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ESTATE TAXATION

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Abstract

In this paper we examine the properties of the optimal linear estate tax in the presence of a complete set of tax instruments available to the social planner. We allow for both types of bequest motives, namely altruistic and accidental. We examine the case for estate taxation which seems to be the strongest (but not impeccable) with accidental bequests. In general, the estate tax is highly sensitive to the relative importance of the two bequest motives.

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1. Introduction

Whether and how to tax estates has long been a highly controversial issue and is still subject to ongoing debate¹. In the US, tax is levied on estates over 675,000 dollars, where the marginal tax rates begin with 37% and rise to 55% for the wealthiest, by all means highly progressive. Advocates of estate tax accentuate its role in redistributing wealth². Opponents (some of them call ardently for complete abolition of the tax³) emphasize the 'hole in the leaky bucket' and raise efficiency arguments as regard to the entailed disincentives: discouraging capital accumulation, reducing incentives to work and bankrupting small (family) firms, inter-alia.

In order to assess the relevance of estate taxation, one should construct an integrated framework that takes all the considerations above into account, thereby balancing the benefits vis-a-vis the costs, while incorporating other tax instruments (notably the income tax) that serve similar purposes and suffer from akin pitfalls. This is the aim of this paper⁴.

Empirical studies indicate that inheritances account for as much as 80% of aggregate wealth (see Kotlikoff and Summers 1981). More conservative estimates (see Davies and Shorrocks 1999) show that 35-45% of total wealth is inherited, hardly negligible. Given that the size distribution of gross estates is highly skewed⁵, it seems,

¹ Paraphrasing Benjamin Franklin's famous adage, the debate on death tax seems to be immortal.

² Historically, inheritance taxation has been first introduced during the Roman-Empire age. The tax collections served for redistribution purposes - to finance retirement benefits to veteran soldiers.

³ The motion in congress has been halted by a veto by President Clinton, likely to be reversed following the change of guards.

⁴ Some works take a preliminary step in this direction. Kopczuk (2000) examines rigorously the case for a non-linear estate tax in the absence of income tax (or other redistributing instruments) in a dynamic setting. Kaplow (2000) and Gale and Slemrod (2001) provide a preliminary view of the major issues. Cremer and Pestieau (2001) introduce a simple static framework of a two-type economy and design the optimal non-linear estate tax. The paper illustrates the distinction between estate and inheritance tax, when descendants differ in innate ability. Inheritance tax is shown to implement desirable sharing rules. The paper briefly discusses the introduction of income taxation and shows that the results remain robust to this modification. The main thrust of the paper is the trade-off between reducing inequality within households and across ones, given informational constraints of the planner with respect to parental wealth and descendants' abilities.

⁵ Gale and Slemrod (2001) indicate that the 89 percent of tax returns with gross estate below \$2.5 million accounted for 53 percent of total gross estates, whereas the 4.1 percent with \$5 million and above accounted for 32 percent of gross estate value.

that estate taxation should play a crucial role in reducing inequality in wealth distribution. The other side of the coin, that is the efficiency cost of estate taxation, is far more contentious. The cost crucially depends on the underlying assumptions regarding the bequest motives.

Bequests could be intended, unintended (accidental) or a hybrid of the two. Intended bequests may derive from either altruism or as a source of payment for services provided by potential heirs. From a tax perspective, both constitute two forms of consumption and should be treated accordingly. If preferences take the Barro-type form of altruism (Barro 1974), then following Chamley (1986), one can conclude that the long run optimal linear tax on inheritances converges to zero. The compound effect of capital (inheritance) tax on future consumption is so large so as to outweigh the potential benefits from redistribution across heterogeneous agents. However, in the context of inheritance, the long run that could well spread over many generations to come might be somewhat irrelevant for practical policy design. We therefore concentrate on the short run that can spread over just a few generations.

The other bequest motive is related to imperfections in annuity markets or precautionary saving against uncertain future health expenses. If life span is uncertain, unintended bequests derive from the inability to purchase actuarially fair annuities in the market. This would be, *prima facie*, the strongest case for estate taxation, for such taxation entails no efficiency cost (see Hurd 1989), and may potentially lead to levying a hundred percent tax on estate (as suggested by the Communist Manifesto, which calls for complete abolition of inheritance rights!).

It is clearly of crucial importance to determine the relative magnitude of the two bequest motives, given the starkly different implications they bear on the design of the optimal estate tax. The literature provides us, however, with mixed empirical evidence⁶. The presence of *inter-vivo* gifts, estate planning and life insurance purchases support an

⁶ As indicated by Pestieau and Poterba (2001): "There is still research controversy about the reasons households make intergenerational transfers, and the importance of altruistic as opposed to unintended bequests." Laitner (2001) indicates that the work to date has yielded ambiguous results.

altruistic motive. Evidence supporting accidental bequest includes, inter-alia, the failure of *Recardian Equivalence*, observed equal inheritances for siblings with different incomes, and the fact that the distribution of consumption within extended families is strongly dependent on the distribution of resources within the extended families.

In the present study we choose a unified framework and allow for both bequest motives. We confine attention to the simple case of a linear tax regime and examine the properties of the optimal estate⁷ tax when other redistributing tax instruments (commodity and income taxation) are available. We also allude to the possibility of discrepancies between the parents' welfare and the social welfare criterion. In particular, such a discrepancy may arise when the social planner assigns a social weight to the offspring's welfare per se, on top of the weight assigned to them through the welfare of their parents.

The organization of the paper is as follows. Section 2 develops a basic analytical model of bequests. The next section examines the case for estate taxation. Section 4 analyzes a case for zero estate taxation. A special case of purely accidental bequests is discussed in section 5. An illuminating example, where an explicit estate tax formula is derived is presented in section 6. Section 7 concludes.

