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CESifo Working Papers

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CESifo Working Paper No. 505

June 2001

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ISSN 1617-9595

CESifo *Working Paper No. 505*
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Abstract

Tax evasion is modeled as a risky activity and integrated into a standard problem of optimal tax design. It is shown that there is a trade off between reducing tax evasion and reducing tax distortion. Thus it is efficient to supplement a broad-based wage tax by a tax on specific consumption if the former is evaded and the latter not. The optimal tax structure can be characterized by an explicit formula.

JEL Classification: H26, H21.

Keywords: Tax evasion, risk-taking, optimal taxation.

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1. Introduction

It is apparent that different taxes are evaded to a different extent, and optimal tax design should clearly take such differences into account. In fact, there is relatively little literature on the choice of an optimal tax structure in an economy where evasion exists. This is rather surprising given that the basic analysis of the positive effects of taxation on evasion goes back to Allingham and Sandmo (1972). In this paper, we analyze how the principles of efficient tax design must be revised to take account of the differential ease with which various taxes can be evaded. We do so in a simple context in which there is a tax applying on a narrow base that is more difficult to evade than one on a broad base. This way of formulating the problem sheds light on the classical issue of when uniform taxation should be supplemented by differential commodity taxes. As well, our analysis makes clear that in the presence of tax evasion, there is a trade-off in optimal tax design. That trade-off is between the efficiency cost of tax distortion and the efficiency cost of tax evasion.

The existing literature on tax design in the presence of tax evasion has two main thrusts. The first of these exploits the idea that the choice of a tax mix can be motivated by tax evasion considerations. If an otherwise ideal tax base can be evaded, obtaining some revenues from a parallel tax base that overlaps to some extent can mitigate the problem. Thus, Boadway, Marchand and Pestieau (1994) analyze the case of a direct tax used for redistributive purposes, and show how the possibility of evasion of that tax can lead to an argument for a commodity tax system, perhaps with differential rates. Cremer and Gahvari (1993) conduct a similar exercise for different commodity taxes. A drawback to these papers is that they introduce tax evasion in a rather crude way by defining an ad hoc cost-of-evasion function. Thus, the fundamental feature of tax evasion — its riskiness to the taxpayer — is suppressed. It is precisely the cost of risk-taking that conditions evasive behavior in the Allingham-Sandmo approach, and one we would expect to be important from a tax design perspective.

The second approach in the literature is to incorporate the possibility of tax evasion into the standard model of optimal redistribution under asymmetric information due to Mirrlees (1971). The emphasis here is on how the inability of the government to monitor income perfectly (unlike in the Mirrlees case) compromises its ability to redistribute. In this case, optimal policy is modeled using the Revelation Principle, following the standard optimal income tax methodology. Both the tax structure and the penalty structure are chosen so that households are induced to reveal their true incomes. Thus, there is no evasion in the optimum, and therefore no costs of risk-taking. The issue of how to design the tax structure to minimize evasion does not arise. See, for example, Cremer and Gahvari (1996), Marhuenda and Ortuno-Ortin (1997), and Chandar and Wilde (1998).[1]

In this paper, there is tax evasion in the optimum, and we incorporate explicitly the cost of risk-taking resulting from the decision to evade taxes. As a result, we are able to identify a fundamental trade-off in efficient tax design between avoiding tax distortions and avoiding tax evasion. It is important to be explicit about the standing of the cost of risk-taking in our analysis. Even though the risk comes about by the representative agent's decision to evade taxes illegally, nonetheless we treat it as a source of welfare loss. It is in fact the government that produces risk opportunities by employing random auditing, a strategy that is necessitated by cost considerations. As in Allingham and Sandmo, individuals respond rationally to these risk opportunities by evading taxes, and in so doing incur a cost of risk-taking. We treat this as an efficiency cost associated with the tax system. It might be argued that fighting evasion activities is an objective the government should pursue as such, and that the costs of risk-taking should not be afforded welfare status on the grounds that they are illegal. This is obviously a matter of judgment. We adopt the position in this paper that the objective of an efficient tax system is to obtain revenue in a way that imposes the least welfare cost on households. Just as tax avoidance reduces household utility by changing consumption patterns, so the risk to which tax evasion activity gives rise is a source of utility loss. It represents a private cost that is socially wasteful.[2] The analysis will make it clear how optimal tax design can mitigate the cost of risk-taking induced by random auditing, albeit at the expense of introducing distortions on the taxpayer's behavior.

This point also sheds some light on the popular call for simplicity in taxation. Taken at face value, it is not clear why simple taxation should be beneficial, given that it works against equity and the efficiency gain is anything but clear. The present paper suggests the following view of simplicity. A simple tax is one for which tax liability is easily calculated and hence

[1] An exception to this is the recent analysis by Boadway and Sato (2000) where the Revelation Principle may fail because of the fact that detection involves errors, either by the taxpayers or by the tax administrators. However, even with the possibility of errors, the Revelation Principle will hold if the government has full freedom to include rewards for truthful reporting in the penalty structure.

[2] This position has also been adopted by Yitzhaki (1987). He argues that tax evasion generates a social cost which adds to the excess burden of tax distortion, although he does not analyze the optimal trade-off between these costs. See also Cowell (1990).

sure, while for one lacking simplicity the tax liability is not obvious and thus uncertain. If a tax lacks simplicity, the taxpayer suffers from bearing a risk with respect to the final tax liability. Optimal tax design should keep such risk small. For this purpose, it may even pay to accept small tax distortions. In this sense, there is a trade-off between tax simplicity and tax efficiency.

Modeling tax evasion as a risky activity clearly complicates matters. To keep the analysis tractable, we adopt simplifications that earlier studies of tax design under evasion could avoid. Unlike Cremer and Gahvari (1993), we assume auditing is exogenous. However, we do specify a fairly flexible formulation of the nature of penalties in case of detection, one that is more general than in the existing literature. Unlike Boadway, Marchand and Pestieau (1994), our analysis ignores the equity objective. Our model is the one of a representative household so focuses solely on efficient tax design. Their analysis, which studies the mix of direct and indirect taxation in a heterogeneous-agent setting, can be viewed as a qualification to Atkinson and Stiglitz (1976). Ours studies deviations from proportionality because of tax evasion and can be viewed as a qualification of Atkinson and Stiglitz (1972).

