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Abstract

When politicians are provided with insufficient incentives by the democratic election mechanism, we show that social welfare can be improved by threshold contracts. A threshold incentive contract stipulates a performance level which a politican must reach in order to have the right to stand for reelection. Read my lips would turn into read my contracts. Reelection thresholds can be offered by politicians during campaigns and do not impair the liberal principle of free and anonymous elections in democracies.

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1 Introduction

Democracies are - and should be - built on the fundamental principles of free and anonymous elections. Consequently, future reelection is uncertain. For instance, voters' preferences at the reelection stage may shift, thus lowering politicians' reelection chances even if they have performed well in the past. Or newly emerging issues during campaigns may influence voting behavior. The randomness of a politician's reelection chances increases further when the benefits from the politician's efforts cannot be measured with sufficient precision or when benefits are affected by external shocks. Consider a reform of the judiciary system as an example for the former, or a labor market reform as an example for the latter.[1]

The randomness of future reelections may not provide politicians with sufficient motivation to devote a socially desirable amount of effort to certain tasks. For example when benefits and thus the efforts of politicians are not perfectly observable or the valuation of the effort allocation changes through shifting voter preferences, neither sanctions for deviations from the socially desirable amount of effort nor rewards for exerting the socially desirable amount of effort will be sufficiently high. Then, the politician has an incentive to choose the effort allocation according to his own preferences.

In this paper we suggest that adding a threshold incentive contract to the election mechanism can increase social welfare without impairing the liberal principles of free and anonymous voting in democracies. A threshold incentive contract stipulates the minimum benefit or performance level a politician has to achieve in order to have the right to stand for reelection. Such threshold contracts may appear to reenforce the problem that the politician is not rewarded for his efforts in the future, since the probability of remaining in office decreases. However, when reelection chances are uncertain, social welfare can be increased through the incentive contract because deviations to lower efforts are punished more heavily. The threshold incentive contract is equivalent to a conditional term limit. Thus, it does not diminish the scope of the fundamental liberal principles of democracies of free and anonymous elections.

[1]The potential benefits in terms of reduction of unemployment rates can be thwarted by negative macroeconomic shocks, which may make it very difficult for voters to assess the performance or the competence of the politician pursuing such reforms.

We consider a model in which an elected politician can exert effort on a public issue such as institutional reforms. The effort creates benefits for the public which are affected by noise, either due to measurement problems or due to shocks. The politician's reelection chances are increasing in the created benefits. Since the politician is assumed to be motivated by holding the office, the uncertainty of the reelection chances does not provide the politician with sufficient motivation to exert socially optimal effort levels. In our paper we allow a court to stipulate a threshold incentive contract which the politician must accept upon election. Such contracts prescribe a level of benefits the politician must achieve to earn the right to stand for reelection. We show that the dual mechanism - threshold incentive contracts and elections - can increase social welfare because it increases the marginal benefit from efforts.

Next, we discuss the possibility that politicians themselves might offer the threshold contracts during their campaigns. We show that optimal contracts are offered if the politicians have the same competence, measured by their marginal costs of exerting effort. If the politicians differ in competence, then the politician with the higher competence will be elected.

Our paper follows the recent proposals to supplement the election mechanism in democracies by incentive contracts. While the existing literature (e.g. Gersbach (1999), Gersbach and Liessem (2000)) introduces incentive contracts which prescribe utility or monetary transfers after a politician has been reelected, the novel element of this paper is the idea of thresholds for reelection.[2] Additional hurdles for reelection can help to motivate politicians to invest in good policies despite the declining reelection probability.

The paper is related to the literature about electoral accountability which was initiated by Barro (1973) and Ferejohn (1986) and recently extended by Persson, Roland and Tabellini (1997). Politicians and voters are assumed to have divergent interests, and elections are a means by which voters control politician misbehavior, since the possibility of reelection induces self-interested politicians to act on behalf of the interests

[2]While in our paper we combine threshold contracts for politicians with the democratic requirements of free and anonymous elections, there is a rapidly growing literature on incentive contracts for central bankers initiated by Walsh (1995a, 1995b) and Persson and Tabellini (1993); for further development see Lockwood (1997), Svensson (1997) and Jensen (1997).

of the electorate. However, that requires backward-looking voting behavior. In our paper, we assume that the voters are only partially backward-looking but use many other criteria in elections. For instance, the competence and personal qualities of a competitor or communication skills and newly emerging issues in campaigns can influence voting and elections (see e.g. Lupia and McCubbins (1998)). When voters are both backward- and forward-looking, we show that reelection thresholds can provide appropriate incentives.

It does not appear very difficult to introduce reelection thresholds in democracies. Threshold contracts would allow politicians to offer voters clear choices. Either a politician sticks to his campaign promise and can stand for reelection or he breaks his promise and that was his last term. In the famous example of when President George Bush announced "read my lips: no new taxes" threshold contracts would have not allowed him to abandon his campaign promise and then stand for reelection. Threshold contracts would have increased the commitment power of the promise if George Bush had wanted to commit himself to no increase of taxes. Another recent example where threshold contracts could have made a difference was the campaign promise of Chancellor Schröder to bring unemployment down to 3.5 million by 2002. With the opportunity of threshold contracts and competition between Schröder and the incumbent Kohl, either Schröder would have stopped short of making such promises or German voters could have been more confident that unemployment will actually decline in 2002.

The paper is organized as follows: In section 2 we outline the model. Section 3 presents the first-best solution. In section 4 we show how the reelection mechanism works. In section 5 we add the threshold incentive contract to the reelection mechanism and indicate the welfare implications. Section 6 gives an example of how the threshold incentive contract works. Section 7 discusses what happens if the politicians themselves offer threshold contracts at the campaign stage. Section 8 concludes.