2. A Basic Model of Bequests

We try to construct as simple a model as possible with just the essential features that are needed to study the effects of estate tax and its relevance. Suppose there are just two generations. The first generation, born in the first period, lives for (at most) two periods and the second generation, born at the beginning of the second period, lives for one period. Without loss of generality, we assume that the size of each cohort is normalized to unity. A member of the first generation works and consumes a single all-purpose good during the first period of life. During the second period, the individual retires and lives from savings. The length of the spell of retirement is uncertain. The

⁷ In our framework the distinction between estate and inheritance tax is irrelevant, since by assumption there is only one descendant for each household.

individual dies at the beginning of the second period with probability $0 < 1-p < 1$ or survives with the complementary probability p for the whole of the second period. Death risk is assumed to be independent across agents. A member of the second generation works and consumes a single all-purpose good during the second period. Each individual is endowed with one unit of time in the first period of life.

The population is assumed to be heterogeneous. Each dynasty, consisting of one member of the first generation (parent) and one member of the second generation (descendant), is characterized by a parameter, we refer to as innate (earning) ability denoted by w ⁸. Following the seminal work of Mirrlees (1971), we assume that innate ability is private information, thus unobservable by the government (social planner) that only observes total income. Innate ability is assumed to be distributed according to the cumulative distribution function $F(w)$ over the support (\underline{w}, \bar{w}) . We assume that for each type of innate ability, a proportion of exactly $1-p$ of the dynasties dies at the beginning of the second period. Thus for each type of ability there is no aggregate risk and therefore no aggregate risk for the economy as a whole.

We assume that parents are altruistic and take into account the utility derived by their offspring. By assuming a Barro-type form of altruism, the allocation of each dynasty is determined by maximization of the discounted expected utility from the point of view of the first generation (the parents). For the sake of simplicity we assume that all dynasties have identical preferences. Thus, innate ability is the only ex-ante parameter that differs across dynasties. Preferences are given by the expected utility function defined over leisure and consumption in both periods and both states of nature ("die" or "survive"):

$$(1) \quad U(l_1, l_3, l_4, c_1, c_2, c_3, c_4) = u(l_1, c_1) + p[v(c_2) + \delta h(l_3, c_3)] + (1-p)\delta h(l_4, c_4),$$

⁸ We thus implicitly assume that abilities of parent and child are identical, reflecting the evidence that intergenerational correlation of incomes may be higher than 0.5 (See Solon 1992).

with $u(\cdot)$, $v(\cdot)$ and $h(\cdot)$ strictly concave, strictly increasing and twice continuously differentiable. We denote by (l_1, c_1) the leisure-consumption bundle consumed by the first generation in the first period. If the first generation survives to the second period, it consumes c_2 and the second generation consumes the bundle (l_3, c_3) ; otherwise the second generation consumes the bundle (l_4, c_4) . We denote by $\delta \geq 0$ the degree of altruism. When $\delta = 0$ there is no altruism and bequests are purely unintended.

We assume that the market for annuities is incomplete, therefore individuals cannot perfectly diversify the risk of uncertain lifetime. To capture this incompleteness at its extreme, we just assume that annuities do not exist.

We start by assuming that the government has a complete set of tax instruments that include consumption tax at each period (t_1^c, t_2^c) , an interest income tax (t^r) , a labor income tax in each period (t_1^l, t_2^l) , uniform lump-sum transfers to the young cohort in each period (τ_1, τ_2) ⁹ and an estate tax (t^e) .

A typical dynasty of type w (representing the wage rate) maximizes the expected utility (1) subject to a set of budget constraints (faced by all generations and in all states of nature) as follows:

In the first period, the parent works, consumes and saves (S):

$$(2) \quad \tau_1 + (1 - t_1^l)w(1 - l_1) = (1 + t_1^c)c_1 + S$$

If she dies at the beginning of the second period, her entire saving is bequeathed to her offspring who faces the following budget constraint:

⁹ The assumption that the transfer is paid to the young cohort only is purely for simplifying the notation. One can also interpret the transfer as a part of a linear labor income tax, which naturally applies to the working cohort (the young cohort in our case).

$$(3) \quad \tau_2 + (1-t^e)S[1 + (1-t^r)r] + (1-t_2^l)w(1-l_4) = (1+t_2^c)c_4$$

We denote by r the interest rate. We assume the rate is fixed, either due to a linear technology or by assuming a small open economy.

If the parent survives to the next period, she then consumes part of her savings and bequeaths the remainder (b):

$$(4) \quad S[1 + r(1-t^r)] = (1+t_1^c)c_2 + b$$

Her offspring then faces the following budget constraint:

$$(5) \quad \tau_2 + (1-t^e)b + (1-t_2^l)w(1-l_3) = (1+t_2^c)c_3$$

Note that b is a purely intended bequest. On the other hand the amount bequeathed by parent when she dies early (namely, her entire savings with the accumulated interest) is only partly unintended.

Maximization of the utility function (1), subject to the budget constraints given by (2)-(5) yields the consumption and leisure demand functions, $c_i(w, t_1^c, t_2^c, t_1^l, t_2^l, t^e, t^r, \tau_1, \tau_2)$, $i=1,2,3,4$, and $l_i(w, t_1^c, t_2^c, t_1^l, t_2^l, t^e, t^r, \tau_1, \tau_2)$, $i=1,2,3,4$. Substituting these functions into (1) yields the indirect utility function $\bar{U}(w, \cdot)$. We also obtain the saving function, $S(w, \cdot)$, and intended bequest function, $b(w, \cdot)$.

Note that the ex-ante plan of the parent generation for her offspring will be fully implemented ex-post by the offspring. Namely, the optimal plan is dynamically consistent. Note further that by assuming that the offspring generation lives for one period only, a more suitable interpretation of b , the intended bequest, would be as an *inter-vivo* gift.

