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LABOR TAXATION IN SEARCH EQUILIBRIUM WITH HOME PRODUCTION

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Abstract

Conventional models of equilibrium unemployment typically imply that proportional taxes on labor earnings are neutral with respect to unemployment as long as the tax does not affect the replacement rate provided by unemployment insurance, i.e., unemployment benefits relative to after-tax earnings. When home production is an option, the conventional results may no longer hold. This paper uses a search equilibrium model with home production to examine the employment and welfare implications of labor taxes. The employment effect of a rise in a proportional tax is found to be negative for sufficiently low replacement rates, whereas it is ambiguous for moderate and high replacement rates. Numerical calibrations of the model indicate that employment generally falls when proportional labor taxes are raised. Progressive labor taxes increase labor market tightness but have ambiguous effects on search effort and employment.

JEL Classification: H24, J22, J64.

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1 Introduction

A popular theme in current policy discussions about labor market reform is that high taxes on labor contribute to high unemployment. Although the claim appears intuitively plausible, it has not received overwhelming support from theoretical and empirical research on unemployment and wage determination. In fact, proportional taxes on labor earnings are *neutral* with respect to unemployment in many conventional models and the empirical research has shown mixed results.[1]

Most of the theoretical models identify the "benefit regime" as the crucial factor that determines how labor taxes affect labor costs and ultimately unemployment. Taxes are neutral as long as they do not affect the after-tax replacement rate, i.e., the relationship between income when unemployed and income when employed. There is in general complete real wage flexibility with respect to changes in labor taxes if unemployment compensation is indexed to the real after-tax consumption wage through a fixed replacement rate. Labor taxes are then borne by labor and there is no effect on labor costs and unemployment.[2]

The potential for employment gains through lower labor taxes hinges on the impact on the replacement rate; there will be an increase in employment only if the tax cut reduces the replacement rate. A benefit regime involving unemployment compensation fixed in *real* terms has this feature. A tax cut induces an increase in the real wage, which implies a decline in the relative compensation of unemployed workers. The tax cut works because it effectively reduces the replacement rate.[3]

The existing literature on taxes and unemployment has paid little attention to income sources other than labor earnings and unemployment benefits.[4] A shortcut is to allow for *exogenous* income or utility components,

[1] The large empirical literature involves numerous studies of the relationships between labor costs and labor taxes. Tyrväinen (1995), Gruber (1997), Jackman et al (1996) and Nymoen and Rodseth (1999) are examples with somewhat conflicting results. Other studies have investigated whether taxes help explain the evolution of unemployment over time and differences across countries; see for example Layard et al (1991), Elmeskov et al (1998), Nickell (1998), Madsen (1998), Phelps (1994) and Scarpetta (1996).

[2] This result holds in models with unions, as in Johnson and Layard (1986) or Layard et al (1991), as well as in models with bargaining between the firm and the individual worker, as in Pissarides (1990). The result also holds in various efficiency wage models.

[3] Pissarides (1998) presents a number of simulation results that illustrate the quantitative importance of the benefit regime for the effects of changes in labor taxes.

[4] A remarkable exception is Edmund Phelps, who in a series of contributions has emphasized the role of wealth and nonwage income in the theory of unemployment. See, for example, Phelps (1994), Phelps and Zoega (1998), and Hoon and Pelps (1996, 1997).

such as income from the "informal sector", income from home production or a fixed value of leisure; see, for example, Bovenberg and van der Ploeg (1998) and Mortensen (1994). Tax cuts will bring about a fall in unemployment provided that (*i*) the additional income sources are unresponsive to changes in the real wage, and (*ii*) more prevalent among unemployed than among employed workers. The reason for this result is, again, that the tax cut reduces the effective replacement rate by inducing a proportionally bigger increase in labor income than in total unemployment compensation.

The purpose of this paper is to examine the effects of labor taxes on labor market outcomes in a model of equilibrium unemployment where the worker's income from home production is *endogenously* determined. To this end a search equilibrium framework along the lines of Pissarides (1990) is extended to allow for home production.[5] Time devoted to home production is taken to be a choice variable for unemployed individuals who allocate their time between job search and home production. The effective replacement rate – inclusive of income from home production – is endogenous in this environment, irrespective of whether unemployment benefits are indexed to labor earnings or fixed in real terms.

For simplicity, we focus on a one-sector economy where the good produced in the household is a perfect substitute to the market-produced good. The model is thus not designed to shed light on the effects of sectoral *tax differentiation*, where the differentiation may depend on the degree of substitutability between market and nonmarket goods. The existing literature on taxation and household production has been primarily concerned with the case for tax differentiation. The contributions in this field include papers by Sandmo (1990), Fredriksen et al (1995), Sorensen (1997), Kolm (2000) and Kleven et al (2000). Sandmo and Kleven et al consider economies with competitive labor markets, whereas the other three papers allow for unemployment due to real wage rigidities.

Kolm's model is richer than ours in some dimensions and more restrictive in others. Her model features two market sectors, with one of them producing goods that are perfect substitutes to the goods produced at home. Job search is ignored, however, which implies that the opportunity cost of home production is zero for the unemployed worker. A corner solution is then obtained where the unemployed worker allocates all available time to home production. Another difference is that Kolm's analysis is partial equilibrium

These models imply neutrality of the labor tax in the *long run*, i.e., once wealth has adjusted.

[5] The seminal paper on the microeconomics of home production is Gronau (1977).

in the sense that there is no link between bargained wages and general labor market conditions.

Models of home production are in some sense observationally equivalent to models with endogenous leisure. Indeed, "for any model with home production, there is a model without home production, but with different preferences, that generate the same outcome for market quantities" (Benhabib et al, (1991), p 1170). The motivation for introducing home production in the present analysis is the desire to build a simple general equilibrium model that encompasses two empirically relevant predictions: (i) hours of work are decreasing in the labor tax rate, and (ii) equilibrium unemployment is independent of the level of productivity. The second of those predictions can also be generated by a model with endogenous leisure, provided that the utility function is of the Cobb Douglas variety; see for example Fredriksson and Holmlund (2001). The Cobb Douglas representation of preferences is restrictive for the purposes of this paper, however, as it implies that hours worked do not respond to after-tax wages.

The next section of the paper presents the basic model, where it is assumed that work-hours are determined through bargaining between the firm and the worker. Section 3 turns to the effects of changes in labor taxes in the basic model. The main analytical result is that a rise in a proportional tax reduces equilibrium employment as long as the replacement rate is zero or close to zero. We also consider progressive taxes and derive conditions under which an increase in progressivity raises labor market tightness. Section 4 anlyzes the effects of tax policies under the assumtion that work hours are determined by the individual employed worker. The results are broadly similar to those obtained when work-hours are subject to bargaining.

