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HUMAN CAPITAL FORMATION, INCOME INEQUALITY AND GROWTH

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Abstract

The paper studies the determinants of income distribution and growth in an overlapping generations economy with heterogenous households. Our framework has the following main features: (1) heterogeneity of consumers with respect to wealth and parental human capital; (2) intergenerational transfers are accomplished via investment in the education of the younger generation. Heterogeneity in income results from the distribution of human capital across individuals in a nondegenerate way. The human capital production is affected by the 'home-education', provided by the parents, as well as the 'public-education' which is provided equally to all young individuals of the same generation. Due to investments in human capital our economy is an endogenous growth model. First, we explore the effects of technological improvements in the human capital process, upon the distribution of income at each date along the equilibrium path. Second, we study the impact of such technogical progress on growth and relate these results to the income distribution inequality. Third, we provide numerical simulations to quantify the effect of changes in the parameters of the model. Simulation results include exact Gini coefficients and tax rate on labor determined endogenously through majority voting.

JEL Classification: D9, E2, F2, J2.

Keywords: Human Capital, income distribution, endogenous growth.

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1 Introduction

Endogenous growth models have attracted tremendous attention in economics in the last two decades. The main emphasis has been on the role played by human capital as an engine to growth [see, e.g., Becker and Tomes (1986), Lucas (1988), Azariadis and Drazen (1990)]. As was demonstrated in various ways in this literature various engogenous growth models provide an extremely efficient analytical tools in studying issues related to growth, convergence and distribution of income in equilibrium [see, e.g., Loury (1981), Tamura (1991), Glomm and Ravikumar (1992), Eckstein and Zilcha (1994), Fischer and Serra (1996), Fernandez and Rogerson (1998), Marrewijk (1999), Galor and Moav (2000), Viaene and Zilcha (2001)]. A central feature in all these studies is the way in which the evolution process of human capital is modelled. The production function of human capital is a complex process since education and learning occur in various ways; thus, the accumulation of human capital or skills depends not only on parents, the 'environment', teachers and schools and investment in education, but also on technology and culture. However, the processes of human capital formation used in economic models concentrate, for tractability reasons, on very few parameters [see, e.g., Jovanovic and Nyarko (1995)].

The aim of this paper is to study a certain process of human capital accumulation and to explore some of its implications for income distribution and growth in an overlapping generations economy. Education/training lies in the heart of our process and it is composed of two parts: The parental role which takes place at 'home', mainly during the period of 'youth', and the 'out of home' schooling, or the 'public part' where, in most cases, is provided by the government and influenced by the 'environment'. Home education is provided by the close family and it is carried out through parental tutoring, social interaction, learning devices available at home (such as computors), etc. In this case the human capital of parents and the time they dedicate to teaching/ tutoring play an important role. The public part includes formal education in schools, public expenditure in schooling, the 'outside' social interactions and other activities like the media etc.

It is well established in many studies by economists (and sociologists) that education plays a significant role in shaping the income distribution and in shaping the growth process. We observe in the recent decades increasing awareness of governments in the education process and, consequently, in enhancing investments to promote human capital skills. In recent years, as the information technology advances and computors are being integrated into the learning technology, we are witnessing some important technological progress in the process of human capital formation. In this paper we shall investigate the effects of various kinds of technological improvements on the intragenerational distribution of income and growth. We shall distinguish between technological progress which affects mostly the 'home-component' of the education process vs. technological improvement which affects mainly the 'public-component' of schooling and learning. The government in our education process has two main tasks: first, in organizing the public provision of education and determining the 'level' of public schooling and, second, in financing the public provision of education via taxes on wage income. We shall not attempt in this paper, except in our numerical simulations, to study the process which determines the 'level of public schooling', but rather take it as given in each period. Clearly, given the initial distribution of human capital (and of income) some democratic process will lead to certain decisions, based on the principle that education is provided equally to the younger generation, while the taxes paid by each individual to finance public education depend on his level of income.[1]

We consider an overlapping generations economy which produces a single good using two types of production factors: physical capital and human capital. It starts at date 0 with some given initial distribution of human capital and physical capital stock. Due to investments in human capital of the younger generation, the economy exhibits endogenous growth.[2] Each individual lives for three periods: the 'youth' period in which no economic decisions are made but education is acquired, the 'working period' where this individual earns wage income, and the 'retirement period' in which only consumption takes place. Intergenerational transfers in our economy take place only in the form of investment, made by parents, in educating their offspring and in the provision of public education. When looking at the

[1] It was shown by Glomm and Ravikumar (1992) that majority voting results in a public educational system as long as the income distribution is negatively skewed. Cardak (1999) strenghens this result by considering a voting mechanism where the median preference for education expenditure, rather than median income household, is the decisive voter.

[2] As in Lucas (1988), Azariadis and Drazen (1990), Benhabib and Farmer (1994), Fischer and Serra (1996), van Marrewijk (1999) and others, production is constrained by education and work experience.

effects of technological changes in human capital formation we find that in some cases a more equal intragenerational income distribution coincides with higher output, while in other cases certain technological improvements enhance growth but make income distribution less equal. Basically, in this work we point out that the way in which technological progress effects the process of human capital accumulation matters: If improvements occur mainly in 'home-education' we find that growth increases while equality in income distributions declines. On the other hand, when the technological improvement affects mostly the 'public-education' then we witness higher growth and more equality in income distribution.[3]

The remainder of the paper is organized as follows. The next section presents a process of human capital formation which is part of an OLG model with altruistic heterogenous agents and characterizes the equilibrium of a closed economy. Section 3 studies the effects of changes in the educational technology and externalities on intragenerational income distributions. Section 4 considers the same counterfactuals but focuses on growth. Section 5 presents numerical simulations of a dynamic general equilibrium model with heterogenous agents. Section 6 concludes the paper.

