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Abstract

In this paper, we show how time-varying unemployment benefits can generate equilibrium wage dispersion in an economy in which identical firms post wages and homogeneous workers search for acceptable offers. We allow for matching frictions and for free entry and exit of vacancies, and we model time-varying unemployment benefits in a simple and natural way. We characterize the equilibrium, and we derive the comparative statics effects of changes in the unemployment compensation system on the equilibrium wage distribution and the unemployment rate.

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1 Introduction

Job search theory must explain not only the optimal behavior of job seekers but also the origin of "the variety of wage offers which is supposed to motivate the behavior of job seekers in the market." (Rothschild, 1973, p. 467) The need to consider job search theory in an equilibrium framework, that is, with both sides of the market explicitly modeled, is especially compelling in light of Diamond (1971), which suggests that in a wage-posting model with homogeneous workers and firms, search costs, no matter how small, will lead to an equilibrium wage distribution that is degenerate at the monopsony wage.

In this paper, we exploit a common feature of labor markets, namely, time-varying unemployment compensation, to overcome the Diamond paradox. Specifically, we consider an economy in which newly unemployed workers initially receive unemployment benefits at rate b. Eventually, if a worker does not find and accept a job in the meantime, the unemployment benefit falls to a lower level, s. The duration of high-benefit receipt is treated as an exponential random variable. This representation of time-varying unemployment compensation enables us to derive a two-point equilibrium wage distribution in a simple stationary setting. We derive this equilibrium distribution using a wage-posting model of sequential search. We allow for free entry and exit of jobs and for matching frictions in the sense that the rate at which unemployed workers and vacant jobs contact one another depends on overall labor market tightness. The use of a matching function to determine the job contact rate for workers (and the worker contact rate for jobs) is relatively unusual in wage-posting models, which typically assume a fixed contact rate. Our model can thus be viewed as a combination of the wage-posting and job-matching (e.g, Pissarides 2000) traditions.

The contribution of our model is to provide a new foundation for equilibrium wage dispersion, but the intuition underlying wage dispersion in our model is one that recurs in equilibrium search theory. In wage-posting models with homogeneous firms, wage dispersion can arise - but need not necessarily do so - if workers have a distribution of reservation wages. For example, with a two-point distribution of reservation wages, firms may be indifferent between offering a high wage and offering a low wage. The tradeoff is that high-wage jobs are more often filled than low-wage jobs are; on the other hand, when filled, low-wage jobs generate more profit.[1]

There are several ways to generate a distribution of reservation wages. Albrecht and Axell (1984) simply assume that workers are *ex ante* heterogenous with respect to the value of leisure, but generating reservation wage dispersion when workers are *ex ante* homogeneous is more difficult. The standard approach to this problem is that of Burdett and Mortensen (1998), who use on-the-job search – an employed job seeker's reservation wage is simply his or her current wage.[2] In our model, reservation wage dispersion is generated by time-varying unemployment benefits. Even though they are identical *ex ante*, the worker who is receiving b can afford to be choosier about the jobs that he or she will accept (has a higher reservation wage) than can the worker who is receiving s.

The insight that time-varying unemployment benefits generate a distribution of reservation wages is an old one. Mortensen (1977) and Burdett (1979) initially developed the idea that in the presence of a (deterministic) time limit on unemployment compensation, a worker's reservation wage will fall as his or her elapsed unemployment duration gets closer to the time limit. This individual problem of nonstationary search has now been analyzed in considerable generality, in particular by van den Berg (1990). Our contribution is to incorporate the basic insight from this literature into an equilibrium wage-posting model. To do this in a tractable way, we abstract from the nonstationarity that complicates the individual problem. Specifically, we model the time at which the benefit falls as an exponential random variable with parameter $1/\lambda$; equivalently, we assume the event that triggers the fall from b to s occurs at Poisson rate λ.[3] This allows us to do our equilibrium analy-

[1] Another way to generate equilibrium wage dispersion with *ex ante* homogenous agents is to build a model in which high-wage firms suffer less turnover than low-wage firms do. Burdett, Lagos, and Wright (2000) do this by allowing workers to pursue "crime opportunities." The idea is that workers in high-wage firms are less likely to take up such opportunities (and, if caught, leave the firm involuntarily) than are workers in low-wage firms.

[2] There are other ways to generate reservation wage dispersion among *ex ante* identical workers. For example, in Albrecht, Axell and Lang (1986) searchers draw wage and price offers independently and simultaneously. A searcher who draws a low price is willing to accept a lower wage, i.e., has a lower reservation wage, than is a searcher who draws a high price.

[3] Our model can be interpreted as one in which the search activity of unemployed workers is imperfectly monitored by a government agency. Suppose unemployed workers are punished by a reduction in their benefits from b to s when found to be putting forth

sis in a stationary framework while focusing on the most important aspect of time-varying unemployment compensation, namely, that after some point the benefit falls.

The next section of this paper presents our model. Section 3 gives the equilibrium of the model, while Section 4 discusses the comparative statics. Although the model can be solved analytically, we present a numerical example in Section 5 to illustrate the properties of the model. Section 6 contains conclusions.

2 The Model

We consider a continuous-time model in which *ex ante* homogeneous workers are infinitely-lived. The measure of workers is fixed and normalized to 1. The decision that workers make is whether or not to accept job offers. Jobs are likewise *ex ante* homogeneous. The decision that a firm (job owner) makes is whether the job should be in the market (entry/exit) and what wage to post when the job is vacant. The measure of jobs in the market (vacancies plus filled jobs) is endogenous. Both workers and firms discount the future at the rate r.

2.1 Workers

At any moment, a worker is either unemployed or employed. When unemployed, a worker receives the (income-equivalent) value of leisure or home production, h, plus unemployment compensation. When initially unemployed, a worker receives b and then moves to the lower level s at Poisson rate λ. Thus, when unemployed, the worker's income, y, can equal either $b + h$ or $s + h$. When employed, a worker's income is the wage that he or she is paid; that is, $y = w$.

