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CESifo Working Papers

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Rainer Niemann

CESifo Working Paper No. 557

September 2001

CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de
ISSN 1617-9595

CESifo *Working Paper No. 557*
*September 2001*

# TAX RATE UNCERTAINTY AND INVESTMENT BEHAVIOR

## Abstract

This paper deals with the effects of tax rate uncertainty on risk-neutral and risk-averse investment behavior. We analyze effects of stochastic tax rates on both real and financial investment. It emerges that under risk neutrality as well as under risk aversion, increased tax rate uncertainty has an ambiguous impact on investment behavior, depending on the investor's utility function and the investment project's structure of cash flows and depreciation deductions. The popular view that tax policy uncertainty depresses real investment is rejected. Tax neutrality in the light of tax policy uncertainty is defined more precisely. Neutrality results for the Johansson-Samuelson tax and the cash flow tax can be widely confirmed for tax rate uncertainty.

JEL Classification: G31, H25, H21.

*Rainer Niemann*
*University of Tübingen*
*Department of Economics*
*Mohlstr. 36*
*72074 Tübingen*
*Germany*
*Rainer.niemann@uni-tuebingen.de*

# 1 Introduction

Tax policy is one of the favorite topics of every newly elected government. As evidence from many OECD-countries indicates, this leads to frequent changes in tax bases, tax rates and sometimes to major shifts in the coordination of corporate and individual taxation[1]. A variety of tax parameters may be subject to changes, but the most obvious one is certainly the statutory tax rate.

A popular view states that uncertain tax policy complicates investment planning and depresses investment. While the former is undoubtedly correct, recent research in public economics proves that the latter is not necessarily true. In some cases, depending on the tax parameter under consideration and the underlying stochastic process, tax policy uncertainty might in fact encourage real investment. In other cases, investment may be discouraged. For a precise analysis, it is necessary to take tax effects on financial investment into account because it serves as a yardstick for real investment in all models of capital budgeting. Unfortunately, most studies leave either the tax rate or its influence on financial investment aside and therefore cannot identify the influence of tax rate uncertainty on the relative advantage of real compared to financial investment.

There is a wide-ranging literature concerning the effects of different uncertain tax parameters on investment behavior. Investment tax credits (ITCs) were originally designed to promote real investment, hence it should be examined whether this objective can be achieved in the light of stochastic ITCs. Hassett and Metcalf (1999) use a model with an output price following a geometric Brownian motion and an uncertain ITC to explain the effects of tax policy uncertainty on aggregate investment. They conclude that tax policy uncertainty tends to delay investment under a countinuous-time random walk, but increases the capital stock under a Poisson jump process while the discount rate is assumed deterministic. In a real options model, investment effects of ITC uncertainty modeled by a jump-diffusion process are analyzed by Agliardi (2001). The result that tax policy uncertainty delays investment may be questionable due to disregarding taxes in the discount factor. Without special distribution assumptions, Watson (1992) discusses saving and investment in a dynamic portfolio model, taking taxes on financial investment into account. Under some conditions, tax rate

[1] The most recent example is Germany's tax system. In 2001, personal and corporate income tax rates were reduced and the imputation system was replaced by a classical system with partial shareholder relief.

uncertainty tends to lower risk-taking. Pointon (1998) focuses on changing imputation tax rates following a Poisson process with a discount factor unaffected by tax rate changes. A constant discount factor despite changing tax rates is used by Auerbach and Hines (1988), too. Alm (1988) distinguishes between tax base and tax rate uncertainty to derive welfare effects of uncertainty. Skinner (1988) computes the additional excess burden of uncertain compared with certain tax policy. Bizer and Judd (1989) use a dynamic general equilibrium approach with a constant discount factor, but do not report general results concerning the efficiency costs of random tax policy.

Since tax effects on financial investment are widely neglected in the literature, this article's objective is to analyze the impact of tax rate uncertainty on real *and* financial investment as perceived by risk-neutral and risk-averse investors. In order to focus on tax rate uncertainty effects, the tax base and all non-tax parameters are assumed deterministic.

The remainder of the paper is organized as follows. Section 2 presents an investment model taking stochastic tax rates into account. Investment decisions under tax rate uncertainty and risk neutrality will be discussed in section 3 which also includes considerations on tax neutrality. Section 4 deals with the impact of tax rate uncertainty in case of risk-averse investors as well as neutral taxation under risk aversion. Section 5 concludes.

## 2 Model design

Assuming a perfect capital market under certainty, individual investment decisions are based on the net present value (NPV) criterion[2]. In the absence of taxation, a real investment project will be carried out if its NPV is positive, which is equivalent to a situation in which the future value at time $T$ of real investment exceeds the one of financial investment[3]:

$$-I_0 + \sum_{t=1}^{T} \frac{\pi_t}{(1+i)^t} > 0 \Leftrightarrow \sum_{t=1}^{T} \pi_t \, (1+i)^{T-t} > (1+i)^T \qquad (1)$$

with $I_0 = 1$: initial outlay
$i =$ const.: pre-tax interest rate
$T < \infty$: time horizon
$\pi_t$, $t = 1, \ldots, T$: pre-tax cash flow at time $t$.

[2] To abstract from a real options context, it is supposed that there is no timing flexibility. Investment is a "now-or-never" rather than a "now-or-later" decision.

[3] In the remainder, initial outlay is normalized to unity. Without loss of generality, cash flows are adjusted accordingly.

This criterion is based on an implicit comparison of a real investment project with a financial investment that yields the interest rate $i$ on the perfect capital market. A modified NPV criterion applies after integrating deterministic taxes. The tax base at time $t$ consists of cash flow $\pi_t$ less depreciation deductions $d_t$. Assuming a constant and proportional tax rate $\tau$ and a complete and immediate loss offset, the post-tax cash flow is given by $\pi_t^\tau = (1-\tau)\ \pi_t + \tau\, d_t$. Changing the yardstick for real investment, interest payments from financial investment are subject to tax, too. Hence, the net-of-tax interest rate is defined by $i^\tau = (1-\tau)\ i$. Inserting yields the NPV after taxes which is again positive if and only if the future value of the real investment project exceeds the one of financial investment:

$$-1 + \sum_{t=1}^{T} \frac{(1-\tau)\ \pi_t + \tau\, d_t}{[1 + i\ (1-\tau)]^t} \ > \ 0 \tag{2}$$

$$\Leftrightarrow \sum_{t=1}^{T} [(1-\tau)\ \pi_t + \tau\, d_t]\ [1 + i\,(1-\tau)]^{T-t} \ > \ [1 + i\,(1-\tau)]^T \tag{3}$$

In order to focus on the effects of uncertainty, the assumption of a constant tax rate has to be relaxed first. With a deterministically changing tax rate $\tau_t$ $(t = 1, \ldots, T)$, the discount factor becomes time-dependent and the above criterion changes to:

$$\sum_{t=1}^{T} [(1-\tau)\ \pi_t + \tau\, d_t] \prod_{s=t+1}^{T} [1 + i\,(1-\tau_s)] > \prod_{s=1}^{T} [1 + i\,(1-\tau_s)]\,. \tag{4}$$

Introducing tax rate uncertainty largely complicates the analysis because it affects both the real investment project under consideration and financial investment and thus implies measuring with a variable yardstick. For assessing an investment project it is not sufficient to compare two single future values. Rather, it is necessary to compute the future values of real and financial investment for each possible state of the stochastic process modeling the tax uncertainty[4]. Since the difference of future values might be of ambiguous sign under different states, expected utility has to be computed to yield an unambiguous individual criterion to compare real and financial investment.

To restrict the model's complexity, it is assumed that all tax uncertainty is summarized in a single stochastic process representing the tax rate $\tau$. Changes in the tax base are not

[4]It will be shown that there are in general many more "states of the process" at time $t$ than possible tax rates.

modeled explicitly. All other parameters used in the model, i.e., cash flows, depreciation deductions and the pre-tax interest rate, are assumed deterministic and constant.

Tax rate uncertainty can be modeled by various stochastic processes. Since tax rates typically undergo discrete jumps at the beginning of fiscal years, a process in discrete time with a discrete state space, i.e., a binomial process, seems appropriate. A binomial process is characterized by either an upward or a downward movement of the state variable at each time instance. In the following context, the additive (or arithmetic) variant of a binomial process is used, which means that the tax rate $\tau$ will be either raised or reduced by a constant $\Delta$ in each period. Therefore, the tax rate at time $t+1$ is

$$\tau_{t+1} = \begin{cases} \tau_t + \Delta & \text{with probability } p \\ \tau_t - \Delta & \text{with probability } 1-p \end{cases} \tag{5}$$

with $p$: probability for a tax rise
$\Delta \geq 0$: amount of tax rate change
$\tau_t$: proportional tax rate at time $t$.