The paper is organized as follows. Section 2 presents a simple model of tax evasion, whose features are chosen so that in the absence of evasion, income taxation is efficient. Section 3 analyzes household behavior, with respect to both the choice of commodities and tax evasion. Section 4 considers the problem faced by government and derives a general result on the efficiency of introducing a distortionary tax instrument specifically in response to income tax evasion. Section 5 analyzes in detail the problem when the labor supply decision is suppressed. In this case the trade-off becomes most transparent. Section 6 illustrates the structure of optimal taxation for a special case. Section 7 summarizes and draws some conclusions. Proofs are relegated to the Appendix.

2. A Simple Model with Tax Evasion

Households in this economy are identical, allowing us to abstract from redistributive considerations. The representative household supplies labor L and consumes two consumption goods—C, a composite numeraire commodity, and X, a specific commodity that can be taxed. Producer prices of C and X are fixed at unity and p respectively, and the wage rate w is also normalized to unity by choice of labor units. Good X can be subject to an excise tax at the per unit rate a, and labor income is taxed at the proportional rate t. We assume for convenience that the numeraire C is untaxed, although we could carry out the analysis assuming that the general tax is on consumption rather than labor income, provided it is possible to evade either tax. There is no possibility of a lump-sum tax by assumption, since in this simple economy it would dominate all other taxes (unless it too could be evaded). By its nature, the tax on X cannot be evaded. For example, it might be on a commodity, like petrol, whose transactions can be readily monitored. However, labor income taxation can be evaded (or equivalently, a general consumption tax could be evaded).

Let q be the proportion of labor income L that is either not reported or is earned in the underground economy at the going wage rate.[3] The income tax t then applies to reported income $(1-q)L$. Evaded income qL may be detected in part or in whole. For the part that is detected, the household bears a pre-determined penalty. To reflect the full range of outcomes that can occur, we adopt a very general formulation for the amount of income that is detected and the penalty imposed. Let $\tilde{z}$ be the stochastic share of evaded income that accrues to the household, where a tilde is used to indicate stochastic variables. Assuming there are no variable costs of detection, the share $1-\tilde{z}$ of a given household's income is confiscated by the tax authorities. It is assumed that the distribution of $\tilde{z}$ is given, reflecting the simplifying assumption that both the probability structure of detection and the penalty are exogenous. As a working assumption, we assume that there is no aggregate risk to the government — the proportion of evaded income detected in the economy as a whole is known. This implies that the only risk associated with tax evasion is that borne by individual taxpayers, a risk that is explicitly taken into account in our analysis.[4] This formulation covers the standard case in which detection of an evading individual reveals all of the income that has been evaded. In this case, nature takes on two states as in Sandmo (1981), Yitzhaki (1987) and Cowell (1990). In one state, tax evasion is successful and the household collects qL from its shadow activity. In the other state, all tax evasion is detected and a penalty is imposed. If the fine is a proportion f of the evaded income, $\tilde{z}$ takes on two values, unity and $1-f$.

Given L, q and $\tilde{z}$ as well as the income tax t, household net or disposable income is

[3] In a more general analysis, we could allow the wage to differ in the market and underground sectors, perhaps compensating for the risk associated with illegal activity. That would complicate the analysis considerably and obscure the point we are trying to make. By the same token, we could allow there to be some evasion of commodity X, though less than for labor income. We have adopted extreme assumptions to make the analysis as clear as possible.

$$\tilde{W}L = [(1-q)(1-t) + q\tilde{z}]L = [(1-t) + q\tilde{\Delta}]L \tag{1}$$

where $\tilde{\Delta} \equiv \tilde{z} - (1-t)$ is the (stochastic) net rate of return per unit of evaded income. Given disposable income $\tilde{W}L$, the household's budget constraint can be expressed as

$$\tilde{C} + P\tilde{X} = \tilde{W}L. \tag{2}$$

where $P = p + a$ is the consumer price of X. Note that since income is stochastic, so is the consumption of goods C and X.

To simplify our analysis and facilitate comparison with the no-evasion case, we assume that utility is additively separable in L and homothetic in C and X. As is well known, under these conditions, uniform taxation of consumption of labor income is optimal. That is, $t > 0, a = 0$ (Atkinson and Stiglitz 1972, Sandmo 1974). Moreover, we want to assume that the household is risk averse, so we adopt the following cardinal representation of utility:

$$U(\Phi(C, X)) - D(L) \tag{3}$$

where $\Phi(C, X)$ — an index of real consumption — is linear homogenous, $U(\cdot)$ is increasing and strictly concave $(U' > 0 > U'')$, and the disutility of labor function $D(L)$ is increasing and strictly convex $(D', D'' > 0)$. Note that the homotheticity of utility in C and X implies that the ratio C/X depends solely on the relative consumer price P and not on income. We exploit that characterization in what follows. Households are assumed to obey the expected utility hypothesis so solve the following problem:

$$\underset{\tilde{C}, \tilde{X}, L, q}{\text{Max}} \; EU(\Phi(\tilde{C}, \tilde{X})) - D(L) \quad \text{s.t. } (2) \tag{4}$$

The government obtains revenue from three sources — the income tax t, the excise tax a, and its share $1 - \tilde{z}$ of evaded income. Government expected revenue per capita can be written:

$$(1-q)tL + a\overline{X} + (1-\overline{z})qL \tag{5}$$

[4] Alternatively, we could assume that the government is risk-neutral and cares only about expected revenue, though this begs the question of how the government acting as a agent of the taxpayers sheds risk. Our analysis and results apply fully in either case.

where $\overline{X} = E\widetilde{X}$ and $\overline{z} = E\widetilde{z}$ are expected outcomes. With no aggregate risk, these are certain, but as mentioned our analysis applies more generally if the government is risk-neutral.

Decisions and events take place sequentially in this economy, and it is useful to be explicit about them. As is usual in optimal tax analysis, the government chooses its policies first, anticipating household behavior. The government is assumed to be able to commit to its announced policies. Households then act in two steps. In the first, they choose labor supply *L* and the proportion *q* of their income to report. Detection then occurs, so taxes and penalties are paid and actual *z* is determined. Given *z*, disposable income is known, and in the second step households decide how to allocate it between C and *X*. In our analysis, it is useful to treat these two steps sequentially. In fact, since under homotheticity *C/X* depends only on *P*, it is not crucial to assume that the state of detection is revealed before the household budget is allocated: the household chooses the same *C/X* ratio regardless of the amount of disposable income $\widetilde{W}L$ that is revealed. However, it is convenient for purely pedagogical purposes to suppose that household consumption is determined ex post.

