A joint Initiative of Ludwig-Maximilians-Universität and Ifo Institute for Economic Research

CESifo Working Papers

START-UP INVESTMENT WITH SCARCE VENTURE CAPITAL SUPPORT*

Vesa Kanniainen
Christian Keuschnigg

CESifo Working Paper No. 439

March 2001

CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Tel.: +49 (89) 9224-1410
Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de

* Drafts of the paper were presented at the University of Saarland, the European Finance Association, Meeting 2000 in London, and University of Freiburg. We appreciate helpful comments by seminar participants, and in particular by D. Bös, J. Eichberger, T. Gehrig, W.L. Megginson and P. Strobel.

CESifo *Working Paper No. 439*
March 2001

START-UP INVESTMENT WITH SCARCE VENTURE CAPITAL SUPPORT

Abstract

Start-up entrepreneurs are often commercially inexperienced. In giving managerial advice, venture capitalists can importantly enhance the success of innovative but highly risky ventures. The supply of experienced venture capitalists is not easily increased, however. When the rate of business formation accelerates, the incumbent venture capitalists tend to include more firms in their portfolio which dilutes the quality of advice, making project risks excessively high in the short-run. The supply of advisory capacity eventually becomes more elastic as new venture capitalists are attracted to the industry. Company portfolios then tend to become more focused again, the quality of advice is restored and the risk of business failure declines.

Keywords: Venture capital, company portfolio, managerial advice, economic rents

JEL Classification: D82, G24, G32, L19.

Vesa Kanniainen
University of Helsinki
Department of Economics
P.O.Box 54
00014 University of Helsinki
Finland
Kanniain@valt.helsinki.fi

Christian Keuschnigg
University of St. Gallen
(IFF)
Varnbuelstraße 19
9000 St. Gallen
Switzerland
christian.keuschnigg@unisg.ch

1 Introduction

The advent of the internet economy and the technological revolutions in telecommunications, biology and other sciences has led to unprecedented rates of business formation. In starting up new firms, pioneering entrepreneurs have been a major driving force of the growth of knowledge-intensive industries. Innovative projects can be highly profitable but tend to be rather risky as well. In fact, complete business failure is quite common among start-ups. Entrepreneurs face several barriers in starting a new firm. As compared to the start-up investment costs, their own resources tend to be limited and they require outside finance. Typically, they look back on an engineering or science career but are commercially inexperienced. Their superior technological knowledge and proprietary information makes it difficult for outside financiers to evaluate the project and monitor its progress. Having no own track record and no collateral, traditional bank finance is often not available. Instead, this void is filled by venture capitalists. They are specialized in financing innovative, but highly risky projects. The rise of venture capital (VC), including private business angels and VC companies, is a rather new phenomenon.[1] While venture capital in the US amounted to no more than 2-5 billion USD annually in the early 90s, recent estimates put it at almost 60 billion in 1999. Lagging behind for a long time, Europe is now catching up as well. In 1999, its venture capital was 25 billion Euros, five times what it was in the early 90s.[2]

Business angels and VC companies not only provide financial resources,[3] but also support firms with their special business expertise, making success more likely.[4] Given the often limited business competence of the founding entrepreneur, the VC advice in building

[1] For our purposes, we treat these two institutions as identical. In reality, business angels often are the early sources of finance while VC funds engage themselves in later stages of business development.

[2] See statistics of the European and US VC associations on www.evca.com and www.nvca.org.

[3] Sahlman (1990) and Gompers (1995) produced illuminating stylized facts on venture capital. For a recent evaluation, cf. Kaplan and Strömberg (2000). Chan, Siegel and Thakor (1990) and Hellman (1998) provided important analytical work.

[4] Cf. Repullo and Suarez (1999), Casamatta (1999) and Schmidt (1999). The role of public policy in venture capital finance is discussed in Keuschnigg and Nielsen (2000).

business relations, hiring the right personnel and marketing the product etc. becomes a key input. The managerial expertise and industry knowledge of the financiers is thus a critical ingredient of a healthy VC sector. There are probably few industries where experience matters as much as in VC investing. Such competence rests on own experience and active business involvement in the respective industry. It cannot be acquired in short order, nor is it easily transferable to other persons.[5] As Gompers and Lerner (1999, p.4) put it: *"Not only is it difficult to raise a new venture capital fund without a track record, but the skills needed for successful venture capital investing are difficult and time-consuming to acquire".* It is expected that the limited supply of experienced venture capitalists (VCs), rather than the availability of capital, is the truly scarce factor in launching young innovative firms. The scarcity and quality of specialized management skills of the financiers might importantly influence the success of newly established industries.

The existing literature on VC finance has focused on the relationship between financiers and entrepreneurs and has studied how financial contracts can be arranged to provide the appropriate incentives conducive to joint success. Our work has a rather different focus. We ask how the scarcity of the managerial resource affects the success of start-ups. We explore the implications for the quality of advice when there are many risky projects in search of funds but a full-fledged VC industry is not yet developed. When entrepreneurs with promising ideas are abundant but the supply of experienced VCs is limited, rents will usually be abnormally high. VCs will be tempted to include a larger number of start-ups in their portfolio.[6] Managerial advice then tends to be stretched too thin over numerous firms, reducing the VC's value added in each single portfolio company and raising the risk

[5] In a rapidly changing business environment, such competence cannot be permanent either and it easily depreciates. After a long and optimistic uprise of information technologies, the prospects disappeared in some areas like internet trade, resulting in failures and bankruptcies. Stock prices of many technology companies recently turned down as the success expectations of some firms proved to be fatally wrong.

[6] A VC is typically involved in several start-up firms and operates a pool of companies, see Gorman and Sahlman (1989), Sahlman (1990), Norton and Tenenbaum (1993) and Reid, Terry and Smith (1997). Building on Kanniainen and Keuschnigg (2000), we explain the optimal number of firms in a VC's portfolio. This issue is ignored in the theoretical literature on VC finance.

of business failure. High rents over a prolonged time will eventually attract additional VCs and ease the shortage in managerial advice. VCs will then focus on fewer firms, advising each one more intensively and thereby keeping the chance of business failure small. Since specialized managerial competence is acquired only through active business experience, the emergence and entry of experienced VCs tends to be a slow process. For this reason, the supply of VCs is presumably rather inelastic in the short run.

The policy debate has traditionally recommended to facilitate access to private equity capital to support entrepreneurship and start-up investment. Indeed, private initiative and government subsidies have lately raised more capital probably than what can fruitfully by used in financing start-up entrepreneurs. What is needed more than ever is informed capital that addresses the commercial inexperience of start-up entrepreneurs and avoids excessive rates of business failure. Indeed, the VC industry has expanded vigorously. One may doubt, however, whether much of it deserves its name in terms of the value added, i.e. the quality of managerial advice, that it actually offers. Informed capital is much scarcer and more difficult to expand than equity finance in the traditional sense. The availability of high quality VC is probably still a considerable bottleneck especially in Europe in launching and successful expansion of highly innovative industries.

The paper illustrates how some important demand and supply side shocks might change the way the industry works. A demand shift, for example, may open up new business opportunities and accelerate the rate of business formation, thereby creating demand for more VC finance. The VC sector may benefit from improved investor knowhow such that they will successfully launch with the same managerial effort a larger fraction of start-ups. With the industry turning more capital intensive, the investment volume needed to complete the start-up phase may become higher. VCs may also find it more costly to provide the required managerial effort, possibly because the market environment becomes more difficult. Apart from these variable costs, the fixed cost of setting up a VC fund may increase on account of more formidable barriers to entry. In all those cases we study how the scarcity of the managerial resource determines the quality of VC investing and shapes the evolution of the industry, both in the short and the long run.

The paper is organized as follows. In section 2, we derive the incentive compatible contract and solve for the optimal managerial advice when a VC finances a portfolio of companies. Section 3 studies the optimal number of portfolio companies and emphasizes the key trade-off between portfolio size and riskiness of projects. Section 4 turns to the industry equilibrium. In the short run, the number of VCs is fixed and each one finances a variable number of firms to satisfy demand for VC support. We discuss how the presence of rents attracts more VCs and then explore the industry equilibrium when free entry makes the supply of VCs elastic and the managerial resource becomes more abundant. Section 5 provides a short summary with some concluding comments.

2 Managerial Advice and Profit Sharing

Basic Assumptions: Our model of venture capital focuses on the managerial contribution of the financier and emphasizes the importance of profit sharing arrangements to realign the incentives of entrepreneurs and financiers for contributing to joint success. We keep the model simple in other respects. All agents are risk neutral. While each VC finances and advises a portfolio of n firms, an entrepreneur pursues only one project or firm. There is no shortage of potential entrepreneurs given that market prices are sufficiently strong to make their risky projects attractive. The number of VCs, in contrast, is fixed in the short run and may adjust only after some time by entry or exit. We abstract from screening or project choice and assume instead that projects have identical stochastic structure ex ante. This will allow us to focus on the symmetric equilibrium.