Workers move from employment to unemployment (worker/job matches break up) at an exogenous Poisson rate δ. The transition rate from unemployment to employment is endogenous and depends on labor market tightness

less search effort than required and that detection of insufficient search effort occurs at Poisson rate λ. If all workers choose to put forth less than the required effort – and this is a plausibe assumption, given that workers are homogeneous –, then this "sanctions" model is equivalent to our model with time-varying benefits. That is, our model can be interpreted as a (simplified) equilibrium version of Abbring, van den Berg, and van Ours (2000).

and on worker choice. Specifically, we assume a constant returns to scale contact function, $M(u, v) = m(\theta)u$, where u is the unemployment rate, v is the measure of vacant jobs, and $\theta = v/u$ represents labor market tightness. The Poisson rate at which an unemployed worker contacts a vacant job is thus $m(\theta)$, and the rate at which a vacancy meets an unemployed worker is $m(\theta)/\theta$. The contact function is increasing in its arguments and satisfies $M(0, v) = M(u, 0) = 0$. These assumptions imply $m(0) = 0$ and $\lim_{\theta \to 0} m(\theta)/\theta = +\infty$, as well as $m'(\theta) > 0$ and $\frac{d[m(\theta)/\theta]}{d\theta} < 0$. Finally, the fact that the offer arrival rate for workers is increasing in θ while the applicant arrival rate for vacancies is decreasing in θ implies the standard elasticity condition, $0 < m'(\theta)\theta/m(\theta) < 1$.

Given any distribution of wage offers across vacancies, $F(w)$, there will be two reservation wages among the unemployed, one for those receiving b and one for those receiving s. Firms have no incentive to offer a wage that is not someone's reservation wage; thus, in equilibrium, at most two wages will be offered. We let w_b denote the reservation wage for workers with unemployment benefit b, w_s the reservation wage for workers with unemployment benefit s, and ϕ the fraction of offers at w_b. Since workers receiving b will reject offers of w_s, not all offers need be accepted in equilibrium.

The higher reservation wage is determined by equating the value of unemployment for those receiving b, $U(b)$, to the value of employment at w_b, $N(w_b)$. Similarly, the lower reservation wage is determined by equating the value of unemployment for those receiving s, $U(s)$, to the value of employment at w_s, $N(w_s)$. The unemployment values are defined by

$$rU(b) = b + h + \phi m(\theta)[N(w_b) - U(b)] + \lambda[U(s) - U(b)] \quad (1)$$

$$rU(s) = s + h + \phi m(\theta)[N(w_b) - U(s)] + (1 - \phi)m(\theta)[N(w_s) - U(s)]. \quad (2)$$

The value for an unemployed worker who is receiving b reflects the fact that only the higher wage offer, w_b, is acceptable. The value for an unemployed worker who is receiving s reflects the fact that such a worker will be less selective; that is, either wage offer will be accepted. Similarly, the employment values are defined by

$$rN(w_b) = w_b + \delta[U(b) - N(w_b)] \quad (3)$$

$$rN(w_s) = w_s + \delta[U(b) - N(w_s)]. \quad (4)$$

Using the reservation wage property, that is,

$$U(b) = N(w_b) \text{ and } U(s) = N(w_s),$$

and substituting in equation (3) yields

$$N(w_b) = \frac{w_b}{r} = U(b).$$

Using equations (1) and (4) and the reservation wage property gives

$$w_s = w_b + \frac{(r+\delta)[w_b - (b+h)]}{\lambda}. \tag{5}$$

Since $w_b > w_s$, equation (5) implies $b + h > w_b$. The intuition is that for a worker employed at w_b, the only possible transition is into the high-benefit unemployment state. Such a transition entails no loss of value. On the other hand, a worker in the high-benefit state faces two possible transitions – into employment at w_b or into the low-benefit unemployment state. The first of these transitions entails no loss of value, but the second does. Thus, to maintain $N(w_b) = U(b)$, the flow utility in the high-benefit unemployment state has to exceed the flow utility when employed at w_b; that is, $b+h > w_b$.

Finally, using equations (1) and (2) and the reservation wage property gives

$$w_b = \frac{[r + \phi m(\theta)](b+h) + \lambda(s+h)}{r + \lambda + \phi m(\theta)} \tag{6}$$

This equation shows that w_b is a weighted sum of the flow utilities of unemployment in the high- and low-benefit states. Multiplying both sides of (6) by $r + \lambda + \phi m(\theta)$ and using $b + h > w_b$ implies $w_b > s + h$. Thus, we have $b + h > w_b > s + h$.

Equations (5) and (6) summarize the worker side of the model and are used below to solve for the equilibrium. It is worth noting that these equations remain valid even when $\phi = 0$ or $\phi = 1$. For example, if $\phi = 1$, equation (5) gives the reservation wage for unemployed workers receiving s. This reservation wage is well-defined even if, in equilibrium, no firm chooses to offer that wage.

2.2 Firms

Jobs are either filled or vacant. A job incurs a flow cost of c whether filled or vacant and produces an output valued at x when filled. Thus, the instantaneous profit for a job paying a wage of w is $-c$ when the job is vacant and $x - w - c$ when the job is filled. There is free entry and exit of vacancies.

Let $\pi(w_b)$ and $\pi(w_s)$ be the values of having vacancies offering w_b and w_s, respectively, and let $J(w_b)$ and $J(w_s)$ be the values of having filled jobs paying w_b and w_s, respectively. As noted above, the rate at which vacant jobs meet unemployed workers is $m(\theta)/\theta$. However, not all unemployed workers will accept w_s. Letting γ denote the fraction of unemployed with reservation wage w_b, we have

$$r\pi(w_b) = -c + \frac{m(\theta)}{\theta}[J(w_b) - \pi(w_b)]$$

$$r\pi(w_s) = -c + \frac{m(\theta)}{\theta}(1-\gamma)[J(w_s) - \pi(w_s)]$$

$$rJ(w_b) = x - w_b - c + \delta[\pi(w_b) - J(w_b)]$$

$$rJ(w_s) = x - w_s - c + \delta[\pi(w_s) - J(w_s)].$$

If $0 < \phi < 1$, that is, if some firms post w_b while others post w_s, then free entry/exit requires that $\pi(w_b) = \pi(w_s) = 0$. This implies

$$c = \frac{m(\theta)}{\theta}\frac{x - w_b - c}{r + \delta} \tag{7}$$

$$c = \frac{m(\theta)}{\theta}(1-\gamma)\frac{x - w_s - c}{r + \delta}. \tag{8}$$

If only w_s is offered, that is, if $\phi = 0$, then $\pi(w_b)$ can be negative, so equation (7) is not relevant. Similarly, if $\phi = 1$, then equation (8) is not relevant.