2 The Model

2.1 The Labor Market

The number of individuals in the economy is fixed and normalized to unity. The individuals are either employed or unemployed, the time horizon is infinite and time is continuous. Employed workers are separated from their jobs at the exogenous rate ϕ. Unemployed workers find new jobs at a rate that depends on their search effort, s, as well as general labor market conditions. If u denotes the number of unemployed workers we can take su to represent the effective number of job searchers in the economy.

The matching process is given by a standard concave and constant-returns-to-scale function that relates the flow of hires, H, to the number of vacancies, v, and the effective number of job searchers, su, i.e., $H(v, su)$. The rate at which the unemployed worker finds a new job is given by $sH(v, su)/su = s\alpha(\theta)$, where $\theta = v/su$ is a measure of labor market tightness and $\alpha(\theta) = H(v, su)/su = H(\theta, 1)$. The rate at which firms fill vacancies is given as $q(\theta) = H(v, su)/v = H(1, 1/\theta)$. Hence, $\alpha(\theta) = \theta q(\theta)$, where $\alpha'(\theta) > 0$ and $q'(\theta) < 0$; the tighter the labor market, the easier for workers to find jobs and the more difficult for firms to find workers. Moreover, note that the elasticity of the expected duration of a vacancy with respect to tightness falls in the unit interval, an implication of constant returns to matching; we have $\eta \equiv -\theta q'(\theta)/q(\theta)$, where $\eta \in (0, 1)$.

The flow equilibrium for the economy can be written as an unemployment equation of the form:

$$u = \frac{\phi}{\phi + s\alpha(\theta)} \tag{1}$$

2.2 Worker Behavior

Individuals are risk neutral, face an exogenous interest rate, r, and derive utility from consumption of the single good in the economy. The good is either purchased from the market or produced at home. The employed worker's time, normalized to unity, is allocated to market work, l, and home production, h^e, i.e., $1 = l + h^e$. The unemployed worker allocates time to search, s, and home production, h^u, i.e., $1 = s + h^u$. The home production function, $z^j = z(h^j)$, $j = e, u$, is increasing and strictly concave.

The employed worker's instantaneous income is given as $I^e = wl + z(h^e) + R$, where w is the real hourly wage and R is a lump sum transfer from the government. The unemployed individual's income derives from home production, the transfer and unemployment benefits (Z^b), i.e., $I^u = z(h^u) + R + Z^b$.

Let U and E denote the expected present values of being unemployed and employed, respectively. The value functions for worker i can be written as follows:

$$rE_i = w_i l_i + z(h_i^e) + R + \phi(U - E_i) \tag{2}$$
$$rU_i = z\left(h_i^u\right) + R + Z^b + s_i\alpha(\theta)\left(E - U_i\right) \tag{3}$$

In a symmetric equilibrium, the utility difference between the expected present values is independent of the transfer and given as:

$$E - U = \frac{I^e - I^u}{r + \phi + s\alpha(\theta)} \tag{4}$$

The employed worker's time allocation is determined through bargaining between the worker and the firm. The unemployed worker chooses search intensity, s_i, to maximize rU_i. The first-order condition for an interior solution takes the form:

$$z'(h_i^u) = \alpha(\theta)(E - U_i) \tag{5}$$

where the left-hand side is the marginal cost of increasing search, which is foregone home production. The right-hand side is the expected marginal return from an increase in search effort.

By making use of (4) and (5) we obtain the partial equilibrium results that an increase in the market wage as well as an increase in labor market tightness reduces the unemployed worker's time in home production (and thus increases search): $\partial h_i^u / \partial w < 0$ and $\partial h_i^u / \partial \theta < 0$. These results are implied by the concavity of the production function and the fact that $E - U$ is increasing in the wage as well as in tightness. Note that the right-hand side of (5) is independent of search effort (time spent in home production), by the envelope theorem. A rise in the wage increases the utility surplus from employment, which encourages search (discourages home production). A rise in tightness increases the marginal return from search, which has similar effects. Also notice that $E - U$ is decreasing in the benefit level, which in turn discourages search effort; hence $\partial h_i^u / \partial Z^b > 0$.

2.3 Firm Behavior

The model of the firm follows Pissarides (1990) with explicit allowance made for hours of work. Let V be the value of an unfilled job and J denote the value of a filled job. The value functions are:

$$rV = -ky + q(\theta)(J - V) \tag{6}$$
$$rJ = yl - w(1+t)l + \phi(V - J) \tag{7}$$

Labor productivity – output per hour – is constant and denoted y. The cost of holding a vacancy is ky, with $k > 0$.[6] t is a proportional payroll tax

[6] The appendix gives a rationalization for this specification of vacancy costs, using a model of a large firm that allocates its workforce between production and recruitment activities.

rate. Invoking the standard free entry condition for vacancies, $V = 0$, we can derive:

$$J = \frac{ky}{q(\theta)} = \frac{yl - w(1+t)l}{r + \phi} \tag{8}$$

which implies a relationship between the "feasible" real wage and labor market tightness, conditional on hours of work and the tax rate:

$$w = y\left(1 - \frac{(r+\phi)k}{q(\theta)l}\right)\frac{1}{1+t} \tag{9}$$

A rise in working time increases the feasible real wage. One can think of this relationship as capturing a productivity effect of longer work-hours. Suppose that the firm has a production department and a personnel department. A rise in work-hours allows the firm to transfer some workers from recruitment activities to production while keeping its total workforce constant. The higher output per employed worker implies a higher feasible real wage.

2.4 Bargaining over Hours and Wages

Wages and work hours are determined in decentralized Nash-bargains between the individual worker and the firm. The Nash bargain thus solves:

$$\max_{w_i, l_i} \Omega(w_i, l_i) = [E_i(w_i, l_i) - U]^{\beta} [J_i(w_i, l_i) - V]^{1-\beta}$$

The first-order conditions with respect to the wage and to work hours are obtained as:

$$w \quad : \quad E - U = \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+t}\right)\frac{ky}{q(\theta)} \tag{10}$$

$$l \quad : \quad E - U = \left(\frac{\beta}{1-\beta}\right)\left(\frac{w - z'(h^e)}{w(1+t) - y}\right)\frac{ky}{q(\theta)} \tag{11}$$

where we have imposed symmetry and the free entry condition $V = 0$, which implies $J = ky/q(\theta)$. These equations imply:

$$z'(h^e) = \frac{y}{1+t} \tag{12}$$

which states that the employed worker's marginal product in home production equals the tax-adjusted marginal product in market work. The allocation of time for the employed worker is independent of labor market tightness; we have $l = l(t)$ and $h^e = 1 - l(t)$ with $l'(t) < 0$. The feasible real wage can then be written as a function of θ and t:

$$w = y\left(1 - \frac{(r+\phi)k}{q(\theta)l(t)}\right)\frac{1}{1+t} \tag{13}$$

which we write as $w = \upsilon(\theta, t)$, where $\upsilon_\theta < 0$ and $\upsilon_t < 0$. It is convenient to combine the expression for the worker's utility surplus, $E - U$, as given by (4), with the feasible real wage in (13) and recognizing $h^e = h^e(t)$. We refer to this expression as the "feasible utility surplus", denoted by $F(\cdot)$:

$$E - U = F(\theta, t) \equiv \frac{I^e(\cdot) - I^u(\cdot)}{r + \phi + s\alpha(\theta)} \tag{14}$$

where $I^e = \upsilon(\theta, t)l(t) + z^e(h^e(t)) + R$, $I^u = z^u(h^u) + Z^b + R$ and $F_\theta < 0$. Note that the right-hand side of (14) is independent of s and h^u, by the envelope theorem. Analogously, we refer to the first-order condition for the bargained wage in (10) as the "bargained surplus":

$$E - U = B(\theta, t) \equiv \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+t}\right)\frac{ky}{q(\theta)} \tag{15}$$

which is increasing in θ. The bargain delivers a worker surplus that is proportional to the value of a filled job, which in equilibrium must equal the expected vacancy cost. A rise in labor market tightness implies that the expected duration of a vacancy, $1/q(\theta)$, increases, which in turn means that the value of filled job rises. The bargain gives the worker a share of the rise in total match surplus.

2.5 Equilibrium

We can characterize the equilibrium of the model by making use of two relationships, namely the feasible surplus, $F(\theta, t)$, and the bargained surplus, $B(\theta, t)$, as illustrated in Figure 1. Since $F_\theta < 0$ and $B_\theta > 0$, the equilibrium is unique and labor market tightness is given as the solution to the equation $\Psi \equiv F(\theta, t) - B(\theta, t) = 0$, i.e.,

$$\Psi \equiv \frac{I^e(\theta, t) - I^u(h^u, Z^b)}{r + \phi + s\alpha(\theta)} - \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+t}\right)\frac{ky}{q(\theta)} = 0 \tag{16}$$

Many of the comparative statics properties of the model are conventional, at least as far as the effects on tightness are concerned. The effect

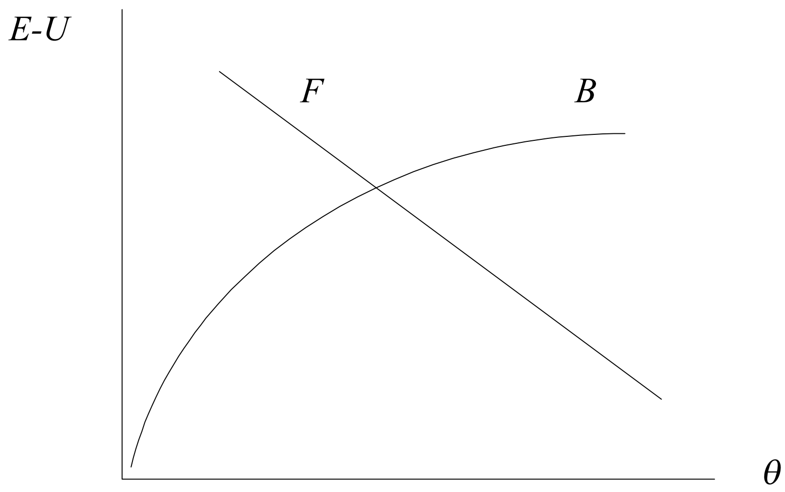

Figure 1: Labor Market Equilibrium

on tightness follows by implicit differentiation of (16), noting that $\Psi_\theta < 0$; the sign of the effect of, say, a rise in the interest rate is thus given by the sign of Ψ_r. It is clear that labor market tightness falls as a response to: (i) a rise in the worker's bargaining power, β; (ii) a rise in the cost of holding a vacancy, k; (iii) a rise in the discount rate, r; (iv) a rise in the separation rate, ϕ; and (v) a rise in the benefit level, Z^b.

The response to changes in labor productivity is somewhat less obvious. We wish to have a model with the realistic property that labor market tightess and unemployment are independent of the level of productivity. Two additional assumptions are introduced to achieve this:

(i) Unemployment benefits are indexed to labor earnings through a fixed replacement rate, i.e., $Z^b = \rho w l$, $\rho \in [0, 1)$.

(ii) The home production functions are given as $z^j = ayf(h^j)$, where a is a positive constant and $j = e, u$. In words: productivity in home production rises along with productivity in market production.

With these assumptions we can state the following result:[7]

Lemma 1: *A uniform increase in labor productivity, i.e., an increase in y, is neutral with respect to labor market tightness, hours of work in the market*

[7] $F_\theta < 0$ still holds since $\rho \in [0, 1)$. Hence $\Psi_\theta < 0$.

and in the household, search effort, and unemployment: $d\theta/dy = dh^e/dy = dl/dy = dh^u/dy = ds/dy = du/dy = 0$.

To prove Lemma 1, note that assumption (ii) together with eq. (12) implies that time allocation for the employed worker is independent of productivity. The feasible real wage implies a relationship of the form $w/y = x(\theta, l(t)) \cdot (1+t)^{-1}$. Substituting into (16) while recognizing that $z^j = ayf(h^j)$ and $Z^b = \rho w(\cdot)\, l(\cdot)$ yield an expression from which y can be eliminated. θ is thus independent of y. Also, using (1), it is clear that unemployment is independent of the level of productivity.

3 The Effects of Taxes

We proceed to investigate the effects of labor taxes, assuming that tax revenues are spent as uniform lump sum grants to each individual in the economy. The utility difference between employed and unemployed workers is thus not affected by the amount of tax revenues raised. The revenue side of the tax system is hence neutral with respect to the real outcomes in the economy and we can examine the effects of varying the tax rates without having to consider how the tax revenues are used. The government's budget restriction, given as $t(1-u)wl = R + u\rho wl$, is always fulfilled through adjustment of the lump sum grant.

3.1 Proportional Taxes

It is clear from (16), recognizing $Z^b = \rho wl$, that the tax rate affects tightness through several routes. First, the tax rate affects the employed worker's utility, given tightness, as given by:

$$\frac{\partial I^e(\cdot)}{\partial t} = l\frac{\partial w}{\partial t} + \left(w - z'(h^e)\right)\frac{\partial l}{\partial t} \tag{17}$$

where the first term is negative and the second positive (since $z'(h^e) = y/(1+t) > w$). The expression simplifies to $\partial I^e(\cdot)/\partial t = -wl/(1+t) < 0$ if (12) and (13) are invoked. The $F(\cdot)$-schedule is shifted to the left and tightness tends to fall. A second effect operates through the benefit level when benefits are indexed to earnings. The higher the tax rate, the lower the benefit level and the higher the feasible utility surplus to the worker. The third effect enters through the bargained surplus. A higher marginal tax reduces the gain to the firm of raising the wage, thus inducing wage moderation and a fall in $E-U$, given tightness. The $B(\cdot)$-schedule shifts to the right and tightness thus tends to rise.

