A joint Initiative of Ludwig-Maximilians-Universität and Ifo Institute for Economic Research

CESifo Working Papers

THE FUNDS CONCENTRATION EFFECT AND DISCRIMINATORY BAILOUT

Ulrich Erlenmaier
Hans Gersbach*

CESifo Working Paper No. 591

October 2001

Presented at CESifo Workshop on Financial Crises and Recovery, Venice, June 2001

CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de
ISSN 1617-9595

* We would like to thank Hans-Jörg Beilharz, Craig Burnside, Mathias Dewatripont, Charles Goodhart, Frank Herrmann, Eva Terberger, Aaron Tornell, seminar participants in Bielefeld, Frankfurt and Heidelberg, and conference participants at the CESifo workshop on Financial Crises 2001 and at the annual meeting of the EEA for helpful comments.

CESifo *Working Paper No. 591*
October 2001

THE FUNDS CONCENTRATION EFFECT AND DISCRIMINATORY BAILOUT

Abstract

In the presence of macroeconomic shocks severe enough to threaten the liquidity or solvency of the banking system, the regulator can rely on the funds concentration effect to save long-term investment projects. Some banks are forced into bankruptcy with the result that other banks obtain more new funds and remain solvent. We investigate two different implementations of the funds concentration effect and the corresponding discriminatory bailout scheme: "random bailout" and "bailout the big ones". While the latter can be problematic in terms of stability, it is superior to the former in terms of welfare and credibility.

JEL Classification: D84, E44, G28.

Keywords: financial intermediation, macroeconomic risk, banking regulation, discriminatory bailout, funds concentration, aggregate liquidity, consistent expectations.

Ulrich Erlenmaier
Alfred-Weber-Institut
University of Heidelberg
Grabengasse 14
69117 Heidelberg
Germany

Hans Gersbach
Alfred-Weber-Institut
University of Heidelberg
Grabengasse 14
69117 Heidelberg
Germany
Gersbach@uni-hd.de

1 Introduction

The frequency and virulence of financial crises has led to serious rethinking concerning the appropriate form of government intervention in financial markets. A major issue is whether such crises can or should be avoided or whether a workout approach is superior to prevention. In particular, it is unclear how financial intermediaries should be regulated when they are subject to large macroeconomic shocks, as has been the case in the recent crisis in Asia.[1]

While in the period before 1970 less intensive competition in banking in connection with interest rate ceilings created oligopoly profits which acted as a buffer against macroeconomic shocks, the present regulatory frameworks are focussed on the prevention of banking crises through cash-asset reserves and risk-sensitive capital requirements. If a banking crisis nevertheless occurs, a variety of approaches are applied. In the most common case of explicit or implicit deposit insurance, the taxpayers' money is used to bail out banks. In some cases, banking crises have been dealt with by closing some banks or by takeovers, which smacks of a discriminatory approach to bailout.[2]

Since the prevention of crises via restricted competition or ex-post bailout with taxpayers' money has costs of its own, and because equity will not always be sufficient to buffer severe macroeconomic shocks,[3] we will focus in this paper on the possibility of discriminatory bailout.

Under discriminatory bailout, the regulator forces only one or a small number of banks into bankruptcy while the remaining banks are allowed to continue with their operations although all banks may be identical with respect to their balance sheet. The rationale for discriminatory bailout can best be understood in an overlapping generation framework where banks invest short-term deposits in long-term productive investment. During the fruition time of the long-term investments, banks need to refinance themselves by taking the deposits from new generations of savers in order to pay back deposits from the old generation.

Suppose that, during the fruition time, new information reveals that the real return on long-term investment is low. This shock will not allow all banks to refinance long-term

[1] See e.g. HELLWIG (1998) and BHATTACHARYA, BOOT, AND THAKOR (1998).

[2] This has happened e.g. during the crisis in Asia and the Swiss regional bank crisis (see RADELET AND SACHS (1998, 1999) and STAUB (1998)).

[3] For example HELLWIG (1995) notes (p. 723): "Given the difficulties of recapitalization after a spell of bad luck - and given the possibility of repeated bad spells - it is not clear what one means in asking a bank to follow a strategy of having more equity as a buffer. More equity at the beginning - certainly! But thereafter?" Moreover, GERSBACH (2001) shows that requiring large equity buffers for banks reduces equity in firms, thereby increasing credit rationing which has negative macroeconomic consequences.

productive investment by taking new deposits, since they cannot credibly promise interest rates that are sufficiently high to attract enough deposits from the new generation. If there is no coordination mechanism that allows depositors to concentrate savings on a fraction of banks, regulatory intervention is desirable.

In order to save both, banks and long-term investment projects, the regulator can rely on the following general-equilibrium effect, which we call *funds concentration effect.* By forcing some illiquid banks into bankruptcy, the share of funds available for the remaining banks will increase, since there are fewer banks competing for new deposits. Moreover, the surviving banks can buy investment projects from bankrupt banks at liquidation value, thus enabling them to credibly offer higher deposit rates to the second generation. The bailout policy of the regulator is discriminatory.

To concentrate on the funds concentration effect of bank closures, we start our analysis with a situation where the banks' insolvency is assumed to result solely from an exogenous macroeconomic shock. As the realization of this shock is not under the control of the banks' managers, it would be most natural to decide randomly about which banks to close (RB). However, closure policies feed back into the banks' strategic behavior and we therefore also consider an other bailout scheme, namely bailout of big banks (BB).

We compare the discriminatory bailout approach with scenarios where banking crises are prevented completely and with the no-regulation case. Moreover, the different implications of the discriminatory bailout schemes with respect to stability, welfare and credibility are analyzed. We identify BB as the preferred bailout scheme if depositors can coordinate on maximum-return assessments. BB dominates RB with respect to welfare and credibility of regulatory actions.

Finally, recognizing that the welfare implication of this paper can only be a first step towards a more complete assessment of the pros and cons of discriminatory bailout, we want to stress that an important aspect of this paper is the provision of a simple analytical framework and a clarification of the major conceptual issues involved. Given the possibility of a macroeconomic shock and discriminatory bailout, deposits are risky. If an individual bank raises deposit rates, it will affect its own bailout probability as well as that of all other banks since the refinancing needs rise accordingly. Therefore, the expected returns for depositors of all banks are influenced by the decision of an individual bank. Moreover, the distribution of deposits among banks will affect expected returns on deposits as well, since some banks have higher refinancing needs under asymmetric distributions than others. These banks might have to offer higher second-period deposit rates than under symmetric distributions, forcing the other banks to offer higher rates as well in order to obtain any savings at all. As expected deposit returns at all banks are affected by individual bank decisions and by depositors' savings decisions, it is not a priori clear whether consistent assessments of depositors' expected returns actually exist. We establish a general existence result for consistent return assessments and also identify the constellations in which such assessments may not exist.

2 Review of the Literature

The role governments should play in managing illiquid banks remains one of the main unresolved issues in banking regulation (see BHATTACHARYA, BOOT, AND THAKOR (1998)). The existing theoretical literature primarily draws on a partial equilibrium point-of-view where systemic consequences are accounted for only by *exogenous* factors. It has been stressed that closure policies have to weigh the costs of bailout (subsidies to uninsured debtholders) with the closure costs (direct bankruptcy costs, externalities). Excessive risk-taking incentives can occur as both costs of bailout and costs of closure. On the one hand, bailout creates moral hazard, as the probability of surviving depends less on the bank's risk choice and more on the regulator's actions. On the other hand, it increases the bank's probability of survival, thus raising the value at stake and, in turn, the bank's incentive to protect it.[4]

Depending on how the different costs are weighed, authors come to different conclusions about the desirability of governmental intervention. While for example HUMPHREY (1986) and SCHWARTZ (1995) advocate a non-interventionist view, the opposite view, namely that in some cases bailing out banks is socially desirable, has been put forward by MISHKIN (1995), SANTOMERO AND HOFFMAN (1998), FREIXAS, PARIGI, AND ROCHET (1998) or CORDELLA AND YEYATI (1999).[5] Our paper gives a new slant to this debate. In our model, closing some banks is necessary so that others can survive without further government intervention. In this sense, putting the funds concentration effect to work is both interventionistic and non-interventionistic.

A further question raised in the literature is how the decision to close a bank should depend on important bank-specific or macroeconomic variables such as the level of uninsured debt on a bank's balance sheet (FREIXAS 1999), the size of a bank (GOODHART AND HUANG 1999) or aggregate investment returns (CORDELLA AND YEYATI 1999). FREIXAS (1999)

[4]See CORDELLA AND YEYATI (1999) for a formalization of the tradeoffs resulting from these two mutually offsetting effects.

[5]For a comprehensive discussion of this issue see GOODHART (1995).

finds that under optimal policies, banks will be closed either if they have a too low *or* a too high level of uninsured debt on their balance sheet. Whether the former or the latter of these policies should be applied depends on the respective dominance of two counteracting effects: the costs of the subsidies to uninsured debt holders on the one hand and the monitoring incentives for debtholders (which are increasing in the level of uninsured debt) on the other hand. CORDELLA AND YEYATI (1999), investigating how closure policies can minimize the risk-taking incentives of banks, find that banks should be bailed out if aggregate investment returns fall below a certain threshold level. The intuition behind their conclusion is that if aggregate returns are high and a bank fails nevertheless, this will signal excessive risk taking, which is discouraged by threatening closure. Bailing out banks in low states of the variable will increase a bank's charter value and therefore decrease risk-taking incentives.

While the conditionality introduced in CORDELLA AND YEYATI (1999) would have no sensible application in the crises scenarios we are mainly interested in,[6] distinguishing between the relative levels of insured deposits and uninsured debt on a bank's balance sheet would be a further useful step for the analysis of bank closure policies in general-equilibrium frameworks.

Finally, GOODHART AND HUANG (1999) provide a framework that justifies a "bail out the big ones" policy as long as risk-taking incentives are not taken into account. If these incentives are important, the optimal rescuing policy may depend on the size of the bank in a non-monotonic way. While GOODHART AND HUANG (1999) derive their results by comparing the costs of bank failure (contagion) and of bailout (rescuing insolvent banks with the taxpayers' money), we stress the following advantages of BB. First, it helps to avoid low-return equilibria. Second, it is more credible ex-post than RB and - in contrary to BS - guarantees the existence of consistent deposit-return assessments. However, BB is subject to self-fulfilling prophecies and hence return assessments might not be unique. Moreover, it might provide risk-taking incentives for big banks.

Besides the analysis of optimal bank closure policies, an important strand of the literature has investigated the regulator's incentives to apply such rules. BOOT AND THAKOR (1993) examined the regulator's incentives to close banks in a manner that results in socially optimal bank portfolio choices. They find that the regulator's optimal bank closure policy is less tight than is socially optimal. The analysis has been extended by ACHARYA AND DREYFUS (1989), FRIES, MELLA-BARRAL, AND PERRAUDIN (1997) and MAILATH AND MESTER (1994). Finally, REPULLO (1999) considers government agencies

[6]While the aggregate-investment indicator would surely indicate that all banks should be rescued in such scenarios, it would still be too costly to do so.

with different objective functions and investigates which of these agencies should make bailout decisions. He finds that central banks should be responsible for dealing with small liquidity shocks, while the deposit insurance agency should deal with large ones. While we do not address institutional design issues - as considered in REPULLO (1999) -, incentives for regulators are briefly discussed during the analysis of the bailout schemes' credibility.

On a conceptual level, this paper is related to the literature in the following respect. Discriminatory bailout can be interpreted as a version of the "*constructive ambiguity*" principle, where regulators have full discretion to let one bank go bankrupt. Two concepts of constructive ambiguity have been discussed in the literature. In FREIXAS (1999) the central bank deciding which banks are to be rescued follows a mixed strategy. In GOODFRIEND AND LACKER (1999) and REPULLO (1999), the bailout policy is not random from the perspective of the central bank but is perceived as such by outsiders that cannot observe the supervisory information that leads to the bailout decision. Our closure policy RB introduces a constructive ambiguity concept similar to FREIXAS (1999) since the regulator will choose to bail out a bank with a certain probability. The BB and the BS concept are subtle mixtures of predetermined bailout (if banks differ in size) and constructive ambiguity (if banks are equal in size). In contrast to FREIXAS (1999), who considers a regulator that follows a mixed strategy when deciding about a *single* bank's bailout, we investigate the whole banking system and motivate constructive ambiguity with aggregate solvency concerns. Therefore, bailout probabilities have to be chosen in a way ensuring that under *all* realizations of the stochastic decision process, the banks that have not been closed will be able to survive. This makes the design of such a policy more demanding.

3 The Model

The model encompasses two overlapping generations; the first generation lives from $t = 0$ to $t = 1$ and the second from $t = 1$ to $t = 2$. Each generation consists of a continuum of households. There is one single physical good in the economy, which can be used for production and consumption. Moreover, there is a number of banks owned and managed by bankers. Banks gather the households' savings and invest them in a production technology.[7] The key features of the model are the following.

[7] For simplicity of representation we do not model bank loans to entrepreneurs.

1. Returns on the production technology are subject to macroeconomic risk.
2. Banks offer *uncontingent* deposit contracts to households, thereby exposing themselves to macroeconomic risk.

We first have to justify why some of the macroeconomic risk remains on the balance sheets of the banks. According to HELLWIG (1998), a bank could in principle reduce its exposure to macroeconomic risk traceable to easily observable indicators such as GDP or interest rates (either by offering state contingent deposit contracts or by transferring risk to third parties via hedging contracts). However, banks bear substantial macroeconomic risk in reality. HELLWIG (1998) offers a detailed account of why this is the case. First, available indicators are only an incomplete measure of exposure to aggregate risk. Second, in practice banks do not conclude contingent deposit contracts for the following reasons: the inflexibility of indexed deposit rates as a risk management tool, the existence of transaction costs, and the market-making role of banks. Moreover, the on-demand clause of deposit contracts may invite runs on banks if repayments are made contingent on the realization of macroeconomic variables such as GDP at a certain point in time. Third, hedging counterparties are often banks themselves and hence our analysis can be applied to the counterparty banks. Moreover, banks that shift their risk to third parties are still exposed to credit risk; this risk is likely to be correlated with the macroeconomic risk they want to insure themselves against.[8]

In order to keep the analysis as simple as possible, we do not focus on the moral hazard of banks or risk aversion of households as further possible explanations for aggregate risk exposure of banks. However, our analysis can be applied to the excessive risk-taking problem, which has been identified as one of the major problems of prudential banking regulation (see e.g. DEWATRIPONT AND TIROLE (1994)). If all banks in the industry undertake portfolio choices with a common macroeconomic risk component that cannot be diversified, regulatory intervention can follow a logic similar to the one outlined in this paper. The additional question emerging in this context is how regulatory bailout schemes affect the banks' risk choices. We will briefly discuss this issue as an extension to our analysis.

[8] GERSBACH (1998) describes two additional scenarios in which banks do not offer contingent deposit contracts. In the first scenario, the regulator can commit to the failure of insolvent banks. Macroeconomic shocks are then borne by risk-neutral entrepreneurs, as long as their inside funds are a sufficient buffer for these shocks. In the second scenario, banking crises are worked out. Banks offer uncontingent deposit rates that can only be paid back when the state of returns is good. Downturn macroeconomic risk is shifted to future generations.