The government (social planner) determines the optimum tax rates so as to maximize some social welfare function, subject to a revenue constraint. For concreteness, we consider a utilitarian social welfare function:

$$(6) \quad W(\cdot) = \int_{\underline{w}}^{\bar{w}} \bar{U}(w, \cdot) dF(w) + \beta \int_{\underline{w}}^{\bar{w}} \bar{h}(w, \cdot) dF(w),$$

where $\bar{h}(w, \cdot) = ph[l_3(w, \cdot), c_3(w, \cdot)] + (1-p)h[l_4(w, \cdot), c_4(w, \cdot)]$ is the expected utility of the second generation of type w . Note that the welfare of the second generation is already incorporated into the social welfare function, W , through the parent's utility, \bar{U} . However, the social welfare may also assign a positive weight to welfare of the offspring generation per se. This is captured by the parameter $\beta \geq 0$. When the social welfare function assigns equal weights to the parent's welfare and the offspring's welfare and counts also the offspring's welfare derived by the parent, then $\beta = 1$. When both count equally, but the offspring's welfare is 'laundered out' of the parent's welfare, then $\beta = 1 - \delta$.

To simplify the notation, we henceforth denote integration (summation) over all dynasties by the expectation operator, so that the social welfare function will be written as:

$$(6') \quad W(\cdot) = E[\bar{U}(w, \cdot)] + \beta E[\bar{h}(w, \cdot)]$$

Note that when $\beta \neq 0$, then the objective of the government (represented by W) does not coincide with the objective of the parents (represented by $E(\bar{U})$).

Naturally, the government can borrow or lend at the prevailing interest rate (r), so that its budget is balanced, in present value terms, over the two periods:

$$(7) \quad t_1^c E(c_1) + t_2^c [pE(c_2) + pE(c_3) + (1-p)E(c_4)] / (1+r) + t^r rE(S) / (1+r) + t_1^l E[w(1-l_1)] \\ + t_2^l E[pw(1-l_3) + (1-p)w(1-l_4)] / (1+r) + t^e \left((1-p)E\{S[1+(1-t^r)r]\} + pE(b) \right) / (1+r) \\ - \tau_1 - \tau_2 / (1+r) \geq 0,$$

where the arguments of the functions are omitted in order to abbreviate the notation. Recall our assumption that there is exactly a proportion p of each type of dynasties (respectively, $1-p$) that survives to the end (respectively, dies at the beginning) of the second period. Thus, the inter-temporal budget constraint is balanced with certainty and not only in expected value.¹⁰

3. The Case for Estate Taxation

Before we turn to find the optimum of the various tax-instruments, (by maximizing (6) subject to (7)), we first address a preliminary issue. Armed with a complete set of tax instruments, we would like to know whether one could do without estate taxation, that is: whether we have enough tax instruments to substitute for estate taxation, irrespective of the structure of preferences.

To proceed, we first consolidate the four budget constraints, (2)-(5), into just two constraints, corresponding to each one of the states of nature ("die" or "survive"). If the parent survives to the end of the second period, then we can substitute for b and S from (2) and (4) into (5) to obtain:

$$(8) \quad \tau_1 + [\tau_2 / (1-t_e)] / [1+r(1-t^r)] + (1-t_1^l)w(1-l_1) + [(1-t_2^l) / (1-t^e)]w(1-l_3) / [1+r(1-t^r)] \\ = (1+t_1^c)c_1 + (1+t_2^c)c_2 / [1+r(1-t^r)] + [(1+t_2^c) / (1-t^e)]c_3 / [1+r(1-t^r)]$$

In the case where the parent dies in the beginning of the second period, we obtain the following budget constraint by substituting (2) into (3):

¹⁰ Note that the independence of death risk assumption would suffice to eliminate aggregate fiscal risk for large populations, applying the law of large numbers.

$$(9) \quad \begin{aligned} & \tau_1 + [\tau_2 / (1 - t_e)] / [1 + r(1 - t^r)] + (1 - t_1^l)w(1 - l_1) + [(1 - t_2^l) / (1 - t^e)]w(1 - l_4) / [1 + r(1 - t^r)] \\ & = (1 + t_1^c)c_1 + [(1 + t_2^c) / (1 - t^e)]c_4 / [1 + r(1 - t^r)] \end{aligned}$$

Close inspection of (8) and (9) reveals that estate tax is not redundant. The effective tax on c_2 is $(1 + t_2^c) / [1 + r(1 - t^r)] \cdot (1 + r) - 1$, whereas the effective tax levied on c_3 and c_4 is $[(1 + t_2^c) / (1 - t^e)] / [1 + r(1 - t^r)] \cdot (1 + r) - 1$. Thus we can see that estate taxation serves to differentiate between consumption of the retired parent (c_2) and her descendant (c_3 or c_4). In general, one would like to preserve the ability to employ differential taxation of consumption goods. This holds true even in a one-dynasty Ramsey setting, where only efficiency considerations apply¹¹. All the more so when dynasties are heterogeneous. In this case one would like to exploit variations in bequest patterns across dynasties to enhance redistribution.

One can observe, though, that interest taxation is redundant, by redefining $\tau_2^* = \tau_2 / (1 + r(1 - t^r))$, $(1 + t_2^{c*}) = (1 + t_2^c) / (1 + r(1 - t^r))$ and $(1 - t_2^{l*}) = (1 - t_2^l) / (1 + r(1 - t^r))$ as the transfer to the young cohort of second generation, the consumption tax in the second period and the labor income tax in the second period, respectively. One can further reduce the set of tax instruments, by setting a single lump-sum transfer per dynasty (τ^*) which is equal to the present value of the two transfers. By virtue of the constant returns to scale assumption, one can choose arbitrarily one of the commodities as an untaxed good. This would eventually leave us with a system of five tax instruments that includes estate tax. As the interest tax is redundant, we simplify the notation by setting $r=0$. The simplified budget constraints (8) and (9) are re-written below:

$$(8') \quad \tau + (1 - t_1^l)w(1 - l_1) + (1 - t_2^{l*})w(1 - l_3) = c_1 + (1 + t_2^c)c_2 + (1 + t_3^c)c_3$$

$$(9') \quad \tau + (1 - t_1^l)w(1 - l_1) + (1 - t_2^{l*})w(1 - l_4) = c_1 + (1 + t_3^c)c_4$$

¹¹ Naturally, in a one-dynasty setting one would drop the lump-sum transfers (namely τ_i , $i=1,2$) to render the optimal tax problem non-trivial.

