CESifo Working Papers

COMMON AGENCY EQUILIBRIA WITH DISCRETE MECHANISMS AND DISCRETE TYPES*

David Martimort
Lars Stole

CESifo Working Paper No. 572

October 2001

CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de
ISSN 1617-9595

* The main results of this paper appeared in a section of an earlier working paper entitled, "Communication spaces, Equilibria Sets and the Revelation Principle under Common Agency," June 1997.

CESifo *Working Paper No. 572*
October 2001

COMMON AGENCY EQUILIBRIA WITH DISCRETE MECHANISMS AND DISCRETE TYPES

Abstract

This paper characterizes the equilibrium sets of an intrinsic common agency game with discrete types and direct revelation mechanisms. After presenting a general algorithm to find the pure-strategy equilibria of this game, we use it to characterize these equilibria when the two principals control activities which are complements in the agent's objective function. Some of those equilibria may entail allocative inefficiency. For the case of substitutes, we demonstrate non-existence of such equilibria with direct mechanisms, but existence may be obtained with indirect mechanisms. Finally, we relax the equilibrium concept and analyze quasi-equilibria. We show that existence is then guaranteed and characterize the corresponding allocations.

JEL Classification: D82, L51.

David Martimort
Université de Toulouse
21 allées de Brienne
Manufacture des Tabacs
Toulouse 31000
France
martimor@cict.fr

Lars Stole
University of Chicago
GSB
1101 E. 58th Street
Chicago, IL 60637
U.S.A.
Lars.stole@gsb.uchicago.edu

1 Introduction

Common agency games under adverse selection have received much attention in the last few years.[3] Contrary to standard monopolistic mechanism design problems which are now quite well known, oligopolistic screening often leads to a complex characterization of the equilibrium allocations. This characterization is complex for two reasons. First, the simple version of the Revelation Principle generally used in monopolistic settings no longer holds. To describe the whole set of equilibrium allocations of such a game, i.e., the whole set of allocations which are implementable through a common agency game, one has either to rely on a Delegation Principle as in Martimort and Stole (2001) or to extend in an appropriate way the type space before using the Revelation Principle as in Epstein and Peters (1996). Second, even in the archetypical environments analyzed by Stole (1991), Martimort (1992, 1996), Martimort and Stole (2000) and Biais, Martimort and Rochet (2000), the description of the equilibrium allocations is hard because it involves solving a pair of differential equations which are not Lipschitzian at a boundary of the type spaces. Except for the variation in competition among principals, these environments are immediate extensions of those used under monopolistic screening. They involve quasi-linear utility functions, a one-dimensional adverse selection parameter distributed continuously on an interval with an everywhere positive density, and each principal's strategy space consisting of the space of continuously differentiable nonlinear schedules. The complexity that the modeler may face in comparing the monopolistic and the oligopolistic screening environments is thus deeply due to the nature of competition in mechanisms.

Much work in incentive theory, however, has studied simpler contracting environments in which the type space is discrete (generally two types) and direct mechanisms are used by the principals. The restriction to discrete type spaces can be justified by invoking the fact that, in the real world, principals find it of value to distinguish only a few subsets of agents,[4] and that, in the theoretical world, much of the economic intuition of self-selection contracts can be understood with only two types. We similarly adopt this two-type restriction in the present paper, but in the context of competitive contracting. In addition to restricting attention to a two-type environment, we also focus on direct communication mechanisms (i.e., contracts which are menus of no more than 2 elements). In the monopoly setting, a restriction to direct mechanisms is not an issue because the Revelation Principle applies and there is no loss of generality. In a common agency framework where screening mechanisms are available to multiple principals, however, such a restriction is, a priori, not meaningful as argued by Peters (1999) and shown with an early and abstract counter-example by Martimort and Stole (1993). Even under oligopolistic screening, however,

[3] See Stole (1991) and Martimort (1992, 1996), and Martimort and Stole (2000).

[4] This is certainly the case for firms using nonlinear pricing since they most of the time offer only a few options to their customers.

the restriction to direct mechanism is economically meaningful and can be justified when menu costs impose that each principal finds it optimal to offer at most a single allocation per subset of identified agents.

Our interest in this paper is to characterize equilibrium allocations in a simple common agency game involving two sellers and one buyer along the lines of those analyzed by Stole (1991) and Martimort (1992, 1996) when one insists on both restrictions above: (i), the finiteness of the type space (and here we focus on the case of two possible types) and, (ii), the set of feasible mechanisms being a priori restricted to direct mechanisms. The motivation for this exercise is twofold. First, from a positive point of view, we want to propose a description of common agency equilibrium in a meaningful and simple environment. For modelers interested in applying the common agency methodology to compare monopolistic and oligopolistic screening environments, it may be of little help to know that a Delegation Principle or an extended version of the Revelation Principle hold in those environments if they are not amenable to a clear description of incentive constraints, at least a description which could be compared to that obtained under monopolistic screening. Second, from a normative point of view, our analysis can be viewed as a first step towards a full characterization of equilibrium allocations in common agency environments. Before enlarging the strategy spaces as requested by the Delegation and the extended Revelation Principles, one may want to know what can be achieved with mechanisms using communication spaces which have the same dimensionality than the set of underlying types.

We start by providing the cooperative benchmark which supposes that the two principals cooperate in their contractual offers and behave as merged entity (Section 2). We then move to the analysis of Nash equilibria of the common agency game with direct mechanisms. Because the most appealing allocations are deterministic, we restrict our attention to non-random direct mechanisms. Additionally, since pure-strategy equilibria also have a natural economic appeal, we further restrict our attention to pure-strategy equilibria and offer an algorithm to compute these equilibria (Section 3). Within the class of direct-communication, common agency games that we analyze, we find that pure-strategy equilibria always exist when the principals control activities of the agent which are complements. We describe the set of those equilibria and show that under competitive contracting there always exists one equilibrium of the common agency game which replicates what can be obtained by a merged principal. However, competitive contracting may also involve a significant efficiency loss for some equilibria which are shown to be asymmetric and the corresponding distortions are characterized (Section 4). In the case of substitutes, existence of a pure-strategy equilibrium with direct mechanisms and truth-telling always fails (Section 5). Non-existence is due to the desire of each principal to offer contracts which induce the agent to lie to the other principal. This phenomenon leads us to analyze more complex strategy spaces with indirect mechanisms (Section 6) which may ensure existence. Alternatively, one may want to keep the same strategy space for the principals but to relax the equilibrium concept. We define the concept of quasi-equilibrium where principals are bound to offer mechanisms which are collectively incentive compatible. We show existence of such quasi-equilibria and we characterize the corresponding allocations (Section 7).[5] All proofs are in an Appendix.

2 The Model and its Benchmarks

We begin with a description of a common agency game between two sellers (principals) selling differentiated products to a common customer. We assume that the buyer has a quasi-linear utility function which is symmetric and concave in q_1 and q_2: $U = -t + u(q_1, q_2, \theta) = -t + \theta(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - \lambda q_1 q_2$, where t is a monetary transfer paid to the principals, q_i the consumption of good i, and θ the valuation for both goods. The parameter $\lambda \in (-1, 1]$ represents the relationship between q_1 and q_2 in the agent's utility function. The two goods are complements when $\lambda < 0$ and substitutes when $\lambda > 0$.[6] The agent gets some reservation utility exogenously normalized at zero if he decides not to consume the two goods. For simplicity, we consider the model of intrinsic common agency in which the agent is forced to consume both goods.[7] The agent's valuation for the good is private information, drawn form the set $\Theta = \{\underline{\theta}, \bar{\theta}\}$ with respective probabilities $1 - \nu$ and ν. Principal P_i's profit is given by $V_i = t_i - C(q_i)$ when he sells quantity q_i of good i at price t_i. We assume that both principals have the same constant marginal cost of supplying the good: $C(q_i) = cq_i$ for $i = 1, 2$. For simplicity, we assume that $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$ is not too large given ν, $\underline{\theta}$, λ and c, thereby guaranteeing a positive consumption for the low type both under cooperation and competition between the sellers.[8] Of course, nothing is specific to this example and a similar framework could equally be developed to model competition between two regulatory bodies or between two lobbying groups trying to influence a common decision-maker.

