \frac{1}{2} \int_{t_0}^{\infty} \{x^T(t) M_E x(t)\} e^{-\theta(t-t_0)} dt \quad (13c)$$

Henceforth, for reasons of convenience, we assume that $t_0 = 0$ and θ is equal to zero (if θ differs from zero, the model could be easily solved following the same procedure used in this paper after a simple transformation of variables[3]).

3 Macroeconomic policy design in the EMU

This section studies alternative modes of policy cooperation in a monetary union as the EMU. We study macroeconomic policy design and macroeconomic adjustment in three alternative macroeconomic policy regimes: (*i*) non-cooperative macroeconomic policies, (*ii*) full cooperation and (*iii*) partial cooperation. The first two regimes are standard in macroeconomic policy analysis. The regimes where subgroups of players form coalitions in which they coordinate their policies but interact in a non-cooperative manner with the players that are not part of the coalition, is not dealt with usually, certainly not in a dynamic context. This is not because such cases would be less interesting or less relevant in practice, but rather because of a lack of analytical tools to analyse such cases. In regimes of partial cooperation, important questions need to be answered, like (*i*) Why certain coalitions arise and others not?, (*ii*) Do these coalitions display stability over time?, (*iii*) How are the gains from cooperation distributed between the members of the coalitions?, (*iv*) How do differences in initial conditions, economic structures and policy preferences affect outcomes in this scenario? In this paper the issues (*iii*) and (*iv*) can be answered whereas some insight can be provided on the coalition formation issue. The stability issue will not be dealt with here.

A study of all three regimes is necessary for a complete insight on macroeconomic policy design in the EMU. The regimes with either fully non-cooperative or fully cooperative policies are clearly the two extreme forms of policy formation. Forms of partial cooperation combine elements of these opposite policy regimes as the following analysis shows.

3.1 The non-cooperative case

In the non-cooperative case players minimise their cost functions (13a-c) with respect to the dynamic law of motion (7) of the system. From Appendix A.1 we find as equilibrium strategies in the non-cooperative open-loop case:

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} a_1 b_1 - \phi_1 K_1 \\ -a_2 b_2 + \phi_1 K_2 \\ -b_1 c_1 d_{1E} + b_2 c_2 d_{2E} + (b_2 c_1 - b_1 c_2) d_{3E} + \phi_3 K_3 \end{pmatrix} s(t) =: H_{nc} s(t) \quad (14)$$

[3] That is, transforming $x(t)$ into $e^{-\frac{1}{2}\theta t}x(t)$ and substituting ϕ_4 by $\phi_4 - \frac{1}{2}\theta$ (see Engwerda *et al.* (1999), p.263, for further details).

where the contents of matrices G and K_i $(i = 1,2,3)$ can be obtained from the Appendix. Then, using these equilibrium controls (14) we obtain the corresponding fiscal players' optimal costs:

$$\begin{aligned} J^{F_i} &= \frac{1}{2}d_i\int_0^{\infty}\{x^T(t)M_{F_i}x(t)\}dt = \frac{1}{2}d_i\int_0^{\infty}\{(s\ f_1\ f_2\ i^E)M_{F_i}(s\ f_1\ f_2\ i^E)^T\}dt = \\ &= \frac{1}{2}d_i\int_0^{\infty}\{s(t)(1\ H_j^T)M_{F_i}\begin{pmatrix}1\\H_j\end{pmatrix}s(t)\}dt = \\ &= \frac{1}{2}d_i(1\ H_j^T)M_{F_i}\begin{pmatrix}1\\H_j\end{pmatrix}\int_0^{\infty}\{s^2(t)\}dt = \\ &= \frac{1}{2}d_i(1\ H_j^T)M_{F_i}\begin{pmatrix}1\\H_j\end{pmatrix}\int_0^{\infty}\{s_0^2e^{-2a_{cl,j}t}\}dt = \\ &= \frac{1}{2}d_i(1\ H_j^T)M_{F_i}\begin{pmatrix}1\\H_j\end{pmatrix}s_0^2\frac{1}{2a_{cl,j}} \qquad i=\{1,2\}\quad j=nc \end{aligned} \tag{15}$$

In the same way we find the costs of the ECB:

$$J^{ECB} = \frac{1}{2}(1\ H_j^T)M_E\begin{pmatrix}1\\H_j\end{pmatrix}s_0^2\frac{1}{2a_{cl,j}} \quad j=nc \tag{16}$$

Furthermore, the resulting closed-loop system is described by the differential equation $\dot{s}(t) = -a_{cl,nc}s(t)$ with $s(0) := s_0$, where the adjustment speed, $a_{cl,nc}$ is obtained as the positive eigenvalue of some related matrix that is defined in the Appendix.

3.2 The cooperative case

In the full cooperative case players minimise a common cost function: $J^C := \tau_1 J^{F_1} + \tau_2 J^{F_2} + \tau_3 J^{ECB}$ subject to (7); τ_i $(i = \{1,2,3\})$ equals the player i's bargaining power with $\tau_1+\tau_2+\tau_3 = 1$. In Appendix A.2 we show that the equilibrium cooperative controls can be written as:

$$\begin{pmatrix} f_1(t)\\ f_2(t)\\ i_E(t)\end{pmatrix} =: H_c s(t), \tag{17}$$

Using these controls the dynamic behaviour of this system is described as: $\dot{s}(t) = -a_{cl,c}s(t)$ with $s(0) =: s_0$, where $a_{cl,c}$ is again a positive eigenvalue of some matrix that is defined in the appendix. The corresponding costs for the players are (15,16) with $j = c$.

3.3 Cases with coalitions of policymakers

Coalition (1,2) To determine the equilibrium open-loop solution for the coalition of the fiscal players that cooperate upon fiscal policies but interact in a non-cooperative way with the ECB, we rewrite the dynamic law of motion (7) as:

$$\dot{s} = \phi_4 s + (-\phi_1\ \phi_2)\begin{pmatrix} f_1(t)\\ f_2(t)\end{pmatrix} + \phi_3 i_E(t) \qquad s(0) := s_0 \tag{18}$$

with cost functions:

$$J^{(1,2)} := \tau_1 J^{F_1} + \tau_2 J^{F_2} \text{ and } J^{ECB} \tag{19}$$

where $\tau_1 + \tau_2 = 1$. In Appendix A.3 it is shown that the equilibrium controls of the fiscal coalition case can be written as:

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} =: H_{(1,2)} s(t) \tag{20}$$

Through the application of these equilibrium controls the dynamic closed-loop expression of the behaviour of the system is described by: $\dot{s}(t) = -a_{cl,(1,2)} s(t)$ with $s(0) := s_0$ and the costs are (15,16) with $j = (1,2)$.

Coalition (1,3) To determine the equilibrium open-loop solution for a coalition of the fiscal authority of country 1 and the ECB that coordinate their policies but act in a non-cooperative fashion with the fiscal authority of country 2, we consider:

$$\dot{s} = \phi_4 s + (-\phi_1 \ \phi_3) \begin{pmatrix} f_1(t) \\ i_E(t) \end{pmatrix} + \phi_2 f_2(t) \qquad s(0) := s_0 \tag{21}$$

with cost functions:

$$J^{(1,3)} := \tau_1 J^{F_1} + \tau_2 J^{ECB} \text{ and } J^{F_2} \tag{22}$$

where $\tau_1 + \tau_2 = 1$. Introducing a redefinition of $x(t)$ corresponding with this (1,3) coalition form:

$$\tilde{x}(t) := \begin{pmatrix} s(t) \\ f_1(t) \\ i_E(t) \\ f_2(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} x(t) =: P_{(1,3)} x(t) \tag{23}$$

we can, basically, use the algorithm we used in the (1,2) coalition case to determine the equilibrium controls (see Appendix A.4)

$$\begin{pmatrix} f_1(t) \\ i_E(t) \\ f_2(t) \end{pmatrix} =: H_{(1,3)2} s(t). \tag{24}$$

Using these optimal controls, the dynamic closed-loop expression of the system is described by: $\dot{s}(t) = -a_{cl,(1,3)} s(t)$ with $s(0) := s_0$, where $a_{cl,(1,3)}$ is obtained as the eigenvalue of some matrix. The costs for the players are (15,16) with $j = (1,3)$.

Coalition (2,3) Finally, we consider the equilibrium open-loop solution for a coalition of the fiscal authority of country 2 and the ECB that coordinate their policies but act in a non-cooperative fashion with the fiscal authority of country 1. To determine this equilibrium we proceed analogous to the previous case. That is, we rewrite (7) as:

$$\dot{s} = \phi_4 s + (\phi_2 \ \phi_3) \begin{pmatrix} f_2(t) \\ i_E(t) \end{pmatrix} - \phi_1 f_1(t) \qquad s(0) := s_0 \tag{25}$$

with cost functions:

$$J^{(2,3)} := \tau_1 J^{F_2} + \tau_2 J^{ECB} \text{ and } J^{F_1} \tag{26}$$

where $\tau_1 + \tau_2 = 1$. Now, introducing the permutation of $x(t)$:

$$\tilde{x}(t) = \begin{pmatrix} s(t) \\ f_2(t) \\ i_E(t) \\ f_1(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} x(t) =: P_{(2,3)} x(t) \tag{27}$$

we find the equilibrium controls in a similar way again like in the (1,2) coalition case. The equilibrium controls can be written as:

$$\begin{pmatrix} f_2(t) \\ i_E(t) \\ f_1(t) \end{pmatrix} =: H_{(2,3)} s(t). \tag{28}$$

The dynamic closed-loop expression of the system is then again described by: $\dot{s}(t) = a_{cl,(2,3)} s(t)$ with $s(0) := s_0$, where $a_{cl,(2,3)}$ is a positive eigenvalue of some matrix. The optimal costs are then obtained analogously as (15,16) with $j = (2,3)$.

3.4 The symmetric case

In this section we consider the model described in the previous sections under the assumption of symmetry of country 1 and 2. In that case one can obtain theoretical results. The outcomes of this analysis are not only interesting on their own, but may be also helpful in analysing the properties of the non-symmetric model. We make the following assumptions w.r.t. the various parameters:

$\alpha_1 = \alpha_2 =: \alpha$; $\alpha_{1E} = \alpha_{2E} = \alpha$; $\beta_1 = \beta_2 =: \beta$; $\beta_{1E} = \beta_{2E} = \beta$; $\chi_1 = \chi_2$; $\xi_1 = \xi_2$; $\gamma_1 = \gamma_2$; $\rho_1 = \rho_2$; $\delta_1 = \delta_2$; $\eta_1 = \eta_2$ $\lambda_1 = \lambda_2$ and $\kappa_1 = \kappa_2$.

Furthermore we introduce for notational convenience the following parameters: $a := a_1$, $e := \frac{\rho_1}{k_1} a_2$, $c := c_1$; $b := b_1$ $d := d_1$; $g := \frac{\chi_1}{d}$ and $g_E := \frac{\chi_E}{d}$.
Then, the dynamics are given by the state equation,

$$\dot{s} = \phi_4 s - \phi_1 f_1 + \phi_1 f_2; \ s(0) = s_0 \tag{29}$$

Whereas the performance criteria reduce to:

$$J^i = \frac{1}{2}d\int_0^\infty \{x^T(t)M_i x(t)\}dt,\ i := \{F_1; F_2; ECB, A; ECB, N\} \tag{30}$$

with

$$M_{F_1} := \begin{pmatrix} b^2 & ab & be & -bc \\ ab & a^2+g & ae & -ac \\ be & ae & e^2 & -ce \\ -bc & -ac & -ce & c^2 \end{pmatrix}; M_{F_2} := \begin{pmatrix} b^2 & -be & -ab & bc \\ -be & e^2 & ae & -ce \\ -ab & ae & a^2+g & -ac \\ bc & -ce & -ac & c^2 \end{pmatrix};$$

$$M_{E,N} := \frac{1}{2}\begin{pmatrix} 2b^2 & b(a-e) & -b(a-e) & 0 \\ b(a-e) & a^2+e^2 & 2ae & -c(a+e) \\ -b(a-e) & 2ae & a^2+e^2 & -c(a+e) \\ 0 & -c(a+e) & -c(a+e) & 2c^2+2g_E \end{pmatrix};$$

and

$$M_{E,A} := \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & (a+e)^2 & (a+e)^2 & -2c(a+e) \\ 0 & (a+e)^2 & (a+e)^2 & -2c(a+e) \\ 0 & -2c(a+e) & -2c(a+e) & 4c^2+4g_E \end{pmatrix}.$$

3.5 The various equilibrium strategies.

The non-cooperative case

From Section 3 we immediately conclude that, provided it exists, the non-cooperative equilibrium strategy is given by

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i^E(t) \end{pmatrix} = -G^{-1}\begin{pmatrix} ab - \phi_1 K_1 \\ -ab + \phi_1 K_2 \\ 0 \end{pmatrix} s(t). \tag{31}$$

In Appendix B it is shown that in fact for both performance criteria of the ECB the equilibrium strategies coincide. Therefore, in both models all cost functions for the fiscal players yield the same outcome and the adjustment speed is the same. Furthermore it is shown that although in principle the number of equilibria may vary between zero and two, generically (in particular if $0 < e < a$), there will be a unique equilibrium. It turns out that the analysis performed by Engwerda *et al.* in (2001) (where the nominal interest rate was treated as an exogenous parameter) yields the same results as here for the non-cooperative case. Therefore, one may consult Table 2 and Figure 1 in that paper for a summary of the different situations that can occur. Introducing $u_{4nc} := a(a-e)+g$, we have that

$$K_1 = K_2 = K = \frac{2b^2 g}{-2\phi_4 u_{4nc} - b\phi_1(3a-e) + \sqrt{(-2\phi_4 u_{4nc} - b\phi_1(3a-e))^2 + 8gb^2\phi_1^2}}. \tag{32}$$

As a consequence,

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} ps(t), \tag{33}$$

with $p = -\frac{ab-\phi_1 K}{u_{4nc}}$. The closed-loop system is then given by

$$\begin{aligned} \dot{s}(t) &= (\phi_4 - 2\phi_1 p)s(t) \\ &= \frac{b\phi_1(a+e) - \sqrt{(-2\phi_4 u_{4nc} - b\phi_1(3a-e))^2 + 8gb^2\phi_1^2}}{2u_{4nc}} s(t) \\ &= : -a_{cl,nc} s(t);\ s(0) = s_0. \end{aligned} \tag{34}$$

with $s(0) = s_0$. Note that the parameter $-a_{cl,nc}$, which determines the convergence speed of the closed-loop system, coincides with the one obtained for the non-cooperative game in Engwerda *et al.* (2001). The corresponding cost for the players is

$$J^{F_1} = J^{F_2} = \frac{1}{4}\frac{d}{a_{cl,nc}}\{(b + p(a-e))^2 + p^2 g\} s^2(0) \tag{35}$$

$$J^{ECB,N} = \frac{1}{4}\frac{d}{a_{cl,nc}}(b + p(a-e))^2 s^2(0) \text{ and } J^{ECB,A} = 0. \tag{36}$$

<u>The cooperative case</u>

Here we assume that $\tau_1 = \tau_2 = \tau$ and $\tau_3 = 1 - 2\tau$, where $0 \le \tau \le \frac{1}{2}$.
First, we consider the performance criterion (9b). Substitution of the parameters shows (see Appendix B) that the unique cooperative equilibrium strategy in that case equals,

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = -R^{-1} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} (\frac{1}{2}b(a-e) - \phi_1 K)s(t) \tag{37}$$

Introducing $u_{4c} := 2\tau g + (a-e)^2$, we have (see Appendix B) that

$$K = \frac{\phi_4 u_{4c} + 2\phi_1 b(a-e) + u_{4c} a_{cl,cN}}{4\phi_1^2}. \tag{38}$$

Here $a_{cl,cN}^2$, the squared value of the closed-loop system parameter, is $\phi_4^2 + 4\phi_1 b \frac{\phi_4(a-e)+b\phi_1}{u_{4c}}$, which can be rewritten as $\phi_4^2 \frac{2\tau g}{u_{4c}}$ (using the fact that $\phi_4(a-e) = -2b\phi_1$). As a consequence,

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} p_{cN} s(t), \tag{39}$$

with $p_{cN} = \frac{-b(a-e)+2\phi_1 K}{u_{4c}}$.
The corresponding cost for the players is given by (35,36), with p replaced by p_{cN} and $a_{cl,nc}$ by $a_{cl,cN}$.
In a similar way (see Appendix B again for details) we obtain for the aggregate performance

criterion that the squared value of the closed-loop system parameter if the equilibrium strategies are applied, $a^2_{cl,cA}$, is $\frac{\phi_4^2 g}{g+(a-e)^2}$. Introducing $u_{4cA} := g + (a-e)^2$, the optimal feedback gain K is in this case $\tau\frac{\phi_4 u_{4cA}+2\phi_1 b(a-e)+u_{4cA}a_{cl,cA}}{2\phi_1^2}$. It is then easily verified that the corresponding equilibrium strategies are,

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} p_{cA}s(t), \tag{40}$$

with $p_{cA} := -\frac{\tau(b-a)-\phi_1 K}{\tau u_{4cA}}$.
Note that both $a_{cl,cA}$ and p_{cA} are independent of τ. As a consequence, the corresponding cost for the fiscal players (obtained by substituting p_{cA} for p and $a_{cl,cA}$ for $a_{cl,nc}$ in (35,36), is independent of τ too. Furthermore it is easily verified that the cost for the ECB are zero again in this case.