2 The Model

2.1 Human Capital Formation

Consider an overlapping generations economy with a continuum of consumers in each generation, each lives for three periods. During the first period each child gets education/training, but takes no economic decisions. Individuals are economically active during a single working period which is followed by the retirement period. We assume no population growth, hence population is normalized to unity. At the beginning of the 'working period', each parent gives birth to one offspring. Agent or consumer is characterized by his/her family name $\omega \in [0,1]$. Denote by Ω the set of families in each generation:

[3] The role of human capital accumulation on income distribution was thoroughly studied by many researchers [see, e.g., Loury (1981), Becker and Tomes (1986), Galor and Zeira (1993), Fernandez and Rogerson (1998), Viaene and Zilcha (2001)]. Recent empirical findings regarding the claim that growth enhances equality in the income distribution are inconclusive [see, e.g., Persson and Tabellini (1994), Forbes (2000)], a fact which is also obtained only in our theoretical work.

Ω is time independent since there is no population growth. Denote by μ the Lebesgue measure on Ω.

Agents are endowed with two units of time in their second period, one being inelastically supplied to labor, while the other is allocated between leisure and time invested in generating human capital of the offspring. The motivation for parental tutoring is the utility parents derive from the future lifetime income of their child. Besides self-educating their own child, parents also pay (by taxes) for formal education, to enhance the human capital of their child. Consider generation t, i.e., all individuals ω born at the outset of date t, denoted G_t , and denote by $h_{t+1}(\omega)$ the level of human capital of family ω's child. We assume that the production function for human capital is composed of two components: informal education provided by the parents at home and public education provided by 'teachers' and the social environment. Informal education depends on the time allocated by the parents to this purpose, denoted by $e_t(\omega)$, and the 'quality of tutoring' represented by the parent's human capital level $h_t(\omega)$. The time allocated to schooling by the public education system is denoted by e_{gt}, and we assume that the human capital of the teachers determine the 'quality' of this contribution of 'public education' to the formation of human capital. We assume that for some constants $\beta_1 > 1$, $\beta_2 > 1$, $\upsilon > 0$ and $\eta > 0$, the evolution process of a family's human capital is given as follows:

$$h_{t+1}(\omega) = \beta_1 e_t(\omega) h_t^{\upsilon}(\omega) + \beta_2 e_{gt} \overline{h}_t^{\eta} \tag{1}$$

where the average human capital of 'teachers' is the *average* human capital of generation t, denoted $\overline{h}_t$. This can be justified if we assume that the individuals engaged in education in each generation, called 'teachers', are chosen randomly from the population of that generation. The parameters υ and η measure the intensity of the externalities derived from parents' and society's human capital respectively. The constants β_1 and β_2 represent the efficiency of informal and formal education: β_1 is affected by the home environment while β_2 is affected by facilities, the schooling system, neighborhood, social interactions, organization, etc. A similar human capital formation process to this one has been used in Eckstein and Zilcha (1994).

Statistical offices of international organizations compile large lists of indicators that describe and compare educational achievements across countries. While these features vary from country to country and thus there may not be a single theory that characterizes all the observed developments, three main common elements have inspired our framework of analysis [see, e.g., Park (1996), Burnhill, Garner and McPherson (1990)]. First, the production function for human capital given in (1) exhibits the property that individuals from a below-average families have a greater return to human capital investment derived from public schooling than those from above-average human capital families. Also, the effort, and therefore cost, of acquiring human capital for the younger generation is smaller for societies endowed with relatively higher levels of human capital [see, e.g., Tamura (1991), Fischer and Serra (1996)]. Second, an important difference between our process of human capital accumulation and most cases discussed in the literature is the representation of the private and the public inputs via time in the production of human capital. Our approach suggests that the *time spent learning* , coupled with the human capital of the instructors, and not the expenditures on education should be the relevant variables in this process. This distinction is important since in a dynamic framework the cost of financing a similar level of human capital fluctuates with relative factor rewards. Third, in our setting, human capital accumulation includes, besides parental tutoring, either public or private education. To see the difference, consider the lifetime income of individual ω , denoted by $y_t(\omega)$. Since the human capital of a worker is observable and constitutes the only source of income, it depends on the effective labor supply:

$$y_t(\omega) = w_t(1 - \tau_t)h_t(\omega) \tag{2}$$

where w_t is the wage rate in period t and τ_t is the tax rate on labor income. Under the public education regime the taxes on incomes are used to finance education costs of the young generation. Making use of (1) and (2), balanced government budget means:

$$\int_\Omega w_t e_{gt}\overline{h}_t d\mu(\omega) = \int_\Omega \tau_t w_t h_t(\omega) d\mu(\omega)$$

or equivalently,

$$e_{gt} = \tau_t \tag{3}$$

that is, the tax rate on labor is equal to the proportion of the economy's effective labor used for public education. In contrast, under a decentralized system, namely under *private education regime*, both $\tau_t(\omega)$ and $e_{gt}(\omega)$ are decision variables of agents and the individual's budget constraint on private education is:

$$\tau_t(\omega)w_t h_t(\omega) = w_t e_{gt}(\omega)\overline{h}_t$$

where the level of teachers' instruction is chosen freely from the market but their average human capital is the same as the economy's. Aggregate resources invested in education then become:

$$\int_\Omega e_{gt}(\omega)d\mu(\omega) = \frac{1}{\overline{h}_t}\int_\Omega \tau_t(\omega)h_t(\omega)d\mu(\omega), \tag{4}$$

which, in this case, depend upon the distribution of human capital in each date. Here, we do not consider the private education regime but focus instead on public education only.

2.2 Equilibrium

Production in this economy is carried out by competitive firms that produce a single commodity, using effective labor and physical capital. This commodity serves for consumption and also as an input in production. There is a full depreciation of the physical capital. The per-capita human capital in date t , h_t , (not including the human capital devoted to formal education) is an input in the production process. In particular we take the aggregate production function to be:

$$q_t = F(k_t, (1 - e_{gt})h_t) \tag{5}$$

where k_t is the capital stock and $(1 - e_{gt})h_t = (1 - \tau_t)h_t$ is the effective human capital used in the production process. F(·,·) is assumed to exhibit constant returns to scale, it is strictly increasing, concave, continuously differentiable and satisfies $F_k(0, (1 - \tau_t)h_t) = \infty$, $F_h(k_t, 0) = \infty$, $F(0, (1 - \tau_t)h_t) = F(k_t, 0) = 0$.