We begin with two benchmarks for comparison: The full information contract and the

[5] An alternative route would be to look for mixed-strategy equilibria and prove existence using possibly the techniques of Dasgupta and Maskin (1986). Their theorems apply when the strategy spaces are closed subsets making them not directly useful in the case here since the set of contracts is a priori unbounded.

[6] When $\lambda = 0$ the two goods are unrelated. When $\lambda = 1$, the goods are perfect substitutes; i.e., $U = t + \theta Q - \frac{1}{2}Q^2$, where $Q = q_1 + q_2$.

[7] See Martimort and Stole (2000) for a similar model where we discuss the difference between intrinsic and delegated common agency. In the latter case, the agent can choose to refuse one of the contract he is offered. On possible motivation for this focus on the intrinsic common agency game is that the buyer and the sellers are all units of the same firm and that trade between those units is mandatory as it is the case for some practices of transfer pricing within the firm.

[8] A sufficient condition is that $\nu\Delta\theta \leq (1 - \nu - \lambda^2)(\underline{\theta} - c)$, though except for Proposition 2, this is much stronger than necessary.

second-best contract obtained when the principals cooperate under asymmetric information and behave as a merged principal.

The full information first-best levels of outputs are obviously given by:

$$q^{FB}(\theta) = \frac{\theta - c}{1+\lambda}, \quad \forall \theta \in \Theta.$$

Moreover, the agent gets zero rent whatever his type:

$$U^{FB}(\theta) = 0, \quad \forall \theta \in \Theta.$$

Let us now move to the standard case of monopolistic screening where a merged principal offers the contract.

Proposition 1 *: The collusive second-best levels of outputs are given by:*

$$q^C(\bar{\theta}) = \frac{\bar{\theta} - c}{1+\lambda},$$

$$q^C(\underline{\theta}) = \frac{\underline{\theta} - c}{1+\lambda} - \left(\frac{\nu}{1-\nu}\right)\frac{\Delta\theta}{1+\lambda}.$$

The cooperative transfers paid by each type for those quantities are given by:

$$t^C(\bar{\theta}) = u(q^C(\bar{\theta}), q^C(\bar{\theta}), \bar{\theta}) - 2\Delta\theta q^C(\underline{\theta}),$$

$$t^C(\underline{\theta}) = u(q^C(\underline{\theta}), q^C(\underline{\theta}), \underline{\theta}).$$

The high valuation agent gets a positive information rent:

$$U^C(\bar{\theta}) = -t^C(\bar{\theta}) + u(q^C(\bar{\theta}), q^C(\bar{\theta}), \bar{\theta}) = 2\Delta\theta q^C(\underline{\theta}).$$

The low valuation agent gets zero information rent:

$$U^C(\underline{\theta}) = -t^C(\underline{\theta}) + u(q^C(\underline{\theta}), q^C(\underline{\theta}), \underline{\theta}) = 0.$$

This proposition is standard and well-known from the monopolistic screening literature. Although the high valuation agent consumes an efficient amount of both goods at the optimal contract under centralized contracting, the low valuation agent's consumption is distorted downwards under asymmetric information. Such a distortion reduces indeed the cost of the incentive constraint of a high valuation agent willing to mimic a low valuation one.

3 Finding Pure-Strategy Equilibria

We now turn our attention to the analysis of the non-cooperative subgame perfect equilibria of the common agency contracting game.

This game unfolds as follows. First, the principals P_i (for $i = 1, 2$) non-cooperatively offer their direct revelation mechanisms $\{t_i(\hat{\theta}_i), q_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$; second, the agent accepts or refuses both offers; and third, the agent chooses within each menu by sending a private report $\hat{\theta}_i$ to principal P_i.

We begin by defining a non-random pure-strategy equilibrium for our specific common agency setting, which requires no restriction on the communication space between each principal and the agent.[9] Unless stated otherwise, we will use the term *equilibrium* to denote this specific notion.

Definition 1 *: In a non-random, pure-strategy equilibrium of the common agency game with communication spaces M_i ($i = 1, 2$), each principal P_i offers a deterministic contract, $\{t_i(m_i), q_i(m_i)\}_{m_i \in M_i}$, and the agent does not randomize among the messages he sends to the principals.*

Before proceeding to a systematic investigation of the pure-strategy equilibria of the common agency game with direct communication (where $\Theta = M_i$), we propose an algorithm which helps to characterize the best-response of a principal to any pure-strategy mechanism offered by his rival.[10]

For any mechanism $\{t_2(m_2), q_2(m_2)\}_{m_2 \in M_2}$ offered by P_2, there is no loss of generality in looking for P_1's best-response within the class of direct revelation mechanisms of the form $\{t_1(\hat{\theta}_1), q_1(\hat{\theta}_1)\}_{\hat{\theta}_1 \in \Theta}$. Any payoff that P_1 can achieve when he offers a mechanism with some general communication space M_1 can be also achieved with such a direct revelation mechanism. Here, we simply apply the Revelation Principle for *a given* non-random mechanism offered by P_2.[11]

However, different mechanisms offered by P_2 may affect differently the agent's incentives to misreport to P_1. To capture this effect mathematically, we define the agent's indirect utility function vis à vis P_1 as:

$$\hat{U}^1(q_1, \theta) \equiv \max_{\{m_2 \in M_2\}} \quad u(q_1, q_2(m_2), \theta) - t_2(m_2).$$

[9] In Section 6 we explore indirect mechanisms, for example.

[10] See also Martimort and Stole (2000) for a general use of this algorithm in the case of a continuum of types.

[11] The argument above relies on the fact that we focus on pure-strategy equilibria between the two principals with the agent not mixing among messages in the mechanisms he receives from either principal. Allowing for mixed strategies alters the analysis; see footnote **??** below.

The indirect utility function gives the value of the agent's utility whatever his own type and his consumption of good 1 once he has communicated an optimal message to P_2 given this type and this quantity. Moreover, for a given indirect utility function, P_1's problem is identical to the standard principal-agent contracting problem under monopolistic screening. P_2's contract can be ignored except for its effect on this indirect utility function. The allocation $\{(t_1(\bar{\theta}), q_1(\bar{\theta})); (t_1(\underline{\theta}), q_1(\underline{\theta}))\}$ chosen, at a best response, by P_1 is solution to the following program:

$$\max_{\{(\bar{t}_1,\bar{q}_1),(\underline{t}_1,\underline{q}_1)\}} \nu(\bar{t}_1 - c\bar{q}_1) + (1-\nu)(\underline{t}_1 - c\underline{q}_1)$$

subject to

$$\hat{U}^1(\bar{q}_1, \bar{\theta}) - \bar{t}_1 \geq \hat{U}^1(\underline{q}_1, \bar{\theta}) - \underline{t}_1 \quad (1)$$

$$\hat{U}^1(\underline{q}_1, \underline{\theta}) - \underline{t}_1 \geq \hat{U}^1(\bar{q}_1, \underline{\theta}) - \bar{t}_1 \quad (2)$$

$$\hat{U}^1(\bar{q}_1, \bar{\theta}) - \bar{t}_1 \geq 0 \quad (3)$$

$$\hat{U}^1(\underline{q}_1, \underline{\theta}) - \underline{t}_1 \geq 0, \quad (4)$$

where $\hat{U}^1(\cdot)$ is the indirect utility function corresponding to the contract offered by P_2.

The first two constraints are the incentive compatibility constraints of the high and the low valuation agents, respectively; the last two constraints are their participation constraints. We will use this program throughout when computing the levels of outputs of the pure-strategy equilibria for the different communication games we consider in this paper.[12] We already note that different message spaces M_2 correspond to possibly different indirect functions $\hat{U}^1(q_1, \theta)$ and therefore to possibly different best-responses by P_1.[13]

For further references, it is useful to express (1) in the case of direct mechanisms. In a pure-strategy equilibrium, the agent chooses to tell the truth to both principals. For the high valuation agent, this means that we must have $\hat{U}^1(\bar{q}_1, \bar{\theta}) - \bar{t}_1 = u(\bar{q}_1, \bar{q}_2, \bar{\theta}) - \bar{t}_1 - \bar{t}_2$. The incentive compatibility constraint (1) may thus take different expressions depending on P_2's offer and the optimal reports that the high valuation agent makes to P_2 conditionally on lying to P_1. Two possible variations of this constraint are:

$$-\bar{t}_1 - \bar{t}_2 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) \geq -\underline{t}_1 - \underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \bar{\theta}) \quad (5)$$

[12] Note that this way of proceeding is not as straightforward in the case of a mixed-strategy equilibria. In a such case, if P_2 randomizes over a distribution of mechanisms, the agent's indirect utility function vis à vis P_1 becomes a random function. In a mixed-strategy equilibrium, P_1 must take into account this randomness at the time of offering his own contract program, in particular including randomness into the constraint set. In the case of random pure-strategy equilibria, our method of determining optimal contracts is relatively unchanged: the agent's indirect utility function will not be a random function as the agent chooses a contract allocation before any randomness is resolved and so P_1's program is unchanged.