To restrict the analysis to cases where the tax rate is in the relevant interval $]0;1[$, the following conditions are assumed to be satisfied:

$$\begin{aligned} & 0 < \tau_t < 1 \quad \forall\, t = 1, \ldots, T \\ \Leftrightarrow \quad & 0 < \tau_0 < 1 \ \wedge \ \tau_0 + T \cdot \Delta < 1 \ \wedge \ \tau_0 - T \cdot \Delta > 0. \end{aligned} \tag{6}$$

The tax rate process can be represented by a binomial tree. Since the amounts of upward and downward movements are both time-invariant and equal to $\Delta$, the binomial tree's branches always recombine. At time $t$, the number of possible tax rates is $t+1$.

In order to focus on the investment effects of changing tax rate uncertainty, the stochastic process should be mean-preserving, i.e., its expected value should be constant despite varying degrees of uncertainty. This eliminates possible effects of a trend in tax rates on investment behavior. In the following context, mean-preservation will be achieved by equating the probabilities for a tax rise and a tax reduction: $p = 1 - p = \frac{1}{2}$. The expected tax rate in period $t$ calculated using information at time 0 therefore equals the initial tax rate $\tau_0$ since the number of tax reductions during the time interval $]0, t]$, $N$, follows a symmetric binomial distribution:

$$
\begin{aligned}
P\,(N=n) &= \binom{t}{n}\, p^n\, (1-p)^{t-n} && (7)\\
\mathrm{E}_0\,[N] &= t\,p = \frac{1}{2}\,t && (8)\\
\mathrm{V}_0\,[N] &= t\,p\,(1-p) = \frac{1}{4}\,t && (9)\\
\tau_t &= \tau_0 + t\,\Delta - 2\,N\,\Delta && (10)\\
\mathrm{E}_0\,[\tau_t] &= \tau_0 + t\,\Delta - 2\,\mathrm{E}_0\,[N]\,\Delta = \tau_0 + t\,\Delta - \frac{1}{2}\,2\,t\,\Delta = \tau_0. && (11)
\end{aligned}
$$

with $\mathrm{E}_t\,[\cdot]$: expected value of a random variable based on information at time $t$
$N$: number of tax reductions during the time interval $]0,t]$
$\mathrm{V}_t\,[\cdot]$: variance of a random variable based on information at time $t$.

Growing tax rate uncertainty is represented by a rising $\Delta$. The variance of the tax rate is

$$\mathrm{V}_0\,[\tau_t] = (-2\Delta)^2\;\mathrm{V}_0\,[N] = t\,\Delta^2. \qquad (12)$$

Like the expected tax rate, the expected values of the net-of-tax interest rate and the post-tax cash flow at time $t$ are independent of $\Delta$:

$$
\begin{aligned}
\mathrm{E}_0\,[i_t^\tau] &= \mathrm{E}_0\,[(1-\tau_t)\;i] = (1-\tau_0)\;i && (13)\\
\mathrm{E}_0\,[\pi_t^\tau] &= \mathrm{E}_0\,[(1-\tau_t)\;\pi_t + \tau_t\,d_t] = (1-\tau_0)\;\pi_t + \tau_0\,d_t. && (14)
\end{aligned}
$$

Although the expectations of the current tax-dependent variables are unaffected by the degree of tax-rate uncertainty, this does not hold for the expected future value of real or financial investment. The reason is that future values (or present values) not only depend on the current state of the stochastic process, but on the path the process will take (has taken) through the entire binomial tree[5]. This fact largely complicates the analysis because there are $2^T$ possible paths through a binomial tree of depth $T$ rather than just $T+1$ possible values of the state variable at time $T$. Thus, the decision problem's complexity is exponential rather than linear with respect to $T$ and cannot be represented by a binomial tree. Instead, a binary tree branching at each point of time at each possible state is needed. In contrast to a binomial tree, a binary tree in general does not recombine.

[5] As a simple two-period example with identical final state variables, $(1+i\;(1-\tau_0-\Delta))\cdot(1+i\;(1-\tau_0)) \neq (1+i\;(1-\tau_0+\Delta))\cdot(1+i\;(1-\tau_0))$ for $\Delta \neq 0$.

# 3 The impact of uncertain tax rates on investment under risk neutrality

Since the payoffs from real as well as financial investment are subject to tax, there is no risk-free asset (cf. Watson (1992), p. 685). Consequently, the impact of tax rate uncertainty has to be computed separately for real and financial investment. By evaluating both effects, it can be analyzed whether the tax rate uncertainty encourages or discourages investment.

In this section, investors are assumed risk-neutral. In this case, maximization of expected utility as an individual decision criterion is equivalent to maximization of expected future wealth. Thus, the decision criterion is given by:

$$\frac{\mathrm{E}_0\left[W_T^R\right] - \mathrm{E}_0\left[W_T^F\right]}{\mathrm{E}_0\left[W_T^F\right]} = \frac{\mathrm{E}_0\left[W_T^R\right]}{\mathrm{E}_0\left[W_T^F\right]} - 1 \rightarrow \text{max!} \tag{15}$$

with $W_T^F$: future wealth from financial investment
$W_T^R$: future wealth from real investment.

The economic interpretation of eq. (15) is that real investment is carried out if and only if its expected future value exceeds the one of financial investment.

## 3.1 Financial investment

The future wealth resulting from financial investment has to be computed for each of the $2^T$ possible realizations of the tax rate process. The probabilities for a rise and for a reduction of the tax rate are both $\frac{1}{2}$, so each possible path has probability $\frac{1}{2^T}$. The expected future value of 1 \$ initial financial investment in the general case is:

$$\mathrm{E}_0\left[W_T^F\right] = \frac{1}{2^T} \sum_{j \in J} \prod_{t=1}^{T} \left[1 + i\left(1 - \tau_t^j\right)\right] \tag{16}$$

with $J$: index set of possible realizations of the tax rate process.

To limit complexity, the two-period case ($T = 2$) is presented in detail. In this case, eq. (16) reduces to:

$$\begin{aligned} \mathrm{E}_0\left[W_2^F\right] &= \frac{1}{4}[\,[1+i\,(1-(\tau_0+\Delta))]\,[1+i\,(1-(\tau_0+2\Delta))] \\ &\quad +[1+i\,(1-(\tau_0+\Delta))]\,[1+i\,(1-\tau_0)] \\ &\quad +[1+i\,(1-(\tau_0-\Delta))]\,[1+i\,(1-\tau_0)] \\ &\quad +[1+i\,(1-(\tau_0-\Delta))]\,[1+i\,(1-(\tau_0-2\Delta))]\,] \\ &= [1+i\,(1-\tau_0)]^2+i^2\,\Delta^2. \end{aligned} \tag{17}$$

Somewhat counterintuitive, this expression increases with rising tax rate uncertainty, which means that the degree of uncertainty influences even risk neutral investors. The same result holds for more than two periods[6]:

$$\mathrm{E}_0\left[W_T^F\right] = [1+i\,(1-\tau_0)]^T + f_T^F\,(i,\Delta,\tau_0) \tag{18}$$

with $f_T^F \equiv f_T^F\,(i,\Delta,\tau_0)$: uncertainty-dependent term in expected future wealth from financial investment.

These terms are increasing and convex in $\Delta$. Everything else equal, financial investment is encouraged by increasing tax rate uncertainty if investors are risk-neutral.

## 3.2 Real investment

Analogous to financial investment, future wealth resulting from real investment has to be calculated for each of the $2^T$ possible realizations of the stochastic process. This case is more complicated because cash flows $\pi_t$ and depreciation deductions $d_t$ are not supposed to remain constant over time. Post-tax cash flows are re-invested at the current net-of-tax interest rates until time $T$. In the general $T$-period case, expected future wealth of real investment is:

$$\mathrm{E}_0\left[W_T^R\right] = \frac{1}{2^T}\sum_{j\in J}\sum_{t=1}^{T}\left[\left[\left(1-\tau_t^j\right)\pi_t+\tau_t^j\,d_t\right]\prod_{s=t+1}^{T}\left[1+i\left(1-\tau_s^j\right)\right]\right]. \tag{19}$$

[6]Expected future wealth has been computed for time horizons of up to $T = 10$. In each case, expected future wealth of financial investment is increasing and convex in $\Delta$. Unfortunately, an expression for the generalized time horizon $T$ could not be proved. Uncertainty-dependent terms for $T = 2, \ldots, 10$ are included in appendix A.