The coalition form (1,2)

Here we assume that $\tau_1 = \tau_2 = \frac{1}{2}$. Substitution of the parameters shows (see Appendix B) that the unique equilibrium strategies coincide for both performance criteria of the ECB and that it is given by,

$$\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} p_{(1,2)}s(t) \tag{41}$$

where $p_{(1,2)} = -\frac{b(a-e)-2\phi_1 K}{g+(a-e)^2}$. Introducing $u_{4,(1,2)} := g + (a-e)^2$, we have that

$$K = -\frac{gb^2}{u_{4,(1,2)}\phi_4 + 2b\phi_1(a-e) - u_{4,(1,2)}a_{cl,(1,2)}}. \tag{42}$$

Here $a^2_{cl,(1,2)}$, the squared value of the closed-loop system parameter, is $\phi_4^2 + 4\phi_1 b\frac{\phi_4(a-e)+b\phi_1}{u_{4,(1,2)}}$ which can also be rewritten as $\phi_4^2\frac{g}{u_{4,(1,2)}}$. Furthermore, the corresponding cost for the players is given by (15,16), with p replaced by $p_{(1,2)}$ and $a_{cl,nc}$ by $a_{cl,(1,2)}$. It is easily verified that the aggregate cost function for the ECB is again zero.

The coalition form (1,3) and (2,3)

If the coalition (1,3) occurs (or its symmetric counterpart (2,3)) the ECB is directly involved in the game (i.e. the common interest rate differs in general from zero). As a consequence the theoretical formulae become much more involved. Therefore they are omitted here.

3.6 Some general conclusions

First, we summarize the conclusions w.r.t. the number of equilibria that may appear in the game.

Theorem 1:
For the cooperative and (1,2) coalition the game has always a unique equilibrium. If $e < a$

the non-cooperative game also has a unique equilibrium. If $e \geq a$ the number of equilibria may vary between zero and two (see Engwerda *et al.* (2001, table 2) for details). $\square$

We will restrict in the rest of this section to the case that $e < a$ and will assume, moreover, that as well $-\phi_4$ as a are positive. For a broad class of realistic model parameters these assumptions hold. As a consequence, the non-cooperative game has a uniquely defined equilibrium. Furthermore, unless stated otherwise, we will restrict our analysis to the non-cooperative, the cooperative and the coalition (1,2) form.
Two striking things we observe from the previous section are that $f_1 = -f_2$ and that the ECB does not influence the game, neither in a direct way (i.e. $i^E = 0$) nor in an indirect way (i.e. via its parameters). These statements do not hold for the coalition (1,3). There, the fiscal instruments differ and the ECB uses its instruments actively to reach its goals. The symmetry assumptions are crucial too, if they are dropped the ECB gets also actively involved into the game.

Since we have explicit formulae for the various cost functions we can exploit these to derive some further general conclusions. Our first observation is that the convergence speed of the closed-loop system satisfies some nice properties:

Lemma 2:
i) $a_{cl,cN} \leq a_{cl,(1,2)}$.
ii) $a_{cl,nc} \leq a_{cl,cN}$ if $\tau \geq \frac{1}{3}$.
iii) $a_{cl,i}(g)$ is a monotonically increasing function with $a_{cl,i}(0) = 0$ and $a_{cl,i}(\infty) = -\phi_4$, $i = nc, c, (1,2)$.
iv) $a_{cl,cN} \leq a_{cl,cA}$. $\square$

The proof can be found in Appendix C.
With respect to the performance criteria we first note that the cost functions for the fiscal players are the same in the coalition case, with the ECB considering an aggregate performance criterion, and the (1,2) coalition (in which case the performance criterion considered by the ECB does not play any role as we already noted above). In other words, the fiscal players are indifferent between these modes of play. The proof of this property is most easily seen by substituting $\tau = \frac{1}{2}$ into the formulae we derived for the "aggregate" coalition case (as we already noted there, in fact the cost are independent of τ. Since it is easily verified that various formulae coincide for $\tau = \frac{1}{2}$, this immediately shows the correctness of the claim for an arbitrarily chosen τ).

Our next results concern the performance criterion (9b). We show, amongst others, that the ECB will prefer a noncooperative above a cooperative mode of play if the cooperation parameter τ becomes large and that the fiscal players will prefer a partial coalition above a cooperative mode of play. The proof is again deferred to the Appendix C. We used the notation $sgn(a)$ here to denote the sign of variable a.

Lemma 3:
i) $sgn(J^E_{nc} - J^E_{cN}) = sgn(a_{cl,nc} - a_{cl,cN})$.
ii) $J^{F_i}_c \geq J^{F_i}_{(1,2)}$. $\square$

From lemma 2.ii) we have that if, e.g., $\tau \geq \frac{1}{3}$, always $J^{F_i}_c \geq J^{F_i}_{(1,2)}$. A more detailed analysis shows that if $\tau = 0$, $J^{F_i}_c < J^{F_i}_{(1,2)}$, and therefore it is easily seen from the proof of lemma 2.ii) that there is always a threshold ω^* such that for all $\tau \geq \tau^*$, $J^{F_i}_c \geq J^{F_i}_{(1,2)}$ and for all $\tau < \tau^*$,

$J_c^{F_i} < J_{(1,2)}^{F_i}$.
Now, consider the case that $\tau \geq \tau^*$. Since aggregate performance is minimized in the cooperative situation and according lemma 2.ii) the ECB's cost are higher in this situation than in the non-cooperative case, the cost of the fiscal players will be less in the cooperative mode of play than in the non-cooperative case. A similar reasoning shows that since $J_c^{F_i} \geq J_{(1,2)}^{F_i}$, the cost of the ECB in the coalition (1,2) mode of play will always be larger than in the cooperative case. Stated differently, we see that under this assumption the ECB will always prefer the non-cooperative mode of play, whereas the fiscal players prefer the coalition (1,2) form. So, summarizing, we have:

Theorem 4:
Assume that the ECB considers the performance criterion (9b). Then, if $\tau \geq \tau^*$, the cooperative mode of play is unsustainable. Here, $\tau^* < \frac{1}{3}$. □

4 A simulation study

4.1 Some coalition formation terminology

In order to obtain some insight into the question which coalition might be realized and which are less plausible, we introduce some terminology. Each of the five policy regimes outlined in the above subsections is called a *coalition form* and each group of two or more players that cooperate in a coalition form a *coalition*. We say that a certain coalition form is *supported by player i*, if player i has no incentive to deviate from this coalition form. If a coalition form has a coalition, then we say that this coalition form is *internally supported* if all players in the coalition support the coalition form. A coalition form is called *externally supported* if all players outside the coalition support the coalition form. If a coalition form is both externally and internally supported, then we will call this coalition form *sustainable*, that is, in such a coalition form no player has an incentive to deviate (leave this coalition form). Finally, we call a coalition form *unsustainable* if all players have an incentive to deviate from this coalition form. So, as well the players inside as outside the coalition can improve by joining another coalition form.
Note that a coalition form which is internally supported is in principle viable. One reason why such a coalition form might not be realized is that e.g. side-payments take place. Here we will ignore these issues. A similar remark holds w.r.t. the unsustainable coalition form. Such a coalition form is in principle not viable, this contrary to a coalition form which is partially supported (i.e. supported by not all players in the coalition). Such a coalition form might be viable, but this typically depends on what other coalition forms have to offer for all the different player(s). So, this requires a more detailed description of the negotiation process, something we will not go into here. The notions introduced above will in particular be used in the simulation study.

4.2 Numerical simulations

In this section we consider the differential game on macroeconomic stabilisation in the EMU that was set up in Section 2, using simulations of a stylised example. We analyse seven different simulations divided over four scenarios: (i) two symmetric cases: a baseline case in which all structural and preference parameters are the same in both countries and the ECB (i.e. the case treated in Section 4), a symmetric case with alternative, more realistic preference parameters of the ECB (i.e. the ECB's preference parameters are assumed to be opposite to those of the governments), (*ii*) two asymmetric cases where countries differ in monetary and fiscal policies transmissions, (*iii*) two asymmetric cases where countries differ in degrees of openness and competitiveness, (*iv*) an asymmetric case where countries differ in bargaining powers in case they enter coalitions.

In the symmetric baseline case, the countries are equally weighted in the ECB's loss function and the following values for the structural model parameters are used[4]: $\gamma = 0.4$, $\delta = 0.2$, $\rho = 0.4$, $\eta = 1$, $\kappa = 1$, $\lambda = 1$ and $\xi = 0.25$. The initial state of the monetary union economy is $s_0 = 0.05$ (implying an initial disequilibrium of 5 % in competitiveness between the two countries). Concerning the preference weights in the objective functions of the players, the following values have been assumed: $\alpha = 2$, $\alpha_E = 2$, $\beta = 5$, $\beta_E = 5$, $\chi = 2.5$ and $\theta = 0.15$. In the second symmetric simulation, an ECB that cares more about inflation than output is assumed: $\alpha_E = 2.5$, $\beta_E = 1$. In the analysis of asymmetric policy transmissions, we analyse the effects of different monetary policy transmissions between countries and the effects of different fiscal policy transmissions. Therefore, in the monetary transmission analysis the first country has a smaller output semi-elasticity of the real interest rate ($\gamma_1 = 0.4$) than the second country ($\gamma_2 = 0.8$). In the fiscal transmission analysis the first country has a higher output elasticity of the fiscal deficit ($\eta_1 = 1$) than the second country ($\eta_2 = 0.6$). In the third scenario different degrees of openness and competitiveness are considered. First, we assume that the first country is less open ($\rho_1 = 0.4$) than the second country ($\rho_1 = 0.8$). After, we assume that the first country's output elasticity of competitiveness is lower ($\delta_1 = 0.2$) than that of the second country ($\delta_2 = 0.4$). Finally, different bargaining powers are assumed according to the following scheme: $\tau^c = \{3/6, 1/6, 2/6\}$, $\tau^{12} = \{3/4, 1/4\}$, $\tau^{13} = \{3/5, 2/5\}$, $\tau^{23} = \{1/3, 2/3\}$. In this way our analysis contributes to the important discussion about the effects of EMU in the case where countries differ in their structural characteristics.

Outcomes are analysed for all the five different equilibria outlined in Sections 3 and 4: case 1 is the non-cooperative equilibrium, case 2 is the full cooperative equilibrium, and cases 3 to 5 are partial cooperative equilibria (case 3 refers to the fiscal coalition (1,2), and cases 4 and 5 to the coalitions between the ECB and a fiscal player, i.e. (1,3) and (2,3)).

1. *A symmetric EMU*

In this first example, starting point is a situation where countries are symmetric and where the fiscal players and the monetary player feature symmetric preferences, $\alpha_{1E} = \alpha_{2E} = \alpha$; $\beta_{1E} = \beta_{2E} = \beta$. In Table 1 the resulting losses in the five different cases of this symmetric baseline case are given. In this symmetric case we recognise the features that we have derived analytically in Section 4 for the case of the loss function (9b) of the ECB: adjustment speed (measured by the size of the a_{cl}s) is fastest under fiscal cooperation and the Pareto case in an unsustainable equilibrium. Moreover, the government's coalition is internally supported. In the case the ECB is using aggregate variables in its objective function, we know from the analysis in the previous section that the cost in the Pareto and fiscal coalition form coincide. Both are sustainable in this case, whereas the coalitions (1,3) and (2,3) are unsustainable. Note that both these last coalitions are supported by the ECB for the loss function (9b).

Table 1 - Cost functions (multiplied by 1,000)[5]

[4]See Engwerda *et al.* (2000) for a similar simulation set up.

[5]In all tables, rows 4 to 6 indicate the pay-off of the coalition weighted by the bargaining powers (yielding

	Nash		Pareto		(1,2)		(1,3)		(2,3)	
	Aggr.	*Nat.*	*Aggr.*	*Nat*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*
J^{F_1}	0.3596	0.3596	0.3032	0.3396	0.3032	0.3032	0.3634	0.4062	1.9002	2.1695
J^{F_2}	0.3596	0.3596	0.3032	0.3396	0.3032	0.3032	1.9002	2.1695	0.3634	0.4062
J^{ECB}	0	0.1675	0	0.2016	0	0.3750	0.0036	0.1248	0.0036	0.1248
J^C	–	–	0.2021	0.2936	–	–	–	–	–	–
$J^{(1,2)}$	–	–	–	–	0.3032	0.3032	–	–	–	–
$J^{(1,3)}$	–	–	–	–	–	–	0.1835	0.2655	–	–
$J^{(2,3)}$	–	–	–	–	–	–	–	–	0.1835	0.2655
a_{cl}	0.1007	0.1007	0.1162	0.1035	0.1162	0.1162	0.0998	0.0955	0.0998	0.0955

2. *Opposite preferences of the fiscal players and the ECB*

In the second symmetric simulation, an ECB that cares more about inflation than output is assumed: $\alpha_E = 2.5$, $\beta_E = 1$. Since this is more realistic than the previous simulation this will be considered as our benchmark case. Figure 1a displays the adjustment in case the aggregate objective function (9a) of the ECB is used and Figure 1b displays the case where the national objective function (9b) of the ECB is used,

[Insert Figure 1a and 1b here]

The adjustment of intra-EMU competitiveness is given in panel (a). The adjustment of the policy variables are found in panels (b)-(d). The common interest rate, panel (b), only reacts in the case of a coalition with one fiscal policy maker: in that case the common interest rate is partly targeted at the situation in the country with which the ECB has formed a coalition. This leads to a higher interest rate in case a coalition is formed with country 1 and a lower interest rate when a coalition is formed with country 2. Panels (e) and (f) display output in country 1 and 2 in the different cases. The initial disequilibrium in intra-EMU competitiveness implies that country 1 is initially above the long-run equilibrium in country 1 and below the long-run equilibrium in country 2. A comparison of Figure 1a and Figure 1b, shows that there are significant differences even in this symmetric case. In particular, we confirm the earlier mentioned feature that in the second case the monetary policy of the ECB is more sensitive to conditions in individual countries than in the first case.