2 The Model

We consider the voters' problem of trying to motivate an elected politician. The voters and the politician are assumed to be risk-neutral. There are two periods. In the first period, the incumbent has to exert effort e on a task T, which for example could be the reform of the judiciary system. The effort e on task T creates benefits B for the public in the first period.[3] For simplicity, we assume

$$B = e. \tag{1}$$

The voters cannot observe B directly; instead, they receive a noisy signal about the benefits. This refers to a situation when the benefits of political actions are not easily measurable. For example, if the politician works on the reform of the judiciary system, the benefits are widespread and could not be identified in simple quantitative terms. We assume the benefit signal to be given as:

$$b = B + \epsilon = e + \epsilon. \tag{2}$$

Factor ϵ is a random variable with the support $[-a, a]$, distributed with the density function $f(\epsilon)$. We assume $E(\epsilon)$ to be zero. Hence the benefit signal b is distributed with the density function $f(b) = f(e + \epsilon)$ on $[e - a, e + a]$.

The expected utility for the public is denoted by U^P. Upon observing b, U^P is given as

$$U^P = E(B \mid b). \tag{3}$$

E is the expectation about the benefits, evaluated after b has been observed. Given our assumption $b = B + \epsilon$ and thus $E(B \mid b) = b$, U^P is simply given as

$$U^P = b. \tag{4}$$

An alternative interpretation of our model would be that the public does not perceive a signal about their benefits, but that the benefits themselves are affected by an external

[3] Additional benefits may also materialize in the second period, but this has no bearing on our main results.

shock. This would model the situation in which say a politician exerts effort on a labor market reform, but the benefits of the effort are affected by macroeconomic shocks. b now stands for the benefits for the public. Our results are valid for both perspectives on the way in which noise makes it impossible for voters to precisely infer the politicians' effort. We will work with the first interpretation.

The voters make their reelection decision dependent on their expected utility and therefore on the observed signal. From the perspective of the first period, however, the election at the beginning of the second period can be affected by many other factors than the benefit signal. Therefore, reelection is uncertain for the politician when he decides on his engagement. We assume that reelection chances can be summarized by a continuous probability function $p(b)$ that is known to the politician at the beginning of the first period. $p(b)$ is the probability that the politician will be reelected if the benefit signal b is realized. The reelection probability is assumed to be monotonically increasing in b with support $[\underline{b}, \bar{b}]$. For $b < \underline{b}$ the reelection probability is assumed to be zero, for $b > \bar{b}$ the reelection probability is one. The fact that the reelection scheme is stochastic can be interpreted in several ways. For instance, while some voters may base their decision exclusively on the past performance of the politician (or the performance signal), others may make their reelection decision dependent on other factors, such as leadership and communication skills of the incumbent, or the perceived competence of a competitor emerging at the reelection stage, or on economic circumstances independent of current policies. Voter preferences may also shift, which induces noise at the reelection stage.

The utility of the politician is given by

$$U^A(b, e) = W^1 + q\{e \mid p(b)\}W^2 - C(e). \tag{5}$$

W^1 denotes the utility of the office in period 1, W^2 the discounted utility of the office in period 2 and $C(e)$ the cost of exerting the effort. The utility from holding office may include monetary benefits, such as a fixed wage, and non-monetary benefits, such as prestige or the desire for a statesman-like image. $q\{e \mid p(b)\}$ denotes the politician's expected reelection probability if he exerts the effort level e, and the reelection scheme $p(b)$ holds. The reelection probability is written in expectational form because the created benefit signal is a random variable. For simplicity of exposition, we denote the expected reelection probability $q\{e \mid p(b)\}$ as $q(e)$. Then, the overall expected utility of office in period 2 is given by $q(e)W^2$. The utility W^1 from office in the first period is sunk after the politician has been elected. Thus, it will be neglected in the subsequent analysis. Then, the remaining utility takes the form:

$$U^A(b,e) = q(e)W^2 - C(e). \tag{6}$$

Given the politician's utility, the participation constraint (PC) that the politician wants to stand for reelection amounts to

$$q(e)W^2 - C(e) \geq 0. \tag{7}$$

The politician chooses an effort level that maximizes his utility. Thus, the incentive constraint (IC) is given as:

$$e = \arg\max_e \{q(e)W^2 - C(e)\}. \tag{8}$$

In order to break ties, we assume that a politician who is indifferent between actions will choose those which yield the highest utility for the voters.

For tractability, the cost $C(e)$ of the agent is assumed to be given as follows:

$$C(e) = ce^2. \tag{9}$$

The factor c can be interpreted in two ways. Either it measures the agent's disinclination to provide the effort e, or it could be interpreted as the competence of the politician, with small c meaning high competence, i.e., achieving a certain benefit level does not require much effort cost from the politician.

At the end of the first period, the benefits for the public are realized. The public observes the benefit signal b and the reelection decision takes place.

The overall game is summarized as follows:

Stage 1: Based on his expected reelection chances $q(e)$, the politician exerts his effort on task T.

Stage 2: The benefit from the politician's activity is realized. The public observes the benefit signal b and takes its reelection decision.

3 First-Best Solution

We first characterize the first-best solution, assuming that the public has perfect information about the agent's effort and could commit to a reelection scheme, i.e. the electorate does not depend on $p(b)$ in designing contracts. We assume that the public enforces the socially optimal effort level directly by a contract heavily penalizing any deviation from the effort level prescribed in the contract. Hence the public's problem is to maximize its utility subject to the politician's participation constraint. The participation constraint must be honored by the public, because otherwise the politician would not seek reelection and would not enter into the contract. Since the PC must be taken into account, the reelection probability under a first-best solution must be equal to one. Otherwise, the public could demand a higher effort level, thereby still fulfilling the PC by increasing the reelection probability.

The perfect information assumption yields

$$U^P = B. \tag{10}$$

Hence, the voters' problem is given by

$$\max\{U^P = e\}, \tag{11}$$

$$\begin{aligned} s.t. \quad W^2 - C(e) &\geq 0, \\ e &\geq 0. \end{aligned}$$

A simple Lagrange calculation leads to

Proposition 1
The first-best effort level is given by

$$e^{FB} = \sqrt{\frac{W^2}{c}}. \tag{12}$$

This is the maximum effort level the public can implement; higher effort levels would not satisfy the participation constraint and the politician would not seek reelection and forgo the contract.