In the two-period case, expression (19) simplifies to:

$$\begin{aligned} \mathrm{E}_0\left[W_2^R\right] &= \left[(1-\tau_0)\ \pi_1+\tau_0\, d_1\right]\left[1+i\ (1-\tau_0)\right]+\left[(1-\tau_0)\ \pi_2+\tau_0\, d_2\right] \\ &\quad +i\,\Delta^2\ (\pi_1-d_1)\,. \end{aligned} \tag{20}$$

Again, expected future wealth is a function of tax rate uncertainty $\Delta$. Obviously, the impact is ambiguous and depends on the sign of the first-period tax base $(\pi_1 - d_1)$. If the tax base and thus the tax payment in the first period is positive, rising tax rate uncertainty raises expected future wealth, otherwise, if taxes are reimbursed, expected future wealth is reduced.

Time horizons of more than two periods yield similar results. Expected future wealth can be computed as the sum of compounded post-tax cash flows given the initial tax rate $\tau_0$ and an additional term summarizing the effects of uncertainty[7]:

$$\begin{aligned} \mathrm{E}_0\left[W_T^R\right] &= \sum_{t=1}^{T}\left[(1-\tau_0)\ \pi_t+\tau_0\, d_t\right]\left[1+i\ (1-\tau_0)\right]^{T-t} \\ &\quad +f_T^R\left(d_1,\ldots,d_T,i,\Delta,\pi_1,\ldots,\pi_T,\tau_0\right) \end{aligned} \tag{21}$$

with $f_T^R \equiv f_T^R\left(d_1,\ldots,d_T,i,\Delta,\pi_1,\ldots,\pi_T,\tau_0\right)$: uncertainty-dependent term in expected future wealth from real investment.

## 3.3 Relative impact of tax rate uncertainty

Analytical expressions for expected future values at hand, it is possible to analyze the impact of tax rate uncertainty on the relative advantage of real versus financial investment. To isolate the uncertainty effect, it is necessary to divide the difference of expected future values into a stochastic and a deterministic part. The latter is defined as the difference of future values taking the initial tax rate $\tau_0$ as constant, the former is the difference of the uncertainty-dependent terms:

$$\mathrm{E}_0\left[W_T^R\right]-\mathrm{E}_0\left[W_T^F\right] = \Phi_T+\phi_T^{RN} \tag{22}$$

$$\Phi_T \equiv \Phi_T\left(d_1,\ldots,d_T,i,\pi_1,\ldots,\pi_T\right) = W_T^R\big|_{\Delta=0}-W_T^F\big|_{\Delta=0} \tag{23}$$

$$\phi_T^{RN} \equiv \phi_T^{RN}\left(d_1,\ldots,d_T,i,\pi_1,\ldots,\pi_T\right) = f_T^R-f_T^F \tag{24}$$

[7]Uncertainty-dependent terms for $T = 2,3,4,5$ are given in appendix B. As in the case of financial investment, the uncertainty-dependent terms could not be proved in the general case of $T$ periods.

with $\Phi_T$: deterministic part of the difference of future values at time $T$
$\phi_T^{RN}$: uncertainty-dependent part of the difference of expected future values at time $T$ under risk neutrality.

This separation allows to identify possible effects of tax rate uncertainty even for real investment projects that are beneficial for all tax rates. Although tax rate uncertainty does not alter investment decisions in these cases, it is possible to quantify the change of the relative advantage.

In the two- and the three-period cases, the uncertainty-dependent expressions can be easily elaborated:

$$\phi_2^{RN} = i\,\Delta^2\,(\pi_1 - d_1 - i) \tag{25}$$

$$\phi_3^{RN} = 2\,\Delta^2\,i\,[\pi_1 - d_1\,(1 + i\,(1 - 2\,\tau_0)) + \pi_2 - d_2 - 2\,i\,(1 - (1 - \tau_0)\,(\pi_1 - i))]\,. \tag{26}$$

Interpreting eq. (25), it is obvious that real investment is encouraged in the two-period case if the first-year's tax base $\pi_1 - d_1$ exceeds the interest rate $i$ and is discouraged if it falls short of the interest rate[8]. This simple example shows that the conclusion that tax rate uncertainty always discourages investment cannot be confirmed. Rather, the effect of tax rate uncertainty may take either sign.

Although the expression in the three-period case is more complicated, eq. (26) indicates that the structure of cash flows and depreciation deductions is essential for the way tax rate uncertainty influences investment behavior. As a qualitative result, under linear depreciation deductions, real investment is encouraged by increased tax rate uncertainty if the underlying pre-tax cash flows follow a degressive pattern and discouraged if they follow a progressive pattern. Accordingly, under declining balance depreciations, real investment is less likely to be encouraged by increased uncertainty than under linear depreciation deductions. This does not alter the fact that declining balance depreciation tends to raise expected future value compared with linear depreciation under a complete loss-offset, for a given degree of tax rate uncertainty.

## 3.4 Numerical examples

The following numerical examples demonstrate that all kinds of tax effects are possible. The parameters are: $T = 3,\ i = 0.1,\ \tau_0 = 0.5,\ I_0 = 1,\ d_t = \frac{I_0}{T}$. To satisfy condition (6), the

[8]Note that all cash flows and depreciation deductions are normalized according to $I_0 = 1$.

tax change parameter $\Delta$ can be varied in the interval $\Delta \in [0, \frac{1}{6}[$. The effects of tax rate uncertainty will be demonstrated by computing the expected future values derived from eq. (61) in appendix B for real investment projects with different pre-tax cash flow structures. As a reference for real investment, the expected future value of financial investment is derived from eq. (52). The relative (dis-)advantage of real investment is given by the difference.

The first example is calculated using a pre-tax cash flow associated with real investment of $(\pi_1, \pi_2, \pi_3) = (0.25, 0.25, 0.727)$:

$$\begin{aligned} \mathrm{E}_0\left[W_3^R\right] &= 1.15798 - 0.028333\,\Delta^2 \\ \mathrm{E}_0\left[W_3^F\right] &= 1.157625 + 0.042\,\Delta^2 \\ \mathrm{E}_0\left[W_3^R\right] - \mathrm{E}_0\left[W_3^F\right] &= \Phi_3 + \phi_3^{RN} = 0.000354167 - 0.0703333\,\Delta^2 \end{aligned}$$

Due to the negative sign of $\phi_3^{RN}$, the pattern follows the popular view that increased tax rate uncertainty discourages real investment: Whereas real investment is beneficial under a deterministic tax rate $\tau_0 = 0.5$ ($\Delta = 0$), it becomes disadvantageous when the periodic tax rate change exceeds 7.1 percentage points ($\Delta > 0.07096$).

Using an investment project with the pre-tax cash flow $(\pi_1, \pi_2, \pi_3) = (0.6, 0.4, 0.1825)$, the opposite effect can be observed:

$$\begin{aligned} \mathrm{E}_0\left[W_3^R\right] &= 1.15742 + 0.0786667\,\Delta^2 \\ \mathrm{E}_0\left[W_3^F\right] &= 1.157625 + 0.042\,\Delta^2 \\ \mathrm{E}_0\left[W_3^R\right] - \mathrm{E}_0\left[W_3^F\right] &= \Phi_3 + \phi_3^{RN} = -0.000208333 + 0.0366667\,\Delta^2 \end{aligned}$$

Assuming sufficient tax rate uncertainty ($\Delta > 0.07538$), a risk neutral investor carries out real rather than financial investment.