The sequence of decisions taken by the government and the representative household can then be summarized as follows:

Stage 1 <u>Government policies</u>: Given $\overline{z}$, the government chooses $\{t, a\}$ to maximize its expected revenues subject to a given level of expected utility for households,[5] anticipating how $\{t, a\}$ affects household behavior.

Stage 2 <u>Household labor supply and evasion</u>: The household takes $\{t, a, \widetilde{z}\}$ as given and chooses $\{L, q\}$ to maximize EU, anticipating how disposable income will be allocated to *C* and *X* in Stage 3. This yields household supply functions $L(t, a; \widetilde{z}), q(t, a, \widetilde{z})$.

Stage 3 <u>Household budget allocation</u>: The extent of detection has been revealed so net income *WL* is now given. The household chooses $\{C, X\}$ to maximize real consumption $\Phi(C, X)$ subject to its budget $C + PX = WL$.

[5] The solution to this problem is equivalent to its dual of maximizing expected utility of the representative household subject to a government revenue constraint. For expositional purposes, it is more convenient to proceed as in the text.

The problem is solved by backward induction beginning with Stage 3. The next section treats the two steps in the household budget. In section 4, we turn to government policy.

3. Household Behavior

We begin first with Stage 3 and then go back to Stage 2.

Stage 3: Budget Allocation

The detection state and therefore *z* have been revealed. Given *WL* from Stage 2, the representative household's budget allocation problem, using budget constraint (2), is:

$$\underset{X}{\text{Max}} \quad \Phi(WL - PX, X) \tag{S3}$$

The first-order condition is:

$$\Phi_X = P\Phi_C \tag{6}$$

It is well known that for a homothetic function like $\Phi(C, X)$, $\Phi_X / \Phi_C = F(C/X)$; that is, the marginal rate of substitution depends only on *C/X* regardless of the level of disposable income. Defining $c \equiv C/X$ as the consumption ratio, (6) may be written $F(c) = P$. Inverting this expression, we obtain

$$c = F^{-1}(P) \equiv f(P)$$

Given that $\Phi(C, X)$ is linear homogeneous, it is straightforward to show that:[6]

$$\frac{dc}{dP} = f'(P) = -\frac{\Phi_C^2}{\Phi\Phi_{CC}}\Big|_{c=f(P)} > 0\,. \tag{7}$$

Moreover, linear homogeneity also implies that:[7] $\Phi_C(C, X) = \Phi_c(c,1) = \Phi_c(f(P),1) \equiv \varphi(P)$.

Differentiating this by *P*, we obtain $\varphi'(P) = \Phi_{CC}f' = -\Phi_C{}^2/\Phi$. Therefore, for later reference,

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[6] By Euler's Theorem, $X\Phi(c,1) = C\Phi_C(c,1) + X\Phi_X(c,1)$. Condition (6) may be written $\Phi_C(c,1) = P\Phi_X(c,1)$, or using the previous equation, $\Phi(c,1) = (P + c)\Phi_C(c,1)$. Differentiating with respect to *P* and *c*, we obtain (7).

$$\frac{\varphi(P)}{\varphi'(P)} = -\frac{\Phi}{\Phi_C} = -\frac{\Phi_X}{\Phi_C} - c = -(P + f(P)). \quad (8)$$

Using this definition of $\varphi(P)$, the household's consumption level, given the optimal choice of $\{C, X\}$ at this stage, can be written:

$$\Phi(C, X) = C\Phi_C + X\Phi_X = \varphi(P)(C + PX) = \varphi(P)WL$$

Thus, $\varphi(P)WL$ can be thought of as a maximum value function resulting from Stage 3.

Stage 2: Labor Supply and Evasion

At this stage, the detection state has not yet been revealed, so disposable income $\tilde{W}L$ is stochastic. Using the anticipated outcome of Stage 3, the problem of the household is:

$$\underset{L,q}{\text{Max}} \quad EU\left(\varphi(P)\tilde{W}L\right) - D(L) \quad \text{(S2)}$$

Assuming an interior solution (see below), the first-order conditions with respect to *q* and *L* may be written, after some simplification:

$$E\left[U'(\cdot)\tilde{\Delta}\right] = 0 \quad (9)$$

$$\varphi(P)(1-t)E\left[U'(\cdot)\right] = D'(L) \quad (10)$$

The second-order conditions are:

$$E\left[U''(\cdot)\tilde{\Delta}^2\right] < 0 \quad (11)$$

$$\varphi^2(P)(1-t)E\left[U''(\cdot)\tilde{W}\right] < D''(P) \quad (12)$$

Two comparative static properties of the household's problem are worth noting. Suppose that the utility function exhibits decreasing absolute risk aversion (DARA) so $-U''/U'$ is

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[7] By linear homogeneity, $\Phi(C, X) = X\Phi(c,1)$. Differentiating by *C*, the expression in the text follows.

decreasing in its argument, and increasing relative risk aversion (IRRA) so $-\varphi(P)WLU''/U'$ is increasing. Then, the following properties apply:

$$E\left[U''(\cdot)\tilde{\Delta}\right] \geq 0 \tag{13a}$$

$$E\left[U''(\cdot)\tilde{\Delta}\tilde{W}\right] \leq 0\,. \tag{13b}$$

Equation (13a) follows from DARA, while (13b) follows from IRRA.[8]

In the above discussion, we have assumed an interior solution for q. Given that the household is risk averse, this will only be the case if the return from evasion is better than a fair bet, that is, $\bar{z} > 1-t$. A positive value for q will be chosen if and only if this condition is satisfied (Sandmo 1981, (18), or Yitzhaki 1987, (4)). If $\bar{z} \leq 1-t$, it does not pay to bear the risk of tax evasion, so q is set optimally at zero.[9] For $q = 0$, $\tilde{W} = W = 1-t$, so the individual faces no risk. In this case, the problem is the standard optimal commodity tax one with two goods and leisure. As mentioned above, for a utility function of the form (3), the optimal tax on goods is proportional, so $a = 0$. In what follows we assume $\bar{z} > 1-t$ and focus on interior solutions with $q \in (0,1)$.