Table 2 gives the resulting welfare losses that the players incur in this example. A detailed look at the figures shows that the same conclusions hold as in the previous case w.r.t. the coalition formation issue.

Table 2 - Cost functions (multiplied by 1,000)

as cost J^k for coalition k) of the corresponding cooperative players, while J^C indicates the pay-off of the grand coalition. The columns *Aggr.* and *Nat.* indicate optimal values and absolute values of eigenvalues when the ECB's loss functions are (9a) and (9b), respectively.

	Nash		Pareto		(1,2)		(1,3)		(2,3)	
	Aggr.	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*
J^{F_1}	0.3596	0.3596	0.3032	0.3062	0.3032	0.3032	0.3634	0.4111	1.9002	2.4289
J^{F_2}	0.3596	0.3596	0.3032	0.3062	0.3032	0.3032	1.9002	2.4289	0.3634	0.4111
J^{ECB}	0	0.0378	0	0.0721	0	0.0846	0.0036	0.0314	0.0036	0.0314
J^C	–	–	0.2021	0.2282	–	–	–	–	–	–
$J^{(1,2)}$	–	–	–	–	0.3032	0.3032	–	–	–	–
$J^{(1,3)}$	–	–	–	–	–	–	0.1835	0.2212	–	–
$J^{(2,3)}$	–	–	–	–	–	–	–	–	0.1835	0.2212
a_{cl}	0.1007	0.1007	0.1162	0.1124	0.1162	0.1162	0.0998	0.0937	0.0998	0.0937

Comparing Tables 1 and 2 we note two major observations. First, all equilibria based on aggregate variables are identical which means that under symmetric goverments' preferences varying ECB's preferences do not matter. Second, the optimal costs and eigenvalues belonging to the aggregate case are identical for the Pareto and the (1,2) coalition form.

3. *Asymmetric monetary policy transmission*

In this example asymmetric monetary transmission is analysed: the base setting of case 2 is assumed, except that the first country has a smaller output semi-elasticity of the real interest rate ($\gamma_1 = 0.4$) than the second country ($\gamma_2 = 0.8$). This example nicely illustrates the discussion about the effects of a common monetary policy in a situation where countries differ in the transmission of monetary policy. Figures 2a and 2b graph the resulting adjustments,

[Insert Figure 2a and 2b here]

In this asymmetric setting, the adjustment and policy strategies are no longer symmetric in both countries. The ECB now reacts in all strategic settings as it objective functions imply that its optimal strategy instrument is sensitive to any asymmetry, in particular in the case where national variables dominate the ECB preferences.

Table 3 shows the losses in this case where we immediately observe that there is no sustainable coalition neither for aggregate variables nor for national variables. We further observe that for both welfare loss functions the Pareto coalition form is supported by player 2, whereas player 1 supports the governments' coalition form. Furthermore we see that both other partial equilibria forms are unsustainable. Note that the fiscal coalition is now only second best for country 2. Because of its stronger exposure now to the monetary policy of the ECB, country 2 would prefer the ECB to be included into the policy cooperation. However, the ECB is not benefiting from full cooperation, it incurs a relatively large cost by joining this coalition form (in the aggregate case). Therefore one might expect a (1,2) coalition here. For the other performance criterion things are less clear. Another point we like to note is that, when governments cooperate, the coalition cost is higher than under the benchmark case (i.e. Table 2).

Table 3 - Cost functions (multiplied by 1,000)

	Nash		Pareto		(1,2)		(1,3)		(2,3)	
	Aggr.	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Agr.*	*Nat.*
J^{F_1}	0.3593	0.3799	0.3311	0.3528	0.3036	0.3196	0.3452	0.5537	1.9640	3.0381
J^{F_2}	0.4101	0.3745	0.2738	0.2548	0.3276	0.2985	2.2414	1.7762	0.4058	0.3306
J^{ECB}	0.0005	0.0347	0.0084	0.0804	0.0001	0.0860	0.0036	0.0403	0.0123	0.0658
J^C	–	–	0.2044	0.2293	–	–	–	–	–	–
$J^{(1,2)}$	–	–	–	–	0.3156	0.3091	–	–	–	–
$J^{(1,3)}$	–	–	–	–	–	–	0.1744	0.2970	–	–
$J^{(2,3)}$	–	–	–	–	–	–	–	–	0.2091	0.1982
a_{cl}	0.0995	0.0993	0.1156	0.1116	0.1166	0.1163	0.0996	0.0913	0.0980	0.0903

4. *Asymmetric fiscal policy transmission*

Not only differences in the transmission of monetary policy are likely to prevail under EMU, also different transmissions of fiscal policies are likely to be present. In this example, we analyse the consequences of such differences in fiscal policy transmission. To do so, assume that the first country has a higher output elasticity of the fiscal deficit ($\eta_1 = 1$) than the second country ($\eta_2 = 0.6$). In that case the following adjustment patterns result,

[Insert Figure 3a and 3b here]

Optimal policies and adjustment are much different from the base case and the case with asymmetric monetary policy transmission. With its instrument being less effective at stabilising domestic conditions, country 2 has to use its instrument with a larger intensity compared to the symmetric case 2. However, this use is also costly for the players as is seen in Table 4 where country 2's losses are typically higher than in the base case. Coalition form (1,3) is now sustainable for the national performance case and, since the additional cost in the aggregate case between the Pareto and (1,3) coalition form is only minor, one might also expect that the (1,3) coalition form will be chosen by the players in the aggregate case. This conclusion is strengthened by the fact that if the fiscal players would join the Pareto coalition (which is prefered by the ECB) then both countries would even be better of in forming a fiscal coalition. Note that the (1,2) and (2,3) coalition forms are unsustainable in this case.

Table 4 - Cost functions (multiplied by 1,000)

	Nash		Pareto		(1,2)		(1,3)		(2,3)	
	Aggr.	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*
J^{F_1}	0.4988	0.3190	0.3388	0.3162	0.3316	0.2941	0.2992	0.2664	2.3920	2.4809
J^{F_2}	0.3159	0.5235	0.3737	0.4146	0.3696	0.4253	2.4378	3.4130	0.4555	0.4832
J^{ECB}	0.2062	0.1717	0.0111	0.2232	0.0294	0.2712	0.0117	0.1698	0.0941	0.1934
J^C	–	–	0.2412	0.3180	–	–	–	–	–	–
$J^{(1,2)}$	–	–	–	–	0.3506	0.3597	–	–	–	–
$J^{(1,3)}$	–	–	–	–	–	–	0.1555	0.2181	–	–
$J^{(2,3)}$	–	–	–	–	–	–	–	–	0.2748	0.3383
a_{cl}	0.1080	0.1103	0.1260	0.1187	0.1250	0.1262	0.1173	0.1092	0.1045	0.1031

5. *Asymmetric degree of openness*

In the third scenario different degrees of openness and competitiveness are considered. These type of asymmetries affect directly the interaction between both countries as they impact upon the transmission of intra-EMU trade in the model. First, we assume that the first country is less open ($\rho_1 = 0.4$) than the second country ($\rho_2 = 0.8$). The adjustment dynamics that result are displayed in Figure 4a and Figure 4b.

[Insert Figure 4a and 4b here]

Since imports from country 2 stabilise output in country 1, country 1 is adversely affected in principal from this change, compared to the base case and it needs to use its fiscal instrument more actively. The adjustment burden for country 2 on the other hand is alleviated by the larger stabilization problems in country 1. This is also seen in the resulting losses that are displayed in Table 5. Investigating Table 5, we observe that the less open economy features the higher stabilisation cost (with the exception of coalition (1,3)). The fiscal coalition form is supported by country 2, whereas the Pareto coalition form is supported by country 1. So, both countries are in principle interested in cooperation with eachother, but one country likes to include the ECB in this cooperation too, whereas the other dislikes this idea. Since the ECB also supports the Pareto coalition form in the aggregate case this coalition form might ultimately be favoured by the players in that case. Note that the (1,3) coalition form is again unsustainable. This is also the case for the (2,3) coalition form if the aggregate performance is considered. Finally we note that the ECB supports the (2,3) coalition.

Table 5 - Cost functions (multiplied by 1,000)

	Nash		Pareto		(1,2)		(1,3)		(2,3)	
	Aggr.	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*
J^{F_1}	0.3909	0.4267	0.2968	0.3247	0.3090	0.3341	0.8324	0.3708	6.1928	3.1776
J^{F_2}	0.3782	0.3422	0.2367	0.2101	0.2185	0.1971	1.0896	2.2075	0.5302	0.6624
J^{ECB}	0.0044	0.0229	0.0016	0.0695	0.0098	0.0792	0.2763	0.0382	3.0762	0.0225
J^{C}	–	–	0.1784	0.2014	–	–	–	–	–	–
$J^{(1,2)}$	–	–	–	–	0.2637	0.2656	–	–	–	–
$J^{(1,3)}$	–	–	–	–	–	–	0.5544	0.2045	–	–
$J^{(2,3)}$	–	–	–	–	–	–	–	–	1.8032	0.3425
a_{cl}	0.0943	0.0943	0.1146	0.1111	0.1141	0.1143	0.0889	0.0935	0.1190	0.0811

6. *Asymmetric degree of competitiveness*

Next, we assume that the first country's output elasticity of competitiveness is lower ($\delta_1 = 0.2$) than that of the second country ($\delta_2 = 0.4$). Such an asymmetry in the sensitiveness to competitive pressures has quite a dramatic impact as Figures 5a and 5b show,

[Insert Figure 5a and 5b here]

In this case there are also marked differences between the cases where the ECB objectives are governed by aggregate variables and where it is governed by national variables. In this case all coalition forms are not internally supported. Coalitions (1,3) and (2,3) are not eternally supported, while the coalition (1,2) is externally supported. Interestingly, the aggregate coalition form (1,2) is internally supported by player 1 and externally supported by the ECB. A coalition between the ECB and player 1 is, however, out of order. Which coalition ultimately will result is unclear here. For the national preference case things are more or less reversed, player 1 and the ECB prefer here the Nash solution and player 2 supports a cooperative mode of play (Pareto).

Table 6 - Cost functions (multiplied by 1,000)

	Nash		Pareto		(1,2)		(1,3)		(2,3)	
	Aggr.	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Aggr.*	*Aggr.*	*Nat.*
J^{F_1}	0.6828	0.2634	0.4189	0.3307	0.3207	0.3112	1.4917	0.8834	1.0647	2.5967
J^{F_2}	0.6362	1.3413	0.8633	0.9549	0.9845	1.0203	11.124	8.1502	3.2270	1.1729
J^{ECB}	0.8424	0.1075	0.0412	0.1949	0.0212	0.1901	3.5577	0.3134	0.4547	0.2628
J^C	–	–	0.4412	0.4935	–	–	–	–	–	–
$J^{(1,2)}$	–	–	–	–	0.6526	0.6658	–	–	–	–
$J^{(1,3)}$	–	–	–	–	–	–	2.5247	0.5984	–	–
$J^{(2,3)}$	–	–	–	–	–	–	–	–	1.8408	0.7178
a_{cl}	0.1143	0.1153	0.1413	0.1355	0.1411	0.1421	0.1667	0.0959	0.0894	0.1074

7. *Asymmetric bargaining powers*

Finally, different bargaining powers are assumed according to the following scheme: $\tau^c = \{3/6, 1/6, 2/6\}$, $\tau^{12} = \{3/4, 1/4\}$, $\tau^{13} = \{3/5, 2/5\}$, $\tau^{23} = \{1/3, 2/3\}$, implying that in a coalition country 1 has three times as many votes as country 2 and 1,5 as many votes as the ECB, whereas the ECB has two times as many votes as country 2. This asymmetric bargaining power case leads to the following adjustment dynamics,

[Insert Figure 6a and 6b here]

In this case player 1 supports the (1,2) coalition form. Furthermore, The (1,3), (2,3) aggregate and Pareto national coalition forms are unsustainable. Since none of the coalitions is supported by more than one player, the Nash outcome might be the ultimate outcome in this case. Therefore, comparing the results of Table 7 with that of that of Table 2, we observe that the introduction of asymmetric bargaining powers crucially change the results of the game. The asymmetry increases the cost of the country with the smaller bargaining power as its importance in a coalition is reduced, while it decreases the costs of the other country. To put it in a general way: more asymmetric bargaining powers reduce the probabilities of coalitions -and therefore of policy cooperation- as policies will be biased towards the needs of the stronger player(s), and the smaller players are less likely to stay in such 'asymmetric' coalitions.

Table 7 - Cost functions (multiplied by 1,000)

	Nash		Pareto		(1,2)		(1,3)		(2,3)	
	Aggr.	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*	*Aggr.*	*Nat.*
J^{F_1}	0.3596	0.3596	0.2832	0.2528	0.2771	0.2405	0.3634	0.3822	1.7926	2.3255
J^{F_2}	0.3596	0.3596	0.3240	0.3702	0.4091	0.4410	1.9002	2.4368	0.3924	0.4865
J^{ECB}	0	0.0378	0.0044	0.0785	0.0545	0.1203	0.0036	0.0365	0.0044	0.0257
J^C	–	–	0.2039	0.23399	–	–	–	–	–	–
$J^{(1,2)}$	–	–	–	–	0.3431	0.3438	–	–	–	–
$J^{(1,3)}$	–	–	–	–	–	–	0.1835	0.2093	–	–
$J^{(2,3)}$	–	–	–	–	–	–	–	–	0.1984	0.2561
a_{cl}	0.1007	0.1007	0.1160	0.1121	0.1210	0.1214	0.0998	0.0947	0.0993	0.0920

Conclusion

Macroeconomic policy cooperation is a crucial issue in a highly integrated economic and political union such as the European Union. We have argued that in the EMU - which introduced a common monetary policy and restrictions on national fiscal policies - increases even more the need for macroeconomic policy cooperation. To study the effects of policy cooperation in the EMU we compared the effects of five alternative policy regimes in a stylized model of the EMU: (i) non-cooperative monetary and fiscal policies, (ii) partial coordination and (iii) full coordination.

Using numerical examples, we illustrated the complex effects that are produced by the various coalitions. Moreover the sustainability of a certain type of coalition and its implications for the optimal strategies and the resulting macroeconomic adjustment, was seen to be highly sensitive to initial settings of preferences and the structural model parameters. We found that the cooperation is often efficient for the fiscal players and, moreover, that the fiscal players' cooperation (against the ECB) leads to a Pareto improvement for them. On the other hand, in many simulations full cooperation does not induce a Pareto improvement for the ECB, while the governments' coalitions imply a considerable loss for the ECB compared to the non-cooperative and full cooperative cases. That is the Pareto form is often unsustainable. This was also shown theoretically for the symmetric case. In the cases that the ECB cooperates with one government against the other, it gains a considerable Pareto-improvement but both governments lose. Therefore, in the experiments done in this paper a kind of dualism arises between the cooperative solutions and the non-cooperative one. More specifically, the stronger the asymmetry of the bargaining powers is, the less probability of coalitions among players becomes.

Considering current European discussions, it is found that the ECB has a rational to pursue an institutional design that does not enforce cooperation and let to the monetary authority a high degree of independence. Therefore, the ECB will try to promote fixed rules for European policy targets. On the other hand, governments will pursue a design based on cooperation that leave them independent in cooperating their policies with the monetary policy of the ECB.

