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SPEED LIMIT POLICIES: THE OUTPUT GAP AND OPTIMAL MONETARY POLICY

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Abstract

In a standard New Keynesian model, a myopic central bank concerned with stabilizing inflation and changes in the output gap will implement a policy under discretion that replicates the optimal, timeless perspective, precommitment policy. By stabilizing output gap changes, the central bank imparts inertia into output and inflation that is absent under pure discretion. Even a fully optimizing (i.e., non-myopic) central bank operating in a discretionary policy environment achieves better social outcomes if it focuses on inflation and changes in the output gap than are achieved under inflation targeting.

JEL Classification: E42, E52, E58.

Keywords: monetary policy, inflation targeting, targeting regimes.

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1 Introduction

Recent work on the design of monetary policy reflects a general consensus on the appropriate objectives of monetary policy. As articulated by Svensson, "... there is considerable agreement among academics and central bankers that the appropriate loss function both involves stabilizing inflation around an inflation target and stabilizing the real economy, represented by the output gap" (Svensson 1999a). Such a loss function forms a key component of "The Science of Monetary Policy" (Clarida, Galí, and Gertler 1999), and has been widely used in recent work on policy design (e.g., McCallum and Nelson 1999, 2000, Jensen 2001, Svensson and Woodford 2000, Vestin 2000, Nessén and Vestin 2000, Söderström 2001, and McCallum 2001). Woodford (1999a) has derived the assumptions under which a quadratic loss function in inflation and the output gap is the correct approximation to the utility of the representative agent.

Despite this apparent agreement about the objectives of policy, it is not clear that stabilizing inflation and level of the output gap are the objectives actually pursued in the conduct of policy. In justifying interest rate increases during 2000, the press releases from the Federal Open Market Committee emphasized the *growth* in output relative to the *growth* in potential rather than the output gap itself (the *level* of output relative to potential).[1] In remarks at the Wharton Public Policy Forum in April 22, 1999, Fed Governor Edward M. Gramlich also describes monetary policy in terms of a focus on demand growth relative to growth in potential output:

> "Solving a standard model of the macroeconomy, such a policy would effectively convert monetary policy into what might be called 'speed limit' form, where policy tries to ensure that aggregate demand grows at roughly the expected rate of increase of aggregate supply, which increase can be more easily predicted."[2]

Growth in demand relative to growth in potential is equal to the *change* in the output gap. The purpose of this paper is to examine what role changes in the output gap – a speed limit policy in Gramlich's words – should play in the design of monetary policy.

Gramlich's comments suggest measurement error is one factor favoring a speed limit policy. If the growth rate of potential is measured more accurately than its level, first differencing the log level of the estimated gap will reduce the variance of the remaining measurement error. I ignore this attribute of a speed limit policy to focus on an aspect of such policies that has not previously been identified. In a standard, forward-looking New Keynesian model, I show that a completely myopic central bank who acts with discretion to minimize a one period loss function in the variability of inflation and the change in the output gap will replicate the socially optimal policy outcomes of a central bank able to precommit. Pure discretion, in which the central bank minimizes the social loss function but is unable to precommit, leads to inefficient stabilization in the

[1] For example, following rate increases during the first half of 2000, the FOMC stated on Feburary 2 that "The Federal Open Market Committee voted today to raise its target for the federal funds rate by 25 basis points to 5-3/4 percent. .... The [Federal Open Market] Committee remains concerned that over time, increases in demand will continue to exceed the growth in potential supply." On May 16, the FOMC's press release stated that "The Federal Open Market Committee voted today to raise its target for the federal funds rate by 50 basis points to 6-1/2 percent. .... Increases in demand have remained in excess of even the rapid pace of productivity-driven gains in potential supply..."

[2] Gramlich went on to note, ".. the monetary authority is happy with the cocktail party temperature at present but moves against anything that increases its warmth. Should demand growth threaten to outrun supply growth (the party to warm up), the seeds of accelerating inflation may be planted and monetary policy should curb the growth demand by raising interest rates."

face of cost shocks (Woodford 1999). It is this inefficiency that is removed if the central bank myopically follows a speed limit policy.

The reason for this surprising result can be traced to Woodford's demonstration that an optimal precommitment policy involves inertia when expectations are forward looking. By imparting inertia into policy actions, the central bank's current actions directly affect the public's expectations of future inflation. A central bank concerned only with social loss but operating under discretion will fail to introduce any inertia. When the central bank strives to stabilize the change in the output gap, however, the lagged output gap becomes an endogenous state variable. This introduces inertia into monetary policy, even under discretion. If the central bank places the same weight on stabilizing the change in the gap as society places on output gap stabilization, then the myopic central bank acting with discretion imparts exactly the optimal degree of inertia into its policy actions.

While the assumption of myopic behavior is not realistic, this result suggests that there may be an important role for the change in the output gap in policy design. At the very least, it suggests that a closer examination of the role of the output gap as a policy objective is called for. To carry out this examination, I employ a parameterized New Keynesian model and evaluate a speed limit policy against other policy regimes. I find that a policy based on targeting the change in the output gap dominates inflation targeting unless inflation adjustment is predominately backward looking. And while optimal inflation targeting involves appointing a weight-conservative central banker who values inflation stability more highly than does society, society can do even better by appointing a *liberal* central banker who highly values stability in output gap changes.

The next section sets out the basic model and, as a benchmark, derives the fully optimal commitment and discretionary policies. The calibrated values of the model's parameters are discussed, and the asymptotic social loss function is evaluated under both precommitment and discretion. As Jensen (2001) and McCallum and Nelson (2000) have previously shown, precommitment achieves a lower value of the loss function than does discretion.

Section 3 demonstrates that the precommitment equilibrium can be achieved under a central bank that myopically minimizes a loss function that depends on inflation and the change in the output gap. This result does not carry over to the case of a fully optimal discretionary central bank, but numerical simulations help define the parameters of the model that govern whether a speed limit policy dominates pure discretion.

Section 4 introduces inflation persistence into the model. Previous research (Rudebusch 2000) has shown that the presence of lagged inflation in the inflation adjustment equation can affect the ranking of alternative policy rules. I compare a speed limit policy to pure discretion as the relative weight on lagged inflation varies. If policy delegation also includes setting the weight the central bank places on its output objective, as in Rogoff (1985) and Jensen (2001), a gap change objective assigned to a liberal central bank dominates assigning the social loss function to a conservative unless inflation is largely backward looking in nature.

Section 5 extends the model to allow for stochastic fluctuations in potential output. This extended model is then used to compare a variety of alternative targeting regimes, including income growth targeting and nominal income growth targeting. In general, a speed limit policy dominates the alternatives of inflation targeting or nominal income growth targeting. Conclusions are summarized in section 6.

2 The basic model under precommitment and discretion

The basic New Keynesian model consists of two equilibrium relationships: an aggregate demand condition that links output and the real interest rate (an "expectational IS curve"), and an inflation adjustment equation. Clarida, Galí, Gertler (1999), Woodford (1999, 2000), McCallum and Nelson (1999), Svensson and Woodford (1999, 2000), among others, have popularized this simple model for use in monetary policy analysis. Its foundations are discussed in Walsh (1998).

The aggregate demand relationship is derived from the first order Euler condition for the representative household's optimal consumption choice problem. Assuming constant relative risk aversion and separability between consumption and leisure, the Euler condition can be approximated around the steady-state as

$$y_t = \mathrm{E}_t y_{t+1} - \sigma \left(R_t - \mathrm{E}_t \pi_{t+1} \right) + u_t \tag{1}$$

where y is output, π is the inflation rate, R is the nominal interest rate, and u is a stochastic disturbance. The parameter σ is equal to the steady-state ratio of consumption to output times the household's elasticity of intertemporal substitution (the inverse of the coefficient of relative risk aversion). All variables are expressed as percent deviations around the steady-state. If output demand arises from consumption and government purchases, then u_t includes $g_t - \mathrm{E}_t g_{t+1}$, where g is the percent deviation of government purchases around the steady-state.

The second component of the model is an inflation adjustment equation. Most recent analyses have employed the Calvo specification of staggered price adjustment, but Roberts (1995) shows that other basic models of price adjustment lead to a similar specification (see also Walsh 1998). With sticky prices, firms must base their pricing decisions on real marginal costs and their expectations of future price inflation. As a consequence, current inflation is given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t \tag{2}$$

where x is the output gap, defined as the difference between actual output and the flexible price equilibrium level of output.[3] The cost shock e_t is assumed to be a white noise process.[4] An appendix available from the author provides a detailed derivation of equations (1) and (2) from their microeconomic foundations.