In the public education model, agent ω at time t maximizes the following lifetime utility:

$$\max_{e_t, s_t} u_t(\omega) = c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} y_{t+1}(\omega)^{a_3} \left[1 - e_t(\omega)\right]^{\alpha_4} \tag{6}$$

subject to

$$c_{1t}(\omega) = y_t(\omega) - s_t(\omega) \geq 0 \tag{7}$$

$$c_{2t}(\omega) = (1 + r_{t+1})s_t(\omega) \tag{8}$$

$$w_t = F_h(k_t, (1 - e_{gt})h_t) \tag{9}$$

$$(1 + r_t) = F_k(k_t, (1 - e_{gt})h_t) \tag{10}$$

$$k_{t+1} = \int_\Omega s_t(\omega)d\mu(\omega) \tag{11}$$

where the income $y_t(\omega)$ is defined by (2) and human capital $h_{t+1}(\omega)$ is given by (1). The $\alpha_i's$ are known parameters and $\alpha_i > 0$ for $i = 1, 2, 3, 4$; $c_{1t}(\omega)$ and $c_{2t}(\omega)$ denote, respectively, consumption in first and second period of the individual's life; $s_t(\omega)$ represents savings; leisure is given by $(1-e_t(\omega))$; $(1 + r_t)$ is the interest factor at date t.

The offspring's income, given by $y_{t+1}(\omega)$, enters parents' preferences directly and represents the motivation for parents' tutoring and formal education expenditure. Eq. (7) is individual ω's budget constraint. Eqs. (9) and (10) are the clearing conditions on factor markets. Condition (11) is a market clearing condition for physical capital, equating the aggregate capital stock at date $t + 1$ to the aggregate savings at date t.

After substituting the constraints, the first-order conditions that lead to the necessary and sufficient conditions for optimum are:

$$\frac{c_{1t}}{c_{2t}} = \frac{\alpha_1}{\alpha_2(1 + r_{t+1})} \tag{12}$$

$$\frac{\alpha_4}{(1 - e_t(\omega))} = \frac{\beta_1\alpha_3(1 - \tau_{t+1})w_{t+1}h_t^v(\omega)}{y_{t+1}(\omega)}, \qquad \text{if} \quad e_t(\omega) > 0 \tag{13}$$

$$\geq \qquad \text{if} \quad e_t(\omega) = 0. \tag{14}$$

The last equation allocates the unit of non-working time between leisure and the time spent on education by the parents. The latter, $e_t(\omega)$, increases with the parents' human capital h_t^v and the wage, net of taxes, at the future date. Eq. (13) establishes a negative relationship between types of education, that is, public education substitutes for parental tutoring as τ_{t+1} increases. Hence, for each individual there exists a particular value of the tax rate such that $e_t(\omega) = 0$. That is, when the marginal utility of leisure is larger than the net future wage obtained from a marginal increase in the human capital of the younger generation derived from parental tutoring. From (7), (8) and (12) we also obtain:

$$c_{1t}(\omega) = \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right) y_t(\omega) \tag{15}$$

$$s_t(\omega) = \left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right) y_t(\omega) \tag{16}$$

3 Income Distribution

Income distribution is a key economic issue and its importance is forcing economists and policymakers to improve their understanding of its underlying determinants. Evidence of a rise in income inequality has been observed in a large number of OECD countries. There is a widely held belief that this rise is driven by events like progress in information technology, integration of world trade and financial markets. Others believe that social norms are crucial determinants of earnings inequality instead. The focus of this section is to consider the inequality in the intragenerational income distribution, in equilibrium, and relate it to the various parameters of our dynamic model.

We shall use the relations that we derived in the previous section to obtain an expression for income at date $t+1$, $y_{t+1}(\omega)$. To that end isolate $y_t(\omega)$ in (13) and make use of (1), (2) and (3) to obtain:

$$y_{t+1}(\omega) = \left(\frac{\alpha_3}{\alpha_3 + \alpha_4}\right) (1 - \tau_{t+1}) w_{t+1} \left[\beta_1 h_t^v(\omega) + \beta_2 e_{gt} \overline{h}_t^{\eta}\right] \tag{17}$$

Eq. (17) determines income at the future date in terms of the net wage at date $t+1$, the parents' and society's level of human capital at date t, the current education input ($\tau_t = e_{gt}$) and the externalities in education. Note that in this framework there is no direct dependence of incomes across generations. Likewise, it is useful to derive the evolution of human capital from the first order conditions. Making use of (13), the human capital of a dynasty given by (1) can be rewritten as follows:

$$h_{t+1}(\omega) = \left(\frac{\alpha_3}{\alpha_3 + \alpha_4}\right)\left[\beta_1 h_t^v(\omega) + \beta_2 \tau_t h_t(\omega) \overline{h}_t^{\eta-1}\right] \tag{18}$$

Let X and W be two random variables with values in a bounded interval in $(-\infty, \infty)$ and let m_x and m_w denote their respective means. Define $\widehat{X} = X/m_x$ and $\widehat{W} = W/m_w$. Denote by F_x and F_w the cumulative distribution functions of $\widehat{X}$ and $\widehat{W}$, respectively. Let $[a, b]$ be the smallest interval containing the supports of $\widehat{X}$ and $\widehat{W}$.

Definition: *F_x is more equal than F_w if, for all $t \in [a, b]$,* $\int_a^t [F_x(s) - F_w(s)]ds \leq 0$.

Thus, F_x is more equal than F_w if F_x *dominates* in the second-degree stochastic dominance F_w.This definition, due to Atkinson (1970), is equivalent to the requirement that the Lorenz curve corresponding to X is everywhere above that of W. We say that X is *more equa*l than W if the c.d.f. of $\widehat{X}$ and $\widehat{W}$ satisfy: F_x is more equal than F_w. Henceforth the relation X is more equal than W is denoted $X \gg W$. We say that X is equivalent to W, and denote this relation by $X \approx W$, if $X \gg W$ and $W \gg X$.