4. The Government Problem

In Stage 1, the government foresees household behavior as characterized by problem (S2) and the associated first-order conditions (9) and (10), where $c = f(P)$ from Stage 3. Government revenue per capita, given by (5), depends on household choice of $\{L, q\}$ in Stage 2, as well as expected commodity purchases in Stage 3, $E\tilde{X} = \bar{X}$. The latter can be rewritten as follows, using the household budget constraint $\tilde{W}L = \tilde{C} + P\tilde{X} = \left(f(P) + P\right)\tilde{X}$ and (1):

$$E\tilde{X} = \frac{L}{P + f(P)}E\tilde{W} = \frac{L}{P + f(P)}\left[(1-q)(1-t) + q\bar{z}\right]$$

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[8] For example, the proof of (13a) relies on the observation that: $EU''\cdot\tilde{\Delta} = -E\left(-U''/U'\right)U'\cdot\tilde{\Delta}$. By (9) this expression equals zero if $-U''/U'$ is constant. What remains to observe is that $E(-U''/U')U'\cdot\tilde{\Delta}$ is non-positive if $-U''/U'$, being a decreasing function, puts less weight on large values of $U'\cdot\tilde{\Delta}$. Similarly Arrow (1970) shows that (13b) follows from increasing relative risk aversion (IRRA).

[9] This assumes that over-reporting is not rewarded, which is the case in practice.

The government maximizes its revenue per capita subject to some given level of household expected utility and the household choice of L and q. The problem can be written as follows:[10]

$$\underset{\{t,a,L,q\}}{\text{Max}}\left[(1-q)t+(1-\bar{z})q+a\frac{(1-q)(1-t)+q\bar{z}}{p+a+f(p+a)}\right]L \tag{S1}$$

subject to (9), (10) and

$$E\left[U\left(\varphi(p+a)\tilde{W}L\right)\right]-D(L)=u \tag{14}$$

where u is the given level of expected utility.

The solution to this problem yields the optimal — that is, efficient —tax policy $\{t,a\}$. Characterizing that policy is not straightforward. However, we are able to derive the following general result, whose proof is relegated to the Appendix:

Proposition 1: If preferences satisfy DARA and IRRA, it is efficient to set $a > 0$ if $q > 0$.

This is in contrast to the result, mentioned above, that if $q=0$, it is efficient to set $a=0$ and to tax consumption uniformly. Proposition 1 can therefore be interpreted as saying that there is a trade off in the design of efficient taxation. The wage tax is more broadly based but it suffers from tax evasion by assumption. The specific excise tax, on the other hand, cannot be evaded but it imposes an efficiency cost relative to the optimal uniform tax. Increasing a incrementally above zero is welfare-improving because it imposes no first-order efficiency cost. Eventually, the cost of the additional distortion imposed by the specific tax has to be traded off against the cost of evasion. The nature of this trade-off can most easily be seen if we concentrate on the special case of a fixed labor supply. This case simplifies matters considerably because, while the wage tax causes tax evasion, it imposes no distortion.

[10] Note that we are treating L and q as artificial variables by the government, and adding as constraints the household's first-order conditions (9) and (10). Alternatively, we could simply treat the solutions of (9) and (10) — $L(t,a), q(t,a)$ — as being given to the government.

5. Optimal Taxation When Labor Supply is Fixed

If labor supply is given, we may delete constraint (10) from the tax-planner's problem.[11] We then obtain the following proposition, which does not rely on either DARA and IRRA. Again the proof is relegated to the Appendix.

Proposition 2: If labor supply is fixed, it is efficient to set *a > 0* if *q > 0*.

The generality of Proposition 2 is striking. All that it requires is strict risk aversion. However, the explanation of this result is far from trivial. As we shall show, the response of individuals' evasion behavior to changes in *a* and *t* is in fact indeterminate without imposing stronger conditions on the utility function than risk aversion. Therefore, the rationale for the proposition is not simply to fight evasion. Instead, we show that the rationale lies in the trading off of social costs associated with tax evasion on the one hand, and tax distortions on the other.

To see this, we begin with behavioral response. Let *q*(a,*t*) be the solution to problem (S2) when labor supply is fixed: that is, it solves (9). Implicit differentiation of (9) yields:

$$\frac{\partial q}{\partial a} = -\frac{\varphi' EU'' \cdot \tilde{W}\tilde{\Delta}}{\varphi EU'' \cdot \tilde{\Delta}^2} \qquad \text{and} \qquad \frac{\partial q}{\partial t} = -\frac{EU'/\varphi L - (1-q)EU'' \cdot \tilde{\Delta}}{EU'' \cdot \tilde{\Delta}^2}$$

Evasion is non-decreasing in *a* ($\partial q / \partial a \leq 0$) if IRRA, or (13b), applies.[12] The sign of $\partial q / \partial t$ is even less determinate. Even if we assume DARA, or (13a), its numerator cannot be signed without ambiguity.[13] Hence little can be said about the effect on *q* of an exogenous increase of *a* combined with a decrease/increase of *t*.

The interpretation of the solution to the government's problem (S1) involves identifying the effects of utility-compensated changes in the taxes *a* and *t*, since expected utility is being held constant in solving for optimal taxes. However, the reaction of *q* to a compensated increase of *a* is equally indeterminate in sign. Define the household's expected utility resulting from its

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[11] While labor supply might be literally fixed, households will in fact choose a constant labor supply in the special case where $U(\Phi) = \log \Phi$. In this case, the first-order condition (10) reduces to *D'*(*L*) = 1/*L*. This utility function is exploited in the example of the next section.