The relative advantage of real investment remains constant despite changing tax rate uncertainty given the pre-tax cash flow $(\pi_1, \pi_2, \pi_3) = \left(2.1, -\frac{43}{30}, 0.4542\right)$:

$$\begin{aligned} \mathrm{E}_0\left[W_3^R\right] &= 1.15764 + 0.042\,\Delta^2 \\ \mathrm{E}_0\left[W_3^F\right] &= 1.157625 + 0.042\,\Delta^2 \\ \mathrm{E}_0\left[W_3^R\right] - \mathrm{E}_0\left[W_3^F\right] &= \Phi_3 + \phi_3^{RN} = 0.0000166667. \end{aligned}$$

### 3.5 Tax rate uncertainty and tax neutrality

Tax systems that are neutral with respect to investment decisions are well-known under certainty. Special cases are the Johansson-Samuelson tax (cf. Samuelson (1964), Johansson (1969)) and the cash flow tax (cf. Brown (1948)). Under cash flow uncertainty and risk neutrality, both systems are neutral with respect to investment decisions (cf. Niemann (1999) for a proof in a real options context). If the tax rate itself is stochastic, the concept of investment neutrality has to be stated more precisely. Transferring the concept from certainty unamendedly requires the complete ineffectiveness of taxation on investment decisions ("first-order neutrality"). In the context presented here, first-order neutrality conditions are:

$$\frac{\mathrm{d}\,\mathrm{E}_0\left[W_T^R - W_T^F\right]}{\mathrm{d}NPV_R} > 0 \quad \wedge \quad \mathrm{E}_0\left[W_T^R - W_T^F\right]_{NPV_R=0} = 0 \;\; \forall\,\Delta \tag{27}$$

with $NPV_R$: pre-tax NPV of a real investment project.

Interpretating the first condition in eq. (27) means that the decision criterion after stochastic taxes is a strictly increasing transformation of the pre-tax criterion regardless of the degree of tax rate uncertainty. The second condition requires that post-tax indifference occurs exactly in cases of pre-tax indifference.

In contrast, using a concept like "second-order neutrality" means that only the stochastic nature of taxation should not alter investment decisions whereas an impact of deterministic tax parameters might remain. Here, the decision criterion after stochastic taxes is required to be a strictly increasing transformation of the deterministic-tax criterion. Further, indifference after stochastic taxes should occur exactly when indifference after deterministic taxes exists. In our model, second-order neutrality conditions are:

$$\frac{\mathrm{d}\,\mathrm{E}_0\left[W_T^R - W_T^F\right]}{\mathrm{d}NPV_R^\tau} > 0 \quad \wedge \quad \mathrm{E}_0\left[W_T^R - W_T^F\right]_{NPV_R^\tau=0} = 0 \;\; \forall\,\Delta \tag{28}$$

$$NPV_R^\tau = \sum_{t=1}^{T} \frac{(1-\tau_0)\,\pi_t + \tau_0\,d_t}{\left(1 + i\,(1-\tau_0)\right)^t} - 1 \tag{29}$$

with $NPV_R^\tau$: deterministic-tax NPV of a real investment project.

#### 3.5.1 Johansson-Samuelson tax

The Johansson-Samuelson tax ensures neutrality by economic depreciation of all assets making *present* values invariant with respect to tax rates. Since interest income is subject to a Johansson-Samuelson tax, too, *future* values are not tax rate invariant.

In the two-period case, the first-order neutrality of a Johansson-Samuelson tax can be easily demonstrated. Expected future value of real investment is given by eq. (20). Substituting economic depreciation for the depreciation deductions

$$d_1^{JS} = PV_0 - PV_1 = \frac{\pi_1}{1+i} + \frac{\pi_2}{(1+i)^2} - \frac{\pi_2}{1+i} = \frac{(1+i)\ (\pi_1 - \pi_2) + \pi_2}{(1+i)^2} \tag{30}$$

$$d_2^{JS} = PV_1 - PV_2 = \frac{\pi_2}{1+i} - 0 = \frac{\pi_2}{1+i} \tag{31}$$

with $d_t^{JS}$: economic depreciation at time $t$
$PV_t$: pre-tax present value of an investment project at time $t$

yields the expected future value of real investment under economic depreciation:

$$\begin{aligned} \mathrm{E}_0\left[W_2^{R,JS}\right] &= \pi_1 \left[\frac{(1+i\ (1-\tau_0))^2}{1+i}\right] + \pi_2 \left[\frac{(1+i\ (1-\tau_0))^2}{(1+i)^2}\right] \\ &\quad + i^2\,\Delta^2 \left[\frac{\pi_1}{1+i} + \frac{\pi_2}{(1+i)^2}\right] \\ &= \left[(1+i\ (1-\tau_0))^2 + i^2\,\Delta^2\right] PV_0 \end{aligned} \tag{32}$$

with $W_T^{R,JS}$: future value of real investment under economic depreciation.

Identical transformations are applicable in the $T$-period case, so that the expected future value of real investment under a Johansson-Samuelson tax is given by:

$$\mathrm{E}_0\left[W_T^{R,JS}\right] = \left[(1+i\ (1-\tau_0))^T + f_T^F\right] PV_0. \tag{33}$$

According to eq. (18), the same holds for the expected future value of financial investment which by definition has a present value of $PV_0 = 1$:

$$\mathrm{E}_0\left[W_T^{R,JS} - W_T^F\right] = \left[(1+i\ (1-\tau_0))^T + f_T^F\right] NPV_R. \tag{34}$$

Thus, an investment project with a positive (negative) NPV before taxes yields a higher (lower) expected future value than financial investment under any degree of tax rate uncertainty. The decision criterion after uncertain taxation is a strictly increasing transformation of the pre-tax criterion. As a consequence, a Johansson-Samuelson tax can never alter investment decisions despite stochastic tax rates. This result is consistent with the property of tax rate invariance mentioned above.

In contrast, according to eq. (28), a tax system ensuring only second-order neutrality just requires that the decision criterion after uncertain taxation is an increasing transformation of the decision criterion under deterministic taxes given the initial tax rate $\tau_0$. Thus, a vanishing uncertainty-dependent part of the difference of expected future values $\phi_T^{RN}$ is sufficient for second-order neutrality. In the two-period case, this is reached if the first-period depreciation deductions equal cash flow less interest rate[9]:

$$\phi_2^{RN} = 0 \quad \Leftrightarrow \quad d_1 = \pi_1 - i. \tag{35}$$

Second-period depreciation deductions are irrelevant in this case. This implies that second-order neutrality may be achieved by depreciation deductions based on historical cost even for projects with a positive pre-tax NPV.

Of course, second-order neutrality is reached only by chance because a tax system that anticipates its own randomness is quite unrealistic. Nevertheless, it might be desirable to restrict tax effects on deterministic parameters to limit complexity of individual tax planning and to avoid estimations of tax risk. Second-order neutrality enhances a tax system's robustness by ensuring that tax effects occuring at the time of investment are maintained during the economic life of a project.

### 3.5.2 Cash flow tax

Another well-known neutral tax system under certainty is the cash flow tax (cf. Brown (1948)). It is characterized by an immediate write-off of initial outlays and no current depreciation deductions. Since interest income is effectively tax-free, tax rate uncertainty cannot affect future wealth from financial investment:

$$W_T^{F,CF} = (1+i)^T \tag{36}$$

with $W_T^{F,CF}$: future value of financial investment under a cash flow tax.

[9] For a zero pre-tax NPV, this equals economic depreciation in the two-period case.

The expected post-tax cash flow from eq. (14) has to be adopted to the cash flow tax which excludes depreciation deductions:

$$\mathrm{E}_0\left[\pi_t^{\tau,CF}\right] = (1 - \mathrm{E}_0[\tau_t])\ \pi_t = (1 - \tau_0)\ \pi_t. \tag{37}$$

Interest income from re-invested post-tax cash flows is tax-free, so expected future value from one unit of real investment is unaffected by tax rate uncertainty, too:

$$\mathrm{E}_0\left[W_T^{R,CF}\right] = \mathrm{E}_0\left[\sum_{t=1}^{T}(1-\tau_t)\ \pi_t\ (1+i)^{T-t}\right] = (1-\tau_0)\sum_{t=1}^{T}\pi_t\ (1+i)^{T-t}. \tag{38}$$

with $W_T^{R,CF}$: future value of real investment under a cash flow tax.

If the tax shield on initial outlay $\tau_0\, I_0 = \tau_0$ can be immediately re-invested in an identical investment project, in effect $\frac{1}{1-\tau_0}$ project units can be acquired. In this case, total expected future wealth from 1$ initial investment is given by

$$\mathrm{E}_0\left[W_T^{R,CF}\right] = \frac{1}{1-\tau_0}\ (1-\tau_0)\sum_{t=1}^{T}\pi_t\ (1+i)^{T-t} = \sum_{t=1}^{T}\pi_t\ (1+i)^{T-t}. \tag{39}$$

Otherwise, i.e., if the tax shield on initial outlay can only be re-invested at the capital market rate $i$, total expected future wealth from 1$ initial investment would amount to

$$\begin{aligned}\mathrm{E}_0\left[W_T^{R,CF}\right] &= (1-\tau_0)\sum_{t=1}^{T}\pi_t\ (1+i)^{T-t} + \tau_0\ (1+i)^T \\ &= (1+i)^T\ \left[(1-\tau_0)\ NPV_R + 1\right].\end{aligned} \tag{40}$$

In both cases, the cash flow tax is first-order neutral with respect to investment decisions under risk neutrality, regardless of the degree of tax rate uncertainty[10].