Finally, our model allows an alternative interpretation for the banks' exposure to macroeconomic risk. It draws on the uncertainty about the accuracy of the banks' risk management systems rather than on uncontingent deposit contracts. Suppose that banks write contingent contracts that - according to their risk management tools - isolate them from macroeconomic risk. If banks use similar risk management tools, the aggregate uncertainty about future returns can be interpreted as aggregate uncertainty with respect to the accuracy of the contingencies in the deposit contracts: risk management tools may overestimate production returns in one macroeconomic scenario while they underestimate them in an other. This leaves the banking system exposed to systematic risk.[9]

3.1 Technology

We assume that there is a long-term technology that pays a random return of R_2 units of the good in $t = 2$ for each unit invested in $t = 0$. If liquidated in $t = 1$, returns are zero.[10] Production returns in $t = 2$ are subject to aggregate risk. Two different realizations of R_2 are possible. In the first state, occurring with probability p_l, we have low returns: $R_2 = r_{2l}$. In the second state, with probability $p_h := (1 - p_l)$, we have high returns $R_2 = r_{2h}$. The realization of the aggregate productivity shock is revealed in $t = 1$ and will be observed by all market participants. We assume (a) constant returns to scale and (b) that investment at arbitrary scale is possible.

3.2 Banks

The need for financial intermediation can arise for several reasons (see BHATTACHARYA AND THAKOR (1993) for a comprehensive overview). We take this need for granted and do not model it explicitly here. A special feature of our model is that banks finance long-term investments with short-term saving contracts. In contrast to the standard DIAMOND AND DYBVIG (1983) framework, there is no risk of consumption timing for the first generation in our model. The individuals of the first generation know that they will never see the fruits of their long-term investments. However, there is an aggregate production risk that makes consumption uncertain in the second period. The economic problem lies in

[9] This view is for example substantiated by SHIN (1999), who suggests that the risk management tools of financial institutions tend to heavily underestimate risk during episodes of market turbulence since they do not take into account the endogeny of future market outcomes (i. e. the fact that outcomes depend on their own actions and that of other market participants).

[10] In an extended version of the paper (ERLENMAIER AND GERSBACH 2001) we relax this assumption and show that our analysis also applies for positive liquidation values .

enabling both generations to participate in the benefits of a risky long-term investment though only the second generation will see the returns of the investment.

In $t = 0$ there are n banks, denoted by $B_1, ..., B_n$. They are long-living institutions enabling both generations to participate in long-term investments. Banks offer deposit contracts at deposit rates d_1^i to the first generation and receive an amount D_1^i of deposits $(i = 1, .., n)$; all deposits are invested in the production technology. In $t = 1$ banks have to pay back their debt $d_1^i D_1^i$ to first-generation depositors. To obtain new funds, they offer deposit contracts to the second generation at deposit rates d_2^i. After banks have received their second-period deposits, two cases can occur for each individual bank. First, it has raised enough funds from second-generation depositors to pay back its debt; in this case it receives investment returns in $t = 2$ and pays back its second-period depositors. If returns are not sufficient to service all depositors in $t = 2$, investment proceeds are uniformly distributed among depositors. Second, the bank cannot raise enough funds; in this case it has to declare bankruptcy, and the investments are liquidated. First-period depositors of such banks receive only the bank's cash, i.e. the savings of second-period depositors if there are any. Second-period depositors receive nothing.

We complete the description of the banking sector by assuming (a) that banks are owned by risk-neutral bankers[11] who live for three periods and consume in $t = 2$, and (b) that bank managers maximize expected bank profits and hence internalize losses that accrue to depositors in case their claims cannot be fully served.

3.3 Households

There are two overlapping generations of consumers (first and second generation), each consisting of a continuum of households living for two periods. They are risk-neutral but want to smooth consumption over time.[12] We denote the individual saving function that describes how much funds household h in generation g $(g = 1, 2)$ is willing to deposit with banks by s_{gh}. $s_{gh}(\cdot)$ is assumed to be an increasing function of the *expected* return paid on bank deposits, which we denote by u.

Note that since in both periods some banks might not be able to fully pay back their debts to depositors, both generations of households have to assess the expected returns

[11]Note that for the sake of tractability we have excluded the possibility of issuing equity. We could allow for equity as long as bank reserves cannot buffer losses completely in the event of negative macroeconomic shocks.

[12]The assumption of risk-neutrality is made for convenience and tractability as in Bernanke and Gertler (1988) and Kijotaki and Moore (1997).

paid by each bank given first-period deposit rates (for the first generation) and given the first-period allocation and second-period deposit rates (for the second generation). We denote the resulting aggregate saving function for generation g as $S_g(\cdot)$ and assume that S_g is continuous and strictly increasing in u. $S_g(u)$ can be represented as an integral of the saving density function $s_{gh}(u)$ over an interval on the real line (without loss of generality $[0,1]$), each point on the interval representing one household: $S_g(u) = \int_0^1 s_{gh}(u)\, dh$. We will refer to this representation when using the expressions "*full measure of savings*" and "*zero measure of savings*" later on. A certain bank has obtained the full measure of savings if it has not attracted all depositors but if the integral of the saving density function over all the banks' depositors is equal to the integral over all households ("full-measure bank"). If a bank has attracted some depositors but the integral of the saving density function over the banks' depositors is zero, then we say that the bank has obtained a zero measure of savings ("zero-measure bank"). In the sequel we will use functions of the type $S(u) = au^\alpha$ with $a, \alpha \in (0, \infty)$ as an example for the saving functions of both generations.

Finally, note that the saving functions S_g for deposits can be interpreted as a result of a portfolio decision. Deposits may only be one of several saving possibilities[13] that are imperfect substitutes. In this case, the expected-return elasticity of deposits can be quite high.

3.4 Example

Throughout the paper we will use the example presented in table 1 to illustrate our results. Note that $\bar{R}_2$ denotes the expected investment return $p_l r_{2l} + p_h r_{2h}$.

$S_1(u) = u$	$p_l = 0.2$	$r_{2l} = 1.03$	$\bar{R}_2 = 1.18$
$S_2(u) = 1.07 \cdot u$	$p_h = 0.8$	$r_{2h} = 1.22$	

Table 1: Example A.

3.5 Regulatory Policy

We will derive the necessity of regulation precisely in sections 4.1 and 5.1. For the time being, note that it will result from the following reasoning. In the case of low production

[13] The others are not modeled explicitly but enter the model via the specification of the saving functions.

returns it might not be possible for all banks to refinance in $t = 1$ since they cannot credibly offer sufficiently high second-period deposit rates. Nevertheless, it might be possible for a fraction of the banks to refinance if depositors concentrated their savings on these banks. Without regulation though, depositors have no possibility of coordinating their savings on such a fraction of banks; equilibria in which no bank is able to refinance can therefore not be excluded. We will consider two types of regulatory scenarios designed to avoid these problems. The first one (prudential banking) ensures that the whole banking system is able to refinance in both states of production returns by encouraging banks to offer low deposit rates in the first period. The second one (discriminatory bailout) allows for situations where the banking system is not able to refinance itself. The regulator solves the coordination problem of depositors by closing a fraction of banks in order to make sure that the others can survive. Closing some banks will have two effects: first, it will reduce the amount of second-period deposits needed by the banking system; second, by taking over investment projects of closed banks, surviving banks can offer higher returns on deposits. In this section we describe the different regulatory approaches formally.

3.5.1 Bailout Schemes

Suppose that there are $m \leq n$ banks in $t = 1$ that have received deposits. The regulator observes the realization r_2 of the macroeconomic shock, i.e. the future prospects of aggregate production returns. The banking system is able to refinance if and only if

$$S_2(r_2/d_1^{\max}) \geq \sum_{i=1}^{n} d_1^i D_1^i. \tag{1}$$

$d_1^{\max}$ is the highest deposit rate that has been offered by a bank in the first period. Note that $r_2/d_1^{\max}$ is the highest return that *all* banks can credibly offer to the second generation in $t = 1$ (because $d_2^i d_1^i$ cannot exceed r_2). If refinancing condition (1) holds, then all banks can survive (for example, if a uniform deposit rate of $r_2/d_1^{\max}$ is offered to second-period depositors) and the regulator will not intervene. Consequently all banks will be allowed to compete for second-generation deposits in this case. In the following we will use the matrix $\mathbf{\Delta} := (\Delta_i)_{i=1}^n$ with $\Delta_i := (\Delta_{iD}, \Delta_{iI})$ to summarize deposits and investments of the banks after the regulatory decision. Δ_{iD} denotes the obligations to first-period depositors and Δ_{iI} denotes the units of investment projects that a bank holds. Hence, if (1) holds, deposits and investments are given by $\Delta_i = (d_1^i D_1^i, D_1^i)$ for $i = 1, .., n$ since the regulator has not stepped in.

If condition (1) does not hold, then not all banks will be able to refinance themselves because new funds at the largest credible uniform deposit rate are less than the aggregate obligations of the banking system.[14] In this case the regulator will close a certain number $(m-k)$ of banks and additionally eliminate a fraction $(1-b)$ of the surviving banks' deposits. Depositors whose deposits have been eliminated will loose their claims on the bank. The bailout schemes differ with respect to the manner in which the subset of surviving banks, which we denote by $\mathcal{B}^+$, is determined. Under **random bailout (RB)**, $\mathcal{B}^+$ is chosen by *randomly* drawing k banks (out of the m banks which have received any deposits). Under **prudential banking (PB)**, the regulator also applies RB but additionally imposes a penalty P on all banks that had to be closed. P is assumed to be so high that a bank strategy with a positive probability of leading to P will always be eschewed in favor of any strategy that does not involve the possibility of insolvency, including exiting from the market.[15] While the surviving banks are chosen randomly under RB and PB, banks are ordered with respect to the amount of first-period deposits they have gathered under **bail out the big ones (BB)**:

$$D_1^{\tau(1)} > \cdots > D_1^{\tau(\underline{k})} = \cdots = D_1^{\tau(k)} = \cdots = D_1^{\tau(\bar{k})} > \cdots > D_1^{\tau(m)}.$$

The set $\mathcal{B}^+$ will contain the banks $B_{\tau(1)}, ..., B_{\tau(\underline{k}-1)}$ and another $k-(\underline{k}-1)$ banks which are chosen randomly from the set $\{B_{\tau(\underline{k})}, ..., B_{\tau(\bar{k})}\}$. [16]

The investment projects of closed banks are distributed among surviving banks in proportion to the amount of deposits they have gathered. Hence, after regulatory intervention, the balance sheet of a surviving bank i consists of obligations $bd_1^i D_1^i$ to first-period depositors and of $b_I D_1^i$ units of investment projects where[17]

$$b_I = b_I(\mathcal{B}^+) := \frac{\sum_{i=1}^n D_1^i}{\sum_{i\in\mathcal{B}^+} D_1^i}. \tag{2}$$

[14]If higher deposit rates would be credibly offered by some banks, then at least the bank that has offered $d_1^{\max}$ in $t=0$ would not be able to receive any deposits.

[15]Note that by offering very unfavorable deposit rates a bank can always ensure that it will never become insolvent, regardless of the behavior of the other banks.

[16]In previous versions (Erlenmaier and Gersbach 2001) we have also considered the reversed pecking order, i.e. bail out the small ones (BS). It is shown that BS raises severe stability problems. In this version we therefore do not further consider this bailout scheme.

[17]Note that we use $\mathcal{B}^+$ not only to denote the set of surviving banks but also to denote the set of indices $i_1, ..., i_k$ that identify the surviving banks. Note also that if all banks in $\mathcal{B}^+$ have only a zero measure of deposits on their balance sheet, then the denominator of equation (2) is zero. In this case investment projects are uniformly distributed among all banks in $\mathcal{B}^+$.

Hence

$$\Delta_i := \begin{cases} (bd_1^i D_1^i \,,\, b_I D_1^i) & \text{if i} \in \mathcal{B}^+ \\ (0 \,,\, 0) & \text{else.} \end{cases}$$

Recall now that regulatory policy must ensure that all remaining banks are able to pay back their first-period depositors with the savings of the second generation. Note that if a bank i receives exactly the amount of second-period deposits that it needs to service its obligations (i.e. $D_2^i = bd_1^i D_1^i$), then it will be able to credibly offer a deposit rate $r_2 b_I/(bd_1^i)$ to its second-period depositors; hence the rate $r_2 b_I/(bd_1^{\max})$ can be offered by *all* surviving banks, and the total amount of second-period savings that can be attracted is at least $S_2\Big(r_2 b_I/(bd_1^{\max})\Big)$. Since b/b_I is equal to the fraction of overall deposits which have not been eliminated (we denote this fraction by q), we conclude that all remaining banks will be able to refinance if

$$S_2\Big(\frac{r_2}{qd_1^{\max}}\Big) \geq qd_1^{\max} \sum_{i=1}^{n} D_1^i.$$

The highest possible fraction $\bar{q}$ of first-period deposits that can be bailed out under the constraint that the surviving banks shall be able to refinance is therefore given as solution of the equation[18]

$$S_2\Big(\frac{r_2}{qd_1^{\max}}\Big) = qd_1^{\max} \sum_{i=1}^{n} D_1^i. \tag{3}$$

Note that under BB we have

$$q = \frac{b \sum_{i=1}^{k} D_1^{\tau(i)}}{\sum_{i=1}^{n} D_1^i}. \tag{4}$$

Hence, k and b can be chosen to ensure that $q = \bar{q}$. Obviously there is more than one combination of k and b that leads to $q = \bar{q}$. It is therefore important to note that allowing the regulator to additionally eliminate a fraction $(1-b)$ of all the surviving banks' balance sheets only serves technical purposes.[19] In principle we do not allow for the balance sheets of all banks to be scaled down arbitrarily without disruptive consequences for the banks when continuing their operations and thus for the economy. If such an arbitrary scale-

[18]Note that the left-hand side of the equation is decreasing while the right-hand side is strictly increasing in q which, together with the fact that inequality (1) does not hold, implies that there is a unique solution $\bar{q} \in [0, 1]$ of equation (3).

[19]This assumption allows us to avoid discontinuities (see page 25).

down were possible, an alternative implementation of the funds concentration effect would be to scale down the balance sheets of all banks without closing any of them completely. But under severe macroeconomic shocks (in which our major interest lies), the scale-down needed would most likely disrupt the banks' operations and thus shrinking all banks simultaneously is no viable alternative. Therefore, under all bailout schemes we will try to choose b as high as possible. Hence, using equation (4) we determine k and b under BB by

$$k = \min\Big\{ l \in \mathbb{N} \mid \sum_{i=1}^{l} D_1^{\tau(i)} \geq \bar{q} \sum_{i=1}^{n} D_1^i \Big\}, \tag{5}$$

$$b = \Big(\bar{q} \sum_{i=1}^{n} D_1^i\Big) / \Big(\sum_{i=1}^{k} D_1^{\tau(i)}\Big). \tag{6}$$

Contrary to BB, the fraction q of bailed out depositors under RB is in general *not* determined by the choice of k and b, since it is not clear which banks will be chosen to survive. To determine k under RB, we must therefore take into account that the fraction of remaining deposits should not exceed $\bar{q}$, regardless of which banks have been chosen. The worst case that can be thought of in terms of remaining deposits is that - as under BB - the k largest banks have been chosen to survive. Hence, to ensure that the fraction of bailed out deposits is equal to $\bar{q}$ in this case, k and b are determined as under BB, i.e. according to equations (5) and (6).