[12] The non-negativity of *∂q/∂a* is a result that can be traced back to Domar and Musgrave (1944). They showed that taxing the return from risky investments might encourage risk-taking. In the present context, the measure of risk taking is *q*, and the net return to risk taking is $\varphi L\tilde{\Delta}$. Since $\varphi'(p+a) < 0$, the return is reduced by increasing the tax rate *a*.

optimization problem as[14] $V(a,t) \equiv E\left[U\left(\varphi(p+a)\tilde{W}L\right)\right]$, where $\tilde{W} = (1-q(a,t))(1-t) + q(a,t)\tilde{z}$. Compensating the household for an increase in *a* requires the following decrease in *t*:

$$\left.\frac{dt}{da}\right|_{V=\text{const}} = -\frac{\partial V/\partial a}{\partial V/\partial t} = \frac{(1-t)\varphi'}{(1-q)\varphi}$$

where we have used (9) in the last step. Using this expression, the utility-compensated effect of a change in *q* can be obtained by differentiating $q(a,t)$ with respect to *a* to give, after simplifying:

$$\left.\frac{dq}{da}\right|_{V=\text{const}} = \frac{\partial q}{\partial a} - \frac{\partial q}{\partial t}\frac{\partial V/\partial a}{\partial V/\partial t} = -\frac{\varphi'}{\varphi}\left[q + \frac{1-t}{(1-q)\varphi L}\frac{EU'}{EU''\cdot\tilde{\Delta}^2}\right] \tag{15}$$

Note that the two terms in brackets are of opposing signs. We can summarize the above results in the following proposition:

Proposition 3: Both uncompensated and compensated changes in policies $\{a,t\}$ have ambiguous effects on tax evasion *q*.

We must look elsewhere for the intuition of the general result of Proposition 2. To do so, we investigate the social benefit and social cost of an increase in the excise tax *a*. The social benefit consists of its effect on the cost of risk-taking induced by the wage tax, while its social cost is the standard tax distortion. These two effects are as follows.

i) Benefit of the excise tax: reduced cost of risk

Tax evasion gives rise to private risk. Its cost is the maximum premium the household would be willing to pay for getting rid of the risk. This risk premium $\Pi = \Pi(a,t)$ is implicitly defined by setting

$$U(\varphi LE\tilde{W} - \Pi) = EU(\varphi L\tilde{W}).$$

For a utility-compensated increase in *a*, the right-hand side remains constant. Hence

$$\left.\frac{d\Pi}{da}\right|_{V=\text{const}} = \left.\frac{d}{da}(\varphi LE\tilde{W})\right|_{V=\text{const}} = \frac{\bar{z}-(1-t)}{p+a+f}\,\frac{1-t}{1-q}\,\frac{EU'}{EU''\cdot\tilde{\Delta}^2} < 0 \tag{16}$$

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[13] For constant absolute risk aversion, $\partial q/\partial t$ has the expected positive sign.

[14] Note than we can ignore the disutility of labor in this case since *L* is being held constant.

Thus, a utility compensated increase in *a* reduces the cost of private risk borne by the household. This is the benefit of the excise tax.

ii) Cost of the excise tax: increase in tax distortion

The excise tax distorts consumption choice in Stage 3 of the household's problem. Recall that the maximum value function from Stage 2 was $\varphi(P)WL$, where at this stage disposable income was already determined. Let *C(P,u)* and *X(P,u)* be compensated demand functions obtained from the dual to the Stage 3 problem. They are obtained as solutions of

$$\Phi(C, X) = u \quad \text{and} \quad \frac{\Phi_X(C, X)}{\Phi_C(C, X)} = P.$$

Since Φ is linear homogenous, the compensated demand for *X* is given by

$$X(P,u) = u / \Phi(f(P),1)$$

where *f(P)* has been defined before. The substitution effect is clearly negative,

$$\frac{\partial X(P,u)}{\partial P} = -uf'\Phi_C/\Phi^2 = -Xf'\Phi_C/\Phi \underset{(8)}{=} -Xf'/(p+a+f) < 0 \qquad (17)$$

Equation (17) is derived from the Stage 3 problem when detection, and therefore disposable income have been determined. From the point of view of the effect of government policy, it is the ex ante effect that is relevant since policies are undertaken before the outcome of detection is revealed. Given the assumed linear homogeneity property of the sub-utility function $\Phi(C, X)$, the substitution effect associated with expected sub-utility $\bar{u} = \varphi LE\tilde{W}$, or $\partial X(P,\bar{u})/\partial P$, takes the same form as (17). The utility-compensated effect of *a* — with the compensation taking the form of a reduction in *t* — is then given by:

$$\frac{d}{da} X(p+a, \varphi LE\tilde{W})\Big|_{V=\text{const}} = \frac{\partial X}{\partial P} + \frac{\partial X}{\partial u}\frac{d}{da}(\varphi LE\tilde{W})\Big|_{V=\text{const}}$$

$$= \frac{\partial X}{\partial P} + \frac{1}{\Phi}\frac{d\Pi}{da}\Big|_{V=\text{const}} < 0. \qquad (18)$$

We are now able to define the social cost of tax distortion. For a marginal and utility compensated increase of *a* the marginal social cost of distortion is

$$-a\,\frac{d}{da}\,X(p+a,\varphi LE\tilde{W})\Big|_{V=\text{const}} > 0\,.$$

Therefore, while an increase in *a* reduces the cost of risk-taking, it also increases the tax distortion.

Optimality in the choice of the excise tax *a* involves trading off the benefit of a reduced cost of risk-taking against the cost of an increase in the tax distortion on consumption. In fact, as is shown in the Appendix, at an optimum the trade-off can be made explicit as follows:

Proposition 4: A necessary condition for the optimal choice of $\{a,t\}$ satisfies

$$a\frac{dX}{da}\Big|_{V=\text{const}} = \frac{1}{\varphi}\,\frac{d\Pi}{da}\Big|_{V=\text{const}}\,. \tag{19}$$

(Division by φ is needed to transform units of sub-utility in units of income.)