[13] For ease of notation, we will leave throughout the dependence of $\hat{U}^1$ on P_2's contract implicit.

when $\hat{U}^1(\underline{q}_1, \bar{\theta}) - \underline{t}_1 = u(\underline{q}_1, \underline{q}_2, \bar{\theta}) - \underline{t}_1 - \underline{t}_2$, (i.e., the agent lies to both principals) and

$$-\bar{t}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) \geq -\underline{t}_1 + u(\underline{q}_1, \bar{q}_2, \bar{\theta}) \tag{6}$$

when $\hat{U}^1(\underline{q}_1, \bar{\theta}) - \underline{t}_1 = u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - \underline{t}_1 - \bar{t}_2$ (i.e., the agent lies to only P_1).

Finally, still in equilibrium, $\hat{U}^1(\underline{q}_1, \underline{\theta}) - \underline{t}_1 = u(\underline{q}_1, \underline{q}_2, \underline{\theta}) - \underline{t}_1 - \underline{t}_2$. Hence, the low-valuation agent's participation constraint (4) becomes

$$-\underline{t}_1 - \underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) \geq 0. \tag{7}$$

4 Characterization of Pure-Strategy Direct Equilibria with Complements

We now turn to the analysis of the specific case where q_1 and q_2 are complements in the agent's utility function, i.e., $-1 < \lambda < 0$. First, we characterize the set of direct communication equilibria of the common agency game.

Proposition 2 *: Assume that q_1 and q_2 are complements ($\lambda \in (-1, 0]$), the cooperative outcome can be implemented as a non-cooperative pure-strategy equilibrium of the direct communication common agency game. There exists a set of equilibria which entail a symmetric output allocation given by the collusive second-best levels*

$$q^C(\theta), \quad \forall \theta \in \Theta.$$

In these equilibria, the principals receive transfers such that (5) and (7) are both binding. The following constraints are also satisfied for the transfers offered by P_1:

$$u(q^C(\underline{\theta}), q^C(\bar{\theta}), \bar{\theta}) - u(q^C(\underline{\theta}), q^C(\underline{\theta}), \bar{\theta}) < t_1(\bar{\theta}) - t_1(\underline{\theta}) < u(q^C(\bar{\theta}), q^C(\bar{\theta}), \bar{\theta}) - u(q^C(\underline{\theta}), q^C(\bar{\theta}), \bar{\theta})$$

and a similar inequality holds for the transfers offered by P_2. The high and the low valuation agent both get the same information rent as in the cooperative outcome.[14]

In short, even under competitive contracting, the cooperative outcome can still be implemented. This result can be intuitively understood by returning to the definition of the indirect utility function $\hat{U}^1(q_1, \theta)$. When a high valuation agent chooses a high consumption from P_1 he also has an incentive to consume a large quantity from P_2 since the two goods are complements. We thus have $\hat{U}^1(\bar{q}_1, \bar{\theta}) = -\bar{t}_2 + u(\bar{q}_1, \bar{q}_2, \bar{\theta})$. Similarly,

[14]Note that there will typically be a continuum of possible different divisions of transfers for the lowest type among the principals. We focus on those divisions which are such that each principal gets a positive expected payoff.

when a high valuation agent chooses a low consumption from P_1, he also has an incentive to consume less from P_2. This case arises when $\bar{t}_2 - \underline{t}_2$ is sufficiently large so that $-\bar{t}_2 + u(\underline{q}_1, \bar{q}_2, \bar{\theta}) < -\underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \bar{\theta})$. P_2 charges then a high price to the high valuation agent so that consuming a low quantity $\underline{q}_1$ makes him eager to also claim he has a low valuation to P_2. We have then $\hat{U}^1(\underline{q}_1, \bar{\theta}) = -\underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \bar{\theta})$. Writing the incentive compatibility constraint (1) for this high valuation agent with the indirect utility function yields (5). This *global incentive compatibility constraint* is exactly the same as if the principals were cooperating and therefore, no distortion is entailed by their non-cooperative behavior.

Of course, if only this global incentive constraint is relevant, the sum of the transfers obtained by both principals can be determined just as under centralized contracting with a merged principal. However, the flexibility in designing the individual transfers received by each principal can be used to insure that, following a deviation, each principal realizes that only the global incentive constraint (5) is relevant. The private incentives that each principal faces when he wants to induce information revelation by the high valuation agent are then aligned with the incentives of the merged principal.

There still exists a whole array of possible transfer differentials $\bar{t}_i - \underline{t}_i$ which are consistent with such an equilibrium (see Figure 1). In all those equilibria, the same symmetric output allocation is realized.

The non-cooperative implementation of the cooperative outcome is striking and contrasts with the continuum-of-types analysis developed in Stole (1991) and Martimort (1992,1996). There, it was shown that the non-cooperative behavior between the principals leads always to more inefficiencies than the cooperative outcome. The key difference is that those papers assume that the agent's valuation is continuously distributed on an interval. With discrete types, there is always some leeway in specifying transfer differentials $\bar{t}_i - \underline{t}_i$ so that the cooperative outcome can still be implemented with competitive contracting. This leeway disappears with a continuum of types since the slope of the nonlinear prices that each principal offers in equilibrium is then exactly pinned down by the agent's incentive compatibility constraint.

We now move to the analysis of inefficient equilibria.

Proposition 3 *: Assume that q_1 and q_2 are complements and that $\lambda^2 \leq 1-\nu$. Then there exists two sets S_1 and S_2 of inefficient pure-strategy asymmetric equilibria of the direct communication common agency game. Set 1 can be indexed by the equilibrium output $q_1^A(\underline{\theta}) \in [\tilde{q}_1(\underline{\theta}), q_1^C(\underline{\theta})]$ that P_1 gives to an inefficient agent with*

$$\tilde{q}_1(\underline{\theta}) \equiv \frac{\underline{\theta} - c}{1+\lambda} - \frac{\nu(1-\nu-\lambda^2-\lambda)}{(1-\nu)(1-\nu-\lambda^2)(1+\lambda)}\Delta\theta,$$

$$q_2^A(\underline{\theta}) = \underline{\theta} - \lambda q_1(\underline{\theta}) - \frac{\nu}{1-\nu}\Delta\theta,$$

$$q_i^A(\bar{\theta}) = q^{FB}(\bar{\theta}) \quad \forall i \in \{1,2\}.$$

In these asymmetric equilibria, the principals receive transfers such that (5), (6) and (7) are all binding. The following constraint is also satisfied by P_2's transfers:

$$u(q_1^A(\underline{\theta}), q_2^A(\bar{\theta}), \bar{\theta}) - u(q_1^A(\underline{\theta}), q_2^A(\underline{\theta}), \bar{\theta}) = t_2^A(\bar{\theta}) - t_2^A(\underline{\theta}) < u(q_1^A(\bar{\theta}), q_2^A(\bar{\theta}), \bar{\theta}) - u(q_1^A(\bar{\theta}), q_2^A(\underline{\theta}), \bar{\theta}).$$

The high valuation agent gets a positive information rent which is strictly smaller than at the cooperative outcome:

$$U^A(\bar{\theta}) = -t_1^A(\bar{\theta}) - t_2^A(\bar{\theta}) + u(q_1^A(\bar{\theta}), q_2^A(\bar{\theta}), \bar{\theta}) = \Delta\theta(q_1^A(\underline{\theta}) + q_2^A(\underline{\theta})) < U^C(\bar{\theta}).$$

The low valuation agent gets zero information rent:

$$U^A(\underline{\theta}) = -t_1^A(\underline{\theta}) - t_2^A(\underline{\theta}) + u(q_1^A(\underline{\theta}), q_2^A(\underline{\theta}), \underline{\theta}) = 0.$$

Set 2 of asymmetric equilibria is obtained by permuting the roles of principals 1 and 2.