2.3 Steady-State Conditions

In steady-state, the measures of workers in each possible state must be constant through time. We will use two steady-state conditions to derive expressions for γ and u.

Workers can be classified into three categories – employed, unemployed and receiving b, and unemployed and receiving s. The measure of employed is $1 - u$, the measure of unemployed receiving b is γu, and the measure of unemployed receiving s is $(1-\gamma)u$. Since the measure of workers is normalized to one, we need only equate inflows and outflows for two of these states. We work with the two unemployment states.

The condition that equates the flows into and out of the high-benefit unemployment state is

$$\delta(1-u) = [\phi m(\theta) + \lambda]\gamma u. \tag{9}$$

Workers flow into this state from employment at rate δ; workers flow out of this state either back into employment (at rate $\phi m(\theta)$) or into the low-benefit unemployment state (at rate λ). The comparable equation for the low-benefit unemployment state is

$$\lambda\gamma u = m(\theta)(1-\gamma)u;$$

that is,

$$\lambda\gamma = m(\theta)(1-\gamma). \tag{10}$$

3 Equilibrium

A steady-state equilibrium is a vector $\{w_b, w_s, \phi, \theta, \gamma, u\}$ such that

(i) $U(b) = N(w_b)$ and $U(s) = N(w_s)$,

(ii) No wage other than w_b or w_s is offered, and one of the following is satisfied:

(a) $0 < \phi < 1$ and $\pi(w_b) = \pi(w_s) = 0$

(b) $\phi = 0$ and $\pi(w_s) = 0$ but $\pi(w_b) \leq 0$

(c) $\phi = 1$ and $\pi(w_b) = 0$ but $\pi(w_s) \leq 0$

(iii) the steady-state conditions (9) and (10) hold.

The first condition states that workers search optimally given the wage offer distribution, $\{w_b, w_s, \phi\}$.[4] The second condition states that firms optimize with respect to their wage offers in the sense that no wage is offered that is not some worker's reservation wage and with respect to their entry/exit decisions.

[4]Since all offers at w_b are accepted, whereas only a fraction $1-\gamma$ of offers at w_s are accepted, the equilibrium distributions of wages offered and of wages paid are not the same. The relationship between the two distributions can be derived from the condition that the flows of workers into and out of high-wage employment must be the same. (Equivalently, one can use the condition that the flows into and out of low-wage employment are the same.) Let η denote the fraction of employed workers who are paid w_b. The steady-state condition is then $\phi m(\theta)u = \delta\eta(1-u)$. Using equations (9) and (10) to eliminate u, this implies $\eta = \phi(\frac{\lambda + m(\theta)}{\lambda + \phi m(\theta)})$.

There are three types of equilibria to consider – equilibria in which only the low wage is offered ($\phi = 0$), equilibria in which only the high wage is offered ($\phi = 1$), and equilibria with wage dispersion ($0 < \phi < 1$). We focus on equilibria with wage dispersion and briefly discuss the corner solutions ($\phi = 0$ and $\phi = 1$) at the end of this section.

Equilibrium with wage dispersion is defined by equations (5)-(10). These equations can be solved recursively. First, we repeat equation (5), which gives w_s as a function of w_b and parameters:

$$w_s = w_b + \frac{(r+\delta)(w_b - b - h)}{\lambda}. \tag{11}$$

Second, equation (10) gives γ as a function of θ and parameters, namely,

$$\gamma = \frac{m(\theta)}{\lambda + m(\theta)}. \tag{12}$$

Third, equation (9) gives u as a function of ϕ, θ, γ, and parameters, namely,

$$u = \frac{\delta}{\delta + \gamma(\phi m(\theta) + \lambda)}. \tag{13}$$

Substituting the expression for w_s into equation (8) leaves three equations (6, 7, and 8) in the remaining endogenous variables w_b, θ, and ϕ. These equations can be rearranged to find w_b and ϕ as functions of θ and parameters and to give an equation in which θ is expressed solely in terms of the parameters of the model. Specifically, the resulting equations are

$$w_b = b + h - c\theta \tag{14}$$

$$\phi = \frac{\lambda(b-s) - c\theta(r+\lambda)}{c\theta m(\theta)} \tag{15}$$

$$c\theta m(\theta) - (r+\delta)c\theta + (x - c - b - h)m(\theta) = 0. \tag{16}$$

The derivation of equations (14)-(16) is given in Appendix 1. Equation (16) has a unique solution for θ so long as $x - c - b - h \leq 0$.[5] Using equations

[5] To see this, rewrite equation (16) as

$$cm(\theta) - (r+\delta)c = -(x - c - b - h)m(\theta)/\theta.$$

The left-hand side, which is monotonically increasing in θ, equals $-(r+\delta)c$ when $\theta = 0$ and tends to infinity as $\theta \to \infty$. The right-hand side is monotonically decreasing in θ (if and only if $x - c - b - h < 0$), tends to infinity as $\theta \to 0$ and tends to zero as $\theta \to \infty$. Thus, when $x - c - b - h < 0$, this equation has a unique solution. When $x - c - b - h = 0$, the unique solution to (16) is the θ that solves $m(\theta) = r + \delta$.

(11)-(15) then gives a unique solution for the other endogenous variables.