Note that the estate tax (t^e) is implicitly defined by $(1+t_3^c) = (1+t_2^c)/(1-t^e)$. Note further that c_1 is chosen as the untaxed good, without loss of generality.

4. A Case for Zero Estate Taxation

4.1 A Special Case: Zero Estate Taxation

The literature of optimal taxation has dealt extensively with the question of the relevance of commodity taxation. A well-known result in the literature is that a proportional wage tax is equivalent to a uniform excise tax. Differentiated commodity taxation introduces wedges between consumer and producer price ratios of the consumption goods. These distortions are on top of the distortions that stem from the wedge between the consumer and producer price ratios of the group of consumption goods and leisure. In our context that focuses on equity considerations, one may justify such non-uniform taxation on the basis of observed variation across different households in the allocation of income across consumption goods. Otherwise, labor income is a "sufficient statistics", and one can do without non-uniform commodity taxation.

Weak separability between the group of the consumption goods and leisure and *homotheticity* with respect to the consumption goods imply, indeed, that the allocation of income across consumption goods is independent of innate ability. Thus, a linear labor income tax comprised of a flat rate and a lump-sum transfer suffices to maximize social welfare; see, for instance, Deaton (1981) and Atkinson and Stiglitz (1972).

We turn next to examine whether this result extends to our formulation. Note that the estate tax in our formulation serves to distinguish between contemporaneous consumption of the old and the young generations in period two.

We ask whether the assumptions about *homotheticity* and *weak separability* stated above, imply that commodity taxation including estate taxation becomes redundant. In

order to be in line with the above-mentioned studies, we assume in this section that no externalities exist, that is $\beta = 0$ (see equation (6) above).

Let us first consider the special case of purely intended bequest. That is, assume first that $p = 1$. In this case we are left with a single budget constraint for each dynasty, given by (8'). If $U(\cdot)$ satisfies *homotheticity* and *weak separability* between the group of consumption goods (namely: c_1, c_2 , and c_3) and the group of l_1 and l_3 ¹², then, indeed, commodity taxation becomes redundant, for in this case all types of dynasties obtain identical linear Engel curves for all consumption goods.

4.2 The General Case: Nonzero Estate Taxation

We turn next to examine the case of uncertain lifetime, which allows for accidental bequest. Note that each dynasty is seeking to maximize (1) subject to budget constraints (8') and (9'). Forming a *Lagrangian*, and denoting by α and μ the *Lagrange* multipliers for (8') and (9'), respectively, we obtain the following seven first-order conditions with respect to $c_1, c_2, c_3, c_4, l_1, l_3$ and l_4 :

$$(10) \quad \partial U / \partial c_1 = \partial u / \partial c_1 = (\alpha + \mu)(1 + t_1^c)$$

$$(11) \quad \partial U / \partial l_1 = \partial u / \partial l_1 = (\alpha + \mu)w(1 - t_1^l)$$

$$(12) \quad \partial U / \partial c_2 = p\partial v / \partial c_2 = \alpha(1 + t_2^c)$$

$$(13) \quad \partial U / \partial c_3 = p\delta \cdot \partial h / \partial c_3 = \alpha(1 + t_3^c)$$

$$(14) \quad \partial U / \partial l_3 = p\delta \cdot \partial h / \partial l_3 = \alpha w(1 - t_2^l)$$

$$(15) \quad \partial U / \partial c_4 = (1 - p)\delta \cdot \partial h / \partial c_4 = \mu(1 + t_3^c)$$

$$(16) \quad \partial U / \partial l_4 = (1 - p)\delta \cdot \partial h / \partial l_4 = \mu w(1 - t_2^l)$$

To render commodity taxation (including the estate tax) redundant, the marginal propensity to consume any consumption good must be constant (with respect to income -

¹² The utility function takes the following form: $U(l_1, l_3, F(c_1, c_2, c_3))$, where $F(\cdot)$ is *homothetic*.

total expenditure) and independent of w (see Deaton 1979). It turns out that this would generally not be the case, even when $U(\cdot)$ satisfies the *homotheticity* and *weak separability* assumptions. To see this, divide (12) by (15) to obtain the familiar first-order condition with respect to c_2 and c_4 :

$$(17) \quad \frac{\partial U / \partial c_2}{\partial U / \partial c_4} = \frac{\alpha(1+t_2^c)}{\mu(1+t_3^c)}$$

Indeed, the left-hand side of equation (17) does not depend on leisure if $U(\cdot)$ is weakly separable. With the *homotheticity* assumption, it follows that the marginal propensities to consume are indeed constant in income (total expenditure). However, because the term α / μ on the right-hand side of (17) is not generally independent of w , it follows that the marginal propensities are not generally independent of w . Hence commodity taxation is not generally uniform (that is, redundant in the presence of labor income tax).

To see why α / μ generally varies with w , recall that α is the *Lagrange* multiplier of the budget constraint in the "survive" state of nature. That is, α is the private marginal utility of income in that state. Similarly, μ is private marginal utility of income in the "die" state of nature. Because there is no perfect insurance market (that is, there is no perfect annuity market), then α is not generally equal to μ , and the ratio α / μ could well depend on w ¹³.

Note further that estate tax plays also a role in intra-dynasty redistribution (in addition to its role in inter-dynasty redistribution). Even with a single dynasty (or, more generally, many dynasties with Parallel linear Engel curves, as in Deaton 1979), the estate tax will generally not be equal to zero. Estate tax serves in this case to correct for the incompleteness of the insurance market by smoothing consumption across the two states of nature ('die' and 'survive').