In starting a firm, the entrepreneur gives up an income $w \geq 0$, which she could earn elsewhere. A project or firm is either successful and yields a return $R > 0$, or is a failure and yields nothing. A successful project results in one unit of output which is, for simplicity, a perfect substitute for the output of other projects. Hence, the value of a successful project is equal to the common market price in the respective industry. Each project requires a uniform start-up investment of $I > 0$. The probability of success is independent across projects. It depends on effort $e_i \in \{0, 1\}$ of the entrepreneur which can be either high or low, and on advice $a_i \geq 0$ that the VC allocates to the i-th project:

$$P(e_i, a_i) = e_i p(a_i), \qquad p''(a_i) < 0 < p'(a_i), \qquad p(a_i) < 1, \qquad A = \sum_{i=1}^{n} a_i. \qquad (1)$$

We assume diminishing returns to advice per project over the relevant range of a_i. Because of the assumption of symmetry, $A = an$. The special form of the success probability implies that entrepreneurial effort is the critical input. Active managerial consulting further adds value and enhances survival chances only if the entrepreneur's effort is high.

Expanding her total consulting activity is increasingly costly to the financier. We assume an increasing and convex cost of the VC's advising effort $\gamma c(A)$ with $c'(A) > 0$, $c''(A) > 0$ and $\gamma > 0$. Given discrete effort choice, the entrepreneur's effort cost is simply $l(e) = \{0, \beta\}$, $\beta > 0$. The subsequent analysis will be greatly simplified in assuming isoelastic functional forms:

$$p(a) = \alpha \cdot \frac{a^{1-\theta}}{1-\theta}, \qquad c(A) = \frac{A^{1+\varepsilon}}{1+\varepsilon}, \qquad 0 < \theta < 1, \qquad \varepsilon > 0. \qquad (2)$$

We interpret the parameter α as reflecting the VC's experience and industry knowledge. We take it for granted that a more experienced VC endowed with superior knowhow is more productive in advising her portfolio companies and achieves with the same effort a higher survival probability than a novice. The productivity parameter α could also reflect the presence of incubators and other specialized infrastructure designed to promote innovative business start-ups.

Venture Capital Investing: Since entrepreneurs have no own resources, the VC must finance the entire start-up cost of each project in exchange for a profit share $1 - s_i$. She also provides managerial advice to the entrepreneur and thereby further adds value to the project. The VC's activities are decomposed into a sequence of decisions which is illustrated by the following time line:

pay fixed entry cost c_f	choose n and pay I	sign contract s_i, b_i	supply effort and advice, e_i, a_i, A	risk resolved

Anticipating the long-run industry analysis in section 5, a VC first has to decide whether to pay a fixed entry cost c_f to establish the VC firm. She then chooses the number of projects that she wants to finance and pays the fixed start-up cost for each one. Next, she proposes the terms of the equity contract including an equity share and, possibly, a base salary b_i for the entrepreneur, anticipating how these provisions determine incentives for her own and the entrepreneur's effort.[7] Given the terms of the contract, both parties next choose their effort levels which determines the survival probability. The level of effort is private information, is not verifiable and cannot be contracted. The fact that the contract is fixed prior to effort choice creates a double sided moral hazard problem. Since effort is costly, the entrepreneur may be tempted to shirk if her profit share is too low. Shirking results in a complete loss for sure. Our formulation also assumes that the VC cannot commit to an effort level ex ante, and will choose the extent of advice dependent on the profit share. Finally, risk is resolved and payments are made.

We derive the financier's decisions by means of backward solution. She maximizes expected profits,

$$\pi^F = \max_{s_i, b_i} \sum_{i=1}^{n} \left[e_i p(a_i)(1 - s_i) R - b_i - I \right] - \gamma c(A) - c_f, \tag{3}$$

subject to participation and incentive compatibility constraints,

$$PC^E : \quad \pi_i^E = e_i p(a_i) s_i R - l(e_i) + b \geq w, \qquad i = 1, ..., n \tag{3.i}$$

$$IC^E : \quad p(a_i) s_i R - \beta \geq 0, \qquad i = 1, ..., n \tag{3.ii}$$

$$IC^F : \quad \{a_i\} = \arg\max \textstyle\sum_i \left[e_i p(a_i)(1 - s_i) R \right] - \gamma c(A). \tag{3.iii}$$

[7] In practice some, though not all, of the contracts allow for a fixed compensation. In Keuschnigg and Nielsen (2000) where entrepreneurs are assumed risk averse, the fixed payment is needed for insurance reasons. Here, it arises if the entrepeneur's outside opportunity is strictly positive. None of our key results are affected if opportunity costs are normalized to zero and $b_i = 0$.

Condition (3.i) is the participation constraint of entrepreneurs arising from their occupational choice. In opting for an entrepreneurial career, she gives up her alternative wage income w. The VC contract must be generous enough to make entrepreneurship a worthwhile option, $\pi_i^E \geq w$. Conditions (3.ii) and (3.iii) reflect the *ex post* incentive constraints. Given that the initial investments are sunk and the contracts are already specified, agents choose effort to maximize the remaining income that is still at their discretion. The minimum profit share s_i that satisfies (3.ii) and makes entrepreneurs willing to provide high effort, $e_i = 1$, depends on the success probability and, thus, on the extent of managerial advice by the VC. On the other hand, shirking by the entrepreneur destroys any return to the VC's advisory effort in (3.iii). Thus, the actions are interactive.

Effort and Advice: The necessary conditions for optimal advice are[8]

$$\Omega_i \equiv e_i p'(a_i)(1 - s_i) R - \gamma c'(A) = 0, \qquad i = 1, \ldots, n. \tag{4}$$

The efforts of the VC and all n entrepreneurs are determined simultaneously. If $e_i^* = 1$, then the VC provides a positive level of advice, $a_i^* > 0$. If the entrepreneur shirks, $e_i^* = 0$, the return to advice is negative, and the VC would not want to waste any effort, $a_i^* = 0$. Our assumptions on the success probability make efforts complementary.[9]

Optimal managerial advice as well as the entrepreneur's effort depend on the agreed profit shares. Anticipating how profit sharing shapes incentives for joint efforts, the VC chooses s_i to maximize her profits. Suppose the entrepreneur's share is sufficiently high such that her incentive constraint (3.ii) is slack. Applying the envelope theorem in (3), $\frac{d\pi^F}{ds_i} = -p(a_i^*) R < 0$. The VC thus raises her profits by cutting the entrepreneur's profit share. She will do so until she hits the IC^E constraint which must therefore be binding in optimum. Conditions (3.ii) and (4) jointly determine the optimal profit shares and

[8]The second order conditions can be characterized more precisely in terms of the Hessian matrix in the usual way. We have $\Omega_{ii} \equiv e_i p''(1 - s_i) R - \gamma c'' < 0$. All cross derivatives are identical and negative as well, $\Omega_{ij} \equiv -\gamma c'' < 0$.

[9]Complementarity of efforts is also considered by Repullo and Suarez (1999), while Casamatta (1999) assumed perfect substitutability.

managerial advice such that entrepreneurial effort is assured to be high. Since all projects are identical ex ante, we may concentrate on the symmetric solution. Figure 1 which is developed fully in appendix A, depicts the entrepreneur's and the financier's incentive constraints, $E(s)$ and $F(s)$, respectively. In general, there may be several intersections. In this case, the profit maximizing solution is the lowest s and, correspondingly, the highest a as in point A of figure 1.

To identify the effects of project value and other parameters on profit shares and managerial advice, we log-linearize the model in the neighborhood of the symmetric equilibrium. The hat notation indicates logarithmic differentials such as $\hat{a} \equiv da/a$. Using (2), the linearized incentive constraints (3.ii) and (4) are

$$IC^E : \hat{s} = -\hat{R} - \hat{\alpha} - (1-\theta)\,\hat{a}, \qquad IC^F : (\theta + \varepsilon)\,\hat{a} = \hat{R} + \hat{\alpha} - \frac{s}{1-s}\hat{s} - \hat{\gamma} - \varepsilon\hat{n}. \quad (5)$$

The entrepreneur's share may be cut if her incentives are strengthened by a higher project value and a higher success probability on account of more advice, $\hat{p} = \hat{\alpha} + (1-\theta)\,\hat{a}$. The financier, in turn, advises more in response to rising project values. Higher effort cost and a smaller equity share $1-s$ of the VC discourage the consulting activity. Note, in particular, that the VC cuts back on her advice if she expands her company portfolio. Taking account of the simultaneity in (5), the equilibrium adjustment of advice is

$$\hat{a} = \frac{1}{(1-s)\,\Psi}\left[\hat{R} + \hat{\alpha} - (1-s)\,(\hat{\gamma} + \varepsilon\hat{n})\right], \qquad \Psi \equiv \theta + \varepsilon - \frac{s\,(1-\theta)}{1-s} > 0. \quad (6)$$

According to (5), a higher project value directly raises the VC's incentives for consulting. In reducing risk, more advice allows to cut the incentive compatible profit share s. The implied increase in the financier's share reinforces her incentives for consulting which then allows for a further reduction in s. When this cycle converges, the total effect is positive, $\Psi > 0$, and exceeds the direct effect.[10] If consulting effort becomes more costly, the overall managerial activity A and the advice per firm a decline. The solution (6) points to a fundamental trade-off that a VC faces in financing additional start-up firms:

[10] The condition $\Psi > 0$ is equivalent to $E(s)$ being steeper than $F(s)$ at point A of figure 1.

