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CESifo Working Papers

TREND AND CYCLES IN U.S. REAL GDP

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CESifo Working Paper No. 607

November 2001

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ISSN 1617-9595

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Abstract

In this paper an Unobserved Components Model is employed to decompose U.S. real GDP into trend and cycle components. The main findings are that there exist three cycles with a period of about two, five and 13 years, respectively, and that the long-run development during the last 50 years can be represented by a segmented linear trend with a break in the drift rate in the early seventies. A further result is a remarkable decrease in the volatility of the cycle component and the recursive residuals over the last two decades.

JEL Classification: C22, E20.

Keywords: trend, cycle, unobserved components models, output gap.

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1 Introduction

A precise and reliable decomposition of the observed time series of real GDP in its trend and cycle components is a crucial task in applied business cycle research and a prerequisite for rational decisions in monetary and fiscal policy. The most popular approach to tackle this problem is the application of some variant of ad hoc filters (e.g., the Hodrick-Prescott or the Baxter-King filter). As is well known, ad hoc filters have a couple of disadvantages as they are typically independent of the time series under analysis (see Maravall 1995).

In this study, an Unobserved Components (UC) Model is used for extracting the trend and cycle components from U.S. real GDP. As Harvey/Jaeger (1993) argue, this class of models provide a useful framework since they "are explicitly based on the stochastic properties of the data". UC-models have a long history in time series econometrics (see Nerlove/Grether/Carvalho 1995) and exhibit a number of advantages. They are based on interpretable and well-defined models for the individual components, are very flexible in accommodating peculiar features of the time series, deliver "optimal" forecasts and can be scrutinized by rigorous tests.

Unobserved Components Models have been used for a time series analysis of U.S. real GDP by Watson (1986), Clark (1987), and Harvey/Jaeger (1993), among others. The novel feature of this study is an investigation whether the total cycle component can be broken up into several subcycles with different lengths and a search for a most simple representation of the trend.

The plan of the paper is as follows: In section 2, we present the basic framework of Unobserved Components Models. Section 3 provides the empirical results for U.S. real GDP from the first quarter of 1950 to the second quarter of 2001. The final section contains a short summary and some concluding remarks.

2 An Unobserved Components Model for quarterly data

Unobserved Components Models are based on the assumption that an observed time series y_t can be decomposed into several components which have an economic interpretation. In the following, we decompose the logarithm of seasonally adjusted real GDP into the unobserved components trend T, cycle C, and the irregular I:

(1) $y_t = T_t + C_t + I_t$.

The trend component represents the long-run development of GDP and is specified as a random walk with a possibly time-varying drift m_t:

(2) $T_t = T_{t-1} + m_{t-1} + e_t$.

e_t is a white noise variable with mean zero and variance s_e^2.

The drift rate m_t is allowed to vary over time and is also defined as a random walk:

(3) $m_t = m_{t-1} + g_2 \, DD_t + x_t$.

DD_t is a dummy variable which can take the values 0 or 1. If it is set to 1 in a specific period, the drift rate shows a jump and the level a kink. The drift impulse x_t is a white noise variable with variance s_x^2.

The cycle component is specified as the sum of M subcycles:

(4) $C_t = \sum_{i=1}^{M} C_{t,i}$.

Each subcycle is specified as a vector AR (1) process:

(5) $$\begin{pmatrix} C_{t,i} \\ C^*_{t,i} \end{pmatrix} = r_i \begin{pmatrix} \cos l_i & \sin l_i \\ -\sin l_i & \cos l_i \end{pmatrix} \begin{pmatrix} C_{t-1,i} \\ C^*_{t-1,i} \end{pmatrix} + \begin{pmatrix} k_{t,i} \\ k^*_{t,i} \end{pmatrix}.$$

The period of subcycle i is $2p / l_i$. The damping factor r_i with $0 < r_i < 1$ ensures that $C_{t,i}$ is a stationary ARMA (2,1) process with complex roots in the

AR-part. The total cycle C_t follows an ARMA (2M, 2M-1) process with restricted MA-parameters (for details see Harvey 1989).

The irregular component comprises a deterministic and a stochastic component:

(6) $I_t = g_0\, DI_t + u_t$.

