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CONTRACTUAL EXTERNALITIES AND COMMON AGENCY EQUILIBRIA

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Abstract

This paper characterizes the equilibrium sets of an intrinsic common agency game with direct exter-nalities between principals both under complete and asymmetric information. Direct externalities arise when the contracting variable of one principal affects directly the other principal's payoff. Out-of-equilibrium messages are used by principals to precommit themselves to distort their strategic behavior. We characterize pure-strategy symmetric equilibria arising in such games under complete information and show their multiplicity. We then introduce asymmetric information to refine the set of feasible conjectures. We show that a unique equilibrium may be selected by conveniently perturbing the information structure. Both under complete and asymmetric information, we show that the equilibrium outputs of the intrinsic common agency game are also equilibrium outputs of the delegated common agency game, although the two games differ in terms of the distribution of surplus they involve.

JEL Classification: D82, L51.

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1 Introduction

In many economic contexts several principals control a common agent and each of those principals is directly affected by the actions imposed on the common agent by the other principals through their bilateral contracts with the agent. Examples of such *direct externalities* abound and cover many different settings like competition under nonlinear pricing, tax competition between different jurisdictions, regulatory oversight by rival agencies, market-makers dealing with common liquidity traders on financial markets, competition on insurance or loan markets among insurers or lenders, and lastly competition between retailers dealing with a common manufacturer.

In standard models corresponding to the examples described above, a decision consists of a transfer and an output. Decision rules can thus be identified with price-output pairs (i.e., nonlinear price schedules) and the analysis of nonlinear prices in common agency environments becomes an interesting theoretical question. To address this broad issue, this paper investigates the equilibria of common agency games in a simple economic environment where principals compete through nonlinear schemes. This meaningful economic environment involves two competing retailers (the principals) producing perfect substitutes with intermediate goods which are bought from a common manufacturer (the agent).

Importantly, two different kinds of externalities are at play simultaneously in our model. First, each principal's action (the intermediate output he requests from the manufacturer) affects the agent's utility function (the cost of the retailer) and there are thus *indirect contractual externalities* as in Stole (1991), Martimort (1992 and 1996) and Mezzetti (1997). Second, and contrary to this earlier common agency literature, each principal's action also affects the other principal's utility function since both principals share the markets for the final good they produce. Hence, there are now also *direct contractual externalities* between the principals. The contractual activity of principal 2 directly affects principal 1's utility function.

In such a framework, and following the tradition of standard principal-agent models, we characterize the set of implementable allocations, i.e., the set of allocations obtained as pure-strategy Nash equilibria of the common agency game with deterministic mechanisms. We find a remarkably large set of such equilibria under *complete information* between the principals and the common agent they control. A whole range of outputs is sustainable as equilibrium outcomes of the common agency game in which principals compete through nonlinear prices. This multiplicity arises from the existence of out-of-equilibrium messages in the equilibrium tariffs offered by both principals. Including these out-of-equilibrium messages in their tariffs, the principals are able to *precommit* themselves to distort their strategic behavior on the final market through the contract they respectively offer to the agent. This commitment value of contracts has no equivalent in a monopolistic screening environment and can only be achieved through extending mechanisms with out-of-equilibrium options. Those options are used as implicit veto threats to prevent other principals from deviating from a given allocation.

Under complete information, common agency games are thus plagued with multiple equilibria. When contractual activities are *complements* in the agent's utility function we demonstrate that any output between the classical monopoly and the Cournot outcomes can be sustained in equilibrium because out-of-equilibrium messages tend to *soften* competition between the principals. When contractual activities are instead *substitutes*, any output between the Cournot and the Bertrand outcomes can now be sustained as out-of-equilibrium messages tend to *harden* competition between the principals. Strikingly, the Cournot outcome always emerges as the unique symmetric equilibrium of the common agency game where principals use *direct revelation mechanisms.* By suppressing out-of-equilibrium messages, those mechanisms eliminate the principals' precommitment ability. Common agency environments offer therefore a significant failure of the Revelation Principle when applied to the agent's underlying type space.[1]

Given the multiplicity of equilibrium outcomes which exist under complete information, we *rationalize* a unique choice from the set of equilibrium conjectures by explicitly introducing asymmetric information between the principals and their common agent. Given that asymmetric information is the most often heard motivation for justifying the use of complex nonlinear schemes, this approach has a strong argument in its favor.[2]

To introduce asymmetric information, we assume that some parameter of the agent's cost function is unknown to both principals. We first derive the symmetric and differentiable pure-strategy equilibria of this common agency game when the distribution of the cost parameter has a fixed and finite support. Adverse selection introduces incentive compatibility constraints which restrict the slope of the equilibrium tariff at all equilibrium outputs. Indeed, by enlarging the set of outputs which are chosen along the equilibrium path, asymmetric information reduces the set of off-the-equilibrium-path behaviors which are consistent with an equilibrium. Compared to the

[1]Epstein and Peters (1996) demonstrate that a larger, universal message space exists for which the revelation principle is valid, but constructing such universal message spaces is currently an obstacle in common agency models.

[2]Our study of nonlinear pricing games is actually more general than it first appears, and has something to say about equilibria in large communication-mechanism common agency games. To see this, we can use a variation of the taxation principle, following the approach in Martimort and Stole (1999); a similar idea is independently developed in Peters (2001). There, we demonstrate that although the revelation principle cannot easily be applied to common agency environments (i.e., it would require a universal message space along the lines developed in Epstein and Peters (1996)), it is not difficult to use an extension of the taxation principle from agency theory – what we call the delegation principle – to characterize the set of equilibria from all message games. Specifically, we show that any equilibrium in a any common-agency communication-mechanism game is also an equilibrium to a game in which each principal chooses a menu of distributions of allocations to offer the agent (from some set of menus restricted to correspond with the original message space limitations) and the agent chooses an element from each principal's offered menu. The main restriction in our analysis over general communication mechanism games, therefore, is to limit the principals to offering menus with only deterministic outcomes (i.e., nonlinear price-quantity schedules) rather than allowing for more general menus of distributions (i.e., nonlinear price-quantity lottery schedules). We are not aware of any equilibrium generated by lotteries that is not also generated by nonlinear prices, but at present we cannot state that this restriction is without loss of generality. A secondary restriction is that the nonlinear pricing equilibrium gives rise to smooth quantity allocations as a function of type. Given that monotonicity of outputs is a typical requirement of incentive compatibility, and that such monotonicity implies that nonlinear pricing schedules are almost everywhere smooth, this restriction is not overly strong in our opinion.

case of complete information, the equilibrium set shrinks significantly when nonlinear tariffs must satisfy incentive compatibility constraints at all equilibrium outputs.

Even when adding asymmetric information on a bounded support, a full range of equilibrium conjectures remains feasible. Asymmetric information with a distribution of the adverse selection parameter having a bounded support falls short of selecting a unique equilibrium even though it significantly restricts the equilibrium set. However, asymmetric information may offer a quite powerful selection device when the support of the distribution of the private information parameter increases without bound. We show that a unique *robust* equilibrium survives this procedure. Because all outputs arise in equilibrium with some probability, and the incentive constraints completely tie down the relationship between outputs and marginal prices, the multiplicity of conjectures consistent with equilibrium behavior disappears.

Section 2 presents our common agency model. We discuss there the direct contractual externalities between rival principals. Section 3 analyzes the case of complete information and shows that a large set of equilibria are sustained with general nonlinear prices. We compare also this set to the equilibrium obtained with simple direct revelation mechanisms. Section 4 extends the analysis to delegated agency games and shows the outputs are the same as under the intrinsic common agency setting of section 3. Section 5 provides the analysis of our common agency game under asymmetric information. We discuss how asymmetric information between the principals and their common agent may help to select among all previous equilibria. Section 6 selects a unique equilibrium conjecture by perturbing the information structure, and section 7 demonstrates the robustness of the selection in delegated agency games. Section 8 concludes.

2 A Common Agency Model with Direct Externalities

We begin with a description of a common game between two retailers (the principals P_i for $i = 1, 2$) selling perfect substitutes on a final market. Each unit of final good q_i is produced with a one-to-one production function from one unit of an intermediate good i. Those two intermediate goods are produced by a single manufacturer (the agent A).

This common agent has a quasi-linear utility function which is symmetric and concave in (q_1, q_2):

$$U = y - C(q_1, q_2, \theta)$$

where y is a monetary transfer and q_i is the production of intermediate good i. $C(\cdot)$ is the common agent's cost function which is symmetric and convex in (q_1, q_2), increasing in q_1 and q_2 and such that the standard sorting assumptions $C_{i\theta} > 0$ for $i = 1, 2$ are both satisfied. For technical reasons, we also assume that $C_\theta(\cdot)$ is convex in (q_1, q_2). θ is a parameter reflecting the cost efficiency of this common agent.

Whenever possible, we state our results with the greatest generality by keeping a general expression for the cost function. However, for tractability and to insure the sufficiency of some arguments, we will sometimes assume that $C(\cdot)$ is quadratic. In this case, we will have:

$$C(q_1, q_2, \theta) = \theta(q_1 + q_2) + \frac{1}{2}(q_1^2 + q_2^2) + \lambda q_1 q_2.$$

The parameter $\lambda \in]-1, 1]$ represents the relationship between q_1 and q_2 in the agent's utility function. The intermediate goods are *complements* in the agent's utility function when $\lambda < 0$ and *substitutes* when $\lambda > 0$.[3] The case of complements is obtained, for instance, when the production of both final goods requires access to a common network.[4] The case of substitutes is obtained instead when final goods are produced from a common input which must be allocated between the two lines of production. This last setting can be viewed as a model of intra-brand competition or vertical contracting along the lines of Hart and Tirole (1990), O'Brien and Shaeffer (1992) and McAfee and Schwartz (1994). Alternatively, with substitutes, and slightly relabeling variables, the setting we analyze can be viewed as a split-award auction along the lines of Anton and Yao (1989).[5] In such procurement models, producers share the final market for efficiency reasons but offer nonlinear bid schedules to a common buyer who decides how much to buy from each sellers.

The common agent gets some reservation utility exogenously normalized to zero if he decides not to produce both intermediate goods. In a first step of our analysis, we consider a model of *intrinsic common agency* in which the agent produces simultaneously either both intermediate goods or none. In a second step of the analysis, we check that the equilibrium outputs of the intrinsic common agency game still emerge under the more complicated *delegated common agency*, i.e., when the agent may choose to work with only one of the competing principals. In this case, the agent has an outside option which is type-dependent and determined by the contract of the sole principal with whom he may choose to work.

The agent's efficiency parameter is his private information. It is drawn from the set $\Theta = [\underline{\theta}, \bar{\theta}]$ according to a common knowledge distribution $F(\cdot)$ with positive density $f(\cdot)$. This distribution satisfies the monotone hazard rate property, $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$ for all $\theta \in \Theta$.

Inverse demand on the market for the homogeneous final good is denoted by $P(Q)$ with $P'(\cdot) < 0$, $P''(\cdot) \leq 0$. $Q = q_1 + q_2$ is the total output produced by the principals. We will sometimes assume that $P(\cdot)$ is linear and, in this case, $P(Q) = a - Q$.

To simplify, and without loss of generality, we assume that principal P_i produces a quantity q_i at no cost. Principal P_i's profit is thus given by:

$$V_i(t_i, q_i, q_{-i}) = P(q_i + q_{-i})q_i - t_i$$

[3]When $\lambda = 0$ the two intermediate goods are unrelated. When $\lambda = 1$, the intermediate goods are perfect substitutes; i.e., $U = y - (\theta Q + \frac{1}{2}Q^2)$, where $Q = q_1 + q_2$.

[4]This can be viewed as a highly stylized model of competition between telephone companies on the market for long-distance calls when the latter companies have access to a local loop controlled by a third company.

[5]See also Wilson (1979) and Bernheim and Whinston (1986) for related models of share auctions.

when he buys a quantity q_i of intermediate i at price t_i and P_{-i} puts a quantity q_{-i} on the final market.

Finally, we assume that principal P_i can only contract with the common agent on activity q_i. For instance, P_i has neither the auditing rights nor the monitoring technologies to observe and, thus, contract on q_{-i}. In the same way, P_i cannot contract on P_j's contract because the latter contract is not observable by P_i. Anti-trust rules may also forbid such reciprocal contracting in the industrial organization setting we analyze here.[6]

Typically, a *pure strategy deterministic contract* between P_i and A is a nonlinear schedule $t_i(q_i)$ mapping the agent's choice of output into the transfer paid by principal P_i.[7] Applying a generalization of the taxation principle, Martimort and Stole (1999) show that there is no loss of generality in considering this class of deterministic nonlinear prices if the modeler is interested in common agency equilibria with deterministic mechanisms. The essence of this Taxation Principle is that extending nonlinear prices to allow further messages on top of quantity choices is of no additional value. From the agent's point of view, communication-per-se is of no value, only the payoff-relevant consequences of such communication. Hence, communication can be replaced with a decentralized menu of choices that are undominated. For example, in the seemingly more general game in which offered nonlinear price schedules depend upon the messages sent by the agent, we could replace the mechanism with the upper envelope of transfers without any strategic consequences for either the principals or the agent. Since pure-strategy equilibria have a natural economic appeal, we further restrict our attention to pure-strategy equilibria in deterministic nonlinear prices.[8]

Note that the production contracted upon by P_{-i} with the common agent enters directly into P_i's objective function. We thus have an instance of *direct contractual externalities.* This novelty of the modeling in the common agency literature under adverse selection stands in sharp contrast with the models developed by Stole (1991) and Martimort (1992). In these latter papers, we both assumed that contractual externalities between the principals occur only because the agent's marginal utility for contractual activity i depends also on contractual activity $j \neq i$, however that this activity, j, does not enter directly into P_i's utility function.[9]

The *intrinsic common agency game* unfolds as follows:

[6] Katz (1987) shows in the related context of delegation games between principal-agent pairs that such reciprocal conditioning of contracts may imply the nonexistence of a contract equilibrium.

[7] These mechanisms are deterministic since the agent chooses an output and not a distribution of outputs among which the principal could randomize. Not only do these randomizations seem unrealistic in the real world and hardly observed in the case of nonlinear pricing, but they also require that judicial courts can enforce these randomizations. This may be quite problematic since any deviation away from the randomization which has been contracted upon can only be detected statistically after many such realizations of the contract. Such detections are instead not feasible in the one-shot relationships that we analyze here.

[8] Nonetheless, even within this class of equilibria, we find that out-of-equilibrium messages play a significant role in describing the set of equilibrium outcomes. It is noteworthy that because mixed-strategy equilibria are also limits of pure-strategy equilibria of games of incomplete information between the principals, the pure-strategy equilibria of such Bayesian common agency games could be obtained using the same techniques as in the present paper.

[9] Laffont and Martimort (1997) call direct (resp. indirect) externalities *type 2* (resp. *type 1*) *externalities.*

1. First, nature draws θ. This parameter is be known only by the common agent under the case of *asymmetric information* or by all players under *complete information*.

2. Principals non-cooperatively offer nonlinear price schedules.

3. The common agent accepts or refuses both contracts.

4. If he refuses, the common agent gets his reservation utility normalized at zero. If he accepts, he chooses an amount of intermediate good to produce and receives the corresponding transfer.

In the case of a *delegated common agency game* that we analyze also below, stage 3 of the timing above is replaced by

3'. The common agent chooses to accept both, one, or none of the contracts.