A Special Case: $\boldsymbol{\rho = 0}$

To determine the net effect on tightness it is useful to begin with a special case where the replacement rate is zero, in which case the induced effect on tightness does not appear. Implicit differentiation of (16) yields:

$$sign\ \frac{d\theta}{dt} = sign\ \{z^e - z^u\} \tag{18}$$

To sign $d\theta/dt$ we thus need to determine the sign of the term $z^e - z^u$. In other words, how does time in home production differ between employed and unemployed workers? We can state the following results:

Lemma 2: *The allocation of time in equilibrium involves $h^e < h^u$ and hence $l > s$ and $z^e < z^u$, provided that the production functions are of the form $z^j = ayf(h^j)$, $j = e, u$.*

Note that the equality $h^e = h^u$ would require equality between the employed worker's marginal product in home production and the marginal return to search, i.e., $y/(1+t) = \alpha\,(E-U)$. However, it can be shown that the inequality $y/(1+t) > \alpha\,(E-U)$ holds, implying that unemployed workers spend more time in home production than those who are employed. The proof is given in the Appendix.

Lemma 2 in conjunction with (18) thus imply $d\theta/dt < 0$; a tax increase reduces labor market tightness. To obtain the effect on unemployment we need to consider how search effort is affected. Use the first-order condition for optimal search in (5) together with the Nash rule in (10) to obtain:

$$z'(h^u) = \theta(t)\left(\frac{\beta}{1-\beta}\right)\left(\frac{ky}{1+t}\right) \tag{19}$$

The right-hand side of (19) is the equilibrium marginal return to search, or the shadow wage of home production for an unemployed worker. The shadow wage is increasing in labor market tightness. A tax hike thus increases home production – reduces search – both directly and indirectly through $\theta(t)$. The effect on unemployment is obtained by differentiation of eq. (1), recognizing $\alpha = \alpha(\theta(t))$ and $s = s(\theta(t); t)$. The result is $du/dt > 0$. It is also straigtforward to show that a tax increase produces an unambiguous decline in the real wage. Use $w = v(\theta(t), t)$ and differentiate to obtain $dw/dt < 0$ (see Appendix).

The results for the special case with $\rho = 0$ is summarized in the following proposition:

Proposition 1: *A tax increase has the following effects on labor market tightness, real wages, home production, work-hours, search and unemployment*: $d\theta/dt < 0$, $dw/dt < 0$, $dh^e/dt > 0$, $dh^u/dt > 0$, $dl/dt < 0$, $ds/dt < 0$, *and* $du/dt > 0$.

The General Case: $\boldsymbol{\rho \geq 0}$

Tax changes will in general induce changes in the benefit level, which in turn influence the overall effects of taxes. The derivative of interest is:

$$sign\ \frac{d\theta}{dt} = sign\ \left(z^e - z^u + \rho \xi_t^l \left(\frac{yl(t)}{1+t} \right) \right) \tag{20}$$

where $\xi_t^l \equiv -\partial \ln l / \partial \ln(1+t) > 0$ is the elasticity of hours of work with respect to the payroll tax rate, as implied by (12). The inequality $z^e < z^u$ is no longer suffcient to guarantee a negative sign since the third term in (20) is positive. The more sensitive hours are with respect to the tax rate and the higher the replacement rate, the more likely the possibility that this "benefit effect" dominates. In general, the effect on tightness is ambiguous.

Calibration

We proceed to a numerical calibration of the model. The matching function is taken to be Cobb Douglas, i.e., $H = m(su)^\eta (v)^{1-\eta}$; it is straightforward to show that this implies $-\theta q'(\theta)/q(\theta) = \eta$. We also assume that the worker's share of the total match surplus equals the elasticity of matching with respect to unemployment, i.e., $\beta = \eta$; this is the so called Hosios-condition that implies that the search equilibrium outcome is efficient under certain conditions (Hosios, 1990). We set $\beta = \eta = 0.5$.

The home production functions are of the form:

$$z^j = ay \left(h^j \right)^b \tag{21}$$

for $j = e, u$ and $b < 1$. The unemployed worker's time in home production is obtained by invoking eq. (19). The day is taken as time unit, y is normalized to 100 and the separation and interest rates are set to $\phi = 0.25/365$ and $r = 0.10/365$. The parameters k, a, b and m were chosen so as to obtain "reasonable" values of ξ_t^l and 5 percent unemployment for a base run with $t = 0.25$. We set $\rho = .30$, which is an average replacement rate for OECD-countries (see Elmeskov et al, 1999). The implied elasticity of hours with respect to tax rates appear reasonable in light of the empirical studies on labor supply. We have $\xi_t^l \in [.24, .43]$ as we proceed from $t = .25$ to $t = .55$. It should be recognized, however, that conventional labor supply studies presumes that working time is at the worker's discretion.[8] The implied partial equilbrium (θ-constant) elasticity of the exit rate from unemployment with respect to benefits, evaluated at the equilibrium with $t = .25$, is in a region where the empirical estimates typically fall: we have $\partial \ln(s\alpha)/\partial \ln \rho \approx -.6$.[9]

We also report the effects on the tax revenues, $T = t(1-u)wl$, the effective replacement rate, I^u/I^e, and the steady state output, inclusive of home production but net of vacancy costs:

$$Q = (1-u) \cdot (yl + z^e) + u \cdot z^u - su\theta \cdot ky \tag{22}$$

A measure of the marginal social cost of raising taxes – or marginal excess burden – is given by $\Delta Q/\Delta T$, the change in total output per dollar of additional tax revenues.

Table 1 shows the results of the calibrations. Tax increases produce modest increases in unemployment at low initial tax rates and substantial effects at high rates. A comparison with the estimates reported in the recent study by Elmeskov et al (1998) is useful. This study, based on pooled data for 19 OECD countries for the period 1983-95, suggests that a rise in the overall tax rate by 10 percentage points would raise the unemployment rate by slightly more than one percentage point. These results are broadly in line with the simulation results for intermediate tax rates in Table 1.

The effective replacement rate increases from 55 to 79 percent as tax rates are increased from 25 to 55 percent. There are no Laffer-effects, i.e., higher tax rates do produce higher tax revenues. The marginal excess burden is modest for low tax rates but substantial for high rates. We have $\Delta Q/\Delta T = -.16$ for tax increases from 25 to 35 percent and $\Delta Q/\Delta T = -1.16$ for increases from 45 to 55 percent

[8] The recent survey by Blundell and MaCurdy (1999) reports estimates of the wage elasticity of labor supply centered around 0.10 for males and around 0.7 for females.