The deterministic component $g_0\, DI_t$, where DI_t is a 0-1 dummy variable, captures outliers which reflect identifiable events and u_t reflects temporary shocks which are modeled as a stochastic variable. u_t is assumed to be a white noise variable with variance s_u^2.

It is assumed that all disturbances are normally distributed and are independent of each other. This is the usual assumption to assure the identification of the parameters (see, e.g., Watson (1986)).

Estimation of the model parameters is carried out by maximum likelihood in the time domain. The initial values for the stationary cycle components are given by the unconditional distribution and for the nonstationary trend and drift components by a diffuse prior. The filtered and smoothed values of the unobserved components are generated by the Kalman filter.

The unobserved components shown in figure 1 and figure 2 are the values from a fixed interval smoother which utilizes all information in the sample (for details see Harvey 1989).

3 Empirical Analysis

In our empirical analysis, we use quarterly data for seasonally adjusted U.S. real GDP from the first quarter of 1950 to the second quarter of 2001 (Source: Bureau of Economic Analysis).

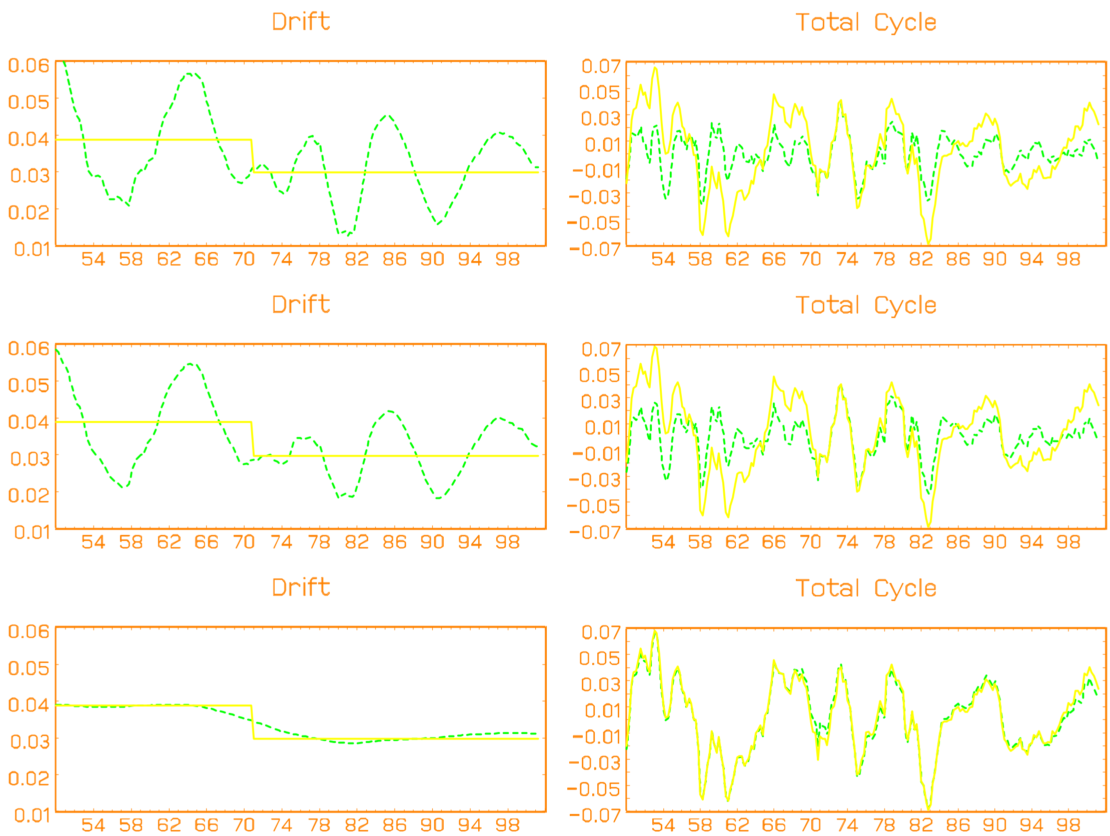
Since the estimated variance of the level shock, s_e^2, was zero in all models considered in this paper, we only present the results for the restricted models where s_e^2 was set to zero.