Of course, this procedure only makes sense when the solution for ϕ given by equation (15) falls in $(0, 1)$. For some parameter configurations, no firms will offer the higher wage. In this case, the equilibrium is given by equations (5)-(6) and (8)-(10) together with $\phi = 0$. For other parameter configurations, no firms will offer the lower wage. In this case, the equilibrium is given by equations (5)-(7) and (9)-(10) together with $\phi = 1$.

One would like to know which parameter configurations imply $\phi = 0$, which lead to wage dispersion, and which imply $\phi = 1$. Since the model is one with several free parameters, it is impossible to answer this question in general. However, we can address this issue by considering the effects of varying a single parameter, holding all others constant. The most intuitive parameter to vary is x, the flow output from filled jobs. Given any set of fixed values for the other parameters of the model (subject only to obvious restrictions such as $b > s$, $r > 0$, etc.), we can show that for x sufficiently close to zero, no equilibrium exists. The intuition is that for x sufficiently small, it is not worth posting a vacancy at the lower wage, even if that vacancy can be filled arbitrarily quickly. For somewhat larger values of x, the equilibrium is one in which $\phi = 0$. As x increases further, it becomes worthwhile for some (but not all) firms to post the higher wage; that is, for a third interval of x, equilibrium entails wage dispersion. Finally, for x sufficiently large, $\phi = 1$, as it is no longer worthwhile to incur the "delay cost" implied by posting w_s.

4 Comparative Statics

We now address the basic comparative statics associated with time-varying unemployment compensation. Specifically, what are the effects of changes in b, s, and λ on unemployment and the equilibrium wage distribution? We concentrate on the case in which there is wage dispersion. Table 1 gives the qualitative comparative statics results. The derivations are in Appendix 2. With the exception of $\partial u/\partial b$, all of these effects are unambiguous.

Table 1
Basic Comparative Statics

	θ	u	ϕ	γ	w_b	w_s
b	$+$	?	$-$	$+$	$-$	$-$
s	0	$+$	$-$	0	0	0
λ	0	$-$	$+$	$-$	0	$+$

At first glance, some of these qualitative comparative statics results seem counterintuitive. For example, all else equal, one would expect an increase in b to increase the reservation wage of the unemployed who are receiving b. To get the correct intuition, it is necessary to focus on the equilibrium effects.

Consider the effect of an increase in b. Workers prefer to cycle between employment at the high wage and unemployment at the high benefit level, that is, to avoid employment at the low wage and unemployment at the low benefit level. An increase in b makes unemployed workers in the high-benefit state more willing to accept a lower value of w_b to avoid the low-benefit state than they were before the increase in b. A similar intuition holds for the negative effect of b on w_s. Unemployed workers receiving s are more eager than they were before the increase in b to get back to the high-benefit unemployment state. This makes them more willing to accept low-wage employment and so reduces their reservation wage. In fact, w_s must fall by more than w_b does. The reason is that the increase in b has countervailing effects on w_b. On the one hand, the direct effect of the increase in b is to make high-benefit unemployment more attractive. If this were the only effect, w_b would increase. On the other hand, as indicated above, the increase in b makes workers in the high-benefit unemployment state more eager to avoid the low-benefit unemployment state, so w_b falls. The second effect dominates. The effect of an increase in b on w_s is, however, unambiguously negative.

With the fall in both w_b and w_s, firms have an incentive to open more vacancies. Because w_s falls by more than w_b, entry at the low wage is particularly attractive; thus, ϕ decreases. With a smaller fraction of vacancies at the high wage, there is an increase in the fraction of unemployed who are receiving b; that is, γ increases. The zero-value condition for high-wage jobs implies that $m(\theta)/\theta$ must fall (and, accordingly, θ must increase) to offset the decrease in w_b. The matching rate for low-wage firms has to fall by even more than the corresponding rate for high-wage firms to maintain zero value for low-wage vacancies. That is, $[m(\theta)/\theta](1-\gamma)$ must fall by more than $m(\theta)/\theta$ does or, as argued above, γ must increase. Finally, the ambiguous effect on unemployment is a result of two offsetting factors. Job offers arrive faster (θ increases) than they did before the increase in b, but relatively fewer of these offers are acceptable (ϕ decreases).

Next, consider the effects of an increase in s. Before the increase in s, workers receiving b were just indifferent between remaining unemployed and accepting a job offering w_b. The fact that s increases does not affect the marginal attractiveness of employment at w_b relative to receipt of b; this is why increasing s has no effect on w_b. Since w_b is unaffected by s, zero value for high-wage vacancies implies no change in the rate at which these vacancies meet unemployed workers; that is, θ is unaffected by the increase in s. The fact that $\partial w_s/\partial s = 0$ is perhaps more surprising. It results from the balance of two effects. On the one hand, the increase in s makes low-benefit unemployment relatively more attractive than employment at the low wage, which would lead to an increase in w_s. On the other hand, however, equation (15) makes it clear that, with $\partial\theta/\partial s = 0$, ϕ must fall with the increase in s. This means that the low-benefit unemployed is less likely to find a high-wage job and causes the value of unemployment at s to fall. These two effects on $U(s)$ balance, so w_s remains unchanged. Since neither w_s nor θ are affected by a change in s, zero value for low-wage vacancies implies that changes in s do not affect γ. Finally, a fall in ϕ, all else equal, implies relatively fewer acceptable offers for unemployed workers at the high benefit level; that is, the average duration of unemployment rises and with it, u increases.

The final comparative statics effects are those of λ. The intuition behind $\partial w_b/\partial\lambda = 0$ is similar to that underlying $\partial w_b/\partial s = 0$; increasing λ does not affect the marginal attractiveness of employment at w_b relative to unemployment at b. As noted above, there is a direct link between w_b and θ via the zero-value condition for high-wage vacancies. If w_b is unaffected by the increase in λ, then neither will θ be. Increasing λ does, however, change the tradeoff between employment at w_s and unemployment at s. As λ increases, the choice between staying unemployed forever at s versus reentering employment at w_s shifts in favor of the former. An increase in w_s is required to restore the balance. An increase in λ, *ceteris paribus,* reduces the expected duration of high-benefit unemployment and so decreases γ, the fraction of unemployed who are in the high-benefit state. In addition, since w_s increases with λ while θ is unaffected, γ must fall in order to maintain zero value for low-wage vacancies. The positive effect of λ on ϕ can also be understood as a side effect of the increase in w_s. With w_b constant, as w_s increases, the composition of vacancies shifts towards the higher wage offer. The increase in ϕ in conjunction with the increase in λ makes the unemployed accept job offers more quickly, and this is why the unemployment rate falls with λ.