4 The Reelection Mechanism

In this section we explore the sub-game perfect equilibria of the game if only the reelection mechanism is at work. The politician chooses his effort according to the incentive constraint (IC) as:

$$e = \arg\max_{e}\{q(e)W^2 - ce^2\}.$$

The expected reelection probability $q(e)$ is given by

$$q(e) = \int_{e-a}^{e+a} p(b)f(b-e)db. \tag{13}$$

Note that $p(b)$ is zero for $b < \underline{b}$ and the reelection is sure for $b \geq \bar{b}$. Therefore, the expected reelection probability $q(e)$ has different forms for the cases $e-a < \underline{b}$, $e-a > \underline{b}$, etc., which we will address when necessary. We obtain:

Proposition 2

Under the reelection scheme $p(b)$ only three effort choices can occur:

(i) $e = 0$ *(lower corner solution),*

(ii) $e = \bar{b} + a$ *(upper corner solution),*

(iii) $e^{int} = \dfrac{\partial q(e)}{\partial e}\dfrac{W^2}{2c}$ *(interior solutions).*

Proof of proposition 2:

According to the IC, the politician chooses the effort level which maximizes his utility under the reelection scheme $p(b)$.

First, we observe $U^A(e) < U^A(\bar{b}+a)$ for all $e > \bar{b}+a$. An effort level $e = \bar{b}+a$ guarantees reelection, because the benefit signal $\bar{b}$ is reached with certainty. It is obvious that the politician would never choose an effort level greater than the one required for sure reelection: effort levels greater than the one required would imply higher costs without additional benefits. Therefore, we can restrict the problem to

$$\max_{e}\{U^A(e)\}; \quad e \in [0; \bar{b} + a].$$

Either there is a corner solution, i.e., $e = 0$ or $e = \bar{b} + a$, or there exists an interior solution.

In the interior solutions, the politician chooses his effort level according to the IC. The first-order condition implies

$$\frac{\partial q(e)}{\partial e} W^2 - 2ce = 0,$$

and the politician exerts the effort[4]

$$e^{int} = \frac{\partial q(e)}{\partial e} \frac{W^2}{2c}.$$

■

The effort levels emerge from the incentive compatibility constraint of the politician. Under the possible solutions, the politician chooses the one which maximizes his utility. Let e^* be the solution of the politician's maximization problem, i.e., the global maximum

$$e^* = \arg\max\{U^A(e)\}.$$

We will now explore the efficiency of the reelection mechanism.

In most cases, the reelection mechanism creates inefficiencies compared to the first-best solution. The first-best solution would be implemented if $\bar{b} = e^{FB} - a$ and the politician chooses the upper corner solution. The first-best solution requires a reelection probability of one because otherwise the PC would be violated. Therefore, none of the interior solutions which imply $q(e) < 1$ can implement the first-best effort level and the upper corner solution is the only solution in which first-best could be reached. Moreover, if $\bar{b} \neq e^{FB} - a$ then first-best cannot be reached either because the politician can secure his reelection with an effort smaller than e^{FB} or because the politician will not be reelected with certainty even if he exerts the socially desirable amount of effort and thus his PC is violated.

[4]Note that multiple interior solutions can exist.

In a next step we will give a more general picture of the circumstances under which the reelection mechanism is relatively efficient and creates high efforts and of the circumstances under which this is not the case.

First of all, the reelection mechanism works best and creates the highest effort if the politician chooses the upper corner solution. We will now show the conditions under which this is likely to happen. Let e^j, $j = 1...k$ denote any other effort level of the interior or lower corner solutions. Then, the politician chooses the upper corner solution if

$$W^2 - c(\bar{b}+a)^2 \geq \int_{e^j-a}^{e^j+a} p(b)f(b-e)dbW^2 - c(e^j)^2 \geq 0 \quad \text{for all } j$$

and thus if

$$\left(1 - \int_{e^j-a}^{e^j+a} p(b)f(b-e)db\right) W^2 \geq c(\bar{b}+a)^2 - c(e^j)^2 \geq 0 \quad \text{for all } j.$$

Thus, for the politician to adopt the upper corner solution, the loss through higher costs has to be outweighed by the gain in expected reelection probability. The costs of exerting the effort $e = \bar{b} + a$ decrease in a (the bounds of the density function of the noise) and $\bar{b}$. The gain in expected reelection probability is high if $p(b)$ has a high gradient.[5]

We now derive conditions for a high effort in the interior solution. Therefore we write the effort e^{int} as

$$e^{int} = \frac{\partial \int_{-a}^{a} p(e+\epsilon)f(\epsilon)d\epsilon}{\partial e} \frac{W^2}{2c}.$$

Using the rules for differentiation of parameter integrals,[6] this can be written as

$$e^{int} = \int_{-a}^{a} \frac{\partial p(e+\epsilon)}{\partial e} f(\epsilon)d\epsilon \frac{W^2}{2c}.$$

which can finally be transformed to

[5]Moreover, one can show that the gain is high and thus the corner solution more likely to be adopted, when the benefit signal has a small variance.

[6]Note that e and ϵ are the two independent variables and $b = e + \epsilon$.

$$e^{int} = \int_{e-a}^{e+a} \frac{\partial p(b)}{\partial e} f(b-e)db \frac{W^2}{2c}. \tag{14}$$

Thus, e^{int} increases the higher the gradient of the reelection scheme is and the lower the variance of the benefit signal is.[7] Additionally, the effort level in the interior solution depends on the benefits of holding the office and on the costs of exerting the effort.

The effort of the politician could be improved by committing to an adequate reelection mechanism. However, one of the fundamental principles of modern liberal democracy is free, anonymous and uncommitted voting at every election. Therefore, the stochastic element in $p(b)$ is a precondition for our analysis.

5 Threshold Incentive Contracts

In this section, we explore whether the introduction of a threshold incentive contract leads to a superior solution without impairing the liberal democracy principle of free and anonymous voting. We assume that there is an independent institution, for example a court, which has the same utility function as the voters and which has the right to decide whether or not the politician is allowed to stand for reelection. The reelection decision is given as follows: The court announces a threshold signal $\hat{b}$ at the beginning of the first period. If the benefit signal realized at the end of the first period is smaller than $\hat{b}$, the politician cannot stand for reelection. If the benefit signal realized is equal to or higher than $\hat{b}$, the politician can stand for reelection. Then, the usual democratic election process with free and anonymous voting takes place. Thus, a hierarchy of incentive contracts and elections is formed. First, the decision is taken whether the politician has the right to stand for reelection, then the usual reelection mechanism takes place.