Recall that the government faces no risks (there is no aggregate risk). Thus, it faces a single budget constraint. However, each dynasty faces two budget constraints; one for each state of nature. In essence the estate tax takes advantage of the government single budget constraint to shift income between the individual two budget constraints (i.e., two states of nature). Such a shift is made possible because c_3 is generally not equal to c_4 (even though they are taxed at the same rate with an estate tax). Appendix A provides a one-dynasty example, which illustrates this point.

5. Purely Accidental Bequest: A Hundred Percent Estate Tax?

We turn next to examine the most obvious case for levying estate tax, namely the case of no altruism where bequests are purely accidental. Formally we look at the case where $\delta = 0$.

The intuition suggests that because individuals are utterly egoistic (by assumption), estate taxation entails zero deadweight loss, for bequests are purely accidental. A case for imposing one hundred percent tax rate on estate is thus established, following the vision of Carl Marx who called for complete abolition of the institute of inheritance in his venerable Communist Manifesto. In what follows we show that this is far from being a forgone conclusion.

Because parents are assumed indifferent with respect to the well being of their descendants, they always set the amount they intentionally bequeath to zero (ruling out negative bequests). Thus, a typical parent of type w is seeking to solve the following program:

$$(18) \quad \max_{l_1, c_1, c_2} \{u(l_1, c_1) + pv(c_2)\}$$

$$\text{Subject to: } \tau_1 + (1 - t_1^l)w(1 - l_1) = (1 + t_1^c)c_1 + c_2,$$

¹³ Note that with a single budget constraint (as when $p=1$, or when there is a perfect market for annuities)

where, with no loss of generality, we choose c_2 as the untaxed good, that is we set $t_2^c = 0$.

Following our earlier notation, the amount inherited by a typical agent of type w born at the second period is either zero with probability p , or $S(w, \cdot) = c_2(w, \cdot)$ with the complementary probability $(1-p)$.

A typical descendant of type w who has not inherited is solving the following program:

$$(19) \quad \max h(l_3, c_3)$$

$$\text{Subject to: } \tau_2 + (1-t_2^l)w(1-l_3) = c_3$$

A typical descendant of type w who inherited S is solving the following program:

$$(20) \quad \max h(l_4, c_4)$$

$$\text{Subject to: } (1-t^e)S + \tau_2 + (1-t_2^l)w(1-l_4) = c_4$$

We denote by α and μ the *Lagrange* multipliers associated with the budget constraints in (19) and (20), respectively.

In our case it does not make sense to assume that the social planner takes into account the welfare of the offspring generation only through the welfare of the parent generation, because the parents do not care at all about their offspring. Here we make the natural assumption that the offspring welfare per-se is included in the social welfare function. Formally, we assume that $\beta > 0$. The social planner is then solving the following program (we follow the notation used above):

the right-hand side of equation (17) is indeed independent of w .

$$(21) \quad \max_{\tau_1, \tau_2, t_1^c, t_1^l, t_2^l, t^e, t_1^c} \{E[\bar{U}] + \beta E[\bar{h}]\}$$

Subject to:

$$t_1^c E[c_1] + t_1^l E[w(1-l_1)] + t_2^l E[pw(1-l_3) + (1-p)w(1-l_4)] + t^e E[(1-p)c_2] - \tau_1 - \tau_2 = 0$$

Formulating the *Lagrangean* one derives the first-order conditions for the optimal tax rates. We concentrate below on the first-order conditions for the estate tax (t^e) and the lump-sum transfer to the second generation (τ_2):

$$(22) \quad -(1-p)E[\mu c_2] + \lambda [t_2^l \frac{\partial E[w(1-\bar{l})]}{\partial t^e} + E[(1-p)c_2]] \geq 0 \quad (\text{With equality when } t^e < 1),$$

where $\bar{l} = pl_3 + (1-p)l_4$ is the expected leisure of the young generation and λ denotes the *Lagrange* multiplier (of the single constraint, which is the government budget constraint in present value),

$$(23) \quad E[p\alpha + (1-p)\mu] + \lambda [t_2^l \frac{\partial E[w(1-\bar{l})]}{\partial \tau_2} - 1] = 0.$$

Following Tuomala (1985), we can employ these conditions to obtain a simple expression for the estate tax rate. First, divide (22) by (23) to obtain:

$$(24) \quad \frac{-(1-p)E[\mu c_2]}{E[p\alpha + (1-p)\mu]} \geq \frac{[t_2^l \frac{\partial E[w(1-\bar{l})]}{\partial t^e} + E[(1-p)c_2]]}{[t_2^l \frac{\partial E[w(1-\bar{l})]}{\partial \tau_2} - 1]}$$

Now suppose that we change t^e and simultaneously adjust τ_2 in order to keep the government budget constraint balanced. Then, by totally differentiating the constraint with respect to t^e , we get:

$$(25) \quad \left. \frac{\partial \tau_2}{\partial t^e} \right|_{\text{RevenueConst}} = - \frac{[t_2^l \frac{\partial E[w(1-\bar{l})]}{\partial t^e} + E[(1-p)c_2]]}{[t_2^l \frac{\partial E[w(1-\bar{l})]}{\partial \tau_2} - 1]}$$

Note further that:

$$(26) \quad \left. \frac{\partial E[w(1-\bar{l})]}{\partial t^e} \right|_{\text{RevenueConst}} = \frac{\partial E[w(1-\bar{l})]}{\partial t^e} + \frac{\partial E[w(1-\bar{l})]}{\partial \tau_2} \cdot \left. \frac{\partial \tau_2}{\partial t^e} \right|_{\text{RevenueConst}}$$

Suppose a corner solution for the estate tax, that is $t^e = 1$. In this case $\alpha = \mu$, because wealth is equalized across the two states of nature. Then, equations (24)-(26) yield the following expression:

$$(27) \quad E[c_2] - E[\mu c_2] / E[\mu] \geq - \frac{t_2^l}{(1-p)} \left. \frac{\partial E[w(1-\bar{l})]}{\partial t^e} \right|_{\text{RevenueConst}}$$

Inequality (27) is a necessary condition for a corner solution. The left-hand side of (27) is $-Cov[c_2, \mu] / E[\mu]$. By virtue of the strict concavity of $h(\cdot)$ and by assuming that c_2 is rising in w , it follows that the left-hand side of (27) is positive.