In the case where $\bar{t}_2 - \underline{t}_2$ is sufficiently small, P_2 charges a low marginal price to the high valuation agent so that even if he consumes a low quantity $\underline{q}_1$ he still claims he has a high valuation to P_2. We have then $\hat{U}(\underline{q}_1, \bar{\theta}) = -\bar{t}_2 + u(\underline{q}_1, \bar{q}_2, \bar{\theta})$. Writing the incentive compatibility constraint (1) for this high valuation agent with the indirect utility function yields therefore (6). This *local incentive compatibility constraint* is exactly the same as if P_1 was taking into account that P_2 has independently already obtained information on the agent. Everything happens therefore as if P_1 had now to obtain information from a coalition made of P_2 and the agent.

We have represented on Figure 2, the values of the transfers in these asymmetric equilibria.

Since, none of the low valuation agent's incentive constraints is binding in equilibrium, $\hat{U}^1(\underline{q}_1, \underline{\theta}) = -\underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \underline{\theta})$ and (4) translates to (7), with an equality at the equilibrium. From P_1's point of view, everything happens thus as if inducing information revelation from the high valuation type requires leaving a payoff $-\bar{t}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta})$ to the high valuation agent which, using (6) and (7), is at least equal to $u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - u(\underline{q}_1, \underline{q}_2, \underline{\theta}) + \underline{t}_2$. With our specification of the agent's utility function,

$$u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - u(\underline{q}_1, \underline{q}_2, \underline{\theta}) = \underline{q}_1(\Delta\theta - \lambda\Delta q_2) \tag{8}$$

where $\Delta q_2 = \bar{q}_2 - \underline{q}_2 > 0$. Had the principals instead cooperated in their contract offers, inducing information revelation from the high valuation type would require leaving a payoff $-\bar{t}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta})$ to the high valuation agent which, using (5) and (7) is least $u(\underline{q}_1, \underline{q}_2, \bar{\theta}) - u(\underline{q}_1, \underline{q}_2, \underline{\theta}) + \underline{t}_2 - \bar{t}_2$. With our specification of the agent's utility function,

$$u(\underline{q}_1, \underline{q}_2, \bar{\theta}) - u(\underline{q}_1, \underline{q}_2, \underline{\theta}) = \Delta\theta(\underline{q}_1 + \underline{q}_2). \tag{9}$$

Comparing (8) and (9), it appears clearly that reducing the output $\underline{q}_1$ offered to a low valuation agent is more valuable in the first case than in the second since $\Delta\theta - \lambda\Delta q_2 > \Delta\theta$ when P_2 offers a monotonic contract such that $\Delta q_2 > 0$.[15] As a result, P_1 further reduces the consumption of a low valuation agent below what he would do at the cooperative contracts. By complementarity, both consumptions of the low valuation agent are in equilibrium below the cooperative outcome.

There exists in fact a continuum of such asymmetric equilibrium quantities coming from the fact that P_1's objective function has a kink at $q_1(\underline{\theta})$. Indeed, in such an equilibrium, the high valuation agent is indifferent between revealing or lying about his type to P_2 when he chooses to claim he has a low valuation to P_1. Starting from this equilibrium output, $q_1(\underline{\theta})$, which is lower than the cooperative outcome, $q^C(\underline{\theta})$, P_1 does not want to induce a further downward distortion in $\underline{q}_1$. For these deviations, the agent prefers indeed to claim he has also a low valuation to P_2. The incentive compatibility constraint that is satisfied is the global one and $q_1(\underline{\theta})$ remains the best of such deviations since P_1's objective function is concave in $\underline{q}_1$ over the interval $[0, q^C(\underline{\theta})]$. It is also clear that P_1 does not want to distort $\underline{q}_1$ further upward. For these deviations, the agent prefers to claim he has also a high valuation to P_2. The incentive compatibility constraint that is now satisfied is the local one and a concavity argument as above shows that $q_1(\underline{\theta})$ itself is the best of such upward deviations.

Finally, it is interesting to note that asymmetric equilibria are characterized by downwards distortions of the productions which are comparable to those arising for all symmetric equilibria obtained in the case of a continuum of types.[16] The economic reason underlying those distortions is the same in both cases. Given the cooperative optimal contract which could be offered by a merged principal, the way that those transfers are split between the two principals may be such that a principal may have an individual incentive to deviate and offer an alternative contract which induces revelation by the high valuation agent of his type to this principal at a smaller cost from his own point of view. This is obtained by reducing further the production offered to a low valuation agent and decreasing the payment made by this agent. By doing so, the deviating principal exerts a negative externality on the non-deviating one who, by complementarity, must also reduce the output offered to a low valuation agent. In equilibrium, this negative externality finally leads to an overall excessive reduction in the volume of trade. The only difference between the discrete and the continuum cases is that, in the latter, both principals have an incentive to deviate from the cooperative contracts and this leads to symmetric equilibria where both principals distort downwards the productions they offer to the agent. In the discrete case, only one of the principal has an incentive to deviate away from the

[15] We show in the Appendix that, in equilibrium, this monotonicity is guaranteed.

[16] See Sole (1991) and Martimort (1992).

cooperative outcome and this leads to asymmetric equilibria.

5 Non-Existence of a Pure-Strategy Direct Equilibrium with Substitutes

With substitutes, the picture is strikingly different.

Proposition 4 *: When q_1 and q_2 are substitutes (i.e., $\lambda \in (0, 1]$), there does not exist a pure-strategy equilibrium in the direct-communication, common agency game.*

The intuition for this result goes as follows: First, assume that P_1 offers a separating contract. As long as this contract does not reverse the Spence-Mirrlees property of the indirect utility of the agent vis à vis P_2, the latter principal has an incentive to raise the output $\bar{q}_2$ he offers to a high valuation agent to make him consume little of good 1 by claiming to P_1 that he has a low valuation. P_2 makes some profit at the expense of P_1 by proposing such an upward deviation to the agent. Of course, P_1 is willing to do the same and there cannot be a pure-strategy equilibrium with separating contracts. Second, there cannot be an equilibrium with both principals inducing full pooling. Indeed, suppose that P_2 offers a pooling contract, then P_1 would takes this offer as given and would offer himself a separating allocation as we show in the Appendix. With substitutes, each principal is thus willing to *"corner"* the other one and there does not exist an equilibrium in which the principals use pure-strategies.[17]

Given this rather disappointing result, on may want to either extend the strategy space available the principals or to relax the equilibrium concept to insure existence.

6 Existence with Indirect Mechanisms: An Example

The non-existence result obtained in the case of direct mechanisms contrasts sharply with what can sometimes be done when message spaces with each principal are conveniently extended. In particular, existence of a pure-strategy equilibrium may no longer be a problem. To show this result, we provide an instructive counter-example. Consider the case where the two suppliers are selling perfectly substitutes.[18] We begin by describing

[17]Myerson (1982) has shown that a truthful equilibrium in contracts may not exist in the case of competing hierarchies by using an abstract example which is closely related to our model.

[18]Importantly, note that the agent is forced to consume both goods in our context. Hence, our focus is only on how the competition between the two suppliers shifts the cost-price margin towards zero. We do not allow the agent to refuse to play one of the given mechanisms. This assumption may be more relevant in a regulatory context than in a competing sellers setting but we choose to keep this interpretation to be coherent with our earlier exposition.

an equilibrium of an indirect communication common agency game.

Proposition 5 *: When q_1 and q_2 are perfect substitutes (i.e., $\lambda = 1$), there exists a pure-strategy equilibrium of the indirect communication common agency game in which principals compete through nonlinear prices, $\{t_i(q_i)\}$ defined over the whole real line, such that:*

- *each principal offers a two-part tariff:*[19]

$$t_i(q_i) = cq_i + a_i, \quad \forall q_i$$

with $a_1 + a_2 = \frac{(\theta - c)^2}{2}$;

- *the agent always chooses the first-best total consumption and if he splits equally his consumption between the two principals:*[20]

$$q^S(\theta) = q^{FB}(\theta) = \frac{\theta - c}{2}, \quad \forall \theta \in \Theta;$$

- *only the high valuation agent gets a positive information rent:*

$$U^S(\bar{\theta}) = 2\Delta\theta q^{FB}(\theta),$$

$$U^S(\underline{\theta}) = 0.$$

In this equilibrium, the profit of each principal is independent on the output he is selling and represents only a fraction of the first best surplus obtained by the low-valuation agent. With perfect substitutes, the principals are making zero profit at the margin. The ability of the agent to choose any possible consumption bundles within the two schedules offered by the principals helps him to play one principal against the other to erode their individual market power. As a result, the overall surplus is the same as in the first-best. Only the distribution of this surplus between the principals and the agent differs. Indeed, because of our assumption of intrinsic common agency, the high valuation agent can only get a fraction of this overall surplus and the low valuation type always gets zero.