The overall game is summarized as follows:

[7] One can show that the variance of the benefit signal influences the outcome as follows: the higher the variance of the benefit signal is, i.e. the lower $f(b-e)$, the less impact the design of the reelection mechanism has. If the variance is very large then it makes no difference whether $p(b)$ has a high gradient or not because the expected reelection probability remains approximately the same.

Stage 1: A court prescribes a threshold signal $\hat{b}$ that the politician has to reach if he wants to stand for reelection. The required signal is known to the politician. Voters have a stochastic reelection scheme $p(b)$.

Stage 2: The politician exerts his effort on task T.

Stage 3: The benefit from the politician's activity is realized. The public and the court observe the benefit signal b. If $b < \hat{b}$, the politician leaves office and does not stand for reelection. If $b \geq \hat{b}$ the politician stands for reelection and the reelection procedure takes place.

As the incentive contract is at work and the court announces $\hat{b}$, the expected reelection probability for a given effort changes to

$$q(e,\hat{b}) = \int_{e-a}^{e+a} p(b)f(b-e)db - \int_{e-a}^{\hat{b}} p(b)f(b-e)db. \tag{15}$$

The last term measures the decline of the expected reelection probability due to the threshold contract. If $e - a < \hat{b}$, then $q(e,\hat{b}) < q(e)$, because the expected reelection probability for some signals is now zero. In this case, the expected reelection probability can be directly written as

$$q(e,\hat{b}) = \int_{\hat{b}}^{e+a} p(b)f(b-e)db. \tag{16}$$

The utility for the politician under the dual mechanism is denoted by $U^A(e,\hat{b})$ and is given by

$$U^A(e,\hat{b}) = q(e,\hat{b})W^2 - ce^2$$

We now explore the consequences of the threshold incentive contract. First, we examine how effort levels under the IC are affected by threshold incentive contracts. In a next step, we derive the optimal incentive contract. Then, we characterize the conditions under which the incentive contract strictly improves welfare.

Proposition 3

The threshold incentive contract weakly improves the effort levels chosen under the incentive constraint.

Proof of proposition 3:

First, we rewrite the maximization problem of the politician under the hierarchy of incentive contracts and elections as

$$\max_e \{U^A(e,\hat{b})\}; \quad e \in [0; \max[\bar{b}+a, \hat{b}+a]]$$

As we know, three cases can occur. The lower corner solution remains the same with $e = 0$, but the upper corner solution changes into

$$e = \hat{b} + a$$

for $\hat{b} > \bar{b}$. Hence, the effort level in the upper solution is higher with a threshold incentive contract, or remains the same for $\hat{b} \leq \bar{b}$.

Regarding the interior solutions, the politician chooses his effort level according to the new incentive constraint as

$$e = \arg\max\{q(e,\hat{b})W^2 - ce^2\},$$

which yields the following effort level in the first order condition

$$e^{int}(\hat{b}) = \frac{\partial q(e,\hat{b})}{\partial e} \frac{W^2}{2c}.$$

This can be written as

$$e^{int}(\hat{b}) = \left[\frac{\partial q(e)}{\partial e} - \frac{\partial \left[\int_{e-a}^{\hat{b}} p(b) f(b-e) db \right]}{\partial e} \right] \frac{W^2}{2c}.$$

Without a threshold incentive contract the interior solutions were

$$e^{int} = \frac{\partial q(e)}{\partial e} \frac{W^2}{2c}.$$

Because of

$$\frac{\partial \left[\int_{e-a}^{\hat{b}} p(b) f(b-e) db \right]}{\partial e} < 0,$$

the interior solutions are larger under the threshold incentive contract if $q(e, \hat{b}) < q(e)$, otherwise the effort level remains the same in both scenarios.

■

The proposition indicates that the threshold incentive contract increases the upper corner solution and the interior solutions if an adequate threshold signal is stipulated. In the upper corner solution, the effort level is raised by choosing a threshold signal $\hat{b} > \bar{b}$. In this case the effort level yielding sure reelection is $e = \hat{b} + a$. For the interior solutions, the effort level can be increased by choosing a threshold signal $\hat{b}$ for which $q(e, \hat{b}) < q(e)$. In this case the cut-off of the reelection probability in the presence of threshold incentive contracts increases marginal reelection chances and thus the marginal utility from exerting effort.

We now examine what threshold signal $\hat{b}$ should be required by the court in order to obtain a second-best solution. We denote the possible corner and interior solutions under the threshold incentive contract by $e^j(\hat{b})$, $j = 1, ..., k$.[8] Let $e^*(\hat{b})$ be the solution of the politician's maximization problem, i.e., the global maximum

$$e^*(\hat{b}) = \arg\max\{U^A(e^j(\hat{b}), \hat{b})\}. \tag{17}$$

Note that $e^*(-a)$ is equal to the effort level e^* chosen when only the reelection mechanism is at work. We state

Proposition 4

The court chooses the threshold signal $\hat{b}^*$ as

$$\hat{b}^* = \arg\max\{e^*(\hat{b})\} \quad s.t. \quad U^A(e^*(\hat{b}^*), \hat{b}^*) \geq 0$$

Proof of proposition 4:

The optimal threshold signal $\hat{b}^*$ should be chosen to maximize the effort level e and thus to maximize the benefits for the public.

$e^*(\hat{b})$ is the effort level that the politician chooses subject to the threshold signal $\hat{b}$. Hence, $e^*(\hat{b})$ has to be maximized over $\hat{b}$. The participation constraint $U^A(e^*(\hat{b}^*), \hat{b}^*) \geq$

[8] We assume that there is a finite number of interior solutions.

0 has to be satisfied, because otherwise the politician would not seek reelection.

■

In the next proposition, we establish a sufficient condition for the dual mechanism to strictly improve welfare.

Proposition 5

(i) If $U^A(e^*(-a)) = 0$, *then* $e^*(\hat{b}^*) = e^*(-a)$;

(ii) If $U^A(e^*(-a)) > 0$ *and* $e^*(-a) = e^{int}(-a)$, *then* $e^*(\hat{b}^*) > e^*(-a)$.