Lemma 1 ***(Dilution of Advice)*** *Increasing the number of portfolio companies dilutes the quality of advice and thereby raises the individual project risk.*

Optimal Contract: The optimal contract consisting of an equity share and a fixed payment is determined recursively. To ensure the entrepreneur's full effort, her share must be sufficiently high. It is determined jointly with the level of advice by the two incentive constraints. Substitute (6) back into the entrepreneur's constraint in (5) yields

$$\hat{s} = \frac{1}{\Psi}\left[(1-\theta)(\hat{\gamma}+\varepsilon\hat{n}) - (1+\varepsilon)\left(\hat{R}+\hat{\alpha}\right)\right]. \tag{7}$$

The intuition is immediately revealed upon inspection of the entrepreneur's incentive constraint. A higher project value directly strengthens the entrepreneur's incentives for high effort, allowing the VC to squeeze the profit share s. When the effort cost γ increases, the VC advises less. As the expected revenue of the project declines on account of a higher downside risk, she must offer a higher profit share to enlist the entrepreneur's full effort. For the same reason, an expansion of the company portfolio forces the VC to raise the entrepreneurs' profit shares.

The VC chooses the minimum profit share that suffices to enlist the entrepreneur's full effort. With (3.ii) binding, the participation constraint (3.i) reduces to $b_i \geq w$. Since the base salary directly reduces profits, the VC is left to compensate the entrepreneur for her foregone outside opportunities, but no more than that.[11] We summarize the results of this section by

Proposition 1 ***(Advice and Profit Sharing)*** *(a) Advice per firm rises with return R and productivity α, but falls with the number of companies n and the VC's effort cost γ. (b) The entrepreneur's profit share falls with return R and productivity α but rises with portfolio size n and effort cost γ.*

[11] This is similar to Strobel (2000). Instead of an optimal portfolio, he discusses how the market is split between VCs and banks. In practice, $b_i > 0$ appears to be somewhat untypical which could be taken as an indication that the opportunity cost is considered insignificant.

3 How Many Portfolio Companies?

The number of portfolio companies is optimal when the contribution of the marginal firm to expected overall profits is zero. Adding another firm to the portfolio shifts down the financier's incentive constraint as is illustrated in Figure 1. Expanding the portfolio thus dilutes advice per firm, see Lemma 1, because the overall effort cost is increasing progressively with total managerial activity[12] $A = an$. In adding another firm, the VC thus raises project risk for all firms in her portfolio. Figure 1 shows that she must therefore offer a higher profit share to all the entrepreneurs that she finances to enlist their full effort. The erosion of her own equity share thus impairs profits from her inframarginal firms and eventually offsets the extra profit added by the marginal project.

These arguments can be made precise. Substituting $b_i = w$, differentiating (3) and imposing symmetry yields $\pi_n \equiv \frac{d\pi}{dn} = [p(a)(1-s)R - w - I] - a\gamma c'(na) - np(a)R\frac{\partial s}{\partial n}$. Although a larger portfolio dilutes advice, the marginal effect on profits is zero by the *envelope theorem* applied to (3.iii). The square bracket indicates the contribution of an extra firm to VC profits. The second term reflects the additional effort cost from extending managerial support to the marginal firm. The last term captures the profit destruction effect. Having more firms leads the VC to advise each one less which erodes survival chances. To preserve incentives in face of higher risk, the VC must cede a higher profit share to all her partners. Use the elasticity noted in (7) together with (3ii) to rewrite the last term, replace c' by (4), use $ap' = (1-\theta)p$ from (2), and finally replace pR by β/s from (3.ii) to obtain

$$\pi_n \equiv \frac{d\pi}{dn} = [\theta p(a)(1-s)R - w - I] - \frac{(1-\theta)\beta\varepsilon}{\Psi(s)} = 0, \qquad \pi_{nn} < 0. \tag{8}$$

The optimal number of firms to be advised is implicitly determined by (8) since the incentive compatible equity share and the intensity of advice depend on n. By (6) and (7), n diminishes a but raises s. Since $\Psi'(s) = -\frac{1-\theta}{(1-s)^2} < 0$, both terms in π_n decline

[12] Parameter ε in (6) reflects the convexity of the effort cost function. If effort cost were linear, i.e. $\varepsilon = 0$, advice per firm would no longer be diluted.

with n, thereby fulfilling the sufficient condition. Figure 2 illustrates the solution for the optimal number of firms, and the implied equity share s, by using $pR = \beta/s$ from (3.ii) and writing the square bracket in (8) as $z_1(s) = \theta\beta(1-s)/s - w - I$ and the last term as $z_2(s)$. The profit destruction effect becomes ever more severe as more firms are added to the portfolio. With small n, on the other hand, the VC advises rather intensively and can appropriate a large profit share without losing the entrepreneur's effort. Marginal benefits (net of effort cost) of expanding the portfolio are then relatively high. They rapidly decline as more firms are included and the equity shares of all the entrepreneurs in the portfolio have to be raised. Appendix A develops figure 2 in more detail, taking account also of the border conditions implied by our functional forms, and proves

Proposition 2 ***(Optimal Portfolio)*** *A unique optimal number of portfolio companies exists,* $0 < n^* < \infty$.

If effort cost were linear ($\varepsilon = 0$), advice and profit share in (6) and (7) would be independent of n. The profit destruction effect would disappear, making marginal benefits a constant $\pi_n = \theta p(a)(1-s)R - w - I \gtrless 0$ and leaving the portfolio problem indeterminate. If, on the other hand, the success probability were linear ($\theta = 0$), a would fall and s would increase in n as before. In this case, however, the benefit of an extra firm (net of marginal effort) would be unambiguously negative, making $\pi_n < 0$ in (8). The optimal number of firms would be driven to one, if that were still profitable. For portfolio size to be well determined, both curvatures are needed. With decreasing returns to advice, it is better to advise more firms where a small amount of advice can make a big difference, rather than concentrating on a single firm only where the marginal effect of advice rapidly becomes less effective in raising the success probability. With a convex cost function, however, the VC's effort cost increases progressively as more firms are added. Advice is easily stretched too thin. Risks inevitably increase, forcing the VC to cede a higher profit share to the entrepreneurs which erodes her own profits. This profit destruction effect eventually makes a further expansion of the portfolio unattractive.

The comparative static analysis exploits the fact that (8) determines the equity share autonomously, see (A.10). We then use (7) to read off the implied number of firms. Appendix A proves[13]

Proposition 3 ***(Effects on Portfolio)*** *The optimal number of start-up firms in the VC portfolio increases with project value R and managerial productivity α but declines with effort cost γ as well as start-up investment cost I,*

$$\hat{n} = \zeta_R \left(\hat{\alpha} + \hat{R}\right) - \zeta_\gamma \hat{\gamma} - \zeta_I \hat{I}, \tag{9}$$

where ζ are positive coefficients defined in (A.10).

Figure 3 illustrates the solution by the intersection of the contract line given by (7) and the portfolio condition given by (8). The latter is horizontal since it autonomously solves for the profit share s^* when (3.ii) is taken account of. By (A.9), $d\pi_n/ds < 0$, an equity share larger than s^* impairs the VC's net marginal benefits from adding an extra firm and makes her consolidate the portfolio, see the directional arrows in figure 3. By Lemma 1, she advises more intensively when she is concentrating on fewer firms and thereby reduces the risk of project failure. Smaller risk allows to cut the entrepreneur's incentive compatible profit share. Moving along the contract line to the South West eventually stops the incentive to consolidate the portfolio as the profit share approaches s^*. A larger start-up cost I shifts down the horizontal portfolio condition as noted in the line following (A.9) because it directly weakens the marginal benefits of portfolio expansion in (8). The initial equity share is now too high. The VC reduces the number of portfolio companies and offers a smaller profit share to her entrepreneurs in exchange for more advice and lower risk. Moving along the contract line to the South West then leads to the new intersection point. An increase in project value R or in managerial productivity α shifts down the contract line but leaves the horizontal portfolio line unaffected. A higher project value

[13] The effects of w and I are qualitatively identical throughout all stages of the analysis. For this reason, we can spare an analysis of the entrepreneur's outside option.

strengthens incentives for advice. Reducing risk allows the VC to offer a smaller profit share s and still enlist the entrepreneur's full effort. With a larger share of her own, she is keen on funding more firms. In adding more firms, she moves North East along the new contract line until the dilution of advice forces her to raise the equity share again to s^*.[14] A higher effort cost γ shifts up the contract line and leads the VC to fund and advise a smaller number of portfolio companies.