The right-hand side of (27) is a product of two factors. The first factor, that is $-t_2^l / (1-p)$, depends on the sign of the labor income tax in the second period. The second factor is the change in aggregate effective labor supply of the second generation caused by a change in the estate tax, which is accompanied by an adjustment of the transfer, τ_2 , which keeps the government revenues intact. Note that with non-altruistic parents such a change in t^e and τ_2 is fully contained within the second generation.

Suppose that the labor income tax rate is positive. Then if aggregate labor supply decreases when the estate tax rate is raised slightly, the sign of the right-hand side of (27) is positive. Thus, we can not state unequivocally that the optimum suggests imposing a hundred percent tax on estate. The result depends on the magnitude of the income effect.

The interpretation is as follows. Notwithstanding having no substitution effect hence zero excess burden (by virtue of the accidental bequest motive), estate tax does have an income effect. Redistribution may affect aggregate labor supply and while obtaining enhanced redistribution directly, it might result in less redistribution indirectly (by reducing the extent of redistribution obtained by other tax instruments - notably income tax). The marginal benefits should outweigh the costs to warrant an increase in the estate tax.

Note that when either labor income tax is zero in the optimum (an anomaly in the presence of variation in earning abilities) or aggregate labor supply is unaffected by the change in the estate tax cum transfer, then the estate tax should be set to unity in the optimum. In this case the adjusted change in t^e amounts to a mean-preserving concentration in the distribution of the wealth. When aggregate labor supply drops and the labor income tax is positive, the benefits derived from enhanced redistribution of inherited wealth may be outweighed by costs due to poorer redistribution of wealth deriving from labor income. Appendix B provides a numerical example of a case in which the optimal estate tax is lower than one hundred percent.

Redistribution via estate taxation need not only interfere with redistribution via other tax instruments, but may also hinder its own *Pigouvian* role. To see that, suppose that we extend the model to a third period. Suppose, that each member of the second generation survives to the third period with probability p , but can not purchase an annuity to diversify the risk. Each allocates consumption across the two periods (second and third) while, being non-altruistic, not accounting for the positive externality exerted on the third generation. Namely, the strictly positive probability of leaving bequest accidentally to the next generation. Thus, aggregate saving is too low relative to the

socially desirable level. The induced income effect of the estate tax could potentially discourage agents from saving to retirement. In order to provide a *pigouvian* subsidy one needs to reduce the estate tax below the one hundred percent level.

6. An Example

In the last section of the paper we turn to the general setting of the optimal tax problem. We allow for both intended and accidental bequest and for a complete set of tax instruments and turn to characterize the properties of the optimal estate tax.

Solving the model for the most general setting is not tractable and yields no insights. Rather than doing that, we choose a special case of additive logarithmic preferences, which allows us to derive the optimal estate tax rate explicitly.

Consider a simplified version of the two period model discussed above. We assume that the only source of heterogeneity across dynasties is coming from differences in the initial endowment of the first generation. We denote by w the wealth endowment of a typical dynasty, given in c_1 terms. We henceforth simplify and drop leisure from the utility function. To stay in line with Mirrlees (1971), we assume that c_1 is unobservable by the government (social planner). The expected utility derived by each dynasty takes the following functional form:

$$U(c_1, c_2, c_3, c_4) = \ln(c_1) + p[\ln(c_2) + \delta \ln(c_3)] + (1-p)\delta \ln(c_4)$$

In this case, an explicit formula for the optimal estate tax can be derived (for details see appendix C):

$$(28) \quad t^e = \frac{(1-p) - \beta}{\delta + (1-p)}$$

We turn next to interpret (28).

[1] Consider first the no-externality case, namely: $\beta = 0$. The optimal estate tax does not depend on the properties of the distribution of types. Estate tax serves for intra-ability redistribution only (between the 'lucky'-die state and the 'unlucky'-survive state), deriving from the imperfection in the market for annuities, resulting in ex-post differences amongst ex-ante identical dynasties. This is captured by the first term in the numerator of (28). Therefore, when $p \rightarrow 1$ and there is no lifetime uncertainty, this term vanishes and $t^e = 0$. Because in our case marginal propensities to consume from wealth are constant and independent of types (see appendix C), then when externality is absent and lifetime is certain (or, alternatively, agents can purchase actuarially fair annuities in the market), estate tax is optimally set to zero. A uniform consumption tax just suffices to obtain the socially desirable allocation.

[2] The estate tax can also serve as a *Pigouvian* instrument to correct for the externality, captured by β . For instance, when $p = 1$, the estate tax is purely *Pigouvian*; and when the externality is positive (namely, $\beta > 0$) the estate tax turns into a *Pigouvian* subsidy.

[3] In general the sign of the optimal estate tax depends on the magnitude of the two terms in the numerator of (28), namely the redistributive component and the *Pigouvian* one. When the former is larger the tax rate will be positive. When the second is larger (in absolute value) there will be an estate subsidy.

[4] As the accidental bequest motive becomes stronger, (namely, $1-p$ rises), other things being equal, the optimal estate tax is rising, as expected.

[5] However, one cannot conclude, when agents become more altruistic (namely, δ rises), as intuition suggests, that the optimal estate tax decreases. This depends on the relative magnitude of the two components discussed above. When the externality is relatively small (namely, $\beta < 1-p$), then this conclusion is true. But, when the

externality motive is relatively strong (namely, $\beta > 1 - p$), the conclusion is false: as δ rises, t^e rises (the estate subsidy falls).