The final aspect of the model specification is the social loss function. As is standard in this literature, this is taken to be a function of inflation and output gap variability:

$$L_t = (1-\beta)\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda x_{t+i}^2\right] \tag{3}$$

This specification reflects the widespread agreement over the objectives of monetary policy alluded to by Svensson (1999b, 1999c). Woodford (1999a) discusses the conditions under which equation (3) can be interpreted as an approximation to the utility of the representative agent.

2.1 Precommitment

A central bank able to precommit to a policy rule chooses a path for current and future inflation and the output gap to minimize the social loss function (3) subject to the inflation adjustment equation (2). Letting $2\psi_{t+i}$ denote the Lagrangian multiplier associated with the period $t+i$ inflation adjustment equation, the central bank's problem is to minimize

$$(1-\beta)\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) + 2\psi_{t+i}\left(\pi_{t+i} - \beta\pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}\right)\right]$$

The first order conditions for this problem are

$$\pi_t + \psi_t = 0 \tag{4}$$

$$\mathrm{E}_t\left(\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}\right) = 0 \qquad i \geq 1 \tag{5}$$

$$\mathrm{E}_t\left(\lambda x_{t+i} - \kappa\psi_{t+i}\right) = 0 \qquad i \geq 0 \tag{6}$$

Equations (4) and (5) reveal the dynamic inconsistency that characterizes the optimal precommitment policy. At time t, the central bank sets $\pi_t = -\psi_t$

[3] This simple inflation adjustment equation has been criticized on several grounds. Estrella and Fuhrer (1998) argue it implies implausible inflation dynamics, while Fuhrer (1997) and Rudebusch (2000b) find that lagged inflation is much more important than the forward looking expectational variable implied by theory. On this last point, Galí and Gertler (1999) argue that the poor empirical performance of equations such as (2) arises from the use of the output gap in place of the theoretically correct real marginal cost. Erceg, Henderson, and Levin (2000) allow for both price and wage stickiness. In section 4 below, equation (2) is modified to include lagged inflation.

[4] This assumption is modified in section 5.

and promises to set $\pi_{t+1} = -\left(\mathrm{E}_t\psi_{t+1} - \psi_t\right)$. But when period $t+1$ arrives, a central bank that reoptimizes will again obtain $\pi_{t+1} = -\psi_{t+1}$ as its optimal setting for inflation, since the first order condition (4) updated to $t+1$ will reappear. Defining policy under commitment as the solution to (4)–(6) implies a choice for π_{t+1}, π_{t+2}, ... that the central bank knows it will not wish to implement. As McCallum and Nelson (2000) note, this "behavior seems highly implausible..."

An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (5) and (6) for all periods, including the current period. Woodford (1999) has labeled this the "timeless perspective" approach to precommitment. One can think of such a policy as having been chosen in the distant past, and the current values of the inflation rate and output gap are the values chosen from that earlier perspective to satisfy the two conditions (5) and (6). McCallum and Nelson (2000) provide further discussion of the timeless perspective and argue that this approach agrees with the one commonly used in many studies of precommitment policies.

There is a third approach to defining a commitment policy that warrants mention, since it represents the natural extension of the approach used in the non-forward looking models employed in the traditional Barro and Gordon literature. In the model consisting of equations (1) and (2), the only state variable is the current cost-push shock realization e_t. The logic employed in the Barro-Gordon literature defined commitment policies as the choice of a rule expressing the policy instrument as a function of the current state. In the present case, it would correspond to the choice of a rule of the form $x_t = be_t$ that minimizes the loss function subject to equation (2). Woodford (1999) shows, however, that such a policy is suboptimal when expectations are forward-looking. A fully optimal precommitment policy will display inertia.

The definition of the optimal precommitment policy used in this paper is that of the timeless perspective approach. Combining (5) and (6), inflation and the output gap satisfy

$$\pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1}) \tag{7}$$

for all $i \geq 0$ under the optimal precommitment policy.

The impact of a cost shock on inflation and the output gap under optimal precommitment can be obtained by calibrating equations (2) and (7) and numerically solving them. Three unknown parameters appear in the model: β, κ, and λ. The discount factor, β, is set equal to 0.99, appropriate for interpreting the time interval as one quarter. A weight on output fluctuations of $\lambda = 0.25$ is used. This value is also employed by Jensen (2001) and McCallum and Nelson (2000). The parameter κ captures both the impact of a change in real marginal cost on inflation and the co-movement of real marginal cost and the output gap. McCallum and Nelson (2000) characterize the empirical evidence as consistent with a value for the impact of the output gap on inflation (κ) in the range [0.01, 0.05]. Roberts (1995) reports higher values, and Jensen (2001) sets $\kappa = 0.1$. I

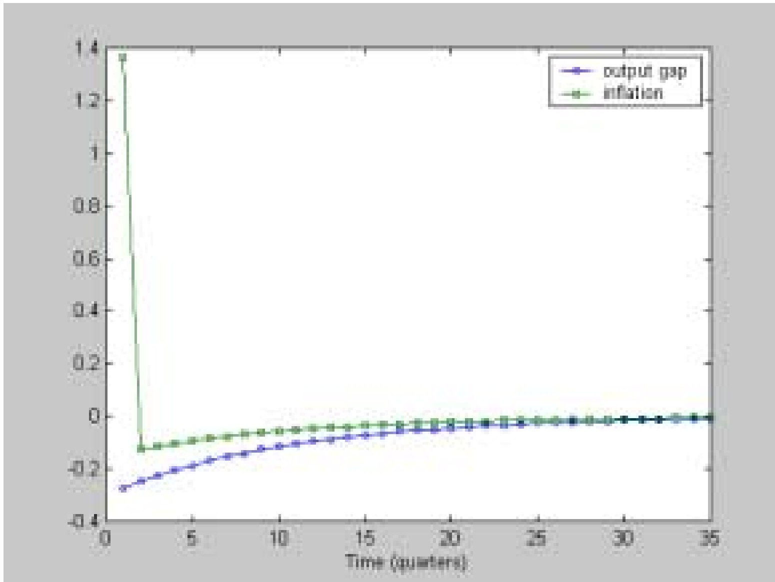

Figure 1: Responses to a Temporary Cost Push Shock under the Optimal Precommitment Policy

set $\kappa = 0.05$ as the baseline value, but results are reported for both larger and smaller values.

Figure 1 shows the response of inflation and the output gap to a transitory cost push shock under the optimal precommitment policy. Despite the fact that the shock itself has no persistence, the output gap displays strong, positive serial correlation. By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in $\mathrm{E}_t\pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.

2.2 Optimal discretion

In contrast to the case of precommitment, a central bank that operates in a discretionary policy regime takes expectations as given. The central bank may recognize that expectations of future inflation depend, through the public's process for forming expectations, on the current state. But in the present model, the state is simply the exogenous shock e_t. Thus, the central bank in a discretionary environment cannot affect the public's expectations of future inflation and so treats these as given in choosing its policy for period t. The policy problem reduces to the simple one-period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to

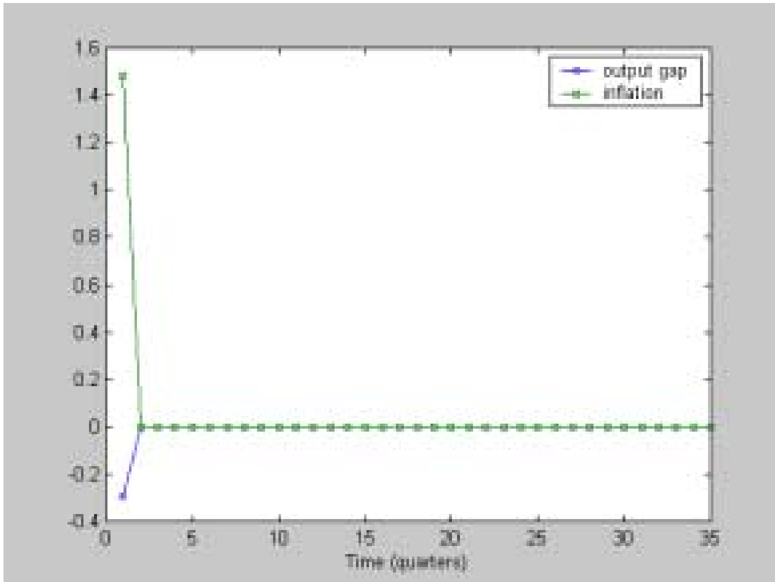

Figure 2: Responses to a Temporary Cost Push Shock under the Pure Discretionary Policy

(2), with expectations given. The first order conditions under discretion imply

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right) x_t \tag{8}$$

In a discretionary policy regime with the central bank acting to stabilize inflation and the output gap, the equilibrium inflation and output gap are determined by equations (2) and (8). Figure 2 shows the impulse response of inflation and the output gap to a cost shock. The figure, which should be compared with Figure 1, reveals that both inflation and the output gap return to baseline just one period after a positive inflation shock under a discretionary policy regime. None of the persistence generated by the optimal precommitment policy occurs under discretion. A temporary cost shock moves the output gap below zero and inflation above zero, but only for a single period.