4 Venture Capital in Equilibrium

We now investigate how VCs react when market conditions for start-up investment change. A demand shift for industry output may boost market prices, making business start-ups more profitable. Higher start-up investment costs I may slow down the rate of business formation. VCs may find it more costly to put up the required effort, i.e. γ increases. The productivity α of VCs in providing advice may also increase, possibly because government provides specialized infrastructure that aims to facilitate business creation, or because VCs are more experienced. Finally, we consider entry barriers to VC investing in terms of a fixed cost c_f of setting up a fund. Before analyzing the consequences of various shocks, we first address the role of rents in attracting new VC investors which gives rise to contrasting short- and long-run equilibria.

4.1 How Rents Promote Entry

Successful VC investing requires much experience and rather specialized knowhow as well as detailed industry knowledge, all of which are difficult to acquire in short order. These requirements represent formidable entry barriers which make the short-run supply of VCs inelastic. Only gradually will VC activity become more widespread through entry of knowledgeable financiers. We therefore take both a short- and long-run perspective.

[14] In the end, advice per firm is reduced by $\hat{a} = -\hat{R}/(1-\theta)$ as can be checked by substituting (9) into (6). This keeps expected revenue pR constant and satisfies (3.ii) for a fixed s.

Keeping the number N of VCs fixed in the short run, we investigate how they adjust their portfolios to changing market conditions. The fixed supply of VCs may create important rents that should attract other VCs to pay the entry cost c_f. As ever more VCs set up a fund, the expansion of the industry will eventually erode profit opportunities and make further entry unprofitable. A zero profit condition then determines the number of VCs in the long run. The questions of how many VCs a given industry is able to sustain, and how entry affects the nature of VC investing, have eluded an analysis in the earlier literature.

Given sufficiently many VCs, we may abstract from strategic interactions and assume that they behave as competitive price-takers. Large numbers also make the individual success probability coincide with the fraction of start-ups that actually succeed in equilibrium. In assuming that a successful project yields one unit of output, we interpret R as the endogenous market-clearing price of output. We then show how the market price feeds back on the quality of managerial advice, risk, profit sharing and portfolio size. When the market is flooded with the extra output by ever more start-ups, profit opportunities will surely be exhausted at some point. To capture this aspect, we introduce a demand function, $D = \phi R^{-\eta}$ where $\phi > 0$ is a demand shift parameter.[15] Some of the comparative static results will depend on whether the price elasticity is smaller or larger than one. Demand for new products tends to be rather price elastic, however. We therefore keep with the case of $\eta > 1$. In industry equilibrium, supply must equal demand,

$$\phi R^{-\eta} = p\left(a\right) nN, \qquad \eta > 1. \tag{10}$$

With N VCs in the industry and each one funding a portfolio of n firms, nN projects get started but only a fraction p of them actually survives the start-up phase. With each successful firm supplying one unit of output, aggregate supply is pnN.

To identify how the short-run equilibrium with a fixed number of VCs is affected by

[15] We keep the demand side deliberately simple and abstract from market uncertainty, network externalities and other aspects of demand for innovative goods. Our focus is on the nature of VC investing. The demand function conveniently models how profits disappear as VC investing expands.

certain supply and demand shocks, we log-linearize (10) at the initial equilibrium position,

$$\hat{\phi} - \eta \hat{R} = (1 - \theta)\, \hat{a} + \hat{\alpha} + \hat{n} + \hat{N}. \tag{11}$$

The comparative statics of the short-run equilibrium is determined by three simultaneous equations (11), (9) and (6) in three unknowns, $\hat{R}$, $\hat{n}$, and $\hat{a}$. Replacing $\hat{n}$ and $\hat{a}$ in (11), appendix B computes how various demand and supply side shocks create excess demand. The market price must then adjust to equilibrate the market clearing condition (11). On the supply side, an increase in the market price induces VCs to expand the number of firms in their portfolio which boosts supply by $\hat{n} = \zeta_R \hat{R}$. Although a higher price tends to encourage managerial support, the dilution of advice on account of a larger number of portfolio firms dominates and makes projects more risky. With fewer firms surviving the start-up phase, supply contracts. Substituting $\hat{n} = \zeta_R \hat{R}$ into (6) gives $\hat{p} = (1 - \theta)\, \hat{a} = \frac{1-\theta}{(1-s)\Psi} \left[\hat{R} - (1 - s)\, \varepsilon \hat{n} \right] = -\hat{R}$. Finally, a higher price reduces demand by $-\eta \hat{R}$. Add up and note that a higher price eliminates excess demand at a rate $-\lambda \hat{R}$ with $\lambda = \eta - 1 + \zeta_R > 0$. Whenever an exogenous shock creates excess demand, the output price increases to restore market clearing and feeds back on VC activity to determine the equilibrium level of advice, profit sharing, portfolio size, and short-run rents. Appendix B derives the comparative static results that are summarized in Table 1 where the columns correspond to equations (B.1,3,4,5,9,10).

Given the required experience in managing and funding start-up companies, new VC funds cannot be established in short order to accommodate an accelerating rate of business formation. With a fixed number of VCs, positive demand shocks should make VC finance rather profitable and create rents in the short run. The last column of table 1 indicates how rents derive from various shocks. With sustained profit opportunities, new VCs should appear and promote the expansion of the industry. According to (B.1), entry spoils market prices which, in turn, cuts into profits as noted in (B.8) and makes further entry increasingly unattractive. Entry stops when profit opportunities are fully exhausted. In solving for the long-run equilibrium with free entry, we first derive in (C.1) the market price which satisfies the zero profit condition (B.8). We then investigate how

Table 1: Fixed Number of VCs

	$\lambda\hat{R}$	$\lambda\hat{a}$	$\lambda\hat{n}$	$\hat{s}$	$\lambda\hat{\pi}$
$\hat{\phi}-\hat{N}$	1	$-\frac{1}{1-\theta}$	ζ_R	0	$(1-s)\,npR$
$\hat{\alpha}$	$-\zeta_R$	$-\frac{\eta-1}{1-\theta}$	$(\eta-1)\,\zeta_R$	0	$(\eta-1)\,(1-s)\,npR$
$\hat{\gamma}$	ζ_γ	$-\frac{1}{1-\theta}\zeta_\gamma$	$-\,(\eta-1)\,\zeta_\gamma$	0	$-\,(\eta-1)\,\gamma c$
$\zeta_I\hat{I}$	$\frac{\theta-s+(1-s)\theta\varepsilon}{(1-s)\Psi}$	$\frac{1+(1-s)\varepsilon\eta}{(1-s)\Psi}$	$-\eta-\frac{(1-\theta)}{(1-s)\Psi}$	$-\frac{I}{\Omega\zeta_I}$	$-?$
$\hat{c}_f$	0	0	0	0	$-\lambda c_f$

such price adjustment affects managerial advice, profit sharing, and portfolio size. Finally, we substitute the price effect into the temporary equilibrium condition (B.1) and obtain the number of VCs that are sustained in the free entry equilibrium. Table 2 summarizes the formal comparative static analysis in appendix C.

Table 2: Free Entry of VCs

	$\hat{R}$	$\hat{a}$	$\hat{n}$	$\hat{s}$	$\hat{N}$
$\hat{\phi}$	0	0	0	0	1
$\hat{\alpha}$	-1	0	0	0	$\eta-1$
$\hat{\gamma}$	$\frac{1-\theta}{1+\varepsilon}$	$\frac{-1}{1+\varepsilon}$	0	0	$\frac{-(\eta-1)(1-\theta)}{1+\varepsilon}$
$\hat{I}$	$\frac{(\theta-s+\delta)I}{(\theta+\delta)(1-s)pR}$	$?$	$?$	$\frac{-I}{\Omega}$	$-?$
$\hat{c}_f$	$\frac{c_f}{(1-s)npR}$	$\frac{-c_f}{(1+\varepsilon)\gamma c}$	$\frac{c_f\zeta_R}{(1-s)npR}$	0	$\frac{-\lambda c_f}{(1-s)npR}$

4.2 Demand Shock

The intuition for the competitive industry equilibrium is best developed in considering a demand shift since it does not directly interfere with individual VC decisions. The immediate effect is an increase in prices by $\hat{R}=(\hat{\phi}-\hat{N})/\lambda$ where $\hat{N}=0$ in the short run, see (B.1) and the first line of table 1. The individual financier responds by adding firms to her portfolio. With a larger number of firms to attend, she advises each single entrepreneur less intensively and accepts a higher failure rate. The effect of more risk on profit sharing exactly cancels with the effect of a higher market price.