The basic model contains no deterministic intervention variable. The implications for the drift rate, i.e. the local growth rate of the trend component, and for the cycle component are shown in figure 1 (dotted lines) for a specification with one, two or three subcycles, respectively. If we allow for at most two subcycles, the drift rate exhibits a clear cyclical pattern (with a period of about ten to thirteen years). This is not in accordance with the theoretical concept of a trend which should have no predictable cyclical behavior. The cyclical phenomenon in the drift rate only disappears when three subcycles are included in the model.

It is interesting to note a striking coincidence between the estimated drift rate and the cycle for the models with one or two subcycles and the corresponding series generated by the Hodrick-Prescott filter with a smoothing parameter of 1600 (the graphs are available from the author). This result was also found by Harvey/Jaeger (1993) who suggest "that the HP filter is tailor-made for extracting the business cycle component from US GNP". In light of our results with the model containing three subcycles, this conclusion seems to be unwarranted: Both the HP filter (at least with the standard value of 1600 for the smoothing parameter) and Unobserved Components Models with only one or two cycle components attribute too much of the variation in GDP to the trend component.

In the preferred model with three subcycles (dotted curve in the left bottom graph in figure 1), the evolution of the smoothed drift rate suggests that the most important change is a slowdown between the mid sixties and the late seventies. If one has the suspicion that this break occurred at a more sharply defined time period it is possible to capture this effect by specifying a drift intervention (DD in equation (3)). Searching over all possible dates between the first quarter of 1969 and the last quarter of 1974 yields the highest value of the likelihood function for a break in the first quarter of 1971. As soon as one includes a deterministic drift intervention, the estimated variance s_x^2 of the stochastic drift shock in equation (3) is zero in all models. This result holds irrespective of the date for the assumed break in the interval between 1969 and 1974. The results presented for the models with a deterministic break are based on a restricted specification where s_x^2 is a priori set to zero.

Figure 1: Drift and total cycle component for different UC models



Notes:

The top panel gives the drift rate and the total cycle for models with one cyclical component, the middle panel for models with two subcycles, and the bottom panel for models with three subcycles.
The solid lines represent the model with a break in the drift rate in 1971:1, the dotted lines the model with a purely stochastic drift rate.
The drift rate is expressed as a yearly rate.

Table 1: Summary Statistics

	log Lik	AIC	SIC
1 Subcycle			
with drift intervention	660.1	-1,306.3	-1,283.1
without intervention	658.2	-1,302.4	-1,279.1
2 Subcycles			
with drift intervention	667.7	-1,315.4	-1,282.1
without intervention	663.3	-1,306.5	-1,273.4
3 Subcycles			
with drift intervention	673.3	-1,320.6	-1,277.5
without intervention	667.9	-1,309.8	-1,266.7

Notes:

The break in the drift rate takes place in 1971 : 1.
logLik denotes the value of the maximized log likelihood function, AIC the Akaike Information Criterion, and SIC the Schwarz Information Criterion.

For a given number of subcycles, all model selection criteria (logLik, AIC, SIC) select the model with a deterministic slope intervention (see Table 1). When we restrict our attention to this model type, likelihood-ratio tests and the Akaike criterion prefer a specification with three subcycles. As is well known, the SIC criterion penalizes additional parameters more heavily than the other criteria. Consequently, it selects the model with just one subcycle. However, since the autocorrelation function of the recursive residuals (the one-step prediction errors) is much worse for this specification than in case of a model with three subcycles, we choose the latter as our preferred model.

Table 2 presents the estimated parameters for three versions of the model with three subcycles. Version 1 represents a purely stochastic model without any deterministic intervention dummy. Version 2 contains the drift intervention in the first quarter of 1971. Since the Jarque-Bera test statistic indicates a significant deviation of the recursive residuals from normality, we took a closer look at the residuals and identified four outliers (defined as values absolutely greater than 2.6 times the standard error). Version 3 is the model with the deterministic break, amended by four impulse interventions DI (equation (6)). The estimated values for the other model parameters are practically not affected by the inclusion of the dummies.

The diagnostics do not reject this model: The Jarque-Bera test statistic is 2.0, all of the first 12 auto-correlation coefficients for the recursive residuals are not significant at the 5 % significance level and the CUSUM-test shows no sign of mis-specification.