Proof: see Appendix.

In the next step we explore whether the dual mechanism improves social welfare if the politician has chosen one of the corner solutions under the reelection mechanism alone.

Proposition 6

Suppose $U^A(e^*(-a)) > 0$. *Then*

(a) $e^*(\hat{b}^*) > e^*(-a)$ *if* $e^*(-a) = 0$ *and* $p(a)f(a) - \int_{-a+\xi}^{a} p(\epsilon)f'(\epsilon)d\epsilon \geq 0$;

(b) $e^*(\hat{b}^*) > e^*(-a)$ *if* $e^*(-a) = \bar{b}+a$ *and* $p(a)f(a)W^2 - \int_{-a+\xi}^{a} p(\bar{b}+a+\epsilon)f'(\epsilon)d\epsilon W^2 - 2c(\bar{b}+a) \geq 0$.

Proof: see Appendix.

In the proofs we show that the dual mechanism strictly improves social welfare if $U^A(e^*(-a)) > 0$ and $e^*(-a)$ is an interior solution. Under certain conditions, the dual mechanism also improves social welfare if $e^*(-a)$ is one of the corner solutions and $U^A(e^*(-a)) > 0$. The reasoning runs as follows: We first show that $U^A(e^*(-a)) > 0$ is a necessary condition for the PC to be satisfied in a solution $e^*(\hat{b}) > e^*(-a)$. Further we show that it is always possible to set a threshold signal $\hat{b}$ for which $q(e^*(-a), \hat{b}) < q(e^*(-a), -a)$ and the politician does not choose an effort $e^*(\hat{b}) < e^*(-a)$. Then, as we recall from proposition 3, the effort in the interior solution is increased because of the increasing marginal utility. Thus, if $e^*(-a) = e^{int}(-a)$, social welfare is strictly improved for $U^A(e^*(-a)) > 0$. Regarding the corner solutions, the effort can only be increased through the dual mechanism if the marginal gain of reelection chances outweighs the marginal increase in costs.

In the following, we give conditions under which the welfare improvement through the dual mechanism is significant.

Obviously, social welfare is maximal if the first-best effort level can be reached. This is possible if the threshold signal can be set as $\hat{b} = e^{FB} - a > \bar{b}$ and the politician chooses the upper corner solution.

Regarding the interior solutions, social welfare can always be strictly improved. Note that the threshold incentive contract can improve the effort in the interior solution in two ways: First, as we have seen in proposition 3, the effort can be continuously increased; second, the incentive contract can induce a jump from one solution to another. To illustrate the latter case, suppose there are two interior solutions $e^1(-a)$ and $e^2(-a)$ with $e^1(-a) < e^2(-a)$ and $U^A(e^1(-a)) > U^A(e^2(-a))$, but the utility difference is small. Then, there is the possibility that the dual mechanism changes the utility in such a way that $U^A(e^2(\hat{b})) > U^A(e^1(\hat{b}))$ and thus the politician will choose the higher effort level. We now discuss how a continuous rise of $\hat{b}$ yields a strong increase in the effort level. Therefore, we undertake a comparison between the efforts chosen in the interior solution under the dual mechanism and under the reelection mechanism alone. Under the dual mechanism $e^{int}(\hat{b})$ is chosen as

$$e^{int}(\hat{b}) = \frac{\partial \left(\int_{e-a}^{e+a} p(b) f(b-e) db - \int_{e-a}^{\hat{b}} p(b) f(b-e) db \right)}{\partial e} \frac{W^2}{2c} \tag{18}$$

As shown in the proof of proposition 3, the difference between the efforts lies in the second term. Clearly, the influence of the threshold incentive contract is larger, the higher the influence of the second term is in comparison to the first, i.e., the smaller the marginal reelection probability was under the reelection mechanism alone.

In the next section, we give an example of how the dual mechanism works.

6 Example

We illustrate the working of the dual mechanism of elections and incentive contracts with an simple example. We assume that the politician's effort is perfectly observable by the public and thus

$$b = e.$$

As before the first-best solution is given by

$$e = \sqrt{\frac{W^2}{c}}.$$

Further, we assume the reelection mechanism $p(b)$ to be

$$p(b) = \begin{cases} 0 & \text{for } b \leq \underline{b}, \\ \gamma + \phi b & \text{for } \underline{b} \leq b \leq \bar{b}, \\ 1 & \text{for } b \geq \bar{b}, \end{cases}$$

with $\bar{b} \leq e^{FB}$ and $\underline{b} \geq 0$ and $\gamma + \phi\bar{b} = 1$.

Because the politician's effort is perfectly observable, $p(b)$ denotes the probability that the politician will be reelected if he exerts effort e and thus $q(e) = p(b)$. The politician's incentive constraint implies that the politician chooses the effort that maximizes his utility and is given as

$$e = \arg\max\{q(e)W^2 - ce^2\}.$$

The participation constraint is satisfied if

$$q(e)W^2 - ce^2 \geq 0.$$

According to the incentive constraint, three possible solutions can occur:

(i) $e = 0$ (lower corner solution),

(ii) $e = \bar{b}$ (upper corner solution),

(iii) $e^{int} = \phi\frac{W^2}{2c}$ (interior solution).

The effort levels $e = 0$ and $e = \bar{b}$ are the lower and the upper corner solutions. The politician does not exert an effort level higher than $\bar{b}$ because of $q(e) = 1$ for all $b \geq \bar{b}$. Thus, with the effort $\bar{b}$ reelection is sure. The interior solution e^{int} climbs in the gradient of the reelection probability, ϕ, in the utility of holding office in period 2, W^2, and declines in the costs of exerting the effort.

Because of

$$\frac{\partial^2 U^A}{\partial e^2} = \frac{\partial^2((\gamma + \phi b)W^2 - ce^2)}{\partial e^2} = -2c < 0,$$

only one utility maximum exists; it is given through the interior solution. The upper corner solution $e = \bar{b}$ is only chosen for $e^{int} \geq \bar{b}$. Further, the lower corner solution $e = 0$ is chosen if the PC is not satisfied for any effort higher than $e = 0$. In all other cases, the politician chooses e^{int}. Obviously, as ϕ increases, the probability rises that $e^{int} \geq \bar{b}$ and that the politician will choose effort $\bar{b}$.