More valuable projects boost profits in (B.8), $\hat{\pi} = (1 - s)\, npR\hat{R}$, and thereby attract new financiers. With ever more VC backed projects flooding the market, entry depresses the market price. In the long-run equilibrium characterized in table 2, the effect on the market price must be zero to eliminate profits and stop further entry. The temporary equilibrium condition (B.1) then shows that the demand shift is offset by entry of $\hat{N} = \hat{\phi}$ new VCs to keep prices at their initial level. A demand shift affects VC investing exclusively via the price channel but has otherwise no direct implications for managerial advice, portfolio size and profit sharing in the long run.

Proposition 4 ***(Demand Shift)*** *(a) With a fixed supply of VCs, a demand shift boosts market prices and leads to more portfolio firms but dilutes advice per firm and raises risk. With constant profit shares, a higher price boosts rents of incumbent VC firms.*
(b) With free entry, the demand shift is fully accommodated by entry of VCs with all other variables returning to the initial values.

Proposition 4 demonstrates an important adjustment pattern. As the rate of business formation accelerates on account of a demand shock, short-run VC investing can expand only if incumbent VCs finance more firms. With a fixed number of VCs, the quality of VC finance must first deteriorate on account of Lemma 1. As advice gets diluted, the rate of business failure is excessive in the short run. As more VCs enter the market and set up a fund, each individual one is able to consolidate her portfolio and concentrate on a smaller number of firms which she advises more carefully. Only in the long run is the quality of VC investing restored again and the project risk is reduced.

4.3 VC Experience

Experienced VCs are more successful in adding value to their firms and score a higher success rate with the same level of effort. As more start-ups turn out successful, market supply expands by $\hat{\alpha}$ in (11). More sophisticated VCs also like to include a larger number of firms in their portfolio, $\hat{n} = \zeta_R \hat{\alpha}$ in (9). Although more experience itself encourages further advisory effort, the dilution of advice due to portfolio expansion dominates, leading to a larger failure rate and thereby retarding start-up investment. Using (A.11) and (B.2), (6) yields $\hat{p} = (1-\theta)\,\hat{a} = \frac{1-\theta}{(1-s)\Psi}\left[\hat{\alpha} - (1-s)\,\varepsilon \hat{n}\right] = -\hat{\alpha}$. Since the first and second terms exactly cancel in (11), supply expands by $\hat{n} = \zeta_R \hat{\alpha}$ and the market price must fall as in table 1. Declining project values on account of market saturation feed back on VC activity and dampen the expansion of portfolios in (9) to give a more moderate overall effect as in table 1. The quality of advice declines and makes start-up investment riskier. Other things equal, an exogenous increase in the success probability on account of higher VC experience allows to cut the incentive compatible profit share as in (7). Interestingly, the dilution of advice on account of larger VC portfolios and the erosion of project values require to increase the profit share to an extent that exactly cancels the direct effect.[16] In short-run equilibrium, the profit share remains constant.

More sophisticated financiers succeed to bring a larger fraction of start-ups to the market. A higher success rate directly boosts VC profits but on the other hand depresses the market price. The net effect on short-run profits is shown to be positive in (B.9). As $\hat{N} = (\eta - 1)\,\hat{\alpha}$ new VCs emerge, prices must fall even further, i.e. by a total amount of $\hat{R} = -\hat{\alpha}$ in the long run.[17] Although more experience encourages, for given project values, larger portfolios, more intensive advice and smaller equity stakes for entrepreneurs, such incentives are completely offset in the long run by the erosion of market value. The net effects in (9), (6) and (7) depend on $\hat{R} + \hat{\alpha} = 0$ in the long run. We summarize:

[16] In fact, the profit share is solved autonomously by the portfolio condition, see (A.9), and the number of firms in (9) is read from (7). Figure 3 illustrates.

[17] Table 1 reveals a short-run price effect equal to $\lambda \hat{R} = -\zeta_R \hat{\alpha} - \hat{N}$ with $\hat{N} = 0$ instantaneously. Upon entry of $\hat{N} = (\eta - 1)\,\hat{\alpha}$ new VCs, the long-run price effect is $\hat{R} = -\hat{\alpha}$ as in table 2.

Proposition 5 ***(VC Experience)*** *(a) With a fixed supply of VCs, superior knowhow erodes market prices, expands portfolio size, dilutes advice per firm and thereby raises risk. With constant profit shares, more experience creates rents in VC investing.*
(b) With free entry, an increase of investor productivity attracts more VCs and reduces the market price. All other variables return to their initial values.

4.4 Cost of Effort

VC investing may become a more difficult business due to government regulations and red tape, unpredictable and rapidly changing market conditions, and other reasons. For a given number of portfolio companies, VCs will then find it more costly to achieve a desired success rate. Quite intuitively, higher effort cost leads financiers to consolidate their portfolio, $\hat{n} = -\zeta_\gamma \hat{\gamma}$ in (9), and to cut back on advice per firm as well. On the other hand, the quality of advice benefits from VCs concentrating on fewer firms. Substituting $\hat{n} = -\zeta_\gamma \hat{\gamma}$ into (6) and using $\zeta_\gamma = 1/\varepsilon$ as given in (A.10), the net effect on managerial support is seen to be exactly zero, $(1-\theta)\,\hat{a} = -\frac{1-\theta}{\Psi}(\hat{\gamma} + \varepsilon \hat{n}) = 0$. Consequently, VC investing contracts on account of smaller company portfolios, leading to an increase in market price and project values. The feed back from a higher price in itself encourages portfolio expansion at the cost of advice and, thereby, dampens the consolidation. In the end, each VC finances fewer firms and also provides less managerial support to each one, see table 1. Again, the equilibrium profit share remains unaffected due to offsetting influences. Since VCs tolerate higher project risk by cutting back advice, entrepreneurs ask for a higher profit share. On the other hand, a higher equilibrium price makes them accept a lower profit share according to (3.ii). The two effects cancel in (7).

Even though a higher market price strengthens profits, the direct effect of higher effort costs dominates and results in smaller profit margins. The subsequent exit of VCs leads to a supply contraction which magnifies the initial price increase.[18] While higher effort cost per se causes VCs to advise a smaller number of start-up firms as indicated by

[18] Substituting $\hat{N}$ from Table 2 into (B.1) and using (A.10) and (B.2), we have $\lambda \hat{R} = \zeta_\gamma \hat{\gamma} - \hat{N} = \frac{1-\theta}{1+\varepsilon}\lambda\hat{\gamma}$.

(9), more profitable projects on account of rising market prices lead them to restore the initial portfolio size, leaving the number of portfolio companies unchanged in the long run. With the dilution of advice effect being eliminated, the reduction in managerial support is still negative but less pronounced than in the short run. Reflecting the adjustment of managerial advice, start-up investment is excessively risky in the short run and moderately more risky in the long run. The incentive compatible equity share of entrepreneurs is affected neither instantaneously nor after entry is completed.

Proposition 6 ***(Cost of Effort)*** *(a) With VCs in fixed supply, increasing effort cost boosts the market price and results in smaller company portfolios and less advice per firm. Profit sharing remains constant but VC profits decline.*
(b) With free entry, higher effort cost leads some VC firms to exit which magnifies the short-run price increase. The initial portfolio size is restored and advice is cut back more moderately. Profit sharing remains constant as well.

4.5 Start-up Investment Cost

Start-up investment may become more capital intensive. When financiers must incur higher start-up costs, they respond by cutting the number of portfolio companies. As they concentrate on fewer firms, managerial advice per firm is higher and thereby helps to contain the failure rate. With lower risk, the entrepreneurs' profit share can be reduced. The net effect on industry supply of smaller portfolios but larger survival rates is negative and boosts the market price as in table 1. VC investing is likely to be less profitable in the short run. Since entrepreneurs are assumed to have no own funds, the VC must finance the entire start-up cost. On the other hand, higher market prices and a larger own profit share props up profits. This effect, however, is unlikely to dominate.

Responding to diminishing rents, part of the VCs will leave the sector, see (B.10) and (C.3). Exit further boosts the market price as in table 1 until the remaining investors break even. Project values in the free entry equilibrium must thus exceed short-run values.

Via this effect, exit of VC firms makes the remaining investors expand their portfolios at the cost of less intensive advice which reverses the short-run adjustment to higher start-up cost in table 1. For this reason, the net effects on managerial advice and portfolio size in the long-run equilibrium become ambiguous in table 2 [see (C.4) and (C.6)].

Proposition 7 ***(Start-up Cost)*** *(a) Larger investment costs make VCs consolidate their company portfolios and reduce risk by advising more intensively each firm. The market price rises. With more valuable and less risky projects, entrepreneurs extract smaller profit shares. Despite of strong market prices, higher start-up costs are likely to diminish short-run rents in VC investing.*
(b) In the long run, exit of VC firms further boosts the market price. The short-run adjustment in portfolio size and managerial advice are reversed and become ambiguous. Entrepreneurs obtain smaller equity shares.