7. Conclusions

Estate/inheritance taxation is highly controversial for myriad reasons. The primary motivation for using it derives from a conventional wisdom that it attains further redistribution while entailing a relatively mild efficiency cost. The strongest case for using inheritance tax is when (following Hurd 1989) imperfection in the market for annuities and uncertain lifetime give rise to accidental bequest. Prima facie, this seems to be the case of zero efficiency cost.

This paper examines the properties of an optimal linear estate tax in the presence of a complete set of tax instruments. We show that generally estate tax is not redundant and further show, that in the presence of lifetime uncertainty and incomplete markets for annuities, the regular conditions for rendering commodity taxation redundant in the presence of an optimal linear income tax are not anymore sufficient.

Contrary to conventional wisdom, we show that in the case of purely accidental bequest, the optimum estate tax should not be generally set to one hundred percent, due to an income effect. The reason is that such a tax can interfere with redistributive effects of other instruments and clash with the *Pigouvian* motive for estate taxation.

We further show that contrary to intuition, it may well be true that estate tax should be raised when parents become more altruistic.

In general we conclude that the relevance of estate taxation depends to a large extent on the relative importance of the two bequest motives, still a matter of deep controversy in the empirical literature.

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Appendix A

The Role of Estate Taxation in Intra-Ability Redistribution

Consider a simplified version of the two period framework introduced in the paper. Suppose that all dynasties are endowed with an equal initial wealth given by $w > 0$ denominated in first-period consumption terms (c_1), chosen as the *numeraire* good. We further simplify by assuming that there is no other source of income, therefore dropping leisure from the utility function.

The expected utility is then given by:

$$(A1) \quad U(c_1, c_2, c_3, c_4) = u(c_1) + pv(c_2) + \delta[ph(c_3) + (1-p)h(c_4)]$$

We assume first that the government is using two tax instruments: a uniform excise tax on consumption goods (t) and a lump-sum transfer given in the beginning of the first period (τ). Without loss of generality, we choose c_1 as the untaxed good.

The typical dynasty is faced with two budget constraints. The budget constraints for the 'survive' and 'die' state of nature are given, respectively, by:

$$(A2) \quad w + \tau = (1+t)(c_2 + c_3) + c_1$$

$$(A3) \quad w + \tau = (1+t)c_4 + c_1$$

A typical dynasty is seeking to maximize the expected utility in (A1) subject to the constraints in (A2) and (A3). Solving for the optimal consumption choices and substituting back into the expected utility (given by (A1)) yield the indirect utility function $\bar{U}(t, \tau)$.

The government is faced with the following budget constraint:

$$(A4) \quad t[c_2 + pc_3 + (1-p)c_4] - \tau = 0$$

Suppose that no externalities exist, that is $\beta = 0$. A utilitarian social planner is then seeking to maximize $\bar{U}(t, \tau)$, the indirect utility function, subject to the constraint in (A4), by choosing the optimal tax instruments.

By forming a *Lagrangian*, one can derive the following two first order conditions (with respect to t and τ):

$$(A5) \quad -\alpha(c_2 + c_3) - \mu c_4 + \lambda[pc_2 + pc_3 + (1-p)c_4 + t(p \frac{\partial c_2}{\partial t} + p \frac{\partial c_3}{\partial t} + (1-p) \frac{\partial c_4}{\partial t})] = 0,$$

$$(A6) \quad \alpha + \mu + \lambda[t(p \frac{\partial c_2}{\partial \tau} + p \frac{\partial c_3}{\partial \tau} + (1-p) \frac{\partial c_4}{\partial \tau}) - 1] = 0,$$

where α and μ are the multipliers of constraints (A2) and (A3), respectively, in the individual optimum; and λ is the multiplier of the government revenue constraint (A4).

Comparing (A2) and (A3) yields that $c_2 + c_3 = c_4$. Thus by setting $t = \tau = 0$ and $\lambda = \alpha + \mu$, the two first-order conditions (A5 and A6) and the government revenue constraint (A4) are satisfied.

In a single-dynasty framework there is only case for intra-dynasty redistribution. A uniform consumption tax can not distinguish between the two states of nature. Thus, there is no justification to introduce distortions when one can not attain enhanced redistribution. Hence, the uniform consumption tax is set to zero, in the optimum.

By introducing estate taxation one can distinguish between c_3 and c_4 (namely, the two states of nature). By doing so, one can enhance social welfare. We turn to illustrate the point. All we need to show is that by introducing estate taxation one can enhance social welfare relative to the no-taxation equilibrium (the optimum in the absence of estate taxation).

We denote by t_2 the consumption tax levied on second period consumption. Note that this would be the effective tax levied on the old cohort during the second period. We denote by t_3 the effective tax imposed on consumption of the young cohort during the second period. Note that estate tax serves to distinguish between the contemporaneous consumption of the two overlapping generations. It is implicitly defined by: $1+t_3 = (1+t_2)/(1-t^e)$.