With direct mechanisms, the indirect utility function of the agent no longer exhibits smooth behavior. As we show in the proof of Proposition 4, a small increase in the output offered through this direct mechanism by P_1 to the high valuation agent may trigger a

[19]One word of caution is in order here. We do not restrict the principals to those two-part tariffs in the first place but obtain these at the equilibrium within the larger class of nonlinear prices.

[20]The agent is indifferent between whom he consumes from and many other splitting of consumptions are equilibrium outcomes. We take this particular splitting to keep the same formula as in the case of differentiated goods.

discontinuous change in the report made to P_2 by this agent. Indeed, the high valuation agent reduces his consumption of good 2 by a large amount. In turn, this reduction increases P_1's profit by a strictly positive amount. This discontinuity in each principal's payoff when they consider increasing the output they offer to the high valuation type leads to the nonexistence of the pure-strategy equilibrium.

Instead, in the present example of an indirect mechanism, there are now enough out-of-equilibrium messages contained in P_2's nonlinear price so that any small change in the quantity consumed from P_1 also triggers a small change in the consumption made from P_2. The agent's indirect utility function becomes smooth and this smoothness ensures the existence of the pure-strategy equilibrium.

It is striking to note that this pure-strategy equilibrium with extended communication with both principals looks very much like a mixed-strategy equilibrium. For all transfer-output pairs offered by a principal in this equilibrium, the profit made on both types is the same.

7 Quasi-Equilibrium

Another way of obtaining existence is to relax the equilibrium concept. The non-existence stressed above comes from the fact that, in a direct communication game with substitutes, each principal wants the agent to lie to the other. We can avoid this problem by imposing a priori that the set of incentive compatible pairs of contracts be collectively agreed by the two principals. Then, each principal can only deviate within this set of *collectively incentive-compatible contracts.*

Definition 2 *: A pair of deterministic direct mechanisms,* $\{t_i(\hat{\theta}_i), q_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta_i}$ *for* $i \in \{1, 2\}$ *is collectively incentive-compatible if and only if the following incentive compatibility constraints are always satisfied:*

$$-t_1(\theta) - t_2(\theta) + u(q_1(\theta), q_2(\theta), \theta) \geq -t_1(\hat{\theta}_1) - t_2(\hat{\theta}_2) + u(q_1(\hat{\theta}_1), q_2(\hat{\theta}_2), \theta) \quad \forall (\theta, \hat{\theta}_1, \hat{\theta}_2) \in \Theta^3. \tag{10}$$

The difference with a purely non-cooperative approach is that each principal must offer contracts which ensures that the agent will always tell the truth not to only to him but also to the other principal. Note that this set of collectively incentive compatible contracts is strictly smaller than the set of contracts which would be incentive compatible for a merged principal since, in this latter case, the agent is forced to send the same reports to both principals and necessarily $\hat{\theta}_1 = \hat{\theta}_2$ on the right-hand-side of (10).

Definition 3 *: A pure-strategy quasi-equilibrium of the common agency game is pair of deterministic direct mechanisms,* $\{t_i(\hat{\theta}_i), q_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta_i}$ *for* $i \in \{1, 2\}$ *which is collectively incentive-compatible, such that each principal* P_i *is on a best response to the contract offered by the other.*

Having defined a quasi-equilibrium being now defined, we can show its existence and characterize the corresponding allocation in the case of substitutes.

Proposition 6 *: When* q_1 *and* q_2 *are substitutes, there exists a unique pure-strategy quasi-equilibrium of the direct communication game with monotonic output schedules. It such that*

$$q^Q(\bar{\theta}) = \frac{\bar{\theta} - c}{1 + \lambda}$$

and

$$q^Q(\underline{\theta}) = \frac{\underline{\theta} - c}{1 + \lambda} - \left(\frac{\nu}{1 - \nu + \lambda}\right)\frac{\Delta\theta}{1 + \lambda}.$$

In this quasi-equilibrium, the only binding incentive constraints are the local incentive compatibility constraints for each principal:

$$u(q^Q(\underline{\theta}), q^Q(\bar{\theta}), \bar{\theta}) - u(q^Q(\underline{\theta}), q^Q(\underline{\theta}), \bar{\theta}) = t^Q(\bar{\theta}) - t^Q(\underline{\theta}).$$

In a quasi-equilibrium, everything happens, from P_1's point of view, as if the inducing information revelation from the high valuation type requires to leave a payoff $-\bar{t}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta})$ to the high valuation agent which, using (6) and (7), is at least equal to $u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - u(\underline{q}_1, \underline{q}_2, \underline{\theta}) + \underline{t}_2$. With our specification of the agent's utility function,

$$u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - u(\underline{q}_1, \underline{q}_2, \underline{\theta}) = \underline{q}_1(\Delta\theta - \lambda\Delta q_2) \tag{11}$$

where $\Delta q_2 = \bar{q}_2 - \underline{q}_2 > 0$ and now $\lambda > 0$. Reducing the output $\underline{q}_1$ offered to a low valuation agent is now less valuable than under cooperation since $\Delta\theta - \lambda\Delta q_2 < \Delta\theta$ when P_2 offers a monotonic contract such that $\Delta q_2 > 0$. As a result, P_1 increases the consumption of a low valuation agent above what he would do at the cooperative contracts. By complementarity, both consumptions of the low valuation agent are in equilibrium below the cooperative outcome. With substitutes, each principal exerts a positive externality on the other and in a quasi-equilibrium, the volume of trade is greater than under cooperation.

It should be stressed that this kind of distortions are exactly the same as in the case of a continuum of types where existence is guaranteed.[21]

[21] See Stole (1991) and Martimort (1992).

The motivation for restricting the set of feasible deviations to the set of collectively incentive compatible contracts comes from the fact that we want to limit the the possibility that either of the principals induces the agent to lie to the other. In a sense, this restriction is a minimal one. If an equilibrium allocation with direct truthful mechanisms exists, it must be such that the incentive constraints (10) are all satisfied by the equilibrium contracts. Otherwise, we would have a contradiction with the agent's equilibrium behavior. Hence, if a pure strategy equilibrium in the direct communication game exists when each principal is allowed to deviate freely, the same equilibrium allocation should also be obtained when each principal is restricted to deviate within a smaller set, the set of collectively incentive compatible contracts.[22] As a direct consequence of this latter remark, we immediately get the following.

Proposition 7 *: When q_1 and q_2 are complements, the set of pure-strategy quasi-equilibria of the direct communication game is the same as the set of subgame perfect equilibria.*

References

Dasgupta, P. and E. Maskin, 1986, "The Existence of Equilibrium in Discontinuous Economic Games 2: Applications," *Review of Economic Studies*, 53: 27-42.

Martimort, D., 1992, "Multi-Principaux avec Anti-Selection," *Annales d'Economie et de Statistiques*, 28: 1-38.

Martimort, D., 1996, "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory," *RAND Journal of Economics*, 27: 1-31.

Martimort, D. and L. Stole, 2000, "Contractual Externalities and Common Agency Equilibria," mimeo Universit de Toulouse.

Martimort, D. and L. Stole, 2001, "The Revelation and Delegation Principles in Common Agency Games," *forthcoming Econometrica.*

Myerson, R., 1982, "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10: 67-81.

[22]In an abstract setting, Myerson (1982) analyzes competition between principal-multi agents hierarchies where agents are specific to a given principal. He shows that there may not exist a truth-telling equilibrium in pure strategies, defines and proves existence of a quasi-equilibrium as a pair of mechanisms such that each principal maximizes his expected payoff within the class of *safe* mechanisms. Any deviation that a principal can make to increase his payoff is unsafe in the sense that arbitrary small perturbations of the mechanism offered by the other principal render infeasible (i.e., not incentive compatible) the deviation. One can check that our definition of a quasi-equilibrium coincide with that of Myerson (1982).