Throughout this section we shall assume that public provision of education is determined by the government, say by elections or other social decision mechanism, and it is equal to e_{gt} in date t and financed by taxing labor income at a fixed rate $\tau_t (= e_{gt})$. In the sequel we shall assume that $v \leq 1$ and that $\eta \leq 1$. Now we show that higher provision of public education reduces inequality in the distribution of income in each generation.

Proposition 1 *In the above economy let $h_0(\omega)$ be the initial human capital distribution. Increasing the public provision of education results in a more equal intragenerational income distribution in each date.*

This result may not be surprising since the public education is provided equally to all the young individuals (of the same generation), while it is financed by a flat tax rate on wage income. However, its importance lies in the fact that it is proved in an equilibrium and that it holds in all future periods.

Proof. Let us consider Eq.(18) for $t = 0$. Since $h_0(\omega)$ is given, $h_0^v(\omega)$ and $\overline{h}_0$ are fixed. By raising e_{g0} the distribution of the human capital for generation 1, $h_1(\omega)$ becomes more equal. This follows from Lemma 2 in Karni and Zilcha (1994). Moreover, we claim from (18) that the average human capital in generation 1 increases as well. Increasing e_{g0} will result in higher $h_1(\omega)$ for all ω and higher level of $\overline{h}_1$. Moreover, it also implies that $h_1^v(\omega)$ will have a *more equal* distribution [see, Shaked and Shanthikumar (1994), Theorem 3.A.5].

Now, let us consider $t = 1$. Increasing e_{g1} will imply the following facts: $h_1^v(\omega)$ becomes more equal and $\beta_2 e_{g1}\overline{h}_1^{\eta}$ is larger than its value before we increased the levels of public education. Using (18) and the same Lemma as before we obtain that $h_2(\omega)$ becomes more equal. This process can be continued for $t = 3, 4, ...,$ which establishes our claim. ■

Consider some technological change that affects the production of human capital. We say that the provision of public education becomes *more efficient* if, in the human capital process (1), β_2/β_1 becomes *larger* without lowering neither β_1 nor β_2. We say that the *private* provision of education becomes *more efficient* if, in the process (1), β_1/β_2 becomes *larger* while neither β_1 nor β_2 declines. A technological improvement in the production of human capital may result in higher efficiency in home education or in public education, or be *neutral*; namely, if the ratio β_2/β_1 remains unchanged while both parameters increase. Let us consider now the effects of each type of technological improvement in the education process on intragenerational income inequality.

Proposition 2 *Consider the above economy. A technological improvement in the production of human capital, given by equation (1), results in:*

(a) If public provision of education becomes more efficient the intragenerational distribution of income becomes more equal in all periods.

(b) If the private provision of education becomes more efficient income inequality becomes larger in all periods.

(c) If the technological improvement is neutral the inequality in income distribution remains unchanged at period 1 but declines for all periods afterwards.

Proof. Let the initial distribution of human capital $h_0(\omega)$ be given. Compare the following two equilibria from the same initial conditions: One with the human capital formation process given by (1) and another with the same process but β_2 is replaced by a larger coefficient $\beta_2^* > \beta_2$. Clearly, we keep β_1 unchanged. Let us rewrite eq. (17) as follows:

$y_{t+1}(\omega) = C_t[h_t^v(\omega) + \frac{\beta_2}{\beta_1} e_{gt}\overline{h}_t^\eta]$

$y_{t+1}^*(\omega) = C_t^*[h_t^{*v}(\omega) + \frac{\beta_2^*}{\beta_1} e_{gt}\overline{h^*}_t^\eta]$

where C_t and C_t^* are some positive constants. Since $h_0(\omega)$ is fixed at date $t = 0$ we find [using once again the Lemma from Karni and Zilcha (1994)] that $\frac{\beta_2^*}{\beta_1} > \frac{\beta_2}{\beta_1}$ imply that $y_1^*(\omega)$ is more equal to $y_1(\omega)$. We also derive that $h_1(\omega)$ are lower than $h_1^*(\omega)$ for all ω and , hence, $\overline{h}_1 < \overline{h}_1^*$. By (18), using the same argument as in the last proof, $h_1^{*v}(\omega)$ is more equal than $h_1^v(\omega)$ and $\frac{\beta_2^*}{\beta_1} e_{g1}\overline{h}_1^{*\eta} > \frac{\beta_2}{\beta_1} e_{g1}\overline{h}_1^\eta$, hence $h_2^*(\omega)$ is more equal than $h_2(\omega)$. This same argument can be continued for all dates $t = 3, 4, 5, ...$ which completes the proof of part (a) of this Proposition. The proof of part (b) follows from the same types of arguments using the fact that if $\beta_1 < \beta_1^*$ then $\frac{\beta_2}{\beta_1} > \frac{\beta_2}{\beta_1^*}$ and, hence, $h_1(\omega)$ is more equal than $h_1^*(\omega)$ and $\overline{h}_1 > \overline{h}_1^*$. This process leads, using similar arguments as before, to $y_t(\omega)$ more equal than $y_t^*(\omega)$ for all periods t. Consider now the claim in part (c). From (18) we see that inequality in the distribution of $h_1(\omega)$ remains unchanged even though all levels of $h_1(\omega)$ increase due to this technological improvement. In particular, $\overline{h}_1$ increases. Now, since inequality of $h_1^v(\omega)$ did not vary but the second term in the RHS of (18) has increased due to the higher value of $\overline{h}_1$, we obtain more equal distribution of $h_2(\omega)$. Now, this argument can be used again at dates 3, 4, ..., which completes the proof. ■

Let us consider now another type of a change in the "home-component" of the production of human capital and its economic implications in equilibrium. Observe the process represented by (1). Let us vary the parameters v and η, which relate to the role played by human capital of the parents or the 'environment'. Since we assume that $v \leq 1$ and $\eta \leq 1$ let us consider the effect that lower values will have on the inequality in income distributions in equilibrium.