Appendix A - The non-cooperative, cooperative and coalition equilibria

1. The non-cooperative case

With $A := \phi_4$, $B_1 := -\phi_1$, $B_2 := \phi_2$ and $B_3 := \phi_3$ the system is described by

$$\dot{s}(t) = As(t) + B_1 f_1(t) + B_2 f_2(t) + B_3 i_E(t); s(0) = s_0,$$

and with $x := (s\ f_1\ f_2\ i_E)$ the performance criterion of player i can be rewritten as $\frac{1}{2}\int_0^\infty x(t)M_i x(t)dt$, $i = F_1, F_2, E$.
Then, the noncooperative Nash solution is found as follows (see Engwerda *et. al.* (1999) for details):

1) Factorize M_i as follows $M_i =: \begin{pmatrix} Q_i & P_i & L_i & S_i \\ P_i^T & R_{1i} & N_i & T_i \\ L_i^T & N_i^T & R_{2i} & V_i \\ S_i^T & T_i^T & V_i^T & R_{3i} \end{pmatrix}$, where all entries are scalars.

2) Calculate $G := \begin{pmatrix} R_{11} & N_1 & T_1 \\ N_2^T & R_{22} & V_2 \\ T_3^T & V_3^T & R_{33} \end{pmatrix}$

and $M := \begin{pmatrix} -A & 0 & 0 & 0 \\ Q_1 & A^T & 0 & 0 \\ Q_2 & 0 & A^T & 0 \\ Q_3 & 0 & 0 & A^T \end{pmatrix} + \begin{pmatrix} B \\ -\tilde{P}_1 \\ -\tilde{P}_2 \\ -\tilde{P}_3 \end{pmatrix} G^{-1} \begin{pmatrix} P_1^T & B_1^T & 0 & 0 \\ L_2^T & 0 & B_2^T & 0 \\ S_3^T & 0 & 0 & B_3^T \end{pmatrix}$. Here $B := (B_1\ B_2\ B_3)$ and $\tilde{P}_i := (P_i\ L_i\ S_i)$.

3) Calculate the positive eigenvalue(s) of M. If $a_{cl,nc}$ is a positive eigenvalue and $v =: (v_0\ v_1\ v_2\ v_3)^T$ a corresponding eigenvector then, generically (see Engwerda *et. al.* (1999) for details), the equilibrium strategies are $\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} := -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \\ S_3^T + B_3^T K_3 \end{pmatrix} s(t)$, where $K_i := \frac{v_i}{v_0}$. Using this equilibrium strategies the resulting closed-loop system is described by $\dot{s}(t) = -a_{cl,nc}s(t),\ s(0) = s_0$.

2. The cooperative case

To determine the cooperative strategies for this model we have to consider: $J^C := \tau_1 J^{F_1} + \tau_2 J^{F_2} + \tau_3 J^{ECB}$ with $\tau_1 + \tau_2 + \tau_3 = 1$. τ_i measures here the relative strength of player i in the game. Introducing $\mu_1 := d_1\tau_1$, $\mu_2 := d_2\tau_2$ and $\mu_3 := \tau_3$, then $J^C = \frac{1}{2}\int_0^\infty \{x^T(t)M_C x(t)\}dt$, where $M_{C,i} := \mu_1 M_{F_1} + \mu_2 M_{F_2} + \mu_3 M_{E,i},\ i = A, N$.
With the notation of Appendix A.1 the unique equilibrium strategies are then obtained as follows (see e.g. Lancaster *et. al.* (1995)):

1) Factorise matrix $M_{C,i}$ as $\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}$, where Q is a scalar; S a 1×3 matrix and R a 3×3 matrix.

2) Calculate the Hamiltonian matrix $Ham := \begin{pmatrix} -(A - BR^{-1}S^T) & BR^{-1}B^T \\ Q - SR^{-1}S^T & (A - BR^{-1}S^T)^T \end{pmatrix}$.

3) Determine the positive eigenvalue $a_{cl,c}$ of Ham and its corresponding eigenvector $v =: (v_0\ v_1)^T$. Calculate $K := \frac{v_1}{v_0}$.

Then the unique equilibrium strategies are $\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} := -R^{-1}(S^T + B^T K)s(t) := H_c s(t)$, and the resulting closed-loop system satisfies $\dot{s}(t) = -a_{cl,c}s(t),\ s(0) = s_0$.

3. Coalition (1,2)

Introducing $M_{(1,2),c} := \mu_1 M_{F_1} + \mu_2 M_{F_2}$ where $\mu_1 := \tau_1 d_1$ and $\mu_2 := \tau_2 d_2$, the cost $J^{(1,2)} := \frac{1}{2}\int_0^\infty \{x^T(t) M_{(1,2),c} x(t)\} dt$. Let $B_1 := (-\phi_1 \ \phi_2)$; $B_2 := \phi_3$; $B := (B_1 \ B_2)$ and $A := \phi_4$. Then the noncooperative Nash equilibrium strategies for this game are obtained as follows:

1) Factorise matrix $M_1 := M_{(1,2),c}$ and $M_2 := M_{E,j}$ as: $\begin{pmatrix} Q_i & P_i & L_i \\ P_i^T & R_{i1} & N_i \\ L_i^T & N_i^T & R_{i2} \end{pmatrix}$, where Q_i, L_i, R_{i2} are scalars; P_i, N_i^T are 1×2 matrices and R_{i1} are 2×2 matrices, $i = 1, 2$.

2) Calculate $G := \begin{pmatrix} R_{11} & N_1 \\ N_2^T & R_{22} \end{pmatrix}$ and

$$M_{(1,2)} := \begin{pmatrix} -A & 0 & 0 \\ Q_1 & A^T & 0 \\ Q_2 & 0 & A^T \end{pmatrix} + \begin{pmatrix} B \\ -(P_1 \ L_1) \\ -(P_2 \ L_2) \end{pmatrix} G^{-1} \begin{pmatrix} P_1^T & B_1^T & 0 \\ L_2^T & 0 & B_2^T \end{pmatrix}.$$

3) Determine the positive eigenvalue(s) of $M_{(1,2)}$. If $a_{cl,(1,2)}$ is a positive eigenvalue and $v =: (v_0 \ v_1 \ v_2)$ a corresponding eigenvector, then (generically) an equilibrium strategy is $\begin{pmatrix} f_1(t) \\ f_2(t) \\ i_E(t) \end{pmatrix} := -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix} =: H_{(1,2)} s(t)$, where $K_i := \frac{v_i}{v_0}$. The resulting closed-loop system is then $\dot{s}(t) = -a_{cl,(1,2)} s(t)$, $s(0) = s_0$.

4. Coalition (1,3)

First note that the inverse of the permutation matrix $P_{(1,3)}$ is $P_{(1,3)}^T$ so that $x(t) = P_{(1,3)}^T \tilde{x}(t)$. Then, with $M_i^{(1,3)} := P_{(1,3)} M_i P_{(1,3)}^T$, $i = \{F_1, F_2, E\}$ (i.e. the matrices obtained from M_i by replacing the third and the fourth rows with the corresponding columns of this matrix) we find $M_{(1,3),c} := \tau_1 d_1 M_{F_1}^{(1,3)} + \tau_2 M_E^{(1,3)}$, so that $J_{(1,3)} = \frac{1}{2}\int_0^\infty \{\tilde{x}^T(t) M_{(1,3),c} \tilde{x}(t)\} dt$.
Next, introduce $B_1 := (-\phi_1 \ \phi_3)$; $B_2 := \phi_2$ and $B := (B_1 \ B_2)$.
Then, apply step 1) and 2) of the algorithm described in the above appendix A.3 to find a corresponding matrix $M_{(1,3)}$.
Determine the positive eigenvalue(s) of $M_{(1,3)}$. If $a_{cl,(1,3)}$ is a positive eigenvalue of this matrix and $v =: (v_0 \ v_1 \ v_2)$ a corresponding eigenvector, then (generically) an equilibrium strategy is $\begin{pmatrix} f_1(t) \\ i_E(t) \\ f_2(t) \end{pmatrix} := -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix} =: H_{(1,3)} s(t)$, where $K_i := \frac{v_i}{v_0}$. The resulting closed-loop system is then $\dot{s}(t) = -a_{cl,(1,3)} s(t)$, $s(0) = s_0$.

5. Coalition (2,3)

The equilibrium strategies for this coalition are obtained similar to the previous algorithm outlined above in Appendix A.4. Note that in this case $x(t) = P_{(2,3)}^T \tilde{x}(t)$, so that with $M_i^{(2,3)} := P_{(2,3)} M_i P_{(2,3)}^T$, $i = \{F_1, F_2, E\}$ and $M_{(2,3),c} := \tau_1 d_1 M_{F_2}^{(2,3)} + \tau_2 M_E^{(2,3)}$, we find that $J^{(2,3)} =$

$\frac{1}{2}\int_0^{\infty}\{\tilde{x}^T(t)M_{(2,3),c}\tilde{x}(t)\}dt$. Introducing $B_1 := (\phi_2\ ,\phi_3)$; $B_2 := -\phi_1$ and $B := (B_1,\ B_2)$ we can proceed then similar as in the above algorithm to first find a corresponding matrix $M_{(2,3)}$. The equilibrium strategies are then again found by determining from this matrix a positive eigenvalue $a_{cl,(2,3)}$ and its corresponding eigenvector $v =: (v_0\ v_1\ v_2)$. Introducing $K_i := \frac{v_i}{v_0}$ (generically) an equilibrium strategy is $\begin{pmatrix} f_2(t) \\ i_E(t) \\ f_1(t) \end{pmatrix} := -G^{-1}\begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix} =: H_{(2,3)}s(t)$, and the resulting closed-loop system is $\dot{s}(t) = -a_{cl,(2,3)}s(t)$, $s(0) = s_0$.

Appendix B - The symmetric case elaborated

The non-cooperative case

First, we consider the performance criterion (9b), $J^{ECB,N}$. To determine the equilibrium strategies we have to calculate the eigenvalues and eigenvectors of the corresponding matrix M (see appendix A.1). By substitution of the various parameters we obtain

$$G := \begin{pmatrix} a^2+g & ae & -ac \\ ae & a^2+g & -ac \\ -\frac{1}{2}c(a+e) & -\frac{1}{2}c(a+e) & c^2+g_E \end{pmatrix} \tag{1}$$

Elementary calculations show that the determinant of G, det, equals $u_{1nc}u_{4nc}$, where $u_{1nc} := ag_E(a+e) + g(c^2+g_E)$ and $u_{4nc} := a(a-e)+g$. Moreover, $det * G^{-1}$ equals

$$\begin{pmatrix} g(c^2+g_E)+\frac{1}{2}a(2ag_E+c^2(a-e)) & \frac{1}{2}a(c^2(a-e)-2eg_E) & acu_{4nc} \\ \frac{1}{2}a(c^2(a-e)-2eg_E) & g(c^2+g_E)+\frac{1}{2}a(2ag_E+c^2(a-e)) & acu_{4nc} \\ \frac{1}{2}c(a+e)u_{4nc} & \frac{1}{2}c(a+e)u_{4nc} & u_{4nc}(a^2+ae+g) \end{pmatrix}.$$

Consequently, matrix M satisfies

$$det * M = \begin{pmatrix} -\phi_4 det - 2ab\phi_1 u_{1nc} & \phi_1^2 u_{1nc} & \phi_1^2 u_{1nc} & 0 \\ b^2 g u_{1nc} & \phi_4 det + b\phi_1 u_{2nc} & b\phi_1 g u_{3nc} & 0 \\ b^2 g u_{1nc} & b\phi_1 g u_{3nc} & \phi_4 det + b\phi_1 u_{2nc} & 0 \\ b^2 g u_{1nc} & \frac{1}{2}b\phi_1(a-e)u_{1nc} & \frac{1}{2}b\phi_1(a-e)u_{1nc} & \phi_4 det \end{pmatrix},$$

where we used the shorthand notations $u_{2nc} := a^3 g_E + \frac{1}{2}ac^2 g + agg_E - e^2 ag_E - \frac{1}{2}c^2 ge$ and $u_{3nc} := -\frac{1}{2}c^2 e - g_E e + \frac{1}{2}ac^2$. Note that $u_{2nc} + gu_{3nc} = (a-e)u_{1nc}$, a relationship which is useful in elaborating details. The structure of matrix $det * M$ is:

$$M1 = \begin{pmatrix} \tilde{a} & \tilde{c} & \tilde{c} & 0 \\ \tilde{b} & \tilde{d} & \tilde{e} & 0 \\ \tilde{b} & \tilde{e} & \tilde{d} & 0 \\ \tilde{b} & \tilde{f} & \tilde{f} & \tilde{g} \end{pmatrix}$$

The eigenvalues of $M1$ are $\tilde{g}$, $\tilde{d}-\tilde{e}$, $\frac{1}{2}(\tilde{a}+\tilde{d}+\tilde{e}) - \frac{1}{2}\sqrt{(\tilde{a}-\tilde{d}-\tilde{e})^2 + 8\tilde{b}\tilde{c}}$ en $\frac{1}{2}(\tilde{a}+\tilde{d}+\tilde{e}) + \frac{1}{2}\sqrt{(\tilde{a}-\tilde{d}-\tilde{e})^2 + 8\tilde{b}\tilde{c}}$. Note that the eigenvector corresponding to the eigenvalue $\tilde{d}-\tilde{e}$ is

$(0\ 1\ -1\ 0)^T$, which therefore does not satisfy the additional requirements for generating an equilibrium. Furthermore, $\tilde{g} < 0$. Consequently, the game has at most two different equilibria. Given the parametric restrictions, it is easily verified that if $r := \frac{\rho_1}{k_1} < 1$ (which implies that $0 < e < a$) $\tilde{a} + \tilde{d} + \tilde{e}$ is negative and $\lambda := \frac{1}{2}(\tilde{a} + \tilde{d} + \tilde{e}) + \frac{1}{2}\sqrt{(\tilde{a} - \tilde{d} - \tilde{e})^2 + 8\tilde{b}\tilde{c}} > 0$. So, under this assumption there is a unique equilibrium. A more detailed look at the eigenvalues shows that they coincide with the relevant eigenvalues (with $\mu = 1$ and $\theta = 0$) reported in Engwerda *et al* (2001). So, the results obtained there apply here. In particular we have that whenever there is only one appropriate positive eigenvalue, this eigenvalue is given by λ. The corresponding eigenvector is: $\begin{pmatrix} -(\tilde{e} + \tilde{d} - \lambda)/\tilde{b} \\ 1 \\ 1 \\ -2(\tilde{f} - \tilde{d} - \tilde{e} + \lambda)/(\tilde{g} - \lambda) \end{pmatrix}$.
Substitution of the corresponding parameters from M shows that

$$\lambda \ : \ = -\frac{1}{2}b\phi_1(a+e)u_{1nc} + \frac{1}{2}\sqrt{(-2\phi_4 det - b\phi_1(3a-e)u_{1nc})^2 + 8gb^2\phi_1^2u_{1nc}^2} \qquad (2)$$

$$= \frac{1}{2}u_{1nc}\{-b\phi_1(a+e) + \sqrt{(-2\phi_4 u_{4nc} - b\phi_1(3a-e))^2 + 8gb^2\phi_1^2}\} \qquad (3)$$

Consequently, the eigenvalue of M we are looking for is $\frac{\lambda}{det}$ and $K := K_1 = K_2 =$

$$\frac{2b^2g}{-2\phi_4u_{4nc} - b\phi_1(3a-e) + \sqrt{(-2\phi_4u_{4nc} - b\phi_1(3a-e))^2 + 8gb^2\phi_1^2}}. \qquad (4)$$

Using this, the rest of the claims follow straightforwardly.

Next, consider the aggregate performance criterion $J^{ECB,A}$. Substitution of the various parameters into appendix A.1 shows that, except for the entries (4,1), (4,2) and (4,3) which are now zero, matrix M in step ii) of the algorithm coincides with the matrix M we determined above for the national performance case. Therefore, it is easily verified that the equilibrium strategies coincide. As a consequence, the resulting closed-loop systems coincide too.