In this latter case, a common agency equilibrium arises if the agent gets more utility by taking both contracts than by taking only a contract offered by only one of the principals.

3 Equilibria of the Intrinsic Common Agency Game under Complete Information

3.1 Multiplicity of Equilibria and the Role of Out-Of-Equilibrium Messages

For tractability and to achieve a complete characterization of the set of common agency equilibria under complete information, we shall focus on the class of twice-differentiable nonlinear prices defined over a sufficiently large domain of quantity, and we restrict our attention to symmetric equilibria; from here on, when we use the phrase *symmetric, differentiable equilibrium*, we imply precisely these restrictions.

In this initial section, we explore settings of complete information in which both principals know the value of θ. Hence, we are interested in equilibria in which, for a fixed θ, the principals play a game in designing optimal nonlinear price schedules to offer the agent. Hence, transfer functions will have a second argument to account for the observed and verifiable type of the agent: $t_i(q_i, \theta)$. When we consider incomplete information, this dependence is removed.

For further reference, we define the following output allocations:

- The *Bertrand* symmetric output, $q^b(\theta)$, satisfies:

$$P(2q^b(\theta)) = C_1(q^b(\theta), q^b(\theta), \theta);$$

For this outcome, the price of one unit of final good equals its marginal cost of production.

• The *Cournot* symmetric output, $q^c(\theta)$, satisfies:

$$P(2q^c(\theta)) + q^c(\theta)P'(2q^c(\theta)) = C_1(q^c(\theta), q^c(\theta), \theta).$$

The marginal revenue of each competing retailer equals then the marginal cost of production. As we will see in Proposition 2 below, in a constrained game in which the competing retailers are not allowed to offer anything other than a single output and corresponding price, the Cournot outcome emerges for familiar reasons. In an unconstrained setting, the Cournot outcome need not emerge.

• The *monopoly* symmetric output, $q^m(\theta)$, satisfies:

$$P(2q^m(\theta)) + 2q^m(\theta)P'(2q^m(\theta)) = C_1(q^m(\theta), q^m(\theta), \theta).$$

The marginal revenue made by a coalition of both competing principals equals the marginal cost of production. This is the outcome achieved had the principals being cooperating in their contractual offers to the common agent.

It is immediate to check that $q^m(\theta) < q^c(\theta) < q^b(\theta)$ as a result of a greater exercise of market power on the final good market when one varies outcomes from Bertrand, to Cournot, and then to monopoly.

To determine retailer 1's optimal choice of transfer, $t_1(q_1, \theta)$, let us take as given the nonlinear price $t_2(q_2, \theta)$ offered by P_2 to the common agent in an equilibrium of the intrinsic common agency game. This allows us to define the agent's optimal choice of q_2, given q_1 is supplied to retailer P_1.

$$q_2^*(q_1, \theta) \equiv \arg\max_{q_2} \ t_2(q_2, \theta) - C(q_1, q_2, \theta).$$

Assuming that the maximand $t_2(q_2, \theta) - C(q_1, q_2, \theta)$ is sufficiently regular such that its optimal solution is uniquely characterized by a first-order condition, we can define $q_2^*(q_1, \theta)$ by the expression:[10]

$$t_2'(q_2^*(q_1, \theta)) = C_2(q_1, q_2^*(q_1, \theta), \theta) \quad \forall \quad (\theta, q_1).^{11} \qquad (1)$$

Under complete information, P_1's best response is to implement a production q_1 and a transfer $t_1 = t_1(q_1, \theta)$ which are solution to the following problem:

$$max_{\{t_1, q_1\}} P(q_1 + q_2^*(q_1, \theta))q_1 - t_1$$

[10] We will check ex post to make show that the agent's program is indeed concave, so that providing that the transfer functions are defined over a broad domain to exclude corner solutions, the first-order approach is valid.

[11] This first-order condition states that, at the equilibrium point, the nonlinear schedule offered by a principal is equal to the common agent's marginal cost with respect to output for this principal. Even if its spirit is somewhat similar, this first-order condition should not be confused with the *"truthfulness"* requirement imposed in other common agency games under complete information by Bernheim and Whinston (1986) and Grossman and Helpman (1994). These latter authors analyze settings where the contracting abilities of each principal are similar, i.e., each final producer can contract on the whole vector of intermediate goods (q_1, q_2). The agent's action choice is then defined by a first-order condition similar to (1) when the common agent has a continuum set of actions to choose from. For differentiable equilibria, the slopes of the nonlinear schedule $t_2(\cdot)$ offered by principal P_2 at the equilibrium point reflects his marginal utility with respect to all contracting variables. Hence, the denomination *"truthful"*.

subject to

$$t_1 + t_2(q_2^*(q_1,\theta),\theta) - C(q_1, q_2^*(q_1,\theta),\theta) \geq 0, \tag{2}$$

where $q_2^*(q_1,\theta)$ is defined through the first-order condition (1). (2) is the agent's participation constraint which stipulates that the agent has to accept both contracts (rather than none) in the intrinsic common agency game.

The following proposition characterizes the full-information pure-strategy equilibrium of our intrinsic common agency game with deterministic and twice-differentiable nonlinear tariffs.

Proposition 1 *: Under complete information, a necessary condition for an output $q(\theta)$ to belong to the set of symmetric, differentiable equilibrium outputs of the intrinsic common agency game are:*

- *$q(\theta) \in [q^c(\theta), q^b(\theta)]$ when the intermediate goods are substitutes in the agent's utility function;*
- *$q(\theta) \in [q^m(\theta), q^c(\theta)]$ when the intermediate goods are complements in the agent's utility function.*
- *the agent gets zero rents.*

When $C(\cdot)$ is quadratic, the above conditions are sufficient (i.e., for any $q(\theta)$ satisfying the above boundaries, there exist transfer functions which implement these outputs in equilibrium.)

Proposition 1 establishes that the intrinsic common agency game under complete information is plagued with a large set of equilibria. The reason for this multiplicity is simple. Let us fix θ which is known to the principal. By offering a smooth nonlinear price schedule *for this value of* θ , the principal P_i not only controls the agent's equilibrium production of intermediate good q_i but also how the agent behaves *around* this equilibrium point following any unexpected contractual offer made by principal P_{-i}. This extra control of the agent's behavior off-the-equilibrium path changes the degree of the principals' competition on the final market. Out-of-equilibrium messages play the role of implicit veto threats to prevent either principal from inducing the agent to produce a different output than that conjectured in equilibrium.[12]

• **The First-order Approach:** To understand the exact origin of this multiplicity and in particular the extra control that a principal can have on the agent by using extended nonlinear prices, it is useful to introduce more notation and to first discuss the *first-order approach* used in deriving these equilibria of the intrinsic common agency game. To characterize the common agent's behavior with a first-order condition, we first assume that the nonlinear tariff $t_{-i}(q_{-i})$ offered by P_{-i}

[12] Of course, if θ is not observable, a distinct nonlinear price schedule cannot be offered for each θ, and hence we may expect this multiplicity to be significantly curtailed when θ is information privately held by the agent. The corresponding analysis will be done in Section 4.

is differentiable and second that it is concave. Henceforth, we focus *a priori* on common agency equilibria with nonlinear tariffs which satisfy these conditions, and then we check ex post that the derived equilibrium tariffs do indeed justify our working assumption.

In fact, the first-order approach is valid for all output q_i, i.e., both for outputs on and off the equilibrium path. This requires implicitly that the tariff $t_{-i}(q_{-i})$ be conveniently extended for outputs which may lie outside the set of equilibrium outputs. Under complete information, quadratic nonlinear prices defined over the whole real lines will in fact be enough to validate this first-order approach as shown in the Appendix.[13]

• **The Case of Substitutes:** With substitutes, any degree of competition between the Cournot and the Bertrand outcomes can be implemented as an equilibrium outcome. The intuition for the fact that competition between the principals raises outputs is rather straightforward. Indeed, since the common agent can always substitute away production of intermediate good 1 against production of intermediate good 2, each principal pays at the margin too much for the intermediate good he wants the agent to produce. In equilibrium, the agent increases the production of both intermediate goods with respect to a situation where the principals would have cooperated in making their contractual offers. Too much final output is placed on the market. The contractual externality between the principals is positive in marginal returns to output, resulting in excessive activities in both relationships.

The intuition for the multiplicity of equilibria is also easy to grasp. Different equilibrium outputs correspond to different slopes of the symmetric nonlinear equilibrium tariff around these equilibrium points. Indeed, by offering a nonlinear price schedule which is conveniently extended outside the set of equilibrium outputs, P_2 can control P_1's incentives to capture the final market. This is done at the contracting stage by stipulating with extra messages how the agent's production of intermediate good q_1 should evolve when P_1 deviates from his equilibrium offer. (1) shows that $t_2(\cdot)$ can be used by P_2 as an implicit threat to veto any desire of P_1 to increase his own production.

In a sense, each principal aims at becoming the Stackelberg leader on the final market by using the first stage of contracting with the agent. Various degrees of convexity of these equilibrium nonlinear prices however, correspond thus to different *conjectures* on the degree of competition which actually arises on the final market.

Equilibrium outputs which are close to the upper bound of the equilibrium set correspond to the least cooperative outcomes and to the *flattest* tariffs around the equilibrium point. The agent's incentives to substitute the production of intermediate good q_1 against q_2 are then exacerbated if P_1 deviates and tries to exert more of his market power by reducing his own production of the

[13] This first-order approach with nonlinear prices was first used in Martimort (1992) in a common agency model with no direct externality and asymmetric information. In this setting also, constructing these extensions turns out to be necessary when concavity of the social surplus function is not sufficiently great. Interestingly, Klemperer and Meyer (1989) in a related model of competition in supply functions also use extended supply functions to validate a quite similar first-order approach.

final good. The upper bound of the equilibrium set thus corresponds to the Bertrand outcome. Equilibrium outputs which are close to the lower bound of the equilibrium set correspond instead to the least competitive outcomes and the *steepest* schedules around the equilibrium point. This lower bound corresponds to the classical Cournot outcome. In this case, principal P_i has *passive conjectures* about how the agent changes his production of intermediate good q_{-i} following any unexpected offer he receives from P_i. Interestingly, this Cournot outcome is the most cooperative outcome achieved through bilateral contracting. We show in the Appendix that imposing the optimality of the agent's decision problem, together with the first-order conditions of the principals' programs, forces at least this minimal amount of competition between the principals.

• **The Case of Complements:** With complements, any degree of competition between the monopoly and the Cournot outcomes can now be implemented as an equilibrium outcome. Equilibrium outputs which are close to the upper bound of the equilibrium set correspond again to the least cooperative outcomes. However, the equilibrium nonlinear price is now *rather steep* around the equilibrium point. Indeed, the agent's incentives to reduce the production of intermediate q_i are exacerbated if principal P_{-i} tries to contract his own production of final good. The upper bound of the equilibrium set corresponds now to the Cournot outcome. Equilibrium outputs which are close to the lower bound of the equilibrium set correspond to the most cooperative outcomes and the flattest schedules around the equilibrium point which is consistent with the global concavity of the agent's problem. The corresponding lower bound is thus the monopoly solution which maximizes the aggregate payoff of the organization.

Under complete information, we obtain the striking result that decentralized contracting with two competing principals may involve no loss of generality. One equilibrium of the common agency game with direct externalities implements the same outcome as under a cooperative behavior. The lack of coordination between the principals can be circumvented by conveniently specifying the out-of-equilibrium messages of the individual tariffs. Of course, decentralized contracting may also involve a significant loss of profit from the principals' point of view if one is interested in the whole set of equilibrium outcomes of the common agency game. Indeed, the contractual externalities on the marginal returns to output between the principals are now negative resulting, generally, in excessively low activities in both bilateral relationships.

The next corollary summarizes the discussions above on the slope of the nonlinear equilibrium schedule in the case of a quadratic cost function.

Corollary 1 *: Under complete information and when $C(\cdot)$ is quadratic, the symmetric equilibrium nonlinear price $t(q,\theta)$ is such that:*

- *when intermediate goods are substitutes $t''(q(\theta),\theta) = 1-\lambda \quad$ if $q(\theta) = q^b(\theta)$ and $t''(q(\theta),\theta) = -\infty \quad$ if $q(\theta) = q^c(\theta)$.*

- *when intermediate goods are complements* $t''(q(\theta),\theta) = -\infty$ *if* $q(\theta) = q^c(\theta)$ *and* $t''(q(\theta),\theta) = 1+\lambda$ *if* $q(\theta) = q^m(\theta)$.

More precisely, we can easily parameterize the equilibrium outputs in the case where $P(\cdot)$ is linear and $C(\cdot)$ is quadratic by the value of the second derivative of the symmetric nonlinear price $t(q,\theta)$ at the equilibrium point $q(\theta)$.

Corollary 2 *: Under complete information, when* $C(\cdot)$ *is quadratic, the symmetric equilibrium outputs are such that:*

$$q(\theta)P'(2q(\theta)) + P(2q(\theta)) = \theta + (1+\lambda)q(\theta) + \frac{\lambda q(\theta)P'(2q(\theta))}{1-\nu}, \tag{3}$$

where $\nu \in]-\infty, 1-\lambda]$ *in the case of substitutes, and where* $\nu \in [1+\lambda, 1]$ *in the case of complements.*

The symmetric nonlinear price $t(q,\theta)$ *which implements the equilibrium output* $q(\theta)$ *above can be taken to be quadratic and:*

$$t(q,\theta) = t(q(\theta),\theta) + t'(q(\theta),\theta)(q - q(\theta)) + \frac{\nu}{2}(q-q(\theta))^2, \tag{4}$$

where $t(q(\theta),\theta) = \theta q(\theta) + \frac{(1+\lambda)}{2}q^2(\theta)$ *and* $t'(q(\theta),\theta) = \theta + (1+\lambda)q(\theta)$.

This quadratic specification of the nonlinear price which implements a given output will be particularly useful in specifying the equilibrium under delegated common agency.

3.2 Equilibrium without Out-Off Equilibrium Messages

Interestingly, the Cournot outcome, which is a bound of the equilibrium set both under substitutes and complements, can also be implemented as an equilibrium outcome of a common agency game played with *direct revelation mechanisms* in this complete information setting.

A deterministic *direct revelation mechanism* offered by principal P_i is a pair $\{t_i(\theta), q_i(\theta)\}$ stipulating a monetary transfer and a given quantity of intermediate good i. These mechanisms involve no extra transfer-output pair on top of that actually used in equilibrium. In the case of complete information, the agent's report on his type is trivial since the type space is reduced to a singleton. The nonlinear prices corresponding to these direct mechanisms consist thus of a single price-quantity pair.

Proposition 2 *: The Cournot output* $q^c(\theta)$ *is the unique equilibrium of the intrinsic common agency game under complete information with direct revelation mechanisms.*

This result is not very surprising in view of previous discussions. With a direct revelation mechanism, principal P_{-i} cannot control the agent's behavior off the equilibrium path for any unexpected contractual offer made by P_i. The common agent does not change his production of intermediate q_{-i} following such an unexpected offer. Conjectures about the agent's behavior off the equilibrium path are *passive*. These passive conjectures implement the same outcome as what is obtained with extended nonlinear prices which are very steep around the equilibrium point. Indeed, with very steep nonlinear prices, the agent does not change his production for P_{-i} when P_i deviates.