Proposition 3 *Consider the process of production of human capital given by (1). Then,*

(a) Comparing two economies which differ only in this parameter v. The economy with the lower v will have more equality in the intragenerational income distribution in all periods.

(b) Comparing two economies which differ only in the parameter η. The economy with the lower value of η will have less equality in the income distribution in all periods.

Proof. Assume, without loss of generality, that $h_0(\omega) \geq 1$ for all ω. Since the two economies have the same initial distribution of human capital $h_0(\omega)$ the process that determines $h_1(\omega)$ differs only in the parameter v. Denote by $v^* < v \leq 1$ the parameters, then it is clear that $[h_0(\omega)]^{v^*}$ is more equal than $[h_0(\omega)]^v$ since it is attained by a strictly concave transformation [see, Theorem 3.A.5 in Shaked and Shanthikumar (1994)]. Likewise, the human capital distribution $h_1^*(\omega)$ is more equal than the ditribution $h_1(\omega)$. This implies that $y_1^*(\omega)$ is more equal than $y_1(\omega)$. Now we can apply the same argument to date 1: the distribution of $[h_1^*(\omega)]^{v^*}$ is more equal than that of $[h_1(\omega)]^v$, hence, using (18) and the above reference, we derive that the distribution of $[h_2^*(\omega)]^{v^*}$ is more equal than that of $[h_2(\omega)]^v$. This process can be continued for all t.

When we lower the value of η, keeping all other parameters constant, we basically lower the second term in (18), $[\overline{h}_0]^\eta$, while $[h_0(\omega)]^v$ remains unchanged. By Lemma 2 in Karni and Zilcha (1994) we obtain that the distribution of $h_1(\omega)$ becomes less equal. This can be continued for $t = 2$ as well since it is easy to verify that $[\overline{h}_1]^\eta$ decreases while $[h_1(\omega)]^v$ becomes less equal. This process can be extended to $t = 2, 3, ...$, which complete the proof. ■

Consider two similar economies which differ only in the initial distributions of human capital: one economy has higher levels of human capital but the same inequality of human capital distributions. Can we compare these two economies with their equilibrium intragenerational income distributions over time? The next proposition provides an answer.

Proposition 4 *Consider two economies which differ only in their initial human capital distributions, $h_0(\omega)$ and $h_0^*(\omega)$. Assume that $h_0^*(\omega) > h_0(\omega)$ for all ω , but $h_0^*(\omega) \approx h_0(\omega)$, namely, these two distributions have the same* *level of inequality. Then, the equilibrium from $h_0^*(\omega)$ will have more equal intragenerational income distributions at all dates t, $t = 1, 2, 3, ...$*

Note that this result indicates that the initial distribution of human capital matters, hence a country that starts with higher levels of human capital, not necessarily more equal, has a better chance to maintain more equality in its future income distributions.

Proof. Observe the following two equations used in the proof of Proposition (2):

$y_{t+1}(\omega) = C_t[h_t^v(\omega) + \frac{\beta_2}{\beta_1} e_{gt} \overline{h}_t^{\eta}]$

$y_{t+1}^*(\omega) = C_t^*[h_t^{*v}(\omega) + \frac{\beta_2}{\beta_1} e_{gt} \overline{h}_t^{*\eta}]$

Since h_0 and h_0^* are equally distributed, the same holds for $h_0^v(\omega)$ and $[h_0^*(\omega)]^v$, since $v \leq 1$. Moreover, since $\overline{h}_0 < \overline{h}_0^*$ we obtain that $h_1^*(\omega)$ is more equal than $h_1(\omega)$ [again, see Lemma 2 in Karni and Zilcha (1994)]. It is easy to verify from (18) that $h_1(\omega)$ are lower than $h_1^*(\omega)$ for all ω. In particular we obtain that $[h_1^*(\omega)]^v$ is more equal than $[h_1(\omega)]^v$ [see Theorem 3.A.5 in Shaked and Shanthikumar (1994)]. Also we have $[\ \overline{h}_1]^\eta <\ [\ \overline{h}_1^*]^\eta$. This implies, using (18), that $h_2^*(\omega)$ is more equal than $h_2(\omega)$. As in our earlier proofs it is easy to see that this process can be continued to generalize this to all periods. ■

Let us consider now the variation over time of the inequality in the distributions of income. We shall demonstrate that under the assumption that the tax rate is the same for all levels of income, inequality declines over time; namely, in our framework the inequality in income distribution at date $t + 1$ is smaller than the inequality in income distribution at date t .

Proposition 5 *If the same tax rate applies to all levels of income, along the equilibrium path the inequality in intragenerational income distribution at date $t + 1$ is smaller than the inequality in the distribution of income at date t.*

Obviously, this model ignores other types of intergenerational transfers, besides provision of education, such as monetary transfers from parents to children. Existence of such transfers may affect the above result.

Proof. Let us show first that in each generation individuals with higher level of human capital choose at the optimum higher level of time to be allocated for private education of their offspring. To see this let us derive from the first order conditions, using some manipulation, the following equation:

$$1 - [1 + \frac{\beta_1 \alpha_4}{\alpha_3}] e_t(\omega) = \frac{\alpha_4 \beta_2}{\alpha_3} e_{gt} \overline{h}_t^{\eta} [h_t^{-v}(\omega)] \tag{19}$$

which demonstrates that higher $h_t(\omega)$ implies higher level of $e_t(\omega)$. Let us show that such a property generates less equality in the ditribution of $y_{t+1}(\omega)$ compared to that of $y_t(\omega)$. It is useful however, to apply (18) for this issue. In fact it represents the period $t+1$ income $y_{t+1}(\omega)$ as a function of the date t income $y_t(\omega)$ via the human capital evolution. Define the function $Q : R \to R$ such that $Q[h_t(\omega)] = h_{t+1}(\omega)$ using (18). This monotone increasing function satisfies: $Q(x) > 0$ for any $x > 0$ and $\frac{Q(x)}{x}$ is decreasing in x. Therefore [see, Shaked and Shanthikumar (1994)], the human capital distribution $h_{t+1}(\omega)$ is more equal than the ditribution in date t, $h_t(\omega)$. This implies that $y_{t+1}(\omega)$ is more equal than $y_t(\omega)$. ■

4 Endogenous Growth

In the last few decades economists have shown great interest in the impact of income inequality on economic growth. The main empirical findings indicate that the conjecture of a negative effect holds (see, e.g. Persson and Tabellini (1994)). More recent evidence differs depending on the sample period, the sample of countries and on whether time-series or cross-section estimation techniques are used (see, e.g., Forbes (2000)). The aim of this section is to explore the relationship between inequality and growth in this framework. Our explanation will be based on the extent of externalities in the process of human capital accumulation.