3.5.2 Bailout Schemes: The Symmetric Case

In this section we illustrate the working of the bailout schemes for an arbitrary symmetric first-period allocation (d_1, D_1) where all banks have offered the same deposit rate d_1 and received the same amount of first-period deposits D_1. In this case all bailout schemes will proceed in the same way. First, $\bar{q}$ is determined as the solution of a simplified version of equation (3):

$$qnd_1D_1 = S_2\Big(\frac{r_2}{qd_1}\Big). \tag{7}$$

Figure 1 illustrates the solution of equation (7) for example A, which will be used for all illustrations unless otherwise indicated. Second, to achieve a fraction $\bar{q}$ of bailed out deposits with b as a high as possible, we choose $k = \lceil nq \rceil$ and $b = (nq)/\lceil nq \rceil$.[20] The k

[20]Note that $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

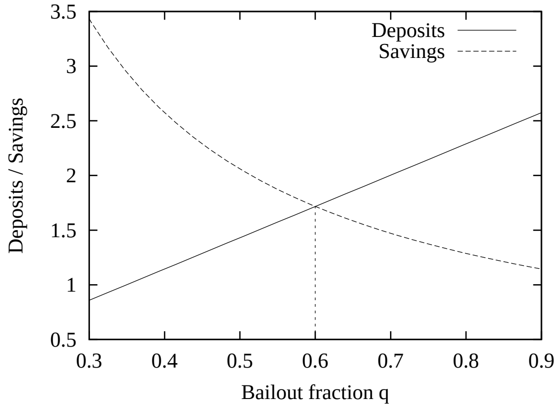

Figure 1: Bailed out first-period Deposits qnd_1D_1 and second-period savings $S_2(r_2/qd_1)$ as functions of q ($nD_1 = 2.7$ and $d_1 = 1.05$, example A).

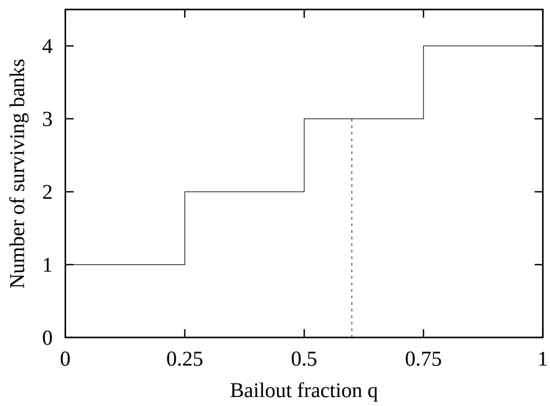

Figure 2: Number k of surviving banks as function of q.

banks that will survive are chosen randomly under all schemes since all banks have raised the same amount of first-period deposits; hence the bailout probability of each single deposit is equal to the fraction $\bar{q}$ of bailed out deposits.

As an example consider the case $n = 4$. Figure 2 depicts the function $q \to \lceil nq \rceil$. If e.g. $\bar{q} = 0.6$, then $\lceil n\bar{q} \rceil = 3$ and one bank will be closed. Moreover, $b = 1.8/3 = 0.6$ and 40% of each surviving bank's deposits are eliminated.

3.5.3 Bailout Schemes: The Asymmetric Case

If the first-period deposit distribution is asymmetric (i.e. if not all banks have received the same amount of deposits), the schemes RB and BB will generally produce different regulatory decisions; under BB, always a fraction $\bar{q}$ of depositors is bailed out and the bailout probabilities for deposits depend on the size of the bank at which the deposits are held. Under RB, in contrary, the bailout fraction can be lower than $\bar{q}$ and the bailout probability of each deposit is given by $(k - 1 + b)/m$.

We start, however, with an important case of asymmetric deposit distribution where RB and BB produce the same regulatory decision: the case where one bank has obtained all deposits and the other banks none. In this case we have $k = 1$ and $b = \bar{q}$ implying that the bailout probability of each deposit is equal to the fraction $\bar{q}$ of bailed out deposits.

As an example illustrating the differences between BB and RB, consider a deposit distribution as depicted in figure 3 and suppose that as above $\bar{q} = 0.6$. To determine k, note that the sum of all first-period deposits is 10. Since $5/10 < 0.6$ and $(5+2)/10 > 0.6$, we have $k = 2$ and $b = 10 \cdot 0.6/(5+2) \approx 0.85$. Under BB, the regulator will therefore close bank 4 and either bank 2 or bank 3; the choice between those two banks is performed randomly with each bank having a probability of 0.5 to survive. After that a fraction $(1 - b) = 0.15$ of the surviving banks' deposits is eliminated. Hence, the bailout probability for deposits at bank 4 is zero, it is $0.5 \cdot 0.85 = 0.425$ for deposits with banks 2 and 3, and it is given by 0.85 for deposits at banks 1. Under RB, in contrast, the bailout probability is equal to $(2 - 1 + 0.85)/4 = 0.462$ for all deposits. Moreover, the fraction of bailed out deposits is 0.6 if bank 1 and bank 2 have been chosen to survive while it drops to $[0.85 \cdot (2+1)]/10 = 0.255$ if banks 3 and 4 have been chosen.

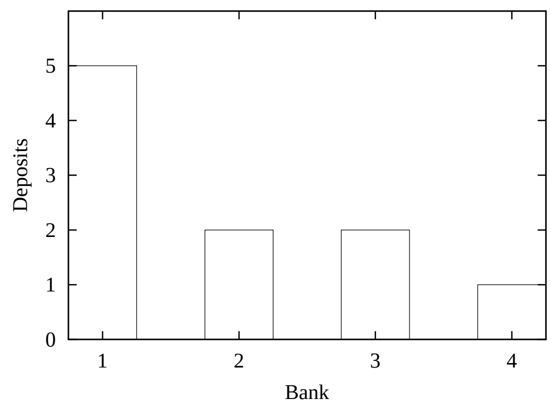

Figure 3: Example for a first-period deposit distribution.

Finally, consider the case where one bank has obtained the full measure of savings and the other $(n-1)$ have obtained zero measures of savings. As for the case where only one bank has received any deposits, we obtain $k = 1$ and $b = \bar{q}$. But while under BB the $(n-1)$ small banks are closed and the bailout probability for the depositors of the big bank is $\bar{q}$, the bailout probability under RB drops to $\bar{q}/n$. Moreover, with probability $(n-1)/n$, the full measure of deposits is eliminated.

3.5.4 Bailout the Big Ones: The Case of Zero-Measure Banks

Concluding the description of the bailout schemes, we note that BB will slightly differ from the procedure described above in case that some banks have only gathered a zero measure of deposits in $t = 0$. Note that under BB such banks will always be closed if the refinancing condition (1) is not fulfilled. Hence, when determining $\bar{q}$ in such a case, the first-period deposit rates offered by zero-measure banks do not have to be taken into account. In such a situation, the regulator will therefore define $d_1^{\max}$ as the maximum first-period deposit rate offered by positive-measure banks and will close all zero-measure banks that have offered deposit rates higher than $d_1^{\max}$. The BB bailout scheme described in the previous sections will then be applied to the positive-measure banks and to the remaining zero-measure banks.

3.6 Summary: Sequence of Events

We now summarize the sequence of events.

1. **Banks offer first-period deposit rates**.
   In $t = 0$ banks simultaneously offer their first period-deposit rates d_1^i $(i = 1, .., n)$. $\mathbf{d_1} = (d_1^i)_{i=1}^n$ denotes the vector of all first-period deposit rates.

2. **Households (first generation) assess expected returns and make their saving decisions.**
   First-generation households make assessments $\mathbf{u_1} = (u_1^i)_{i=1}^n$ about the expected returns that will be paid on deposits by each bank. Based on these assessments, they decide on the amount of savings they want to deposit with each bank. We denote the vector of all first-period deposits by $\mathbf{D_1} = (D_1^i)_{i=1}^n$. Finally, banks invest the deposits obtained in the production technology.

3. **Regulatory policy.**
   The regulator observes the realization of the productivity variable r_2. If the refinancing condition (1) is fulfilled, the regulator will not intervene. In this case the deposits and investments of bank i are given by $\Delta_i = (d_1^i D_1^i, D_1^i)$. If condition (1) is not fulfilled, then one of the bailout schemes will be applied and some banks will be closed. The set of surviving banks is denoted by $\mathcal{B}^+$. Investment projects of closed banks are distributed among surviving banks in proportion to the amount of first-period deposits they have gathered. Deposits and investments of bank i after regulatory policy are given by $\Delta_i = (0, 0)$ if it has been closed and by $\Delta_i = (b d_1^i D_1^i, b_I D_1^i)$ if it has survived.

4. **Surviving banks offer second-period deposit rates**.
   Surviving banks simultaneously offer their second-period deposit rates $d_2^i(\mathbf{\Delta})$ $(i \in \mathcal{B}^+)$. The vector of all second-period deposit rates is denoted by $\mathbf{d_2} = (d_2^i)_{i \in \mathcal{B}^+}$.

5. **Households (second generation) assess expected returns and make their saving decisions.**
   Second-generation households make assessments $\mathbf{u_2} = (u_2^i)_{i \in \mathcal{B}^+}$ about the expected returns that will be paid on deposits by each bank. Based on this assessment, they decide on the amount of savings they want to deposit with each bank. The vector of all second-period deposits is denoted by $\mathbf{D_2} = (D_2^i)_{i \in \mathcal{B}^+}$.

6. **Surviving banks pay their second-period depositors back.**
   In $t = 2$ surviving banks receive returns from investments and pay their second-period depositors back. Profits are consumed by managers.

We call steps 4 -6 the second-period subgame of the intermediation game.

3.7 Equilibrium Concept

In order to derive the subgame-perfect equilibrium of the game described in section 3.6, some subtle points have to be taken into account. In particular we need to discuss how the households' return assessments can be derived. Two issues are important in this respect.

First, given an assessment $\mathbf{u_g}$ by households in generation g ($g = 1, 2$) about expected deposit returns, the deposit distribution $\mathbf{D_g} = \mathbf{D_g}(\mathbf{u_g})$ is derived from the households' utility maximization. We use $\mathcal{B}_g^{\max}$ to denote the subset of all banks that are assessed to pay the maximum expected return $u_g^{\max}$ among all banks for generation g. The banks in $\mathcal{B}_g^{\max}$ will receive all the savings of the households: $\sum_{i \in \mathcal{B}_g^{\max}} D_g^i(\mathbf{u_g}) = S(u_g^{\max})$ and $D_g^i(\mathbf{u_g}) = 0$ for all $i \notin \mathcal{B}_g^{\max}$.

Since depositors are indifferent with regard to all banks in $\mathcal{B}_g^{\max}$, it is unclear how deposits are distributed among these banks. We will assume that if two banks are in $\mathcal{B}_g^{\max}$, they will receive the same amount of deposits if all of their characteristics are identical.[21] This means that indifferent depositors will randomize among their preferred banks with equal probability and independently of each other.

Second, the households' assessments have to be consistent. In order to give a precise definition of consistency, we use $\mathbf{U_1}(\mathbf{d_1}, \mathbf{D_1})$ to denote the vector of expected returns on first-period deposits resulting from the allocation $(\mathbf{d_1}, \mathbf{D_1})$ and from regulatory policy. Furthermore, given the matrix $\mathbf{\Delta}$ of deposits and investments after regulatory policy and given second-period deposit rates $\mathbf{d_2}$ and deposit distribution $\mathbf{D_2}$, we can define the resulting vector of expected second-period returns as $\mathbf{U_2}\Big(\mathbf{\Delta}, \mathbf{d_2}, \mathbf{D_2}\Big)$. We will show in section 4.1 that the functions $\mathbf{U_1}(\cdot)$ and $\mathbf{U_2}(\cdot)$ are well defined for all entries i with $i \in \mathcal{B}_g^{\max}$, i.e. for banks that are assessed to pay the maximum expected returns on deposits. However, if $i \notin \mathcal{B}_g^{\max}$ then bank i will receive no deposits and exit the market; it is therefore unclear whether the assessment was correct in the first place. To deal with this problem, we introduce a so called "*zero-measure test*". We calculate the expected

[21] In $t = 0$, banks are identical if they have offered the same first-period deposit rates and in $t = 1$ they are identical if their balance sheets are identical and if they have offered the same second-period deposit rate.

returns for each bank resulting from a deposit distribution $\hat{\mathbf{D}}_{\mathbf{g}}(\mathbf{u}_{\mathbf{g}})$. $\hat{\mathbf{D}}_{\mathbf{g}}(\mathbf{u}_{\mathbf{g}})$ differs from $\mathbf{D}_{\mathbf{g}}(\mathbf{u}_{\mathbf{g}})$ only in one respect, namely that banks $i \notin \mathcal{B}_g^{\max}$ receive a zero measure of savings instead of no savings at all:

$$\hat{D}_g^i(\mathbf{u}_{\mathbf{g}}) := \begin{cases} D_g^i(\mathbf{u}_{\mathbf{g}}) & \text{if i} \in \mathcal{B}_g^{\max} \\ \text{zero measure} & \text{else.} \end{cases}$$

Definition 1 (Consistent assessments)
Given first-period deposit rates $\mathbf{d_1}$, an assessment $\mathbf{u_1}$ is consistent if and only if

$$\mathbf{U_1}\Big(\mathbf{d_1}, \hat{\mathbf{D}}_{\mathbf{1}}(\mathbf{u_1})\Big) = \mathbf{u_1}.$$

Given the matrix $\mathbf{\Delta}$ of post-regulation deposits and investments, and given second-period deposit rates $\mathbf{d_2}$, an assessments $\mathbf{u_2}$ is consistent if and only if

$$\mathbf{U_2}\Big(\mathbf{\Delta}, \mathbf{d_2}, \hat{\mathbf{D}}_{\mathbf{2}}(\mathbf{u_2})\Big) = \mathbf{u_2}.$$

Note that we will only consider different assessments for two banks if they are different with regard to at least one of their characteristics.

Consistent assessments mean that depositors make optimal saving decisions[22] and that expected returns are equal to returns generated when depositors distribute themselves among the preferred banks. Whether or not consistent assessments exist will be discussed at length in the next section. If more than one consistent assessment exists, we apply the Pareto selection criterion and assume that the assessment which generates the highest returns will be realized. We therefore define:

Definition 2 (Optimal assessments)
An assessment $\mathbf{u_g}$ $(g = 1, 2)$ is called optimal if it is consistent and if u_g^{max} is at least as high as the maximum expected return resulting from any other consistent assessment.

We will see that under regulation the best assessment and the corresponding deposit distribution are always unique. We conclude this section by summarizing our equilibrium concept. Note that, since banks are identical ex-ante, we constrain ourselves to the analysis of symmetric equilibria.

[22] I.e. savings decisions that lead to the highest expected returns, given the deposit rates offered by the banks.

Definition 3 (Equilibrium concept)
For any given regulatory policy, a symmetric subgame-perfect Bayesian equilibrium is a set consisting of first-period deposit rates $\mathbf{d_1} = (d_1, .., d_1)$, *assessments* $\mathbf{u_1} = (u_1, ..., u_1)$, *a deposit distribution* $\mathbf{D_1} = (D_1, ..., D_1)$, *reaction functions* $\mathbf{d_2} = \mathbf{d_2}(\mathbf{\Delta})$ *that assign a vector of second-period deposit rates* $\mathbf{d_2}$ *to each possible set* $\mathbf{\Delta}$ *of post-regulation deposits and investments, and a second-period deposit distribution* $\mathbf{D_2} = (D_2, .., D_2)$. *This set has to fulfill the following conditions:*

1. *Given* $\mathbf{\Delta}$, *second-period deposit rates* $\mathbf{d_2}(\mathbf{\Delta})$ *constitute an equilibrium in the subgame.*
2. *The second-period subgame equilibrium is symmetric, i.e. banks that are identical in* $t = 1$ *offer the same second-period deposit rate.*
3. *The strategies* $\left(\mathbf{d_1}, \mathbf{d_2}(\cdot)\right)$ *constitute a subgame-perfect Bayesian Nash equilibrium in the entire game.*
4. *Assessments are optimal.*

The equilibrium concept is a subgame-perfect Bayesian Nash equilibrium involving two subtleties. First, individual deposit decisions have no influence on return assessments since the contribution of each single depositor to overall deposits has zero measure. However, the *distribution* of deposits matters. Second, different deposit distributions for the same vector of deposit rates can imply different probabilities for bank defaults, which feeds back into the return assessments. Both subtleties raise considerable problems for the determination of return assessments. These problems will be addressed in the following section.