Even if direct mechanisms may seem quite appealing in a complete information environment, the comparison of Propositions 1 and 2 confirms the unavailability of the Revelation Principle (at least a version of it with deterministic mechanisms). The set of equilibrium outcomes achieved with indirect mechanisms differs significantly from the set of equilibrium outcomes achieved when both principals are restricted to use direct mechanisms. This latter result confirms a finding of Martimort and Stole (1998) where we analyze less structured games between competing principals.[14] More precisely, this comparison shows that focusing on this particular class of mechanisms involves a quite important loss of generality even in meaningful economic environments.[15]

3.3 Relationship with the Literature

• **Direct versus Indirect Contractual Externalities:** The multiplicity of equilibria obtained with indirect mechanisms and complete information *only* arises because we have considered a setting with direct externalities between the principals. Indeed, in the framework of Stole (1991) and Martimort (1992 and 1996) which involves no direct externalities, the equilibrium outcome under complete information is the same whether principals offer nonlinear prices or only direct mechanisms.

To confirm this result, let us consider a slightly modified version of the present framework in which principals are monopolies on segmented markets. The inverse demand on each market is now $P(q_i)$. Let us now also define the monopoly symmetric output, $\tilde{q}^m(\theta)$, as:

$$P(\tilde{q}^m(\theta)) + \tilde{q}^m(\theta)P'(\tilde{q}^m(\theta)) = C_1(\tilde{q}^m(\theta), \tilde{q}^m(\theta), \theta).$$

This output maximizes the aggregate payoff of the coalition made of both principals and the agent.

Proposition 3 *: (Stole (1991) and Martimort (1992)) Under complete information and without direct externalities, the unique pure-strategy symmetric equilibrium of the intrinsic common agency game achieves the monopoly outcome,* $q(\theta) = \tilde{q}^m(\theta)$.

[14] See also Peck (1996) for a related example in the case of a multiprincipal-multiagent model.

[15] The fundamental reason for the failure of the Revelation Principle is that direct mechanisms are, by definition, unable to convey information on how a principal would like the agent to react to a deviation made by the other principal. In the vocabulary of Epstein and Peters (1996), direct mechanisms based on the agent's report on his information only are unable to convey *market information*.

Without direct externalities, the equilibrium contract offered by P_1 to the common agent takes into account that the latter optimally adapts his production of intermediate good q_1 to any change in q_2 induced by P_2's deviation. A small change in q_2 away from the equilibrium value has now only a second-order effect on the aggregate payoff of the coalition between the principal P_1 and the agent since output q_2 does not affect directly P_1's payoff. Hence, P_1 has no incentive to use the agent's behavior as an implicit veto threat against any possible deviation by P_2.

On the contrary, in the case of direct externalities, a small change in q_2 away from the equilibrium value has also a first-order effect on the payoff of the coalition made of P_1 and A through the change in $q_1^*(q_2, \theta)$ it induces. Out-of-equilibrium messages help the principals to construct an equilibrium where they behave more (resp. less) aggressively on the final good market than with direct mechanisms when this externality is positive (resp. negative).

• **Vertical Contracting:** In the whole literature on vertical contracting, a single principal deals with several agents.[16] Each agent is directly affected by the trades between the principal and the other agents. The distribution of bargaining which is assumed by this literature is exactly the mirror image of that made in the present paper. Instead of having the principals willing to extract the common agent's rent, the agent (in our framework) offers a set of bilateral contracts to the principals and is willing to push them to their reservation values normalized at zero. A bilateral contract between P_i and A stipulates both a transfer t_i and a trade q_i. With private offers, the agent cannot credibly commit to the monopoly trades which would maximize the aggregate payoff of the overall coalition. Henceforth, each principal P_i must form beliefs about what offers are made to his competitors when he contemplates an agent's deviation (t_i, q_i) away from the equilibrium value (t_i^*, q_i^*). McAfee and Schwartz (1994) analyze *passive beliefs* which are such that the principal P_i infers nothing from this deviation on what is the contractual agreements signed between other principals and the agent. Those contracts remain equal to their equilibrium values (t_{-i}^*, q_{-i}^*). Interestingly, passive beliefs play the same role in the vertical contracting environment as in our common agency game. In both cases, they generate the same Cournot outcome.

• **Delegation and Competition between Principal-Agent Pairs:** Our findings on the multiplicity of equilibria of the common agency game with nonlinear prices are clearly related to the literature on delegation in competing principal-agent pairs with publicly observable contracts analyzed in Katz (1987), Ferschtman, Judd and Kalai (1991) and Kühn (1997) among many others. This literature also shows that a large set of equilibria can be sustained when competing principals try to influence the behavior of their respective agents before agents play a game on their behalf. The differences with our common agency game are threefold: first, under common agency, the contracts do not need to be publicly observable to get a multiplicity of equilibrium outcomes. Second, in common agency games, the interaction between the principals' contracts comes directly from the fact that the common agent's utility function depends on both contractual activities. Tracing

[16] See Segal (1999) who synthesizes much of the results of the literature on vertical contracting.

out how the agent changes his behavior in response to an unexpected contractual offer amounts to solving a simple decision problem. In delegation games, the behavior of the non-cooperating agents following a deviation by any principal is instead obtained from the continuation Nash equilibrium played by the agents. In particular, the principals need not suffer from direct externalities to get a multiplicity of equilibria. All that matters for this multiplicity is that the overall payoff of a given principal-agent pair depends on the contracting decision taken in other pairs through a direct externality affecting agents. Indeed, a small change in q_2 away from its equilibrium value has a first-order effect on the payoff of the coalition made of P_1 and A_1 through the change in $q_1^*(q_2, \theta)$ it induces. Out-of-equilibrium messages can cause the principals to behave more or less aggressively depending on the sign of this externality.

4 Equilibria of the Delegated Common Agency Game under Complete Information

We now move to the case where the agent can choose to take only one of the equilibrium contracts offered by the principal.

We are going to first characterize the best response of P_1 to a given nonlinear tariff $t_2(\cdot, \theta)$ offered by P_2. Under delegated agency, we must consider the possibility that the agent only takes the contract $t_2(\cdot, \theta)$. Suppose he does so, then he gets a utility:

$$U_2(\theta) = max_{q_2} \;\; t_2(q_2, \theta) - C(0, q_2, \theta).$$

To compute this expression, we need to know the expression of $t_2(\cdot)$ for all outputs and in particular for those which are different from the equilibrium output.

Principal P_1's best response to a given nonlinear tariff $t_2(\cdot)$ offered by P_2 is obtained as a solution to the following problem:

$$max_{\{t_1, q_1\}} P(q_1 + q_2^*(q_1, \theta))q_1 - t_1$$

subject to (2) and

$$t_1 + t_2(q_2^*(q_1, \theta)) - C(q_1, q_2^*(q_1, \theta), \theta) \geq U_2(\theta). \tag{5}$$

(5) is the new agent's participation constraint which stipulates that the agent prefers to accept both contracts rather than only P_2's contract.[17]

Which participation constraint is binding in the problem above affects the transfer t_1 paid by P_1 at a best response and thus the equilibrium distribution of surplus. It does not affect the output chosen in a best response, however.

[17] One may initially entertain the possibility that P_1 may want to offer a contract which will induce the agent to exclusively contract with P_1. This would require

$$\max_{q_1} \;\; t_1(q_1, \theta) - C(q_1, 0, \theta) \geq \max\left\{\max_{q_2} \;\; t_2(q_2, \theta) - C(0, q_2, \theta), \max_{q_1, q_2} \;\; t_1(q_1, \theta) + t_2(q_2, \theta) - C(q_1, q_2, \theta)\right\}.$$

Proposition 4 *: Under complete information and when $C(\cdot)$ is quadratic, any equilibrium output $q(\theta)$ of the intrinsic common agency game with twice differentiable nonlinear prices is also an equilibrium output of the delegated common agency game.*

In the corresponding symmetric equilibrium of the delegated common agency game, the agent gets zero rent when intermediate goods are complements and a strictly positive rent when intermediate goods are substitutes.

Under delegated common agency, the agent has the extra option of playing one principal against the other. With complements, this outside option is not binding and what matters for assessing the distribution of surplus between the three agents is the possibility that the agent rejects both contracts. With substitutes instead, this outside option is now binding. Even though we are under complete information, the common agent obtains some strictly positive rent. The extra option has thus strong distributive consequences but no allocative impact since the set of equilibrium outputs in the two games are the same.

5 Equilibria of the Intrinsic Common Agency Game under Asymmetric Information

It may seem a priori odd to use a whole nonlinear price schedule to control the agent in a world of complete information and no uncertainty. The standard motivation for looking at those complex schemes relies generally on the fact that the principal is unable to discriminate among the different possible types of the agent in a private information self-selection setting. Therefore, we now turn to the case of asymmetric information in the framework of our intrinsic common agency game.

5.1 Computing Best-Responses

Before proceeding to a systematic investigation of the pure-strategy equilibria of the common agency game under asymmetric information, we propose a general algorithm which helps to characterize the best-response of a principal to any pure-strategy nonlinear contract offered by his rival.

For any nonlinear price $t_2(\cdot)$ offered by P_2, there is indeed no loss of generality in looking for P_1's best-response within the class of direct revelation mechanisms of the form $\{t_1(\hat{\theta}), q_1(\hat{\theta})\}$ where $\hat{\theta}$ is the agent's report to P_1. Any payoff that P_1 can achieve when he offers a mechanism with some

If these constraints were binding in a symmetric equilibrium, then it must be the case that

$$\max_{q_1} \; t_1(q_1, \theta) - C(q_1, 0, \theta) = \max_{q_2} \; t_2(q_2, \theta) - C(0, q_2, \theta).$$

Hence, all profits would be bidded away to the agent in such a candidate equilibrium. Given this outcome, it would be optimal for one of the principals to deviate and offer a contract which would induce the agent to commonly contract with the two principals. Hence, the relevant constraints are those in (5) above.

general communication space can also be achieved with such a direct revelation mechanism. Here, we simply apply the standard Revelation Principle to determine the outputs implemented by P_1 at his best response to *a given* deterministic nonlinear price $t_2(\cdot)$ offered by P_2. Of course, in equilibrium, nonlinear prices must be best responses to each other. To validate the first-order approach, we extend the nonlinear schedules offered by both principals for out-of-equilibrium outputs.[18]

However, different nonlinear prices offered by P_2 affect differently the agent's incentives to produce for P_1 and therefore P_1's incentives to distort his consumption of intermediate good 1 for informational reasons. In other words, for a given coalition between a principal and the agent, the trade-off between extraction of the agent's informational rent and maximization of the aggregated payoff of this coalition depends on the other contract signed by the agent.

To capture this effect mathematically in a clear manner, let us thus define *the agent's indirect utility function* vis-à-vis P_1 as:

$$\hat{U}^1(q_1, \theta) = max_{q_2} \;\; t_2(q_2, \theta) - C(q_1, q_2, \theta). \tag{6}$$

This indirect utility function gives the maximal payoff of an agent with type θ when his production for P_1 is q_1 and when he chooses his output q_2 optimally. When $t_2(\cdot)$ is defined over the whole real line, differentiable and sufficiently concave, $q_2^*(q_1, \theta)$ is again defined by the first-order condition (1). Note that different nonlinear tariffs $t_2(q_2)$ correspond to different indirect functions $\hat{U}^1(q_1, \theta)$ and therefore to possibly different best responses by P_1. For notational ease, we leave the dependence of $\hat{U}^1(q_1, \theta)$ on P_2's contract implicit.

For a given indirect utility function, finding P_1's best response to $t_2(\cdot)$ is a task which is by now standard from the methodology of single principal-agent optimal contracting problems. The standard implementability conditions must be satisfied by this contractual best response. The only difference with standard contracting problems comes from the fact that P_1 suffers from the direct externality exerted by P_2's contracting. However, from a technical point of view, the difficulty is to ensure that P_1's problem is concave since, again this concavity is endogenous and depends on the price schedule $t_2(\cdot)$ offered by P_2. We show in the Appendix that this concavity is ensured when the support of the distribution of types is small enough and $C(\cdot)$ is quadratic, but also, when the symmetric equilibrium schedule $t(\cdot)$ is quadratic. This latter case arises, for example, when $F(\cdot)$ is uniform, $P(\cdot)$ linear and $C(\cdot)$ is quadratic and for the equilibrium we select in Proposition 7.

For convenience, let us also define the $U(\theta)$ as the common agent's informational rent:

$$U(\theta) = max_{\hat{\theta}} \;\; t_1(\hat{\theta}) + \hat{U}^1(q_1(\hat{\theta}), \theta). \tag{7}$$

[18]Note that, under asymmetric information, the active portion of the nonlinear price schedule associated with the equilibrium direct mechanisms $\{t_i(\theta), q_i(\theta)\}$ is of the form $t_i(q_i^{-1}(q_i))$ and is defined only over the domain $Q_i = \{q_i | q_i = q_i(\theta) \text{ for some } \theta\}$. Using these limited, non-extended nonlinear prices when computing best responses would require the modeler to compute the principal's benefit of making a subset of types with non-zero measure bunch on the corner of the tariff offered by his rival. Such analysis would require giving up (1) as a characterization of the best choice of the agent for any output chosen in the rival's nonlinear price. Stole (1999) does so in the case of indirect externalities and shows that these extensions may not be needed.

It turns out that the implementability conditions of P_1's best response can be expressed more easily in terms of the informational rent-output pair $\{U(\theta), q_1(\theta)\}$ rather than in terms of the transfer-output pair $\{t_1(\theta), q_1(\theta)\}$. The following lemma is standard in the self-selection literature.

Lemma 1 *: If $\hat{U}^1_{1\theta}(q_1, \theta) \leq 0$ for all (q_1, θ), a pair $\{U(\cdot), q_1(\cdot)\}$ is implementable if and only if, for all $\theta \in \Theta$, the following two conditions are satisfied:*

- *first-order condition,*
$$\dot{U}(\theta) = \hat{U}^1_\theta(q_1(\theta), \theta); \tag{8}$$
- *second-order condition,*
$$q_1(\theta) \text{ is non-increasing.} \tag{9}$$

The informational rent-output pair $\{U(\theta), q_1(\theta)\}$ chosen at a best response by P_1 is therefore solution to the following program:

$$\max_{\{U(\theta), q_1(\theta)\}} \int_{\underline{\theta}}^{\bar{\theta}} (P(q_1(\theta) + q_2^*(q_1(\theta), \theta))q_1(\theta) + t_2(q_2^*(q_1(\theta), \theta)) - C(q_1(\theta), q_2^*(q_1(\theta), \theta), \theta) - U(\theta))f(\theta)d\theta$$

subject to (8)-(9) and

$$U(\theta) \geq 0 \quad \text{for all } \theta \in \Theta \tag{10}$$

which is the agent's participation constraint. We will use this program throughout when computing the best response of each principal.

Importantly for what follows, the standard single-crossing or Spence-Mirrlees property, $\hat{U}^1_{1\theta}(q_1, \theta) \leq 0$, which is usually assumed to obtain a well-behaved monotonic solution to principal P_1's problem, can no longer be postulated a priori. Instead, the implicit dependence of $\hat{U}^1(q_1, \theta)$ on P_2's contract implies that this single-crossing property is endogenous and may or may not arise at the equilibrium of the common agency game. Nevertheless, we focus in what follows on pure strategy equilibria where this single-crossing property emerges in both indirect utility functions vis-à-vis either principal.