# 4 The impact of uncertain tax rates on investment under risk aversion

Under risk aversion, investors maximize expected utility from future wealth $W_T^F$ or $W_T^R$, respectively. The utility function $U(W_T)$ is assumed to be twice continuously differentiable, increasing, strictly concave and time-invariant.

[10]It should be noted that the cash flow tax is not neutral with respect to investment timing when tax rates are deterministically changing. However, in the context presented here, there is no timing decision.

Future wealth in the $2^T$ possible states can be computed for real and financial investment as demonstrated in the previous section. Expected utility is given by

$$\mathrm{E}_0\left[U\left(W_T^k\right)\right]=\frac{1}{2^T}\sum_{j\in J}U\left(W_{T,j}^k\right),\quad k=F,R. \tag{41}$$

The impact of tax rate uncertainty on investment behavior will be measured by certainty equivalents (CEs) which are defined as

$$\mathrm{CE}_0\left(W_T^k\right)=U^{-1}\left(\mathrm{E}_0\left[U\left(W_T^k\right)\right]\right),\quad k=F,R \tag{42}$$

with $\mathrm{CE}_t\left(\cdot\right)$: certainty equivalent based on information at time $t$.

Equivalent to maximization of expected utility, the investor maximizes an investment project's certainty-equivalent post-tax NPV:

$$\frac{\mathrm{CE}_0\left(W_T^R\right)-\mathrm{CE}_0\left(W_T^F\right)}{\mathrm{CE}_0\left(W_T^F\right)}=\frac{\mathrm{CE}_0\left(W_T^R\right)}{\mathrm{CE}_0\left(W_T^F\right)}-1\rightarrow\max! \tag{43}$$

A project is carried out if (43) is positive. Since tax rates are the only source of uncertainty the pre-tax decision criterion is still an investment project's deterministic pre-tax NPV.

Due to the utility function's non-linearity, closed-form solutions are unlikely. Accordingly, we have to focus on numerical simulations.

## 4.1 Financial investment

Even in the two-period case and for simple utility functions like logarithmic or power utility it is not possible to isolate the effects of varying $\Delta$ analytically. In contrast to risk neutrality, our numerical simulations in the three-period case indicate that the effect of tax rate uncertainty on expected utility from financial investment is ambiguous, depending on the utility function. The table included in appendix C denotes the CEs of future value of financial investment using different utility functions. Depending on the utility function, the CE of financial investment in $T=3$ can increase, decrease or remain (almost) constant if tax rate uncertainty $\Delta$ is increased.

## 4.2 Real investment

Using identical examples, the results from risk neutrality concerning real investment can be widely confirmed in a qualitative way. Under increased tax rate uncertainty, CEs of real investment can increase, decrease or remain constant. Of course, these results depend on the utility function under consideration as well as the structure of cash flows and depreciation deductions. Applying different (strictly concave) utility functions on identical projects, CEs of real investment may rise or fall. For comparison with risk neutrality the three examples calculated with the parameters $i = 0.1$; $\tau_0 = 0.5$; $d_t = \frac{1}{T}$; $T = 3$; and the pre-tax cash-flows $(\pi_1, \pi_2, \pi_3) = (0.25, 0.25, 0.727)$; $(0.6, 0.4, 0.1825)$; $\left(2.1, -\frac{43}{30}, 0.4524\right)$ already used under risk neutrality are included in appendix D.

## 4.3 Relative impact of tax rate uncertainty

Since the impact of increased tax rate uncertainty on CEs of both real and financial investment is ambiguous, it is necessary to isolate the uncertainty effect on the relative advantage. Like in section 3.3, the difference between CEs of real and financial investment is divided into a deterministic and a stochastic component:

$$\phi_T^{RA} = \mathrm{CE}_0\left[W_T^R\right] - \mathrm{CE}_0\left[W_T^F\right] - \Phi_T \tag{44}$$

with $\phi_T^{RA}$: uncertainty-dependent part of the difference of CEs at time $T$ under risk aversion,

where $\Phi_T$ is defined as in eq. (23). Again, the numerical results in appendix E indicate that $\phi_T^{RA}$ may take either sign, and unambiguous statements concerning effects of tax rate uncertainty cannot be made. In most cases, increased tax rate uncertainty tends to discourage real investment, but there are examples of rising as well as falling cash flows in which real investment is encouraged by tax rate uncertainty, depending on the utility function.

## 4.4 Tax rate uncertainty and tax neutrality under risk aversion

Under risk aversion, first-oder neutrality is defined by the following conditions:

$$\frac{\mathrm{d}\left[\mathrm{CE}_0\left(W_T^R\right) - \mathrm{CE}_0\left(W_T^F\right)\right]}{\mathrm{d}NPV_R} > 0 \;\wedge\; \mathrm{CE}_0\left(W_T^R\right) - \mathrm{CE}_0\left(W_T^F\right)\Big|_{NPV_R=0} = 0 \quad \forall\,\Delta, \tag{45}$$

i.e., the post-tax decision criterion is again required to be a strictly increasing transformation of the pre-tax criterion and pre-tax and post-tax indifference must occur in identical situations. Accordingly, second-order neutrality is satisfied if

$$\frac{\mathrm{d}\left[\mathrm{CE}_0\left(W_T^R\right)-\mathrm{CE}_0\left(W_T^F\right)\right]}{\mathrm{d}NPV_R^\tau}>0 \quad \wedge \quad \mathrm{CE}_0\left(W_T^R\right)-\mathrm{CE}_0\left(W_T^F\right)\big|_{NPV_R^\tau=0}=0 \quad \forall\,\Delta, \tag{46}$$

i.e., if the investment criterion after uncertain taxes is a strictly increasing transformation of the criterion after deterministic taxes and if post-tax indifference occurs in identical situations regardless of the degree of uncertainty.

Analytical expressions for CEs not at hand, the derivation of neutral tax systems becomes impossible. Therefore, we will restrict the discussion of neutral tax systems to the Johansson-Samuelson tax and the cash flow tax.

### 4.4.1 Johansson-Samuelson tax

Under certainty, the Johansson-Samuelson tax leaves present values unchanged even with deterministically changing tax rates. This property can be used in the context of stochastic tax rates applied to each possible path of the tax rate process. Future value of financial investment for path $j$ is defined by

$$W_{T,j}^F=\prod_{t=1}^{T}\left[1+i\left(1-\tau_t^j\right)\right],\ j=1,\ldots,2^T. \tag{47}$$

If the pre-tax present value of real investment for a given path $j\in J$ of tax rate realizations is $PV_0=c=const.$, then deductibility of economic depreciation ensures that the post-tax future value for path $j$ is $W_{T,j}^R=c\cdot W_{T,j}^F$. This property holds for all paths $j\in J$ of the tax rate process. Thus, expected utility at time $T$ is given by

$$\mathrm{E}_0\left[U\left(W_T^R\right)\right]=\frac{1}{2^T}\sum_{j\in J}U\left(W_{T,j}^R\right)=\frac{1}{2^T}\sum_{j\in J}U\left(c\,W_{T,j}^F\right) \tag{48}$$

for real investment and by

$$\mathrm{E}_0\left[U\left(W_T^F\right)\right]=\frac{1}{2^T}\sum_{j\in J}U\left(W_{T,j}^F\right) \tag{49}$$

for financial investment, respectively. Since marginal utility is assumed strictly positive the Johansson-Samuelson tax maintains its first-order neutrality irrespective of risk aversion even under tax rate uncertainty. In the special case of homogeneous utility functions the certainty-equivalent post-tax NPV equals the pre-tax NPV:

$$
\begin{aligned}
\mathrm{CE}_0\left(W_T^R\right) &= U^{-1}\left[\frac{1}{2^T}\, c^\eta \sum_{j\in J} U\left(W_{T,j}^F\right)\right] \qquad (50)\\
&= U^{-1}\left[c^\eta\, \mathrm{E}_0\left[U\left(W_T^F\right)\right]\right] = c\, \mathrm{CE}_0\left(W_T^F\right)\\
&\Rightarrow \frac{\mathrm{CE}_0\left(W_T^R\right)}{\mathrm{CE}_0\left(W_T^F\right)} - 1 = c - 1 = NPV_R
\end{aligned}
$$

with $c > 0$: constant
$\eta$: degree of homogeneity $[U(c\,W) = c^\eta\, U(W)]$.