4 Equilibria in the Second Period and Consistent Assessments

In this section we first solve the second-period subgame and then analyze the existence of consistent assessments in the first period. All proofs in this and the next sections are deferred to the appendix.

4.1 Equilibria in the Second Period

Recall that a surviving bank i in $t = 1$ has $bd_1^i D_1^i$ first-period deposits and $b_I D_1^i$ units of investment projects. If the refinancing condition (1) holds or if no regulation is applied in $t = 1$, then $b = b_I = 1$ and all banks that have received any deposits in $t = 0$ compete for second-period deposits. If, on the other hand, condition (1) does not hold and regulation is applied, then $b \leq 1$, $b_I > 1$, and the regulator closes all banks outside of $\mathcal{B}^+$.[23] The surviving banks' profits in both cases are given

$$\Pi_2^i := \begin{cases} r_2 b_I D_1^i - (1-b) d_1^i D_1^i - d_2^i D_2^i & \text{if } D_2^i \geq b d_1^i D_1^i \\ -d_1^i D_1^i & \text{else.} \end{cases}$$

To analyze the second-period subgame equilibrium we define $\tilde{d}_1^{\max} := \max_{i \in \mathcal{B}^+}\{d_1^i\}$, $\bar{d}_2^* := S_2^{-1}\left(\sum_{i \in \mathcal{B}^+} b d_1^i D_1^i\right)$ and

$$\mathcal{E}_2^* := \left\{ d_2^i = \bar{d}_2^*, \;\; D_2^i = b d_1^i D_1^i \;\; (i \in \mathcal{B}^+) \right\}.$$

Note that $\bar{d}_2^*$ is the lowest deposit rate that generates enough second-period deposits for all surviving banks to refinance and that $\mathcal{E}_2^*$ is the (potential) second-period equilibrium where all surviving banks offer $\bar{d}_2^*$.

We start with the analysis of the no-regulation case. This case is only presented to derive the necessity of regulation. We will restrict ourselves in this case to first-period constellations where all banks have offered the same deposit rate d_1 and therefore have received the same amount D_1 of deposits. Note that this implies that $\tilde{d}_1^{\max} = d_1$ and $\bar{d}_2^* = S_2^{-1}(m d_1 D_1)$ where m is the number of banks that have received any deposits in $t = 0$.

Proposition 1 (No-regulation case)
Suppose that banks have offered the same deposit rate d_1 and therefore have received the same amount D_1 of deposits in $t = 0$. Then the following statements hold:

(i) If $r_2/d_1 \geq \bar{d}_2^*$, then $\mathcal{E}_2^*$ is an equilibrium. Moreover, from the point of view of the banks, $\mathcal{E}_2^*$ Pareto-dominates all other possible equilibria.

(ii) If $r_2/d_1 < \bar{d}_2^*$, then there is no equilibrium where all banks can refinance themselves. Moreover, in all symmetric second-period equilibria, where all banks offer the same deposit rate d_2, no bank can refinance itself and we have $D_2^i = 0$ for all $i \in \mathcal{B}^+$.

[23]Note that if no bank has been closed by the regulator, then $\mathcal{B}^+$ simply denotes the set of all banks that have received any deposits in $t = 0$.

Intuitively statement (i) stems from the following reasoning. First, deviations from $\mathcal{E}_2^*$ are not profitable since higher deposit rates increase repayment obligations; deviation to lower deposit rates either leads to the loss of all second-period deposits to the other banks or to the concentration of all savings on the deviating bank, which both takes the deviating bank's profits down to zero. Second, $\mathcal{E}_2^*$ Pareto-dominates all other equilibria, since equilibria with higher deposit rates lead to an increase in repayment obligations and because in equilibria with lower deposit rates no bank will receive any deposits. The mechanism leading to the latter observation is also responsible for the second part of statement (ii) and can be explained as follows.

Assume that the refinancing condition were fulfilled for $\tilde{m}$ banks ($\tilde{m} < m$), i.e. that

$$\frac{r_2}{d_1} \geq S_2^{-1}\Big(\tilde{m} d_1 D_1\Big).$$

If depositors could manage to deposit their savings only with a subset of $\tilde{m}$ banks, these banks would be able to refinance. But since all banks are identical, depositors cannot coordinate to deposit with a particular subset of banks; rather they would randomize independently between banks and, by the law of the large numbers, every bank would receive the same amount of savings, which is not enough to refinance. This in turn implies that none of the banks will receive *any* savings. Discriminatory bailout solves this coordination problem by closing some of the banks so that the remaining ones can raise enough new funds to refinance.

Of course there can be asymmetric constellations where one bank is able to refinance. Imagine the case where there are only two banks and $r_2/d_1 \geq S_2^{-1}(d_1 D_1)$. If one bank offers u_*, defined as the positive solution of $u = r_2 D_1 / S_2(u)$, and the other bank offers a lower deposit rate, the depositors' coordination problem is solved, since they know that the bank that has offered u_* can pay strictly higher returns. Without regulation, however, there is a severe coordination problem, because both of them would like to be the bank that is able to pay depositors back. Therefore, in our analysis of the no-regulation case in section 5.1 we will assume that no bank will receive any second-period savings if $r_2/d_1 < \bar{d}_2^*$. On the other hand, if $r_2/d_1 \geq \bar{d}_2^*$, we assume that $\mathcal{E}_2^*$ is played which - according to proposition 1 - can be justified by the Pareto selection criterion.

We now turn to the case where regulation (PB, RB or BB) ensures that $r_2/(q\tilde{d}_1^{\max}) \geq \bar{d}_2^*$. The regulation case will be analyzed in the general setting where banks may have offered different deposit rates in $t = 0$. We have already derived that under symmetric first-period allocations, banks will offer second-period deposit rates that are just sufficient to attract enough second-period savings to pay back their obligations to first-period depositors. Deviations to lower deposit rates can be excluded as the bank which has offered lower deposit rates will receive no second-period savings. Under asymmetric first-period constellations, however, deviations to lower deposit rates could be profitable for big banks, since the smaller non-deviating banks cannot cope with all second-period savings alone. This in turn could lead to non-existence of equilibria or to equilibria where not all banks can refinance (despite the fact that all banks would be able to refinance if offered deposit rates were high enough). To avoid these problems, we assume that the regulator imposes a **lower bound on second-period deposit rates** (**LBD**), i.e. she guarantees that no banks offers a deposit rate lower than $\bar{d}_2^*$.[24] This ensures that refinancing of all banks indeed occurs in equilibrium under asymmetric first-period constellations as the next proposition indicates.

Proposition 2 (Regulation case)
Suppose that regulation ensures that $r_2/(q\tilde{d}_1^{max}) \geq \bar{d}_2^*$ and that LBD is applied in $t = 1$. Then $\mathcal{E}_2^*$ is a second-period equilibrium. Moreover, from the point of view of the banks, $\mathcal{E}_2^*$ Pareto-dominates all other possible equilibria.

Regulation LBD ensures that banks do not undercut the rate $\bar{d}_2^*$. Moreover, banks that have offered higher deposit rates than $\bar{d}_2^*$ have higher repayment obligations than under $\mathcal{E}_2^*$. This implies that deviations from $\mathcal{E}_2^*$ are not profitable and that all other possible equilibria are Pareto-dominated by $\mathcal{E}_2^*$.

Throughout the paper we will assume that under regulation, banks will play $\mathcal{E}_2^*$ which can be justified by the Pareto criterion. In this case, second-period deposits just suffice to cover the refinancing needs of the banks and we can describe expected returns on first-period deposits of bank i as $u_1^i = (p_l q_l^i + p_h q_h^i) d_1^i$ $(i = 1, ..., n)$. q_l^i (q_h^i) denotes the bailout probability for bank i in the case of low (high) production returns.

4.2 Consistent Assessments in the First Period

In this section we analyze the existence and uniqueness of consistent assessments in the first period, assuming that one of the regulatory schemes is applied.[25] Consider a situation where there are two groups of banks, $\mathcal{B}_l$ and $\mathcal{B}_h$, that have offered first-period deposit rates d_{1l} and d_{1h} $(d_{1l} \leq d_{1h})$ respectively. Note that the assessments and deposits for all banks in $\mathcal{B}_l$ and for all banks in $\mathcal{B}_h$ must be identical. Note also that this scenario includes two

[24] Again we could think that a high enough penalty is imposed in case that banks do not follow the regulatory requirement.

[25] The no-regulation case will be summarized in section 5.1.

important cases that need to be considered in order to analyze symmetric equilibria of the intermediation game: first, the symmetric *non-deviation case* where all banks offer the same first-period deposit rate $d_{1l} = d_{1h} = d$ and second, the *deviation case* in which one bank deviates to a lower or a higher deposit rate. In the non-deviation case all banks are in one group (without loss of generality in $\mathcal{B}_h$) while in the deviation case either the deviating bank is in $\mathcal{B}_h$ while the the non-deviating banks are in $\mathcal{B}_l$ or vice versa.

We will now examine expected first-period returns on deposits in the non-deviation case and in the deviation case if one group of banks receives all deposits.[26] We denote the deposit rate in the non-deviation case and the deposit rate offered in the group of banks that has received all savings in the deviation case by d. The corresponding return assessment is denoted by u and the bailout probability when productivity is low (high) is denoted by q_l (q_h). Using equation (7) we observe that in both cases u can only be consistent if it solves the system $\mathcal{S}(d)$ that consists of the equations

$$\begin{aligned} u &= (p_l q_l + p_h q_h) d & (8) \\ q_l &= \min\left\{ \frac{1}{dS_1(u)} S_2\left(\frac{r_{2l}}{q_l d}\right) \, , \, 1 \right\} & (9) \\ q_h &= \min\left\{ \frac{1}{dS_1(u)} S_2\left(\frac{r_{2h}}{q_h d}\right) \, , \, 1 \right\} & (10) \end{aligned}$$

and of the constraints $q_l > 0$ and $q_h > 0$. Note that refinancing condition (1) will hold in both states of production returns if and only if $dS_1(d) \leq S_2(r_{2l}/d)$. We denote the highest first-period deposit rate at which this is the case by $d_{\mathbf{L}}$. Hence, $d_{\mathbf{L}}$ is the unique solution of the equation

$$S_2^{-1}\Big(dS_1(d)\Big) = \frac{r_{2l}}{d}.$$

Moreover, we use $\underline{u} := \min\{u | S(u) \geq 0\}$. The next lemma is crucial for the analysis of consistent first-period assessments.

Lemma 1
Suppose that $d > \underline{u}$. Then the system $\mathcal{S} = \mathcal{S}(d)$ has a unique solution which we denote by $(\bar{u}_d, \bar{q}_{l,d}, \bar{q}_{h,d})$. Moreover, this solution has the following properties:

(i) $\bar{u}_{(\cdot)}$, $\bar{q}_{l,(\cdot)}$ *and* $\bar{q}_{h,(\cdot)}$ *are continuous functions of* d.

(ii) $\bar{u}_d = d$ *for* $d \leq d_{\mathbf{L}}$ *and* $\bar{u}_d < d$ *for* $d > d_{\mathbf{L}}$.

[26] We will see later (in propositions 3 and 4) that we do not have to consider the case where one bank deviates and both the deviating bank and the non-deviating banks receive deposits.

(iii) $\bar{q}_{l,d} < 1$ *for* $d > d_{\mathbf{L}}$; $\bar{q}_{l,d}, \bar{q}_{h,d} > 0$ *for all* d.

(iv) $\bar{q}_{l,(\cdot)}$ *is strictly decreasing in* d *for all* $d \in \mathcal{D}_M$ *where*

$$\mathcal{D}_M := \Big\{ d \mid dS_1(\bar{u}_d) \le S_2(r_{2h}/d) \text{ and } d > d_{\mathbf{L}} \Big\}.$$

The existence and uniqueness of a solution for $\mathcal{S}$ is derived from a fixed-point argument: the right-hand-side of equation (8) is a decreasing function of u (since bailout probabilities q_l and q_h are decreasing in u) while the left-hand-side is strictly increasing. This implies existence and uniqueness of $\bar{u}_d$ because of the continuity of the bailout probabilities as functions of u. Figure 4 illustrates the solution of $\mathcal{S}$. Note that u and d are represented by the percentage points by which they exceed 1, i.e. $u = 1.06$ is represented by 6. This scale will be used for u and d in all following illustrations.

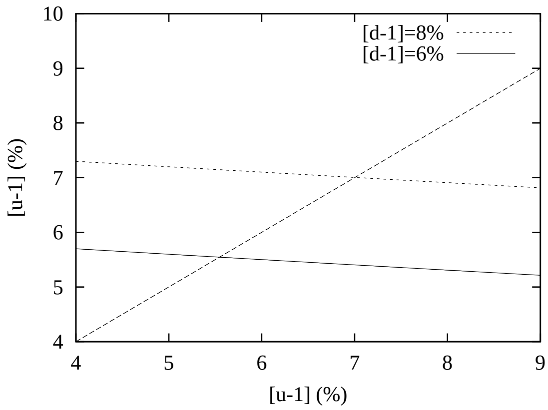

Figure 4: The left-hand and the right-hand side of equation (8) as functions of u for different values of d (example A) Note that u is represented by the percentage points by which it exceeds 1.

Note that at this point our technical device that allows for a fraction $(1-b)$ of the surviving banks' deposits to be eliminated guarantees the continuity of the bailout probabilities as functions of u and thus the existence of a solution for $\mathcal{S}$. If only entire banks could be closed, the bailout probabilities would not be continuous in u. Discontinuities would appear for all u where a marginal higher value of u requires to close an additional bank: in such points bailout probability would fall by $1/n$. Hence, consistent assessments might not exist for some values of d.

In the next two propositions we characterize consistent and optimal assessments. Note that in the non-deviation case where all banks have offered the same first-period deposit rate d_1, assessments for expected returns of banks are denoted by u_1. In the deviation case there are two groups of banks, $\mathcal{B}_l$ and $\mathcal{B}_h$, that have offered different first-period deposit rates d_{1l} and d_{1h} respectively ($d_{1l} < d_{1h}$). Here we denote the corresponding assessments by u_{1l} and u_{1h} respectively.

Proposition 3 (Consistent assessments: Non-deviation case)
If all banks have offered the same first-period deposit rate d_1, *then* $\mathbf{u_1} = (\bar{u}_{d_1}, ..., \bar{u}_{d_1})$ *is the only consistent assessment under all bailout regimes.*

Proposition 4 (Consistent assessments: Deviation case)
In the deviation case only the following types of assessments can be consistent:

a) $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ *b)* $u_{1h} < u_{1l} = \bar{u}_{d_{1l}}$ *c) assessments of the type* $u_{1l} = u_{1h}$.