5.2 Equilibria Set under Intrinsic Common Agency

For further references, we define:

- The *virtual Cournot* symmetric output as $\tilde{q}^c(\theta)$ such that:

$$P(2\tilde{q}^c(\theta)) + \tilde{q}^c(\theta)P'(2\tilde{q}^c(\theta)) = C_1(\tilde{q}^c(\theta), \tilde{q}^c(\theta), \theta) + \frac{F(\theta)}{f(\theta)}C_{1\theta}(\tilde{q}^c(\theta), \tilde{q}^c(\theta), \theta).$$

• The *virtual monopoly* symmetric output as $\tilde{q}^m(\theta)$ such that:

$$P(2\tilde{q}^m(\theta)) + 2\tilde{q}^m(\theta)P'(2\tilde{q}^m(\theta)) = C_1(\tilde{q}^m(\theta), \tilde{q}^m(\theta), \theta) + 2\frac{F(\theta)}{f(\theta)}C_{1\theta}(\tilde{q}^m(\theta), \tilde{q}^m(\theta), \theta).$$

The first output schedule corresponds to standard Cournot outcome when costs have been replaced by virtual costs to capture the effect of informational asymmetries between the principals and their common agent. The second output schedule corresponds to the monopoly outcome when those virtual costs are counted twice.[19] Because there is no information rent for $\underline{\theta}$, we have $\tilde{q}^c((\underline{\theta})) = q^c(\underline{\theta})$ and $\tilde{q}^m((\underline{\theta})) = q^m(\underline{\theta})$. Also, because there is no corresponding notion of virtual Bertrand because no information rents are captured by either firm.

We can now state the following proposition which describes the set of common agency symmetric equilibria of the game under asymmetric information.

Proposition 5 *: Assuming the concavity of the principals' problem and the agent's single-crossing property, a necessary condition for an output schedule $q(\theta)$ to be implemented in a symmetric equilibrium of the intrinsic common agency game is that it satisfies the following differential equation:*

$$\dot{q}(\theta) = -\frac{C_{1\theta}(q(\theta), q(\theta), \theta)\left(P(2q(\theta)) + q(\theta)P'(2q(\theta)) - C_1(q(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)}C_{1\theta}(q(\theta), q(\theta), \theta)\right)}{C_{12}(q(\theta), q(\theta), \theta)\left(P(2q(\theta)) + 2q(\theta)P'(2q(\theta)) - C_1(q(\theta), q(\theta), \theta) - 2\frac{F(\theta)}{f(\theta)}C_{1\theta}(q(\theta), q(\theta), \theta)\right)}, \tag{11}$$

with the appropriate boundary conditions below:

- *when intermediate goods are substitutes, $q(\underline{\theta}) \in [\tilde{q}^c(\underline{\theta}), q^b(\underline{\theta})]$.*
- *when intermediate goods are complements, $q(\bar{\theta}) \in [\tilde{q}^m(\bar{\theta}), \tilde{q}^c(\bar{\theta})]$.*

When $C(\cdot)$ is quadratic and $F(\cdot)$ satisfies the monotone hazard rate property, the principals' programs are concave, the agent's single-crossing property holds, and these necessary conditions are also sufficient for characterizing equilibrium outputs.

Moreover, only the least efficient type $\bar{\theta}$ makes zero informational rent in any equilibrium of the intrinsic common agency game.

Under asymmetric information, the slope of the symmetric equilibrium nonlinear price $t(q)$ is defined at any equilibrium point $q(\theta)$ in such a way that the following first-order condition characterizes the agent's choice for all θ:

$$t'(q(\theta)) = C_1(q(\theta), q(\theta), \theta). \tag{12}$$

[19]This output schedule would be implemented by the principals' non-cooperative behavior if they were both contracting on the *whole* production Q on the final market and not on their respective productions. This double informational distortion which enters the virtual costs summarizes the double rent extraction phenomenon which arises in this context.

By specifying an adverse selection problem around any value of θ, the slope of the equilibrium schedule in the neighborhood of this value θ is completely defined. The convexity of the equilibrium schedule at an equilibrium point is a priori not as *free* as under complete information. Differentiating (12) with respect to θ yields the following expression defining the second derivative of the equilibrium nonlinear price at an equilibrium point:

$$t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta) = \frac{C_{1\theta}(q(\theta), q(\theta), \theta)}{\dot{q}(\theta)} + C_{12}(q(\theta), q(\theta), \theta). \tag{13}$$

The convexity of the nonlinear price at any equilibrium point which, from the discussion in Section 3, describes the degree of competition between the principals, is thus completely determined by how equilibrium output evolves in the neighborhood of this equilibrium point.

The slope of this output schedule is itself determined by two forces playing simultaneously. First, as in standard one-principal-agent models, each principal P_i wants to reduce the amount of intermediate good q_i he requests from the agent in order to limit the latter's informational rent. This incentive distortion depends on the distribution of the agent's types through its hazard rate. Second, as under complete information, P_i has also strategic incentives to increase his profit on the final market by strategically employing the agent against P_{-i}.

As a result of the principals' incentives to reduce the amount of intermediate goods they request from the common agent for informational reasons, the equilibrium set is now *strictly* within two boundaries. These boundaries (respectively, $\tilde{q}^c(\theta)$ and $\tilde{q}^m(\theta)$ under complements, and $\tilde{q}^c(\theta)$ and $q^b(\theta)$ under substitutes) correspond to the limits of the equilibrium sets obtained under complete information (Proposition 1) for the most efficient type $\underline{\theta}$ but not elsewhere. Asymmetric information therefore reduces significantly the equilibrium set. These output boundary results are summarized in the next proposition.

Proposition 6 *: The output schedules $q(\theta)$ implemented in any symmetric differentiable equilibrium of the common agency game are such that:*

- *when intermediate goods are substitutes, $\tilde{q}^c(\theta) < q(\theta) < q^b(\theta)$ $\forall \theta \in (\underline{\theta}, \bar{\theta}]$.*
- *when intermediate goods are complements, $\tilde{q}^m(\theta) < q(\theta) < \tilde{q}^c(\theta)$ $\forall \theta \in [\underline{\theta}, \bar{\theta})$.*

As under complete information, a symmetric equilibrium nonlinear price is also relatively steep around the Cournot outcome and flat far away from this point. We check this for the case of a quadratic cost function.

Corollary 3 *: Assume that $C(\cdot)$ is quadratic, the symmetric equilibrium nonlinear price $t(q)$ is such that:*

- *when intermediate goods are substitutes* $t''(q(\underline{\theta})) = 1 - \lambda \quad$ *if* $q(\underline{\theta}) = q^b(\underline{\theta})$ *and* $t''(q(\underline{\theta})) = -\infty \quad$ *if* $q(\underline{\theta}) = q^c(\underline{\theta})$.
- *when intermediate goods are complements* $t''(q(\bar{\theta})) = -\infty \quad$ *if* $q(\bar{\theta}) = \tilde{q}^c(\bar{\theta})$ *and* $t''(q(\bar{\theta})) = 1 + \lambda \quad$ *if* $q(\bar{\theta}) = \tilde{q}^m(\bar{\theta})$.

This Corollary confirms that asymmetric information is in fact still consistent with as many conjectures about the agent's behavior as available under complete information once the support of the distribution of θ remains bounded. In the case of substitutes (resp. complements), this flexibility comes from the freedom of equilibrium conjectures available at the lowest (resp. highest) type of the distribution. Once, this choice for a boundary point of (11) is made, the equilibrium schedule is fully pinned down by the incentive compatibility constraint (12).[20]

6 Equilibrium Selection under Intrinsic Common Agency with a Uniform Distribution

The goal of this section is to understand which of the equilibrium conjectures available under complete information can be *rationalized* by introducing convenient perturbations of the information structure. The rationalization we use is that a focal equilibrium of the common agency game under complete information should be the limit of equilibria of a game where principals have only an imperfect knowledge of the adverse selection parameter θ. More precisely, the slope of the complete information equilibrium nonlinear price which is selected should be derived from equilibrium behavior in the neighborhood of this point once a complete distribution of the adverse selection parameter has been stipulated. In this case, the out-of-equilibrium outputs included in a nonlinear equilibrium price under complete information become equilibrium outputs for some particular type of the agent in a model with asymmetric information. As shown in Section 4, asymmetric information on a bounded support fails short of selecting a unique equilibrium even if it helps to reduce the equilibrium set. To further refine the set of reasonable conjectures which can sustain an equilibrium under complete information, we now consider perturbations of the information structure consisting in expanding infinitely the support of the distribution of types. The selected equilibrium should be robust to such extensions of the type space. Our definition of a *robust* equilibrium is that the corresponding output schedule $q(\theta)$ should remain an equilibrium when the spread of the distribution increases without bound. While we proceed by focusing on

[20] Note that the multiplicity of symmetric equilibria obtained under asymmetric information and direct externality has fundamentally the same origins as under complete information: the desire of the principals to precommit themselves. This should be contrasted with the multiplicity of equilibria which emerges in the case of complements and no direct externalities. Stole (1991) and Martimort (1992) show indeed that this multiplicity comes from the fact that the differential equation characterizing the equilibrium output schedules is not Lipschitz and has a singularity for the lowest type of the distribution.

uniform distributions, this is primarily for tractability. The intuition that, as the support of the agent's type space increases the degrees of freedom in the equilibrium set is reduced, is more general.

The next proposition gives conditions ensuring that such a robust equilibrium exists.

Proposition 7 *: Assume that $P(\cdot)$ is linear, $C(\cdot)$ is quadratic, and that $F(\cdot)$ is uniform on the interval $[\underline{\theta}, \bar{\theta}]$. Denote by $q(\theta|\underline{\theta}, \bar{\theta})$ the output schedule corresponding to any symmetric equilibrium of the common agency game when the support of type is $[\underline{\theta}, \bar{\theta}]$. Then, we have:*

- *When intermediate goods are substitutes (resp. complements) $q(\theta|\underline{\theta}, \bar{\theta})$ is an equilibrium schedule of the common agency game when the support of type is restricted to $[\underline{\theta}, \bar{\theta}']$ (resp. $[\underline{\theta}', \bar{\theta}]$) for any $\bar{\theta}' < \bar{\theta}$ (resp. $\underline{\theta}' > \underline{\theta}$). This allows to simply denote these solutions by $q(\theta|\underline{\theta})$ (resp. $q(\theta|\bar{\theta})$).*

- *When intermediate goods are substitutes (resp. complements), there exists a $q^*(\theta)$, which is an equilibrium output function from the common agency game, such that as $\bar{\theta}$ increases to $+\infty$ (resp. as $\underline{\theta}$ decreases to $-\infty$)[21], we have:*

$$\lim_{\bar{\theta} \to +\infty} E[q(\theta|\underline{\theta}) - q^*(\theta)|\underline{\theta}, \bar{\theta}] = 0$$

$$(\textit{ resp.} \quad \lim_{\underline{\theta} \to -\infty} E[q(\theta|\bar{\theta}) - q^*(\theta)|\underline{\theta}, \bar{\theta}] = 0,)$$

where $E[\cdot|\underline{\theta}, \bar{\theta}]$ denotes the expectation operator with respect to a uniform distribution over $[\underline{\theta}, \bar{\theta}]$ and where $q^*(\theta)$ is the output schedule of an equilibrium of the common agency game which holds for any possible support $[\underline{\theta}, \bar{\theta}]$. Both with substitutes and complements, $q^*(\theta)$ is the unique symmetric equilibrium of the common agency game under asymmetric information having a linear output schedule:

$$q^*(\theta) = q^*(\underline{\theta}) - \beta^*(\theta - \underline{\theta})$$

where $\beta^* > 0$ and $q^*(\underline{\theta}) \in (q^c(\underline{\theta}), q^b(\underline{\theta}))$

The logic of the selection device used above is to look at the family of solutions of (11) when the distribution for θ has an increasingly larger support. We further assume that the distribution is uniform to keep unchanged the value of the hazard rate even when the support of this distribution is extended. Then, any solution of (11) on a given interval remains an equilibrium output for a perturbed common agency game obtained when the spread of the distribution increases. However, as shown in Proposition 7, the equilibrium set of outputs shrinks as the spread of the distribution increases. Indeed, both in the cases of complements and of substitutes, we show that any solution

[21] Of course, to extend the spread of the distribution one must be ready to consider negative cost parameters or infinite ones. Those two perturbations are thus only meaningful when the demand intercept a is sufficiently large.

of (11) converges in expectation to the *unique* linear solution $q^*(\theta)$ to (11). This latter equilibrium is the only one which survives these extensions of the support of the distribution.[22]

Asymmetric information on an unbounded support makes any output into an equilibrium output for some type even if the probability of this particular types becomes quite small as the spread of the distribution increases. In the limiting case of an infinitely large support, only a single conjecture is consistent with equilibrium behavior.

For all types, the equilibrium schedule $q^*(\theta)$ is *strictly* within the set of equilibria described in Proposition 6. The only equilibrium under complete information which can be rationalized as being the limit of asymmetric information equilibria when the support of the distribution increases without bound entails neither the *maximal amount of cooperation* nor the *maximal amount of competition* between the principals.

To fix ideas, let us take the case of perfect substitutes, i.e., $\lambda = 1$. We find that $\beta^* = \frac{1}{3}$ and

$$q^c(\underline{\theta}) = \frac{a - \underline{\theta}}{5} < q^*(\underline{\theta}) = \frac{2(a - \underline{\theta})}{9} < q^b(\underline{\theta}) = \frac{a - \underline{\theta}}{4},$$

where a is the demand intercept. Finally, the conjectures about how the agent adapts his production of intermediate good 2 for any unexpected change in the production of intermediate good 1 is just *the mean* of the passive conjectures held under Cournot competition ($\frac{\partial q_2^*(q_1,\theta)}{\partial q_1} = 0$) and the very reactive conjectures held under Bertrand competition ($\frac{\partial q_2^*(q_1,\theta)}{\partial q_1} = -1$), namely:

$$\frac{\partial q_2^*(q_1, \theta)}{\partial q_1} = \frac{1}{t''(q_2^*(q_1, \theta)) - 1} = -\frac{1}{2}.$$

Because the equilibrium schedule $q^*(\theta)$ is linear in θ, it is invertible. Let denote by $\theta^*(q)$ its inverse. We have:

$$\theta^*(q) = \underline{\theta} + \frac{q^*(\underline{\theta}) - q}{\beta^*}$$

or

$$\theta^*(q) = \bar{\theta} + \frac{q^*(\bar{\theta}) - q}{\beta^*}.$$

Moreover, the slope of the symmetric equilibrium schedule $t^*(\cdot)$ at any point $q^*(\theta)$ satisfies

$$t^{*'}(q^*(\theta)) = \theta + (1 + \lambda)q^*(\theta)$$

or

$$t^{*'}(q) = \theta^*(q) + (1 + \lambda)q$$

for any q in the range of $q^*(\cdot)$. Substituting,

$$t^{*'}(q) = \bar{\theta} + (1 + \lambda)q^*(\bar{\theta}) + \left(1 + \lambda - \frac{1}{\beta^*}\right)(q - q^*(\bar{\theta})). \tag{14}$$

[22]Note that $q^*(\cdot)$ being linear, the nonlinear equilibrium schedule $t(q)$ is quadratic.

Consistently with our extensions of the support of types over the whole real line, this linear expression of $t^{*'}(q)$ will be also extended even for outputs which may lie outside the range of $q^*(\cdot)$.