4.6 Entry Cost in Venture Capital

Finally, more rapid innovation, faster industry restructuring, government regulations and other uncertainties can increase the cost of setting up a VC fund and can also inflate general overhead expenses for market studies and read tape. Such cost is not specifically related to any individual start-up company but must be covered by revenues from the entire portfolio of firms. Since fixed costs are sunk at later stages of the VC cycle, they cannot directly affect managerial advice, profit sharing and portfolio size in the short run. In cutting profits, however, inflated fixed costs restrain entry of financiers. Fewer VC backed investments drive up the market price until VCs break even again. From (B.1) we infer that the industry sustains a smaller number of VCs, i.e. $\hat{N} = -\lambda\hat{R}$, see Table 2. By (9), VCs expand their portfolio of firms when projects become more valuable, $\hat{n} = \zeta_R\hat{R}$. Even though a higher price encourages advice, the larger portfolio dilutes advice to an extent that managerial support per firm declines.[19] Projects become more risky. Profit

[19] Substituting $\hat{n} = \zeta_R\hat{R}$ into (6), we get $\hat{a} = \frac{\hat{R}-(1-s)\varepsilon\hat{n}}{(1-s)\Psi} = -\frac{\hat{R}}{1-\theta}$ as in Table 2 (use B.2).

sharing remains invariant since increased risk and higher project value exactly offset each other in affecting incentives of entrepreneurs.

Proposition 8 ***(Entry Cost)*** *(a) With a fixed supply of VCs, higher entry cost reduces profits without any other effect on VC investing.*
(b) In the long run, entry costs lead to exit of VC firms. The subsequent price increase expands portfolio size but dilutes advice and raises risk. Profit shares remain constant.

5 Conclusions

This paper proposed a model of start-up investment and entrepreneurship that emphasizes the scarce supply of managerial capacity in venture capital finance. The supply of venture capitalists may expand on two margins, either by existing funds financing and advising more firms or by entry. In view of the specialized knowhow and experience required in venture capital investing, we contrasted a short-run equilibrium with a fixed number of venture capitalists and a long-run equilibrium with free entry and elastic supply. The upshot is that, in the short run, venture capital investing can expand only if the existing funds finance more start-ups each. When the total cost of managerial effort increases progressively, however, a larger number of portfolio companies tends to dilute the managerial support available to each individual firm, making start-ups more risky. When the number of venture capitalists remains fixed in the short run, the incumbent financiers may enjoy sizeable rents which will eventually attract new suppliers. In the competitive, long-run equilibrium with free entry and zero profits, the increased number of venture capitalists allows each one to consolidate her portfolio and focus advice on a smaller number of firms. The quality of venture capital investing is restored again. For this reason, a demand shift first results in inferior quality of venture capital support and a rather high rate of business failure. Only when the industry expands by entry of new financiers, the start-up firms will receive more intensive managerial advice, making them avoid overly high rates of business failure. Apart from a demand shift, we have investigated a number of other shocks that might importantly change the way the industry operates. In all cases, the key trade-off between portfolio size and intensity of advice per firm distinguishes the short- and long-run nature of venture capital investing. Our analysis suggests that economic policy should pay more attention on the supply of managerially experienced venture capitalists rather than merely subsidizing equity capital. Scarce supply of managerial support for innovative start-ups may be more of a bottleneck limiting the expansion of innovative industries than the supply of financial resources.

Appendix

A Proofs of Propositions 2 and 3

Constraints: We first show how the solution of a and s of (3.ii) and (4) depends on n. Since we must impose $p \leq 1$, the form of $p(a)$ in (2) implies an upper limit $\bar{a}$ for advice. By the same argument, (3.ii) implies a minimum profit share $\underline{s}$,

$$\bar{a} \equiv [(1-\theta)/\alpha]^{1/(1-\theta)}, \qquad \underline{s} = \beta/R. \tag{A.1}$$

We thus write $p(a) = (a/\bar{a})^{1-\theta}$. Figure 1 plots the IC^Ecurve in (3.ii) in s, a-space:

$$IC^E: \quad a = E(s) = \bar{a} \cdot (\underline{s}/s)^{1/(1-\theta)}, \qquad E' < 0 < E''. \tag{A.2}$$

This curve hits the upper limit at $\underline{s}$, i.e. $\bar{a} = E(\underline{s})$. Since the profit share cannot exceed one, it is bounded below by $\underline{a} = E(1) = \bar{a}\underline{s}^{1/(1-\theta)}$. The VC's incentive constraint (4) is

$$IC^F: \quad a = F(s) = \left[\frac{(1-s)\,\alpha R}{\gamma n^{\varepsilon}}\right]^{\frac{1}{\theta+\varepsilon}}, \quad F'(s) < 0, \quad F''(s) \gtrless 0. \tag{A.3}$$

This curve satisfies $F(1) = 0$. Since $F''(s) = \frac{-F'(s)(1-\theta-\varepsilon)}{(1-s)(\theta+\varepsilon)}$, it is concave for $1-\theta < \varepsilon$ and convex otherwise. For an interior solution with IC^E binding, we must impose

$$F(\underline{s}) \leq \bar{a} \qquad \Leftrightarrow \qquad (1-\theta)^{\theta+\varepsilon}(n^{\varepsilon}\gamma)^{1-\theta}/\alpha^{1+\varepsilon} \geq (R-\beta)^{1-\theta}. \tag{A.4}$$

For a solution to exist, the incentive constraints in Figure 1 must intersect. By equating $F(s) = E(s)$, we get $H(s) \equiv (1-s)^{1-\theta} s^{\theta+\varepsilon} = [(1-\theta)\beta]^{\theta+\varepsilon} (\gamma n^{\varepsilon})^{1-\theta} / (\alpha R)^{1+\varepsilon} \equiv X$. The H-schedule satisfies $H(0) = H(1) = 0$ and attains a maximum at $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon} < 1$ which follows from $H'(s) = 0$. Existence of a solution requires $H(\bar{s}) > X$,

$$H(\bar{s}) = \frac{(1-\theta)^{1-\theta}(\theta+\varepsilon)^{\theta+\varepsilon}}{(1+\varepsilon)^{1+\varepsilon}} > \frac{[(1-\theta)\beta]^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta}}{(\alpha R)^{1+\varepsilon}}. \tag{A.5}$$

Since $H(s) = X$ has two solutions, the incentive constraints in Figure 1 intersect twice. For the solutions to be admissible, (A.5) and (A.4) must be satisfied simultaneously. Multiplying (A.5) by $(\alpha R)^{1+\varepsilon}/\beta^{\theta+\varepsilon}$ and comparing with (A.4) gives

$$(1-\theta)^{1-\theta}\left(\frac{\theta+\varepsilon}{\beta}\right)^{\theta+\varepsilon}\left(\frac{\alpha R}{1+\varepsilon}\right)^{1+\varepsilon} > (1-\theta)^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta} \geq \alpha^{1+\varepsilon}(R-\beta)^{1-\theta}. \tag{A.6}$$

Choosing R large and β small opens a wide wedge, allowing placement of the middle term in this interval by 'calibrating' appropriate values for γ and n.

Of the two intersection points in Figure 1, A is the solution. All combinations to the north east of the E-schedule are admissible. For any s, the F-curve gives optimal advice according to (4). Points along $F(s)$ and above $E(s)$ all represent Pareto-improvements relative to point B. Applying the envelope theorem to (3), the VC maximizes profit by increasing her own profit share, i.e. by reducing s. She moves along the F-curve to the north west until the entrepreneur's incentive constraint binds at A. Equations (5) linearize the constraints at the solution A. The condition $\Psi > 0$ in (6) reflects the fact that IC^E is steeper than IC^F at A.[20]

Existence and Uniqueness: Write (8) as $\pi_n = z_1(s) - z_2(s)$ where n enters only via its effect on $s(n)$ which is the intersection of (A.2) and (A.3) with the lowest share s. Using $p(a) = \underline{s}/s$ from IC^E, the profit creation and destruction effects, z_1 and z_2, are

$$z_1(s) \equiv \theta\beta\frac{1-s}{s} - w - I, \qquad z_2(s) \equiv \frac{(1-\theta)\beta\varepsilon}{\Psi(s)}. \tag{A.7}$$

[20]Note that $E'(s) = \frac{-a}{(1-\theta)s} < 0$ and $F'(s) = \frac{-a}{(\theta+\varepsilon)(1-s)} < 0$, whence $E'(s) < F'(s) \Leftrightarrow \Psi(s) > 0$.