More specifically, we obtain:

(i) Under the RB or PB bailout scheme, $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ *is the* only *consistent assessment.*

(ii) Under the BB bailout scheme, $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ *is a consistent assessment and* $u_{1h} < u_{1l} = \bar{u}_{d_{1l}}$ *is a consistent assessment if and only if*

$$\left(p_l I_{l,d_{1l},d_{1h}} + p_h I_{h,d_{1l},d_{1h}}\right) d_{1h} < \bar{u}_{d_{1l}}$$

where[27]

$$I_{i,d_{1l},d_{1h}} := 1\left\{S_2(r_{2i}/d_{1h}) \geq dS_1(\bar{u}_{d_{1l}})\right\} \quad (i = l, h).$$

Moreover, an assessment $u_{1l} = u_{1h}$ *is never optimal.*

Proposition 3 follows directly from lemma 1 since expected returns for depositors can be expressed by equations (8) - (10). Moreover, under RB, bailout probabilities for all banks are the same, implying that the banks that have offered the highest deposit rates will always pay the highest returns. Therefore assessments are also unique in the deviation case. Under BB, however, we cannot generally exclude assessments that assign higher expected returns to banks in $\mathcal{B}_l$ despite the fact that those banks have offered lower deposit rates than the banks in $\mathcal{B}_h$. This is due to a *self-fulfilling prophecy* effect caused by BB. Suppose that a bank is assessed to pay higher expected returns than the other banks. This bank will obtain more deposits than the others and hence will be "bigger" in terms of the bailout regime. Under BB it will therefore have a higher bailout probability. This effect can indeed compensate for lower deposit rates. To see why an assessment $u_{1l} = u_{1h}$ cannot be optimal under BB, note that in this case banks in $\mathcal{B}_h$ must be smaller with respect to first-period deposits than banks in $\mathcal{B}_l$; otherwise bailout probability *and* offered deposit rates would be higher for $\mathcal{B}_h$-banks. But this implies that $\mathcal{B}_h$-banks have a lower

[27] Note how the indicator function $1\{\cdot\}$ is defined. $1\{A\}$ is equal to 1 if statement A holds and equal to 0 if statement A does not hold.

bailout probability than in the case where they receive all deposits. The formalization of these arguments leads to statement (ii) in proposition 4.

Propositions 3 allow us to characterize symmetric equilibria under regulation solely in terms of the first-period deposit rate d_1 offered by all banks:

1. Banks offer $\mathbf{d_1} = (d_1, ..., d_1)$ which leads to the assessments $\mathbf{u_1} = (\bar{u}_{d_1}, ..., \bar{u}_{d_1})$ and to the deposit distribution $\mathbf{D_1} = \Big(D_1, ..., D_1\Big)$ where $D_1 = S_1(\bar{u}_{d_1})/n$.

2. The regulator observes the realization r_2 of the aggregate productivity shock and determines $\bar{q}$ as the positive solution of

$$qd_1 S_1(\bar{u}_{d_1}) = S_2(\frac{r_2}{qd_1})$$

   if that solution is lower than 1; otherwise $\bar{q}$ is set equal to 1. k and b are determined by $k = \lceil n\bar{q} \rceil$ and $b = (n\bar{q})/\lceil n\bar{q} \rceil$.

3. A set $\mathcal{B}^+$ of k banks is randomly chosen from all n banks; each bank has the same probability of being chosen. Investment projects of closed banks are uniformly distributed among surviving banks. Deposits and investments of surviving banks are given by

$$\Delta = \Big(bd_1 D_1\, ,\, nD_1/k\Big).$$

4. Banks offer $\mathbf{d_2} = (d_2, ..., d_2)$ where

$$d_2 := S_2^{-1}\Big(\bar{q} S_1(\bar{u}_{d_1})\Big).$$

We will characterize symmetric equilibria by using the short form $\mathcal{E} = (d_1)$.

5 Allocations Under Different Regulatory Approaches

Note that from a $t = 0$ perspective expected profits for bank i are given by

$$\Pi_1^i := -\operatorname{Prob}(A_i)(d_1^i D_1^i + P) + \Big(1 - \operatorname{Prob}(A_i)\Big)\, \mathbb{E}\Big[R_2 b_I D_1^i - (1-b) d_1^i D_1^i - d_2^i D_2^i \mid A_i^c\Big].$$

A_i denotes the eventuality of bank i being closed by the regulator or not being able to meet its obligation in $t = 1$ and A_i^c denotes the complement of A_i, i.e. the possibility of bank i living on until $t = 2$.[28] Remember that P is the penalty imposed by the regulator under A_i. While $P > 0$ under PB, we have $P = 0$ under RB and BB. Note again that under all regulatory approaches, banks are assumed to internalize losses that accrue to depositors. The penalty P under PB will be imposed additionally to any other penalties that might be used to force banks to internalize losses.

5.1 No Regulation

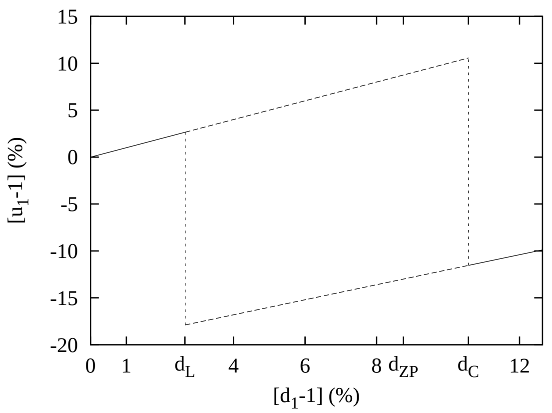

Figure 5: Expected returns u_1 for the first generation as function of the first-period deposit rate d_1 (no-regulation case, example A). d_{ZP} stands for $\tilde{d}_{\mathbf{ZP}}$.

In this section we analyze the no-regulation case to motivate the potential benefits of regulation. Consider a symmetric equilibrium where all banks have offered the same deposit rate d_1 in $t = 0$. Note that according to proposition 1 only three first-period return assessments are possible, namely d_1, $p_h d_1$ and zero. Obviously, if $d_1 \leq d_{\mathbf{L}}$, then only $u_1 = d_1$ is consistent and banks can refinance in both states of production returns.[29] If $d_1 > d_{\mathbf{L}}$, then $u_1 = d_1$ is no longer consistent since under this assessment banks would go bankrupt for $r_2 = r_{2l}$ (because $\bar{d}_2^* > r_{2l}/d$), which would lead to $u < d_1$. Equilibria where banks are correctly assessed to pay zero returns can also be excluded. Hence, the only other possible assessment is $u_1 = p_h d_1$. Such an assessment will be correct if and only if $r_{2h}/d_1 \geq \tilde{d}_2^*(d_1) > r_{2l}/d_1$ where $\tilde{d}_2^*(d) := S_2^{-1}\Big(dS_1(p_h d)\Big)$. Hence, by defining $d_{\mathbf{C}}$ as the unique solution of $\tilde{d}_2^*(d) = r_{2l}/d$, we have derived that without regulation no consistent assessments exist if $d_{\mathbf{L}} < d_1 \leq d_{\mathbf{C}}$.

Using the parameter values from example A, we illustrate the consistent-assessment problem in figure 5: if $d_{\mathbf{L}} < d_1 \leq d_{\mathbf{C}}$, and depositors assume that all banks will survive in both states of production returns, then return assessments are given by the upper (broken) line in figure 5. Actual returns paid are represented by the lower (broken) line; if depositors assume that banks can only refinance in the high state, then assessments are given by the lower line and actual returns paid by the higher line. Finally, if $d_1 \leq d_{\mathbf{L}}$ or $d_1 > d_{\mathbf{C}}$, the solid lines represent the respective consistent return assessments for first-generation

[28] Note that given A_i bank i cannot pay anything to depositors since the liquidation value of the project is zero.

[29] Note that in this case first-period savings amount to $S_1(d)$ and hence $\bar{d}_2^* = S_2^{-1}\Big(dS_1(d)\Big)$ which by definition of $d_{\mathbf{L}}$ is not higher than r_{2l}/d.

depositors.

In order to assess the benefits of regulation we also want to compare expected returns for depositors resulting with and without regulation. For our purposes it is sufficient to observe that banks will not bid deposit rates higher than $d_1 = \tilde{d}_{\mathbf{ZP}}$ which is defined as the unique solution of the equation $\tilde{\pi}(d) = 0$. $\tilde{\pi}(d)$ are the banks' profits per deposit in a symmetric equilibrium $d_1 = d$ if $d > d_{\mathbf{C}}$. They can be described by

$$\tilde{\pi}(d) = -p_l d + p_h \Big(r_{2h} - d\tilde{d}_2^*(d) \Big).$$

Symmetric equilibria with higher deposit rates will not occur since such equilibria would imply negative bank profits (because $\pi(\cdot)$ is strictly increasing in d). Our results are summarized in the following proposition.

Proposition 5

Suppose that banks play a symmetric strategy $\mathbf{d_1} = (d_1, ..., d_1)$ in the first period and that there is no regulation. Then the following statements hold:

(i) If $d_{\mathbf{L}} < d_1 \leq d_{\mathbf{C}}$, then no consistent assessments exist.

(ii) For both generations, the highest possible symmetric equilibrium returns are either achieved if $d_1 = d_{\mathbf{L}}$ or $d_1 = \tilde{d}_{\mathbf{ZP}}$. The corresponding unique first-period assessments are $d_{\mathbf{L}}$ and $p_h\tilde{d}_{\mathbf{ZP}}$ respectively.

(iii) If $\tilde{d}_{\mathbf{ZP}} \leq d_{\mathbf{C}}$, then the highest possible symmetric equilibrium returns are achieved for $d_1 = d_{\mathbf{L}}$.

Note that in example A we have $\tilde{d}_{\mathbf{ZP}} = 1.079 < d_{\mathbf{C}} = 1.105$ and hence statement (iii) applies. Proposition 5 points to the potential benefits of regulation. Without regulation, the existence of consistent assessments is not guaranteed and it can occur that none of the banks is able to refinance in $t = 1$, implying that intermediation services break down completely for the second generation. In the following, we discuss how regulatory approaches can avoid the breakdown of intermediation. In section 5.2, we consider the enforcement of prudential equilibria with $d_1 \leq d_{\mathbf{L}}$, and in section 5.3 we analyze the case of discriminatory closure of some banks in order to allow the others to refinance. Both scenarios also help to avoid the problem of nonexistent assessments, as we have already observed in proposition 3. In section 6 we explicitly compare the no-regulation and the different regulatory approaches with respect to stability and expected returns paid on deposits.

5.2 Prudential Banking

In this section we assume that the regulatory regime forces banks to avoid the possibility of default.

Proposition 6
The unique symmetric equilibrium under prudential banking is $\mathcal{E}_L$.

Obviously, prudential banking can heavily depress deposit rates and investments if a serious productivity shock can occur. Moreover, in the case $r_{2l} < \underline{u}$, intermediation is impossible. In the next sections we therefore examine work-out type regulatory approaches to banking crises and their implications.

5.3 Discriminatory Bailout

In this section we investigate the equilibria that occur under discriminatory bailout. In order to describe the banks' profits under discriminatory regulation schemes, we recall the definition of the set $\mathcal{D}_M$ and additionally introduce the set $\mathcal{D}_H$:

$$\begin{aligned} \mathcal{D}_M &:= \Big\{ d \mid dS_1(\bar{u}_d) \leq S_2(r_{2h}/d) \text{ and } d > d_{\mathbf{L}} \Big\} \\ \mathcal{D}_H &:= \Big\{ d \mid dS_1(\bar{u}_d) > S_2(r_{2h}/d) \Big\}. \end{aligned}$$

The sets $\mathcal{D}_M$ and $\mathcal{D}_H$ refer to a situation where all banks have symmetrically offered a deposit rate $d_1 = d$ in $t = 0$. If $d \in \mathcal{D}_M$, then all banks can refinance in the good state but not in the bad state of production returns while banks cannot refinance in both states for $d \in \mathcal{D}_H$. Moreover, $d_2^*(d) := S_2^{-1}\Big(dS_1(\bar{u}_d)\Big)$ is the second-period deposit rate that - if symmetrically offered by all n banks in $t = 1$ - generates just enough savings for all banks to refinance.

Now consider a potential symmetric equilibrium $\mathcal{E} = (d_1)$, or a deviation d_1^{dev} from such a symmetric equilibrium where the deviating bank receives all savings. Then the expected profits that a bank makes on each unit of deposits are given by $\pi(d_1)$ and $\pi(d_1^{\text{dev}})$ respectively where π is defined by[30]

$$\pi(d) := \begin{cases} \bar{R}_2 - dd_2^*(d) & \text{if } d \leq d_{\mathbf{L}} \\ -p_l(1-\bar{q}_{l,d})d + p_h\Big(r_{2h} - dd_2^*(d)\Big) & \text{if } d \in \mathcal{D}_M \\ -p_l(1-\bar{q}_{l,d})d - p_h(1-\bar{q}_{h,d})d & \text{if } d \in \mathcal{D}_H. \end{cases}$$

[30]Remember that $\bar{R}_2 = p_l r_{2l} + p_h r_{2h}$.

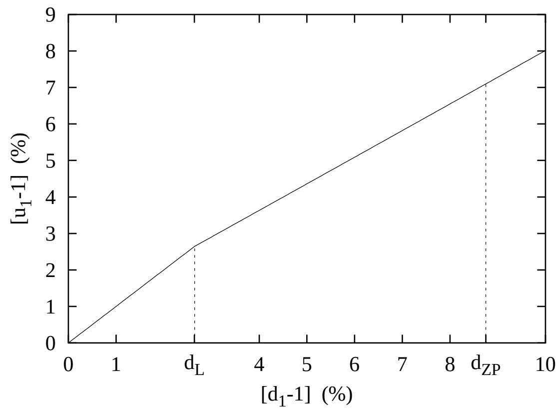

Figure 6: Expected returns u_1 for the first generation as function of the first-period deposit rate d_1 (example A).

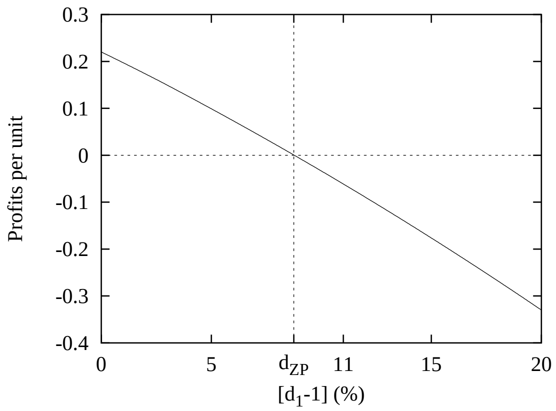

Figure 7: Profits per unit of deposits as function of the first-period deposit rate d_1 (example A).

Lemma 2

$\pi(\cdot)$ is a continuous function of d.

We note that $\pi(d) > 0$ if $d \le d_{\mathbf{L}}$ and that $\pi(d) < 0$ if $d \in \mathcal{D}_H$. Hence, by the continuity of $\pi(\cdot)$, there is a first-period deposit rate d with $\pi(d) = 0$. We will work with the following assumption:

Assumption 1 (UZP)

There is a unique first-period deposit rate d where profits are zero ($\pi(d) = 0$). We denote this deposit rate by $d_{\mathbf{ZP}}$ and further assume that $\pi(d) < 0$ for $d > d_{\mathbf{ZP}}$.

Note that assumption UZP is satisfied if $\bar{u}_{(\cdot)}$ is increasing in d for all $d \in \mathcal{D}_M$ which can be verified for our example saving functions.

Lemma 3

If $S_i(u) = a_i u^{\alpha_i}$ with $a, \alpha \in (0, \infty)$ $(i = 1, 2)$, then $\bar{u}_{(\cdot)}$ is increasing in d for all $d \in \mathcal{D}_M$.

We can now turn to the analysis of the random bailout regime:

Proposition 7

Suppose that UZP holds. Then the unique symmetric equilibrium under random bailout is $\mathcal{E}_{ZP} := (d_{\mathbf{ZP}})$.

$\mathcal{E}_{ZP}$ is the zero-profit equilibrium. Equilibria with higher deposit rates imply negative profits for banks and will thus not be played. Equilibria with lower deposit rates do not exist, since banks will have an incentive to deviate to slightly higher deposit rates thereby collecting all savings. Figures 6 and 7 show expected returns for the first generation and banks' expected profits as functions of the first-period deposit rate d_1 that has been offered.