All of the three versions imply that there exist three cycles in the U.S. real GDP series with a period of 2.2 years, 5.2 years, and 13.1 years, respectively. Figure 2 shows the evolution of the three subcycles and of the total cycle. The short cycle has a marked contribution in the fifties and eighties but is very small in the seventies and nineties. The intermediate cycle shows the highest amplitude between 1970 and 1983 but exhibits remarkably damped waves afterwards. The long cycle displays rather regular sine-like waves with a small “irregularity” around 1975. In this period, the regular cyclical downswing seems to be substituted by the permanent decrease in the long-run growth rate.

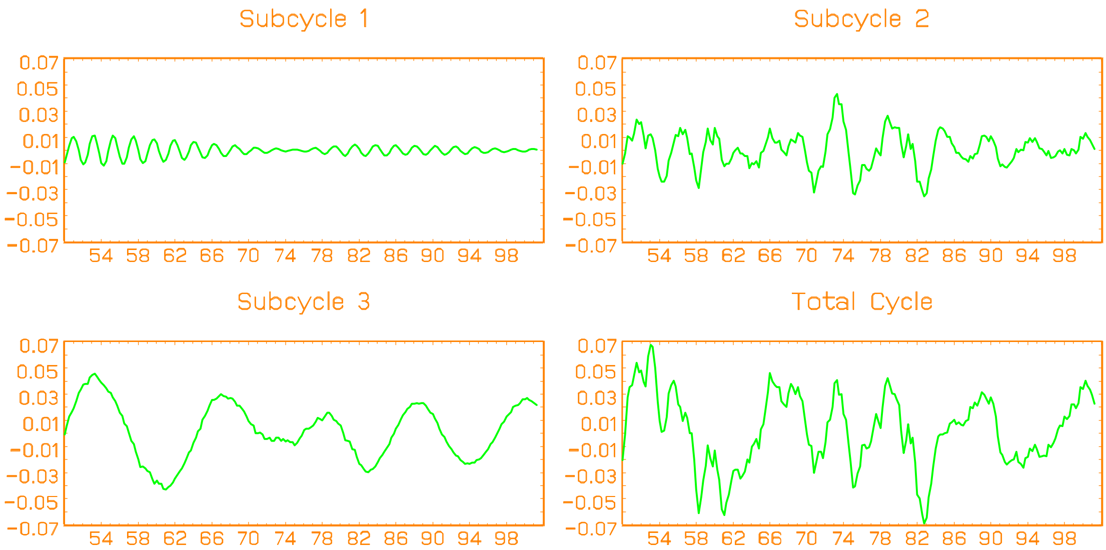
The results provided in this paper also suggest that a very good representation of GDP can be found by specifying a very smooth trend (Rotemberg (1999) argues persuasively in favor of “trends that are as smooth as possible subject to the constraint that the cycles be reasonable behaved.”). The smoothness of the proposed trend is only interrupted in the early seventies when the annualized growth rate drops from 3.9 % to 3.0 %.

Table 2: Parameter estimates for a models with three subcycles

	Version 1		Version 2		Version 3	
Trend						
s_e	-		-		-	
s_x	0,0002	(1,7)	-		-	
DD (71:1)	-		-0,0023	(7,3)	-0,0022	(7,7)
Cycle						
r_1	0,9903	(102,3)	0,9907	(100,7)	0,9899	(105,5)
l_1	0,7161	(62,8)	0,7164	(65,3)	0,7159	(65,4)
s_{k_1}	0,0007	(1,9)	0,0007	(1,7)	0,0008	(2,0)
r_2	0,9202	(28,4)	0,9177	(35,0)	0,9262	(40,2)
l_2	0,3072	(5,9)	0,3016	(7,1)	0,3030	(7,9)
s_{k_2}	0,0059	(2,6)	0,0064	(7,3)	0,0059	(6,9)
r_3	0,9834	(49,3)	0,9892	(93,5)	0,9897	(96,0)
l_3	0,1114	(6,9)	0,1199	(9,8)	0,1201	(10,6)
s_{k_3}	0,0045	(1,3)	0,0035	(2,4)	0,0034	(2,4)
Irregular						
s_u	0,0001	(0,1)	0,0000	(0,0)	0,0000	(0,0)
DI (1958:1)	-		-		-0,0145	(1,8)
DI (1971:1)	-		-		0,0112	(3,8)
DI (1978:2)	-		-		0,0146	(5,5)
DI (1980:2)	-		-		-0,0083	(2,5)
logLik	667,9		673,3		684,1	