The cooperative case

First we consider again the performance criterion (9b). After substitution of the parameters we see that matrix $M_{C,N}$ (see Appendix A.2) is given by

$$M_{C,N} = \frac{1}{2}\begin{pmatrix} 2b^2 & b(a-e) & -b(a-e) & 0 \\ b(a-e) & a^2+e^2+2\tau g & 2ae & -c(a+e) \\ -b(a-e) & 2ae & a^2+e^2+2\tau g & -c(a+e) \\ 0 & -c(a+e) & -c(a+e) & 2(c^2+(1-2\tau)g_E) \end{pmatrix}. \qquad (5)$$

So, with $A := \phi_4$, $B := \phi_1(-1\ 1\ 0)$, $Q := b^2$, $S := \frac{1}{2}b(a-e)(1\ -1\ 0)$ and $R := \frac{1}{2}\begin{pmatrix} a^2+e^2+2\tau g & 2ae & -c(a+e) \\ 2ae & a^2+e^2+2\tau g & -c(a+e) \\ -c(a+e) & -c(a+e) & 2(c^2+(1-2\tau)g_E) \end{pmatrix}$ we can determine now the Hamiltonian of the system Ham. Since all entries of this matrix are scalar, it is easily verified that the positive eigenvalue equals $a_{cl,cN} := \sqrt{(A - BR^{-1}S^T)^2 + BR^{-1}B^T(Q - SR^{-1}S^T)}$ and its corresponding eigenvector $\begin{pmatrix} BR^{-1}B^T \\ (A - BR^{-1}S^T + a_{cl,cN}) \end{pmatrix}$. From this, K immediately results. Apart from the determination of the inverse of R things can be calculated straightforwardly now. Therefore, we conclude this subsection with the exposition of matrix R^{-1} (from which the verification of correctness is left to the reader). Introducing $u_{1c} := (1-2\tau)g_E(a+e)^2 + 2\tau g(c^2+(1-2\tau)g_E)$; $u_{2c} := \frac{1}{4}c^2(a-e)^2 - ae(1-2\tau)g_E$ and $u_{4c} := 2\tau g + (a-e)^2$ the determinant of R is $det := \frac{1}{4}u_{1c}u_{4c}$, and we have

$$det * R^{-1} = \begin{pmatrix} \frac{1}{2}u_{1c} + u_{2c} & u_{2c} & \frac{1}{4}(a+e)cu_{4c} \\ u_{2c} & \frac{1}{2}u_{1c} + u_{2c} & \frac{1}{4}(a+e)cu_{4c} \\ \frac{1}{4}(a+e)cu_{4c} & \frac{1}{4}(a+e)cu_{4c} & \frac{1}{4}((a+e)^2 + 2\tau g)u_{4c} \end{pmatrix} \tag{6}$$

Next, we consider the aggregate criterion (9a). After substitution of the parameters we see that matrix $M_{C,A}$ (see Appendix A.2) is given by

$$\begin{pmatrix} 2\tau b^2 & \tau b(a-e) & -\tau b(a-e) & 0 \\ \tau b(a-e) & \frac{1}{2}\tau(2g + (a-e)^2) + \frac{1}{4}(a+e)^2 & -\frac{1}{2}\tau(a-e)^2 + \frac{1}{4}(a+e)^2 & -\frac{1}{2}c(a+e) \\ -\tau b(a-e) & -\frac{1}{2}\tau(a-e)^2 + \frac{1}{4}(a+e)^2 & \frac{1}{2}\tau(2g + (a-e)^2) + \frac{1}{4}(a+e)^2 & -\frac{1}{2}c(a+e) \\ 0 & -\frac{1}{2}c(a+e) & -\frac{1}{2}c(a+e) & c^2 + (1-2\tau)g_E \end{pmatrix}. \tag{7}$$

From this similar as in the case when the performance criterion (9b) is used, the matrices Q, S and R result. Introducing $u_{1cA} := \frac{1}{2}(a+e)^2(1-2\tau)g_E + \tau g(c^2 + (1-2\tau)g_E)$; $u_{2cA} := -\frac{1}{4}c^2(a+e)^2 + (\tau(a^2+e^2+g) + \frac{1}{4}(1-2\tau)(a+e)^2)(c^2 + (1-2\tau)g_E)$; $u_{3cA} := \frac{1}{4}c^2(a+e)^2 - (2\tau ae + \frac{1}{4}(1-2\tau)(a+e)^2)(c^2 + (1-2\tau)g_E)$ and $u_{4cA} := g + (a-e)^2$ the determinant of R, det, is $\tau u_{1cA}u_{4cA}$ and

$$det * R^{-1} = \begin{pmatrix} u_{2cA} & u_{3cA} & \frac{1}{2}\tau(a+e)cu_{4cA} \\ u_{3cA} & u_{2cA} & \frac{1}{2}\tau(a+e)cu_{4cA} \\ \frac{1}{2}\tau(a+e)cu_{4cA} & \frac{1}{2}\tau(a+e)cu_{4cA} & \frac{1}{2}\tau((a+e)^2 + 2\tau g)u_{4cA} \end{pmatrix}.$$

From this it is easily verified (using the fact that $\phi_1 = -\frac{\phi_4(a-e)}{2b}$) that the Hamiltonian Ham equals $\frac{1}{g+(a-e)^2}\begin{pmatrix} -\phi_4 g & \frac{\frac{1}{2}\phi_4^2(a-e)^2}{\tau b^2} \\ 2\tau b^2 g & \phi_4 g \end{pmatrix}$. Analogous to the case of performance criterion (9b), it follows from this straightforwardly that the positive eigenvalue $a_{cl,cA}$ is $\frac{\phi_4\sqrt{g}}{\sqrt{g+(a-e)^2}}$ and $K = \tau\frac{\phi_4 u_{4cA} + 2\phi_1 b(a-e) + u_{4cA}a_{cl,cA}}{2\phi_1^2}$.

The coalition (1,2)

First we consider again the performance criterion (9b). After substitution of the parameters we see that matrix $M_{(1,2)}$ (see Appendix A.4) is given by

$$M_{(1,2),c} = \frac{1}{2}\begin{pmatrix} 2b^2 & b(a-e) & -b(a-e) & 0 \\ b(a-e) & a^2+e^2+g & 2ae & -c(a+e) \\ -b(a-e) & 2ae & a^2+e^2+g & -c(a+e) \\ 0 & -c(a+e) & -c(a+e) & 2c^2 \end{pmatrix}. \tag{8}$$

Consequently,

$$G := \frac{1}{2}\begin{pmatrix} a^2+e^2+g & 2ae & -c(a+e) \\ 2ae & a^2+e^2+g & -c(a+e) \\ -c(a+e) & -c(a+e) & 2c^2+2g_E \end{pmatrix} \qquad (9)$$

Next, introduce $u_{1,(1,2)} := g_E(a+e)^2 + g(c^2+g_E)$ and $u_{4,(1,2)} := g+(a-e)^2$. Elementary calculations show that the determinant of G, det, equals $\frac{1}{4}u_{1,(1,2)}u_{4,(1,2)}$. Moreover, $det * G^{-1}$ equals

$$\frac{1}{4}\begin{pmatrix} u_{1,(1,2)}+(c^2+g_E)u_{4,(1,2)} & -u_{1,(1,2)}+(c^2+g_E)u_{4,(1,2)} & u_{4,(1,2)}c(a+e) \\ -u_{1,(1,2)}+(c^2+g_E)u_{4,(1,2)} & u_{1,(1,2)}+(c^2+g_E)u_{4,(1,2)} & u_{4,(1,2)}c(a+e) \\ u_{4,(1,2)}c(a+e) & u_{4,(1,2)}c(a+e) & u_{4,(1,2)}(g+(a+e)^2) \end{pmatrix}.$$

Consequently,

$$det * M_{(1,2)} = \begin{pmatrix} -\phi_4 det + \frac{1}{2}\phi_1 b(e-a)u_{1,(1,2)} & \phi_1^2 u_{1,(1,2)} & 0 \\ \frac{1}{4}b^2 g u_{1,(1,2)} & \phi_4 det + \frac{1}{2}\phi_1 b(a-e)u_{1,(1,2)} & 0 \\ \frac{1}{4}b^2 g u_{1,(1,2)} & \frac{1}{2}\phi_1 b(a-e)u_{1,(1,2)} & \phi_4 det \end{pmatrix}. \qquad (10)$$

Therefore, the eigenvalues of $M_{(1,2)}$ are ϕ_4, $a_{cl,(1,2)N}$ and $-a_{cl,(1,2)N}$, with $a^2_{cl,(1,2)N} := \phi_4^2 + 4\phi_1 b\frac{\phi_4(a-e)+b\phi_1}{u_{4,(1,2)}}$. So, there is always a unique equilibrium. The with $a_{cl,(1,2)N}$ corresponding eigenvector is $(-\frac{u_{4,(1,2)}\phi_4+2\phi_1 b(a-e)-u_{4,(1,2)}a_{cl,(1,2)N}}{2gb^2}, \frac{1}{2}, 1)^T$. From which the rest of the conclusions then immediately result.
Next, we consider the aggregate performance case. After substitution of the parameters in the algorithm described in Appendix A.3 we see that matrix G coincides with (9). Consequently we see, after some elementary calculations, that the matrix $M_{(1,2)}$ satisfies $(g+(a-e)^2)M_{(1,2)} =$ $\begin{pmatrix} -\phi_4(g+(a-e)^2)-2\phi_1 b(a-e) & 4\phi_1^2 & 0 \\ b^2 g & \phi_4(g+(a-e)^2)+2\phi_1 b(a-e) & 0 \\ 0 & 0 & \phi_4(g+(a-e)^2) \end{pmatrix}$. From this it is easily deduced that the only relevant positive eigenvalue $a_{cl,(1,2)A}$ coincides with $a_{cl,(1,2)N}$. Moreover, it is also easily verified that the corresponding K and strategies coincide with the case where the performance criterion (9b) is used.

Appendix C

Proof of Lemma 2:
i) To show that $a_{cl,cN} \le a_{cl,(1,2)N}$ we note that

$$\begin{aligned} a^2_{cl,(1,2)N} - a^2_{cl,cN} &= \phi_4^2\frac{g}{g+(a-e)^2} - \phi_4^2\frac{2\tau g}{2\tau g+(a-e)^2} \\ &= \phi_4^2\frac{g(a-e)^2(1-2\tau)}{(g+(a-e)^2)(2\tau g+(a-e)^2)} \\ &\ge 0. \end{aligned}$$

ii) First note that, using the equality $\phi_4(a-e) = -2b\phi_1$, $a_{cl,nc}$ can be rewritten as

$\frac{\frac{1}{2}\phi_4(a-e)(a+e)+\|\phi_4\|\sqrt{(\frac{1}{2}(a+e)(a-e)+2g)^2+2g(a-e)^2}}{2(a(a-e)+g)}$. Consequently,

$$\begin{aligned} a^2_{cl,cN} - a^2_{cl,nc} &= -\phi_4^2\{\frac{\frac{1}{2}(a^2-e^2)^2+4gu_{4,nc}}{4u^2_{4,nc}} - \frac{2\tau g}{u_{4,c}}\} + \phi_4^2\frac{(a^2-e^2)\sqrt{(\frac{1}{2}(a^2-e^2)+2g)^2+2g(a-e)^2}}{4u^2_{4,nc}} \\ &=: -t_1+t_2. \end{aligned}$$

Since $t_2 \geq 0$, it is obvious that if t_1 in the above expression is negative also $a^2_{cl,cN} - a^2_{cl,nc} \geq 0$. Next assume that t_1 is positive. Then, $a^2_{cl,cN} - a^2_{cl,nc} \geq 0$ if and only if $t_2^2 - t_1^2 \geq 0$. Elementary calculations show that

$$t_2^2 - t_1^2 = \frac{\frac{1}{2}g(a-e)^3\phi_4^4}{u^2_{4,nc}u^2_{4,c}}\{\tau(a-e)(a+e)^2 + 2(1-\tau)g(a(3\tau-1)+e(1+\tau))\}.$$

Obviously, this last expression is positive, if $\tau \geq \frac{1}{3}$, which concludes the proof.
iii) For the non-cooperative case the proof can be found in Engwerda *et al.* (2001). The proof of the other two cases is found by straightforward differentiation.
iv)

$$\begin{aligned} a^2_{cl,cA} - a^2_{cl,cN} &= \frac{g\phi_4^2}{g+(a-e)^2} - 2\tau\frac{g\phi_4^2}{2\tau g+(a-e)^2} \\ &= \frac{\phi_4^2}{g+(a-e)^2}\frac{g}{2\tau g+(a-e)^2}(a-e)^2(1-2\tau) \\ &\geq 0. \end{aligned}$$

□

Proof of Lemma 3:
i) From the cost functional (1) we have that

$$J_c^E - J_{nc}^E = \frac{1}{4}\frac{ds^2(0)}{a_{cl,cN}a_{cl,nc}}\{a_{cl,nc}(b+p_c(a-e))^2 - a_{cl,cN}(b+p_{nc}(a-e))^2\}$$

Since, $p_i = \frac{\phi_4 + a_{cl,i}}{2\phi_1}$ and $\phi_4(a-e) = -2b\phi_1$ we have

$$\begin{aligned} sgn(J_c^E - J_{nc}^E) &= sgn(a_{cl,nc}(2b\phi_1 + (\phi_4+a_{cl,cN})(a-e))^2 - a_{cl,cN}(2b\phi_1 + (\phi_4+a_{cl,nc})(a-e))^2) \\ &= sgn((a_{cl,cN} - a_{cl,nc})a_{cl,nc}a_{cl,cN}(a-e)^2) \\ &= sgn(a_{cl,cN} - a_{cl,nc}). \end{aligned}$$

ii) From (1) we have that

$$J_c^{F_i} - J_{(1,2)}^{F_i} = \frac{1}{4}\frac{ds^2(0)}{a_{cl,c}a_{cl,(1,2)}}\{a_{cl,(1,2)}((b+p_{cN}(a-e))^2 + p^2_{cN}g) - a_{cl,cN}((b+p_{(1,2)}(a-e))^2 + p^2_{(1,2)}g)\}$$

Using again the facts that $p_i = \frac{\phi_4+a_{cl,i}}{2\phi_1}$ and $\phi_4(a-e) = -2b\phi_1$ we have that $sgn(J_c^{F_i} - J_{(1,2)}^{F_i})$ can be rewritten as

$$\begin{aligned} & sgn(a_{cl,(1,2)}(a^2_{cl,cN}(a-e)^2 + (\phi_4+a_{cl,cN})^2g) - a_{cl,cN}(a^2_{cl,(1,2)}(a-e)^2 + (\phi_4+a_{cl,(1,2)})^2g)) \\ = \; & sgn(\phi_4^2g(a_{cl,(1,2)} - a_{cl,cN}) + ((a-e)^2+g)(a_{cl,(1,2)}a^2_{cl,cN} - a_{cl,cN}a^2_{cl,(1,2)})) \\ = \; & sgn((a_{cl,cN} - a_{cl,(1,2)})(-\phi_4^2g + ((a-e)^2+g)a_{cl,(1,2)}a_{cl,cN})). \end{aligned}$$