Having derived this result, we now set out to examine whether and how allocations are affected if the regulator follows BB instead of RB. We have already indicated that BB can lead to self-fulfilling prophecy effects when first-period deposit rates are set asymmetrically. Banks that have offered lower deposit rates can consistently be assessed to pay higher returns than banks that have offered higher deposit rates. To present our results we introduce the following tie-breaking rule:

(TR) *If depositors receive the same expected returns when depositing with non-deviating banks as when depositing with the deviating bank, they choose the non-deviating ones.*

Moreover, we introduce the function

$$\Pi^{\text{dev}}(d) := \max\Big\{\pi(\tilde{d})S(\bar{u}_{\tilde{d}}) \;:\; \bar{u}_{\tilde{d}} > \bar{u}_d\Big\},$$

which describes the maximum profits that can be obtained when deviating from a symmetric equilibrium where all banks have offered a first-period deposit rate d. Finally we distinguish the following cases for the relationship between expected equilibrium returns for the first generation and offered first-period deposit rates:

1. There is a deposit rate $d_{\mathbf{UH}}$ ($d_{\mathbf{L}} \le d_{\mathbf{UH}}$) such that $\bar{u}_{(\cdot)}$ is strictly increasing in d for $d < d_{\mathbf{UH}}$ and strictly decreasing for $d_{\mathbf{UH}} < d \le d_{\mathbf{ZP}}$. (**UID**)
2. There is a deposit rate $d_{\mathbf{UL}}$ ($d_{\mathbf{L}} \le d_{\mathbf{UL}}$) such that $\bar{u}_{(\cdot)}$ is strictly decreasing in d for $d < d_{\mathbf{UL}}$ and strictly increasing for $d_{\mathbf{UL}} < d \le d_{\mathbf{ZP}}$. (**UDI**)

These cases are illustrated in figures 8 and 9. Note that under UID (UDI), both constellations are possible: (a) $d_{\mathbf{UH}} < d_{\mathbf{ZP}}$ ($d_{\mathbf{UL}} < d_{\mathbf{ZP}}$) and (b) $d_{\mathbf{UH}} \ge d_{\mathbf{ZP}}$ ($d_{\mathbf{UL}} \ge d_{\mathbf{ZP}}$).

Proposition 8
Suppose that the assumption UZP holds and that TR is applied. Then the following holds under bail out the big ones:

(i) $\mathcal{E} = (d)$ *is an equilibrium for each deposit rate* $d \in \mathcal{U}_{\max} := \operatorname{argmax} \bar{u}_{(\cdot)} \,|_{\pi(d) \ge 0}$.

(ii) Under UID we obtain that $\mathcal{E}_{UH} := \Big(\min\{d_{\mathbf{UH}}, d_{\mathbf{ZP}}\}\Big)$ *is the unique symmetric equilibrium.*

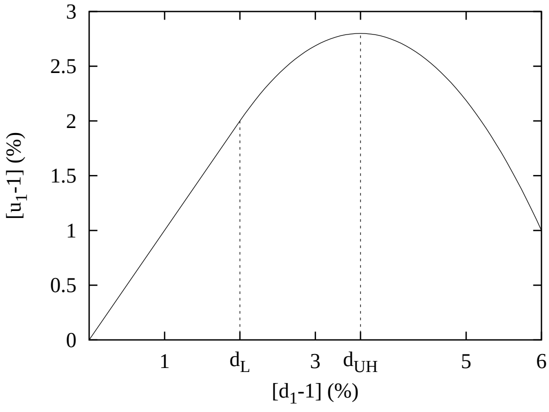

Figure 8: The case UID.

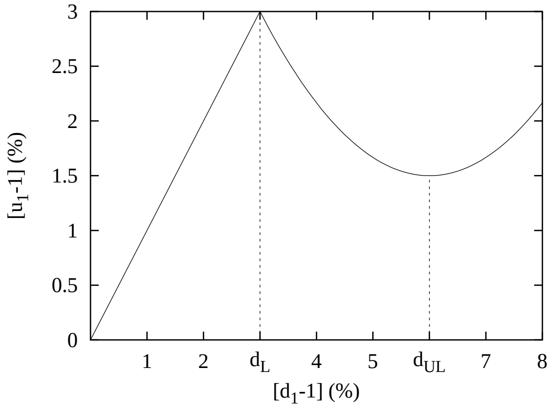

Figure 9: The case UDI.

(iii) Under UDI we obtain:

- *If $d_{\mathbf{L}} > \bar{u}_{d_{\mathbf{ZP}}}$, then $\mathcal{E}_L$ is the unique symmetric equilibrium.*
- *If $d_{\mathbf{L}} < \bar{u}_{d_{\mathbf{ZP}}}$, then $\mathcal{E}_{ZP}$ is an equilibrium and $\mathcal{E}_L$ is an equilibrium if and only if $\Pi^{dev}(d_{\mathbf{L}}) \leq \pi(d_{\mathbf{L}})S(d_{\mathbf{L}})/n$. No other equilibria exist.*

The next corollary is an immediate consequence of proposition 8. It is concerned with the cases where $\bar{u}_{(\cdot)}$ is strictly increasing (**UI**) or strictly decreasing (**UD**) for $d_{\mathbf{L}} < d \leq d_{\mathbf{ZP}}$.

Corollary 1

Suppose assumption UZP holds and that TR is applied. Then under bail out the big ones we obtain:

(i) Under UI, $\mathcal{E}_{ZP}$ is the unique equilibrium.

(ii) Under UD, $\mathcal{E}_L$ is the unique equilibrium.

What is the economic intuition behind the results in proposition 8? Let us first turn to statement (i). Under BB, maximum expected return equilibria are supported, since even if banks deviate to higher deposit rates, depositors can consistently assess non-deviating banks as paying higher returns, thereby securing maximum expected returns. This is not possible under RB. Let us now turn to the interesting case of statement (ii) where $d_{\mathbf{UH}} < d_{\mathbf{ZP}}$. Equilibria with higher deposit rates are not possible, because banks would deviate to lower rates, and depositors would switch to the deviating banks since they can guarantee higher expected returns. Again this is made possible by the self-fulfilling prophecy effect of BB. Lower deposit rates are not possible because banks will deviate to higher rates. Statement (iii) can be explained by the same reasoning.

6 Comparison

In this section we compare the three regulatory scenarios (prudential banking, random bailout and bail out the big ones) and the no-regulation scenario. Our comparison is concerned with three issues: fragility issues, credibility issues and expected returns. For two points of the analysis we have relied on simulation results: first, for the determination of the shape of $\bar{u}_{(\cdot)}$ as function of d; second, for the comparison of expected returns in the $\mathcal{E}_L$ and the $\mathcal{E}_{ZP}$ equilibrium.

We will focus on what we call the "normal case", namely the case where $\bar{u}_{(\cdot)}$ is strictly increasing in d. The label "normal" is justified by the fact that $\bar{u}_{(\cdot)}$ has this property for our family of example saving functions and because $\bar{u}_{(\cdot)}$ has behaved in this way for a wide range of other numerical examples.

6.1 Stability Issues

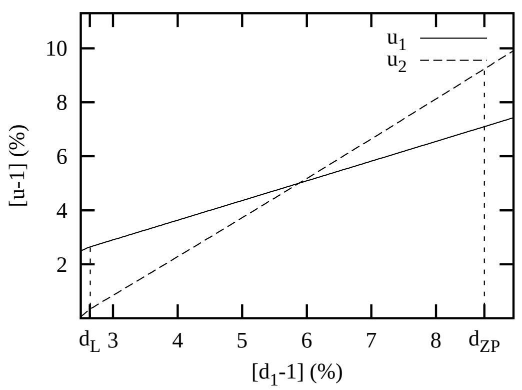

Figure 10: Expected returns for both generations as function of the first-period deposit rate d_1 (example A).

Regulation improves the stability of intermediation. First, the existence of second-period equilibria is guaranteed under regulation. Moreover, these equilibria can be ranked by banks according to the Pareto criterion. Without regulation, neither are guaranteed. Second, under regulation the nonexistence of consistent assessments in the first period, which may occur without regulation (see proposition 5), can be avoided.

The crucial question in comparing the stability across regulatory schemes is whether the proposed coordination mechanism for depositors return assessments works if there is more than one consistent assessment. We have assumed that if there is more than one consistent assessment, then depositors choose the assessment that promises the highest expected returns (optimal assessment). If this is the case, all regulatory regimes yield the same stable result, namely unique assessments and a unique equilibrium: $\mathcal{E}_{ZP}$ (under RB and BB) and $\mathcal{E}_L$ (under PB). The result for PB and RB is independent of whether the optimal-assessment criterion holds or not. The stability of the BB regime on the other hand depends upon it heavily: if it does not hold, uniqueness is not guaranteed (see proposition 4).

6.2 Return Issues

Recall that in the normal case we have to compare the equilibria $\mathcal{E}_L$ (implemented by prudential banking and possibly implemented without regulation), $\tilde{\mathcal{E}}_{ZP}$ (possibly implemented without regulation) and $\mathcal{E}_{ZP}$ (implemented by RB and BB). The expected returns for the first generation (u_1) and for the second generation (u_2) in the different equilibria are presented in table 2.

	First Generation	Second Generation
$\mathcal{E}_L$	$d_{\mathbf{L}}$	$d_2^*(d_{\mathbf{L}})$
$\tilde{\mathcal{E}}_{ZP}$	$p_h\tilde{d}_{\mathbf{ZP}}$	$p_l\underline{u} + p_h\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}})$
$\mathcal{E}_{ZP}$	$p_l\bar{q}_{l,d}d_{\mathbf{ZP}} + p_h d_{\mathbf{ZP}}$	$p_l S_2^{-1}\Big(\bar{q}_{l,d}d_{\mathbf{ZP}}S_1(\bar{u}_{d_{\mathbf{ZP}}})\Big) + p_h d_2^*(d_{\mathbf{ZP}})$

Table 2: Expected returns under different equilibria

The most important question is whether regulation can improve expected returns for both generations. We observe that returns $u_i(\mathcal{E}_{ZP})$ in $\mathcal{E}_{ZP}$ are higher for both generations than returns $u_i(\tilde{\mathcal{E}}_{ZP})$ in $\tilde{\mathcal{E}}_{ZP}$ $(i = 1, 2)$. This is stated in the next proposition:

Proposition 9
$u_i(\mathcal{E}_{ZP}) > u_i(\tilde{\mathcal{E}}_{ZP})$ for $i = 1, 2$.

Proposition 9 implies that regulation can improve welfare. On the other hand, it is not clear whether $\mathcal{E}_{ZP}$ also delivers higher returns than $\mathcal{E}_L$. Obviously, $u_1(\mathcal{E}_{ZP}) > u_1(\mathcal{E}_L)$, but the effect for the second generation is ambiguous, since $\bar{q}_{l,d}d_{\mathbf{ZP}}$ might be smaller than $d_{\mathbf{L}}$ and hence might offset the effect that $d_2^*(d_{\mathbf{L}}) < d_2^*(d_{\mathbf{ZP}})$. However, in all simulation exercises $\mathcal{E}_{ZP}$ also improves returns for the second generation compared to $\mathcal{E}_L$. Hence, in these cases discriminatory bailout improves expected returns for both generations compared to PB and compared to the non-regulation case. As illustration we show in figure 10 expected returns for the first and the second generation under discriminatory bailout as function of offered first-period deposit rates (for example A). The returns resulting under $\mathcal{E}_L$ and $\mathcal{E}_{ZP}$ are presented in table 3.[31]

[31]Note that the equilibrium $d_1 = \tilde{d}_{\mathbf{ZP}}$ does not exist in the no-regulation case for example A. Hence the highest possible equilibrium returns are achieved under $\mathcal{E}_L$.

	$[d_1 - 1](\%)$	$[u_1 - 1](\%)$	$[u_2 - 1](\%)$	q_l
$\mathcal{E}_L$	2.64	2.64	0.34	1
$\mathcal{E}_{ZP}$	8.75	7.00	9.24	0.92

Table 3: First-period deposit rates, expected returns, and fraction of bailed out depositors for example A.

6.3 Credibility Issues

The issue of credibility obviously only has a bearing on the three regulatory schemes. The most important difference with respect to the credibility of those schemes is the *out-of-equilibrium* strategy that is required. While the credibility of PB first of all depends on the credibility of the penalties that have to be applied (which will not be taken up here), the credibility of BB and RB depends on the impact of the respective out-of-equilibrium closure rules.

In section 3.5.3 we have already illustrated that while the maximum fraction of depositors is always bailed out under BB, under RB it might be necessary to bail out a significantly lower fraction of first-period deposits than would be possible. This occurs if the deposit distribution is very unequal. The necessity to commit to lower-than-possible bailout fractions might well reduce the credibility of the RB scheme. Agents might expect the regulator to abandon RB and bail out more depositors if an asymmetric deposit distribution occurs. As mentioned above, this kind of credibility problem does not occur under BB.

6.4 Extensions

In an extended version of this paper Erlenmaier and Gersbach (2001) we explore several extensions to the framework discussed here. In particular, it is shown that cases where $\bar{u}_{(\cdot)}$ is not increasing in d can occur and that for these cases BB dominates RB with respect to expected returns. We also indicate that RB may provide less incentives for excessive risk-taking than BB.

7 Conclusion

We have attempted to provide a general-equilibrium analysis of the funds concentration effect and the corresponding regulatory bailout schemes in an overlapping-generations framework. We have found that bail out the big ones (BB) dominates random bailout (RB) and prudential banking with respect to expected returns on deposits and with respect to credibility.

A Appendix

A.1 Proofs for Section 4

Proof of proposition 1.

(i) Step 1: $\mathcal{E}_2^*$ is an equilibrium.

First, we note that if $d_2^i = \bar{d}_2^*$ is played ($i \in \mathcal{B}^+$), the assessment $u_2^i = \bar{d}_2^*$ ($i \in \mathcal{B}^+$) is optimal and only the deposit distribution $D_2^i = d_1^i D_1^i$ ($i = 1, .., n$) is consistent with this assessment. Second, we have to show that deviations from $d_2^i = \bar{d}_2^*$ are not profitable. Deviations to higher deposit rates can be excluded since they raise repayment obligations. If a bank deviates to $d_2^{\text{dev}} < \bar{d}_2^*$, it is not possible for all banks to receive a positive measure of deposits.[32] But then either all or none of the non-deviating banks receive a positive measure of deposits.[33] Hence, there can only be one case where the deviating bank receives a positive measure of savings, namely if it can attract the full measure of second-period savings. But in this case the deviation cannot be profitable for the following reasons. Depositors only choose to give resources to the deviating bank if returns are at least as high as returns at the non-deviating banks. But if depositors chose to deposit with the non-deviating banks, returns are given by $\min\{u_*, \bar{d}_2^*\}$, where u_* is the positive solution of

$$u = \frac{(m-1)r_2 D_1}{S_2(u)}. \tag{11}$$

If, on the other hand, depositors deposit with the deviating bank, returns cannot be higher than $\min\{u_*^{\text{dev}}, d_2^{\text{dev}}\}$, where u_*^{dev} is the positive solution of equation (11) when $m-1$ is replaced by 1. Hence, the inequality $u_*^{\text{dev}} \geq u_*$ can only be fulfilled if $m = 2$ and $d_2^{\text{dev}} \geq u_*$. But in this case the deviating banks' profits cannot be higher than zero.

(i) Step 2: Pareto-dominance.