While our model is, of course, very stylized, it does offer some predictions of interest. Most European unemployment systems have longer durations of unemployment benefits, more generous unemployment benefits, and social assistance at higher levels when unemployment compensation expires compared to the United States system. This corresponds to a lower λ, a higher b, and a higher s. Our model suggests that such economies should have higher unemployment than the United States. They should also have a smaller fraction of vacancies offering the higher wage, thus longer unemployment durations on average.

5 Numerical Example

In this section, we present a numerical example to illustrate the properties of the model. The example uses the contact function, $m(u, v) = 8\sqrt{u \cdot v}$, that is, $m(\theta) = 8\sqrt{\theta}$, and in the baseline case, we assume that $b = 1$, $s = 0$, $h = 1$, $\lambda = 2$, $x = 2$, $\delta = .2$, $c = .5$, and $r = .05$. The baseline parameter values were chosen with two criteria in mind. First, the parameter values themselves should be reasonable. Second, the values of the endogenous variables that follow from these parameter values should also be reasonable.

Table 1 presents the solution for our baseline case (in row 1) and the comparative statics of changes in b.

Table 1: Comparative Statics for b
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
$x = 2$, $h = 1$, $\delta = .2$, $c = .5$, $r = .05$
$s = 0$ and $\lambda = 2$

b	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
1	1.0317	8.1258	.061140	.80248	.224850	1.4842	1.4197
1.5	2.0447	11.439	.075368	.85065	.077313	1.4777	1.3499
2	3.0546	13.982	.081716	.87486	.040695	1.4727	1.2818

The baseline case generates an unemployment rate of about 6%. The equilibrium value of $\theta = 1.0317$ implies a steady-state measure of vacancies of $v = \theta u = (1.0317)(.06114) = .063078$. The average duration of unemployment is slightly less than one and one half months ($12 \times (1/8.1258) = 1.4768$), while the average duration of a vacancy is similar ($12 \times (1.0317/8.1258) = 1.5236$). As we increase b, labor market tightness increases, i.e., θ rises, as predicted. The fraction of vacancies offering the higher wage (ϕ) falls and the fraction of short-term unemployed (γ) rises. Wages for all workers fall, but, as predicted, the low wage falls by more than the high wage.

Table 2 presents the comparative statics results for changes in s. We fix $b = 1$ so that the results can be compared to those in row 1 of Table 1.

Table 2: Comparative Statics for s
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
$x = 2$, $h = 1$, $\delta = .2$, $c = .5$, $r = .05$
$b = 1$ and $\lambda = 2$

s	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
.1	1.0317	8.1258	.067566	.80248	.177140	1.4842	1.4197
.2	1.0317	8.1258	.075504	.80248	.129420	1.4842	1.4197
.3	1.0317	8.1258	.085551	.80248	.081711	1.4842	1.4197
.4	1.0317	8.1258	.096821	.80248	.033998	1.4842	1.4197

As indicated above, changing s has no effect on θ, γ, w_b, or w_s. As s increases, the fraction of high-wage vacancies falls and the unemployment rate rises. Comparing the effect of an increase in s from .1 to .2 with the increase in b from 1 to 1.5, one can see that in this example the effect of increasing the low unemployment benefit on unemployment is greater than a comparable increase in the high benefit.

Finally, Table 3 presents the comparative statics results for changes in λ. The third row corresponds to the baseline case presented in Table 1. Lower levels of λ correspond to steady states with longer average durations of high unemployment benefit receipt; i.e., the expected period over which the unemployed receive b is longer. As λ falls, the fraction of unemployed receiving b rises (i.e., γ increases), unemployment rises, and the fraction of vacancies offering the higher wage falls. As we argued above, this is because of the adverse effect of a decrease in λ on the lower wage, which is apparent in Table 3.

Table 3: Comparative Statics for λ
Solution with $m(\theta) = 8\theta^{\frac{1}{2}}$
$x = 2$, $h = 1$, $\delta = .2$, $c = .5$, $r = .05$
$b = 1$ and $s = 0$

λ	θ	$m(\theta)$	u	γ	ϕ	w_b	w_s
3	1.0317	8.1258	.045341	.73036	.34035	1.4842	1.4412
2.5	1.0317	8.1258	.051708	.76472	.28260	1.4842	1.4326
2	1.0317	8.1258	.061140	.80248	.22485	1.4842	1.4197
1.5	1.0317	8.1258	.076555	.84417	.16710	1.4842	1.3982
1	1.0317	8.1258	.106290	.89042	.10935	1.4842	1.3553

The point of this example is to illustrate that time-varying unemployment benefits generate equilibrium wage dispersion for a range of parameter values.

The equilibrium is one in which unemployment arises not only because of matching frictions, but also from the rational rejection of wage offers by job seekers in favor of further search. The example also shows that our model can generate a wage distribution with greater density at the low wage than at the high wage; i.e., our wage density is "downward sloping."

6 Concluding Remarks

In this paper, we use as simple a model as possible to show that time-varying unemployment benefits can lead to equilibrium wage dispersion. Some of our assumptions, however, can be relaxed without substantially complicating the model or changing the results.

First, we assume that workers are risk neutral. Since the purpose of unemployment compensation is to insure workers against the risk of joblessness, it might be more satisfactory to assume risk aversion. This can be done by changing flow incomes while unemployed, $b + h$ and $s + h$, into flow utilities, say $\xi(b + h)$ and $\xi(s + h)$, and, similarly, flow incomes while employed, w, into flow utilities, $\xi(w)$, where $\xi'(y) > 0$ and $\xi''(y) < 0$. This would not qualitatively change our results. If one wanted to investigate "optimal unemployment insurance" in a search equilibrium context (e.g., Fredriksson and Holmlund, 2001), it would, of course, be necessary to introduce this complication.