Table 1 compares the asymptotic social loss under commitment and discretion for the baseline parameter values and for larger and smaller values of λ.[5]

[5] The variance of the cost shock is set equal to 0.015; this value affects the absolute magnitude of the loss function but not the relative comparions across regimes since e is the only shock. As $\beta \rightarrow 1$, the asymptotic value of the loss function is calculated as $\sigma_\pi^2 + \lambda\sigma_x^2$, where σ_π^2 and σ_x^2 are the asymptotic variances of inflation and the output gap. In all the models considered in this paper, the linear rational expectations solutions take the form $Z_t = MZ_{t-1} + v_t$

Table 2 reports the standard deviations of inflation and the output gap under precommitment and pure discretion. Under discretion, the output-inflation trade-off is less advantageous. In response to a cost shock, the central bank allows inflation to fluctuation more, and the output gap less, than would be the case under an optimal precommitment policy. As a consequence, the loss from operating in a discretionary policy regime is greatest when inflation stabilization is relatively more important (i.e., as λ becomes smaller).

Table 1: Asymptotic Loss (social loss x 10^4)

	λ			
	0.1	0.25	0.5	1.0
Commitment	1.939	2.055	2.116	2.161
Discretion	2.195	2.228	2.239	2.244
% loss from discretion	13.20%	8.42%	5.81%	3.84%

Table 2: Standard Deviations

	λ			
	0.1	0.25	0.5	1.0
Precommitment				
σ_π	1.335	1.396	1.427	1.450
σ_x	1.252	0.655	0.399	0.243
Pure discretion				
σ_π	1.463	1.485	1.493	1.496
σ_x	0.732	0.297	0.149	0.075

3 Discretion and the change in the output gap

Much of the recent literature on monetary policy design has assumed the central bank can commit to a policy rule, and optimal rules or rules constrained to take simple forms (such as Taylor rules) are evaluated. Less well understood is how the gains of commitment in forward looking models might be obtained even if the central bank must operate with discretion.[6] An exception is Jensen (2001) who considers the optimal assignment of a nominal income growth objective to the central bank (in addition to inflation and output gap objectives).

where v_t is a vector of mean zero, serially uncorrelated innovations. The variance covariance matrix of Z, denoted by Σ_{ZZ}, is obtained from

$$vec(\Sigma_{ZZ}) = [I - (M \otimes M)]^{-1} vec(\Sigma_{vv})$$

where Σ_{vv} is the variance-covariance matrix of v and $vec(X)$ is the vector of stacked columns of a matrix X. The unconditional variances of inflation and the output gap can then be found as $C\Sigma_{ZZ}C'$ for a suitably defined matrix C.

[6] The literature provides numerous possible solutions to the average inflation bias that could arise under discretion in the Baro-Gordon model, including delegation to a conservative central bank (Rogoff 1985), incentive schemes (Walsh 1995), and inflation targets (Svensson 1997).

He numerically calculates the optimal weights on nominal growth and inflation objectives that society should assign to a central bank operating under discretion. Thus, rather than assume the central bank can commit to a simple rule, Jensen evaluates how changing the *objectives* of the central bank might affect output and inflation. This approach parallels that used to develop solutions to the traditional average inflation bias arising under discretion (e.g., Rogoff 1985, Walsh 1995, and Svensson 1997). Similarly, Woodford suggests that adding an interest smoothing objective to the policy maker's loss function can improve outcomes by introducing inertia.

As argued in Walsh (1995), the appropriate starting point is to derive the optimal objectives of the central bank and then to evaluate how these might be implemented through, for example, inflation targeting (Svensson 1997) or nominal income growth targeting. While the use of the Barro-Gordon model, or other backward looking models, suggested that simply ensuring the central bank focuses on inflation and the output gap was sufficient to replicate the optimal commitment policy, this is no longer true when agents are forward-looking. The next subsection shows that, in one special case, the central bank should focus on stabilizing inflation and the change in the output gap – that is, it should follow a speed limit policy. This motivates the closer examination given to a speed limit policy in the following sections .

3.1 Myopic discretion

Consider the case of a myopic central bank, concerned only with minimizing its current period loss function, taking private sector expectations as given. Such a central bank ignores the intertemporal aspects of the policy problem for two reasons. First, because it is operating under discretion, it treats expectations of future inflation and output as given. Second, because it is assumed to act myopically, it ignores the impact its current policy choice may have on future states. To analyze how society would wish such a central bank to act, assume the central bank's loss function can differ from society's loss function given by (3). This simply reflects the fact that societies frequently assign goals to governmental policy making institutions, and these goals can differ from "social welfare" itself. Specifically, the central bank's loss function is modified to take the form

$$(1-\beta)\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) + T(\pi_{t+i}, x_{t+i}; s_{t+i}) \right] \tag{9}$$

where $s_t = \{\pi_{t-1}, x_{t-1}, e_{t-1}, s_{t-1}\}$ is the history of the economy up to date t.

A completely myopic central bank acting under pure discretion solves a single period problem in which it minimizes $\frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + T(\pi_t, x_t; s_t)$, subject to (2), taking the current state s_t and expectations as given. The first order conditions are

$$\pi_t + \frac{1}{2} T_\pi + \varphi_t = 0 \tag{10}$$

$$\lambda x_t + \frac{1}{2}T_x - \kappa\varphi_t = 0 \tag{11}$$

where φ_t is the Lagrangian multiplier on the inflation adjustment equation (2) that constrains the joint behavior of inflation and the output gap. Substituting (11) into (10),

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)x_t - \frac{1}{2}\left(\frac{1}{\kappa}\right)T_x - \frac{1}{2}T_\pi \tag{12}$$

Comparing (12) with (7) shows that if $T_x = -2\lambda x_{t-1}$ and $T_\pi = 0$, (12) becomes identical to (7). Myopic discretion then replicates the inflation and output gap outcomes that occur under the optimal precommitment policy. One choice for T that satisfies these restrictions is obtained by setting $T(x_{t-1}) = -2\lambda x_{t-1}x_t + \lambda x_{t-1}^2 + T_0$, where T_0 is an arbitrary constant. In this case, the loss function of the myopic central bank is

$$\left[\pi_t^2 + \lambda(x_t - x_{t-1})^2\right] + T_0$$

Thus, a myopic central bank operating under discretion will achieve the optimal precommitment policy outcome if it acts to minimize fluctuations in inflation and the *change* in the output gap – a speed limit policy. This result follows immediately when it is recognized that the relationship between inflation and the change in the output gap implied by the optimal precommitment policy and given in equation (7) is identical to the first order condition for a discretionary central bank with a loss function equal to $\pi_t^2 + \lambda\left(x_t - x_{t-1}\right)^2 + T_0$.[7]

If potential output follows a deterministic time trend, then $x_t - x_{t-1}$ is equal to output growth relative to trend.[8] It follows that in this case, a myopic central bank operating under discretion will achieve the optimal precommitment policy outcome if its loss function is a function of inflation variability and the variability of the growth rate of real output relative to trend growth, given by $\pi_t^2 + \lambda(y_t - y_{t-1} - \delta)^2$, where δ is the trend growth rate of potential output.

If the central bank is concerned with changes in the output gap, a natural inertia is introduced into the policy process in a way that mimics the optimal precommitment solution. A positive inflation shock is met with a real contraction that lowers the output gap. If policy actions are completely temporary, as they are under pure discretion based on the social loss function, the change in the output gap in the period following the shock will be positive as output rebounds from the temporary contraction. A central bank that is concerned with stabilizing the change in the gap will continue to maintain a contractionary policy to dampen this increase in the gap.