[9] Layard et al (1991) summarize the empirical work by the claim that "the basic result is that the elasticiy of the expected duration of unemployment with respect to benefits is generally in the range $0.2 - 0.9$ depending on the state of the labor market and the country concerned..." (p 255).

Table 1. The Effects of Tax Increases (Proportional Taxes).
Parameters: $\beta = \eta = .5$, $y = 100$, $k = .667$, $a = .5$, $b = .6$,
$m = .01939$, $r = .10/365$, $\phi = .25/365$, $\rho = .3$

	$t = \mathbf{.25}$	$t = \mathbf{.35}$	$t = \mathbf{.45}$	$t = \mathbf{.55}$
θ	.913	.895	.870	.827
u (%)	5.0	5.7	6.9	10.1
w (*index*)	77.2 (100)	71.5 (92.6)	66.5 (86.1)	62.2 (80.6)
h^e (%)	8.6	10.4	12.5	14.7
h^u (%)	29.8	37.9	48.7	65.3
Q (*index*)	96.8 (100)	96.1 (99.3)	95.1 (98.2)	92.9 (96.0)
T	16.8	21.1	24.4	26.3
I^u/I^e (%)	55.2	61.3	68.8	79.3
$\Delta Q/\Delta T$		$-.16$	$-.30$	-1.16

3.2 Progressive Taxes

It is well known that progressive taxes are conducive to wage moderation in a variety of non-competitive models of wage determination.[10] Wage moderation is also associated with higher employment in the standard bargaining (or efficiency wage) models. We examine whether these results carry over to a search equilibrium model with home production and endogenous search effort. There is no general presumption that the results will carry over, one reason being that the effect on wage setting is not sufficient to determine the effect on employment.

The tax function facing the single firm is taken to be linear and of the form:

$$\Gamma_i = (\mu_0 + \tau w_i)\, l_i \tag{23}$$

where the marginal payroll tax rate is denoted τ and μ_0 captures the non-proportionality of the tax system; $\mu_0 < 0$ – a tax allowance – implies a progressive tax schedule. The tax allowance is taken as given by each firm and worker. We will, however, assume that it is indexed ex post to the general wage level, i.e., $\mu_0 = \mu w$. The hourly wage cost facing a representative firm in equilibrium is thus given as $w_c = (1 + \mu + \tau)w$, where $\mu + \tau \equiv \bar{\tau}$ is the average tax rate. We can then conveniently investigate the effects of an

[10] See for example Lockwood and Manning (1993), Holmlund and Kolm (1995), Koskela and Vilmunen (1996), Sorensen (1999) and Fuest and Huber (2000).

increase in tax progressivity that takes the form of an increase in the marginal tax rate while holding the average tax rate constant. This experiment is, of course, tantamount to simultaneous changes of τ and μ – a rise in τ accompanied by a cut in μ such that $\bar{\tau}$ remains constant.

With bargaining over hours, we have hours determined by $z'(h^e) = y/(1+\tau)$. The relationship determining equilibrium tightness takes the form:

$$\Psi^* \equiv \frac{I^e(w,\tau) - I^u(h^u, \rho wl)}{r+\phi+s\alpha(\theta)} - \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+\tau}\right)\frac{ky}{q(\theta)} = 0 \qquad (24)$$

where $I^e(w,\tau) = wl(\tau) + z^e(h^e(\tau))$. The real wage is obtained from the free entry condition

$$w = \upsilon(\theta,\tau,\bar{\tau}) = y\left(1 - \frac{(r+\phi)k}{q(\theta)l(\tau)}\right)\frac{1}{1+\bar{\tau}} \qquad (25)$$

An increase in progressivity operates through several routes in addition to the wage moderation effect. A higher marginal tax rate affects the worker's utility surplus through hours of work, $l(\tau)$ and $h^e(\tau)$, which in turn influences the feasible real wage, $w = \upsilon(\theta,\tau,\bar{\tau})$. A rise in τ reduces labor supply and increases home production. The net effect on the worker's total income, $I^e = wl + z^e$, is positive since $z'(h^e) > w$; this effect thus tends to *increase* the utility surplus. The decline in labor supply also reduces the feasible real wage, which tends to *reduce* the worker's real income and thereby the utility surplus from employment. There is also an effect that operates through the benefit level. The decline in work-hours induce a fall in the benefit level, which in turn *increases* the returns to employment relative to unemployment.

The net effect on tightness is obtained as:

$$sign\,\frac{d\theta}{dt} = sign\,\left[\left(\frac{y}{1+\bar{\tau}}\right)\frac{\partial l}{\partial \tau}\left[(1-\rho)(1+\tau)-(1+\bar{\tau})\right] + I^e - I^u\right] \qquad (26)$$

This expression can in general take either sign but is positive as long as the tax system is not "too" progressive. Formally, a sufficient condition for $d\theta/dt > 0$ is $\tau \leq (\rho+\bar{\tau})/(1-\rho)$. The condition is obviously fulfilled if (26) is evaluated at initially proportional taxes. Notice the interaction with the replacement rate; the higher the replacement rate, the more likely that labor market tightness is raised by a higher marginal tax rate. Loosely stated, tax progressivity needs to be substantial in order to obtain a negative effect on tightness from a further rise in the marginal tax rate.

4 Hours Determined by the Worker

4.1 The Model

Suppose now that hours are determined by the employed worker who allocates his time so as to maximize rE_i, which is equivalent to maximization of the instantaneous utility. Assuming an interior solution, this yields the familiar "profit maximization" condition:

$$z'(h_i^e) = w_i \tag{27}$$

implying that the marginal productivity of home production equals the real wage. Since the production function is strictly concave, it follows immediately that a rise in the wage causes a reduction in time spent in home production and an increase in time spent in market work: $\partial h_i^e/\partial w_i < 0$ and $\partial l_i/\partial w_i > 0$. The indirect utility function is given as $\hat{I}^e(w_i)$. Notice that a tax increase here affects the indirect utility only through the real wage, since hours are optimally chosen; we have $\partial \hat{I}^e(w)/\partial t = l \cdot (\partial w/\partial t)$ by the envelope theorem. When there is bargaining over work-hours, however, there is also an effect through hours. A small cut in work-hours, given the wage, is beneficial to the worker since the bargaining outcome yields too long hours relative to what the worker prefers; cf. eq. (17) above.

The feasible real wage is now given as:

$$w = y\left(1 - \frac{(r+\phi)k}{q(\theta)l(w)}\right)\frac{1}{1+t} \tag{28}$$

with slope in the (w, θ)-space given by:

$$sign\frac{\partial w}{\partial \theta} = sign\{-1 + \varepsilon^S(\frac{y}{w_c} - 1)\} \tag{29}$$

where $w_c \equiv w(1+t)$ is the wage cost and $\varepsilon^S \equiv wl'(w)/l(w) > 0$ is the wage elasticity of labor supply; note that $y/w_c > 1$ because of hiring costs.