Let us consider first the effect that technological improvement in the production of human capital will have on output in equilibrium. Consider (1) and remember that we call the first term on the RHS, $\beta_1 e_t(\omega) h_t^v(\omega)$, the *home-component*, and the second term, $\beta_2 e_{gt} \overline{h}_t^{\eta}$, the *public-component.* Now we prove:

Proposition 6 *Consider the human capital production process given by (1). The following types of technological improvements result in :*

(a) Increasing the efficiency of the public-component, or increasing η or both, will result in higher output in all periods.

(b) Increasing the efficiency of the home-component, or increasing v or both, will increase output in all dates.

Proof. Let us just sketch the proof of this claim. Any technological improvement, either in the public-component or the home-component, will imply higher human capital stock as of period 1 and on. Since, the initial capital stock is given this will increase the output in date 1 and, hence, the aggregate savings in this period. Thus the output in date 2 will be higher and hence the capital stock to be used as well. This process continues in all coming periods. ■

Corollary 7 *(a) In the following two cases of technological improvement in the home-component we obtain higher economic growth coupled with less inequality in the distributions of income: (i) an increase in β_1 (ii) an increase in v.*

(b) When technological improvement in the public-component occurs, hence either β_2 or η increases, then higher growth is accompanied by more equality in the distribution of incomes.

If we consider the computor-information revolution as a technological improvement in enhancing knowledge, then we ask whether the home-component benefits more than the public- component in the formation process of human capital. We believe that computors and internet has enhanced the home-education considerably, while schools benefit only in a limited manner. Part (a) of our Corollary may provide some explanation to the recent widespread phenomena (mostly during the nineties) that in the OECD countries economic growth is accompanied by increasing inequality in the distribution of income.

Define the growth factor of aggregate labor as:

$$\gamma_t \equiv \frac{\int_\Omega h_{t+1}(\omega)d\mu(\omega)}{\int_\Omega h_t(\omega)d\mu(\omega)} \tag{20}$$

Substitution of (18) in (20) gives use to an alternative expression for γ_t :

$$\gamma_t = \left(\frac{\alpha_3}{\alpha_3+\alpha_4}\right)\left[\beta_1 \frac{\int_\Omega h_t^v(\omega)d\mu(\omega)}{\int_\Omega h_t(\omega)d\mu(\omega)} + \beta_2 \tau_t \overline{h}_t^{\eta-1}\right] \tag{21}$$

Now let us consider the effect of technological improvements in the human capital production on the rate of growth in human capital stock. First we assume technological progress in the home-component.

Proposition 8 *Let $\eta \leq 1$ and $v \leq 1$. Assume that a technological improvement occurs in the home-component of the human capital production process. Then, the growth factor γ_t of the human capital declines in all dates.*

Proof. Assume that we have an increase in the parameter v, while other parameters remain unchanged. We shall apply now the result of Proposition 2. Increasing v (or β_1) will increase human capital distribution's inequality; therefore, $[\overline{h}_t]^{-1}h_t(\omega)$ becomes *more dispersed* (in the sense of mean-preserving spread). Hence, for any strictly concave function its expected value declines; in particular, this implies that $[\overline{h}_t]^{-v} \int h_t^v(\omega)\, d\mu(\omega)$ decreases. Since $v \leq 1$, $\frac{\int_\Omega h_t^v(\omega)d\mu(\omega)}{\int_\Omega h_t(\omega)d\mu(\omega)}\overline{h}_t^{1-v}$ declines as well as $\overline{h}_t^{\eta-1}$ and, by the proof of Proposition 2 , $\overline{h}_t$ increases. Thus, by (21), it is easy to see that as a result of a rise in v we obtain lower values for γ_t. Using the above expression for γ_t it can be shown that the same result holds when we increase β_1. Here, we should apply the fact that multiplying β_1 by a factor $\lambda > 1$ the first term on the RHS of equation (21) declines since $v < 1$, while the second term, which includes $\overline{h}_t^{\eta-1}$ declines since (again using the proof of Proposition 2) $\overline{h}_t$ increases. This completes the proof. ■

5 Numerical Simulations

The aim of this section is to introduce a dynamic computable general equilibrium model with heterogenous agents and to characterize the properties of the equilibria of the model discussed above. In particular, we are interested in establishing the relationship between changes in technology parameters and the growth and distribution of income that can be sustained in equilibrium. Though the effects of most changes in parameters of the model have been described in the preceding propositions, it is important to quantify these effects in various situations. To facilitate the interpretation of our theoretical results the first set of numerical simulations assume that the sequence of τ_t

is exogenously given. Then we allow for the tax rate to be endogenously determined through majority voting.