Second, we neglect the issue of unemployment compensation finance. Unemployment compensation finance, in the form of a balanced budget constraint, could be introduced into our model in several ways. One possibility is to assume that employed workers are taxed to finance unemployment benefits. In this case, the flow income of an employed worker would be $w(1 - t)$ and the balanced budget constraint would be

$$[\eta w_b + (1 - \eta)w_s]t(1 - u) = [\gamma b + (1 - \gamma)s]u,$$

where η is the fraction of employment at the higher wage (cf., footnote 4). The extra variable that goes with this constraint is the tax rate, t. It is similarly straightforward to finance unemployment compensation in our model by having employers pay a wage tax or a lump-sum tax. The approach we follow is equivalent to assuming that unemployment compensation is financed out of general revenues and that the equilibrium effects are small enough to neglect.

A third point is that workers employed at wage w_s would prefer to quit into unemployment if quitters were allowed to receive benefits. We implicitly are assuming that this is not possible. One could assume that quitters are detected with some exogenous probability q and then punished in some way, for example, by being denied any unemployment compensation. As workers are identical, any worker receiving w_s would make the same decision – whether to quit or not to quit. If w_s is to be offered in equilibrium, that common decision has to be to not quit. This places a constraint on the lowest possible value for this wage, which might or might not be binding in equilibrium. This modification would add an efficiency wage flavor to the model, but the basic conclusion that wage dispersion is possible as an equilibrium phenomenon would not change.

There are some other, more substantial, modifications of our model that could generate continuous equilibrium wage dispersion. For example, one might consider a model with both time-varying unemployment compensation and on-the-job search; that is, one could combine our approach with that of Burdett and Mortensen (1998). Relative to the "pure" Burdett and Mortensen model, that is, their model with *ex ante* homogeneous workers and firms, such a combination (i) would generate search unemployment in the sense that some wage offers would be rejected by the unemployed and (ii) could potentially generate an equilibrium wage distribution that is not everywhere upward sloping.

While these extensions are interesting, we feel that the simple model is sufficient for our purpose. That is, it allows us to explore how time-varying unemployment compensation can generate equilibrium wage dispersion, even though both workers and firms are *ex ante* homogeneous. We have thus added to the equilibrium search literature by demonstrating a new approach to overcoming the Diamond (1971) paradox.

References

[1] Abbring, J., G. van den Berg, and J. van Ours, "The Effect of Unemployment Insurance Sanctions on the Transition Rate from Unemployment to Employment," mimeo, 2000.

[2] Albrecht, J. and B. Axell, "An Equilibrium Model of Search Unemployment," *Journal of Political Economy*, 92 (1984), 824-40.

[3] Albrecht, J., B. Axell, and H. Lang, "General Equilibrium Wage and Price Distributions," *Quarterly Journal of Economic*s, 101 (1986), 687-706.

[4] Burdett, K., "Search, Leisure, and Individual Labor Supply," in Lippman, S. and J. McCall (eds.), *Studies in the Economics of Search* (1979) (Amsterdam: North Holland)

[5] Burdett, K., R. Lagos, and R. Wright, "Inequality, Unemployment and Crime," mimeo, 2000.

[6] Burdett, K. and D. Mortensen, "Wage Differentials, Unemployment, and Employer Size," *International Economic Review*, 39 (1998), 257-273.

[7] Diamond, P., "A Model of Price Adjustment," *Journal of Economic Theory*, 3 (1971), 156-68.

[8] Fredriksson, P. and B. Holmlund, "Optimal Unemployment Insurance in Search Equilibrium," *Journal of Labor Economics*, forthcoming 2001.

[9] Mortensen, D., "Unemployment Insurance and Job Search Decisions," *Industrial and Labor Relations Review*, 30 (1977), 505-17.

[10] Pissarides, C., *Equilibrium Unemployment Theory*, 2nd edition (2000) (Basil Blackwell, Oxford).

[11] Rothschild, M. "Models of Market Organization with Imperfect Information: A Survey," *Journal of Political Economy*, 81 (1973), 1283-1308.

[12] van den Berg, G., "Nonstationarity in Job Search Theory," *Review of Economic Studies*, 57 (1990), 255-77.

Appendix 1: Derivation of Equations (14) to (16)

We start by deriving equation (14). From equation (8),

$$w_s = \frac{m(\theta)(1-\gamma)(x-c) - c\theta(r+\delta)}{m(\theta)\,(1-\gamma)}.$$

Equating this to the expression for w_s given by (5) yields

$$\frac{m(\theta)(1-\gamma)(x-c) - c\theta(r+\delta)}{m(\theta)\,(1-\gamma)} = w_b + \frac{(r+\delta)(w_b - b - h)}{\lambda}.$$

Using equation (7),

$$m(\theta)(x-c) = m(\theta)w_b + c\theta(r+\delta).$$

Substitution then gives

$$\frac{-\gamma c\theta(r+\delta)}{m(\theta)\,(1-\gamma)} = \frac{(r+\delta)(w_b - b - h)}{\lambda}.$$

Thus,

$$w_b - b - h = \frac{-\gamma\lambda c\theta}{m(\theta)\,(1-\gamma)} = \frac{-(\frac{m(\theta)}{\lambda+m(\theta)})\lambda c\theta}{m(\theta)\left(\frac{\lambda}{\lambda+m(\theta)}\right)} = -c\theta,$$

which verifies (14).

Next, set the expression for w_b from (14) equal to the one given by (6); that is,

$$b + h - c\theta = \frac{(r+\phi m(\theta))(b+h) + \lambda(s+h)}{r+\lambda+\phi m(\theta)}$$

Solving for ϕ verifies (15).

Finally, from (7)

$$c\theta(r+\delta) = m(\theta)(x - w_b - c).$$

That is,

$$c\theta(r+\delta) = m(\theta)(x - b - h + c\theta - c),$$

which, after rearrangement, verifies (16).