Since the least efficient type's participation constraint is binding in equilibrium, direct integration of (14) yields:

$$t^*(q) = t^*(q^*(\bar{\theta})) + \left(\bar{\theta} + (1+\lambda)q^*(\bar{\theta})\right)(q - q^*(\bar{\theta})) + \left(1 + \lambda - \frac{1}{\beta^*}\right)\frac{(q - q^*(\bar{\theta}))^2}{2}. \tag{15}$$

with $t^*(q^*(\bar{\theta})) = \bar{\theta}q^*(\bar{\theta}) + \frac{(1+\lambda)}{2}(q^*(\bar{\theta}))^2$. Hence, the nonlinear tariff $t^*(\cdot)$ is quadratic in q just as in the case of complete information.

•**Relationship with the Literature:** The reader may have recognized that our technique of introducing large perturbations in the support of the distribution to select a unique equilibrium bears some resemblance with the important work of Klemperer and Meyer (1989). These authors are interested in the analysis of a game where two suppliers of an homogeneous final good compete with supply functions when the demand they face is uncertain. In our context, this would amount to envision a game where principals compete by offering demand functions $q_i(p)$ to the common agent when there is some uncertainty on its marginal cost. The difference with our model are thus threefold. First, we have considered competition in nonlinear prices instead of demand functions. Different quantities are thus bought by a principal at different prices. This is not the case with demand functions. Second, we have explicitly modeled the choice of the common agent instead of relying on an exogenous Walrasian auctioneer. Third, and quite importantly, we assume adverse selection instead of ex ante uncertainty. With cost uncertainty at the time of contracting, the agent's participation constraint is active at the *ex ante* stage and the conflict between incentive and participation constraints disappears.

Let us thus consider that both principals still compete with nonlinear prices $t_i(q_i)$ but that contracting takes now place at the ex ante stage. Formally, we can show that the[23] necessary and sufficient condition (when $C(\cdot)$ is quadratic) satisfied by an equilibrium schedule $q(\theta)$ *under ex ante uncertainty* is that $q(\theta)$ is a solution to the following differential equation:

$$\dot{q}(\theta) = -\frac{C_{1\theta}(q(\theta), q(\theta), \theta)\left(P(2q(\theta)) + q(\theta)P'(2q(\theta)) - C_1(q(\theta), q(\theta), \theta)\right)}{C_{12}(q(\theta), q(\theta), \theta)\left(P(2q(\theta)) + 2q(\theta)P'(2q(\theta)) - C_1(q(\theta), q(\theta), \theta)\right)}, \tag{16}$$

with the following initial condition:

- when intermediate goods are substitutes, $q(\underline{\theta}) \in [q^c(\underline{\theta}), q^b(\underline{\theta})]$.
- when intermediate goods are complements, $q(\bar{\theta}) \in [q^m(\bar{\theta}), q^c(\bar{\theta})]$.

With ex ante uncertainty, the virtual cost functions which enter both the numerator and the denominator of (11) disappear and are replaced by simple cost functions. The solutions of the differential equation (16) do not depend on the distribution of θ but only on its support.

[23]Proof available upon request.

When the support of the distribution gets larger, one can also select a unique equilibrium by extending the spread of the distribution and this can be done, contrary to what we have done with adverse selection, irrespectively of the distribution of θ which is assumed. This equilibrium is the linear solution:

$$q^{km}(\theta) = q^{km}(\underline{\theta}) - \beta^{km}(\theta - \underline{\theta}).$$

In the case of perfect substitutes ($\lambda = 1$), one can show that $\beta^{km} = \frac{1}{2} - \frac{\sqrt{3}}{6} < \beta^* = \frac{1}{3}$. This clearly shows that once asymmetric information arises at the ex ante contracting stage, concerns for rent extraction matter so that the tendency to reduce output for large θ becomes greater than under ex ante uncertainty.

7 Delegated Common Agency under Asymmetric Information

Under delegated common agency, a new constraint must be added to the principal's problem. This new participation constraint stipulates that the agent's rent under common agency must be greater than the rent $U_2(\theta)$ he would get by taking only the contract $t_2(\cdot)$ offered by P_2.

The informational rent-output pair $\{U(\theta), q_1(\theta)\}$ chosen at a best response by P_1 is therefore solution to the following program:

$$\max_{\{U(\theta),q_1(\theta)\}} \int_{\underline{\theta}}^{\bar{\theta}} (P(q_1(\theta)+q_2^*(q_1(\theta),\theta))q_1(\theta)+t_2(q_2^*(q_1(\theta),\theta)) - C(q_1(\theta),q_2^*(q_1(\theta),\theta),\theta) - U(\theta))f(\theta)d\theta$$

subject to (8)-(9)-(10) and

$$U(\theta) \geq U_2(\theta) \quad \text{for all } \theta \in \Theta \tag{17}$$

which is the new agent's participation constraint under delegated common agency.

Under complete information, we showed that delegated common agency has never any allocative impact but has distributive consequences only in the case of substitutes. Key to our result was the fact that, thanks to the fact that the equilibrium schedule can be taken as quadratic, we were able to compute the utility $U_2(\theta)$ of the agent in case he contracts only with principal P_2 and to show how. Under asymmetric information, the equilibrium schedule of the intrinsic common agency game is not always quadratic, making it harder to assess the role of the new participation constraint. It is only quadratic for the equilibrium schedule $t^*(\cdot)$ selected in Section 6. Instead of doing a complete analysis which would consist in assessing whether any equilibrium schedule $q(\theta)$ of the intrinsic common agency game remains an equilibrium schedule of the delegated common agency game, we are going to focus only on the selected equilibrium $q^*(\theta)$ which corresponds to a quadratic tariff.

Proposition 8 *: Under asymmetric information and when $C(\cdot)$ is quadratic, $P(\cdot)$ linear and $F(\cdot)$ uniform, the equilibrium output $q^*(\theta)$ of the intrinsic common agency game is also an equilibrium output of the delegated common agency game.*

In the corresponding symmetric equilibrium of the delegated common agency game, the least efficient agent gets zero rent when outputs are complements and a strictly positive rent when outputs are substitutes.

8 Conclusion

This paper has contributed to the analysis of common agency games along several lines. First, we have characterized the set of equilibria outcomes of an intrinsic common agency game when there exist direct externalities between the principals. We have shown that Folk Theorem like results hold in such a context. Second, we have shown that the equilibrium outputs of the intrinsic common agency game still holds under delegated common agency. The distribution of surplus may be different. Third, we have isolated the source of this multiplicity of equilibria in both games: the principals' desire to manipulate the agent's behavior even out of the equilibrium. These manipulations can only be achieved with indirect mechanisms stipulating transfers for out-of-equilibrium output choices. As a consequence, direct revelation mechanisms fail to replicate the equilibrium outcomes achieved with indirect mechanisms: a failure of the Revelation Principle in common agency games. Finally, to refine within the set of these possible equilibrium conjectures, we have introduced asymmetric information. We have derived the equilibrium schedules of the common agency game with asymmetric information. Asymmetric information does somewhat restrict the set of symmetric equilibrium outcomes for a given distribution on a finite support. Moreover, we have been able to pin down a unique robust equilibrium only by considering increasingly larger spreads for the uniform distribution of the adverse-selection parameter. This equilibrium output also arises under delegated common agency.

References

Anton, J. and D. Yao, 1989, "Split Awards, Procurement, and Innovation," *Rand Journal of Economics,* 20: 538-552.

Bernheim, D. and M. Whinston, 1986, "Menu Auctions, Resource Allocations, and Economic Influence," *Quarterly Journal of Economics,* 101: 1-31.

Calzolari, Giacomo, "Incentive Regulation of Multinational Enterprises," working paper, January 2001.

Champsaur, P. and J.C. Rochet, 1989, "Multiproduct Duopolists," *Econometrica*, 57: 533-557.

Dasgupta, P., P. Hammond, and E. Maskin, 1979, "The Implementation of Social Choice Rules," *Review of Economic Studies*, 46: 185-216.

Dixit, A., G. Grossman and E. Helpman, 1997, " Common Agency and Coordination," *Journal of Political Economy,* 105: 752-769.

Epstein, L. and M. Peters, 1996, "A Revelation Principle for Competing Mechanisms," *Journal of Economic Theory,* 88: 119-160.

Ferschtman, C., K. Judd, and E. Kalai, 1991, "Observable Contracts: Strategic Delegation and Cooperation." *International Economic Review*, 32: 551-559.

Gibbard, A., 1973, "Manipulation of Voting Schemes," *Econometrica*, 41: 587-601.

Green, J. and J.-J. Laffont, 1977, "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica,* 45: 427-438.

Guesnerie, R., 1995, *A Contribution to the Pure Theory of Taxation*, Cambridge University Press, Cambridge.

Grossman, G. and E. Helpman, 1994, " Protection for Sale," *American Economic Review,* 84: 833-850.

Harris, M. and R. Townsend, 1981, "Resource Allocation under Asymmetric information," *Econometrica*, 49: 33-64.

Hart, O. and J. Tirole, 1990, "Vertical Integration and Market Foreclosure," *Brookings Papers on Economics Activity,* 205-276.

Katz, M., 1987, "Game Playing Agents: Unobservable Contracts as Precommitments," mimeo, University of California at Berkeley.

Klemperer, P. and M. Meyer, 1989, "Supply Function Equilibria in Oligopoly under Uncertainty," *Econometrica,* 57: 1243-1278.

Kühn, K-U., 1997, "Vertical Restraints in a Manufacturer Duopoly: The Case of Nonlinear Pricing," *Rand Journal of Economics*, 28: 37-62.

Laffont, J.J. and D. Martimort, 1997, "The Firm as a Multicontract Organization," *Journal of Economics and Management Strategy*, 6: 201-234.

Laussel, D. and M. Le Breton, 1994, "The Structure of Equilibrium Payoffs in Common Agency," Mimeo, GREQAM, Université d'Aix-Marseille 2.

McAfee, P., 1993, "Mechanism Design by Competing Sellers," *Econometrica*, 61: 1281-1312.

McAfee, P. and M. Schwartz, 1993, "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity and Uniformity,," *American Economic Review*, 84: 210-230.

Martimort, D., 1992, "Multi-Principaux avec Anti-Selection," *Annales d'Economie et de Statistiques*, 28: 1-38.

Martimort, D., 1996, "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory," *RAND Journal of Economics*, 27: 1-31.

Martimort, D. and L. Stole, 1998, "Communication Spaces, Equilibria Sets and the Revelation Principle under Common Agency," mimeo, University of Chicago.

Martimort, D. and L. Stole, 1999, "The Revelation and Taxation Principles in Common Agency Games," mimeo, University of Chicago.

Mezzetti, C., 1997, "Common Agency with Horizontally Differentiated Principals," *Rand Journal of Economics*, 28: 323-345.

Myerson, R., 1979, "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47: 61-73.

Myerson, R., 1982, "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10: 67-81.

O'Brien, D. and G. Shaeffer, 1992, "Vertical Control with Bilateral Contracts," *Rand Journal of Economics*, 23: 299-308.

Palfrey, T., 1992, "Bayesian Implementation," in *Advances in Economic Theory, vol. 1,* ed. J.J. Laffont, 183-323. Cambridge University Press.

Peck, J., 1996, "Competing Mechanisms and the Revelation Principle," Ohio State University, mimeo, July 1996.

Peters, M., 1999, "Common Agency and the Revelation Principle", University of Toronto, mimeo, January 1999.

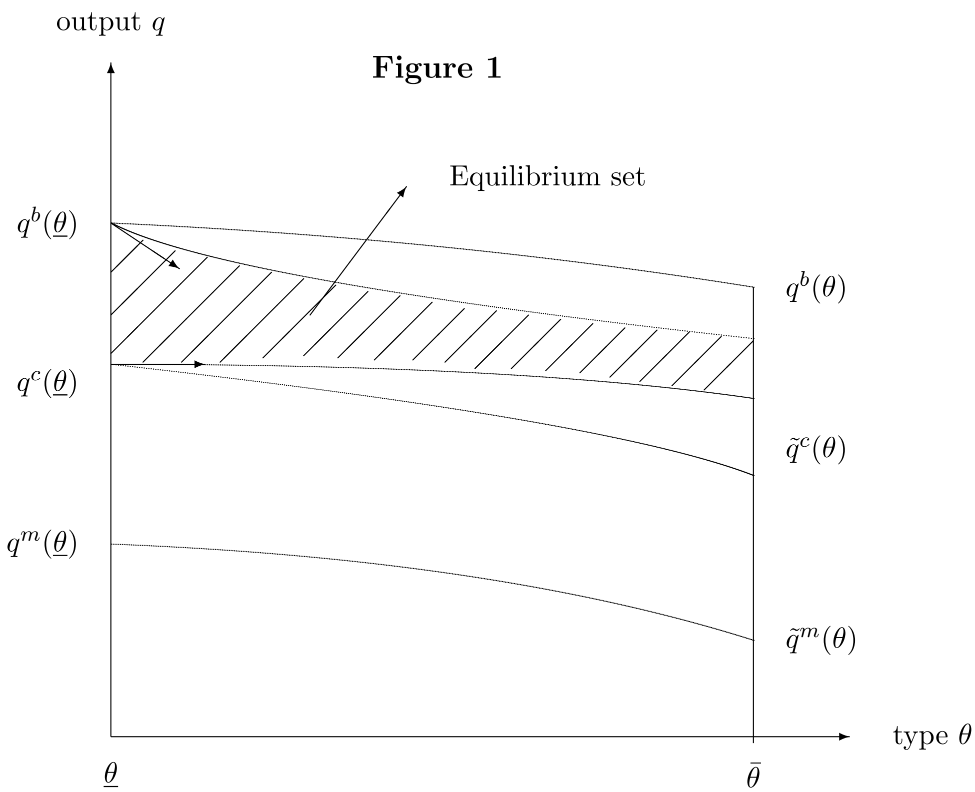
Segal, I, 1999, "Contracting with Externalities," *Quarterly Journal of Economics,* 114: 337-389.

Stole, L., 1991, "Mechanism Design under Common Agency," mimeo, University of Chicago.

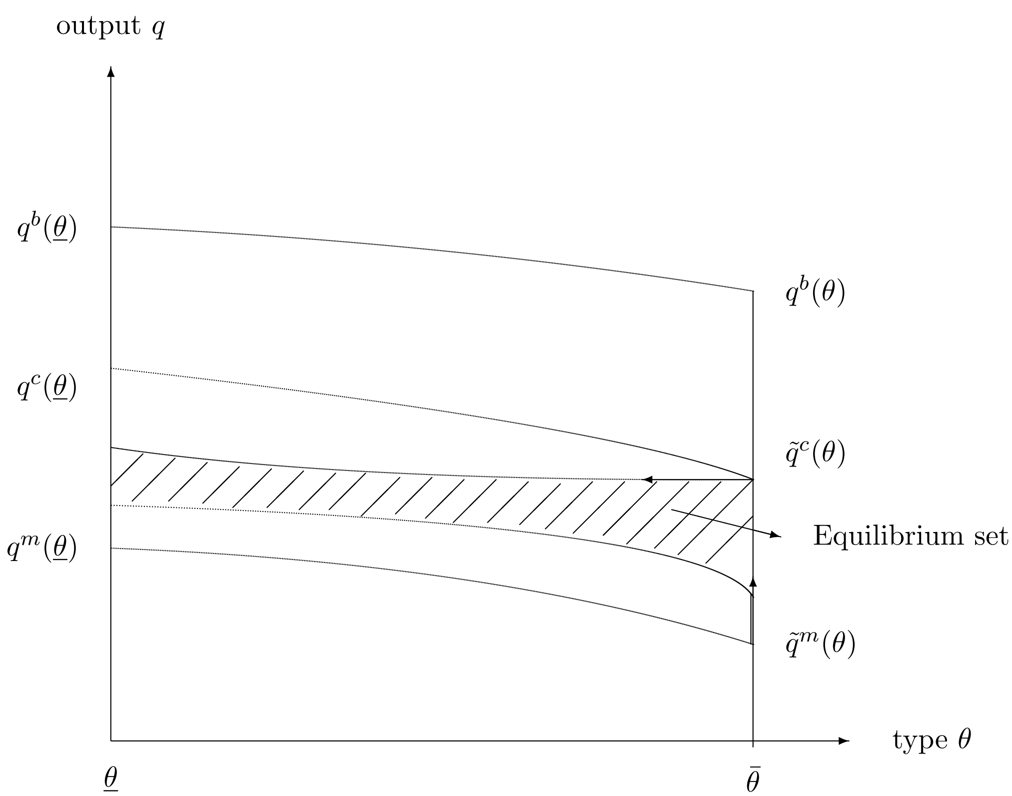
Stole, L., 1999, "Mechanism Design under Common Agency: Theory and Applications," mimeo, University of Chicago.