Peters, M., 1999, "Common Agency and the Revelation Principle," forthcoming, *Econometrica.*

Stole, L., 1991, "Mechanism Design under Common Agency," mimeo, University of Chicago.

Appendix

Proof of Proposition 1: There is no loss of generality in applying the Revelation Principle as there is effectively a single "merged" principal who offers a contract $\{(\bar{t}, \bar{q}_1, \bar{q}_2), (\underline{t}, \underline{q}_1, \underline{q}_2)\}$ which maximizes the sum of expected profit subject to incentive and participation constraints which are respectively:

$$-\underline{t} + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) \geq -\bar{t} + u(\bar{q}_1, \bar{q}_2, \underline{\theta}), \tag{12}$$

$$-\bar{t} + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) \geq -\underline{t} + u(\underline{q}_1, \underline{q}_2, \bar{\theta}), \tag{13}$$

$$-\underline{t} + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) \geq 0, \tag{14}$$

$$-\bar{t} + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) \geq 0. \tag{15}$$

As usual in two-type adverse selection models, (13) and (14) are the only relevant constraints at the optimum. The optimal relaxed cooperative contract is then solution of the following program

$$\max_{\{(\bar{t}, \bar{q}_1, \bar{q}_2); (\underline{t}, \underline{q}_1, \underline{q}_2)\}} (1-\nu)(-c(\underline{q}_1 + \underline{q}_2) + \underline{t}) + \nu(-c(\bar{q}_1 + \bar{q}_2) + \bar{t})$$

subject to (13) and (14). Solving this program and taking into account that the solution of this optimization is symmetric, we find the results in Proposition 1. The assumption that $\Delta\theta$ is sufficiently small (see footnote 8), specifically that $\nu\Delta\theta \leq (1-\nu)(\underline{\theta} - c)$, guarantees that $q_i^C(\underline{\theta}) \geq 0$. It is easy to check that the condition $q_i^C(\bar{\theta}) > q_i^C(\underline{\theta})$ and the assumption $u_{\theta q_i} = 1 > 0$ for $i = 1, 2$, ensures that the omitted constraints (12) and (15) are slack at the optimum. The expressions of the transfers and information rents immediately follow. ■

Proof of Propositions 2 and 3: Suppose that a pure-strategy equilibrium exists and for each principal P_i, let $(\bar{q}_i, \bar{t}_i)$ and $(\underline{q}_i, \underline{t}_i)$ denote the corresponding outputs and equilibrium transfers for respectively the high and the low valuation agent. The proof proceeds in five steps.

Step 1: Monotonic Allocations in Equilibrium

Lemma 1 *: All pure-strategy equilibria of any communication game with complements implement monotonic allocations of outputs:* $\bar{q}_i \geq \underline{q}_i$ *for* $i = 1, 2$*.*

Proof: In any pure-strategy equilibrium, the following incentive compatibility constraints must be satisfied: For the low valuation agent,

$$-\underline{t}_1 - \underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) \geq -\bar{t}_1 - \bar{t}_2 + u(\bar{q}_1, \bar{q}_2, \underline{\theta}), \tag{16}$$

$$-\underline{t}_1 + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) \geq -\bar{t}_1 + u(\bar{q}_1, \underline{q}_2, \underline{\theta}), \tag{17}$$

$$-\underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) \geq -\bar{t}_2 + u(\underline{q}_1, \bar{q}_2, \underline{\theta}), \tag{18}$$

and for the high valuation agent

$$-\bar{t}_1 - \bar{t}_2 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) \geq -\underline{t}_1 - \underline{t}_2 + u(\underline{q}_1, \underline{q}_2, \bar{\theta}), \tag{19}$$

$$-\bar{t}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) \geq -\underline{t}_1 + u(\underline{q}_1, \bar{q}_2, \bar{\theta}), \tag{20}$$

$$-\bar{t}_2 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) \geq -\underline{t}_2 + u(\bar{q}_1, \underline{q}_2, \bar{\theta}). \tag{21}$$

Summing (16) and (19) implies

$$\Delta\theta(\Delta q_1 + \Delta q_2) \geq 0; \tag{22}$$

summing (17) and (20) implies

$$\Delta q_1(\Delta\theta - \lambda\Delta q_2) \geq 0; \tag{23}$$

summing (18) and (21) implies

$$\Delta q_2(\Delta\theta - \lambda\Delta q_1) \geq 0, \tag{24}$$

where we denote $\Delta q_i \equiv \bar{q}_i - \underline{q}_i$. When $\lambda < 0$, it is easy to check that only allocations such that $\Delta q_i \geq 0$ satisfy (22), (23) and (24). ‖

This first Lemma is important since it now allows us to restrict the analysis to pure-strategy equilibria with monotonically increasing allocations.

Step 2: Monotonicity of agent's best response.

• Denote by $q_1^*(\theta)$ the output such that the agent with type θ is indifferent between telling the truth or not to P_2 when he consumes a quantity $q_1^*(\theta)$ of good 1:

$$-\bar{t}_2 + u(q_1^*(\theta), \bar{q}_2, \theta) \equiv -\underline{t}_2 + u(q_1^*(\theta), \underline{q}_2, \theta).$$

• Define $\hat{\theta}_2^*(q_1, \theta) =\equiv \arg\max_{\hat{\theta}_2 \in \{\underline{\theta}, \bar{\theta}\}} -t_2(\hat{\theta}_2) + u(q_1, q_2(\hat{\theta}_2), \theta)$.

Lemma 2 *Assume that P_2 offers a monotonic contract such that $\bar{q}_2 \geq \underline{q}_2$, then $\hat{\theta}_2^*(q_1, \theta)$ is increasing in q_1 and in θ, and $q_i^*(\theta)$ is decreasing in θ.*

Proof: By definition we have: $-t_2(\hat{\theta}_2)+u(q_1, q_2(\hat{\theta}_2), \theta) \leq -t_2(\hat{\theta}_2^*(q_1, \theta))+u(q_1, q_2(\hat{\theta}_2^*(q_1, \theta)), \theta)$ for any $\hat{\theta}_2 \leq \hat{\theta}_2^*(q_1, \theta)$. But, since $\lambda < 0$ and $q_2(\hat{\theta}_2) \leq q_2(\hat{\theta}_2^*(q_1, \theta))$, we have for $q_1' > q_1$

$$-t_2(\hat{\theta}_2) + u(q_1', q_2(\hat{\theta}_2), \theta) \leq -t_2(\hat{\theta}_2^*(q_1, \theta)) + u(q_1', q_2(\hat{\theta}_2^*(q_1, \theta)), \theta)$$

which ensures that $\hat{\theta}_2^*(q_1', \theta) \geq \hat{\theta}_2^*(q_1, \theta)$. Reasoning similarly holding q_1 constant, increasing θ and using $u_{1\theta} > 0$ yields that $\hat{\theta}_2^*(\cdot)$ is increasing in θ. Finally, totally differentiating the defining expression for $q_1^*(\theta)$ with respect to θ and q_1 yields $\frac{dq_1^*(\theta)}{d\theta} = \frac{\theta}{\lambda} < 0$. ‖

Step 3: Monotonicity of P_1's Best-Response Contract: We now prove that P_1's best-response to a monotonically increasing contract offered by P_2 is itself monotonically increasing:

Lemma 3 *: A mechanism offered by P_1 which satisfies incentive constraints (1) and (2) is monotonic, i.e., it satisfies $\bar{q}_1 \geq \underline{q}_1$.*