Evaluating these terms at the lowest admissible equity share (see Figure 1), we get

$$z_1(\underline{s}) \equiv \theta(R-\beta) - w - I, \qquad z_2(\underline{s}) \equiv \frac{(1-\theta)\varepsilon\beta}{\Psi(\underline{s})}. \tag{A.8}$$

Since $\Psi' < 0$, Ψ gets larger for small s, making $z_2(\underline{s})$ comparatively small. In raising R relative to β [see also the discussion of (A.6)], we make $z_1(\underline{s})$ arbitrarily large and $\underline{s}$ small. The effect on $\underline{s}$ also squeezes $z_2(\underline{s})$. With R appropriately set, we thus have $z_1(\underline{s}) > z_2(\underline{s}) > 0$ in Figure 2. Expanding portfolio size n raises the share s on account of the "dilution of advice" effect, see Figure 1. Since $z_1'(s) < 0$ and $z_2'(s) > 0$, the profit creation effect melts down while the profit destruction effect becomes ever larger. In particular, since $\Psi(\bar{s}) = 0$ for $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon} < 1$, $z_2(s) \to \infty$ for $s \to \bar{s}$.[21] With both schedules monotonic, a unique solution n^* exists in the interval $[\underline{s}, \bar{s}]$ corresponding to $[\underline{n}, \bar{n}]$.

Effects on Portfolio Size: In writing (8) as $\pi_n = z_1(s) - z_2(s)$, n enters the optimality condition only via its effect on s, see figure 1 and (7). The sufficient condition is, thus,

$$\frac{\partial \pi_n}{\partial s} = -\frac{1}{s}\Omega < 0, \qquad \Omega \equiv \frac{\beta(\theta+\delta)}{s}, \qquad \delta \equiv \varepsilon \cdot \left[\frac{s(1-\theta)}{(1-s)\Psi}\right]^2. \tag{A.9}$$

Take the differential of the condition $d\pi_n = 0$ and get $\Omega\hat{s} = -I\hat{I}$. Since s is related to n as in (7), we equate this with (7) and get (9) with the coefficients

$$\zeta_R = \frac{(1+\varepsilon)}{(1-\theta)\varepsilon}, \qquad \zeta_\gamma = \frac{1}{\varepsilon}, \qquad \zeta_I = \frac{I\Psi}{(1-\theta)\varepsilon\Omega}. \tag{A.10}$$

B Venture Capitalists in Fixed Supply

Market Price: The comparative statics with fixed N is determined by three simultaneous equations (11), (9) and (6) in three unknowns, $\hat{R}$, $\hat{n}$, and $\hat{a}$, relating to market clearing, portfolio size and managerial advice. We solve the system by using (9) and (6)

[21] Evaluating at $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon}$ gives $E'(\bar{s}) = \frac{-E(\bar{s})(1+\varepsilon)}{(1-\theta)(\theta+\varepsilon)}$ and $F'(\bar{s}) = \frac{-F(\bar{s})(1+\varepsilon)}{(1-\theta)(\theta+\varepsilon)}$. By (A.2) and (A.3), there is one $\bar{n}$ such that $E(\bar{s}) = F(\bar{s})$, implying a tangency solution $E'(\bar{s}) = F'(\bar{s})$ and $\Psi(\bar{s}) = 0$ in Figure 1. The value of $\bar{s}$ corresponds to the maximum of $H(s)$ as noted prior to (A.5).

to replace advice $\hat{a}$ and portfolio size $\hat{n}$ in (11). Using (A.10) and $(1-s)\,\Psi$ as given in (B.2) in various places, we find how the market price $\hat{R}$ eliminates excess demand:[22]

$$\lambda \hat{R} = \hat{\phi} - \hat{N} - \zeta_R \hat{\alpha} + \zeta_\gamma \hat{\gamma} + \frac{\theta - s + (1-s)\,\theta\varepsilon}{(1-s)\,\Psi} \zeta_I \hat{I}. \tag{B.1}$$

The slope of the excess demand function λ, plus some other parameters, are defined as:

$$\lambda \equiv \eta - 1 + \zeta_R > 0, \quad \Psi = \frac{(1-s)(1+\varepsilon) - (1-\theta)}{(1-s)}, \quad (1-s)\,npR = \frac{1+\varepsilon}{1-\theta}\gamma c. \tag{B.2}$$

The last equality uses (2) and (4), $Ac' = (1+\varepsilon)\,c$, $ap' = (1-\theta)\,p$ and $(1-s)\,Rp' = \gamma c'$.

Portfolio Size: Having found the equilibrating market price, we may now substitute back the solution to obtain the implications for portfolio size, managerial advice and profit sharing. Multiplying (9) by λ, substituting (B.1) and using (B.2) yields

$$\lambda \hat{n} = \zeta_R \left(\hat{\phi} - \hat{N}\right) + (\eta - 1)\,\zeta_R \hat{\alpha} - (\eta - 1)\,\zeta_\gamma \hat{\gamma} - \left[\eta + \frac{1-\theta}{(1-s)\,\Psi}\right] \zeta_I \hat{I}. \tag{B.3}$$

Advice and Profit Shares: Using (B.1), (B.3) and (A.10) in (6) gives

$$\lambda \hat{a} = -\frac{1}{1-\theta}\left(\hat{\phi} - \hat{N}\right) - \frac{\eta - 1}{1-\theta}\hat{\alpha} - \frac{1}{1-\theta}\zeta_\gamma \hat{\gamma} + \frac{1 + (1-s)\,\varepsilon\eta}{(1-s)\,\Psi}\zeta_I \hat{I}. \tag{B.4}$$

The profit share is fixed autonomously by the portfolio condition. From (A.9) we have

$$\hat{s} = 0 \cdot \left(\hat{\phi} - \hat{N}\right) + 0 \cdot \hat{\alpha} + 0 \cdot \hat{\gamma} - \frac{I}{\Omega}\hat{I} < 0. \tag{B.5}$$

Venture Capital Profits: By the envelope theorem, we may ignore in (3) how parameters influence profits via a or n since these variables are chosen endogenously:

$$\begin{array}{lll} \frac{d\pi}{ds_i} = -p_i R, & \frac{d\pi}{dR} = \sum_i (1-s_i)\,p_i, & \frac{d\pi}{dI} = -n, \\ \frac{d\pi}{d\gamma} = -c\,(A)\,, & \frac{d\pi}{d\alpha} = \sum_i (1-s_i)\,Rp_i/\alpha, & \frac{d\pi}{dc_f} = -1. \end{array} \tag{B.6}$$

Parameters do affect profits via the share s, however. By (B.5), profit sharing depends on start-up cost exclusively. Higher start-up costs directly cut into profits but also strengthen

[22] Using (A.7) and (B.2), the portfolio condition (8) is $w + I = (\theta - s)(\varepsilon + \theta)\,\beta/(s\Psi)$, implying $\theta - s > 0$.

them by raising the VC's equity share. Using (B.5), (3.ii) and (A.9) including $\delta > 0$, and noting $\theta - s > 0$ from footnote 22, the net effect is negative:

$$\frac{d\pi}{dI} = -n - \frac{npRs}{I} \frac{\hat{s}}{\hat{I}} = -\frac{\theta - s + \delta}{\theta + \delta} \cdot n. \tag{B.7}$$

Defining $\hat{\pi} \equiv d\pi$, we repeat this in elasticity form:

$$\hat{\pi} = (1 - s)\, npR \left(\hat{R} + \hat{\alpha} \right) - \gamma c \hat{\gamma} - \frac{\theta - s + \delta}{\theta + \delta} In\hat{I} - c_f \hat{c}_f. \tag{B.8}$$

Replacing $\hat{R}$ by the equilibrium price effect in (B.1) and using (A.10) and (B.2), we have

$$\lambda\hat{\pi} = (1 - s)\, npR \left(\hat{\phi} - \hat{N} \right) + (\eta - 1)(1 - s)\, npR\hat{\alpha} - (\eta - 1)\, \gamma c \hat{\gamma} - \lambda c_f \hat{c}_f. \tag{B.9}$$

For investment we have, upon using (3.ii) and (A.9), (A.10)

$$\lambda\hat{\pi} = [s - (\eta - 1)(\theta - s) - \lambda\delta] \frac{nI}{\theta + \delta} \hat{I} < 0. \tag{B.10}$$

Higher start-up cost raises the price which feeds back positively on profits. We assume that this feedback does not overturn the directly negative effect of higher cost, i.e. we assume the square bracket to be negative.