Empirical estimates of labor supply elasticities typically fall in a range from zero to unity (cf. footnote 8). If $\varepsilon^S = 1$, the ratio y/w_c must exceed two in order to obtain a positive sign. This possibility is rather remote, however, as it would require unrealistically high vacancy costs. We thus assume $\partial w/\partial\theta < 0$ and write the feasible real wage as $w = \omega(\theta, t)$, with $\omega_\theta < 0$ and $\omega_t < 0$. The higher the tax rate, the lower the feasible wage at a given level of labor market tightness.

For later use we derive an expression for the "θ-constant" wage elasticity with respect to a tax increase:

$$\xi_t^w \equiv -\left(\frac{\partial \ln w}{\partial \ln(1+t)}\right)_{\bar{\theta}} = \frac{1}{1-\varepsilon^S\left[(y/w_c)-1\right]} \tag{30}$$

where $\xi_t^w > 1$. Note that ξ_t^w is increasing in the labor supply elasticity. A higher tax rate reduces the feasible real wage directly as well as indirectly through the induced decline in work-hours and the associated negative productivity effect. The more sensitive work-hours are with respect to a decline in the wage, the sharper the reduction in hours and the stronger the negative impact on the feasible wage.

The unemployed worker's allocation of time between home production and search has already been characterized, i.e., the worker maximizes the value of unemployment. An expression for the worker's "feasible utility surplus" is obtained as:

$$E-U = F(\theta,t) \equiv \frac{\hat{I}^e(\omega(\theta,t)) - I^u(h^u, Z^b)}{r+\phi+s\alpha\left(\theta\right)} \tag{31}$$

where the right-hand side is independent of h^u (and s). Moreover, the worker's surplus is decreasing in tightness, i.e., $F_\theta < 0$

Wages are determined in decentralized Nash bargains between individual firms and workers, recognizing that work hours are determined by employed workers once the wage is set. The employed worker's indirect utility function is given as $\hat{I}^e(w_i)$, with the partial derivative $\partial\hat{I}_i^e/\partial w_i = l_i$. The Nash bargain thus solves:

$$\max_{w_i} \Omega(w_i, l(w_i)) = \left[E_i(w_i) - U\right]^{\beta}\left[J_i(w_i) - V\right]^{1-\beta}$$

The first-order condition for this problem can be written as

$$\beta J_i\left(\frac{\partial E_i}{\partial w_i}\right) + (1-\beta)\left(E_i - U\right)\left(\frac{\partial J_i}{\partial w_i}\right) = 0 \tag{32}$$

where the free entry condition $V = 0$ is imposed. Notice that the gain to the worker of a higher wage is given as $\partial E_i/\partial w_i = l_i/(r+\phi)$ whereas the "gain" to the firm is $\partial J_i/\partial w_i = \left[-(1+t)l_i + (y - w_{ic})l'(w_i\right]/(r+\phi)$. A higher wage has a direct negative effect of the value of the firm but also an offsetting positive effect arising from the fact that the higher wage encourages labor supply. One can think of (32) as an equation that determines the "bargained surplus", $B(\theta,t)$, conditional on labor market tightness and the tax rate. Rewriting slightly we have:

$$E-U = B(\theta,t) \equiv \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+t}\right)\left[1-\varepsilon^S\left(\frac{y}{w_c\left(\theta,t\right)}-1\right)\right]^{-1}\frac{ky}{q(\theta)} \tag{33}$$

$B(\theta, t)$ is positive under the assumption that $\left[1 - \varepsilon^S \left((y/w_c) - 1\right)\right] > 0$, an assumption already made (cf. eq. (29)). Indeed, this is a requirement for an interior solution of the wage bargain. The terms in the square bracket capture the fact that the cost to the firm of a higher wage is declining in the wage elasticity of labor supply.[11] The lower the cost to the firm of raising the wage, the higher the surplus to the worker. Notice also that $B_\theta > 0$ holds since $q'(\theta) < 0$ and $\partial w_c / \partial \theta < 0$.

Suppose that $Z^b = \rho w l$ and $z^j = ayf(h^j)$, $j = e, u$. Equilibrium obtains as $F(\cdot) - B(\cdot) = 0$, i.e.,

$$\Phi \equiv \frac{\hat{I}^e(w) - I^u(h^u, \rho w l)}{r + \phi + s\alpha(\theta)} - \frac{\left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+t}\right)\frac{ky}{q(\theta)}}{1 - \varepsilon^S\left(\frac{y}{w_c} - 1\right)} = 0 \tag{34}$$

where $\hat{I}^e = wl(w) + z^e(h^e(w))$ and $I^u = z^u(h^u) + \rho w l$. $\Phi_\theta < 0$ is assumed.[12]

4.2 Proportional Taxes

Inspection of (34) reveals that a tax increase affects the feasible surplus to the worker by reducing the real wage and thereby the worker's utility from employment: $\hat{I}^e(w) = \hat{I}^e(\omega(\theta, t))$, with $\omega_t < 0$. Another effect works through the benefit level when benefits are indexed to earnings. The bargained surplus is "directly" affected – the $1/(1+t)$ term on the right-hand side of (34) – as well as indirectly through hours of work, recognizing that $y/w_c = [1 - (r + \phi)k/q(\theta)l(w)]^{-1}$ where $w = \omega(\theta, t)$ and $\omega_t < 0$. The elasticity of B with respect to a tax rise takes the form:

$$\zeta \equiv -\frac{\partial \ln B}{\partial \ln(1+t)} = \frac{1 - \varepsilon^S\left[(y/w_c)\,\xi_t^w - 1\right]}{1 - \varepsilon^S\left[(y/w_c) - 1\right]} \tag{35}$$

where the denominator is negative and the numerator can take either sign. Notice that $\zeta < 1$ is implied by the fact that $\xi_t^w > 1$.

To derive the net effect on tightness we begin with the special case with

[11] The effect of a wage increase on the value of the firm can be written as: $\partial J/\partial w = -(1+t)l\left[1 - \varepsilon^S((y/w_c) - 1)\right]/(r + \phi)$.

[12] One can show that the inequality $\Phi_\theta < 0$ holds for sufficiently low values of ρ and/or ε^S. The subsequent comparative statics results are derived under the assumption that ε^S is (locally) constant.