Proof: By adding the incentive constraints, we get $\Delta_\theta \hat{U}^1(\bar{q}_1, \cdot) \geq \Delta_\theta \hat{U}(\underline{q}_1, \cdot)$ where Δ_θ is the difference operator over θ (e.g., $\Delta_\theta \hat{U}(q, \theta) \equiv \hat{U}^1(q, \bar{\theta}) - \hat{U}(q, \underline{\theta})$). Thus, it is sufficient to show that $\Delta_\theta \hat{U}^1(q_1, \theta)$ is continuous and increasing in q_1. Using our definition of $q_1^*(\theta)$, there are three possible regions of q_1 to consider. For $q_1 < q^*(\bar{\theta})$, we have $\hat{\theta}_2^*(q_1, \underline{\theta}) = \hat{\theta}_2^*(q_1, \bar{\theta}) = \underline{\theta}$ and therefore $\Delta_\theta \hat{U}(q_1, \theta) = [q_1 + q_2(\underline{\theta})]\Delta\theta$. For $q_1 \in (q^*(\bar{\theta}), q_1^*(\underline{\theta}))$, we have $\hat{\theta}_2^*(q_1, \underline{\theta}) = \underline{\theta}$ and $\hat{\theta}_2^*(q_1, \bar{\theta}) = \bar{\theta}$, and therefore $\Delta_\theta \hat{U}(q_1, \theta) = q_1[\Delta\theta - \lambda \Delta_\theta q_2(\theta)]$. Finally, for $q_1 > q^*(\underline{\theta})$, we have $\hat{\theta}_2^*(q_1, \underline{\theta}) = \hat{\theta}_2^*(q_1, \bar{\theta}) = \bar{\theta}$ and therefore $\Delta_\theta \hat{U}(q_1, \theta) = [q_1 + q_2(\bar{\theta})]\Delta\theta$. Within all three regions, $\Delta_\theta \hat{U}^1(q, \theta)$ is continuous and increasing in q. Straightforward algebra reveals that $\Delta_\theta \hat{U}^1(q, \theta)$ is continuous at $q_1^*(\underline{\theta})$ and $q_1^*(\bar{\theta})$. ‖

Step 4: Output Best-Responses: Lemma 3 implies that (2) is satisfied whenever (1) is binding at the optimum of P_1's program. To reduce the agent's rent it must also be that (4) is binding. It remains to make precise the expressions of those constraints depending on P_1's offer.

• For a consumption $\underline{q}_1 \leq q_1^*(\bar{\theta})$, the incentive constraint (5) is binding and, using Lemma 2, $\hat{\theta}_2^*(\underline{q}_1, \underline{\theta}) = \underline{\theta}$. After having eliminated transfers, we can rewrite P_1's program as (up to some constant corresponding to P_2's transfer):

$$\max_{\{\bar{q}_1, \underline{q}_1 \leq q_1^*(\bar{\theta})\}} \nu(-c\bar{q}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}))$$

$$+(1-\nu)\left(-c\underline{q}_1 + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) + \frac{\nu}{1-\nu}(u(\underline{q}_1, \underline{q}_2, \underline{\theta}) - u(\underline{q}_1, \underline{q}_2, \bar{\theta}))\right).$$

This yields the following best-responses: For the consumption of the high valuation agent

$$\bar{q}_1 = \bar{\theta} - c - \lambda\bar{q}_2,$$

and for the consumption of the low valuation agent

$$\underline{q}_1 = \max\left\{\underline{q}_1^G, q_1^*(\bar{\theta})\right\},$$

where $\underline{q}_1^G \equiv \underline{\theta} - c - \lambda\underline{q}_2 - \frac{\nu}{1-\nu}\Delta\theta$.

• For a consumption $\underline{q}_1 \geq q_1^*(\bar{\theta})$, the incentive constraint (6) is relevant in P_1's problem. After some manipulations, his objective function becomes (up to some constant corresponding to P_2's transfers):

$$\max_{\{\bar{q}_1, \underline{q}_1 \geq q_1^*(\bar{\theta})\}} \nu(-c\bar{q}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}))$$

$$+(1-\nu)\left(-c\underline{q}_1 + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) + \frac{\nu}{1-\nu}(u(\underline{q}_1, \underline{q}_2, \underline{\theta}) - u(\underline{q}_1, \bar{q}_2, \underline{\theta}))\right).$$

After optimization of P_1's program, we obtain the following best responses:

$$\bar{q}_1 = \bar{\theta} - c - \lambda\bar{q}_2,$$

and

$$\underline{q}_1 = \min\left\{\underline{q}_1^L, q_1^*(\bar{\theta})\right\},$$

where $\underline{q}_1^L \equiv \underline{\theta} - c - \lambda\underline{q}_2 - \frac{\nu}{1-\nu}(\Delta\theta - \lambda\Delta q_2)$.

Step 5: Equilibrium Conditions:

• Consider now a monotonic contract offered by P_2. Monotonicity implies that $\underline{q}_1^L \leq q_1^*(\bar{\theta}) \leq \underline{q}_1^G$, and hence P_1's objective function attains its maximum at a kink at $q_1^*(\bar{\theta})$. Moreover, P_1's optimal contract satisfies $-\bar{t}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}) = -\underline{t}_1 + u(\underline{q}_1, \bar{q}_2, \bar{\theta})$ because the local incentive constraint is binding, and thus we have $\hat{U}^2(\underline{q}_2, \bar{\theta}) = -\underline{t}_1 + u(\underline{q}_1, \underline{q}_2, \bar{\theta})$ for all $\underline{q}_2 \leq \bar{q}_2$, where $\hat{U}^2$ is the indirect utility function of the agent vis à vis P_2. As such, (5) is the relevant incentive compatibility constraint for P_2 when determining his own output best-responses. These best-responses are then given by $\bar{q}_2 = \bar{\theta} - c - \lambda\bar{q}_1$, and $\underline{q}_2 = \underline{q}_2^G \equiv \underline{\theta} - c - \lambda\underline{q}_1 - \frac{\nu}{1-\nu}\Delta\theta$.

• In equilibrium, it is immediate to observe that we must always have $\bar{q}_1 = \bar{q}_2 = q^{FB}(\bar{\theta})$.

• An asymmetric equilibrium is obtained when $\underline{q}_1 = q_1^*(\bar{\theta})$ and $\underline{q}_2 = \underline{q}_2^G$. The largest and smallest implementable values of $\underline{q}_1$ are determined as follows. The highest sustainable value of $\underline{q}_1$ occurs when $\underline{q}_i = \underline{q}_i^G$ for $i = 1, 2$ which is precisely the cooperative quantities,

$q_i^C(\underline{\theta})$. The lowest sustainable value is found by setting $\underline{q}_1 = \underline{q}_1^L$ and $\underline{q}_2 = \underline{q}_2^G$ and checking that the resulting output schedules are monotonic. At such a point, one finds $\underline{q}_1 = \tilde{q}_1(\underline{\theta}) \equiv q^{FB}(\underline{\theta}) - \frac{\nu(1-\nu-\lambda^2-\lambda)}{(1-\nu)(1-\nu-\lambda^2)(1+\lambda)}\Delta\theta$ and $\underline{q}_2 = q^{FB}(\underline{\theta}) - \frac{\nu}{(1-\nu-\lambda^2)(1+\lambda)}\Delta\theta$. Monotonicity is satisfied whenever $q_1^{FB}(\bar{\theta}) - \tilde{q}_1(\underline{\theta}) \geq 0$, which is equivalent to $\frac{(1-\nu-\lambda)}{(1-\nu)(1-\nu-\lambda^2)}\Delta\theta \geq 0$, or more simply $1-\nu \geq \lambda^2$. Finally, our assumption on $\Delta\theta$ small guarantees that $\underline{q}_1 > \underline{q}_2 \geq 0$.

• A symmetric equilibrium is obtained when $q_i(\underline{\theta}) < q_i^*(\bar{\theta})$ for $i = 1, 2$. Then, it is a best response for each principal to consider (5) as the incentive constraint binding in his own program. The equilibrium consumptions for a low valuation agent are the same as under cooperation between the principals.

• Lastly, we need to check that $\underline{q}_i \geq 0$ in the posited equilibria outcomes. The assumption that $\Delta\theta$ is sufficiently small (see footnote 8), specifically that $\nu\Delta\theta \leq (1-\nu-\lambda^2)(\underline{\theta} - c)$, guarantees that, in equilibrium, $\underline{q}_i \geq 0$. ■

Proof of Proposition 4: The proposition proceeds in three steps.