### 4.4.2 Cash flow tax

Well known from deterministic-tax analysis of stochastic cash flows the cash flow tax loses its neutrality property if investors are assumed risk averse. This is also true for stochastic tax rates applied to deterministic cash flows. Regardless whether the tax shield on the inital outlay is immediately re-invested in an identical real investment project or at the capital market rate $i$, a simple two-period example with the parameters $(\pi_1, \pi_2) = (0.3, 0.89)$, $i = 0.1$, $\tau_0 = 0.5$, $\Delta \in (0.05, 0.08)$, $U(W) = \ln W$ disproves first-oder and second-order neutrality of the cash flow tax.

## 5 Summary and conclusion

This article examines the effects of tax rate uncertainty on investment behavior for risk-neutral and risk-averse investors. Since financial investment that always serves as a yardstick for real investment is subject to tax as well as real investment, tax effects on both real *and* financial investment have to be taken into account. In a simple capital budgeting model we compute expected future values and expected utility of deterministic cash flows using a deterministic pre-tax interest rate and stochastic tax rates. Tax rate uncertainty is modeled by an arithmetic binomial process.

Under risk neutrality, maximization of expected utility is equivalent to maximization of expected future value of investment. We find that increased tax rate uncertainty has an ambiguous impact on investment behavior. While it always increases expected future value of financial investment, it may raise or lower expected future value of real investment or leave it unchanged. It is even possible that it raises expected future value of real investment more than the one of financial investment, so that real investment may be encouraged or discouraged, depending on the structure of cash flows and depreciation deductions.

Under risk aversion, maximization of expected utility is equivalent to maximizing the certainty equivalent of future value of investment. Following our numerical results, an ambiguity similar to the one under risk neutrality is observed. Now, certainty equivalents of future values of financial as well as real investment may be raised, lowered or left unchanged by increased tax rate uncertainty. Again, unambiguous investment incentives cannot be obtained.

These results lead to the conclusion that the popular view that tax policy uncertainty depresses real investment has to be rejected. Investment incentives from tax policy uncertainty may take either sign.

Under tax policy uncertainty, tax neutrality has to be defined more precisely. First-order neutrality requires complete ineffectiveness of a tax system while second-order neutrality simply demands ineffectiveness of tax policy uncertainty with possible remaining tax effects from deterministic parameters.

Summarizing, the neutrality results known from certainty and cash flow uncertainty can be confirmed for tax rate uncertainty. Under risk neutrality, the Johansson-Samuelson tax and the cash flow tax maintain their neutrality property in the absence of timing flexibility. In contrast, under risk aversion, the Johansson-Samuelson tax is still neutral whereas the cash flow tax loses its neutrality property.

It should be noted that all statements concerning tax effects and tax neutrality are conditional on the model under examination. Therefore, it should be explored if these results can be confirmed under modified assumptions, e.g. in continuous-time models, for different tax rate processes and in real option-based models of timing of irreversible investment with multiple sources of uncertainty.

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# Appendix

## A Expected future value of financial investment

The uncertainty-dependent terms $f_T^F \equiv f_T^F(i, \Delta, \tau_0)$ of expected future value of financial investment for $T = 2, \ldots, T$ according to eq. (18) are given is the following equations:

$$
\begin{aligned}
f_2^F &= i^2\,\Delta^2 && (51)\\
f_3^F &= 4\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)] && (52)\\
f_4^F &= 10\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)]^2 + 5\,i^4\,\Delta^4 && (53)\\
f_5^F &= 20\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)]^3 + 40\,i^4\,\Delta^4\,[1 + i\,(1-\tau_0)] && (54)\\
f_6^F &= 35\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)]^4 + 175\,i^4\,\Delta^4\,[1 + i\,(1-\tau_0)]^2\\
&\quad +61\,i^2\,\Delta^2 && (55)\\
f_7^F &= 56\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)]^5 + 560\,i^4\,\Delta^4\,[1 + i\,(1-\tau_0)]^3\\
&\quad +768\,i^6\,\Delta^6\,[1 + i\,(1-\tau_0)] && (56)\\
f_8^F &= 84\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)]^6 + 1470\,i^4\,\Delta^4\,[1 + i\,(1-\tau_0)]^4\\
&\quad +4996\,i^6\,\Delta^6\,[1 + i\,(1-\tau_0)]^2 + 1385\,i^8\,\Delta^8 && (57)\\
f_9^F &= 120\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)]^7 + 3360\,i^4\,\Delta^4\,[1 + i\,(1-\tau_0)]^5\\
&\quad +22720\,i^6\,\Delta^6\,[1 + i\,(1-\tau_0)]^3 + 24320\,i^8\,\Delta^8\,[1 + i\,(1-\tau_0)] && (58)\\
f_{10}^F &= 165\,i^2\,\Delta^2\,[1 + i\,(1-\tau_0)]^8 + 6930\,i^4\,\Delta^4\,[1 + i\,(1-\tau_0)]^6\\
&\quad +81730\,i^6\,\Delta^6\,[1 + i\,(1-\tau_0)]^4 + 214445\,i^8\,\Delta^8\,[1 + i\,(1-\tau_0)]^2\\
&\quad +50521\,i^{10}\,\Delta^{10}. && (59)
\end{aligned}
$$

# B Expected future value of real investment

The uncertainty-dependent terms of expected future value of real investment for $T = 2, 3, 4, 5$ according to eq. (21) are given in the following equations

$$
\begin{aligned}
f_2^R &= i\,\Delta^2\,(\pi_1 - d_1) && (60)\\
f_3^R &= 2\,\Delta^2\,i\,(\pi_1 + \pi_2 - d_1 - d_2)\\
&\quad +2\,\Delta^2\,i^2\,[2\,(1-\tau_0)\,\pi_1 - (1 - 2\,\tau_0)\,d_1] && (61)\\
f_4^R &= \Delta^2\,i\,[3\,(\pi_1 - d_1) + 4\,(\pi_2 - d_2) + 3\,(\pi_3 - d_3)]\\
&\quad +\Delta^2\,i^2\,[13\,((1-\tau_0)\,\pi_1 + \tau_0\,d_1)\\
&\quad +7\,((1-\tau_0)\,\pi_2 + \tau_0\,d_2) - 6\,d_1 - 4\,d_2]\\
&\quad +\Delta^2\,i^3\,(1-\tau_0)\,[10\,((1-\tau_0)\,\pi_1 + \tau_0\,d_1) - 3\,d_1]\\
&\quad +5\,\Delta^4\,i^3\,(\pi_1 - d_1) && (62)\\
f_5^R &= \Delta^2\,i\,[4\,(\pi_1 - d_1) + 6\,(\pi_2 - d_2) + 6\,(\pi_3 - d_3) + 4\,(\pi_4 - d_4)]\\
&\quad +\Delta^2\,i^2\,[28\,((1-\tau_0)\,\pi_1 + \tau_0\,d_1) + 22\,((1-\tau_0)\,\pi_2 + \tau_0\,d_2)\\
&\quad +10\,((1-\tau_0)\,\pi_3 + \tau_0\,d_3) - 12\,d_1 - 12\,d_2 - 6\,d_3]\\
&\quad +2\,\Delta^2\,i^3\,(1-\tau_0)\,[22\,((1-\tau_0)\,\pi_1 + \tau_0\,d_1)\\
&\quad +8\,((1-\tau_0)\,\pi_2 + \tau_0\,d_2) - 6\,d_1 - 3\,d_2]\\
&\quad +4\,\Delta^2\,i^4\,(1-\tau_0)^2\,[(1-\tau_0)\,5\,\pi_1 - (1 - 5\,\tau_0)\,d_1]\\
&\quad +\Delta^4\,i^3\,[24\,(\pi_1 - d_1) + 16\,(\pi_2 - d_2)]\\
&\quad +\Delta^4\,i^4\,[40\,\pi_1\,(1-\tau_0) + 8\,d_1\,(5\,\tau_0 - 3)] && (63)
\end{aligned}
$$