$\rho = 0$ and obtain:

$$\begin{aligned} sign\left(\frac{d\theta}{dt}\right)_{\rho=0} &= sign\left\{\zeta\left(z^e - z^u\right) + (\zeta - \xi_t^w)wl\right\} \\ &= sign\{\zeta(I^e - I^u) - \xi_t^w wl\} \end{aligned} \tag{36}$$

Lemma 2 still holds and $I^e - I^u$ must hold to induce labor market participation. From (36) we thus have $d\theta/dt < 0$, irrespective of the sign of ζ. A tax increase reduces labor market tightness. To determine the effect on unemployment we need to consider how search effort is affected. Use the first-order condition for optimal search in (6) together with the Nash rule in (33) to obtain:

$$z'(h^u) = \theta(t)\left(\frac{\beta}{1-\beta}\right)\left(\frac{ky}{1+t}\right)\left[1 - \varepsilon^S\left(\frac{y}{w_c\left(\theta(t), t\right)} - 1\right)\right]^{-1} \tag{37}$$

The right-hand side of (37) – the shadow wage of home production for an unemployed worker – is affected by taxes in a rather complex way, both directly and indirectly through $\theta(t)$. The effect does not appear possible to sign without further assumptions.

In the general case with a positive replacement rate we must recognize the induced changes in the benefit level. A tax cut increases both $\hat{I}^e$ and I^u and the effect on the utility difference between employment and unemployment is generally ambiguous. Define $D \equiv \hat{I}^e - I^u$ and differentiate to obtain:

$$\frac{\partial D}{\partial t} = l\left[1 - \rho(1+\varepsilon^S)\right]\frac{\partial w}{\partial t} \tag{38}$$

where $\partial D/\partial t \lesseqgtr 0$ as $\rho(1+\varepsilon^S) \lesseqgtr 1$. A tax cut increases the utility difference as long as the replacement rate and/or the labor supply elasticity are not too high, i.e., as long as $\rho(1+\varepsilon^S) < 1$. Implicit differentiation of (34) yields:

$$\begin{aligned} sign\frac{d\theta}{dt} &= sign\left\{\zeta\left(z^e - z^u\right) + \left[\zeta\left(1-\rho\right) - \left(1 - \rho\left(1+\varepsilon^S\right)\right)\xi_t^w\right]wl\right\} \\ &= sign\{\zeta(I^e - I^u) - \left[1 - \rho(1+\varepsilon^S)\right]\xi_t^w wl\} \end{aligned} \tag{39}$$

which can take either sign. Sufficient conditions for $d\theta/dt \geq 0$ are $\zeta \geq 0$ and $\rho(1+\varepsilon^S) \geq 1$. Analogous to the case with bargaining over hours, a tax increase may have a substantial negative effect on the benefit level if the replacement rate is high and supply very elastic. Although we cannot rule out the possibility that labor market tightness increases when taxes are raised, there are limits to how high a replacement rate the model can take. The effective replacement rate, I^u/I^e, must be lower than unity to induce market participation. The statutory replacement rate, ρ, must be lower than the effective rate since home production during unemployment exceeds home production during employment. Formally, the inequality $I^u < I^e$ requires $\rho < 1 - (z^u - z^e)/wl$.

Calibration
We have calibrated the model using the functional forms and parameters given in Table 1.[13] Table 2 shows the results. They are very similar to the results shown in Table 1 for the case with bargaining over hours.

Table 2. The Effects of Tax Increases (Proportional Taxes)
Parameters: Se Table 1.

	$t = \mathbf{.25}$	$t = \mathbf{.35}$	$t = \mathbf{.45}$	$t = \mathbf{.55}$
θ	.909	.891	.865	.826
u (%)	5.0	5.6	6.7	9.4
w (*index*)	77.2 (100)	71.5 (92.6)	66.5 (86.1)	62.2 (80.6)
h^e (%)	9.4	11.4	13.7	16.1
h^u (%)	29.4	37.2	47.5	62.5
Q (*index*)	96.6 (100)	95.9 (99.3)	94.8 (98.1)	92.7 (96.0)
T	16.6	20.9	24.1	26.0
I^u/I^e (%)	54.8	60.7	67.8	77.5
$\Delta Q/\Delta T$		−.16	−.34	−1.11

4.3 Progressive Taxes

Equilibrium labor market tightness with progressive taxes is given by a modified version of eq. (34):

$$\Phi^* \equiv \frac{\hat{I}^e(\omega(\theta,\bar{\tau})) - I^u(h^u, \rho wl)}{r + \phi + s\alpha(\theta)} - \frac{\left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+\tau}\right)\frac{ky}{q(\theta)}}{1 - \varepsilon^S\left(\frac{y}{w_c} - 1\right)} = 0 \tag{40}$$

where $w_c = w(1 + \bar{\tau})$ and

$$w = \omega(\theta, \bar{\tau}) = y\left(1 - \frac{(r+\phi)k}{q(\theta)l(w)}\right)\frac{1}{1+\bar{\tau}} \tag{41}$$

[13]Note that the discussion of the comparative statics properties presumed a constant ε^S. However, with a Cobb Douglas production function we have ε^S decreasing in the wage.

Inspection of (40) immediately reveals that labor market tightness is increased by a rise in progressivity, i.e., a rise in the marginal tax rate, τ, with the average tax rate, $\bar{\tau}$, kept constant. This is the wage moderation effect well known from the recent literature: a rise in the marginal payroll tax rate increases the marginal cost to the firm of raising the wage. The (direct) effect on the worker's surplus is neutralized by increases in the tax allowance.

The rise in tightness is associated with a reduction in the real wage, since $\omega_\theta < 0$. This also implies – from eq. (27) – that hours of market work decline, whereas hours allocated to home production among employed workers increase. When the worker determines hours we thus have:

Proposition 2: *A rise in the marginal tax rate, with the average tax rate kept fixed, increases labor market tightness and reduces the real wage. Time devoted to home production among employed workers increases, whereas hours of market work are reduced.*

To determine the effect on unemployment we need to look at the impact on search effort. A rise in the marginal tax rate has two effects on search, noting that the relevant tax rate here is τ:

$$z'(h^u) = \theta(\tau)\left(\frac{\beta}{1-\beta}\right)\left(\frac{ky}{1+\tau}\right)\left[1-\varepsilon^S\left(\frac{y}{w_c}-1\right)\right]^{-1} \tag{42}$$

The right-hand side is the equilibrium marginal returns to search. There is a direct *negative* effect, which is due to the fact that the returns to search have declined. There is also an indirect *positive* effect associated with the increase in tightness. Clearly, it is the changes in the ratio $\theta(\tau)/(1+\tau)$ that matters for the unemployed worker's time allocation.The elasticity of tightness with respect to the marginal tax rate must exceed unity in order to guarantee an unambiguously positive search response to a marginal tax hike. This need not generally be the case, however. The elasticity of tightness with respect to the marginal tax rate may or may not exceed unity and the impact on search is therefore generally ambiguous. This ambiguity carries over to the impact on employment: the favorable impact of the rise in tightness is conceivably offset by a sufficiently strong adverse search response. Some calibrations, not reported, indicate that more progressive (or less regressive) taxes do reduce unemployment.