Wilson, R, 1979, "Auctions of Shares," *Quarterly Journal of Economics,* 93: 675-699.

Figure 1



Case of Substitutes



Case of Complements

Appendix

Proof of Proposition 1 and Corollaries 1 and 2: We take as given the nonlinear price $t_2(q_2, \theta)$ offered by P_2 to the common agent and suppose that $t_2 - C$ is sufficiently concave such that there exists a unique maximizer defined by the first-order condition, (1). (We later check that the agent's objective function is sufficiently concave to justify our working assumption.) Define

$$q_2^*(q_1, \theta) \equiv \arg\max_{q_2} \; t_2(q_2, \theta) - C(q_1, q_2, \theta).$$

Under complete information, P_1's problem is thus:

$$\max_{\{t_1, q_1\}} \quad P(q_1 + q_2^*(q_1, \theta))q_1 - t_1$$

subject to

$$t_1 + t_2(q_2^*(q_1, \theta), \theta) - C(q_1, q_2^*(q_1, \theta), \theta) \geq 0. \tag{18}$$

• Differentiating (1) w.r.t. q_1 yields the following relationship, true for all (q_1, θ):

$$(t_2''(q_2^*(q_1, \theta), \theta) - C_{22}(q_1, q_2^*(q_1, \theta))\frac{\partial q_2^*(q_1, \theta)}{\partial q_1} = C_{12}(q_1, q_2^*(q_1, \theta), \theta). \tag{19}$$

In P_1's problem, (18) is binding since the principal wants to reduce as much as possible the transfer he offers to the agent. Inserting the value of t_1 thus obtained into the maximand, the principal's objective function can be written as a function, $V(q_1, \theta)$, of q_1 and θ. A necessary condition for optimality is the agent's first-order condition (1); we will show that ex post the agent's objective is globally concave when cost is quadratic, hence this condition is also sufficient. Optimizing w.r.t. q_1 yields:

$$\frac{\partial V}{\partial q_1}(q_1, \theta) = q_1 P'(q_1 + q_2^*(q_1, \theta)) \left(1 + \frac{\partial q_2^*(q_1, \theta)}{\partial q_1}\right) + P(q_1 + q_2^*(q_1, \theta))$$

$$-C_1(q_1, q_2^*(q_1, \theta), \theta) - (-t_2'(q_2^*(q_1, \theta), \theta) + C_2(q_1, q_2^*(q_1, \theta), \theta))\frac{\partial q_2^*(q_1, \theta)}{\partial q_1} = 0. \tag{20}$$

From (1) and (19), the RHS above can be simplified and we get for a symmetric equilibrium such that $q_1(\theta) = q_2(\theta) = q(\theta)$ and $t_1(\cdot) = t_2(\cdot) = t(\cdot)$:

$$q(\theta)P'(2q(\theta)) + P(2q(\theta)) = C_1(q(\theta), q(\theta), \theta) - \frac{q(\theta)P'(2q(\theta))C_{12}(q(\theta), q(\theta), \theta)}{t''(q(\theta), \theta) - C_{11}(q(\theta), q(\theta), \theta)}. \tag{21}$$

• In any differentiable equilibrium, it is necessary that the agent's objective function is locally concave at the equilibrium choice. This imposes two conditions on the Hessian of this symmetric problem:

$$t''(q(\theta), \theta) - C_{11}(q(\theta), q(\theta), \theta) \leq 0 \text{ for all } \theta. \tag{22}$$

and

$$(t''(q(\theta), \theta) - C_{11}(q(\theta), q(\theta), \theta)) \geq C_{12}^2(q(\theta), q(\theta), \theta) \text{ for all } \theta. \tag{23}$$

• We use these necessary local concavity conditions to derive the boundaries of the equilibrium sets.

- *The case of substitutes*: Substituting the condition (22) into (21) and noting that $C_{12} > 0$:

$$q(\theta)P'(2q(\theta)) + P(2q(\theta)) \leq C_1(q(\theta), q(\theta), \theta). \tag{24}$$

  $C(\cdot)$ being convex, $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$ ensure that (24) implies that $q(\theta) \geq q^c(\theta)$.

  Inserting (23) into (21) yields:

$$P(2q(\theta)) \geq C_1(q(\theta), q(\theta), \theta). \tag{25}$$

  From $C(\cdot)$ being convex, $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$, (25) implies that $q(\theta) \leq q^b(\theta)$.

- *The case of complements*: Inserting the condition (22) into (21) and noting that $C_{12} < 0$, we get:

$$q(\theta)P'(2q(\theta)) + P(2q(\theta)) \geq C_1(q(\theta), q(\theta), \theta). \tag{26}$$

  From $C(\cdot)$ being convex, $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$, (26) implies that $q(\theta) \leq q^c(\theta)$.

  Substituting (23) into (21) yields:

$$2q(\theta)P'(2q(\theta)) + P(2q(\theta)) \leq C_1(q(\theta), q(\theta), \theta). \tag{27}$$

  From $C(\cdot)$ being convex, $P'(\cdot) < 0$ and $P''(\cdot) \leq 0$, (27) implies that $q(\theta) \geq q^m(\theta)$.

• Note that, from (21), $q(\theta)$ depends only on the second derivative of the symmetric equilibrium tariff $t(q)$ at the equilibrium point. Let us denote by $t''(q(\theta), \theta) = \nu$ this second derivative. varying ν while respecting the concavity of the agent's program, we will show that quadratic nonlinear prices have thus enough flexibility to implement those equilibrium outputs. For those quadratic nonlinear prices and when $C(\cdot)$ is also quadratic, (22) and (23) amount respectively to

$$\nu \leq 1, \tag{28}$$

$$(\nu - 1 + \lambda)(\nu - 1 - \lambda) \geq 0. \tag{29}$$

Global concavity of the agent's problem requires that

$$t''(q_i, \theta) - C_{ii}(q_1, q_2, \theta) \leq 0 \ \text{ for all } \ (q_1, q_2) \ \ i = 1, 2, \tag{30}$$

and

$$(t''(q_1, \theta) - C_{11}(q_1, q_2, \theta))(t''(q_2, \theta) - C_22(q_1, q_2, \theta)) \geq C_{12}(q_1, q_2, \theta) \ \text{ for all } \ (q_1, q_2). \tag{31}$$

Global concavity is guaranteed as soon as these quadratic prices are extended over the whole real line. Note that, since in equilibrium the agent's participation constraint is binding and the agent's objective function is concave, we have $2t(q(\theta), \theta) - C(q(\theta), q(\theta), \theta) = 0 \geq t(q_1, \theta) + t(q_2, \theta) - C(q_1, q_2, \theta)$ for all (q_1, q_2).

• Using (21), equilibrium outputs can be parameterized by ν and, in the case where $C(\cdot)$ is quadratic, we find:

$$q(\theta)P'(2q(\theta)) + P(2q(\theta)) = \theta + (1+\lambda)q(\theta) + \frac{\lambda q(\theta)P'(2q(\theta))}{1-\nu}, \tag{32}$$

where $\nu \in]-\infty, 1-\lambda]$ in the case of substitutes, and where $\nu \in [1+\lambda, 1]$ in the case of complements to insure that (28) and (29) both hold.

When $P(\cdot)$ is also linear, we finally find:

$$q(\theta) = \frac{a-\theta}{4+\lambda\frac{\nu}{\nu-1}} \tag{33}$$

In the case of substitutes ($\lambda > 0$), $\nu \in]-\infty, 1-\lambda]$. Hence, $q(\theta)$ takes values in $[q^c(\theta), q^b(\theta)]$ where $q^c(\theta) = \frac{a-\theta}{4+\lambda}$ and $q^b(\theta) = \frac{a-\theta}{3+\lambda}$. In the case of complements ($\lambda < 0$), $\nu \in [1+\lambda, 1]$ and $q(\theta)$ takes values in $[q^m(\theta), q^c(\theta)]$ where $q^m(\theta) = \frac{a-\theta}{5+\lambda}$.

• Let us check the concavity of each principal's problem. Taking into account that $\frac{\partial q_2^*(q_1,\theta)}{\partial q_1} = \frac{\lambda}{\nu-1}$, we obtain:

$$\frac{\partial^2 V}{\partial q_1^2}(q_1,\theta) = q_1 P''(q_1 + q_2^*(q_1,\theta))\left(1+\frac{\lambda}{\nu-1}\right)^2 + 2P'(q_1+q_2^*(q_1,\theta))\left(1+\frac{\lambda}{\nu-1}\right) - \left(1+\frac{\lambda^2}{\nu-1}\right). \tag{34}$$

The first term is negative thanks to $P''(\cdot) \leq 0$. The second term is negative since, both with complements and substitutes, $\left(1+\frac{\lambda}{\nu-1}\right) > 0$ and $P'(\cdot) < 0$. Taking into account that $\nu < 1$, the third term has the sign of $\nu - 1 + \lambda^2$. With substitutes, $\nu - 1 \leq -\lambda$ and thus $\nu - 1 + \lambda^2 \leq \lambda(\lambda-1) \leq 0$ and the third term on the right-hand-side of (34) is also negative insuring concavity of the principal's problem. With complements, $-\nu + 1 \geq -\lambda$ and thus $\nu - 1 + \lambda^2 \geq \lambda(\lambda+1) \geq 0$ and the third term on the right-hand-side of (34) is still negative insuring again concavity of the principal's problem. □

Proof of Proposition 2: Assume that P_2 offers the direct revelation mechanism $\{t_2(\theta), q_2(\theta)\}$, then P_1's problem is:

$$max_{\{t_1,q_1\}} P(q_1 + q_2(\theta))q_1 - t_1$$

subject to

$$t_1 + t_2(\theta) - C(q_1, q_2(\theta), \theta) \geq 0. \tag{35}$$

Again (35) is binding at the optimum of P_1's problem. Inserting the corresponding value of t_1 into P_1's objective function, observing that the corresponding maximand is concave in q_1 and optimizing with respect to q_1 yields:

$$q_1 P'(q_1 + q_2(\theta)) + P(q_1 + q_2(\theta)) = C_1(q_1, q_2(\theta), \theta). \tag{36}$$

In a symmetric equilibrium, we obtain $q_1(\theta) = q_2(\theta) = q^c(\theta)$. □

Proof of Proposition 3: The proof starts as that of Proposition 1. (20) must be replaced by

$$\frac{\partial V}{\partial q_1}(q_1,\theta) = q_1 P'(q_1)+P(q_1)-C_1(q_1,q_2^*(q_1,\theta),\theta)+(t_2'(q_2^*(q_1,\theta),\theta)-C_2(q_1,q_2^*(q_1,\theta),\theta))\frac{\partial q_2^*(q_1,\theta)}{\partial q_1} = 0. \tag{37}$$

The first-order condition $t_2'(q_2^*(q_1,\theta),\theta) - C_2(q_1,q_2^*(q_1,\theta),\theta)$ characterizes the agent's choice of output. Hence, since in a symmetric equilibrium, conjectures must be correct and thus $q_2^*(q,\theta) = q = \tilde{q}^m(\theta)$. It is routine to check that the same outcome can be implemented with direct mechanisms. □

Proof of Proposition 4: We take as given the nonlinear price $t_2(q_2)$ offered by P_2 to the common agent and compute P_1's best response. We conjecture that this schedule has the same slope and the same curvature as in the case of intrinsic common agency. The difference with the case of intrinsic common agency may come from the nature of the binding participation constraint. This schedule is thus of the form:

$$t_2(q_2,\theta) = t_2(q(\theta),\theta) + t'(q(\theta),\theta)(q_2 - q(\theta)) + \frac{\nu}{2}(q_2 - q(\theta))^2, \tag{38}$$

where again $t'(q(\theta),\theta) = \theta + (1+\lambda)q(\theta)$.

With this expression, we can compute $U_2(\theta)$. Indeed $t_2(q_2,\theta) - C(0,q_2,\theta)$ is strictly concave in q_2 when $\nu < 1$, a condition which holds because of the global concavity of the agent's problem under intrinsic common agency.

Let us denote by $\hat{q}_2(\theta) = \arg\max_{q_2} \quad t_2(q_2,\theta) - C(0,q_2,\theta)$ the optimal output chosen by the agent when he takes only contract $t_2(\cdot)$. This output is obtained through a first-order condition:

$$t_2'(\hat{q}_2(\theta),\theta) = \theta + \hat{q}_2(\theta)$$

or to put it differently

$$\hat{q}_2(\theta) = \left(\frac{1+\lambda-\nu}{1-\nu}\right) q(\theta). \tag{39}$$

We can easily check that $\hat{q}_2(\theta) > q(\theta)$ if and only if $\lambda > 0$.

Given the slope and the curvature of $t_2(\cdot)$ which is the same as under intrinsic common agency, P_1 induces the same output q_1 as under intrinsic common agency. Hence, the symmetric equilibrium outputs are the same under delegated and under intrinsic common agency. The only remaining variable to be found is the value of the intercept $t(q(\theta),\theta)$ of the symmetric equilibrium schedule at the equilibrium point.

With the value of $\hat{q}_2(\theta)$ found above, we can compute:

$$U_2(\theta) = t_d(q(\theta),\theta) + t'(q(\theta),\theta)(\hat{q}_2(\theta) - q(\theta)) + \frac{\nu}{2}(\hat{q}_2(\theta) - q(\theta))^2 - C(0,\hat{q}_2(\theta),\theta).$$

After tedious computations, we get:

$$U_2(\theta) = t_d(q(\theta),\theta) - \theta q(\theta) - \frac{q(\theta)^2}{2}\left(1 - \frac{\lambda^2}{1-\nu}\right). \tag{40}$$

In equilibrium, each principal reduces as much as possible the transfer he offers to the common agent so that one of the participation constraint is binding. Hence, the value of $t(q(\theta),\theta)$ is solution to the equation:

$$2t(q(\theta),\theta) - C(q(\theta),q(\theta),\theta) = \max\left\{0, t(q(\theta),\theta) - \theta q(\theta) - \frac{q(\theta)^2}{2}\left(1 - \frac{\lambda^2}{1-\nu}\right)\right\}. \tag{41}$$

Let us assume that the right-hand side above is 0. Then, we have

$$t(q(\theta),\theta) = \theta q(\theta) + \frac{(1+\lambda)q(\theta)^2}{2}$$

and we must have

$$t(q(\theta),\theta) < \theta q(\theta) + \frac{q(\theta)^2}{2}\left(1 - \frac{\lambda^2}{1-\nu}\right),$$

which holds when $\lambda\left(1 + \frac{\lambda}{1-\nu}\right) < 0$ but $1 + \frac{\lambda}{1-\nu} > 0$ from the global concavity of the agent's problem under intrinsic common agency. Hence, the latter inequality holds when $\lambda < 0$, i.e., for complements. In this case, the agent gets zero rent in equilibrium.