It ought to be stressed that Propositions 2, 3 and 4 go through for any strictly concave utility functions *U*. However, the assumption of exogenous labor supply is critical. If *L* is variable, income effects complicate the analysis. Labor supply reacts to variations in its return, $\varphi\tilde{W}$. A utility-compensated increase in *a* reduces the expected return as shown by (16). It is straightforward to show that a utility-compensated increase of *a* also reduces the variance of labor return, var $(\varphi\tilde{W}) = (\varphi q)^2$ var $\tilde{z}$:

$$\frac{d(\varphi q)}{da}\Big|_{V=\text{const}} = \varphi' q + \varphi\frac{dq}{da}\Big|_{V=\text{const}} \underset{(14)}{=} -\frac{\varphi'(1-t)}{\varphi(1-q)L}\,\frac{EU'}{EU''\cdot\tilde{\Delta}^2} < 0\,.$$

Reduced variance partly compensates for reduced expectation. Reductions in labor remuneration produce two effects with which we need not cope when labor supply is exogenous. First, there is an efficiency loss due to the distorted labor supply. Second, risk tolerance is affected by a wealth effect. The choice of *q* responds to changes in *L*. Hence, it is hardly surprising that Proposition 1 requires more restrictive assumptions than Proposition 2. Nonetheless, the additional assumptions needed (DARA, IRRA) are standard.[15]

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[15] As noted earlier, some might argue that the risk associated with tax evasion should not be treated as a welfare cost, given the illegality of tax evasion. Ignoring the cost of risk is equivalent to setting the right-hand side of (19) to zero. In this case with fixed labor supply, it follows immediately that *a=0* regardless of how risk averse

6. Illustrative Example

The optimal tax rule (19) can be used to compute optimal tax structures. To illustrate, consider the special case with $U(\Phi(C, X)) = \log(\Phi(C, X))$. As noted earlier, labor supply is constant in this case so (19) applies. To simplify the example further, suppose as in the standard case that if evasion is detected, all evaded income is revealed. Let δ be the probability of detection. The penalty for detection is that all under-reported income is confiscated, so $\tilde{\Delta} = \tilde{z} - (1-t) = -(1-t)$ with probability δ, and $\tilde{\Delta} = 1-(1-t) = t$ with probability $1-\delta$. Evaluating the first-order condition (9) of the household's tax evasion problem yields $q = 1 - \delta / t$. Using this, the expected net wage rate becomes

$$E\tilde{W} = (1-t) + (t-\delta)^2 / t \,.$$

Note that neither $E\tilde{W}$ nor q depend on the specific tax rate a in this example. Substituting $E\tilde{W}$ into (19) and using (17) and (18) gives us

$$\frac{aXf'}{P+f} = (aX - LE\tilde{W})\frac{\varphi'}{\varphi} + \left(\frac{aX}{E\tilde{W}} - L\right)\frac{d}{dt}E\tilde{W} \cdot \left.\frac{dt}{da}\right|_{V = \text{const}}$$

Using $\left.\frac{dt}{da}\right|_{V = \text{const}} = \frac{\varphi'(1-t)}{\varphi(1-q)}$ that was derived earlier, as well as (8), and $\delta = t(1-q)$, we obtain, after some straightforward computations:

$$\frac{aX}{L} = S \tag{19'}$$

where S is defined by

$$S \equiv \left[\left(1 - t(1-q^2)\right)f' + \left(1 - t(1-q)\right)q\right]^{-1}\left[\left(1 - t(1-q)\right)q\left(1 - t(1-q^2)\right)\right].$$

The left-hand side of (19') is the optimal specific tax revenue as a share of gross wage income. The right-hand side of (19') depends on a only via $f'(p+a)$. Assuming Cobb-Douglas

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the household is, and therefore how responsive is the household's evasion to the income tax rate. In the more general case with variable labor supply, it turns out that a will deviate from zero in the optimum when the government gives no welfare weight to the risk of evasion.

preferences $\Phi = C^{\gamma} X^{1-\gamma}$ allows us to eliminate the dependence of *S* on *a*. In this special case $f' = \gamma/(1-\gamma)$ so that

$$S \equiv [(1-t)(1-q)\gamma + (1-t(1-q))q]^{-1}[(1-t(1-q))q(1-t(1-q^2))(1-\gamma)].$$

The following diagrams show *S* for selected values of γ = 0.9, 0.95, 0.99 as function of *q* and *t*. Large values of γ indicate small expenditure shares for *X*, 1-γ. For the chosen parameters, the optimal specific tax share *S* takes on values that remain below 5.3 percent. As one would expect, *S* increases monotonically in 1-γ and *q*. The fact that *S* decreases in *t* is counterintuitive and needs to be explained. The explanation is that *q* and *t* are not independent. Only *t* is exogenous, whereas $q = 1 - \delta / t$ depends on *t*. Hence the diagrams cannot be used for comparative statics. If we hold *q* fixed and increase *t*, we implicitly decrease the detection probability δ . This effect obviously compensates for the increase in *t*.

The interesting parameter is *q*. It can be approximated by the size of the shadow economy as percent of GDP. Empirical estimates are surveyed by Schneider and Enste (2000). They strongly depend on the employed estimation method. Estimates for *q* of up to 0.3 and more do not seem to be unrealistic even for OECD countries. The estimates for some developing countries show an underground sector that is nearly three-quarters the size of officially recorded GDP.

[include diagram here]

The optimal share of specific tax revenue, *S = aX/L*.

7. Conclusions

Evading taxes is a risky activity that arises because of government tax enforcement policies: tax evaders run the risk of being detected and punished. The risk is a private one, but it gives rise to a social cost. Unfortunately, unlike with other purely private risks, the risk of tax evasion cannot easily be eliminated. To do so would require implementing taxes that cannot be evaded. That would be undesirable since taxes that cannot be evaded are likely to be narrow ones that impose significant distortions. The government is therefore faced with trading off the costs of risk borne by taxpayers who rationally decide to engage in the tax evasion lottery against the costs of distortion arising from choosing the tax mix so as to reduce the opportunities for evasion. The analysis of this paper shows how these two costs must be balanced in the optimum. The optimal trade-off is illustrated using a specific model in which a broad-based income tax is efficient but prone to evasion, but a narrow-based distortionary excise tax that cannot be evaded is available. In this context, the existence of risk aversion alone is sufficient to warrant introducing the excise tax when income tax evasion is positive. Moreover, the exact scope of tax evasion is irrelevant: the result holds however small the proportion of income that is evaded. The intuition is that introducing the excise tax initially produces a second-order welfare cost, while at the same time inducing a first-order reduction in the private cost of tax evasion.

In formulating our argument, our presumption has been that narrow tax bases may be more difficult to evade than broad ones, and we have interpreted our findings as casting doubt on the standard arguments for uniform commodity or income taxation when appropriate separability conditions apply to utility functions. It is apparent more generally that alternative tax bases will differ not only in their distortionary effect but also in the ease with which they may be evaded. Fully efficient taxation must take account of the incremental effects each type of tax has on both the cost of distortions and the costs of risk-taking. When all taxes can be evaded to some extent, the analytical task is challenging. It would become even more so if households were heterogeneous.