Step 1: Nonexistence of a Separating Pure-Strategy Equilibrium: Consider first the case where this equilibrium is fully separating. In such an equilibrium, it must be that the following holds: $\hat{U}^1(\bar{q}_1, \bar{\theta}) = u(\bar{q}_1, \bar{q}_2, \bar{\theta}) - \bar{t}_2$. Since $\underline{q}_1 < \bar{q}_1$, we must also have $\hat{U}^1(\underline{q}_1, \bar{\theta}) = u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - \bar{t}_2$. From the optimality of P_1's offer, (1) must be binding and therefore: $u(\bar{q}_1, \bar{q}_2, \bar{\theta}) - \bar{t}_1 = u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - \underline{t}_1$. Inserting this expression into the principal's objective function and optimizing with respect to $\bar{q}_1$ yields the first-best consumption for the efficient agent when this maximum is given by the following first-order condition: $\bar{q}_1^0 = \bar{\theta} - c - \lambda\bar{q}_2$, where q_1^0 represents the optimum consistent with the equilibrium requirement that the high-type always consumes $\{\bar{q}_1, \bar{q}_2\}$. In equilibrium, however, the optimality of P_2's offer requires also that (1) is binding and therefore: $u(\bar{q}_1^0, \bar{q}_2, \bar{\theta}) - \bar{t}_2 = u(\bar{q}_1^0, \underline{q}_2, \bar{\theta}) - \underline{t}_2$. Hence, any small upward deviation by P_1 such that $\bar{q}_1' > \bar{q}_1$ entails $\hat{U}^1(\bar{q}_1, \bar{\theta}) = u(\bar{q}_1, \underline{q}_2, \bar{\theta}) - \underline{t}_2$. The best of such deviations has still (1) binding and therefore: $u(\bar{q}_1, \underline{q}_2, \bar{\theta}) - \underline{t}_2 - \bar{t}_1 = u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - \bar{t}_2 - \underline{t}_1$. Inserting this expression into the principal's objective function and optimizing with respect to $\bar{q}_1$ yields a contradiction. Indeed, P_1's objective function is continuous in $\bar{q}_1$ and differentiable on the right-hand-side of $\bar{q}_1^0$. It is easy to check that its derivative is proportional to $\bar{\theta} - c - \bar{q}_1 - \lambda\underline{q}_2$ which is greater than 0 for $\bar{q}_1^0$ when P_2 offers a separating contract. Hence, a small upward deviation in $\bar{q}_1$ raises his profit. This gives a contradiction with the fact that $\bar{q}_1^0$ is at a global optimum of P_1's profit and that, in equilibrium, the high type consumes the high allocation from each principal.

Step 2: Nonexistence of a Pooling Pure-Strategy Equilibrium: This is immediate: If P_2 offers a pooling contract, P_1 deviates and screens across the agent's types.

Step 3: Nonexistence of a Hybrid Pure-Strategy Equilibrium. Suppose that P_2 offers a pooling contract, say (t_2, q_2), and P_1 offers a separating contract. In equilibrium, the optimality of P_1's offer requires that (1) is binding and therefore: $u(\bar{q}_1, q_2, \bar{\theta}) - \bar{t}_1 = u(\underline{q}_1, q_2, \bar{\theta}) - \underline{t}_1$. It also requires that (4) is binding, i.e.: $u(\underline{q}_1, q_2, \underline{\theta}) - \underline{t}_1 - t_2 = 0$. We consider now the incentives of P_2 to offer such a pooling contract. First of all, P_2 must prefer this contract to a deviation in which he offers $(t_2, q_2) = (\underline{t}_2, \underline{q}_2)$ and $(\bar{t}_2, \bar{q}_2)$ where $\bar{q}_2 > q_2$. In this case, we have: $\hat{U}^2(\bar{q}_2, \bar{\theta}) = u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - \underline{t}_1$. The best of such deviations is obtained when (1) is binding, i.e.: $u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - \underline{t}_1 - \bar{t}_2 = u(\bar{q}_1, q_2, \bar{\theta}) - \bar{t}_1 - t_2$. Inserting the expression of the transfer into P_2's objective function and optimizing with respect to $\bar{q}_1$, a necessary condition for such a deviation not to be beneficial is to have:

$$q_2 \geq \bar{\theta} - c - \lambda \underline{q}_1. \tag{25}$$

However, if P_2 finds optimal to offer a pooling contract, it must be that (4) is binding: $u(\underline{q}_1, q_2, \underline{\theta}) - \underline{t}_1 - t_2 = 0$. Optimizing with respect to q_2 yields then:

$$q_2 = \underline{\theta} - c - \lambda \underline{q}_1, \tag{26}$$

a contradiction with (25). ■

Proof of Proposition 5: Assume that P_2 offers a nonlinear schedule such that: $t_2(q_2) = cq_2 + a_2$. We can compute the indirect utility function of the agent vis à vis P_1: $\hat{U}^1_I(q_1, \theta) = \max_{q_2}\ u(q_1, q_2, \theta) - cq_2 - a_2$. Maximizing this concave expression over q_2 and substituting yields $\hat{U}^1_I(q_1, \theta) = -a_2 + cq_1 + \frac{(\theta - c)^2}{2}$. Satisfying (1) and (2) imposes that: $\bar{t}_1 - c\bar{q}_1 = \underline{t}_1 - c\underline{q}_1$. In particular, this implies that P_1 is indifferent between all pairs $(\bar{t}_1, \bar{q}_1)$ and $(\underline{t}_1, \underline{q}_1)$ since he gets the same profit on each. Let that profit be denoted a_1. Moreover, the participation constraints (3) is satisfied when (4) is binding. This yields: $a_1 + a_2 = \frac{(\underline{\theta} - c)^2}{2}$. All direct mechanisms which satisfy these properties can be offered in a best-response of P_1 to the indirect mechanism offered by P_2. Consider the schedule $t_1(q_1) = cq_1 + a_1$. This indirect mechanism supports all possible allocations which can arise as a best-response of P_1. Hence, we have constructed an equilibrium in indirect mechanisms defined over all the real line. Furthermore, when he is given this pair of nonlinear schedules the agent chooses the first-best consumptions. ■

Proof of Propositions 6 and 7: We first characterize the set of collectively incentive compatible mechanisms. We focus as usual on the incentive compatibility constraints of a high valuation agent and check ex post the incentive constraint of a low valuation agent. The set of collectively incentive compatible contracts for $\bar{\theta}$ is characterized by constraints

(19) to (21). Similarly, The set of collectively incentive compatible contracts for $\underline{\theta}$ is characterized by constraints (16) to (18).

When $\lambda > 0$, the local incentive constraints (20) and (21) define a set of transfers which is strictly interior to that defined by the global incentive constraints (19). Indeed, we then have:

$$-\bar{t}_1 - \bar{t}_2 + \underline{t}_1 + \underline{t}_2 \geq u(\bar{q}_1, \underline{q}_2, \bar{\theta}) - u(\bar{q}_1, \bar{q}_2, \bar{\theta}) + u(\underline{q}_1, \bar{q}_2, \bar{\theta}) - u(\bar{q}_1, \bar{q}_2, \bar{\theta})$$

$$> u(\underline{q}_1, \underline{q}_2, \bar{\theta}) - -u(\bar{q}_1, \bar{q}_2, \bar{\theta})$$

when $\bar{q}_i > \underline{q}_i$ for $i \in \{1, 2\}$. Hence, as on Figure 3, this is the local incentive constraint which is binding in each principal's best response to what the other offers. Neglecting the low valuation agent's incentive constraint which has to be checked ex post, the transfers offered by P_1 are such that (20) and (7) are binding. After having eliminated transfers, we can rewrite P_1's program as (up to some constant corresponding to P_2's transfer):

$$\max_{\{\bar{q}_1, \underline{q}_1 \leq q_1^*(\bar{\theta})\}} \nu(-c\bar{q}_1 + u(\bar{q}_1, \bar{q}_2, \bar{\theta}))$$

$$+(1-\nu)\left(-c\underline{q}_1 + u(\underline{q}_1, \underline{q}_2, \underline{\theta}) + \frac{\nu}{1-\nu}(u(\underline{q}_1, \underline{q}_2, \underline{\theta}) - u(\underline{q}_1, \bar{q}_2, \bar{\theta}))\right).$$

This yields the following best-responses: For the consumption of the high valuation agent

$$\bar{q}_1 = \bar{\theta} - c - \lambda \bar{q}_2,$$

and for the consumption of the low valuation agent

$$\underline{q}_1 = \underline{\theta} - c - \lambda \underline{q}_2 - \frac{\nu}{1-\nu}(\Delta\theta - \Delta q_2).$$

For a symmetric quasi-equilibrium, the outputs are finally as in Proposition (6). It is easy to check that the low valuation agent's incentive constraints are all slack. ■