Let us now assume that the right-hand side of (41) is not 0. Then, we have

$$t(q(\theta),\theta) = \theta q(\theta) + \frac{q(\theta)^2}{2}\left(1 + 2\lambda + \frac{\lambda^2}{1-\nu}\right)$$

and we must have

$$t(q(\theta),\theta) > \theta q(\theta) + \frac{(1+\lambda)q(\theta)^2}{2}$$

which holds again when $\lambda\left(1 + \frac{\lambda}{1-\nu}\right) > 0$ but $1 + \frac{\lambda}{1-\nu} > 0$ from the global concavity of the agent's problem under intrinsic common agency. Hence, the latter inequality holds when $\lambda > 0$, i.e., for substitutes. In this case, the agent gets a strictly positive rent in equilibrium. Then, this rent is

$$U_2(\theta) = q^2(\theta)\lambda\left(1 + \frac{\lambda}{1-\nu}\right).$$

□

Proofs of Propositions 5 and 6:

• (8) indicates that $U(\cdot)$ is decreasing because

$$\hat{U}^1_\theta(q_1(\theta),\theta) = -C_\theta(q_1(\theta), q_2^*(q_1(\theta),\theta),\theta) < 0.$$

Hence, the participation constraint (10) is binding only at $\bar{\theta}$. Henceforth:

$$U(\theta) = -\int_\theta^{\bar{\theta}} C_\theta(q_1(z), q_2^*(q_1(z),z),z)dz.$$

Inserting into the principal's objective function and integrating by parts, the objective function to be optimized pointwise becomes a function of q_1 only, namely:

$$V(q_1,\theta) = q_1 P(q_1 + q_2^*(q_1,\theta)) + \hat{U}^1(q_1,\theta) + \frac{F(\theta)}{f(\theta)}\hat{U}^1_\theta(q_1,\theta).$$

Assuming concavity of $V(\cdot)$ with respect to q_1 (this is checked ex post) and optimizing pointwise w.r.t. q_1 yields:

$$q_1 P'(q_1 + q_2^*(q_1,\theta))\left(1 + \frac{\partial q_2^*(q_1,\theta),\theta)}{\partial q_1}\right)$$

$$+P(q_1 + q_2^*(q_1,\theta)) + U^1_1(q_1,\theta) + \frac{F(\theta)}{f(\theta)}U^1_{1\theta}(q_1,\theta) = 0. \tag{42}$$

Using again the Envelope Theorem:

$$\hat{U}^1_1(q_1,\theta) = -C_1(q_1, q_2^*(q_1(\theta),\theta),\theta)$$

$$\hat{U}^1_{1\theta}(q_1,\theta) = -C_{1\theta}(q_1, q_2^*(q_1,\theta),\theta) - C_{12}(q_1, q_2^*(q_1,\theta),\theta)\frac{\partial q_2^*(q_1,\theta)}{\partial q_1}$$

where $\frac{\partial q_2^*(q_1,\theta)}{\partial q_1}$ is well-defined from the fact that $t_2(\cdot)$ is twice differentiable. Inserting into (42) yields for a symmetric equilibrium such that $q_1(\theta) = q_2(\theta) = q(\theta)$:

$$q(\theta)P'(2q(\theta)) + P(2q(\theta)) = C_1(q(\theta), q(\theta),\theta) + \frac{F(\theta)}{f(\theta)}C_{1\theta}(q(\theta), q(\theta),\theta)$$

$$+\frac{C_{12}(q(\theta), q(\theta),\theta)}{t''(q(\theta)) - C_{11}(q(\theta), q(\theta),\theta)}\left(\frac{F(\theta)}{f(\theta)} - q(\theta)P'(2q(\theta))\right). \tag{43}$$

However, $t'(q(\theta)) = C_1(q(\theta), q(\theta),\theta)$ for all θ. Differentiating w.r.t. θ yields:

$$(t''(q(\theta)) - C_{11}(q(\theta), q(\theta),\theta))\dot{q}(\theta) = C_{12}(q(\theta), q(\theta),\theta))\dot{q}(\theta) + C_{1\theta}(q(\theta), q(\theta),\theta).$$

Inserting into (43) yields (11).

• We now establish the bounds on $q(\theta)$:

- *The case of substitutes* ($C_{12} > 0$). We prove that $q(\theta) \in [\tilde{q}^c(\theta), q^b(\theta)]$ for all θ. First we show that if $q(\underline{\theta}) \in [q^c(\underline{\theta}), q^b(\underline{\theta})]$, then the solution to (11) starting from this point remains in the set $[\tilde{q}^c(\theta), q^b(\theta)]$ for all θ.

Suppose that there exists $\hat{\theta}$, the first value of θ greater than $\underline{\theta}$, such that $q(\hat{\theta}) = q^b(\hat{\theta})$. We have then:

$$\dot{q}(\hat{\theta}) = -\frac{C_{1\theta}}{2C_{12}} < \dot{q}^b(\hat{\theta}) = -\frac{C_{1\theta}}{C_{11} + C_{12} - 2P'} < 0.$$

Hence, $q(\theta) > q^b(\theta)$ for $\theta \in (\hat{\theta} - \epsilon, \hat{\theta})$ where ϵ is small enough. A contradiction.

Suppose that there exists $\hat{\theta}$, the first value of θ greater than $\underline{\theta}$, such that $q(\hat{\theta}) = \tilde{q}^c(\hat{\theta})$. We have then:

$$\dot{q}(\hat{\theta}) = 0 > \dot{q}^c(\hat{\theta})$$

when $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$, $C(\cdot)$ and $C_\theta(\cdot)$ are convex. This holds when $C(\cdot)$ is quadratic. Hence, $q(\theta) < \tilde{q}^c(\theta)$ for $\theta \in (\hat{\theta} - \epsilon, \hat{\theta})$ where ϵ is small enough. A contradiction.

Second, we prove that no solution exists with $q(\underline{\theta}) \notin [q^c(\underline{\theta}), q^b(\underline{\theta})]$. Consider (43) evaluated at $\underline{\theta}$. The following two relationships emerge (where the inequalities follow from applying the agent's local second-order condition):

$$q(\underline{\theta})P'(2q(\underline{\theta})) + P(2q(\underline{\theta})) - C_1(q(\underline{\theta}), q(\underline{\theta}), \underline{\theta}) = -q(\underline{\theta})P'(2q(\underline{\theta}))\left(\frac{C_{12}(q(\underline{\theta}), q(\underline{\theta}), \underline{\theta})}{t''(q(\underline{\theta})) - C_{11}(q(\underline{\theta}), q(\underline{\theta}), \underline{\theta})}\right) < 0,$$

$$P(2q(\underline{\theta})) - C_1(q(\underline{\theta}), q(\underline{\theta}), \underline{\theta}) = -q(\underline{\theta})P'(2q(\underline{\theta}))\left(\frac{C_{12}(q(\underline{\theta}), q(\underline{\theta}), \underline{\theta})}{t''(q(\underline{\theta})) - C_{11}(q(\underline{\theta}), q(\underline{\theta}), \underline{\theta})} + 1\right) > 0.$$

Hence, $q^c(\underline{\theta}) \leq q(\underline{\theta}) \leq q^b(\underline{\theta})$.

- *The case of complements* ($C_{12} < 0$). We prove that $q(\theta) \in [\tilde{q}^m(\theta), \tilde{q}^c(\theta)]$ for all θ. First, we show that if $q(\bar{\theta}) \in [\tilde{q}^m(\bar{\theta}), \tilde{q}^c(\bar{\theta})]$, then a solution to (11) going through this point stays within the set $[\tilde{q}^m(\theta), \tilde{q}^c(\theta)]$.

  Suppose that there exists $\hat{\theta}$, the last value of θ smaller than $\bar{\theta}$ such that $q(\hat{\theta}) = \tilde{q}^m(\hat{\theta})$. We have then:

  $$\dot{q}(\hat{\theta}) = -\infty < \dot{\tilde{q}}^m(\hat{\theta}).$$

  But $q(\theta)$ is increasing when $q(\theta) < \tilde{q}^m(\theta)$ by (11), and it cannot be that $q(\theta)$ is a solution to (11) going through $q(\bar{\theta})$ and $q(\theta) \in [\tilde{q}^m(\theta), \tilde{q}^c(\theta)]$ for all θ. A contradiction.

  Suppose next that there exists $\hat{\theta}$, the last value of θ smaller than $\bar{\theta}$, such that $q(\hat{\theta}) = \tilde{q}^c(\hat{\theta})$. We have then:

  $$\dot{q}(\hat{\theta}) = 0 > \dot{\tilde{q}}^c(\hat{\theta})$$

  when $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) > 0$, $C(\cdot)$ and $C_\theta(\cdot)$ are convex. This holds when $C(\cdot)$ is quadratic. Hence, $q(\theta) > \tilde{q}^c(\theta)$ for $\theta \in (\hat{\theta}, \hat{\theta} + \epsilon)$ where ϵ is small enough. A contradiction.

  Second, we prove that no solution exists with $q(\underline{\theta}) \notin [q^m(\underline{\theta}), q^c(\underline{\theta})]$. It is immediate to check that if this were not the case, then (11) implies that $\dot{q}(\theta) > 0$. This further implies that the agent's choice is not incentive compatible (assuming the agent's single-crossing property is satisfied) and thus this solution cannot be part of any common agency equilibrium.

• We now turn to the global concavity of the agent's problem:

- *The case of substitutes*: Using (11), we have:

$$t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta)$$
$$= \frac{C_{12}(q(\theta), q(\theta), \theta)}{q(\theta)P'(2q(\theta) + P(2q(\theta)) - C_1(q(\theta), q(\theta), \theta) - \frac{F(\theta)}{f(\theta)}C_{1\theta}(q(\theta), q(\theta), \theta)}$$
$$\times \left(-q(\theta)P'(2q(\theta)) + \frac{F(\theta)}{f(\theta)}C_{1\theta}(q(\theta), q(\theta), \theta) \right). \quad (44)$$

  Since $q(\theta) > \tilde{q}^c(\theta)$, the R.H.S. above is negative. Moreover, $|t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta)| \geq C_{12}(q(\theta), q(\theta), \theta)$ since $P(2q(\theta)) \geq C_1(q(\theta), q(\theta), \theta)$ when $q(\theta) \leq q^b(\theta)$.

  The fact that $t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta) \leq 0$ and that $|t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta)| \geq C_{12}(q(\theta), q(\theta), \theta)$ proves that the Hessian of the type θ-agent's problem is negative semi-definite at $(q(\theta), q(\theta))$ for any θ and any equilibrium schedule $q(\theta)$. Hence, the agent's objective function is locally concave at this point. To have global concavity of the agent's problem, we need more. This Hessian must be negative semi-definite at all pairs (q_1, q_2). When $C(\cdot)$ is quadratic, the Hessian of the agent's problem is negative semi-definite at all pairs $(q_1 = q_1(\theta), q_2 = q_2(\theta'))$ since $0 \geq -\lambda \geq t''(q_1) - 1$ and $0 \geq -\lambda \geq t''(q_2) - 1$. Thus, (30) and (31) both hold. For outputs which lie outside the range of the equilibrium schedule $q(\theta)$, the equilibrium tariff is extended in a quadratic (and continuously differentiable) way so that these conditions also hold.

- *The case of complements*: Since $q(\theta) < \tilde{q}^c(\theta)$, the R.H.S. of (44) is again negative. Moreover, $|t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta)| > |C_{12}(q(\theta), q(\theta), \theta)|$ since $P(2q(\theta)) + 2q(\theta)P'(2q(\theta)) > C_1(q(\theta), q(\theta), \theta) + 2\frac{F(\theta)}{f(\theta)}C_{1\theta}(q(\theta), q(\theta), \theta)$ when $q(\theta) \geq q^m(\theta)$. The Hessian of the agent's problem is thus definite negative at all pairs (q_1, q_2) when $C(\cdot)$ is quadratic.

  The fact that $t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta) \leq 0$ and that $|t''(q(\theta)) - C_{11}(q(\theta), q(\theta), \theta)| \geq |C_{12}(q(\theta), q(\theta), \theta)|$ proves that the Hessian of the type θ-agent's problem is negative at $(q(\theta), q(\theta))$ for any θ and any equilibrium schedule $q(\theta)$. Hence, the agent's objective function is locally concave at this point. To have global concavity of the agent's problem, we need more. This Hessian must be negative at all pairs (q_1, q_2). When $C(\cdot)$ is quadratic, the Hessian of the agent's problem is definite negative at all pairs $(q_1 = q_1(\theta), q_2 = q_2(\theta'))$ since $0 \geq -\lambda \geq t''(q_1) - 1$ and $0 \geq -\lambda \geq t''(q_2) - 1$. Finally, (30) and (31) both hold. For outputs which lie outside the range of the equilibrium schedule $q(\theta)$, the equilibrium tariff is extended in a quadratic (and continuously differentiable) way so that these conditions also hold.

• We now check that the indirect utility function vis-à-vis either principal satisfies the Spence-Mirrlees single-crossing property, $\hat{U}^i_{i\theta}(q, \theta) \leq 0$ for all q and $i = 1, 2$. In fact, we have, for any equilibrium point $q(\theta')$:

$$\hat{U}^i_{i\theta}(q(\theta'),\theta) = -C_{i\theta}(q(\theta'), q^*_{-i}(q(\theta'),\theta),\theta) - C_{-i\theta}(q(\theta'), q^*_{-i}(q(\theta'),\theta),\theta)\frac{\partial q^*_{-i}(q(\theta'),\theta)}{\partial q}.$$

Omitting arguments, taking into account that $C(\cdot)$ is quadratic, and using symmetry, the RHS above is

$$= -\left(1 + \frac{\lambda\dot{q}(\theta')}{\lambda\dot{q}(\theta') + 1}\right).$$

Using (11), this RHS becomes:

$$\left(\frac{P(2q(\theta')) - C_1(q(\theta'), q(\theta'), \theta')}{q(\theta')P'(2q(\theta')) - \frac{F(\theta')}{f(\theta')}}\right)$$

which is negative since $q(\theta') \leq q^b(\theta')$ for all θ'. Because, $t(\cdot)$ is extended over the whole real line in a quadratic way, the latter inequality holds for all q even those for which there does not exist θ' such that $q = q(\theta')$.