[7] The proposed form of T is not unique. For example, $T_x = 0$ and $T_\pi = p_{t-1}$ also yields (7). This choice is equivalent to price level targeting (Svensson 1999b, Vestin 2000). Ben Friedman has pointed out to me the similarity of a speed limit policy to the derivative corrective factor analyzed by Phillips (1957).

[8] Suppose $\bar{y}_t = \bar{y}_0 + \delta t$. Then, $x_t - x_{t-1} = (y_t - \bar{y}_t) - (y_{t-1} - \bar{y}_{t-1}) = y_t - y_{t-1} - \delta$, where $y_t - y_{t-1}$ is the growth rate of real output.

Vestin (2000) has used a forward looking model of the form given in equations (1) and (2) to study price level targeting under discretion. Previously, Svensson (1999b) had shown that price level targeting had desirable properties in a model with a Lucas-type aggregate supply function. Vestin reaches similar conclusions. Some intuition for these results can be obtained by noting that the first order condition under precommitment, equation (7), would also arise if a myopic central bank operated with discretion to minimize a loss function that depends on output gap variability and price *level* variability. In this case, as Clarida, Galí, and Gertler (1999) have noted, the central bank's first order condition would simply be $p_t = -\left(\frac{\lambda}{\kappa}\right) x_t$. Taking first differences yields (7).

3.2 Output gap changes and optimal discretion

The previous subsection analyzed the policy choice of a myopic central bank. While the assumption of a myopic central bank is unreasonable, the surprising result that such a central bank could deliver the optimal precommitment policy indicates that a speed limit policy might succeed more generally in improving policy by inducing inertial behavior. At a minimum, it suggests the role of stabilizing output gap changes as a policy objective warrants further study. An obvious question is whether similar gains can be achieved if the central bank is not myopic but instead acts to optimize fully under discretion. In this subsection, this issue is addressed.

McCallum and Nelson (2000) discuss two different definitions of optimal discretionary policy in models with forward-looking expectations. Under the first, the central bank treats future expectations of both inflation and the output gap as exogenous when it chooses current inflation and output (subject to the inflation adjustment relationship). Alternatively, the central bank may take as given the *process* through which private agents form their expectations. In this latter case, the central bank recognize that expectational terms such as $\mathrm{E}_t\pi_{t+1}$ will depend on the state variables at time t and that these state variables may be affected by policy actions at time t or earlier.

These two definitions of an optimal discretionary policy were equivalent in the context of the model of the previous section. This was because the state vector under discretion consisted solely of the serially uncorrelated disturbance e_t. Expectations of future inflation were functions of the exogenous process e_t and independent of current discretionary policy actions.

The two alternative definitions of discretionary policy differ once we assume the central bank's loss function involves the change in the output gap. In choosing x_t to affect $x_t - x_{t-1}$, the central bank's policy choice will be a function of x_{t-1}. This introduces the lagged output gap as a state variable, even though the underlying disturbances are serially uncorrelated and there are no other lagged endogenous state variables. Private agents will base their forecasts of future values of x_{t+i} and π_{t+i} on x_{t-1} and e_t. Following McCallum and Nelson and Jensen, it is assumed the central bank recognizes this dependence when it operates with discretion.[9]

Under either the optimal precommitment policy or discretion with a gap change objective, the equilibrium output gap will be a linear function of the lagged gap and the cost shock. Under precommitment, denote this solution for x_t as

$$x_t^c = a_x^c x_{t-1} + b_x^c e_t$$

while under discretion with an output gap change objective, denote the solution as

$$x_t^{gc} = a_x^{gc} x_{t-1} + b_x^{gc} e_t$$

Outcomes under the two alternative policy regimes can be compared by examining the equilibrium values of the coefficients appearing in these two equations.[10] In either case, the equilibrium rate of inflation can be written in terms of the a_x^i and b_x^i coefficients as

$$\pi_t = \left(\frac{\kappa a_x^i}{1-\beta a_x^i}\right) x_{t-1} + \left(1 + \frac{\kappa b_x^i}{1-\beta a_x^i}\right) e_t, \qquad i = c, gc$$

The parameter a_x^c is the solution less than one in absolute value of a quadratic equation that can be written as

$$c(a_x^c) \equiv (1-\beta a_x^c)\left(\frac{1-a_x^c}{a_x^c}\right) = \left(\frac{\kappa^2}{\lambda}\right). \tag{13}$$

In contrast, a_x^{gc} is given by the solution less than one in absolute value of a fourth order polynomial equation that can be written as

$$gc(a_x^{gc}) \equiv (1-\beta a_x^{gc})^3\left(\frac{1-a_x^{gc}}{a_x^{gc}}\right) = \left(\frac{\kappa^2}{\lambda}\right). \tag{14}$$

Only the first factor differs in the definitions of $c(\ )$ and $gc(\ )$. Both $c(\ )$ and $gc(\ )$ are decreasing functions of a_x^i for $0 < a_x^i < 1$. Since $0 < 1-\beta a_x^i < 1$, $(1-\beta a_x^i)^3 < 1-\beta a_x^i$ so it follows that $a_x^{gc} < a_x^c$. Optimal discretionary policy with an output gap change objective imparts some persistence to output, unlike pure discretion, but it imparts less persistence than under the optimal precommitment policy.[11]

While analytical solutions to (13) and (14) are not available, some insights can be gained by inspection. To do so, consider delegating monetary policy to a central bank following a speed limit policy but with a weight λ^{cb} on the change in the output gap objective. Note that (14) can then be rewritten as

$$(1-\beta a_x^{gc})\left(\frac{1-a_x^{gc}}{a_x^{gc}}\right) = \left(\frac{\kappa^2}{\hat{\lambda}}\right) \tag{15}$$

[9] The two definitions also differ when the inflation adjustment equation is modified to allow the cost shock to be serially correlated or to include some weight on lagged inflation, as will be the case in the model of section 4.

[10] Under pure discretion, $x_t^d = b_x^d e_t$ where $b_x^d = -\kappa/(\lambda+\kappa^2)$.

[11] See the appendix available from the author for details.

where $\hat{\lambda} = \lambda^{cb}(1-\beta a_x^{gc})^2$. If $\hat{\lambda} = \lambda$, (13) and (15) imply that $a_x^c = a_x^{gc}$. In this case, discretion under a speed limit policy imparts the same degree of inertia to the gap as optimal precommitment does. $\hat{\lambda} = \lambda$ occurs when $\lambda^{cb} = \lambda/(1-\beta a_x^{gc})^2 > \lambda$; optimal inertia is obtained if the central bank places more weight on its output objective than the social loss function does. A Rogoff "liberal" is required.[12] However, the optimal precommitment policy is not replicated exactly. If $\lambda^{cb} = \lambda/(1-\beta a_x^{gc})^2$ so that $a_x^c = a_x^{gc}$, the output gap reaction to an inflation shock is given by

$$b_x^{gc} = -\left(1-\beta a_x^{gc}\right)\left(\frac{\kappa}{\lambda\left[1+\beta(1-a_x^{gc})\right]+\kappa^2}\right)$$

and $|b_x^{gc}| < |b_x^c|$. Thus, the policy that imparts the correct amount of inertia responds too little to the inflation shock. A speed limit policy that reduced the amount of inertia (lowering a_x^{gc} by appointing a somewhat less liberal central banker) would improve the response to inflation shocks.