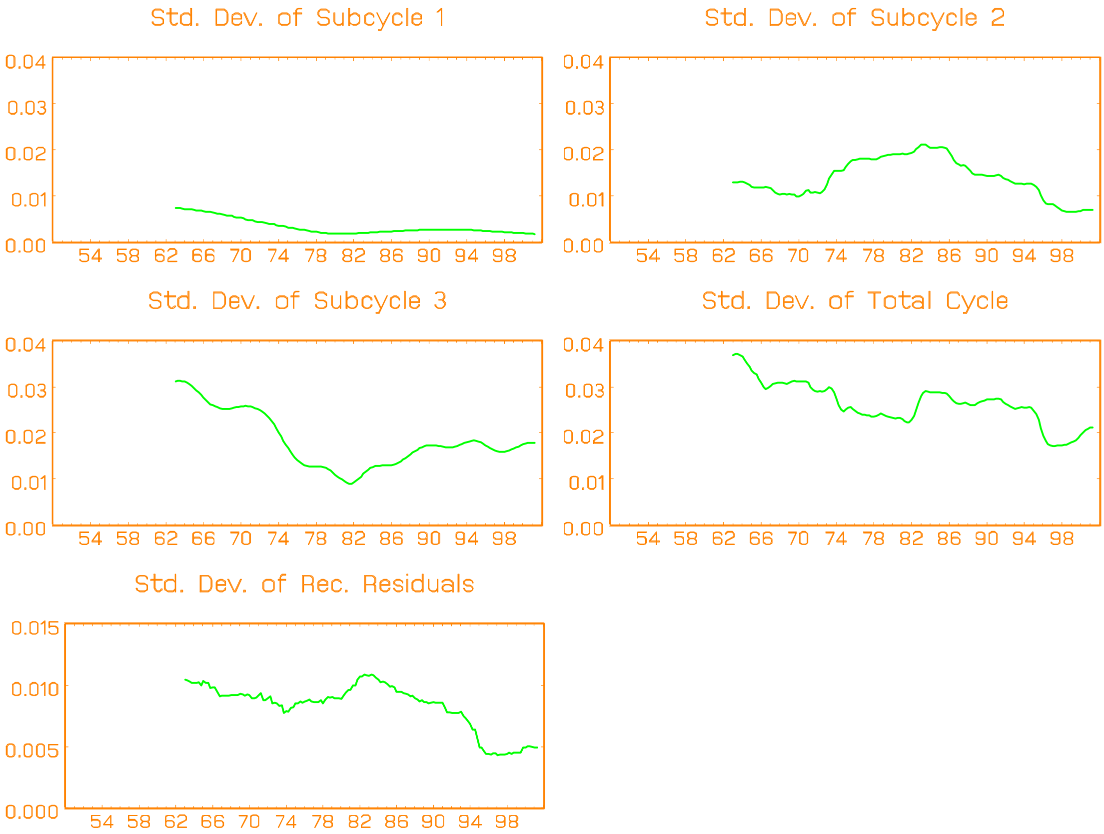
Note: t-values in parentheses.

Figure 2: Three subcycles and the total cycle



An important feature of the U.S. economy discussed in the recent literature (see, e.g., Blanchard/Simon (2001); McConnell/Perez-Quiros (2000)) is the apparent decrease in the volatility of GDP. We analyze this aspect in a somewhat preliminary way using the framework of Unobserved Components Models. To be concrete, we compute rolling standard deviations of the three subcycles, the total cycle, and the recursive residuals by employing a window of 52 quarters (the mean length of the long subcycle). Figure 3 presents the results for the period from 1963 to 2001.