# C Certainty equivalents of future wealth from financial investment

The following certainty equivalents are computed by using the parameters $i = 0.1;\ \tau_0 = 0.5;\ T = 3$:

| Table 1 | (1) | (2) | (3) |
|---|---|---|---|
| $\Delta$ | $U(W) = W$ | $U(W) = \ln W$ | $U(W) = W^{0.75}$ |
| 0 | 1.15763 | 1.15763 | 1.15763 |
| 0.01 | 1.15763 | 1.15762 | 1.15763 |
| 0.02 | 1.15764 | 1.15761 | 1.15763 |
| 0.03 | 1.15766 | 1.1576 | 1.15765 |
| 0.04 | 1.15769 | 1.15757 | 1.15766 |
| 0.05 | 1.15773 | 1.15755 | 1.15768 |
| 0.06 | 1.15778 | 1.15751 | 1.15771 |
| 0.07 | 1.15783 | 1.15747 | 1.15774 |
| 0.08 | 1.15789 | 1.15742 | 1.15778 |
| 0.09 | 1.15797 | 1.15737 | 1.15782 |
| 0.1 | 1.15804 | 1.15731 | 1.15786 |

| | (4) | (5) | (6) |
|---|---|---|---|
| $\Delta$ | $U(W) = W^{0.5}$ | $U(W) = W^{0.25}$ | $U(W) = 1 - e^{-0.75W}$ |
| 0 | 1.15763 | 1.15762 | 1.15763 |
| 0.01 | 1.15763 | 1.15762 | 1.15762 |
| 0.02 | 1.15763 | 1.15762 | 1.15762 |
| 0.03 | 1.15763 | 1.15761 | 1.15761 |
| 0.04 | 1.15763 | 1.1576 | 1.15759 |
| 0.05 | 1.15764 | 1.15759 | 1.15757 |
| 0.06 | 1.15764 | 1.15758 | 1.15755 |
| 0.07 | 1.15765 | 1.15756 | 1.15752 |
| 0.08 | 1.15766 | 1.15754 | 1.15749 |
| 0.09 | 1.15767 | 1.15752 | 1.15745 |
| 0.1 | 1.15768 | 1.15749 | 1.15741 |

Table 1: Certainty equivalents $\mathrm{CE}_0\left[W_3^F\right]$ of future wealth from financial investment for different utility functions.

# D Certainty equivalents of future wealth from real investment

The following examples are computed by using the general parameters already used for financial investment: $i = 0.1$; $\tau_0 = 0.5$; $T = 3$. The cash flows from real investment under consideration are: $(\pi_1, \pi_2, \pi_3) = (0.25, 0.25, 0.727)$; $(0.6, 0.4, 0.1825)$; $\left(2.1, -\frac{43}{30}, 0.4524\right)$.

| Table 2 | (1) | (2) | (3) |
|---|---|---|---|
| $\Delta$ | $U(W) = W$ | $U(W) = \ln W$ | $U(W) = W^{0.75}$ |
| 0 | 1.15798 | 1.15798 | 1.15798 |
| 0.01 | 1.15798 | 1.15796 | 1.15797 |
| 0.02 | 1.15797 | 1.15789 | 1.15795 |
| 0.03 | 1.15795 | 1.15778 | 1.15791 |
| 0.04 | 1.15793 | 1.15762 | 1.15786 |
| 0.05 | 1.15791 | 1.15742 | 1.15779 |
| 0.06 | 1.15788 | 1.15717 | 1.1577 |
| 0.07 | 1.15784 | 1.15687 | 1.1576 |
| 0.08 | 1.1578 | 1.15654 | 1.15748 |
| 0.09 | 1.15775 | 1.15615 | 1.15735 |
| 0.1 | 1.1577 | 1.15572 | 1.1572 |

| | (4) | (5) | (6) |
|---|---|---|---|
| $\Delta$ | $U(W) = W^{0.5}$ | $U(W) = W^{0.25}$ | $U(W) = 1 - e^{-0.75W}$ |
| 0 | 1.15798 | 1.15798 | 1.15798 |
| 0.01 | 1.15797 | 1.15796 | 1.15796 |
| 0.02 | 1.15793 | 1.15791 | 1.1579 |
| 0.03 | 1.15787 | 1.15782 | 1.1578 |
| 0.04 | 1.15778 | 1.1577 | 1.15766 |
| 0.05 | 1.15766 | 1.15754 | 1.15748 |
| 0.06 | 1.15752 | 1.15735 | 1.15726 |
| 0.07 | 1.15736 | 1.15712 | 1.157 |
| 0.08 | 1.15717 | 1.15685 | 1.1567 |
| 0.09 | 1.15695 | 1.15655 | 1.15637 |
| 0.1 | 1.15671 | 1.15622 | 1.15599 |

Table 2: Certainty equivalents $\mathrm{CE}_0\left[W_3^R\right]$ of future wealth from real investment for $(\pi_1, \pi_2, \pi_3) = (0.25, 0.25, 0.727)$.

| Table 3 | (1) | (2) | (3) |
|---|---|---|---|
| $\Delta$ | $U(W)=W$ | $U(W)=\ln W$ | $U(W)=W^{0.75}$ |
| 0 | 1.15742 | 1.15742 | 1.15742 |
| 0.01 | 1.15742 | 1.15742 | 1.15742 |
| 0.02 | 1.15745 | 1.15743 | 1.15744 |
| 0.03 | 1.15749 | 1.15744 | 1.15748 |
| 0.04 | 1.15754 | 1.15745 | 1.15752 |
| 0.05 | 1.15761 | 1.15747 | 1.15758 |
| 0.06 | 1.1577 | 1.1575 | 1.15765 |
| 0.07 | 1.1578 | 1.15753 | 1.15773 |
| 0.08 | 1.15792 | 1.15757 | 1.15783 |
| 0.09 | 1.15805 | 1.15761 | 1.15794 |
| 0.1 | 1.1582 | 1.15765 | 1.15806 |

| | (4) | (5) | (6) |
|---|---|---|---|
| $\Delta$ | $U(W)=W^{0.5}$ | $U(W)=W^{0.25}$ | $U(W)=1-e^{-0.75W}$ |
| 0 | 1.15742 | 1.15742 | 1.15742 |
| 0.01 | 1.15742 | 1.15742 | 1.15742 |
| 0.02 | 1.15744 | 1.15743 | 1.15743 |
| 0.03 | 1.15746 | 1.15745 | 1.15744 |
| 0.04 | 1.1575 | 1.15748 | 1.15747 |
| 0.05 | 1.15754 | 1.15751 | 1.15749 |
| 0.06 | 1.1576 | 1.15755 | 1.15753 |
| 0.07 | 1.15767 | 1.1576 | 1.15757 |
| 0.08 | 1.15774 | 1.15765 | 1.15761 |
| 0.09 | 1.15783 | 1.15772 | 1.15766 |
| 0.1 | 1.15793 | 1.15779 | 1.15772 |

Table 3: Certainty equivalents $\mathrm{CE}_0\left[W_3^R\right]$ of future wealth from real investment for $(\pi_1,\pi_2,\pi_3)=(0.6,0.4,0.1825)$.

| Table 4 | (1) | (2) | (3) |
|---|---|---|---|
| $\Delta$ | $U(W) = W$ | $U(W) = \ln W$ | $U(W) = W^{0.75}$ |
| 0 | 1.15764 | 1.15764 | 1.15764 |
| 0.01 | 1.15765 | 1.15754 | 1.15762 |
| 0.02 | 1.15766 | 1.15722 | 1.15755 |
| 0.03 | 1.15768 | 1.15668 | 1.15743 |
| 0.04 | 1.15771 | 1.15594 | 1.15727 |
| 0.05 | 1.15775 | 1.15498 | 1.15705 |
| 0.06 | 1.15779 | 1.1538 | 1.1568 |
| 0.07 | 1.15785 | 1.15241 | 1.15649 |
| 0.08 | 1.15791 | 1.1508 | 1.15614 |
| 0.09 | 1.15798 | 1.14897 | 1.15573 |
| 0.1 | 1.15806 | 1.14692 | 1.15528 |

| | (4) | (5) | (6) |
|---|---|---|---|
| $\Delta$ | $U(W) = W^{0.5}$ | $U(W) = W^{0.25}$ | $U(W) = 1 - e^{-0.75W}$ |
| 0 | 1.15764 | 1.15764 | 1.15764 |
| 0.01 | 1.15759 | 1.15756 | 1.15755 |
| 0.02 | 1.15744 | 1.15733 | 1.15727 |
| 0.03 | 1.15718 | 1.15693 | 1.15682 |
| 0.04 | 1.15682 | 1.15638 | 1.15617 |
| 0.05 | 1.15636 | 1.15567 | 1.15535 |
| 0.06 | 1.1558 | 1.1548 | 1.15434 |
| 0.07 | 1.15513 | 1.15377 | 1.15315 |
| 0.08 | 1.15436 | 1.15258 | 1.15177 |
| 0.09 | 1.15348 | 1.15123 | 1.15022 |
| 0.1 | 1.1525 | 1.14971 | 1.14848 |

Table 4: Certainty equivalents $\mathrm{CE}_0\left[W_3^R\right]$ of future wealth from real investment for $(\pi_1, \pi_2, \pi_3) = \left(2.1, -\frac{43}{30}, 0.4524\right)$.