C Free Entry of Venture Capitalists

Market Price: Table 1 shows that entry of VCs expands supply and erodes the market price which, in turn, cuts into profits as noted in (B.8). Entry occurs and the price continues to fall until profits are gone. If we start from a zero profit equilibrium, we compute from (B.8) the long-run equilibrium price that sets $\hat{\pi} = 0$. Knowing this price, we invert the temporary equilibrium condition in the first column of table 1 to solve for the implied entry of VCs. Use (B.2) and set $\hat{\pi} = 0$ in (B.8) to get

$$\hat{R} = -\hat{\alpha} + \frac{1 - \theta}{1 + \varepsilon} \hat{\gamma} + \frac{\theta - s + \delta}{\theta + \delta} \frac{I}{(1 - s)\, pR} \hat{I} + \frac{c_f}{(1 - s)\, npR} \hat{c}_f. \tag{C.1}$$

Number of VCs: Entry of VCs squeezes the temporary equilibrium price as in (B.1). With free entry, the long-run price must fall until profits vanish and prevent further entry, see (C.1). Substituting (C.1) into (B.1) and solving for $\hat{N}$ shows how many VCs the industry supports in the long run. Applying the definition of λ as well as (A.10) gives

$$\hat{N} = \hat{\phi} + (\eta - 1)\,\hat{\alpha} - \frac{(\eta - 1)(1 - \theta)}{1 + \varepsilon}\hat{\gamma} - \frac{\lambda c_f}{(1 - s)\,pRn}\hat{c}_f. \tag{C.2}$$

Using (3.ii), (A.9), (A.10) and (B.2), higher start-up investment cost is seen to reduce entry,

$$\hat{N} = [s - (\theta - s)(\eta - 1) - \lambda\delta]\,\frac{nI}{(\theta + \delta)\,\zeta_R \varepsilon \gamma c}\hat{I} < 0, \tag{C.3}$$

where the effect is negative by the argument noted in (B.10).

Portfolio Size: Substitute (C.1) into (9) and note (B.2) as well as (A.9), (A.10) and (3.ii). The effect of start-up investment cost is ambiguous:

$$\hat{n} = 0 \cdot \hat{\alpha} + 0 \cdot \hat{\gamma} + \frac{c_f \zeta_R}{(1 - s)\,npR}\hat{c}_f + \frac{\delta\zeta_R - 1}{(\theta + \delta)(1 - s)\,pR}I\hat{I}. \tag{C.4}$$

Advice and Profit Share: Substitute (9) into (6) and use $1 - (1 - s)\,\varepsilon\zeta_R = \frac{-(1-s)\Psi}{1-\theta}$,

$$(1 - s)\,\Psi\hat{a} = -\frac{(1 - s)\,\Psi}{1 - \theta}\left(\hat{\alpha} + \hat{R}\right) + (1 - s)\,\varepsilon\zeta_I\hat{I}, \tag{C.5}$$

where the effect of $\hat{\gamma}$ cancels. Insert the price change from (C.1), use (A.10) and (B.2), and rearrange:

$$\hat{a} = 0 \cdot \hat{\alpha} - \frac{1}{1 + \varepsilon}\hat{\gamma} - \frac{c_f}{(1 + \varepsilon)\,\gamma c}\hat{c}_f + \frac{(1 - \theta - \delta)\,nI}{(\theta + \delta)(1 + \varepsilon)\,\gamma c}\hat{I}. \tag{C.6}$$

The effect of $\hat{I}$ is again indeterminate. The effect on the profit share is given by (B.5) and is zero except for start-up cost:

$$\hat{s} = 0 \cdot \hat{\alpha} + 0 \cdot \hat{\gamma} + 0 \cdot \hat{c}_f - \frac{I}{\Omega}\hat{I}. \tag{C.7}$$

References

[1] Casamatta, Catherine (1999), *Financing and Advising: Optimal Financial Contracts With Venture Capitalists*, University of Toulouse, mimeo.

[2] Chan, Yuk-Shee, Daniel Siegel and Anjan Thakor (1990), Learning, Corporate Control and Performance Requirements in Venture Capital Contracts, *International Economic Review*, 31, 365-381.

[3] Gompers, Paul A. (1995), Optimal Investment, Monitoring, and the Staging of Venture Capital, *Journal of Finance* 5, 1461-1489.

[4] Gompers, Paul and Josh Lerner (1999), *The Venture Capital Cycle*, Cambridge, Mass.: MIT Press.

[5] Gorman, Michael and William A. Sahlman (1989), What Do Venture Capitalists Do?, *Journal of Business Venturing*, 4, 231-248.

[6] Hellmann, Thomas (1998), The Allocation of Control Rights in Venture Capital Contracts, *Rand Journal of Economics*, 29, 57-76.

[7] Kanniainen, Vesa and Christian Keuschnigg (2000), *The Optimal Portfolio of Start-up Firms in Venture Capital Finance*, CESifo working paper.

[8] Kaplan, Steven N. and Per Strömberg (2000), *Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts*, NBER working paper series 7660.

[9] Keuschnigg, Christian and Soren Bo Nielsen (2000), *Tax Policy, Venture Capital, and Entrepreneurship*, NBER WP No. 7976.

[10] Norton, Edgar and Bernard H.Tenenbaum (1993), Specialization versus Diversification as a Venture Capital Investment Strategy, *Journal of Business Venturing*, 8, 431-442.

[11] Reid, G.C., N.G. Terry and J.A. Smith (1997), Risk Management in Venture Capital Investor-Investee Relations, *European Journal of Finance*, 3, 27-47.

[12] Repullo, Rafael and Javier Suarez (1999), *Venture Capital Finance: A Security Design Approach*, CEPR DP No. 2097.

[13] Sahlman William A. (1990), The Structure and Governance of Venture-Capital Organizations, *Journal of Financial Economics*, 27, 473-521.

[14] Schmidt, Klaus M. (1999), *Convertible Securities and Venture Capital Finance*, CESifo WP No. 217.

[15] Strobel, Peter (2000), *Venture Capital Versus Bank-Finance*, University of Saarland, mimeo.

Figures

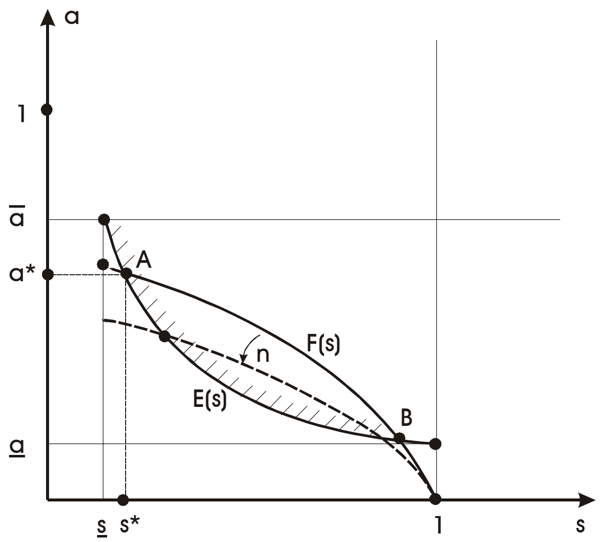

Figure 1: Optimal Advice and Profit Share

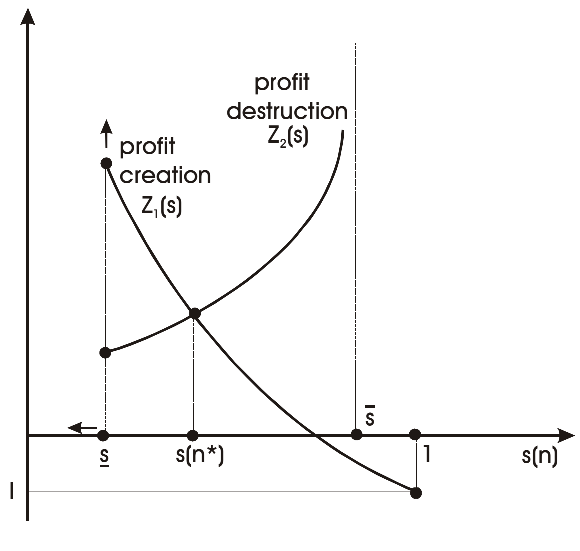

Figure 2: Optimal Number of Firms

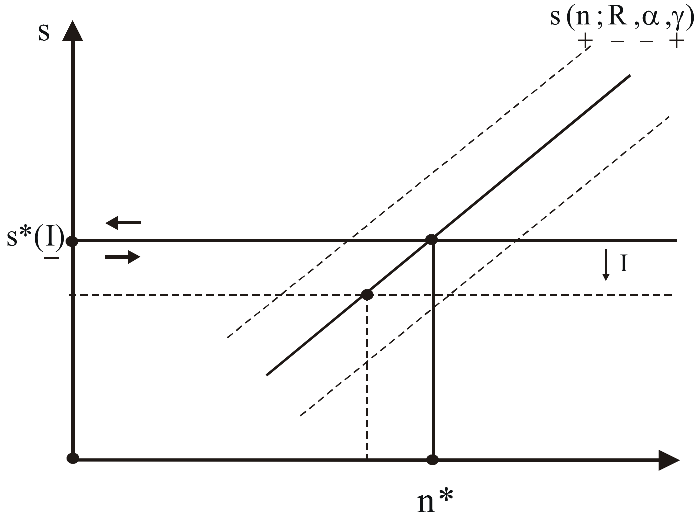

Figure 3:Number of Portfolio Companies