From lemma 2.i) we therefore conclude that $sgn(J_c^{F_i} - J_{(1,2)}^{F_i}) = sgn(\phi_4^2 g - ((a-e)^2 + g)a_{cl,(1,2)}a_{cl,cN})$. Since $a_{cl,(1,2)}^2 = \phi_4^2 \frac{g}{u_{4,(1,2)}}$ and $a_{cl,cN}^2 = \phi_4^2 \frac{2\tau g}{u_{4,c}}$, we finally have

$$\begin{aligned} sgn(J_c^{F_i} - J_{(1,2)}^{F_i}) &= sgn(\phi_4^4 g^2 (1 - \frac{2\tau u_{4,(1,2)}}{u_{4,c}})) \\ &= sgn((1-2\tau)(a-e)^2) \geq 0, \end{aligned}$$

which concludes the proof. □

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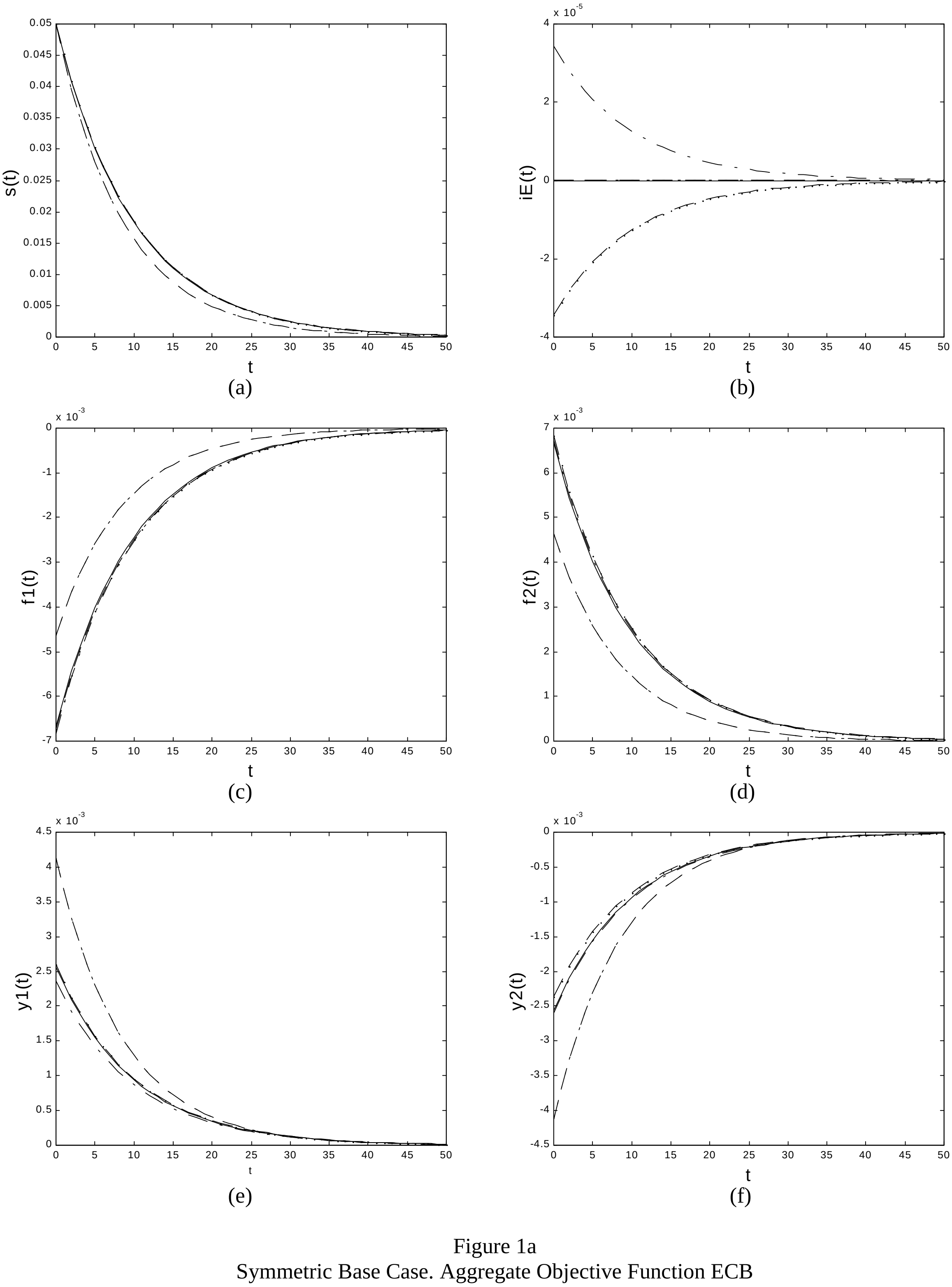

Figure 1a

Symmetric Base Case. Aggregate Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

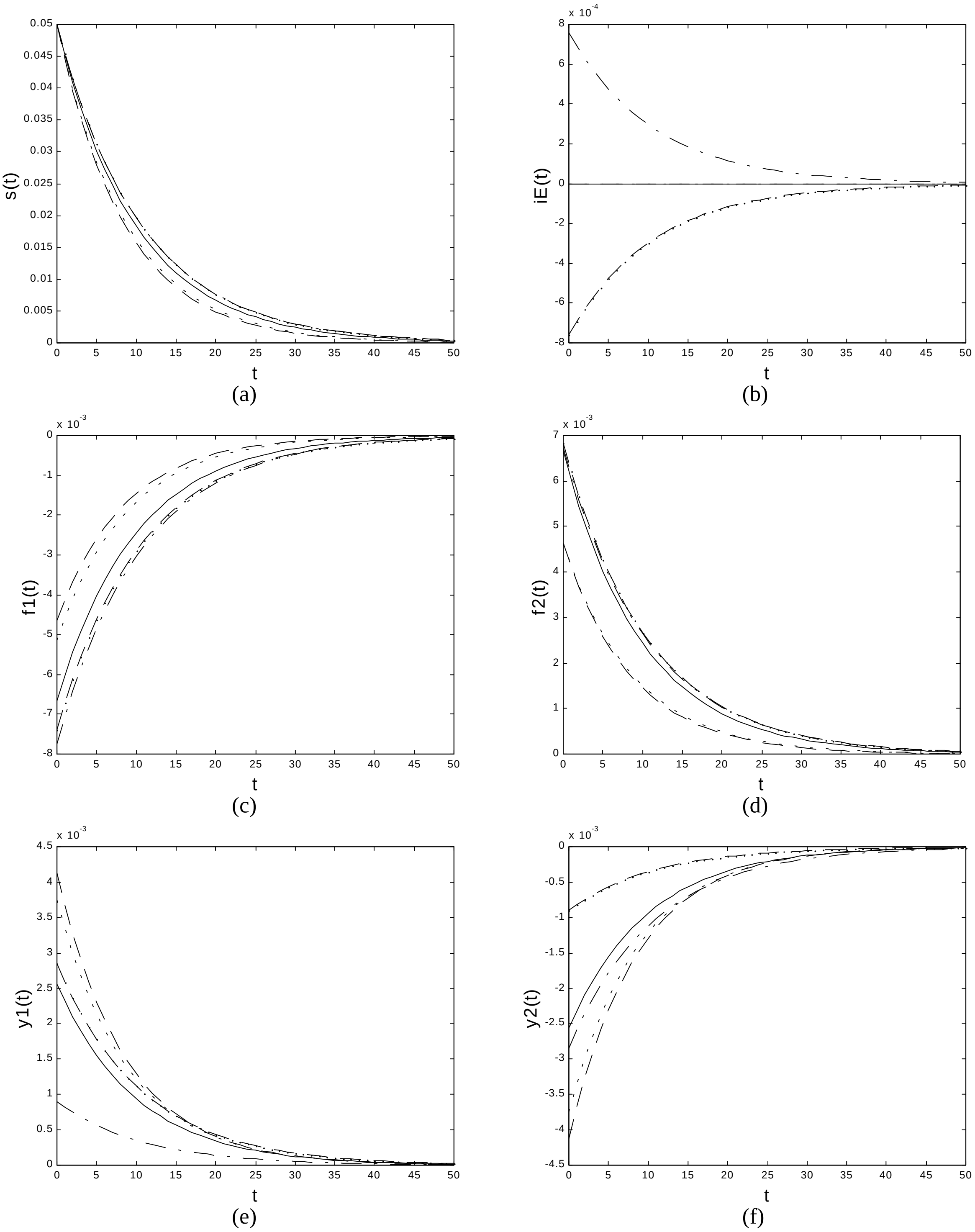

Figure 1b
Symmetric Base Case. National Objective Function ECB
– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

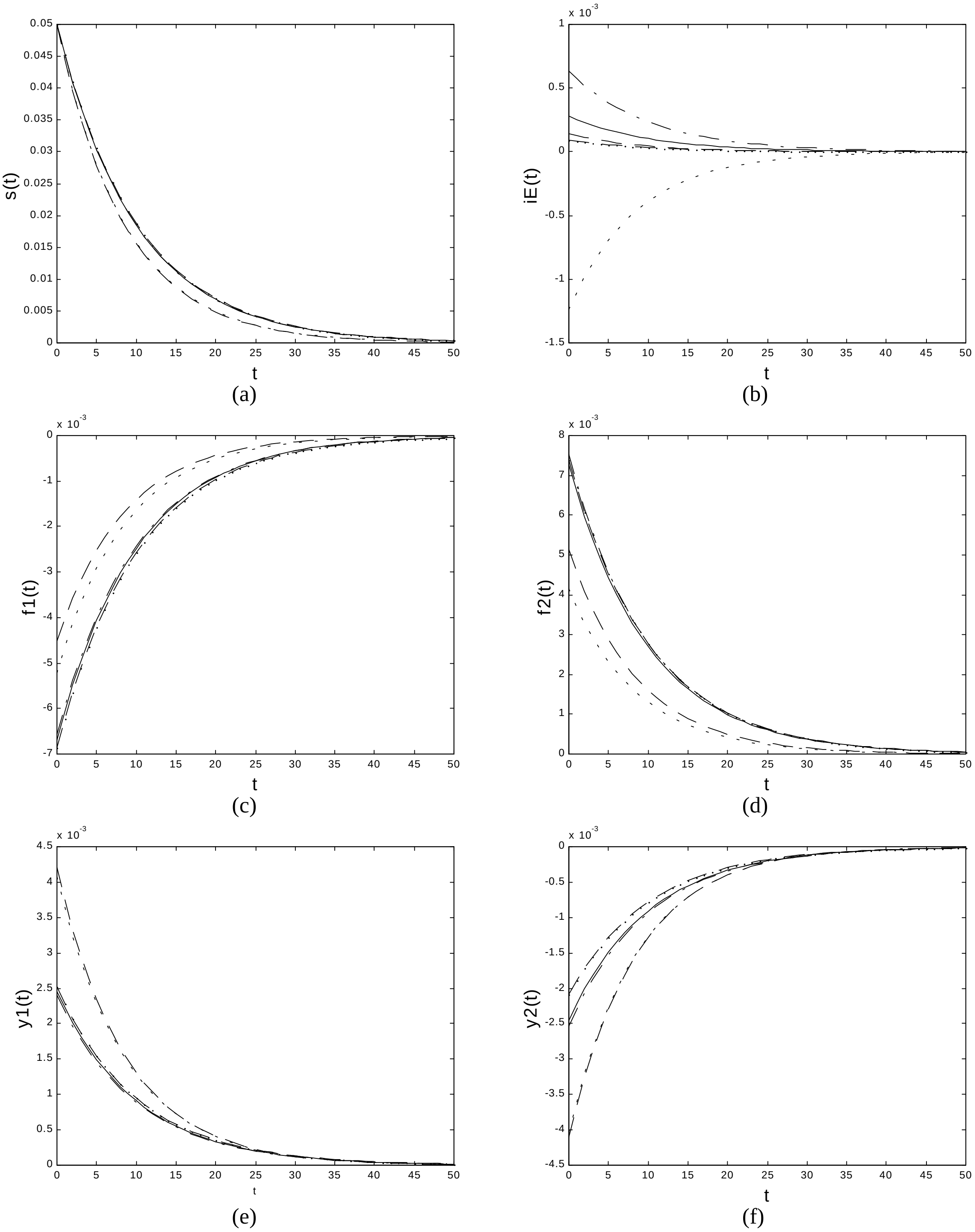

Figure 2a

Asymmetric Case, γ_1 = 0.4, γ_2 = 0.8. Aggregate Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

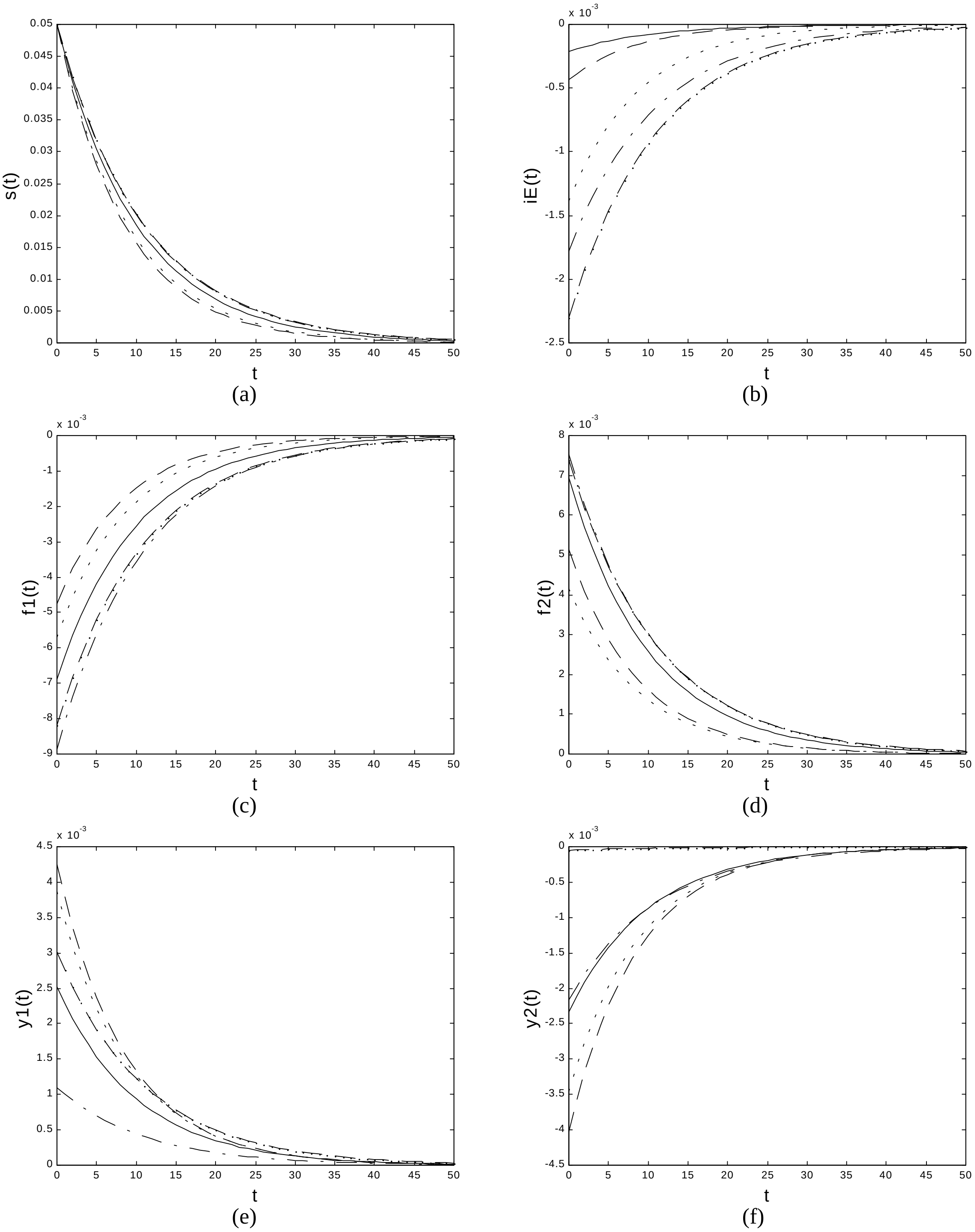

Figure 2b

Asymmetric Case, $\gamma_1 = 0.4$, $\gamma_2 = 0.8$. National Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