3.3 Simulation results

To further evaluate outcomes under discretion, numerical methods are employed to solve the model consisting of equations (1) and (2) under alternative assumptions about the policy regime (commitment versus discretion) and the objective function of the central bank. For simplicity, the disturbance to the aggregate demand relationship (1) is set equal to zero; as is well know, this shock poses no issues of policy design and the nominal interest rate can be used to neutralize its affect on both the gap and inflation. In this case, the model can be written in state space form as

$$\begin{bmatrix} e_{t+1} \\ x_t \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = A \begin{bmatrix} e_t \\ x_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + BR_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[12] The term liberal is used loosely. In the standard analysis following Rogoff (1985), a conservative central banker places less weight on output gap variability relative to inflation variability than does society. Here, the weight refers to the balance between variability in the change in the gap relative to inflation variability. In the present case, λ^{cb} should be scaled by $\sigma^2_{\Delta x}/\sigma^2_x$ where $\Delta x_t = x_t - x_{t-1}$ to obtain the additional inflation variance the central bank following a speed limit policy would accept to reduce the variance of the gap by one unit. Since e is serially uncorrelated, $\sigma^2_{\Delta x}/\sigma^2_x = 2(1-a_x^{gc})$. Hence, the central bank is a liberal if $2(1-a_x^{gc})\lambda^{cb} > \lambda$. Using the definition of λ^{cb}, this becomes $2(1-a_x^{gc})/(1-\beta a_x^{gc})^2 > 1$ which holds for $\beta = 0.99$ and all a_x^{gc}, $0 < a_x^{gc} < 0.99995$. Thus, λ^{cb} does correspond to a Rogoff-type liberal central banker.

where[13]

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{\sigma}{\beta} & 0 & \left(1+\frac{\sigma\kappa}{\beta}\right) & -\frac{\sigma}{\beta} \\ -\frac{1}{\beta} & 0 & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \sigma \\ 0 \end{bmatrix}$$

Define $X_{1t} = [e_t, x_{t-1}]'$, $X_{2t} = [x_t, \pi_t]'$, $\chi_{t+1} = [\varepsilon_{t+1}, 0, 0, 0]'$, and let $Z_t = [X_{1t}, X_{2t}]'$. Then the system can be written compactly as

$$\mathrm{E}_t Z_{t+1} = AZ_t + BR_t + \chi_{t+1} \tag{16}$$

The policy instrument R_t is set to minimize an objective function expressed as

$$L_k = (1-\beta)\mathrm{E}_t \sum \beta^i Z'_{t+i} Q_k Z_{t+i} \tag{17}$$

where Q_k depends on the specification of the single period loss function under policy regime k. Under pure discretion, denoted $k = PD$, this is simply $\pi_t^2 + \lambda x_t^2$ so

$$Q_{PD} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

With an output gap change objective, denoted by GC,

$$Q_{GC} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & -\lambda & \lambda & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Under optimal discretionary policy regime k, the solution for the policy instrument R_t that minimizes (17) subject to (16) takes the form $R_t = -F_k X_{1t}$. Details of the solution procedures are provided in Söderlind (1999).[14]

Table 3 presents the asymptotic loss obtained under the optimal discretionary policy with the central bank minimizing the social loss function (PD) and the optimal discretionary policy under a speed limit policy (GC for gap change). Panels A and B of the table report the asymptotic loss under each policy expressed relative to the outcome under the optimal precommitment policy. Results are reported for various values of the policy preference parameter λ and the output gap elasticity of inflation κ.

[13] It will be convenient to write the system in this form and let ε_{t+1} denote the innovation to the cost shock in period $t+1$ since the model will be extended below to allow $e_{t+1} = \gamma_e e_t + \varepsilon_{t+1}$ with $0 < \gamma_e < 1$. In this case, the first row of the 4×4 matrix on the right A becomes [γ_e 0 0 0].

[14] Numerical calculations were carried using the MATLAB programs of Paul Söderlind.

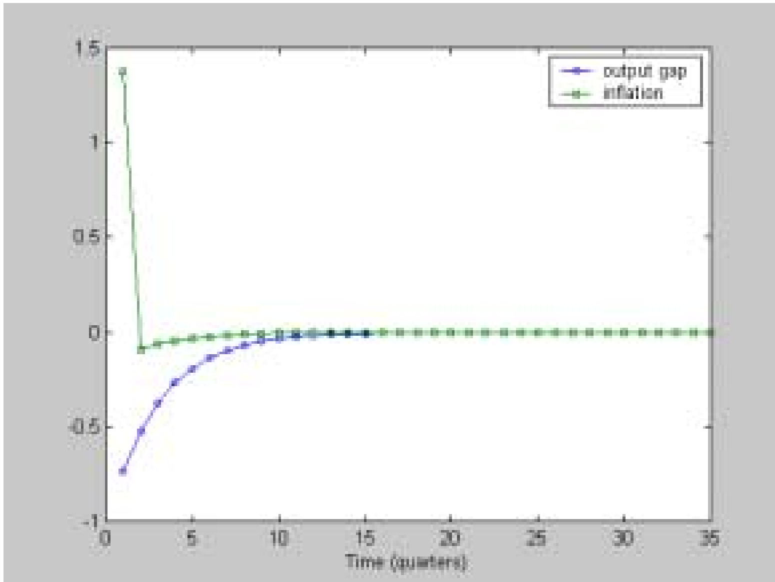

Figure 3: Responses to a Temporary Cost Push Shock under the OGG Policy

For the benchmark parameter values ($\beta = 0.99$, $\lambda = 0.25$, $\kappa = 0.05$), social loss is lower in a discretionary policy environment when the central bank follows a speed limit policy than when it acts to minimize social loss. While the loss is not reduced to what could be achieved under precommitment, shifting to a gap change objective cuts the loss due to discretion by almost 30%. This gain arises from the persistence introduced by the change in the gap objective. Figure 3, which should be compared to Figures 1 and 2 shows that a speed limit policy generates persistence in the face of a temporary cost shock, but that the output gap is much more variable than under the optimal precommitment policy. This suggests that the advantages of GC over PD will fall if society places greater weight on output gap stabilization (i.e., a larger λ). This is verified in Table 3, which shows that the relative performance of pure discretion improves, for given κ (the output gap elasticity of inflation), as λ increases. Only for very small values of κ or values of λ significantly above the baseline value, however, does pure discretion dominate discretion with an output gap change objective.

The greater output gap variability under the GC policy also suggests that, in contrast to the case under pure discretion, policy under an GC objective might be improved if a weight-liberal central bank conducts policy – that is, a central bank who places relatively less weight on its inflation objectives. Such a central bank will produce greater stability in the change in the output gap and generate policy responses that would be closer to those called for under the optimal precommitment policy. This intuition will be verified in the next section.

Table 3: Comparison of Pure Discretion and Speed Limit Policies
A: Pure Discretion (PD) Loss relative to precommitment:

	λ			
	0.1	0.25	0.5	1.0
$\kappa = 0.01$	2.14%	1.03%	0.49%	0.13%
$\kappa = 0.05$	13.20%	8.42%	5.81%	3.84%
$\kappa = 0.1$	23.49%	16.35%	11.87%	8.42%
$\kappa = 0.2$	32.15%	27.30%	21.67%	14.14%

Table 3: Comparison of Pure Discretion and Speed Limit Policies
B: Speed Limit Policy (GC) Loss relative to precommitment

	λ			
	0.1	0.25	0.5	1.0
$\kappa = 0.01$	4.73%	4.09%	3.57%	3.12%
$\kappa = 0.05$	6.29%	6.13%	5.81%	5.37%
$\kappa = 0.1$	6.16%	6.35%	6.24%	6.08%
$\kappa = 0.2$	5.76%	6.04%	6.19%	4.33%

One interesting implication of Table 3 is that under pure discretion the loss relative to optimal precommitment varies much more as the parameter κ varies than it does when there is an output gap change objective. The same is true of variations in the parameter λ. The *GC* policy appears more robust with respect to uncertainty about the slope of the short-run output–inflation trade off and uncertainty about the weight to place on output objectives than pure discretion is.

4 Inflation persistence

The forward looking model employed in the previous sections has been criticized for failing to match the short-run dynamics exhibited by inflation (Estrella and Fuhrer 1998). Specifically, inflation seems to respond sluggishly and to display significant persistence in the face of shocks, while (2) allows current inflation to be a jump variable that can respond immediately to any disturbance. Equation (2) therefore would be unlikely to display the inertial behavior of inflation that is observed in the data (Nelson 1998). This section modifies the inflation adjustment equation in two ways. Subsection 4.1 incorporates endogenous persistence by including the lagged inflation rate in (2). This results in a specification for inflation adjustment that more closely matches that used in recent empirical investigations and is a modification that seems to be necessary if model simulations are to match the time series properties of actual inflation. Subsection 4.2 allows the cost shock to be serially correlated.

4.1 Endogenous persistence

When the inflation adjustment is altered to incorporate a direct effect of lagged inflation on current inflation, equation (2) is replaced with

$$\pi_t = (1-\phi)\beta \mathrm{E}_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t \tag{18}$$

where $\phi \in [0, 1]$ measures the importance of backward looking inertia in the inflation process.