Our task was made much simpler, and our results much sharper, by the various simplifying assumptions we have made. One of the most crucial is that utility be additive separable in labor and homothetic in consumption. This not only ensured that the benchmark case with no evasion was uniform taxation. It also simplified the analysis. The extension to the case where utility is only weakly separable in labor would presumably be straightforward. Propositions 2-4 will clearly continue to apply, since labor is assumed fixed. One may conjecture that Proposition 1 will also apply, since weak separability still ensures that the benchmark case entails uniform taxation. The proofs would certainly be more complicated. However, Propositions 1 and 2 will clearly not be relevant when weak separability and homotheticity are violated. In this case, a will generally be non-zero as part of the Ramsey optimal tax system. One then expects that tax evasion considerations would imply that the optimal value of a

would exceed the Ramsey value—and may even cause it to change sign—but the analysis would be exceedingly complex.

Appendix

We solve for the government's problem (S1), associating the Lagrange multipliers $-\lambda$, υ, and $-\mu$ with the constraints (9), (10), and (14), respectively. The first-order conditions with respect to *t* and *a* gives:

$$\frac{p+f}{p+a+f}\,\frac{1}{\varphi} \;=\; \lambda\left[\frac{EU'}{(1+q)L\varphi} - EU''\cdot\tilde{\Delta}\right] \;+\; \upsilon\left[\varphi(1-t)EU'' + \frac{EU'}{(1-q)L}\right] - \mu EU',$$

$$\frac{p+f+af'}{(p+a+f)^2}\,\frac{(1-q)(1-t)+q\bar{z}}{(1-t)\varphi'} \;=\; \lambda\left[EU''\cdot\tilde{\Delta} + \frac{q}{1-t}EU''\cdot\tilde{\Delta}^2\right]$$

$$-\upsilon\left[\varphi EU''\cdot\tilde{W} + \frac{EU'}{L}\right] + \mu EU'$$

Eliminating $\mu EU'$ and using (8), we obtain

$$\frac{af'}{p+a+f} - \frac{q}{1-t}\left[\bar{z}-(1-t)\right]\frac{p+f-af'}{p+a+f} \tag{20}$$

$$=\quad \lambda\left[\frac{q\varphi}{1-t}EU''\cdot\tilde{\Delta}^2 + \frac{EU'}{(1-q)L}\right] \quad+\quad \upsilon\varphi q\left[\frac{EU'}{(1-q)L} - \varphi EU''\cdot\tilde{\Delta}\right]$$

Taking the partial derivative with respect to *q* yields

$$\left[\bar{z}-(1-t)\right]\frac{p+f}{p+a+f} \;=\; -\lambda\varphi EU''\cdot\tilde{\Delta}^2 + \upsilon\varphi^2(1-t)EU''\cdot\tilde{\Delta} \tag{21}$$

By making use of (21) we are able to write (20) in the form of

$$\frac{af'}{p+a+f}\left\{1+\frac{q}{1-t}\left[\bar{z}-(1-t)\right]\right\} \;=\; \frac{\lambda+\upsilon\varphi q}{(1-q)L}EU'\,. \tag{22}$$

Solving (21) for λ and inserting into (22) yields

$$\frac{af'}{p+a+f}\,\frac{E\tilde{W}}{1-t} \;=\; -\,\frac{p+f}{p+a+f}\,\frac{\bar{z}-(1-t)}{\varphi(1-q)L}\,\frac{EU'}{EU''\cdot\tilde{\Delta}^2}$$

$$+\;\upsilon\varphi\,\frac{EU''\cdot\tilde{\Delta}\tilde{W}}{EU''\cdot\tilde{\Delta}^2}\,\frac{EU'}{(1-q)L}\,. \tag{23}$$

The case of exogenous labor supply is captured by setting $\upsilon \equiv 0$. Proposition 2 follows from (23) by noting

$$\bar{z} > 1-t,\;\; q < 1,\;\; \varphi > 0,\;\; f' > 0,\;\; p+a+f > 0,\;\; EU' > 0, EU''\cdot\tilde{\Delta}^2 < 0\,.$$

Proposition 4 also follows from (23) after setting $\upsilon \equiv 0$. By making use of (16), the right-hand side becomes

$$-\,\frac{p+f}{\varphi L(1-t)}\,\frac{d\Pi}{da}\Big|_{V=\mathrm{const}}\,. \tag{24}$$

Using (17), the left-hand side of (23) is seen to equal

$$-\,\frac{aE\tilde{W}}{X(1-t)}\,\frac{\partial X}{\partial P} \underset{(17)}{=} -\,\frac{aE\tilde{W}}{X(1-t)}\left[\frac{dX}{da}\Big|_{V=const} - \frac{1}{\Phi}\,\frac{d\Pi}{da}\Big|_{V=\mathrm{const}}\right]. \tag{25}$$

Equating (24) and (25) and rearranging gives us

$$\left[\frac{p+f}{\varphi L} + \frac{aE\tilde{W}}{X\Phi}\right]\frac{d\Pi}{da}\Big|_{V=\mathrm{const}} \;=\; \frac{aE\tilde{W}}{X}\,\frac{dX}{da}\Big|_{V=\mathrm{const}}$$

Proposition 4 is then obtained from observing:

$$(19) \quad \Leftrightarrow \quad \frac{(p+f)X}{\varphi LE\tilde{W}} + \frac{a}{\Phi} \;=\; \frac{1}{\varphi} \quad \Leftrightarrow$$

$$(p+f)\varphi + a\varphi \;=\; \Phi \underset{(8)}{=} (p+a+f)\varphi\,.$$

The second equivalence in the chain makes use of the definition $X \equiv U/\Phi = \varphi LE\tilde{W}/\Phi$.