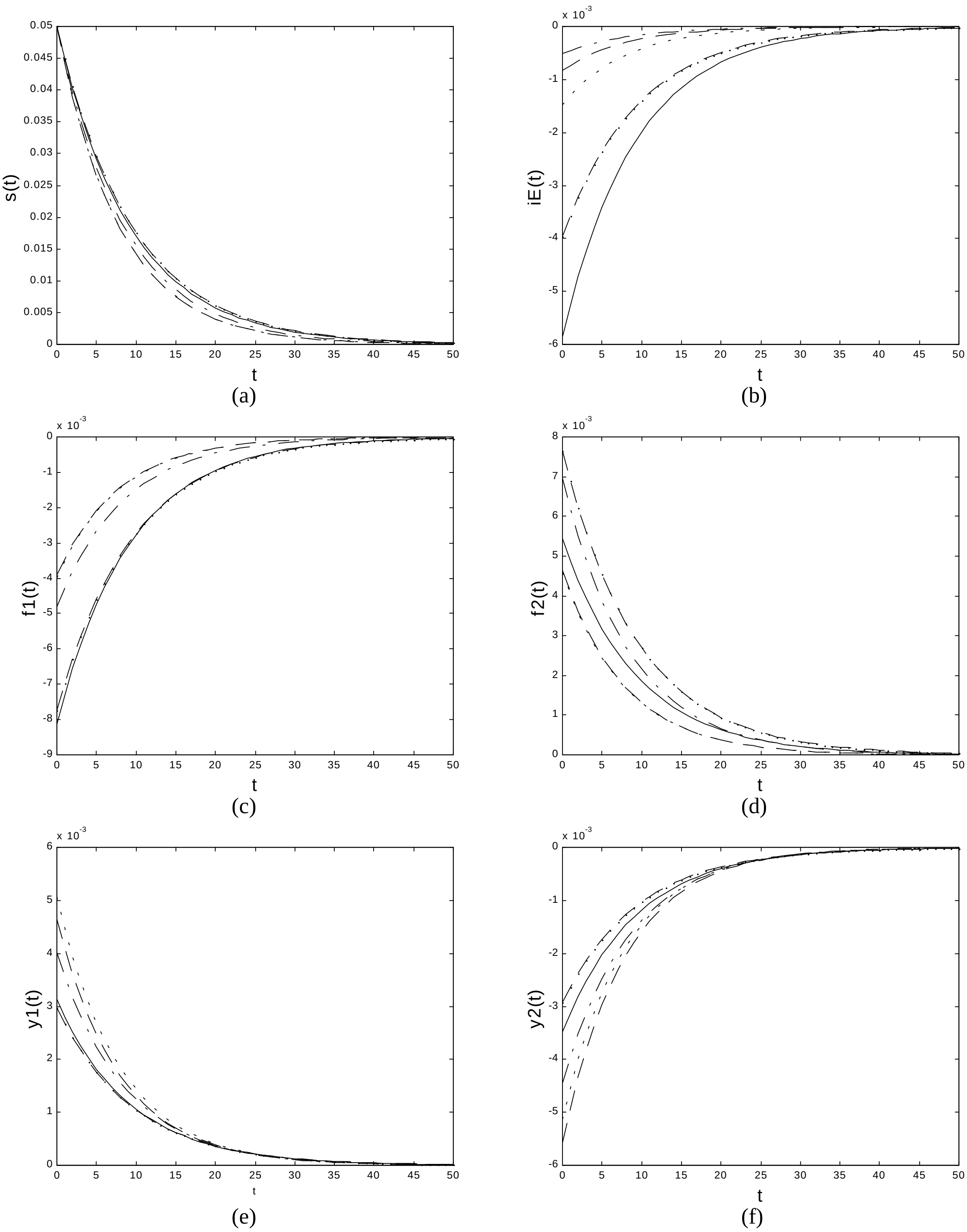

Figure 3a

Asymmetric Case, $\eta_1 = 1$, $\eta_2 = 0.6$. Aggregate Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

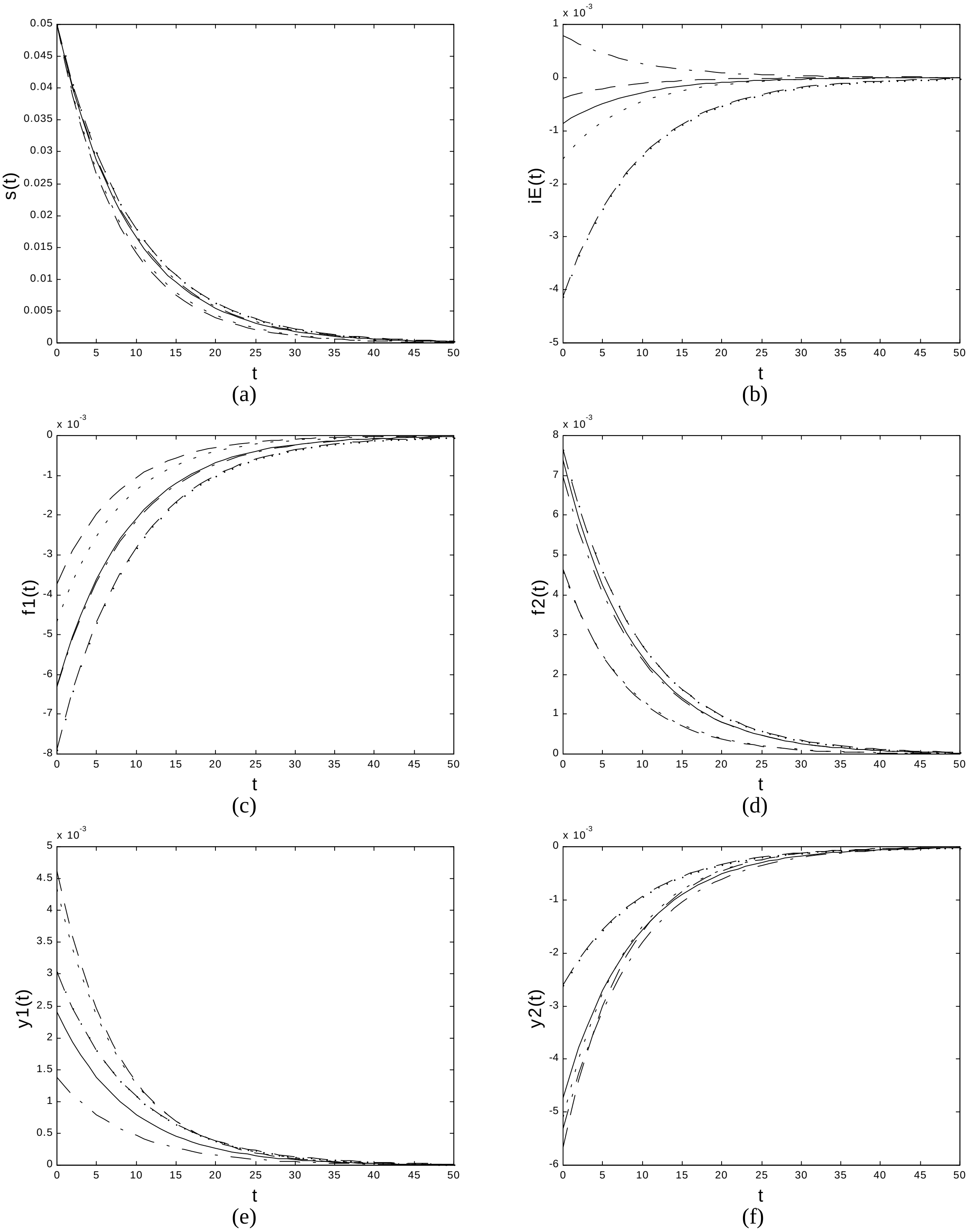

Figure 3b

Asymmetric Case, $\eta_1 = 1$, $\eta_2 = 0.6$. National Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

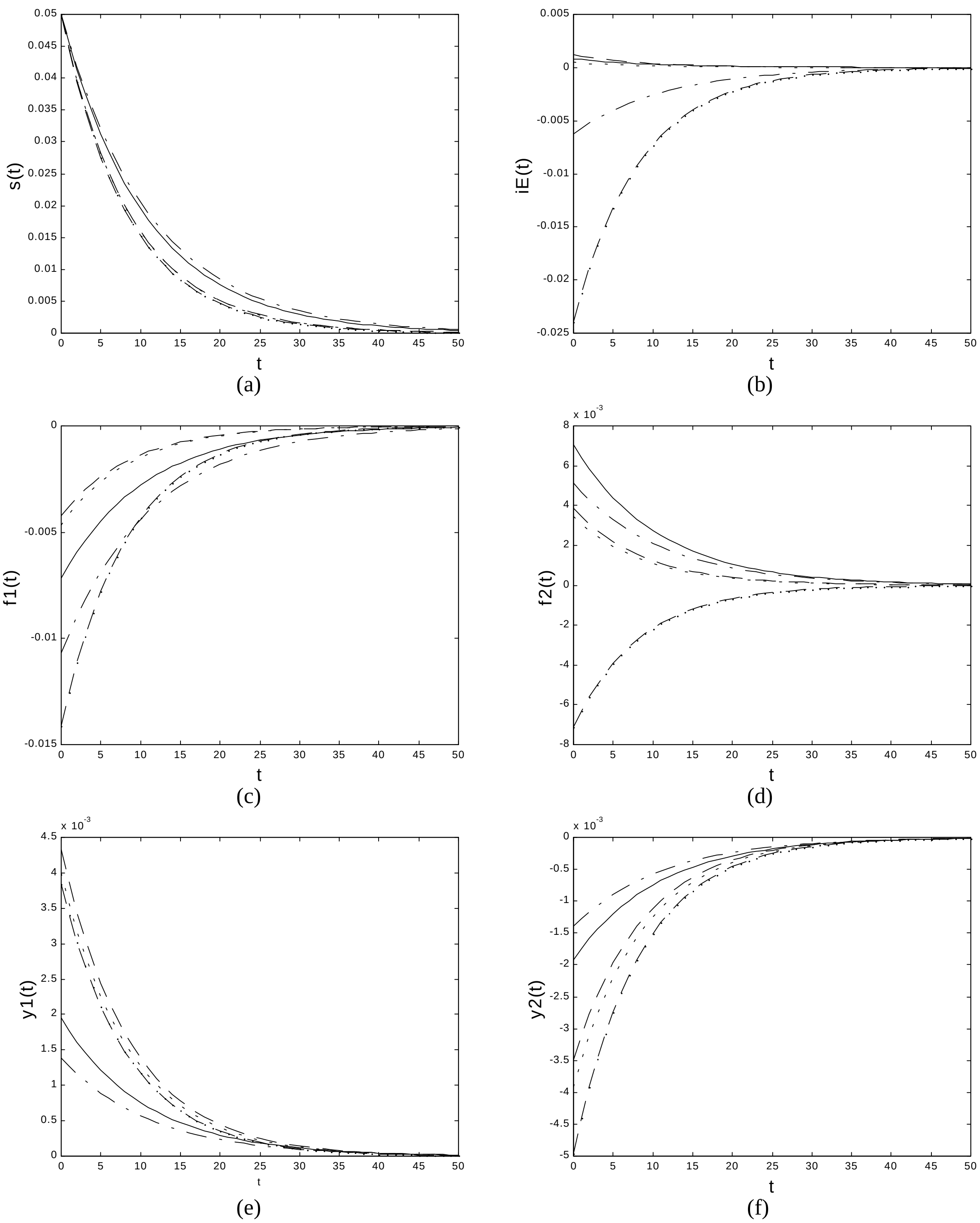

Figure 4a

Asymmetric Case, ρ_1 =0.4, ρ_2 = 0.8. Aggregate Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

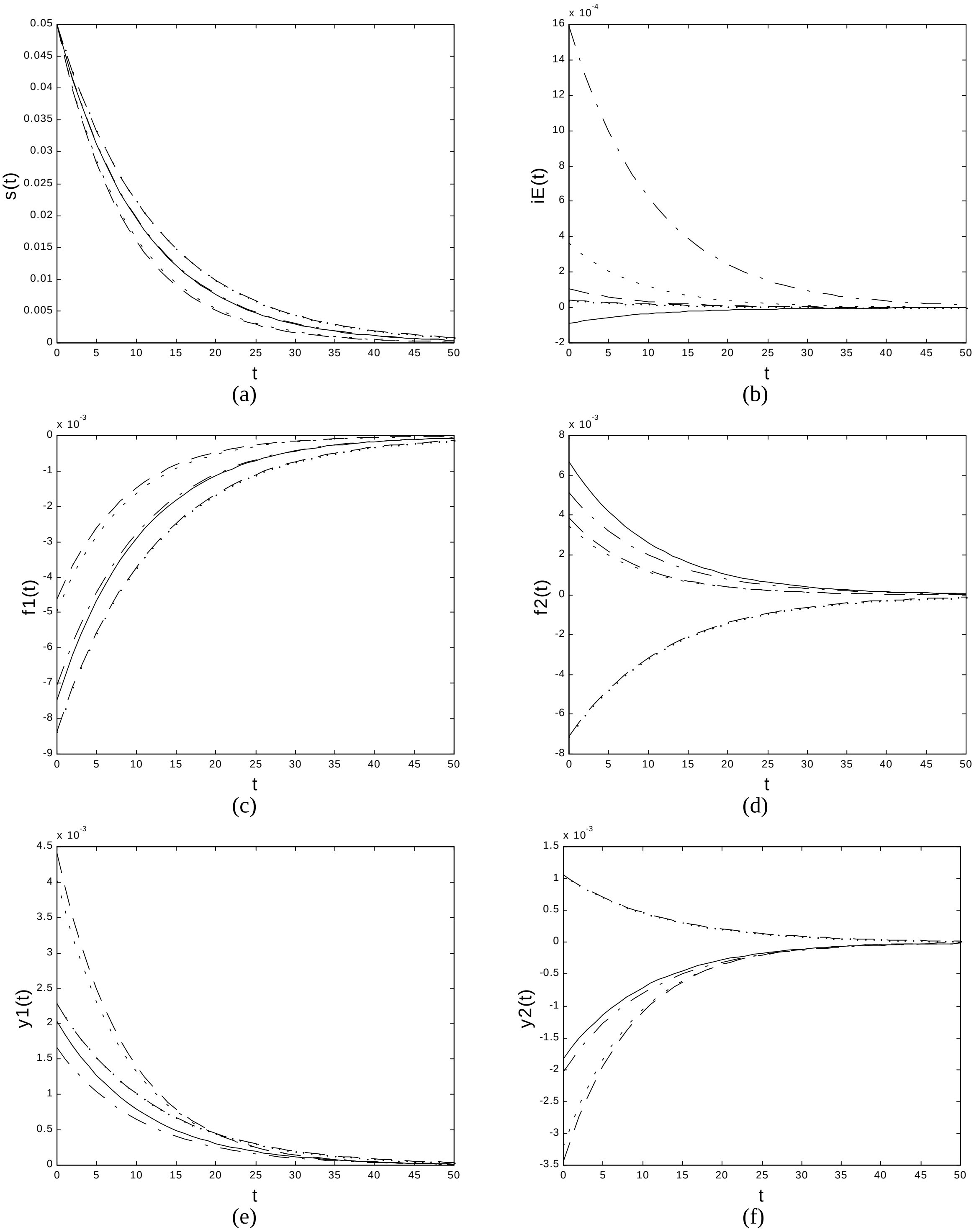

Figure 4b

Asymmetric Case, $\rho_1 = 0.4$, $\rho_2 = 0.8$. National Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