The choice of ϕ can be critical in assessing outcomes under alternative policies. In a backward looking model (i.e., $\phi = 1$), Ball (1999) found evidence that nominal income growth targeting could produce disastrous results. McCallum (1997), however, showed that this was no longer the case when expectations played a role. Rudebusch (2000) reached similar conclusions in his analysis of nominal income targeting, finding that it performed poorly for high values of ϕ.

The appropriate value of ϕ has been the source of controversy in the literature. Rudebusch (2000) estimates an equation that takes the basic form of (18) and concludes that, for the U.S., ϕ is about 0.7. That is, he finds that most weight is placed on the lagged inflation term. This is consistent with Fuhrer (1997) who reports estimates of ϕ close to 1. Galí and Gertler (1999) argue that the coefficient on lagged inflation rate is small when a measure of marginal cost is used in place of the output gap, however. Galí, Gertler, and López-Salido (2001) report a value of 0.3 for Europe. Much of the recent theoretical literature has adopted a value of $\phi = 0$, with only forward looking expectations entering. This was the form used in equation (2) and employed in the previous sections of this paper. Jensen (2001) sets $\phi = 0.3$ in his analysis of nominal income growth targeting, arguing that for policy evaluation it is appropriate to emphasize the role of forward looking expectations. McCallum and Nelson (2000) set $\phi = 0.5$.[15] I follow Jensen in adopting a value of 0.3 as a baseline. However, in this section, I evaluate output gap growth and pure discretion policies for values of ϕ ranging from zero to one. Baseline values of all the parameters are given in Table 4.

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[15] The specifications in both Jensen and in McCallum and Nelson differ slightly from that used in equation (18). Jensen's inflation equation is (using my notation)

$$\pi_t = \beta(1-\phi)E_t\pi_{t+1} + \phi\pi_{t-1} + (1-\phi)\kappa x_t + e_t$$

while McCallum and Nelson assume

$$\pi_t = \beta(1-\phi)E_t\pi_{t+1} + \beta\phi\pi_{t-1} + \kappa x_t + e_t$$

Jensen's specification can be written as

$$\pi_t = (1-\phi)\pi_t^* + \phi\pi_{t-1} + e_t$$

where $\pi_t^* = \beta E_t \pi_{t+1} + \kappa x_t$. This specification can be obtained from the model of Galí and Gertler (1999), where ϕ is the fraction of "rule of thumb" price setters. Note that in this formulation, the output gap has no impact on inflation as $\phi \to 1$. Inflation is then just an exogenous random walk process, and the standard backward looking Phillips curve is not obtained in the limit as all price setters follow the rule of thumb behavior.

Table 4: Baseline parameter values

σ	λ	κ	ϕ	σ_e
1.5	0.25	0.05	0.3	0.015

When $\phi \neq 0$, the lagged inflation rate becomes an endogenous state variable. To solve the model and derive the optimal discretionary policies, the model is written in state space form:

$$\mathrm{E}_t\bar{Z}_{t+1} \equiv \begin{bmatrix} e_{t+1} \\ x_t \\ \pi_t \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \bar{A}\begin{bmatrix} e_t \\ x_{t-1} \\ \pi_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \bar{B}R_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \equiv \bar{A}\bar{Z}_t + \bar{B}R_t + \bar{\chi}_{t+1} \tag{19}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{\sigma}{\beta(1-\phi)} & 0 & \frac{\sigma\phi}{\beta(1-\phi)} & \left(1+\frac{\sigma\kappa}{\beta(1-\phi)}\right) & -\frac{\sigma}{\beta(1-\phi)} \\ -\frac{1}{\beta(1-\phi)} & 0 & -\frac{\phi}{\beta(1-\phi)} & -\frac{\kappa}{\beta(1-\phi)} & \frac{1}{\beta(1-\phi)} \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sigma \\ 0 \end{bmatrix}$$

The loss functions again take the form $L_k = (1-\beta)\mathrm{E}_t \sum \beta^i \bar{Z}'_{t+i}\bar{Q}_k\bar{Z}_{t+i}$ for $k = PD, GC$. As in the previous subsection, the optimal discretionary policy is derived for each loss function. The equilibrium solutions for the output gap and inflation are then used to evaluate the asymptotic social loss.

Figure 4 provides the results of comparing pure discretion to optimal discretion with a gap change objective. The solid line in the figure shows the percentage gain over pure discretion that is obtained by assigning an output gap growth objective as a function of ϕ when the central bank puts a weight λ on its output objective. For all values of $\phi < 0.7$, society gains from assigning an output gap change objective to the central bank. The gain increases as ϕ rises until it peaks at $\phi = 0.5$. It then declines. When inflation is predominately backward looking, $\phi > 0.7$, pure discretion designed to minimize social loss based on the output gap measure leads to a smaller asymptotic loss. This result is not surprising. The presence of forward looking expectations imparts persistence under a precommitment policy that is missing under pure discretion. The *GC* policy imparts greater persistence in a way that captures the persistence under precommitment. When inflation is completely backward looking, however, the distinction between optimal precommitment and optimal discretion disappears. There can be no gain from distorting the central bank's loss function. When inflation is forward looking however, the speed limit policy improves over pure discretion.

So far, only one aspect of policy delegation has been considered – the definition of the appropriate output variable in the central bank's loss function. Policy also depends on the relative weight assigned to the bank's inflation and output objectives, and this may differ from the value of λ that appears in the social loss. Alternative policy regimes can be characterized by the objectives assigned to the central bank and the weights attached to each objective. Alternative regimes defined in this way will be called targeting regimes. Specifically,

> A *targeting regime* is defined by a) the variables in the central bank's loss function (the objectives), and b) the weights assigned to these objectives, with policy implemented under discretion to minimize the expected discounted value of the loss function.[16]

An inflation targeting regime, for instance, will be defined by the assignment of the loss function $\pi_t^2 + \lambda_{IT} x_t^2$ to the central bank, where the weight λ_{IT} is chosen optimally to minimize the asymptotic social loss function. Similarly, an output gap change targeting regime is one in which the central bank's loss function is $\pi_t^2 + \lambda_{GCT} (x_t - x_{t-1})^2$ with λ_{GC} chosen to minimize asymptotic social loss.

A gird search is conducted over values of λ_k to obtain the optimal weight to assign the central bank for the inflation targeting loss function $(\pi_t^2 + \lambda_{IT} x_t^2)$ and the speed limit version of the loss function $(\pi_t^2 + \lambda_{GCT} (x_t - x_{t-1})^2)$. The dashed line in Figure 4 shows the percent gain obtained by shifting from an inflation targeting regime to an output gap change regime when the optimal weight is used. For all ϕ, $\lambda_{IT} < \lambda < \lambda_{GCT}$; that is, under inflation targeting it is optimal to delegate to a conservative central bank ala Rogoff (1985), while with an output gap change objective, it is optimal to delegate to a liberal central bank. The results when the two targeting regimes are compared are qualitatively similar to the gain that was found when the central bank used a weight equal to that in the social loss function (the solid line in the figure). Unless inflation is predominately a backward looking process, a central bank that is concerned with changes in the output gap outperforms an inflation targeting bank, and a liberal central bank with a gap change objective outperforms a conservative central bank that minimizes the social loss function.

4.2 Serially correlated cost shocks

The previous section introduced persistence through the inclusion of lagged inflation in the inflation adjustment equation. An alternative specification is to return to the basic form of the inflation adjustment equation given by equation

[16] This definition of a targeting regimes is consistent with that of Svensson (1999c), who states "By a targeting rule, I mean, at the most general level, the assignment of a particular loss function to be minimize" (p. 617).