If $d_2 > \bar{d}_2^*$, then banks obtain lower profits than in $\mathcal{E}_2^*$ because repayment obligations are higher. If $d_2 < \bar{d}_2^*$, then the amount of overall savings is bounded by $S_2(d_2)$ and hence second-period deposits of a single bank are limited by $S_2(d_2)/m$ (since depositors cannot

[32] If that were the case, all banks would have to be assessed paying the *same* return $u_2 \leq d_2^{\text{dev}}$. But then $S_2(u_2) < md_1D_1$, and hence at least one bank cannot refinance. This implies that $u_2 = 0$ and $S_2(u_2) = 0$ in contradiction to the assumption that all banks receive a positive measure of deposits.

[33] Since the non-deviating banks are identical with respect to all their characteristics, they will receive the same amount of deposits.

coordinate on a subset of banks). But this implies that no bank can refinance and the only consistent assessment is $u_2 = 0$ for all banks, implying that no bank receives any savings.

(ii) The first observation is obvious and the second has already been derived under step 2 in the proof of (i).

□

Proof of proposition 2.

Step 1: $\mathcal{E}_2^*$ is an equilibrium. We observe that if $d_2^i = \bar{d}_2^*$ is played, the assessment $u_2^i = \bar{d}_2^*$ $(i \in \mathcal{B}^+)$ is optimal and only the deposit distribution $D_2^i = bd_1^i D_1^i$ $(i = 1, .., n)$ is consistent with this assessment. Deviations to higher deposit rates would increase repayment obligations and can therefore not be profitable. Deviations to lower rates are excluded by LBD.

Step 2: Pareto-dominance. Equilibria where some banks have offered lower deposit rates than $\bar{d}_2^*$ are excluded by LBD, and all other equilibria are Pareto-dominated by $\mathcal{E}_2^*$ since repayment obligations are higher than under $\mathcal{E}_2^*$.

□

Proof of lemma 1.

Step 1: The system $\mathcal{S}$ has a unique solution.

First, note that if $\bar{u}_d$ is a solution of the system $\mathcal{S}$, then $\bar{u}_d > \underline{u}$. But for all $u > \underline{u}$, equations (9) and (10) have unique, strictly positive solutions, which we denote by $q_l = q_l(d, u)$ and $q_h = q_h(d, u)$ respectively.[34] Figure 11 illustrates the argument by depicting the left-hand and the right-hand side of equation (9) for example A and $d = 1.05$. Figure 11 also illustrates that the functions $q_l(d, \cdot)$ and $q_h(d, \cdot)$ are decreasing in u for fixed d by depicting the solutions of the equation for $u = 1.05$ and $u = 1.07$. Moreover, $q_l(\cdot, \cdot)$ and $q_h(\cdot, \cdot)$ are continuous in d and u since the left and the right-hand side of the equations (9) and (10) are continuous functions of d and u. Inserting $q_l(d, u)$ and $q_h(d, u)$ in equation (8), we obtain an implicit equation for u. As we saw above, we have to restrict the range of this equation to $u > \underline{u}$. Hence the left-hand side of this equation is strictly increasing in u

[34] The left-hand sides of the equations are strictly increasing in q_l and q_h respectively and they take all values in $(0, \infty)$; the right-hand sides are decreasing in q_l and q_h respectively.

and takes all values in $\mathbb{R}$ that are strictly higher than $\underline{u}$; the right-hand side is decreasing in u and higher than $\underline{u}$ if u is close enough to $\underline{u}$.[35] Hence, by the mean value theorem, a unique solution $\bar{u}_d$ of this implicit equation exists. This solution is a continuous function of d since the left and the right-hand side of the equation are continuous functions of d. Finally, inserting $\bar{u}_d$ in q_l and q_h we obtain $\bar{q}_{l,d} := q_l(d, \bar{u}_d)$ and $\bar{q}_{h,d} := q_h(d, \bar{u}_d)$.

Step 2: Proof of statements (i) - (iv)

The continuity of $\bar{u}_{(\cdot)}$ has already been shown in step 1 and the continuity of $\bar{q}_{l,(\cdot)}$ and $\bar{q}_{h,(\cdot)}$ follows directly from the continuity of $\bar{u}_{(\cdot)}$, $q_l(\cdot,\cdot)$ and $q_h(\cdot,\cdot)$. Statements (ii) and (iii) are straightforward.We now need to substantiate (iv), i.e. the monotony of $\bar{q}_{l,(\cdot)}$ for all $d \in \mathcal{D}_M$. Recall that

$$\mathcal{D}_M = \Big\{ d \mid dS_1(\bar{u}_d) \leq S_2(r_{2h}/d) \text{ and } d > d_{\mathbf{L}} \Big\},$$

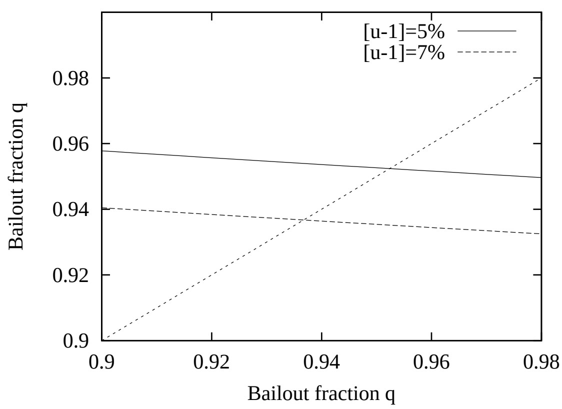

Figure 11: The left-hand and the right-hand side of equation (9) as functions of q (example A, $d = 1.05$).

and note that for $d \in \mathcal{D}_M$ we have

$$\bar{q}_{l,d} = \frac{1}{dS_1(\bar{u}_d)} S_2\Big(\frac{r_{2l}}{\bar{q}_{l,d} d}\Big), \tag{12}$$

$\bar{q}_{h,d} = 1$ and

$$\bar{u}_d = (p_l \bar{q}_{l,d} + p_h) d. \tag{13}$$

Suppose now that $d < \tilde{d}$. If $\bar{u}_d \leq \bar{u}_{\tilde{d}}$, then we obtain $\bar{q}_{l,d} > \bar{q}_{l,\tilde{d}}$ from equation (12). If on the other hand $\bar{u}_d > \bar{u}_{\tilde{d}}$, then $\bar{q}_{l,d} > \bar{q}_{l,\tilde{d}}$ follows from equation (13).

□

Proof of proposition 3.

Since banks are identical, assessments and deposit distribution have to be symmetrical: $\mathbf{u_1} = (u, ..., u)$ and $\mathbf{D_1} = (D, .., D)$ where $D = S(u)/n$. Hence the expected return on first-period bank deposits is given by equations (8) - (10). But we know from lemma 1 that in this case $u = \bar{u}_{d_1}$ is the only consistent assessment.

□

[35] Note that if $u \to \underline{u}$, the right-hand side approaches d, which is assumed to be higher than $\underline{u}$.

Proof of proposition 4.

We denote the share of deposits that the banks in $\mathcal{B}_l$ receive by λ_l . First, note that an assessment $u_{1l} < u_{1h}$ always leads to $\lambda_l = 0$ and to a symmetric distribution of all savings among banks in $\mathcal{B}_h$. Hence, by lemma 1, this assessment is consistent if and only if $u_{1h} = \bar{u}_{d_{1h}}$ and if the assessment passes the zero measure test: if all banks in $\mathcal{B}_l$ receive a zero measure of deposits, then depositors at $\mathcal{B}_l$-banks must receive lower expected returns than $\bar{u}_{d_{1h}}$. Of course, the same is true for the converse assessment $u_{1h} < u_{1l}$. It leads to $\lambda_l = 1$ and $u_{1l} = \bar{u}_{d_{1l}}$ and is consistent if and only if the return paid by zero-measure $\mathcal{B}_h$-banks is smaller than $\bar{u}_{d_{1l}}$.

The assessment $u_{1l} < u_{1h} = \bar{u}_{d_{1h}}$ is consistent under RB, PB and BB, since under all those bailout schemes the bailout probability for depositors at zero-measure banks is never higher than the bailout probability for deposits at positive-measure banks. Furthermore, under PB and RB bailout probabilities are the same for all deposits. Hence, expected returns on deposits of $\mathcal{B}_h$-banks are strictly higher than those on deposits of $\mathcal{B}_l$-banks for any consistent assessment. Finally, under BB, the assessment $u_{1h} < u_{1l} = \bar{u}_{d_{1l}}$ passes the zero-measure test if and only if

$$\left(p_l I_{l,d_{1l},d_{1h}} + p_h I_{h,d_{1l},d_{1h}}\right) d_{1h} < \bar{u}_{d_{1l}}.$$

This follows from the fact that under BB zero-measure banks are closed if they have offered higher first-period deposit rates than all positive-measure banks and if the refinancing condition (1) for the banking system does not hold. Therefore, $\mathcal{B}_l$-bank deposits will be bailed out with the same probability as in the symmetric case where all banks have offered d_{1l}.[36] Hence, expected returns on $\mathcal{B}_l$-bank deposits are equal to $\bar{u}_{d_{1l}}$. The variables $I_{i,d_{1l},d_{1h}}$ ($i = l, h$) indicate whether the overall refinancing condition (1) holds in the low (high) state of production returns, thereby setting the bailout probability of the zero-measure $\mathcal{B}_h$-banks to 0 or 1.

It remains to show that an assessment $u_{1l} = u_{1h}$ cannot be optimal under BB. Hence it remains to analyze assessments of the type $u_{1l} = u_{1h}$ under BB. Note that such assessments can only be consistent if $\lambda_l > n_l/n$ where n_l denotes the number of banks in $\mathcal{B}_l$. Hence, the statement $u := u_{1l} = u_{1h}$ is a consistent assessment is equivalent to the existence of a

[36]See section 3.5.4 for the special treatment of zero-measure banks under BB.

real number $\lambda_l > n_l/n$ with u solving the system $\bar{\mathcal{S}}(\lambda_l)$ described by equations 14 - 19:

$$
\begin{aligned}
u &= (p_l q_{l,l} + p_h q_{h,l}) d_{1l} && (14)\\
u &= (p_l q_{l,h} + p_h q_{h,h}) d_{1h} && (15)\\
q_{i,l} &= \begin{cases} 1 & \text{if } q_i \geq \lambda_l \\ q_i/\lambda_l & \text{else} \end{cases} \quad (i = l, h) && (16)\\
q_{i,h} &= \begin{cases} (q_i - \lambda_l)/(1-\lambda_l) & \text{if } q_i \geq \lambda_l \\ 0 & \text{else} \end{cases} \quad (i = l, h) && (17)\\
q_l &= \min\Big\{ \frac{1}{d_{1h} S_1(u)} S_2\Big(\frac{r_{2l}}{q_l d_{1h}}\Big), 1 \Big\} && (18)\\
q_h &= \min\Big\{ \frac{1}{d_{1h} S_1(u)} S_2\Big(\frac{r_{2h}}{q_h d_{1h}}\Big), 1 \Big\}. && (19)
\end{aligned}
$$

Note that $q_{i,l}$ and $q_{i,h}$ denote the bailout probabilities of banks in $\mathcal{B}_l$ and $\mathcal{B}_h$ respectively ($i = l, h$ denotes the state of production returns). The remaining statements therefore follow from lemma 4.

□

Lemma 4

Suppose that $\underline{u} < d_{1l} < d_{1h}$. *If* $\bar{\mathcal{S}}(\lambda_l)$ *has a solution* u *for arbitrary* $\lambda_l \in (0, 1]$, *then* $u < \bar{u}_{d_{1h}}$.

Proof.

First of all note that a solution of the complete system has to solve the subsystems $\bar{\mathcal{S}}_l$ (consisting of equations 14, 16, 18 and 19) and the subsystem $\bar{\mathcal{S}}_h$ (consisting of equations 15, 17, 18 and 19).

The proof rests on the observation that the sub-system $\bar{\mathcal{S}}_h$ consists of the same equations as the system $\mathcal{S}(d_{1h})$, which has the solution $\bar{u}_{d_{1h}}$. The only difference is that in the system $\mathcal{S}(d_{1h})$, $q_{i,h}$ is replaced by q_i ($i = l, h$) in equation (15). The statement $u < \bar{u}_{d_{1h}}$ therefore follows from the fact that $q_{i,h} \leq q_i$.

To present this argument in a more formal way, we use the index i to indicate both $i = l$ and $i = h$. Note that a solution of $\bar{\mathcal{S}}_h$ can be derived by solving equations (18) and (19) and inserting the solutions in equation (17), which yields $q_{i,h}(u, \lambda_l)$. Since the right-hand side of equation (17) is decreasing in λ_l for arbitrary $q_i \leq 1$, we obtain that $q_{i,h}(u, \cdot)$ is decreasing in λ_l. By inserting $q_{i,h}$ in equation (15) we can therefore conclude that the solution $u(d_{1h}, \lambda_l)$ of the resulting equation is decreasing in λ_l. Hence $u(d_{1h}, \lambda_l) \leq u(d_{1h}, 0) = \bar{u}_{d_{1h}}$. In order to show that the inequality holds strictly for $\lambda_l > 0$, we assume that $u(d_{1h}, \lambda_l) = \bar{u}_{d_{1h}}$ for $\lambda_l > 0$. By inserting $u = \bar{u}_{d_{1h}}$ in equations (18) and (19) we can then see that $q_i = \bar{q}_{i,d_{1h}}$ and therefore that $q_{i,h}(\lambda_l) < \bar{q}_{i,d_{1h}}$. But then insertion into equation (15) would imply that $u(d_{1h}, \lambda_l) < \bar{u}_{d_{1h}}$, in contradiction to our assumption.

□

A.2 Proofs for Section 5

The propositions in section 5 are concerned with symmetric equilibria in $t = 0$. Hence we will either have to analyze the case where all banks offer the same first-period deposit rate d_1 or the deviation case where $(n-1)$ banks offer the same first-period deposit rate d_1 and one bank j offers a different rate d_1^{dev}. We will always denote the assessment for the non-deviating banks by u_1 and that for the deviating bank by u_1^{dev}. Resulting first-period deposits are denoted by D_1 and D_1^{dev} respectively.

Proof of proposition 5.

It only remains to substantiate (ii) and (iii). Statement (ii) is obvious for first-period returns and follows for second-period returns, because they are given by $S_2^{-1}\Big(dS_1(d)\Big)$ if $d_1 \leq d_{\mathbf{L}}$ and by $p_l\underline{u} + p_h\tilde{d}_2^*(d_1)$ if $d_1 > d_{\mathbf{L}}$. Both expressions are increasing in d_1. Statement (iii) follows immediately from (ii) since if $d_{\mathbf{ZP}} \leq d_{\mathbf{C}}$ only symmetric equilibria with $d_1 \leq d_{\mathbf{L}}$ are possible.

□

Proof of proposition 6.

Step 1: $\mathcal{E}_L$ is an equilibrium.

Given $\mathbf{d_1} = (d_{\mathbf{L}}, ..., d_{\mathbf{L}})$, the only consistent assessment is $\mathbf{u_1} = (d_{\mathbf{L}}, ..., d_{\mathbf{L}})$. Hence the equilibrium deposit distribution is $D_1^i = S_1(d_{\mathbf{L}})/n$ $(i = 1, ..., n)$ and each bank's expected profits are given by

$$\Pi_1 = \frac{S_1(d_{\mathbf{L}})}{n} p_h(r_{2h} - r_{2l}) > 0.$$

Consider now a deviation of one bank. Deviation to $d_1^{\text{dev}} < d_1$ leads to $u_1^{\text{dev}} < u_1$ and hence to $D_1^{\text{dev}} = 0$, which cannot be profitable. On the other hand $d_1^{\text{dev}} > d_1$ leads to $u_1 < u_1^{\text{dev}} = \bar{u}_{d_1^{\text{dev}}}$ and hence to $D_1^{\text{dev}} = S_1(\bar{u}_{d_1^{\text{dev}}})$. This implies that the deviating bank cannot refinance in the case of low production returns. By our assumption, the punishment P which is imposed by the regulator in this case would outweigh all possible deviation gains.