We now introduce a threshold incentive contract.

A court announces a threshold signal $\hat{b}$, which the politician has to reach if he wants to stand for reelection. Then, the politician's reelection probability changes to

$$q(e, \hat{b}) = \begin{cases} 0 & \text{for } \; b \leq \max[\underline{b}, \hat{b}], \\ \gamma + \phi b & \text{for } \; \max[\underline{b}, \hat{b}] \leq b \leq \bar{b}, \\ 1 & \text{for } \; b \geq \max[\bar{b}, \hat{b}]. \end{cases}$$

The politician chooses his efforts according to the modified incentive constraint, which now amounts to

$$e = \arg\max\{q(e, \hat{b})W^2 - ce^2\}.$$

The possible solutions, i.e. the possible utility maxima are given as

(i) $e = 0$ (lower corner solution),

(ii) $e = \max\{\bar{b}, \hat{b}\}$ (upper corner solution),

(iii) $e^{int}(\hat{b}) = \max\{\phi \frac{W^2}{2c}, \hat{b}\}$ (interior solution).

The lower corner solution remains the same as before. The upper corner solution can be either $e = \hat{b}$ or $e = \bar{b}$. In the former case, the politician must exert a higher effort to ensure reelection. The interior solution changes into $e = \hat{b}$ for $\hat{b} > \phi W^2/2c$, because the politician would not get reelected by exerting an effort smaller than $\hat{b}$. We have shown that only one utility maximum exists and thus $e = \hat{b}$ in this case is a second-best solution if the utility from exerting $\hat{b}$ is weakly larger than zero and thus the PC is satisfied. We use $e^*(\hat{b})$ to denote the global utility maximum and hence the effort that the politician chooses under the threshold incentive contract.

To derive the optimal threshold signal $\hat{b}^*$ we must ensure that $\hat{b}^*$ maximizes the chosen effort under the IC and that the PC is satisfied.

Thus, the optimal signal $\hat{b}^*$ is chosen as

$$\hat{b}^* = \arg\max\{e^*(\hat{b})\} \quad s.t. \quad U^A(e^*(\hat{b}^*), \hat{b}^*) \geq 0.$$

Obviously, $\hat{b}^* = e^{FB}$ is the solution. The politician will not choose an effort level smaller than e^{FB}, because then he would not be reelected. The participation constraint is satisfied because $U^A(e^{FB}, e^{FB}) = 0$. According to our tie-breaking rule, the politician chooses e^{FB} and not $e = 0$.

In this example, the threshold incentive contract always leads to the first-best solution. It is thus welfare-improving if the politician chooses the interior solution under the reelection mechanism alone, or if $\bar{b} < e^{FB}$ and the politician chooses the upper corner solution. There are two reasons for this result: First, the benefit signal is not noisy. The public and the court observe the benefit signal perfectly and thus the reelection mechanism has the best chance of working. Furthermore, the PC is always satisfied under e^{FB} when there is no noise that could diminish expected reelection probability. Second, the assumption $\bar{b} \leq e^{FB}$ is necessary because otherwise the reelection probability under the first-best effort level is smaller than one and the PC would not be satisfied. In this case, the optimal threshold signal would be the signal which fulfills $U^A(e^*(\hat{b}^*), \hat{b}^*) = 0$.

The threshold incentive contract works as follows: By giving the politician a threshold that he has to reach if he wants to stand for reelection, the court can force the politician to exert a higher effort than he would exert without the threshold. If the benefit signal is not noisy, the first-best effort level always can be reached.

7 Determination of Incentive Contracts

In this section, we explore what happens if the politicians themselves can determine the threshold signal $\hat{b}$. We assume that there is a campaign stage before the first period in which two political candidates denoted by i, j offer threshold signals $\hat{b}_i$, $\hat{b}_j$ to the public which they are willing to accept as threshold incentive contracts for their reelection bids.

The competences of the politicians i, j measured by c_i, c_j are assumed to be known to the voters. The offered threshold signals $\hat{b}_i$, $\hat{b}_j$ are associated with effort levels $e_i^*(\hat{b}_i)$, $e_j^*(\hat{b}_j)$ which the politicians i, j would exert in office. Because of our complete information assumption the voters can derive these effort levels by observing $\hat{b}_i$, $\hat{b}_j$.[9] $\hat{b}_i^*$, $\hat{b}_j^*$ denote the threshold signals which the independent court would require from the politicians i, j for them to have the right to stand for reelection. They are associated with the efforts $e_i^*(\hat{b}_i^*)$, $e_j^*(\hat{b}_j^*)$.

The voters observe the threshold offers and cast their votes. We assume that each politician is elected with a probability of 1/2, if $e_i^*(\hat{b}_i) = e_j^*(\hat{b}_j)$ and $c_i = c_j$. If $c_i > c_j$ and $e_i^*(\hat{b}_i) = e_j^*(\hat{b}_j)$ we assume as a tie-breaking rule that politician j is elected with probability 1.[10] If $e_i^*(\hat{b}_i) > e_j^*(\hat{b}_j)$ then politician i is elected with probability 1.

The structure of the game is summarized as follows:

Stage 1: Two politicians denoted by i, j with competences c_i, c_j offer threshold signals $\hat{b}_i$, $\hat{b}_j$ to the public which they are willing to accept as threshold incentive contracts for their reelection bids.

Stage 2: The voters observe the threshold offers and make their election decisions.

Stage 3: The elected politician exerts his effort on task T.

Stage 4: The benefit from the politician's activity is realized. The public observes the benefit signal b. If $b < \hat{b}_i$, $\hat{b}_j$ respectively, the politician leaves office and does not

[9] Note that the choice of the effort and also the first-best effort levels depend on the competence of the politician.

[10] This tie-breaking rule is not crucial and allows us to avoid the ϵ-framework in characterizing the equilibria.

stand for reelection. If $b \geq \hat{b}_i$, $\hat{b}_j$ respectively, the politician stands for reelection and reelection takes place.