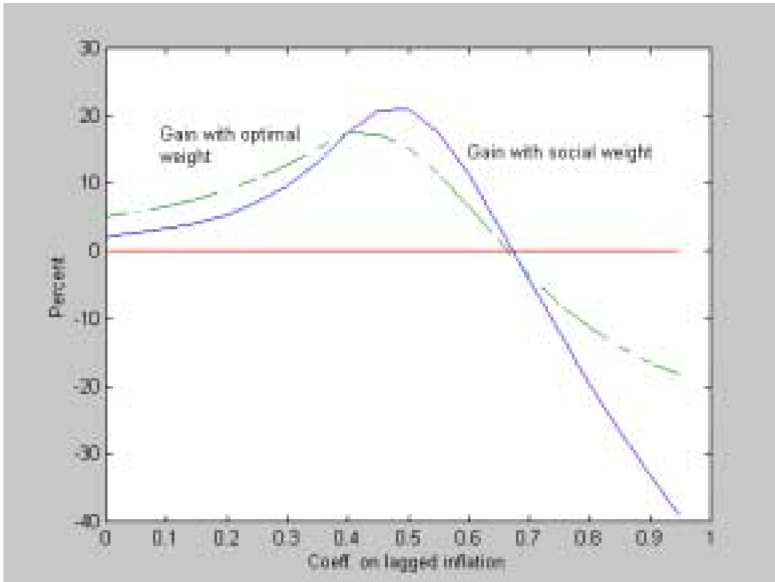

Figure 4: Percent gain from discretion with an output growth objective relative to discretion with a social loss function

(2), that is, with $\phi = 0$, and allow the cost shock to be serially correlated. Clarida, Galí, and Gertler (1999) show that when the social loss function (3) is assigned, there is no role for a conservative central bank when the cost shock is serially uncorrelated. That is, the optimal value of λ_{IT} in this case is just λ. However, when e_t follows the $AR(1)$ process

$$e_t = \gamma_e e_t + \varepsilon_t \tag{20}$$

and $\gamma_e > 0$, there are gains from delegating to a conservative central bank.

Table 5 shows the optimal values of λ_{IT} and λ_{GCT} and the associated asymptotic social loss as a function of γ_e. Serially correlated cost shocks reduce the optimal value of λ_{IT}, making a conservative inflation targeter desirable. In contrast, increased cost shock persistence makes it optimal to delegate to a more liberal central bank under an GCT regime. As the table shows, however, delegation to a liberal central bank assigned inflation and output gap change objectives dominates delegation to a conservative central bank assigned inflation and output gap objectives regardless of the value of γ_e.

Table 5: Optimal Policy Weights and Loss Functions[17]

		Commitment		Inflation Targeting		Output Gap Growth Targeting	
		λ	L^c	λ_{IT}	Social loss	λ_{GCT}	Social loss
	0	0.25	2.055	0.25	2.228	0.65	2.113
γ_e	0.3	0.25	4.253	0.20	4.905	0.95	4.435
	0.6	0.25	15.349	0.10	20.108	1.65	16.273

5 Model extensions and other targeting regimes

When potential output follows a deterministic trend, the change in the output gap is just real output growth relative to trend. In this case, the previous results under the GC and GCT regimes are equivalent to output growth (relative to trend) targeting regimes. When potential output is subject to stochastic shocks, however, output growth policies and policies that focus on the change in the gap will differ. Since policy objectives expressed in terms of inflation and output growth may be more transparent to the public than ones expressed in terms of the change in the gap, this section compares the two policies when potential output follows a persistent $AR(1)$ process.

In addition, a regime of nominal income growth targeting is also analyzed. Jensen (2001) recently reports that nominal income growth targeting may be superior to inflation targeting or to pure discretion. The intuition for this result is that nominal income growth targeting imparts an inertia to policy that is absent under pure discretion, and this inertia allows a nominal income growth targeting regime to achieve outcomes that are closer to those achieved under precommitment. Since this is the same rationale behind the superior performance of a speed limit policy, it is of interest to include nominal income growth targeting in the comparison.

In the previous sections, the basic model could be kept quite simple since only the output gap and inflation were relevant and only cost shocks generated a policy trade off that posed interesting issues of policy design. Under nominal income growth targeting or output growth targeting, however, shocks to potential output will induce policy responses. Thus, to compare outcomes under different delegation schemes, the model needs to be enriched to incorporate other possible disturbances that may affect the economy differently under alternative policy regimes.

5.1 The modified model

Two changes are made to the model of section 4.1 that included the π_{t-1} in the inflation adjustment equation. First, a backward looking element in the form of lagged output is added to the aggregate demand relationship. Expressed in terms of the output gap, this yields

[17] Social loss is times 10^4.

$$x_t = \theta x_{t-1} + (1-\theta)\mathrm{E}_t x_{t+1} - \sigma(R_t - \mathrm{E}_t \pi_{t+1}) + \mu_t \tag{21}$$

where

$$\mu_t = u_t - \bar{y}_t + \theta \bar{y}_{t-1} + (1-\theta)\mathrm{E}_t \bar{y}_{t+1}$$

Equation (21) can be motivated by the presence of habit formation in consumption (Fuhrer 2000). The demand shock u_t is assumed to be serially correlated and follows the $AR(1)$ process

$$u_t = \gamma_u u_{t-1} + \eta_t \tag{22}$$

Second, potential real output is assumed to follow an $AR(1)$ process:

$$\bar{y}_t = \bar{\gamma}\bar{y}_{t-1} + \xi_t \tag{23}$$

The innovation processes η_t and ξ_t are assumed to be white noise, zero mean processes that are mutually uncorrelated and uncorrelated with the cost shock innovation ε_t. The shock ξ_t represents a disturbance to potential output. Noting that $\mathrm{E}_t\bar{y}_{t+1} = \bar{\gamma}\bar{y}_t$, μ_t in equation (21) can be written as $\mu_t = u_t - [1-(1-\theta)\bar{\gamma}]\,\bar{y}_t + \theta\bar{y}_{t-1}$.The model now consists of equations (18), (20), (21), (23), (22), and (23). This makes the model almost identical to the one employed by Jensen (2001).[18]

The new parameters appearing in this extended model are the serially correlation coefficients γ_u and $\bar{\gamma}$, the weight on the lagged output gap in the expectational IS relationship, θ, and the variances of the innovations to demand and potential output. None of these parameters affects policy choice or the social loss under the policies considered earlier. These policies, and the social loss function, involved only the output gap and inflation. The stochastic process followed by potential output did affect equilibrium output but not the output gap or inflation. The structure of the aggregate demand relationship did affect the rule for the nominal interest rate needed to achieve given values of the output gap and inflation, but it did not alter the equilibrium for either the gap or for inflation. This separation will no longer be true for some of the policy structures to be considered below, so we now need to parameterize the complete model. Benchmark values are listed in Table 6.

Table 6: Baseline parameter values for extended model

σ	λ	κ	ϕ	θ	β
1.5	0.25	0.05	0.3	0.5	0.99
σ_e	σ_u	σ_y	γ_e	γ_u	γ_y
0.015	0.015	0.005	0	0.3	0.97

[18] As noted earlier, Jensen's specification of the inflation adjustment equation with lagged inflation differs slightly from the one used here.

5.2 Policy regimes and loss functions

A total of six alternative policy regimes are considered. These differ from one another in terms of the loss function the central bank is assumed to minimize. All six regimes assume that the central bank operates with discretion. Four of the regimes, pure discretion, inflation targeting, output gap change, and output gap change targeting, have already been defined. The two new regimes are output growth targeting and nominal income growth targeting. The regimes and their single period loss functions are defined in Table 7.

Table 7: Alternative policy regimes

Regime name		Loss function
Pure discretion	**PD**	$\pi_t^2 + \lambda x_t^2$
Inflation targeting	**IT**	$\pi_t^2 + \lambda_{IT}^* x_t^2$
Change in gap	**GC**	$\pi_t^2 + \lambda \left(x_t - x_{t-1}\right)^2$
Change in gap targeting	**GCT**	$\pi_t^2 + \lambda_{GCT}^* \left(x_t - x_{t-1}\right)^2$
Output growth targeting	**OGT**	$\pi_t^2 + \lambda_{OGT}^* \left(y_t - y_{t-1}\right)^2$
Nominal income growth targeting	**NIT**	$\pi_t^2 + \lambda_{NIT}^* \left(\pi_t + y_t - y_{t-1}\right)^2$

The nominal income targeting regime, NIT, is defined in a manner consistent with the other targeting regimes – that is, the central bank's objective contains inflation variability and nominal income growth variability, with the weight on nominal income growth chosen optimally. This specification differs from Jensen (2001) who assumes the central bank's loss function is $(1+f)\pi_t^2 + \lambda x_t^2 + \lambda^* \left(\pi_t + y_t - y_{t-1}\right)^2$ where both f and λ^* can be chosen optimally. To maintain closer comparability with the other regimes which all have just one free parameter, I adopt the definition of nominal income growth targeting given in Table 7.

As before, each of the loss functions can be expressed as

$$(1-\beta)\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \hat{Z}_{t+i}' \hat{Q}_k \hat{Z}_{t+i}$$

for a suitably defined matrix $\hat{Q}_k$.