• Let us provide conditions ensuring the concavity of $V(q_1,\theta)$ with respect to q_1. We can rewrite:

$$V(q_1,\theta) = q_1P(q_1 + q^*_2(q_1,\theta)) + t_2(q^*_2(q_1,\theta)) - C(q_1, q^*_2(q_1,\theta),\theta) - \frac{F(\theta)}{f(\theta)}C_\theta(q_1, q^*_2(q_1,\theta),\theta).$$

Thus, in the case where $C(\cdot)$ is quadratic:

$$\frac{\partial V}{\partial q_1}(q_1,\theta) = q_1P'(q_1 + q^*_2(q_1,\theta))\left(1 + \frac{\partial q^*_2(q_1,\theta)}{\partial q_1}\right) + P(q_1 + q^*_2(q_1,\theta))$$

$$-\theta - q_1 - \lambda q^*_2(q_1,\theta) - \frac{F(\theta)}{f(\theta)}\left(1 + \frac{\partial q^*_2(q_1,\theta)}{\partial q_1}\right) \tag{45}$$

and

$$\frac{\partial^2 V}{\partial q_1^2}(q_1,\theta) = q_1P''(q_1 + q^*_2(q_1,\theta))\left(1 + \frac{\partial q^*_2(q_1,\theta)}{\partial q_1}\right)^2 + 2P'(q_1 + q^*_2(q_1,\theta))\left(1 + \frac{\partial q^*_2(q_1,\theta)}{\partial q_1}\right)$$

$$-\left(1 + \lambda\frac{\partial q^*_2(q_1,\theta)}{\partial q_1}\right) + \frac{\partial^2 q^*_2(q_1,\theta)}{\partial q_1^2}\left(q_1P'(q_1 + q^*_2(q_1,\theta)) - \frac{F(\theta)}{f(\theta)}\right). \tag{46}$$

But $\frac{\partial q^*_2(q_1,\theta)}{\partial q_1} = \frac{\lambda}{t''(q^*_2(q_1,\theta))-1}$ where $t(\cdot)$ is a symmetric equilibrium nonlinear price which satisfies the conditions (30) and (31) and thus, both with substitutes and complements, $\left(1 + \frac{\partial q^*_2(q_1,\theta)}{\partial q_1}\right) > 0$ (obtained from (30)) and $\left(1 + \lambda\frac{\partial q^*_2(q_1,\theta)}{\partial q_1}\right) > 0$ (obtained from (31)). The first term on the right-hand-side above is negative thanks to $P''(\cdot) \leq 0$, the second term is also negative thanks to $P'(\cdot) < 0$. The third term is also negative. Hence, the concavity of $V(q_1,\theta)$ with respect to q_1 is ensured when $\frac{\partial^2 q^*_2(q_1,\theta)}{\partial q_1^2}\left(q_1P'(q_1 + q^*_2(q_1,\theta)) - \frac{F(\theta)}{f(\theta)}\right)$ is positive. Note that this latter condition is satisfied if, in equilibrium, $t(\cdot)$ is quadratic since then $\frac{\partial^2 q^*_2(q_1,\theta)}{\partial q_1^2} = 0$. This holds for the linear equilibrium $q^*(\theta) = q^*(\underline{\theta}) - \beta^*(\theta - \underline{\theta})$ discussed in Proposition 7. This condition also holds provided that, for all equilibria and whatever the distribution of types, $t''(\cdot)$ remains almost constant, i.e., when $\bar{\theta}-\underline{\theta}$ is small enough. □

Proof of Corollary 3: Immediate from (13) and Proposition 5. □

Proof of Proposition 7: Under the assumptions of Proposition 7, (11) becomes:

$$\lambda\dot{q}(\theta) = -\frac{a-\underline{\theta}-(4+\lambda)q-2(\theta-\underline{\theta})}{a-\underline{\theta}-(5+\lambda)q-3(\theta-\underline{\theta})}. \tag{47}$$

• The first point of the proposition is easy to obtain. The solutions to the differential equation (47) only depend on their value at the lower bound of the support in the case of substitutes and at the upper bound of the support in the case of complements.

• We look for a linear solution to (47) of the form $q(\theta) = q(\underline{\theta}) - \beta(\theta-\underline{\theta})$. Differentiating (47) yields therefore:

$$\lambda\beta = \frac{(4+\lambda)\beta-2}{(5+\lambda)\beta-3} \tag{48}$$

or β solution to

$$(5+\lambda)\lambda\beta^2 - 4(1+\lambda)\beta + 2 = 0.$$

The only solution to this second degree equation such that the corresponding nonlinear price satisfies both (30) and (31) is:

$$\beta^* = \frac{2(1+\lambda)-\sqrt{2(2+\lambda^2-\lambda)}}{\lambda(5+\lambda)}. \tag{49}$$

Being given this value of β^* on can compute $q^*(\underline{\theta})$ such that:

$$\lambda\beta^* = \frac{a-\underline{\theta}-(4+\lambda)q^*(\underline{\theta})}{a-\underline{\theta}-(5+\lambda)q^*(\underline{\theta})},$$

and one finds

$$q^*(\underline{\theta}) = \frac{(a-\underline{\theta})(1-\lambda\beta^*)}{2-\lambda+\sqrt{2(2+\lambda^2-\lambda)}}.$$

• We now check that all solutions to (47) converge towards the solution above as $\Delta\theta$ increases. We focus on the case of substitutes where we fix $\underline{\theta}$ and enlarges $\bar{\theta}$. The case of complements can be treated similarly by fixing $\bar{\theta}$ and decreasing $\underline{\theta}$.

We first write (47) as a system of autonomous differential equations in q and $\theta-\underline{\theta}$ depending on some parameter t:

$$\lambda\frac{dq}{dt} = -(a - \underline{\theta} - (4+\lambda)q - 2(\theta - \underline{\theta})), \tag{50}$$

$$\frac{d(\theta - \underline{\theta})}{dt} = a - \underline{\theta} - (5+\lambda)q - 3(\theta - \underline{\theta}). \tag{51}$$

A particular solution to this system is obtained with $q_0 = \frac{a-\underline{\theta}}{2+\lambda}$ and $\theta_0 - \underline{\theta} = -\frac{a-\underline{\theta}}{2+\lambda}$.

The general solution to (50)-(51) solves:

$$\lambda\frac{d\tilde{q}}{dt} = (4+\lambda)\tilde{q} + 2\tilde{\theta}, \tag{52}$$

$$\frac{d\tilde{\theta}}{dt} = -(5+\lambda)\tilde{q} - 3\tilde{\theta} \tag{53}$$

where $\tilde{q} = q - q_0$ and $\tilde{\theta} = \theta - \theta_0$.

Differentiating (52) with respect to t and using (52) again and (53) to eliminate $\frac{d\tilde{\theta}}{dt}$ and $\tilde{\theta}$, we get:

$$\lambda\frac{d^2\tilde{q}}{dt^2} - 2(2-\lambda)\frac{d\tilde{q}}{dt} - (2+\lambda)q = 0. \tag{54}$$

The general solution to this second-order differential equation is :

$$\tilde{q} = Ae^{z_1 t} + Be^{z_2 t}, \tag{55}$$

where z_1 and z_2 are roots of $\lambda z^2 - 2(2-\lambda)z - (2+\lambda) = 0$ and one finds:

$$z_1 = \frac{2 - \lambda - \sqrt{2(2 - \lambda + \lambda^2)}}{\lambda},$$

$$z_2 = \frac{2 - \lambda + \sqrt{2(2 - \lambda + \lambda^2)}}{\lambda}.$$

Note that $z_1 < 0$ and $z_2 > 0$. A and B are obtained from the conditions

$$\tilde{q}(0) = q(\underline{\theta}) - q_0 = A + B,$$

where $q(\underline{\theta}) \in [q^c(\underline{\theta}), q^b(\underline{\theta})]$ and from (52)

$$\tilde{\theta}(0) = \underline{\theta} - \theta_0 = \frac{\lambda z_1 - (4+\lambda)}{2}A + \frac{\lambda z_2 - (4+\lambda)}{2}B.$$

Note also that, from (52),

$$\tilde{\theta}(t) = \frac{1}{2}\left(\lambda\frac{d\tilde{q}}{dt} - (4+\lambda)\tilde{q}(t)\right) = \frac{1}{2}\left((\lambda z_1 - (4+\lambda))\, Ae^{z_1 t} + (\lambda z_2 - (4+\lambda))Be^{z_2 t}\right).$$

When t goes to $-\infty$, $\frac{\tilde{q}}{\tilde{\theta}}$ behaves as $\frac{2}{\lambda z_1 - (4+\lambda)}$ which is equal to the slope of the linear solution to (47), i.e., $-\beta^*$ exactly. Finally,

$$\tilde{q}(t) + \beta^*\tilde{\theta}(t) = A\left(1 + \frac{\beta^*}{2}(\lambda z_1 - (4+\lambda))\right)e^{z_1 t} + B\left(1 + \frac{\beta^*}{2}(\lambda z_2 - (4+\lambda))\right)e^{z_2 t}.$$

The first term in the right-hand-side above is equal to zero by definition of β^*. The second goes to zero as t goes to $-\infty$ since $z_2 > 0$. Hence $\tilde{q}(t) + \beta^*\tilde{\theta}(t)$ goes to zero as t goes to $-\infty$. Finally, $q(\theta) + \beta^*(\theta - \underline{\theta})$ converges towards $q^*(\underline{\theta})$ when t goes to $-\infty$.

Hence, whatever the solution to the differential equation (47), $q(\theta|\underline{\theta})$, we have:

$$\lim_{\theta \to \infty} q(\theta|\underline{\theta}) = q^*(\theta).$$

Since, those solutions are independent on the upper bound of the support of the uniform distribution $\bar{\theta}$. It is thus immediate that

$$\lim_{\bar{\theta} \to \infty} E[q(\theta|\underline{\theta}) - q^*(\theta)] = 0.$$

□

Proof of Proposition 8: We take as given the quadratic price $t_d^*(q_2)$ offered by P_2 to the common agent and compute P_1's best response in the case of a delegated common agency game. This schedule $t_d^*(q_2)$ has the same derivative as the symmetric equilibrium tariff $t^*(q_2)$ of the intrinsic common agency game (see equation (15)). Hence, for any output q_2, we have:

$$t_d^{*'}(q_2) = \bar{\theta} + (1+\lambda)q^*(\bar{\theta}) + \left(1 + \lambda - \frac{1}{\beta^*}\right)(q - q^*(\bar{\theta})).$$

With this expression, we can compute $U_2(\theta)$. Indeed $t_2(q_2) - C(0, q_2, \theta)$ is strictly concave in q_2 when $\lambda < \frac{1}{\beta^*}$, a condition which holds as it can be checked by using (49).

Let us still denote by $\hat{q}_2(\theta) = argmax_{q_2} t_2(q_2) - C(0, q_2, \theta)$. This output is thus obtained through a first-order condition:

$$t_d^{*'}(\hat{q}_2(\theta)) = \theta + \hat{q}_2(\theta)$$

or to put it differently

$$\hat{q}_2(\theta) = \frac{q^*(\theta)}{1 - \lambda\beta}. \tag{56}$$

We can easily check that $\hat{q}_2(\theta) > q(\theta)$ if and only if $\lambda > 0$.

Given the slope and the curvature of $t_d^*(\cdot)$ which are the same as under intrinsic common agency, P_1 induces the same output q_1 as under intrinsic common agency in a best response if the agent's participation constraint is only binding at $\bar{\theta}$ and the nature of the binding participation constraint (0 or $U_2(\bar{\theta})$) determines the value of $t_d^*(q(\bar{\theta}))$. In this case, the equilibrium output $q_d^*(\theta)$ remains an equilibrium output of the delegated common agency game. The only remaining variable to be found is the value of the intercept $t(q(\theta))$ of the symmetric equilibrium schedule at the equilibrium point.

We are first going to prove that $U(\theta)$ has a greater slope (in absolute terms) than $U_2(\theta)$ when the principal induces an equilibrium output $q^*(\theta)$ in a best response to $t_d^*(q_2)$. Indeed, we have:

$$\dot{U}(\theta) = -2q^*(\theta)$$

and

$$\dot{U}_2(\theta) = -\hat{q}_2(\theta).$$

Moreover, $|\dot{U}(\theta)| > |\dot{U}_2(\theta)|$ for all θ when $1-2\lambda\beta^* > 0$. This inequality obviously holds when $\lambda < 0$ (complements). Using (49), we can check that this inequality also holds when $\lambda > 0$ (substitutes).

In equilibrium, each principal reduces as much as possible the transfer he offers to the common agent so that one of the participation constraint of the least efficient type is binding. Hence, the value of $t_d^*(q^*(\bar{\theta}))$ is solution to the equation:

$$2t_d^*(q^*(\bar{\theta})) - 2\bar{\theta}q^*(\bar{\theta}) - (1+\lambda)(q^*(\bar{\theta}))^2 = max(0, t_d^*(q^*(\bar{\theta})) + (\bar{\theta} + (1+\lambda)q^*(\bar{\theta}))(\hat{q}_2(\bar{\theta} - q^*(\bar{\theta})) \\ + \frac{1}{2}\left(1+\lambda-\frac{1}{\beta^*}\right)(\hat{q}_2(\bar{\theta}) - q^*(\bar{\theta}))^2 - \left(\bar{\theta}\hat{q}_2(\bar{\theta}) + \frac{(\hat{q}_2(\bar{\theta}))^2}{2}\right)). \tag{57}$$

Let us assume that the right-hand side of (57) is 0. Then, we have

$$t_d^*(q^*(\bar{\theta})) = \theta q^*(\bar{\theta}) + \frac{(1+\lambda)q^*(\bar{\theta})^2}{2}$$

and we must have

$$t_d^*(q^*(\bar{\theta})) + (\bar{\theta} + (1+\lambda)q^*(\bar{\theta}))(\hat{q}_2(\bar{\theta} - q^*(\bar{\theta})) \\ + \frac{1}{2}\left(1+\lambda-\frac{1}{\beta^*}\right)(\hat{q}_2(\bar{\theta}) - q^*(\bar{\theta}))^2 - \left(\bar{\theta}\hat{q}_2(\bar{\theta}) + \frac{(\hat{q}_2(\bar{\theta}))^2}{2}\right) < 0.$$

The left-hand side above is equal to $\frac{\lambda}{2(1-\lambda\beta^*)}(q^*(\bar{\theta}))^2$ which is negative when $\lambda < 0$ (complements). In this case, the binding participation constraint of the least efficient type is 0.

Let us assume that the right-hand side (57) is strictly positive. Then, we have

$$t_d^*(q^*(\bar{\theta})) = 2\bar{\theta}q^*(\bar{\theta}) + (1+\lambda)(q^*(\bar{\theta}))^2 + (\bar{\theta} + (1+\lambda)q^*(\bar{\theta}))(\hat{q}_2(\bar{\theta} - q^*(\bar{\theta})) \\ + \frac{1}{2}\left(1+\lambda-\frac{1}{\beta^*}\right)(\hat{q}_2(\bar{\theta}) - q^*(\bar{\theta}))^2 - \left(\bar{\theta}\hat{q}_2(\bar{\theta}) + \frac{(\hat{q}_2(\bar{\theta}))^2}{2}\right).$$

Then, we must have

$$t_d^*(q^*(\bar{\theta})) + (\bar{\theta} + (1+\lambda)q^*(\bar{\theta}))(\hat{q}_2(\bar{\theta} - q^*(\bar{\theta})) \\ + \frac{1}{2}\left(1+\lambda-\frac{1}{\beta^*}\right)(\hat{q}_2(\bar{\theta}) - q^*(\bar{\theta}))^2 - \left(\bar{\theta}\hat{q}_2(\bar{\theta}) + \frac{(\hat{q}_2(\bar{\theta}))^2}{2}\right) > 0.$$

The left-hand side above is equal to $\frac{\lambda}{2(1-\lambda\beta^*)}(q^*(\bar{\theta}))^2$ which is positive when $\lambda > 0$ (substitutes). In this case, the binding participation constraint of the least efficient type is strictly positive. □