Let us now turn to Proposition 1. In the case of endogenous labor supply, we must determine the sign of υ in (23). For this purpose, take the partial derivative with respect to *L*. Then substitute for λ by making use of (21). The resulting equation is

$$\left[t(1-q)+(1-\bar{z})q+\frac{a}{p+a+f}\,E\tilde{W}\right]+\left[\bar{z}-(1-t)\right]\frac{p+f}{p+a+f}\,\frac{EU''\cdot\tilde{\Delta}\tilde{W}}{EU''\cdot\tilde{\Delta}^2}$$

$$=\upsilon\varphi^2(1-t)EU''\cdot\tilde{\Delta}\,\frac{EU''\cdot\tilde{\Delta}\tilde{W}}{EU''\cdot\tilde{\Delta}^2}+\upsilon\left[D''-\varphi^2(1-t)EU''\cdot\tilde{W}\right]. \tag{26}$$

The first bracketed term on the left-hand side is the government's expected net revenue. It is non-negative by assumption. In order to sign the second term on the left-hand side and the term that appears as the first factor of υ on the right-hand side, we make use of (12). It follows that both these terms are non-negative. Finally, the second-order condition (11b) ensures that the term which appears as second factor of υ on the right-hand side in (26) is positive. We conclude that υ is non-negative just as the factor of υ in (23). This gives us Proposition 2.

References

Allingham, M.G. and A. Sandmo, 1972, Income tax evasion: A theoretical analysis, *Journal of Public Economics* 1, 323-38.

Arrow, K.J., 1970, Essays in the theory of risk-bearing (North-Holland, Amsterdam).

Atkinson, A.B. and J.E. Stiglitz, 1972, The structure of indirect taxation and economic efficiency, *Journal of Public Economics* 1, 97-119.

Atkinson, A.B. and J.E. Stiglitz, 1976, The design of tax structure: Direct versus indirect taxation, *Journal of Public Economics* 6, 55-75.

Boadway, R., M. Marchand and P. Pestieau, 1994, Towards a theory of direct-indirect tax mix, *Journal of Public Economics* 55, 71-88.

Boadway, R. and M. Sato, 2000, The optimality of punishing only the innocent: The case of tax evasion, *International Tax and Public Finance* 7, 641-64.

Chandar, P. and L.L. Wilde, 1998, A General Characterization of Optimal Income Tax Enforcement, *Review of Economic Studies* 65, 165-83.

Cowell, F.A., 1990, Cheating the government. The economics of evasion (MIT Press, Cambridge, Mass.).

Cremer, H. and F. Gahvari, 1993, Tax evasion and optimal commodity taxation, *Journal of Public Economics* 50, 261-275.

Cremer, H. and F. Gahvari, 1996, Tax evasion and optimal general income tax, *Journal of Public Economics* 60, 235-49.

Domar, E.D. and R.A. Musgrave, 1944, proportional income taxation and risk-taking, *Quarterly Journal of Economics* 58, 388-422.

Marhuenda, F. and I. Ortuno-Ortin, 1997, Tax enforcement problems, *Scandinavian Journal of Economics* 99, 61-72.

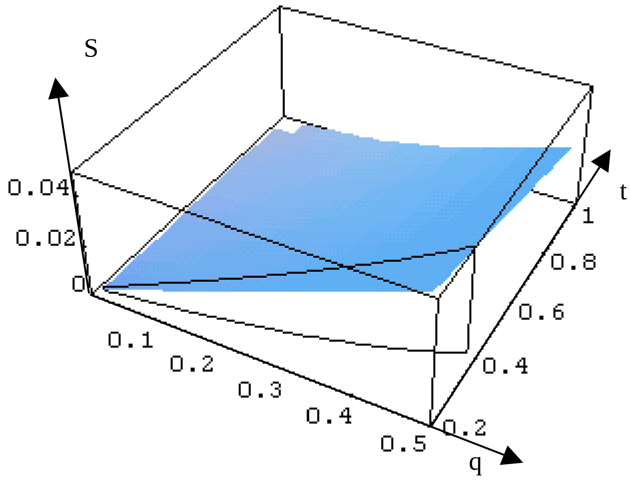
Mirrlees, J.A., 1971, An exploration in the theory of optimum income taxation, *Review of Economic Studies* 38, 175-208.

Sandmo, A, 1974, A note on the structure of optimal taxation, *American Economic Review* 64, 701-706.

Sandmo, A., 1981, Income tax evasion, labour supply, and the equity-efficiency tradeoff, *Journal of Public Economics* 16, 265-288.

Schneider, F. and D.H. Enste, 2000, Shadow economies: size, courses, and consequences, *Journal of Economic Literature* 38, 77-114.

Yitzhaki, S., 1987, On the excess burden of tax evasion, *Public Finance Quarterly* 15, 123-137.



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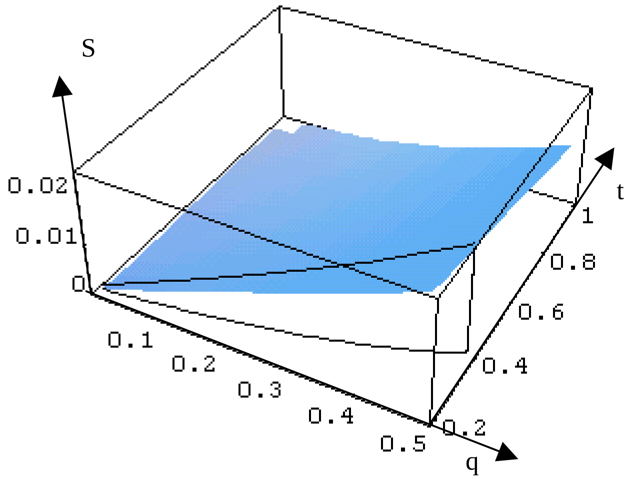

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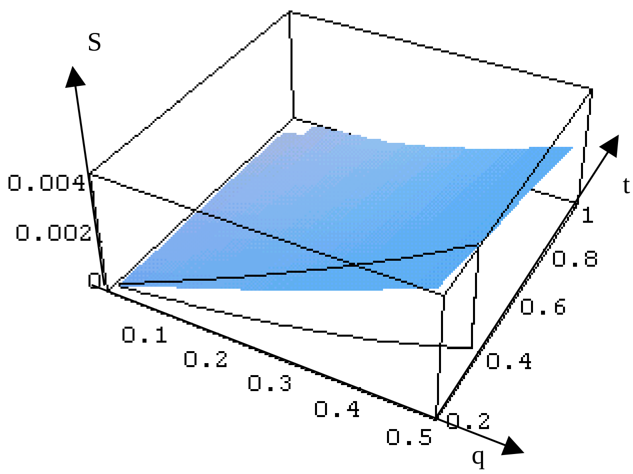

γ = 0.99