5.3 Evaluation

Each of the seven alternative policy regimes is evaluated for the baseline parameters and for several permutations from these baseline values. Results are reported in Table 8 which gives the asymptotic social loss under each regime. For comparison, the loss under the optimal precommitment policy (denoted PC) is also shown. For each column, the social loss under the regime yielding the lowest loss appears in bold.

Table 8: Alternative policy regimes[19]

	(1) Baseline	(2) σ_y= 0.01	(3) $\kappa = 0.01$	(4) $\kappa = 0.1$	(5) $\kappa = 0.2$	(6) $\phi = 0.6$
PC	4.315	4.315	5.335	3.466	2.416	14.376
PD	5.167	5.167	5.515	4.435	3.106	17.150
IT	5.114	5.114	5.505	4.360	3.059	15.597
GC	4.664	4.664	5.508	3.641	2.495	15.207
GCT	**4.457**	**4.457**	**5.354**	**3.594**	2.495	**14.553**
OGT	4.531	4.737	5.584	3.636	2.517	14.607
NIT	5.547	5.836	12.149	3.814	**2.485**	18.076

With the baseline parameter values, targeting the change in the output gap (output gap change targeting) yields the lowest social loss of any of the discretionary regimes. It comes within about 3% of the precommitment loss (4.457 vs. 4.315). Output growth targeting is slightly worse (at 4.531) because shifts in potential output affect policy through their impact on output growth, although such shocks would not induce a response under an optimal precommitment policy. Still, targeting the growth rate of output is the second best discretionary regime and does significantly better than either pure discretion or inflation targeting. Both *GCT* and *OGT* are superior to inflation targeting and nominal income growth targeting.

Column 2 of Table 8 shows the impact of doubling the variance of shocks to potential output. The first five regimes depend only on inflation and the output gap, so none of these are affected by this change. However, policy regimes based on output growth or nominal income growth are affected. Policy based on output growth remains superior to the nominal income based regime in the face of this change.

I next consider alternative values of the output gap elasticity of inflation, κ. For both smaller values of this elasticity (col. 3) and when κ is doubled from the baseline value of 0.05 to 0.1 (col. 4), the *GCT* policy continues to yield the lowest social loss. When κ is doubled again to 0.2, however, nominal income growth targeting emerges as the best policy regime, although even in this case, *GCT* (and simple *GC*) are close seconds. Finally, the last column of Table 8 shows the impact of increasing the weight on lagged inflation in the inflation adjustment equation from 0.3 to 0.6. Again *GCT* yields the lowest value of the loss function..

As we saw earlier, variations in the social weight λ on output gap stabilization can affect the relative performance of pure discretion and output gap change policies. Table 9 reports results for the baseline value ($\lambda = 0.25$) in column 1, columns 2 - 4 show that the change in the gap targeting regime continues to yield the lowest loss for both smaller and larger values of λ, although *NIT* does as well when λ is small.

Table 9: Effect of Alternative Weights on Output Gap Variability [20]

[19] Loss times 10^4.
[20] Loss times 10^4.

	(1) $\boldsymbol{\lambda = 0.25}$	(2) $\boldsymbol{\lambda = 0.1}$	(3) $\boldsymbol{\lambda = 0.5}$	(4) $\boldsymbol{\lambda = 1.0}$
PC	4.315	3.783	4.744	4.890
PD	5.167	4.755	5.416	5.427
IT	5.114	4.718	5.302	5.408
GC	4.664	4.010	4.791	5.426
GCT	**4.457**	**3.924**	**4.770**	**5.006**
OGT	4.531	3.946	4.957	5.463
NIT	5.547	**3.924**	8.206	13.529

To summarize, except for inflation processes that are primarily backward looking or inflation processes that are very sensitive to the output gap, the speed limit policy based on targeting inflation and the change in the output gap dominates the other regimes.[21]

The targeting regimes evaluating in this section are obviously just a subset of the possible regimes that could be examined.[22] The main focus has been on comparing inflation targeting with a speed limit policy, since the former has attracted a great deal of support and the latter seems to reflect some thinking at the Federal Reserve. One obvious alternative is to consider is a hybrid regime that imbeds these two as special cases. For example, suppose the central bank acts to minimize a loss function given by

$$\frac{1}{2}(1-\beta)\sum \beta^i \mathrm{E}_t \left[\pi_{t+i}^2 + +\lambda_1 x_{t+i}^2 + \lambda_2 \left(x_{t+i} - x_{t+i-1}\right)^2\right]$$

where λ_1, the weight on gap variability, and λ_2, the weight on gap change variability, are chosen optimally to minimize social loss. Optimal values of λ_1 and λ_2 were calculated for the baseline parameters and for $\lambda = (0.1, 0.25, 0.5, 1.0)$. In all cases, when λ_1 and λ_2 were restricted to be nonnegative, the optimal value of λ_1 was zero. That is, no role was found for the variability of the output gap once the variability of the change in the gap was included in the objective function.

[21] After the first draft of this paper was circulated, Söderström (2001) added the output gap change objective to his evaluation of alternative targeting regimes. His regimes included money growth targeting, interest rate smoothing, nominal income targeting, and average inflation targeting. Except when inflation was predominately backward looking or the output elasticity of inflation was very large, output gap change targeting yielded the smallest asymptotic loss in his model, results consistent with those found here.

[22] For example, since *NIT* performs poorly for the baseline parameter values (social loss is almost 30% higher than under precommitment and it does worse than even pure discretion), I also evaluated a modified nominal income targeting regime that assumed the central bank's loss function was $\pi_t^2 + \lambda x^2 + \lambda^* \left(\pi_t + y_t - y_{t-1}\right)^2$. This definition of nominal income growth targeting is closer to Jensen's but is harder to evaluate relative to the regimes in Table 7 because it lacks the parallel structure the others share in common. Generally results were similar to those under *NIT* – it performed well for large values of κ and small values of λ. Unlike *NIT*, however, it yielded slightly lower loss than *GCT* when ϕ was large.

6 Conclusions

Previous work on monetary policy in forward looking New Keynesian models has focused on optimal simple rules under the assumption that the central bank is able to commit to a rule. In this paper, I have assumed the relevant policy regime is one of discretion, and the problem faced in designing policy is to assign a loss function to the central bank. The approach is one used by Jensen (2001) to examine nominal income growth targeting and is consistent with the contracting approach employed by Persson and Tabellini (1993), Walsh (1995), and Svensson (1997), although that earlier literature was concerned mainly with the average inflation bias that could arise under discretion.

In a forward looking New Keynesian model it was shown that the optimal, timeless perspective precommitment policy could be achieved by a totally myopic, discretionary central bank if the bank was assigned an output gap change objective rather than an output gap objective. While virtually all the recent literature has assumed that a social loss function dependent on inflation and the output gap is the appropriate objective of policy, discretionary policy with such a social loss function imparts too little persistence to output and inflation. A policy aimed at stabilizing inflation and the change in the output gap (a speed limit policy) imparts the socially optimal degree of persistence when the central bank is myopic.

When the central bank is not myopic but instead optimally chooses policy in a discretionary regime, an output gap change objective no longer produces outcomes that coincide with the optimal precommitment policy. Simulations suggested that a speed limit policy dominates pure discretion except when forward looking expectations are relatively unimportant. Policy regimes based on the change in the gap were also compared to alternative targeting regimes such as inflation targeting and nominal income growth targeting. A policy regime that targets the change in the gap proved superior to other regimes.

Previous authors have considered the introduction of other objectives designed to induce inertia into policy. In Woodford's original discussion of interest rate inertia, he argued that empirical evidence of inertial interest rate behavior reflected the attempt by central banks to influence forward-looking expectations. By committing itself to a rule that induces inertial behavior in the nominal interest rate, current changes in policy generate changes in expected future interest rates. This allows the central bank to influence expected future inflation, improving its trade-off between inflation and output gap variability. Nominal income growth targeting implicitly introduces the lagged value of real output into the state vector and generates some persistence even under a regime of pure discretion. This accounts for the good performance of nominal income growth targeting that Jensen finds. Speed limit policies also induce inertia. An avenue for future work is to investigate the impact of errors in measuring the output gap on the relative performance of different targeting regimes. If such errors are highly serially correlated, the case for a speed limit policy might be further strenghtened.

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