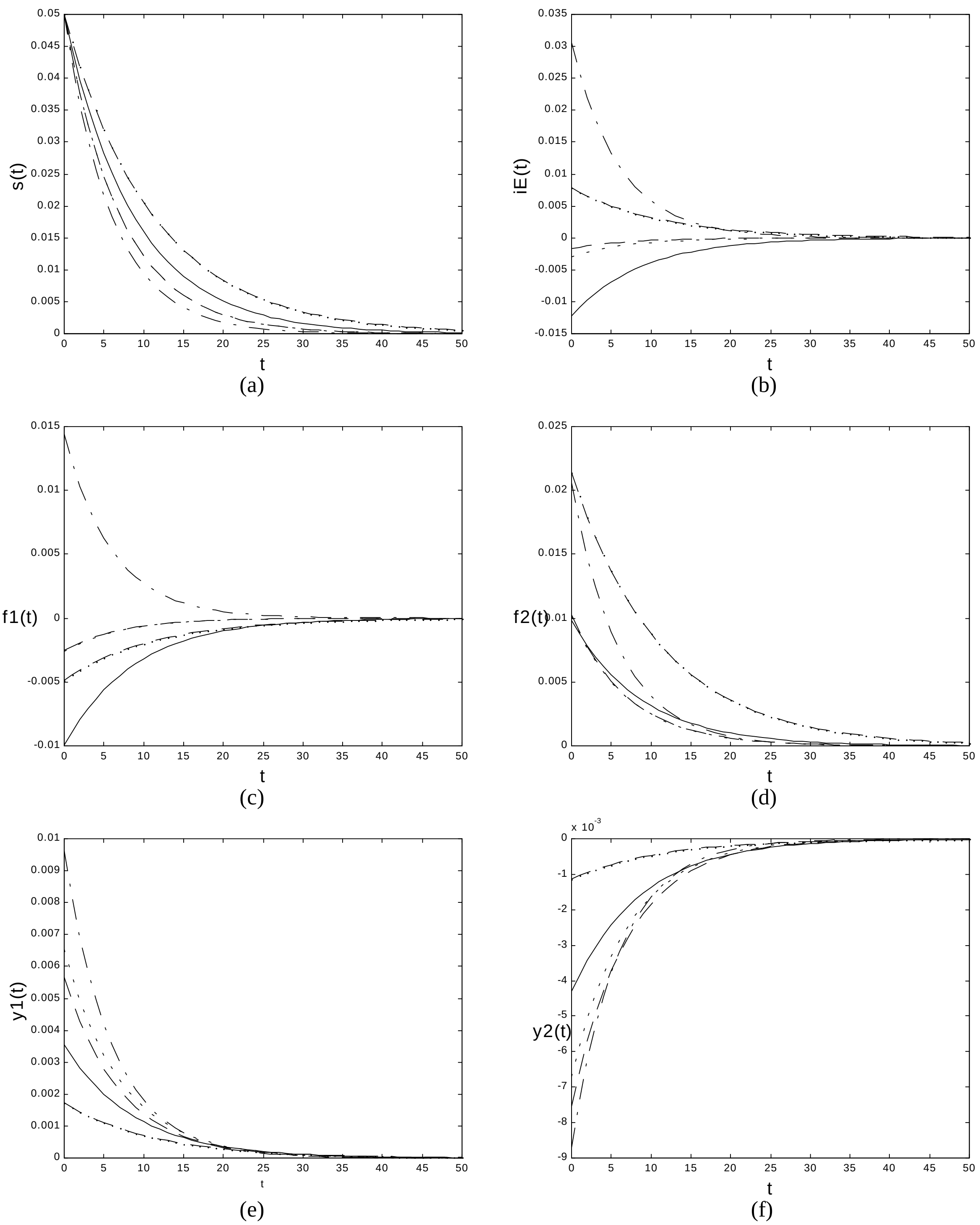

Figure 5a

Asymmetric Case, $\delta_1 = 0.2$, $\delta_2 = 0.4$. Aggregate Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

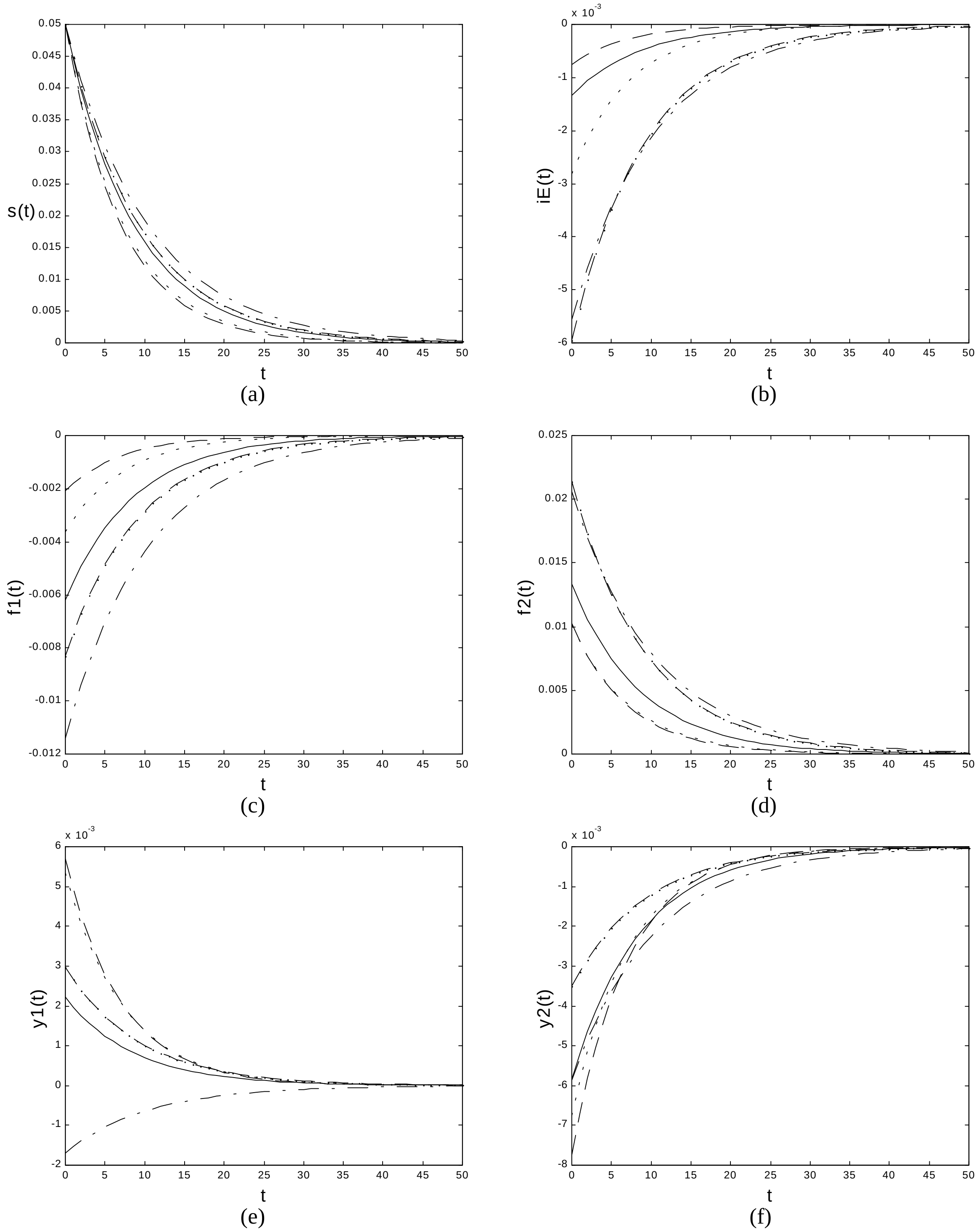

Figure 5b

Asymmetric Case, $\delta_1 = 0.2$, $\delta_2 = 0.4$. National Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

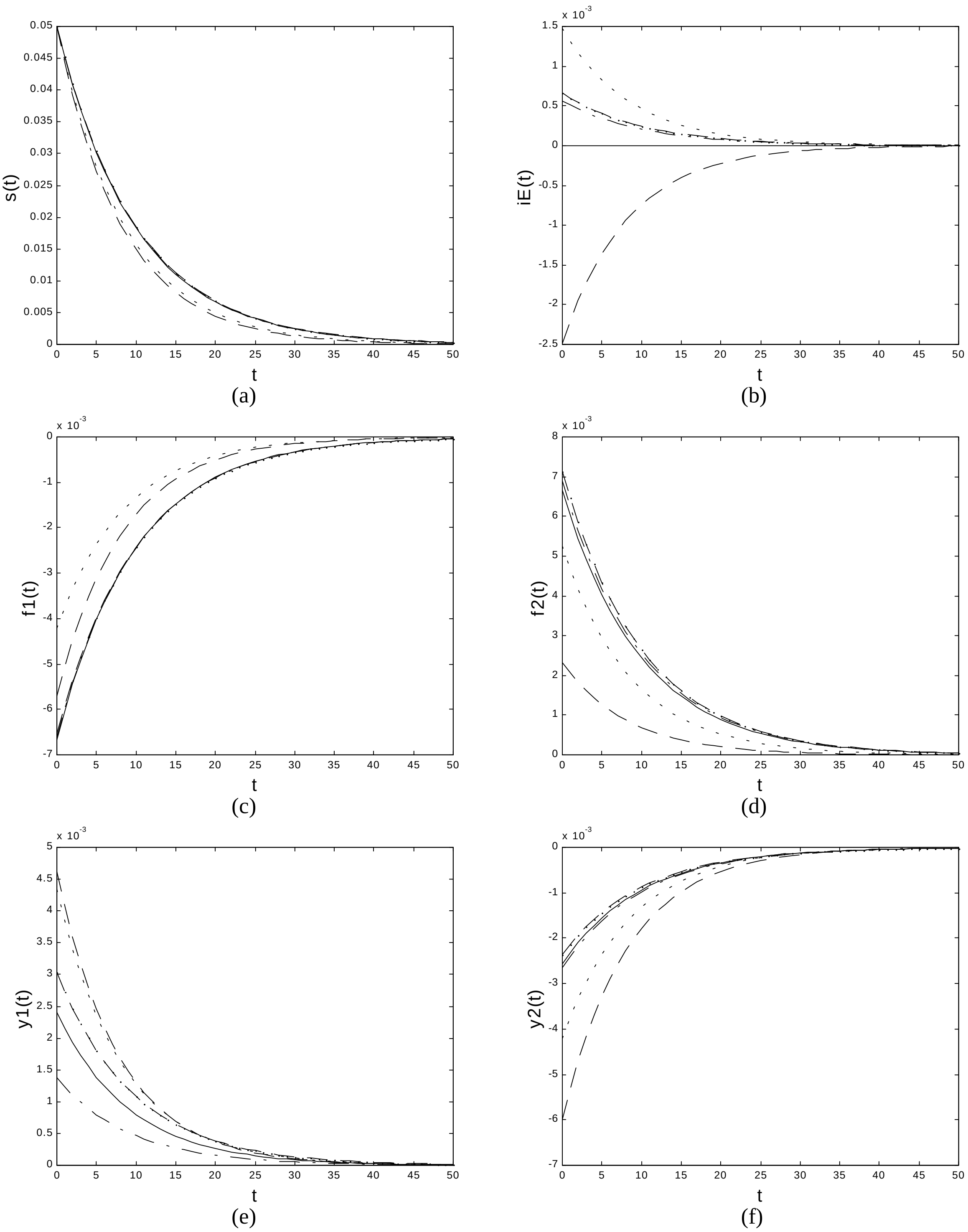

Figure 6a

Asymmetric Bargaining Weights, Aggregate Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), — · — (2,3)

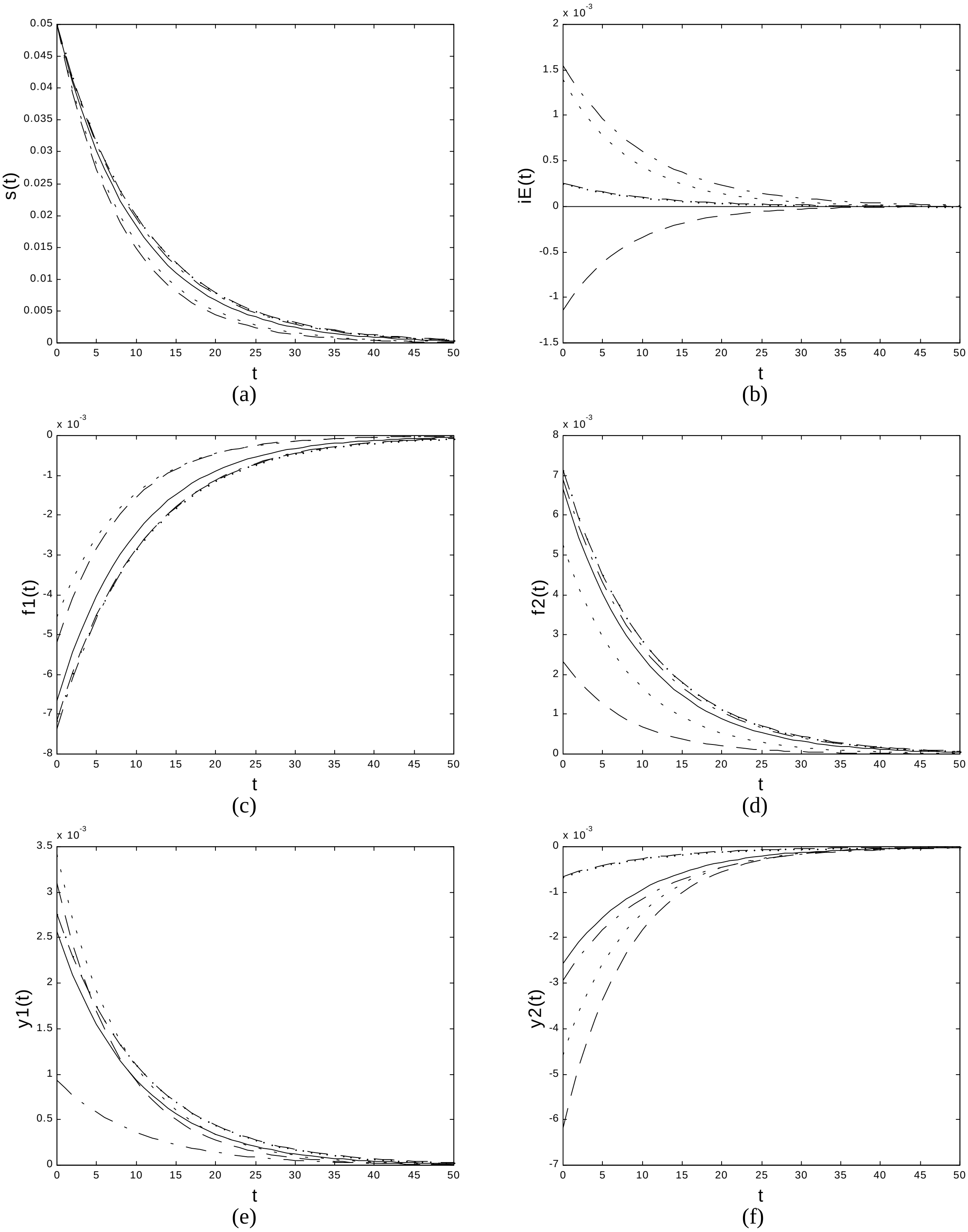

Figure 6b

Asymmetric Bargaining Weights. National Objective Function ECB

– Nash, --- Pareto, …. (1,2), – · –(1,3), – · – (2,3)