Appendix 2: Comparative Statics Derivations

a. Comparative statics for θ: Using (16),

$$\frac{\partial\theta}{\partial b} = \frac{m(\theta)}{c[m(\theta)-(r+\delta)] + m'(\theta)[c\theta + (x-c-b-h)]}.$$

The denominator of this expression is positive since, from (16), we have

$$c[m(\theta)-(r+\delta)] = -[m(\theta)/\theta](x-c-b-h) > 0$$

and

$$c\theta + (x-c-b-h) = c\theta(r+\delta)/m(\theta) > 0.$$

Thus, $\partial\theta/\partial b > 0$. Since neither s nor λ enters into (16), we have $\partial\theta/\partial s = \partial\theta/\partial\lambda = 0$.

b. Comparative statics for w_b: Using (14), $\partial w_b/\partial b = 1 - c(\partial\theta/\partial b)$. That is,

$$\frac{\partial w_b}{\partial b} = \frac{-c(r+\delta) + m'(\theta)[c\theta + (x-c-b-h)]}{c[m(\theta)-(r+\delta)] + m'(\theta)[c\theta + (x-c-b-h)]}.$$

From (16), $c(r+\delta) = \frac{m(\theta)}{\theta}[c\theta + (x-c-b-h)]$, so by substitution

$$\frac{\partial w_b}{\partial b} = \frac{[-\frac{m(\theta)}{\theta} + m'(\theta)][c\theta + (x-c-b-h)]}{c[m(\theta)-(r+\delta)] + m'(\theta)[c\theta + (x-c-b-h)]}.$$

As $m'(\theta)\theta < m(\theta)$ and $c\theta + (x-c-b-h) > 0$ we have $\partial w_b/\partial b < 0$. Since s and λ appear in neither (14) nor (16) it follows that $\partial w_b/\partial s = \partial w_b/\partial\lambda = 0$.

c. Comparative statics for w_s: From (11),

$$\frac{\partial w_s}{\partial b} = \frac{\partial w_b}{\partial b} - \frac{r+\delta}{\lambda} < 0 \text{ and}$$

$$\frac{\partial w_s}{\partial\lambda} = \frac{(b+h-w_b)(r+\delta)}{\lambda^2} = \frac{c\theta(r+\delta)}{\lambda^2} > 0.$$

Finally, $\partial w_s/\partial s = 0$.

d. Comparative statics for ϕ: From (15),

$$\frac{\partial\phi}{\partial b} = \frac{\lambda}{c\theta m(\theta)} + \frac{\partial\phi}{\partial\theta}\frac{\partial\theta}{\partial b},$$

where

$$\frac{\partial\phi}{\partial\theta} = \frac{c\theta m(\theta)[-c(r+\lambda)] - [\lambda(b-s) - c\theta(r+\lambda)][cm(\theta) + c\theta m'(\theta)]}{[c\theta m(\theta)]^2}$$

and

$$\frac{\partial\theta}{\partial b} = \frac{m(\theta)}{c[m(\theta) - (r+\delta)] + m'(\theta)[c\theta + (x-c-b-h)]}.$$

We thus have

$$\frac{\partial\phi}{\partial b} = \frac{\lambda}{c\theta m(\theta)} - \frac{m(\theta)\lambda(b-s) + \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)]}{c\theta^2 m(\theta)(c[m(\theta) - (r+\delta)] + m'(\theta)[c\theta + (x-c-b-h)])}.$$

Multiplying both sides by $c\theta m(\theta)$, the sign of $\partial\phi/\partial b$ is the same as that of

$$\lambda - \frac{m(\theta)\lambda(b-s) + \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)]}{c\theta[m(\theta) - (r+\delta)] + m'(\theta)\theta[c\theta + (x-c-b-h)]}.$$

Since the denominator of the fraction is positive, the sign of $\partial\phi/\partial b$ is the same as that of

$$\lambda c\theta[m(\theta) - (r+\delta)] + \lambda m'(\theta)\theta[c\theta + (x-c-b-h)] - m(\theta)\lambda(b-s) - \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)].$$

From (16), $x - c - b - h = [c\theta(r+\delta)/m(\theta)] - c\theta$; thus, the sign of $\partial\phi/\partial b$ is the same as that of

$$\lambda m(\theta)[c\theta - (b-s)] - \lambda c\theta(r+\delta)[1 - \frac{m'(\theta)\theta}{m(\theta)}] - \theta m'(\theta)[\lambda(b-s) - c\theta(r+\lambda)].$$

The first and third of these three terms are negative by equation (15); specifically, by the condition $\phi > 0$. The second term is negative by $m'(\theta)\theta < m(\theta)$. We thus have $\partial\phi/\partial b < 0$.

Next,

$$\frac{\partial\phi}{\partial\lambda} = \frac{b-s-c\theta}{c\theta m(\theta)} > 0$$

since, again from (15), $b - s - c\theta > 0$.

Finally,

$$\frac{\partial \phi}{\partial s} = -\frac{\lambda}{c\theta m(\theta)} < 0.$$

e. Comparative statics for γ: From (12),

$$\frac{\partial \gamma}{\partial \theta} = \frac{\lambda m'(\theta)}{(\lambda + m(\theta))^2} > 0.$$

Then $\partial\gamma/\partial\lambda = -m(\theta)/[\lambda + m(\theta)]^2 < 0$, and the rest of the derivatives of γ have the same signs as the partials of θ with respect to the various parameters. Specifically, $\partial\gamma/\partial b > 0$ and $\partial\gamma/\partial s = 0$.

f. Comparative statics for u: From (13),

$$\frac{\partial u}{\partial b} = \frac{\partial u}{\partial \phi m(\theta)} \frac{\partial(\phi m(\theta))}{\partial b} + \frac{\partial u}{\partial \gamma}\frac{\partial \gamma}{\partial b},$$

where

$$\frac{\partial u}{\partial \phi m(\theta)} = \frac{-\delta\gamma}{(\delta + \gamma[\phi m(\theta) + \lambda])^2} < 0$$

and

$$\frac{\partial u}{\partial \gamma} = \frac{-\delta(\phi m(\theta) + \lambda)}{(\delta + \gamma[\phi m(\theta) + \lambda])^2} < 0.$$