# E Impact of tax rate uncertainty on the relative advantage of real vs. financial investment

The following tables contain the separated effect of tax rate uncertainty on the relative advantage of real versus financial investment. In accordance with the examples mentioned above, the general parameters are $i = 0.1$; $\tau_0 = 0.5$; $T = 3$, the cash flows from real investment: $(\pi_1, \pi_2, \pi_3) = (0.25, 0.25, 0.727)$; $(0.6, 0.4, 0.1825)$; $\left(2.1, -\frac{43}{30}, 0.4524\right)$. As mentioned in eq. (44), the effect is given by $\phi_T^{RA} = \text{CE}_0\left[W_T^R\right] - \text{CE}_0\left[W_T^F\right] - \Phi_T$.

| Table 5 | (1) | (2) | (3) |
|---|---|---|---|
| $\Delta$ | $U(W) = W$ | $U(W) = \ln W$ | $U(W) = W^{0.75}$ |
| 0 | 0 | 0 | 0 |
| 0.01 | $-7.03333 \cdot 10^{-6}$ | -0.0000193627 | -0.000101156 |
| 0.02 | -0.0000281333 | -0.0000774596 | -0.0000404638 |
| 0.03 | -0.0000633 | -0.000174317 | -0.0000910486 |
| 0.04 | -0.000112533 | -0.000309978 | -0.000161877 |
| 0.05 | -0.000175833 | -0.000484503 | -0.000252957 |
| 0.06 | -0.0002532 | -0.000697973 | -0.000364303 |
| 0.07 | -0.000344633 | -0.000950482 | -0.000495928 |
| 0.08 | -0.000450133 | -0.00124215 | -0.00064785 |
| 0.09 | -0.0005697 | -0.0015731 | -0.00082009 |
| 0.1 | -0.000703333 | -0.00194349 | -0.00101267 |

| | (4) | (5) | (6) |
|---|---|---|---|
| $\Delta$ | $U(W) = W^{0.5}$ | $U(W) = W^{0.25}$ | $U(W) = 1 - e^{-0.75W}$ |
| 0 | 0 | 0 | 0 |
| 0.01 | -0.0000131979 | -0.0000162803 | -0.0000177426 |
| 0.02 | -0.000052795 | -0.000065127 | -0.0000709704 |
| 0.03 | -0.000118801 | -0.000146557 | -0.000159684 |
| 0.04 | -0.000211232 | -0.0002606 | -0.000283884 |
| 0.05 | -0.000330112 | -0.000407294 | -0.000443571 |
| 0.06 | -0.000475469 | -0.000586693 | -0.000638747 |
| 0.07 | -0.000647339 | -0.00079886 | -0.000869413 |
| 0.08 | -0.000845767 | -0.00104387 | -0.00113557 |
| 0.09 | -0.0010708 | -0.00132181 | -0.00143722 |
| 0.1 | -0.0013225 | -0.00163278 | -0.00177437 |

Table 5: Impact of tax rate uncertainty on the difference of certainty equivalents of future wealth from real investment $\phi_3^{RA}$ for $(\pi_1, \pi_2, \pi_3) = (0.25, 0.25, 0.727)$ and financial investment.

| Table 6 | (1) | (2) | (3) |
|---|---|---|---|
| $\Delta$ | $U(W) = W$ | $U(W) = \ln W$ | $U(W) = W^{0.75}$ |
| 0 | 0 | 0 | 0 |
| 0.01 | $3.66667 \cdot 10^{-6}$ | $5.48135 \cdot 10^{-6}$ | $4.12034 \cdot 10^{-6}$ |
| 0.02 | 0.0000146667 | 0.0000219252 | 0.0000164813 |
| 0.03 | 0.000033 | 0.0000493307 | 0.0000370826 |
| 0.04 | 0.0000586667 | 0.0000876966 | 0.0000659239 |
| 0.05 | 0.0000916667 | 0.000137021 | 0.000103005 |
| 0.06 | 0.000132 | 0.000197302 | 0.000148324 |
| 0.07 | 0.000179667 | 0.000268536 | 0.000201882 |
| 0.08 | 0.000234667 | 0.000350721 | 0.000263677 |
| 0.09 | 0.000297 | 0.000443852 | 0.000333708 |
| 0.1 | 0.000366667 | 0.000547924 | 0.000411973 |

| | (4) | (5) | (6) |
|---|---|---|---|
| $\Delta$ | $U(W) = W^{0.5}$ | $U(W) = W^{0.25}$ | $U(W) = 1 - e^{-0.75W}$ |
| 0 | 0 | 0 | 0 |
| 0.01 | $4.57401 \cdot 10^{-6}$ | $5.02768 \cdot 10^{-6}$ | $5.24305 \cdot 10^{-6}$ |
| 0.02 | 0.0000182959 | 0.0000201105 | 0.0000209716 |
| 0.03 | 0.0000411652 | 0.0000452479 | 0.0000471841 |
| 0.04 | 0.0000731813 | 0.0000804389 | 0.0000838777 |
| 0.05 | 0.000114343 | 0.000125682 | 0.000131048 |
| 0.06 | 0.00016465 | 0.000180975 | 0.000188691 |
| 0.07 | 0.000224099 | 0.000246317 | 0.0002568 |
| 0.08 | 0.000292689 | 0.000321704 | 0.000335368 |
| 0.09 | 0.000370419 | 0.000407133 | 0.000424387 |
| 0.1 | 0.000457285 | 0.000502602 | 0.000523846 |

Table 6: Impact of tax rate uncertainty on the difference of certainty equivalents of future wealth of real investment $\phi_3^{RA}$
for $(\pi_1, \pi_2, \pi_3) = (0.6, 0.4, 0.1825)$ and financial investment.

| Table 7 | (1) | (2) | (3) |
|---|---|---|---|
| $\Delta$ | $U(W)=W$ | $U(W)=\ln W$ | $U(W)=W^{0.75}$ |
| 0 | 0 | 0 | 0 |
| 0.01 | 0 | -0.000103271 | -0.0000258172 |
| 0.02 | 0 | -0.000413174 | -0.000103285 |
| 0.03 | 0 | -0.000929982 | -0.00023245 |
| 0.04 | 0 | -0.00165415 | -0.000413394 |
| 0.05 | 0 | -0.00258632 | -0.00064623 |
| 0.06 | 0 | -0.00372732 | -0.000931102 |
| 0.07 | 0 | -0.00507818 | -0.00126819 |
| 0.08 | 0 | -0.00664012 | -0.00165771 |
| 0.09 | 0 | -0.00841456 | -0.0020999 |
| 0.1 | 0 | -0.0104032 | -0.00259506 |

| | (4) | (5) | (6) |
|---|---|---|---|
| $\Delta$ | $U(W)=W^{0.5}$ | $U(W)=W^{0.25}$ | $U(W)=1-e^{-0.75W}$ |
| 0 | 0 | 0 | 0 |
| 0.01 | -0.0000516349 | -0.0000774529 | -0.0000896543 |
| 0.02 | -0.000206578 | -0.000309876 | -0.00035859 |
| 0.03 | -0.000464946 | -0.000697464 | -0.000806727 |
| 0.04 | -0.000826932 | -0.00125054 | -0.00143393 |
| 0.05 | -0.00129281 | -0.00193957 | -0.00224001 |
| 0.06 | -0.00186293 | -0.00279513 | -0.00322473 |
| 0.07 | -0.00253773 | -0.00380796 | -0.0043878 |
| 0.08 | -0.00331773 | -0.00497892 | -0.00572886 |
| 0.09 | -0.00420353 | -0.00630904 | -0.00724752 |
| 0.1 | -0.00519583 | -0.00779948 | -0.00894333 |

Table 7: Impact of tax rate uncertainty on the difference of certainty equivalents of future wealth of real investment $\phi_3^{RA}$ for $(\pi_1, \pi_2, \pi_3) = \left(2.1, -\frac{43}{30}, 0.4524\right)$ and financial investment.