5 Concluding Remarks

The paper has explored various effects of labor taxes in a search equilibrium model of the labor market. An attractive feature of this model is that it conveniently allows for an arguably realistic analysis of the unemployed worker's time allocation problem in an environment where both search and home production are options. Moreover, we can analyze how taxes affect unemployment and hours worked in a unified theoretical framework.

The results confirm that some wellknown neutrality results in the existing literature disappear once we allow for endogenous home production. Higher proportional labor taxes cause higher unemployment for zero or sufficiently low replacement rates in unemployment insurance. The effect on unemployment is ambiguous in general, but the numerical calibrations suggest that tax hikes contribute to higher unemployment.

An natural extension of the analysis would be to develop a model with two market sectors, where one sector produces goods that are close or perfect substitutes to the goods produced at home. Such a model would allow an analysis of the employment and welfare implications of sectoral tax differentiation in a unified general equilibrium framework. It could also be used for comparisons between alternative tax reforms, for example comparisons between the effects of general tax cuts and sectorally differentiated tax cuts. These and other issues are left for future work.

Appendix

A1. On Vacancy Costs

The purpose of this note is to derive the labor demand condition from a model of a "large" firm and show that the chosen specification of vacancy costs – with costs proportional to labor productivity as given by eq. (6) – is consistent with a specific recruitment technology.

Consider a firm that allocates its labor force between production and recruitment activities. Let n_i denote the total number of employees and e_i the number of workers allocated to the production department. Vacancies (v_i) are created according to the "production function"

$$v_i = c(n_i - e_i)l_i \tag{A1}$$

where l_i denotes work-hours and c is a positive parameter. The net change in employment is given by

$$\dot{n}_i = q(\theta)v_i - \phi n_i = q(\theta)\left[c(n_i - e_i)l_i\right] - \phi n_i \tag{A2}$$

where ϕ is the separation rate. In a steady state, the ratio e_i/n_i is given as

$$\frac{e_i}{n_i} = \left(1 - \frac{\phi}{cq(\theta)l_i}\right) \tag{A3}$$

The ratio is increasing in the number of work-hours. A rise in work-hours means that more workers can be recruited by a given number of workers in the personnel department. The firm can thus transfer some workers to the production department without experiencing a decline in its total workforce. This implies a rise in labor productivity, which in turn increases the feasible real wage, i.e., the wage that the firm can offer its workers at zero profits.

Let the firm's profits be given by

$$\pi_i = e_i l_i y - w_c n_i l_i \tag{A4}$$

and use (A3) together with $\pi_i = 0$ to obtain:

$$w_c = y\left(1 - \frac{\phi c^{-1}}{q(\theta)l_i}\right) \tag{A5}$$

This is an equation of the same form as eq. (9) in the main text, with $r = 0$ and $k = 1/c$.

A dynamic formulation yields the same expression, except for an interest factor. Suppose that the firm's objective is to maximize the present discounted value of profits, i.e.,

$$\Pi_i = \int_0^\infty \exp(-rt)[e_i l_i y - w_c n_i l_i]dt \tag{A6}$$

The firm's problem is to maximize Π_i subject to (A2). By standard methods one can establish that the necessary conditions for maximum can be collapsed to:

$$w_c = y\left(1 - \frac{(r+\phi)c^{-1}}{q(\theta)l_i}\right) \tag{A7}$$

which is equivalent to the labor demand condition given by (9) in the main text.

A2. Proof of Lemma 2

To prove Lemma 2, note that $h^e = h^u$ requires that the marginal productivity in home production for the employed worker must equal the marginal return to search for the unemployed individual, i.e., $y/(1+t) = \alpha\,(E-U)$. Moreover, this equality must be invariant to a uniform rise in labor productivity, assuming production functions of the form $z^j = ayf(h^j)$, $j = e, u$. A rise in y increases the employed worker's marginal productivity at home according to

$$\frac{\partial\,[y/(1+t)]}{\partial y} = \frac{1}{1+t} \tag{A8}$$

as implied by (12). The effect on the marginal return to search is given by

$$\frac{\partial\,(\alpha\,(E-U))}{\partial y} = \frac{\alpha}{r+\phi+s\alpha}\left(l\,(1-\rho)\,\frac{\partial w}{\partial y} + af(h^e) - af(h^u)\right) \tag{A9}$$

These expressions make use of the results that tightness and time allocation are invariant to a uniform rise in productivity. By using (9) and evaluating (A9) at $h^e = h^u$, and thus $l = s$, we obtain

$$\left(\frac{\partial\,(\alpha\,(E-U))}{\partial y}\right)_{l=s} = \left(\frac{(1-\rho)\alpha l}{r+\phi+\alpha l}\right)\left(1-\frac{(r+\phi)k)}{q(\theta)l}\right)\frac{1}{1+t} < \frac{1}{1+t} \tag{A10}$$

which implies that the employed worker's marginal productivity at home exceeds the marginal return to search. The inequality $l > s$ must thus hold, and hence $z^e < z^u$, as $h^e < h^u$.

A3. Taxes and Real Wages

To obtain the effect on the real wage we use eq. (13), recognizing $\theta(t)$. The elasticity of the real wage with respect to the tax rate can be written as

$$\frac{d\ln w}{d\ln(1+t)} = \left(-\frac{\partial \ln w}{\partial \ln\theta}\right)\left(-\frac{d\ln\theta}{d\ln(1+t)}\right) + \frac{\partial \ln w}{\partial \ln(1+t)} = \varepsilon^w_\theta \cdot \varepsilon^\theta_t - 1 \tag{A11}$$

noting that $\frac{\partial \ln w}{\partial \ln(1+t)} = -1$ and

$$\varepsilon^w_\theta \equiv -\frac{\partial \ln w}{\partial \ln\theta} = \eta\left(\frac{y}{w_c} - 1\right) > 0 \tag{A12}$$

The elasticity of tightness with respect to the tax rate is obtained from implicit differentiation of (16):

$$\varepsilon^\theta_t \equiv -\frac{d\ln\theta}{d\ln(1+t)} = \frac{z^u - z^e}{wl\cdot\varepsilon^w_\theta + \gamma\,(I^e - I^u)} > 0 \tag{A13}$$

where $\gamma \equiv ((r+\phi)\eta + s\alpha) / (r+\phi+s\alpha)$, $\gamma \in (0,1)$. To show that the real wage declines we need to check that $\varepsilon_\theta^w \cdot \varepsilon_t^\theta < 1$. We obtain:

$$\varepsilon_\theta^w \cdot \varepsilon_t^\theta = \frac{\varepsilon_\theta^w (z^u - z^e)}{\varepsilon_\theta^w (z^u - z^e + I^e - I^u) + \gamma (I^e - I^u)} < 1 \qquad \text{(A14)}$$

which proves the claim.

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