Exogenous Public Education

In our numerical examples we replace (5) by the Cobb-Douglas production $q_t = Ak_t^{\theta}(1-\tau_t)^{1-\theta}h_t^{1-\theta}$, that is $w_t = A(1-\theta)\left(k_t/(1-\tau_t)h_t\right)^{\theta}$ and $(1+r_t) = A\theta\left((1-\tau_t)h_t/k_t\right)^{1-\theta}$. In the baseline case, we assume that the economy is in a steady-state. To characterize the latter, consider Eqs. (2), (11), (16) and the Cobb-Douglas production function to obtain:

$$\frac{k_{t+1}}{k_t} = \frac{(1-\theta)\alpha_2}{\theta(\alpha_1+\alpha_2)}(1+r_t) \tag{22}$$

Making use of (21):

$$\frac{k_{t+1}}{h_{t+1}} = \frac{A\alpha_2(1-\theta)}{(\alpha_1+\alpha_2)}(1-\tau_t)^{1-\theta}(\gamma_t)^{-1}\left(\frac{k_t}{h_t}\right)^{\theta} \tag{23}$$

which describes the dynamic path of the capital-labor ratio of the economy. In the long-run $k_{t+1}/h_{t+1} = k_t/h_t$ is a constant k/h if $\tau_t = \tau$ and $\gamma_t = \gamma$. The time-independence of γ can be obtained by incorporating externalities that yield constant returns to scale to parents' and society's human capital in (1), namely assuming $\upsilon = \eta = 1$. In that case we obtain the long-run capital-labor ratio from (23):

$$\frac{k}{h} = (1-\tau)\left[\frac{\alpha_2(1-\theta)A}{\gamma(\alpha_1+\alpha_2)}\right]^{\frac{1}{(1-\theta)}} \tag{24}$$

From the above equations, we obtain the expression for long-run output and income growth:

$$\frac{q_{t+1}}{q_t} = \frac{\int y_{t+1}(\omega)d\mu(\omega)}{\int_{\Omega} y_t(\omega)d\mu(\omega)} = \frac{\alpha_2(1-\theta)A}{(\alpha_1+\alpha_2)}\left(\frac{h(1-\tau)}{k}\right)^{1-\theta}$$

Substituting (24) in this last expression gives:

$$\frac{q_{t+1}}{q_t} = \gamma \tag{25}$$

Long-run economic growth coincides with the effective labor growth factor γ, regardless of initial conditions. Our model in the stationary state is therefore an AK-type endogenous growth model where all variables grow at the rate $(\gamma - 1)$.

Besides $\upsilon = \eta = 1$, we assume that the other baseline parameters are $k_{-1} = 55.78$, $\tau = 0.2$, $\alpha_1 = \alpha_2 = \alpha_4 = 1$, $\alpha_3 = 2$, $A = 4$, $\theta = .3$ and $\beta_1 = \beta_2 = 1.6$. We consider a discrete number of heterogenous families, namely 11, with a human capital at $t = -1$ taking the values $1, 2, ...11$. The initial endowments in physical and human capital were chosen with two criteria in mind. First, the values of the endogenous variables that follow from these initial conditions and parameter values are long-run values at all dates. Second, the initial heterogeneity in human capital calibrates an exact Gini coefficient close to the European average, namely 0.303. The following formula for the Gini coefficient is used:

$$g_t = \frac{1}{2n^2\overline{y}_t} \sum_{i=1} \sum_{j=1} |y_i - y_j| \tag{26}$$

where n represents the number of families, $\overline{y}_t$ is average income, y_i and y_j are individual incomes.

Given the set of baseline parameters of the model, the equilibrium path of all variables belonging to a particular family is obtained in two steps. First, the human capital of any individual at date t is given by (18). Aggregating the levels of human capital across individuals and equating the aggregate capital stock at date t to the aggregate savings at date $t - 1$ (see 11)), we obtain aggregate production q_t, the equilibrium w_t and $(1 + r_t)$. Upon this information, each individual derives his/her income $y_t(\omega)$ from (2) and summary statistics like the Gini coefficient can be computed. Second, given the time path of wages, marginal returns to physical capital and income of each dynasty, each individual can compute $e_t(\omega)$, $c_{1t}(\omega)$, $c_{2t}(\omega)$, and $u_t(\omega)$.

Column 1 of Table 1 presents the solution for our baseline case (reproduced in all subsequent tables) and the comparative statics of changes in υ and η. In the numerical simulations, given the chosen parameters we solve the model for 200 periods. As patterns emerge within 20 periods we discard the last 180 periods and compute the relevant statistics averaging over the first 10 periods and over the second 10 periods. This table indicates that inequality as measured by Gini coefficients is sensitive to externalities arising from the home component but not to externalities arising from the public part of human capital formation. Decreasing returns in parents' human capital (column 2) reduce inequality substantially, all individuals becoming equal in the long-run. In contrast income divergence is observed with increasing returns (column 3). In column 2, increased equality is obtained at the expense of growth, whether measured in terms of income or human capital.

Table 2 looks at a technological improvement in human capital formation represented here by rises in the $\beta's$. Columns 2 to 4 show that a greater efficiency in education is conducive to growth while not affecting income distributions. A comparison of columns 2 and 3 shows the stronger impact that parental education has on growth.

It is important to note a difference between our theoretical results and those of Tables 1 and 2. The former predict effects on the income distribution as a result of changes in the process of human capital formation which are stronger than those obtained from our numerical simulations. A reason lies in the use of the concept of second order stochastic dominance in most proofs. For example, when looking at income distribution, a mean preserving spread change maintains average income $\overline{y}_t$ unchanged. In contrast, in our numerical simulations, average income $\overline{y}_t$ varies according to the scenarios under consideration. As it enters directly in formula (26) of the Gini coefficient, most results on income distribution differ and become negligible except when dealing with υ.