Step 2: No other equilibria $\mathcal{E} = (d_1)$ with $d_1 \neq d_{\mathbf{L}}$ exist.

If $d_1 < d_{\mathbf{L}}$, then deviation to a slightly higher deposit rate d_1^{dev} ($d_1 < d_1^{\text{dev}} < d_{\mathbf{L}}$) would lead to $u_1 < u_1^{\text{dev}} = \bar{u}_{d_1^{\text{dev}}} = d_1^{\text{dev}}$. Therefore $D_1^{\text{dev}} = S_1(d_1^{\text{dev}}) < S_1(d_{\mathbf{L}})$ and the deviating bank would be able to refinance in both states of production returns. The collection of all savings outweighs the slightly higher interest payment. If $d_1 > d_{\mathbf{L}}$, the only possible assessment is $\mathbf{u_1} = (\bar{u}_{d_1}, ..., \bar{u}_{d_1})$ where $\bar{u}_{d_1} < d_1$. But this implies a positive probability of being closed and suffering the punishment P, which leads to negative expected profits. Of course this is not possible in equilibrium.

□

Proof of lemma 2.

The proof rests on the continuity of the functions $S_i(\cdot)$ $(i = 1, 2)$, $\bar{u}_{(\cdot)}$ and $\bar{q}_{i,(\cdot)}$ $(i = l, h)$. First we prove the continuity for the points $d \in \mathcal{D}_M$. Consider a sequence $(d_n)_{n \in \mathbb{N}}$ with $d_n \to d$ $(n \to \infty)$. If $dS_1(\bar{u}_d) < S_2(r_{2h}/d)$, then $d_n S_1(\bar{u}_{d_n}) < S_2(r_{2h}/d)$ for sufficiently large n. If $dS_1(\bar{u}_d) = S_2(r_{2h}/d)$, we have $\bar{q}_{h,d_n} \to 1$ by equation (10) and $d_2^*(d_n) \to r_{2h}/d$ $(n \to \infty)$. Hence, in both cases we obtain $\pi(d_n) \to \pi(d)$.

For points $d \in \mathcal{D}_H$ the proof is completely analogous. Now suppose that $d \leq d_{\mathbf{L}}$. Again, only the case $d = d_{\mathbf{L}}$ is interesting. If $d_n \to d$, then $q_{l,d_n} \to 1$ by equation (9) and $d_n d_2^*(d_n) \to d_{\mathbf{L}} d_2^*(d_{\mathbf{L}}) = r_{2l}$. Hence $\pi(d_n) \to p_h(r_{2h} - r_{2l})$.

□

Proof of lemma 3.

If $d \in \mathcal{D}_M$, then by equation (9) we obtain

$$q_l = \frac{a_2}{da_1 u^{\alpha_1}} (\frac{r_{2l}}{q_l d})^{\alpha_2}$$

implying that $q_l = cd^{-1}u^{-\tilde{\alpha}_1}$ where

$$c := \Big(\frac{a_2 r_{2l}^{\alpha_2}}{a_1}\Big)^{1/(1+\alpha_2)}$$

and $\tilde{\alpha}_i := \alpha_i/(1+\alpha_i)$ $(i = 1, 2)$. Inserting q_l in equation (8) we find that $\bar{u}_d$ can be described as solution of the equation $F(u, d) = 0$ where

$$F(u,d) := u - p_l c u^{-\tilde{\alpha}_1} - p_h d.$$

But since $F(\bar{u}_d, d) = 0$ we find that $F_u \bar{u}'_d + F_d = 0$ implying that $\bar{u}'_d = -F_d/F_u$.[37] Hence, $\bar{u}'_d > 0$ follows from

$$\begin{aligned} F_d(u,d) &= -p_h \\ F_u(u,d) &= 1 + \tilde{\alpha}_1 p_l u^{-(\tilde{\alpha}_1+1)}. \end{aligned}$$

□

Proof of proposition 7.

The proof follows the same arguments as the proof of proposition 6:

(1) $\mathcal{E}_{ZP}$ is an equilibrium. Deviation to $d_1^{\text{dev}} < d_{\mathbf{ZP}}$ leads to $u_1^{\text{dev}} < u_1$ and hence to $D_1^{\text{dev}} = 0$, which cannot be profitable. On the other hand, $d_1^{\text{dev}} > d_1$ leads to $u_1 < u_1^{\text{dev}} = \bar{u}_{d_1^{\text{dev}}}$ and hence to $D_1^{\text{dev}} = S(\bar{u}_{d_1^{\text{dev}}})$. But since $\pi(d_1^{\text{dev}}) < 0$, deviation profits are negative.

(2) No other equilibria $\mathcal{E} = (d_1)$ with $d_1 \neq d_{\mathbf{ZP}}$ exist. If $d_1 < d_{\mathbf{ZP}}$, then deviation to a slightly higher deposit rate d_1^{dev} $(d_1 < d_1^{\text{dev}} < d_{\mathbf{ZP}})$ leads to $D_1^{\text{dev}} = S(\bar{u}_{d_1^{\text{dev}}})$. By continuity, losses in profits per unit of deposits can be offset by the collection of all savings if $(d_1^{\text{dev}} - d_1)$ is small enough. If $d_1 > d_{\mathbf{ZP}}$, then $\pi(d_1) < 0$, which cannot be the case in equilibrium.

□

Proof of proposition 8.

Considering a deviation $d_1^{\text{dev}} \neq d_1$ from a symmetric equilibrium $\mathbf{d_1} = (d_1, ..., d_1)$, we make the following preliminary remarks:

(1) From proposition 4 we know that only the following two assessments and deposit constellations are possible:

$$\begin{array}{llll} \text{(A1)} & u_1^{\text{dev}} < u_1 & D_1^{\text{dev}} = 0 & D_1 = S_1(\bar{u}_{d_1})/n \\ \text{(A2)} & u_1^{\text{dev}} > u_1 & D_1^{\text{dev}} = S_1(\bar{u}_{d_1^{\text{dev}}}) & D_1 = 0. \end{array}$$

[37] F_u and F_d denote the partial derivatives of F with respect to u and d respectively.

A1 is consistent if $d_1^{\text{dev}} < d_1$ and A2 if $d_1^{\text{dev}} > d_1$. Moreover, if $d_{\mathbf{L}} \leq d_1, d_1^{\text{dev}} \leq d_{\mathbf{ZP}}$, then the following additional consistency conditions hold. A1 is consistent if $\bar{u}_{d_1^{\text{dev}}} \leq \bar{u}_{d_1}$ and A2 if $\bar{u}_{d_1^{\text{dev}}} \geq \bar{u}_{d_1}$. This follows directly from proposition 3 and the fact that (e.g. for A1)

$$\left(p_l I_{l,d_1,d_1^{\text{dev}}} + p_h I_{h,d_1,d_1^{\text{dev}}}\right) d_1^{\text{dev}} \leq p_h d_1^{\text{dev}} < \bar{u}_{d_1^{\text{dev}}} \leq \bar{u}_{d_1}.$$

(2) We can exclude equilibria $\mathcal{E} = (d_1)$ where $d_1 > d_{\mathbf{ZP}}$ because they are negative expected profits equilibria.

(3) We do not have to consider deviations to $d_1^{\text{dev}} \geq d_{\mathbf{ZP}}$ since they cannot be profitable. They lead to zero profits in the case of A1 and to zero or negative profits in the case of A2.

(4) If $d_1 \in \mathcal{U}_{\max}$, then $d_1 \geq d_{\mathbf{L}}$.

Now we turn to the proof of the proposition.

(i) Suppose that $d_1 \in \mathcal{U}_{\max}$. Deviation to $d_1^{\text{dev}} \neq d_1$ with $d_1^{\text{dev}} \leq d_{\mathbf{ZP}}$ cannot be profitable since by TR depositors would always choose to deposit with the non-deviating banks (A1 is consistent and $\bar{u}_{d_1} \geq \bar{u}_{d_1^{\text{dev}}}$).

(ii) We define $\tilde{d} =: \min\{d_{\mathbf{UH}}, d_{\mathbf{ZP}}\}$. From statement (i) we know that $\mathcal{E}_{UH}$ is a Nash equilibrium because $\mathcal{E}_{UH} = (\tilde{d})$ and $\tilde{d} \in \mathcal{U}_{\max}$. No other equilibrium with $d_1 \neq \tilde{d}$ exists, since for $d_1 < \tilde{d}$ deviation to a slightly higher deposit rate $d_1^{\text{dev}} > d_1$ leads to A2 (because $\bar{u}_{d_1^{\text{dev}}} > \bar{u}_{d_1}$) and hence such a deviation is always profitable if $(d_1^{\text{dev}} - d_1)$ is small enough. The same argument applies for $d_1 > \tilde{d}$ if $\tilde{d} = d_{\mathbf{UH}}$. In this case, deviation to slightly lower deposit rates is profitable.

(iii) Case 1 follows from the same arguments as used under (ii). Consider now case 2. $\mathcal{E}_{ZP}$ is a Nash equilibrium according to statement (i). Now turn to the question whether $\mathcal{E}_L$ is an equilibrium. Obviously, only deviations to $d_1^{\text{dev}} > d_{\mathbf{L}}$ with $\bar{u}_{d_1^{\text{dev}}} > d_{\mathbf{L}}$ can be profitable. Hence deviation is profitable if and only if $\Pi^{\text{dev}}(d_{\mathbf{L}}) - \pi(d_{\mathbf{L}})S_1(d_{\mathbf{L}})/n > 0$. Moreover, no other equilibria $\mathcal{E} = (d_1)$ can exist since for $d_{\mathbf{L}} < d_1 < d_{\mathbf{UL}}$, deviation to slightly lower, and for $d_1 < d_{\mathbf{L}}$ and $d_1 \geq d_{\mathbf{UL}}$ deviation to slightly higher deposit rates is profitable by the same arguments as in (ii).

□

Proof of proposition 9.

Suppose that $u_1(\tilde{\mathcal{E}}_{ZP}) \geq u_1(\mathcal{E}_{ZP})$. Then $\tilde{d}_{\mathbf{ZP}} > d_{\mathbf{ZP}}$ (see table 2) and hence $\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) > d_2^*(d_{\mathbf{ZP}})$. But since under $\tilde{\mathcal{E}}_{ZP}$ banks' profits per deposit are given by

$$p_h\Big(r_{2h} - \tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}})\Big),$$

this would imply that those profits are smaller than $\pi(d_{\mathbf{ZP}}) = 0$, which is impossible in equilibrium. To prove that $u_2(\tilde{\mathcal{E}}_{ZP}) < u_2(\mathcal{E}_{ZP})$, we draw on the fact that $\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) \leq d_2^*(d_{\mathbf{ZP}})$.[38] This implies that

$$\begin{aligned} u_2(\tilde{\mathcal{E}}_{ZP}) &= p_l\underline{u} + p_h\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) \\ &\leq p_l\underline{u} + p_h d_2^*(d_{\mathbf{ZP}}) \\ &< u_2(\mathcal{E}_{ZP}). \end{aligned}$$

□

[38]Suppose that $\tilde{d}_2^*(\tilde{d}_{\mathbf{ZP}}) > d_2^*(d_{\mathbf{ZP}})$. Then $\tilde{d}_{\mathbf{ZP}} > d_{\mathbf{ZP}}$ which - as above - would imply that profits per deposits are negative under $\tilde{\mathcal{E}}_{ZP}$.

References

Acharya, F., and J.-F. Dreyfus (1989): "Optimal Bank Reorganization Policies and the Pricing of Federal Deposit Insurance," *Journal of Finance*, 44, 1313–1333.

Bernanke, B., and M. Gertler (1988): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, 14–31.

Bhattacharya, S., A. Boot, and A. Thakor (1998): "The Economics of Bank Regulation," *Journal of Money, Credit and Banking*, 30, 745–770.

Bhattacharya, S., and A. Thakor (1993): "Contemporary Banking Theory," *Journal of Financial Intermediation*, 3, 2–50.

Boot, A., and A. Thakor (1993): "Self-Interested Bank Regulation," *American Economic Review*, 83, 206–212.

Cordella, T., and E. Yeyati (1999): "Bank Bailouts: Moral Hazard vs. Value Effect," IMF Working Paper WP/99/106.

Dewatripont, M., and J. Tirole (1994): *The Prudential regulation of Banks*. MIT Press, Cambridge, Mass.

Diamond, D., and P. Dybvig (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401–419.

Erlenmaier, U., and H. Gersbach (2001): "The Funds Concentration Effect and Discriminatory Bailout," (extended version, June 2001), University of Heidelberg, Mimeo.

Freixas, X. (1999): "Optimal Bail Out Policy, Conditionality and Creative Ambiguity," University of Pompeu Fabra, Mimeo.

Freixas, X., B. Parigi, and J. Rochet (1998): "Lender of Last Resort: A Theoretical Foundation," Universities of Pompeu Fabra, Toulouse and Venice, Mimeo.

Fries, S., P. Mella-Barral, and W. Perraudin (1997): "Optimal Bank Reorganization and the Fair Pricing of Deposit Guarantees," *Journal of Banking and Finance*, 21, 441–468.

Gersbach, H. (1998): "Financial Intermediation, Regulation and the Creation of Macroeconomic Risk," University of Heidelberg, Mimeo.

——— (2001): "Optimal Capital Structure of an Economy," University of Heidelberg, Mimeo.

Goodfriend, M., and J. Lacker (1999): "Limited Commitment and Central Bank Lending," Mimeo.

Goodhart, C. (1995): *The Central Bank and the Financial System.* MIT Press, Cambridge, Mass.

Goodhart, C., and H. Huang (1999): "A Model of the Lender of Last Resort," IMF Working Paper WP/99/39.

Hellwig, M. (1995): "Systemic Aspects of Risk Management in Banking and Finance," *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, 131, 723–737.

——— (1998): "Banks, Markets and the Allocation of Risks in an Economy," *Journal of Institutional and Theoretical Economics*, 154, 328–345.

Humphrey, T. (1986): "The Classical Concept of the Lender of Last Resort," in *Essays on Inflation, 5th ed.*, ed. by T. Humphrey. Federal Reserve Bank of Richmond, Richmond, Va.

Kijotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Mailath, G., and L. Mester (1994): "A Positive Analysis of Bank Closure," *Journal of Financial Intermediation*, 3, 272–299.

Mishkin, F. (1995): "Comments on Systemic Risk," in *Banking, Financial Markets, and Systemic Risk*, ed. by G. Kaufmann. Research in Financial Services, Private and Public Policy, Vol. 7, JAI press Inc., Hampton.

Radelet, S., and J. Sachs (1998): "The Onset of the East Asian Financial Crisis," NBER Working Paper No. 6680.

——— (1999): "What have we learned, so far, from the Asian Financial Crisis?," Working Paper.

Repullo, R. (1999): "Who should Act as Lender of Last Resort? An Incomplete Contracts Model," CEMFI, Madrid, Mimeo.

Santomero, A., and P. Hoffman (1998): "Problem Bank Resolution," Financial Institutions Center, Wharton School, University of Pennsylvania, Working paper.

Schwartz, A. (1995): "Systemic Risk and the Macroeconomy," in *Banking, Financial Markets, and Systemic Risk*, ed. by G. Kaufmann. Research in Financial Services, Private and Public Policy, Vol. 7, JAI Press Inc., Hampton.

Shin, H. (1999): "Risk Management with Interdependent Choice," *Oxford Review of Economic Policy*, 15(3), 52–62.

Staub, M. (1998): "Swiss Regional Bank Crisis," *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, 134, 654–683.