Let $\Psi = \dfrac{\delta}{(\delta + \gamma[\phi m(\theta) + \lambda])^2} > 0$. Then, we have

$$\frac{\partial u}{\partial b} = -\Psi(\gamma \frac{\partial \phi m(\theta)}{\partial b} + (\phi m(\theta) + \lambda)\frac{\partial \gamma}{\partial b}).$$

Next

$$\begin{aligned}\frac{\partial \phi m(\theta)}{\partial b} &= \frac{\lambda}{c\theta} + \{\frac{c\theta[-c(r+\lambda)] - [\lambda(b-s) - c\theta(r+\lambda)]c}{c^2\theta^2}\}\frac{\partial \theta}{\partial b} \\ &= \frac{\lambda}{c\theta} - \frac{\lambda(b-s)}{c\theta^2}\frac{\partial \theta}{\partial b} = \frac{\lambda}{c\theta}(1 - \frac{(b-s)}{\theta}\frac{\partial \theta}{\partial b})\end{aligned}$$

and

$$\frac{\partial \gamma}{\partial b} = \frac{\lambda m'(\theta)}{(\lambda + m(\theta))^2}\frac{\partial \theta}{\partial b} > 0.$$

Thus,

$$\begin{aligned}\frac{\partial u}{\partial b} &= -\lambda\Psi\left(\frac{\gamma}{c\theta}(1-\frac{(b-s)}{\theta}\frac{\partial\theta}{\partial b})+\frac{m'(\theta)(\phi m(\theta)+\lambda)}{(\lambda+m(\theta))^2}\frac{\partial\theta}{\partial b}\right)\\ &= -\lambda\Psi\left(\frac{\gamma}{c\theta}+\frac{\partial\theta}{\partial b}[\frac{m'(\theta)(\phi m(\theta)+\lambda)}{(\lambda+m(\theta))^2}-\frac{\gamma(b-s)}{c\theta^2}]\right).\end{aligned}$$

Since

$$\frac{m'(\theta)(\phi m(\theta)+\lambda)}{(\lambda+m(\theta))^2}=\frac{\gamma m'(\theta)(\phi m(\theta)+\lambda)}{m(\theta)(\lambda+m(\theta))},$$

we have

$$\frac{\partial u}{\partial b}=-\frac{\lambda\gamma}{c\theta}\Psi\left(1+\frac{\partial\theta}{\partial b}[\frac{m'(\theta)(\phi m(\theta)+\lambda)c\theta}{m(\theta)(\lambda+m(\theta))}-\frac{(b-s)}{\theta}]\right).$$

Then,

$$\text{sign }[\frac{\partial u}{\partial b}]=-\text{ sign }[1+\frac{\partial\theta}{\partial b}(\frac{m'(\theta)(\phi m(\theta)+\lambda)c\theta}{m(\theta)(\lambda+m(\theta))}-\frac{(b-s)}{\theta})]$$

Without imposing more restrictions on $m(\theta)$, this latter sign is indeterminate.

Next, since $\frac{\partial\gamma}{\partial s}=\frac{\partial\theta}{\partial s}=0$, we have

$$\frac{\partial u}{\partial s}=\frac{\partial u}{\partial\phi}\frac{\partial\phi}{\partial s}.$$

Since

$$\frac{\partial u}{\partial\phi}=\frac{-\delta\gamma m(\theta)}{(\delta+\gamma[\phi m(\theta)+\lambda])^2}<0$$

and, as shown above, $\frac{\partial\phi}{\partial s}<0$, we have $\frac{\partial u}{\partial s}>0$.

Finally, using $\frac{\partial\theta}{\partial\lambda}=0$, we have

$$\frac{\partial u}{\partial\lambda}=\frac{\partial u}{\partial\phi}\frac{\partial\phi}{\partial\lambda}+\frac{\partial u}{\partial\gamma}\frac{\partial\gamma}{\partial\lambda}-\frac{\delta\gamma}{(\delta+\gamma[\phi m(\theta)+\lambda])^2}.$$

The final term is the direct effect of λ on u. Substitution gives,

$$\begin{aligned}\frac{\partial u}{\partial\lambda} &= -(\frac{\delta\gamma m(\theta)}{(\delta+\gamma[\phi m(\theta)+\lambda])^2})(\frac{b-s-c\theta}{c\theta m(\theta)})+\\ &\quad (\frac{\delta(\phi m(\theta)+\lambda)}{(\delta+\gamma[\phi m(\theta)+\lambda])^2})(\frac{m(\theta)}{(\lambda+m(\theta))^2})-\frac{\delta\gamma}{(\delta+\gamma[\phi m(\theta)+\lambda])^2}.\end{aligned}$$

Or,

$$\frac{\partial u}{\partial \lambda} = -\Psi(\frac{\gamma(b-s-c\theta)}{c\theta} - \frac{m(\theta)(\phi m(\theta)+\lambda)}{(\lambda+m(\theta))^2} + \gamma).$$

The sign of $\frac{\partial u}{\partial \lambda}$ is the same as that of

$$-\frac{\gamma(b-s)}{c\theta} + \frac{m(\theta)(\phi m(\theta)+\lambda)}{(\lambda+m(\theta))^2}.$$

Using equations (12) and (15) for γ and ϕ, the sign of $\frac{\partial u}{\partial \lambda}$ is the same as that of

$$\frac{-\frac{m(\theta)}{\lambda+m(\theta)}(b-s)}{c\theta} + \frac{m(\theta)[\frac{\lambda(b-s)-c\theta(r+\lambda)}{c\theta}+\lambda]}{(\lambda+m(\theta))^2} = \frac{-m(\theta)(b-s)-c\theta r}{c\theta(\lambda+m(\theta))^2} < 0.$$

Thus, $\frac{\partial u}{\partial \lambda} < 0$.