We evaluate the effect of a small rise in t_3 accompanied by an adjustment of the lump-sum transfer, τ , so that the government revenue constraint remains balanced, relative to the no-tax equilibrium. Formally,

$$(A7) \quad \left. \frac{\partial \bar{U}}{\partial t_3} \right|_{\text{RevenueConst}, t_3=0} = -\alpha c_3 - \mu c_4 + (\alpha + \mu) \left. \frac{\partial \tau}{\partial t_3} \right|_{\text{RevenueConst}, t_3=0}$$

$$= -\alpha c_3 - \mu c_4 + (\alpha + \mu)[pc_3 + (1-p)c_4]$$

Thus,

$$(A8) \quad \left. \frac{\partial \bar{U}}{\partial t_3} \right|_{\text{RevenueConst}, t_3=0} > 0 \Leftrightarrow pc_3 + (1-p)c_4 > \frac{\alpha}{\alpha + \mu} c_3 + \frac{\mu}{\alpha + \mu} c_4$$

Note that $c_4 > c_3$. Thus,

$$(A9) \quad \left. \frac{\partial \bar{U}}{\partial t_3} \right|_{\text{RevenueConst}, t_3=0} > 0 \Leftrightarrow (1-p) > \frac{\mu}{\alpha + \mu} \Leftrightarrow (1-p)\alpha > p\mu$$

By dividing (13) by (15) in the text, it follows that:

$$(A10) \quad \frac{h'(c_3)}{h'(c_4)} = \frac{(1-p)\alpha}{p\mu}$$

By concavity of $h(\cdot)$ and because $c_4 > c_3$, $\left. \frac{\partial \bar{U}}{\partial t_3} \right|_{RevenueConst, t_3=0} > 0$.

Thus, estate taxation serves to redistribute across the two states of nature, thereby enhancing social welfare.

Appendix B

Purely Accidental Bequest: Not Necessarily a Hundred Percent Estate tax

Consider an example based on the model presented in the text. Suppose there are only two types of dynasties given by $\bar{w}=410$ and $\underline{w}=210$, with equal share in population. We make the following parametric assumption with respect to the utility functions:

$$(B1) \quad u(c_1, l_1) = \ln(c_1) + \ln(l_1)$$

$$(B2) \quad v(c_2) = 5\ln(c_2)$$

$$(B3) \quad h(c_j, l_j) = \sqrt{c_j} + \sqrt{l_j}, \text{ for } j = 3, 4$$

We further assume that $p=0.7$ and $\beta = 1$. We solve numerically the government maximization program given in equation (21) in the text for the specific parameters chosen, to obtain numerical solution for the optimal tax rates. We summarize the results below:

INSTRUMENT	RATE
τ_1	68.31
τ_2	104.8
t_1^l	0.197
t_2^l	0.167
t^e	0.964
t_1^c	0.471

Close inspection of the table reveals that, indeed, in the presence of a strictly positive labor income tax in the second period, the optimal estate tax is set at $t^e = 0.964$, that is lower than one hundred percent.

A key feature of the model is the assumption we make with respect to the functional form of $h(c, l)$. Note that for the specific functional form given in (B3), the utility function of the second generation satisfies the following property:

$$(B4) \quad \frac{\partial^2 w(1-l)}{\partial I \partial w} > 0,$$

where I denotes the endowment of wealth (say, inherited).

Note that property (B4), a sort of 'agent convexity' requirement, states that individuals with higher ability tend to spend a larger share of their marginal dollar on the consumption good, and correspondingly a lower fraction on leisure. The condition implies that progressive estate taxation will result in a reduction in the aggregate effective labor supply.

Following the analysis in the text, the estate tax is set below the one hundred percent level in the optimum. The reason being interference with the redistributing role of labor income taxation.

Appendix C

Derivation of the Optimal Estate Tax

A typical dynasty of type w is seeking to solve the following program:

$$(C1) \quad \max_{c_1, c_2, c_3, c_4} \{ \ln(c_1) + p[\ln(c_2) + \delta \ln(c_3)] + (1-p)\delta \ln(c_4) \}$$

$$\text{Subject to:} \quad \begin{aligned} w + \tau &= c_1 + c_2(1+t_2) + c_3(1+t_3) \\ w + \tau &= c_1 + c_4(1+t_3) \end{aligned}$$

Note that the estate tax is implicitly defined by:

$$(C2) \quad (1+t_3) = (1+t_2)/(1-t^e)$$

It is easy to verify that the solution to the maximization program in (C1) is given by:

$$(C3) \quad \begin{aligned} c_1 &= \alpha_1(w + \tau) \\ c_2 &= \alpha_2(w + \tau)/(1+t_2) \\ c_3 &= \alpha_3(w + \tau)/(1+t_3) \\ c_4 &= \alpha_4(w + \tau)/(1+t_3) \end{aligned}$$

With $\alpha_1 = 1/(1+p+\delta)$, $\alpha_2 = \alpha_1(p+\delta)/(1+\delta)$, $\alpha_3 = \alpha_2\delta$ and $\alpha_4 = \alpha_2(1+\delta)$.

We turn next to formulate the social planner's program. The social planner is seeking to solve the following maximization program:

$$(C4) \quad \max_{t_2, t_3, \tau} \{ E[\ln(c_1)] + pE[\ln(c_2)] + p(\beta + \delta)E[\ln(c_3)] + (1-p)(\beta + \delta)E[\ln(c_4)] \}$$

$$\text{Subject to: } t_2 p E(c_2) + t_3 [p E(c_3) + (1-p) E(c_4)] - \tau = 0$$

Formulating the *Lagrangian*, using (C3), one can derive the first-order conditions for the optimal tax rates. We concentrate here on the first-order conditions for t_2 and t_3 :

$$(C5) \quad -p/(1+t_2) + \lambda pE(c_2) - \lambda p t_2/(1+t_2)E(c_2) = 0$$

$$(C6) \quad -(\beta + \delta)/(1+t_3) + \lambda[pE(c_3) + (1-p)E(c_4)] - \lambda t_3/(1+t_3)[pE(c_3) + (1-p)E(c_4)] = 0$$

Employing (C5) and (C6) yields the following two expressions:

$$(C7) \quad \lambda E(c_2) = 1$$

$$(C8) \quad \lambda[pE(c_3) + (1-p)E(c_4)] = (\beta + \delta)$$

Dividing (C7) by (C8) and substituting (C3) yields the following:

$$(C9) \quad (1+t_3)/(1+t_2) = (\delta + 1 - p)/(\beta + \delta)$$

By virtue of (C2) it follows that,

$$(C10) \quad t^e = \frac{(1-p) - \beta}{\delta + (1-p)}$$

This completes the derivation.