Majority Voting

Though there is a growing awareness of governments in education, enhancing human capital skills require financial resources to cover the investment. Though the majority of constituents recognize the importance of learning, they are not prepared to contribute financially via income taxes in the same way. To establish the preferences of each individual with respect to $\tau_t(\omega)$ let us compute the reduced-form utility of each agent. Substituting the first order conditions in (6), lifetime utility of agent ω can be rewritten as:

$$u_t(\omega) = \Omega_t (1 - \tau_t(\omega))^{\alpha_1 + \alpha_2} (1 - \tau_{t+1}(\omega))^{\alpha_3}$$

$$(\beta_1 h_t(\omega)^v + \beta_2 \tau_t(\omega) h_t(\omega) \overline{h}_t^{\eta-1})^{\alpha_3 + \alpha_4} \tag{27}$$

where Ω_t groups all parameters and variables like factor rewards which are given to atomistic individuals. Knowing that each agent cannot enforce any tax rate at the future date, i.e. $\tau_{t+1}(\omega)$ is given to him, the maximization of (27) with respect to $\tau_t(\omega)$ gives:

$$\tau_t(\omega) = \frac{(\alpha_3+\alpha_4)}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4)} - \frac{(\alpha_1+\alpha_2)}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4)} \frac{\beta_1 \overline{h}_t^{1-\eta}}{\beta_2 h_t(\omega)^{1-\upsilon}}$$

Table 1 Baseline and Other Specification Externalities

Externalities	(1)	(2)	(3)	(4)	(5)
υ	1	.8	1	1.1	1
η	1	1	.8	1	1.1
Relative factor returns	.47	.325	.406	.809	.556
$(1 + r_t)/w_t$	.47	.330	.394	3.53	.668
Gini coefficient (g_t)	.303	.15	.303	.415	.303
(income)	.303	.03	.303	.633	.303
Growth rate (%)	28.	-1.4	15.3	85.8	43.6
(aggr. output)	28.	.0	13.0	large	63.3
Growth rate (%)	28.	-2.4	14.7	90.9	44.6
(aggr. human capital)	28.	.0	12.9	large	64.5
Parental education (e_t)	.6	.587	.627	.610	.569
(poorest agent)	.6	.578	.636	.635	.516

Table 2 Baseline and Other Specification Efficiency

Efficiency	(1)	(2)	(3)	(4)
β_1	1.6	1.76	1.6	1.76
β_2	1.6	1.6	1.76	1.76
Relative factor returns	.47	.526	.482	.537
$(1+r_t)/w_t$	.47	.528	.483	.540
Gini coefficient (g_t)	.303	.303	.303	.303
(income)	.303	.303	.303	.303
Growth rate (%)	28.	38.2	30.0	40.2
(aggr. output)	28.	38.7	30.1	40.8
Growth rate	28.	38.7	30.1	40.8
(aggr. human capital)	28.	38.7	30.1	40.8
Parental education (e_t)	.6	.606	.593	.6
(poorest agent)	.6	.606	.593	.6

Each agent chooses the optimal $\tau_t(\omega)$ such that the cost of current spending on education (in terms of foregone current and future consumption) is equal to the reward of a marginal increase in the human capital of their children. It is clear that the heterogeneity in $\tau_t(\omega)$ derives from the heterogeneity in human capital. When $\eta \leq 1$ and $v < 1$ below-average agents are willing to pay a tax rate lower than above-average agents. When $\eta = v = 1$, $\tau_t(\omega) = \tau$. In terms of our numerical simulations, the first step produces a vector of $\tau_t(\omega)$ based on (28). Given this vector of individual preferences for education expenditure, we assume that the level of public schooling is obtained at each date through majority voting. Numerically, majority voting boils down to identifying the median voter's preference for public schooling.

Tables 3 and 4 repeat the comparative statics of Tables 1 and 2, now with endogenous public education. The simulation results of these tables establish a substitution in equilibrium between public education and parental education: an increase in τ_t decreases the time spent on parental education e_t and hence, raises leisure. This substitution among types of provision of education has a number of implications, one of which being that growth rates are all positive now. For the rest, results confirm a positive relationship between income inequality and income growth in only one type of scenarios, namely when externalities arising from parents' human capital vary.

6 Conclusion

In our framework a technological change in the aggregate production function will not have an impact on the distribution of income. Therefore, we consider only technological improvements in the human capital accumulation process. As we show, in this case, the effect is ambiguous: it depends on the manner in which it affects the process. It is important to note that introducing intergenerational transfers in our model will modify the results: in such a case, technological progress in the aggregate production function may have different effects on the intragenerational income distributions [see Karni and Zilcha (1994)].

Our theoretical analysis does not depend on the levels of the public provision of education, $\{e_{gt}\}$. The choice of some 'optimal' level of public education requires some social welfare function due to the heterogeneity of the households. However, the majority voting criterion is widely used in economic theory, hence, one can determine this level using the median voter's optimal choice. This has been used in our numerical simulations. This framework can be generalized by introducing an additional redistributive measures by the government, such as social security. This may vary some of our results.

Table 3 Externalities and Median Voter

Externalities	(1)	(2)	(3)	(4)	(5)
υ	1	.8	1	1.1	1
η	1	1	.8	1	1.1
Tax rate (r_t)	.20	.335	.01	0.031	.339
	.20	.358	.0	.0	.480
Relative factor returns	.47	.422	.310	.573	.830
$(1 + r_t)/w_t$	.47	.431	.291	1.85	2.52
Gini coefficient (g_t)	.303	.162	.303	.424	.303
(income)	.303	.044	.303	.656	.303
Growth rate (%)	28.	4.0	11.3	67.8	64.1
(aggr. output)	28.	3.1	6.7	large	large
Growth rate (%)	28.	5.2	7.6	67.7	71.9
(aggr. human capital)	28.	3.2	6.7	large	large
Parental education (e_t)	.6	.522	.666	.657	.485
(poorest agent)	.6	.478	.667	.667	.057

Table 4 Efficiency and Median Voter

Efficiency	(1)	(2)	(3)	(4)
β_1	1.6	1.76	1.6	1.76
β_2	1.6	1.6	1.76	1.76
Tax rate (τ_t)	.20	.16	.236	.20
	.20	.16	.236	.20
Relative factor returns	.47	.485	.522	.537
$(1+r_t)/w_t$	.47	.481	.529	.540
Gini coefficient (g_t)	.303	.303	.303	.303
(income)	.303	.303	.303	.303
Growth rate (%)	28.	35.2	33.1	40.2
(aggr. output)	28.	34.4	34.4	40.8
Growth rate (%)	28.	34.8	34.0	40.8
(aggr. human capital)	28.	34.4	34.4	40.8
Parental education (e_t)	.6	.617	.581	.6
(poorest agent)	.6	.618	.580	.6

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