We now look for sub-game perfect equilibria of the campaigning game.

Proposition 7

(i) If $c_i = c_j$, there exists a unique equilibrium in which both politicians offer the threshold signals $\hat{b}_i^* = \hat{b}_j^*$.

(ii) If $c_i > c_j$, there exists a unique equilibrium in which politician i offers the threshold signal $\hat{b}_i^*$ and politician j offers the threshold signal $\hat{b}_j^o$ with

$$\hat{b}_j^o = \arg\max_{\hat{b}_j} U^A(e_j^*(\hat{b}_j), \hat{b}_j, c_j) \quad s.t. \ e_j^*(\hat{b}_j^o) \geq e_i^*(\hat{b}_i^*).$$

Proof of proposition 7:

First, note that

$$U^A(e_i^*(\hat{b}_i^*), \hat{b}_i^*, c_i), U^A(e_j^*(\hat{b}_j^*), \hat{b}_j^*, c_j) \geq 0$$

since the PC is satisfied if the politicians offer the threshold signals $\hat{b}_i^*$, $\hat{b}_j^*$.

(i) Suppose $c_i = c_j$.

Threshold signal offers $\hat{b}_i^* = \hat{b}_j^*$ are an equilibrium, because a downward deviation by a politician would yield a zero election probability for him.

Threshold signal offers $\hat{b}_i = \hat{b}_j < \hat{b}_i^* = \hat{b}_j^*$ and $\hat{b}_i = \hat{b}_j > \hat{b}_i^* = \hat{b}_j^*$ cannot be an equilibrium. They would induce efforts $e_i^*(\hat{b}_i) = e_j^*(\hat{b}_j) < e_i^*(\hat{b}_i^*) = e_j^*(\hat{b}_j^*)$. A deviation by politician i to a threshold signal corresponding to an infinitesimally higher effort would yield an election probability of one and thus a higher utility for politician i.

Finally, threshold signal offers $\hat{b}_i < \hat{b}_j \leq \hat{b}_j^*$ cannot be an equilibrium either, because politician i could raise his expected utility by choosing a signal $\hat{b}_i = \hat{b}_j$ which would give him a positive election probability.

(ii) Suppose $c_i > c_j$.

We show that threshold offers $\hat{b}_i^*$ with a corresponding effort level $e_i^*(\hat{b}_i^*)$ and $\hat{b}_j^o$ with

$$\hat{b}_j^o = \arg\max_{\hat{b}_j} U^A(e_j^*(\hat{b}_j), \hat{b}_j, c_j) \quad s.t.\ e_j^*(\hat{b}_j^o) \geq e_i^*(\hat{b}_i^*)$$

are an equilibrium. First note that $U^A(e_j^*(\hat{b}_j^o), \hat{b}_j^o, c_j) \geq 0$ since by choosing $\hat{b}_j^o = \hat{b}_j^*$ $U^A(e_j^*(\hat{b}_j^*), \hat{b}_j^*, c_j) \geq 0$. The politician j will not be elected if he deviates to a threshold signal corresponding to $e_j^*(\hat{b}_j) < e_i^*(\hat{b}_i^*)$. Thus he chooses the threshold signal which maximizes his utility under the constraint $e_j^*(\hat{b}_j) \geq e_i^*(\hat{b}_i^*)$. Politician i does not deviate either since he cannot offer a higher utility for voters by selecting other thresholds.

The rest of the proof follows the lines of the proof of (i) and is therefore omitted here.

■

In the proof we have shown that the politicians offer the optimal incentive contracts if they have the same competence. If politician j has a higher competence than politician i, then politician j offers at least a threshold signal $\hat{b}_j$ which yields $e_j^*(\hat{b}_j) = e_i^*(\hat{b}_i^*)$. Thus, he will be elected with certainty. However, different competences of politicians yield inefficiencies in the determination of incentive contracts in the sense that the more competent politician obtains a rent which depends on the size of the competence differential.

The inefficiency increases if the election probability also depends on other factors like the appearance or the communication skills of the competing politicians. Then, a politician who has a high election probability because of these factors has more leeway to choose his incentive contract offer. Thus, the self-determination of incentive contracts is promising if the campaigning is very competitive and problem solving competences are influencing voting behavior.

8 Conclusion

Our analysis suggests that thresholds to reelection could be a viable supplementary mechanism to improve democratic procedures. Of course, there are a variety of practical issues involved in using incentive contracts in politics, as discussed in Gersbach (1999) and in Gersbach and Liessem (2000) for contracts stipulating money transfers. Moreover, one might wonder whether a threshold contract as proposed in this paper may lead to an underinvestment of effort on newly emerging issues, because they are not relevant for the right to stand for reelection. However, voters can punish such a behavior of a politician by not reelecting him. Moreover, one could introduce a clause that the contract is canceled or renegotiated in the case of extra-ordinary events such as a war. Overall, however, the threshold contracts suggested in this paper promise efficiency gains and might be the ones most easily introduced in politics.

Finally, the literature has identified a number of further inefficiencies in the political system(see e.g. the surveys and contributions of Mueller (1989), Drazen (2000), Dixit (1996), Bernholz and Breyer (1993), Buchanan and Tullock (1965), Stiglitz (1989), Persson and Tabellini (1990) and (2000) and by Gersbach and Haller (2001)). How the dual mechanism can be applied for these kinds of inefficiencies is a useful extension. The actual reach of the dual mechanism can only be judged after these avenues have been explored.

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Appendix

Proof of proposition 5:

Proof of (i):

Suppose $U^A(e^*(-a)) = 0$. $e^*(-a)$ maximizes the utility under the reelection mechanism alone and thus

$$U^A(e^*(-a)) \geq U^A(e) \quad \forall e,$$

and because of our tie-breaking rule

$$U^A(e^*(-a)) > U^A(e) \quad \text{for } e > e^*(-a).$$

Then, if $e^*(\hat{b}) > e^*(-a)$, the PC would be violated and the optimal threshold signal is set as $\hat{b}^* = -a$.