Figure 3: Rolling standard deviations



The standard deviation of the short cycle is more or less steadily declining (with a small hump at the end of the eighties), that of the intermediate cycle shows an increase until 1983 and since then a steady decrease (the opposite holds for the long cycle), and the standard deviation of the total cycle clearly shows a negative trend. The most impressive change is for the standard deviation of the recursive residuals. Since 1982, it declined by more than half. This finding of a decrease in the variance of shocks over the last two decades corroborates the results obtained by Blanchard/Simon (2001) and others.

As a last exercise, we compare the NBER chronology of business cycles with the troughs and peaks of the cycle component generated by the UC-model.

Table 3: Dates of troughs and peaks of business cycle

Trough		Peak	
NBER	UC	NBER	UC
-	-	1953 : 2	1953 : 1
1954 : 2	1954 : 2	1957 : 3	1955 : 3
1958 : 2	1958 : 2	1960 : 2	1959 : 2
1961 : 1	1961 : 1	1969 : 4	1966 : 1
1970 : 4	1970 : 4	1973 : 4	1973 : 2
1975 : 1	1975 : 1	1980 : 1	1978 : 4
1980 : 3	1980 : 3	1981 : 3	1981 : 1
1982 : 4	1982 : 4	1990 : 3	1989 : 1
1991 : 1	1993 : 3	-	2000 : 2

Table 3 shows the dates for the troughs and peaks constructed by the NBER (Source: www.nber.org/cycles.html) and those generated by the UC-model. It has to be stressed that the concepts are not identical. The NBER defines a recession as “a period of significant decline in total output, income, employment, and trade”. Recessions start at the peak of a business cycle and end at the trough. In the framework of this paper, a peak (a trough) is a local maximum (minimum) of the cycle component, i.e. the deviation of output from trend.

With the exception of the last cycle the dates of the troughs are identical for both methods. The reason is that in the quarter following a NBER-trough the growth rate of GDP is usually higher than the growth rate of trend, i.e. the growth rate of the cycle component is positive. After the NBER-trough in 1991:1, the growth rates of GDP recovered so weakly that it took two and a half years until the trough of the cycle component was reached. This finding may cast some doubts about the often asserted exceptionally long upswing in the nineties.

In contrast to the troughs, the dates for the peaks are systematically different: The peak of the cycle components leads the NBER peak by one to 15 quarters, with a mean of 5.5 quarters. The reason is that after a peak of the cycle component its growth rate is negative, but lower than the drift rate when measured in absolute terms. As long as this is the case, the growth rate of GDP is still positive. But the probability is very high that after a couple of quarters we will see a recession with a decrease in real GDP. In this sense, observing a decline in the cycle component is a strong indicator for a recession in the near future. It is left to future research to investigate the reliability of this indicator in real time.

4 Summary and Conclusion

Unobserved Components models are a useful framework for analyzing economic time series. The decomposition of U.S. real GDP in trend and cycle components shows that we can characterize this time series as the sum of a segmented linear trend and three cycles. The intermediate cycle with a period of about five years and the long cycle with a period of about 13 years are consistent with some ideas in classical business cycle theory where a “cycle minor” and a “cycle major” were important ingredients (see, e.g., Matthews (1959) or Schumpeter (1939)). A long cycle with a period of 14 years (7 fat and 7 lean years) was already mentioned in the Bible (Genesis, chapter 41).

These findings have an important implication. Ad hoc filters like the Hodrick-Prescott or the Baxter-King filter may produce a misleading representation of output series. In the design of these filters it is often assumed that business cycles have a period between two and about seven years. Implementing such a filter attributes too much of the variability of GDP to the trend component. The danger is that a long upswing (as in the nineties) is erroneously interpreted as a permanent increase in the drift rate and not as part of a long cycle which will inevitably lead to a prolonged period of lower growth rates.

A second major finding of this study is the remarkable decrease in the volatility of the cycle component and of the one-step prediction error in the course of the last twenty years or so. The implications for business cycle research are far from obvious (for some speculative arguments see Blanchard/Simon (2001)). Unobserved Components models may be a useful tool for future research on this topic. It is possible to amend the basic model outlined in section 2 by time-varying variances of the cycle shocks k and to test for a regime shift in a rigorous way. A promising approach is the combination of this extension with the specification of multivariate models for several time series which have common cycles.

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