Proof of (ii):

Since costs and $q(e, \hat{b})$ are continuous in e and $\hat{b}$ respectively, there exist $\delta > 0$ and $\xi > 0$ sufficiently small that $U^A(e^*(-a)+\delta, -a+\xi) \geq 0$. Thus, in principle it is possible to satisfy the PC if effort and the threshold signal are marginally increased.

We proceed in two steps. In a first step we show that for $U^A(e^*(-a)) > 0$ there always exist threshold signals $\hat{b}$ with $q(e^*(-a), \hat{b}) < q(e^*(-a), -a)$ for which the politician does not choose a solution smaller than $e^*(-a)$. Then we show that the effort in the interior solution can always be enlarged if $U^A(e^*(-a)) > 0$.

We first show that for $U^A(e^*(-a)) > 0$, there are always threshold signals $\hat{b}$ for which the politician does not choose an effort level $e < e^*(-a)$ under the dual mechanism.

The solution $e^*(-a)$ satisfies $U^A(e^*(-a)) \geq U^A(e)$ for all e and thus

$$\int_{e^*(-a)-a}^{e^*(-a)+a} p(b)f(b-e)dbW^2 - c(e^*(-a))^2 \geq \int_{e-a}^{e+a} p(b)f(b-e)dbW^2 - ce^2 \quad \text{for } e^*(-a) \neq e.$$

Then, for the threshold signal $\hat{b} = e^*(-a) - a$ and for $e < e^*(-a)$:

$$\int_{e^*(-a)-a}^{e^*(-a)+a} p(b)f(b-e)dbW^2 - c(e^*(-a))^2 > \int_{e^*(-a)-a}^{e+a} p(b)f(b-e)dbW^2 - ce^2,$$

because the introduction of an incentive contract diminishes expected reelection probability and thus the utility for a given effort level. Thus, threshold signals $\hat{b} = e^*(-a) - a + \xi$ with ξ sufficiently small exist, such that for $e < e^*(-a)$

$$\int_{e^*(-a)-a+\xi}^{e^*(-a)+a} p(b)f(b-e)dbW^2 - c(e^*(-a))^2 \geq \int_{e^*(-a)-a+\xi}^{e+a} p(b)f(b-e)dbW^2 - ce^2.$$

Thus there exist threshold signals with $q(e^*(-a), \hat{b}) < q(e^*(-a))$ for which the politician does not choose an effort smaller than $e^*(-a)$.

Suppose $e^*(-a) = e^{int}(-a)$. For sufficiently small ξ, the politician will again choose the same interior solution. Then, the effort is enlarged for a threshold signal $\hat{b} = e^*(-a) - a + \xi$ because of proposition 3, which implies that the cut-off of the reelection probability through the threshold incentive contract increases marginal utility of the politician from exerting effort.

■

Proof of proposition 6:

We first show (a).

Suppose $e^*(-a) = 0$. Then, the effort can be enlarged if for threshold signals $\hat{b} = -a+\xi$, $\xi > 0$ and an effort level $\delta > 0$

$$U^A(0, -a+\xi) \leq U^A(\delta, -a+\xi).$$

The condition can be rewritten as

$$\int_{-a+\xi}^{a} p(b)f(b)dbW^2 \leq \int_{-a+\xi}^{\delta+a} p(b)f(b-\delta)dbW^2 - c\delta^2.$$

Thus, we obtain

$$\int_{a}^{\delta+a} p(b)f(b-\delta)dbW^2 + \int_{-a+\xi}^{a} p)b)[f(b-\delta) - f(b)]dbW^2 - c\delta^2 \geq 0.$$

This is equivalent to

$$\int_{a-\delta}^{a} p(\delta+\epsilon)f(\epsilon)d\epsilon W^2 + \int_{-a+\xi-\delta}^{a-\delta} p(\delta+\epsilon)[f(\epsilon) - f(\epsilon+\delta)]d\epsilon W^2 - c\delta^2 \geq 0.$$

By taking the derivatives at $\delta = 0$ we obtain a sufficient condition as

$$p(a)f(a) - \int_{-a+\xi}^{a} p(\epsilon)f'(\epsilon)d\epsilon \geq 0.$$

We next prove (b).

Suppose $e^*(-a) = \bar{b} + a$. Then, the effort can be enlarged if for threshold signals $\hat{b} = e^*(-a) - a + \xi$, $\xi > 0$ and an effort level $e^*(\hat{b}) = e^*(-a) + \delta$, $\delta > 0$

$$U^A(\bar{b} + a, e^*(-a) - a + \xi) \leq U^A(\bar{b} + a + \delta, e^*(-a) - a + \xi).$$

The condition can be rewritten as

$$\int_{\bar{b}+\xi}^{\bar{b}+2a} p(b)f(b - (\bar{b} + a))dbW^2 - c(\bar{b} + a)^2 \leq \int_{\bar{b}+\xi}^{\bar{b}+\delta+2a} p(b)f(b - (\bar{b} + a + \delta))dbW^2 - c(\bar{b} + a + \delta)^2.$$

Thus we obtain

$$\int_{\bar{b}+2a}^{\bar{b}+\delta+2a} p(b)f(b - (\bar{b} + a + \delta))dbW^2 + \int_{\bar{b}+\xi}^{\bar{b}+2a} p(b)[f(b - (\bar{b} + a + \delta)) - f(b - (\bar{b} + a))]dbW^2$$
$$-c(\bar{b} + a + \delta)^2 + c(\bar{b} + a)^2 \geq 0.$$

This is equivalent to

$$\int_{a-\delta}^{a} p(\bar{b} + a + \delta + \epsilon)f(\epsilon)d\epsilon W^2 + \int_{\bar{b}+\xi-(\bar{b}+a+\delta)}^{\bar{b}+2a-(\bar{b}+a+\delta)} p(\bar{b} + a + \delta + \epsilon)[f(\epsilon) - f(\epsilon + \delta)]d\epsilon W^2$$
$$-c(\bar{b} + a + \delta)^2 + c(\bar{b} + a)^2 \geq 0.$$

By taking derivatives at $\delta = 0$, a sufficient condition is

$$p(a)f(a)W^2 - \int_{\xi-a}^{a} p(\bar{b} + a + \epsilon)f'(\epsilon)d\epsilon W^2 - 2c(\bar{b} + a) \geq 0.$$

■