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CESifo Working Papers

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CESifo Working Paper No. 659 (3)

January 2002

Category 3: Social Protection

Presented at CESifo Norvegian Seminar, June 2001

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ISSN 1617-9595

CESifo *Working Paper No. 659*
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Abstract

Work requirements can make it easier to screen the poor from the non-poor.They can also affect future poverty by changing the poors' incentive to invest in their income capacity. The novelty of our study is the focus on long term poverty. We find that the argument for using work requirements as a screening device is both strengthened and weakened with long term poverty, and that the possibility of using work requirements weakens the incentives to exert effort to escape poverty. We also show that the two incentive problems, to screen poverty and deter poverty, are interwoven; the fact that the poor can exert an effort to increase their probability of being non-poor in the future makes it easier to separate the poor from the non-poor in the initial phase of the program. Finaly we show that if it is possible to commit to a long term poverty alleviation program it is almost always optimal to impose some work requirements on those that receive transfers.

JEL Classification: D82, I38.

Keywords: long-term poverty, ratchet effect, moral hazard, screening.

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1 Income transfers and incentive problems

When funds are made available to alleviate poverty, a welfare administrator faces at least two challenges. The first question he or she (but 'she' hereafter) needs to address is how to channel these funds to those in real need of them. This is a screening or sorting problem, and ignoring it leads to unnecessarily large outlays, in the form of transfers flowing to people not in need of support. At the same time, there are many reasons why a person may live below the poverty line. One reason is that he or she (but 'he' hereafter) has not exerted su±cient e®ort to increase his skill level. If poor people can (to some extent) in°uence their future earnings capacity, there is also a potential moral hazard problem the welfare administrator needs to keep in mind: welfare assistance policies might discourage the poor to invest in their future earnings capacity.

Welfare assistance can be granted in several ways, depending on what the welfare administrator can observe and on the instruments at hand: as subsidy schemes, means-testing, in-kind transfers. In this paper, we focus on workfare programs{that is, program that make transfers contingent on the acceptance of a work requirement{and evaluate how successful these are both at screening and at solving the moral hazard problem when people happen to remain poor during a longer time.

We are not the ¯rst to evaluate workfare programs in the light of these considerations. Most notably, it has been addressed in a formal model by Besley and Coate (1992). The novelty of our study is the focus on long-term poverty. We let individuals' income opportunities be correlated over time. This assumption adds a new dimension to the poverty alleviation problem, since it enables the welfare administrator to collect information about peoples' income opportunities as time passes. Potential welfare claimants might understand this and adjust their behavior accordingly.

To get a rough idea of how the dynamics in°uence the costs and bene¯ts of using workfare, consider the problem of targeting the poor. Let there be two groups of individuals in society, one with a low income potential, we call them L-individuals, and one with a high income potential, we call them H-individuals. The government wants to guarantee everyone a minimum income z, which is higher than the income L earns in the market, but lower than the income H earns. H-individuals may nevertheless claim bene¯ts intended for the poor, since the welfare administrator cannot observe a person's income opportunities. It is to prevent such fraudulent behavior that workfare may be used. Requiring welfare recipients to work c hours in the public sector to qualify for transfers, makes it costly for those with a relatively high earning capacity to join the program. Every hour spent in a public sector job could alternatively be used in the private sector, and since an H-person has a relatively high income potential this loss is relatively high. The negative e®ect of workfare is that a work requirement reduces the poor's market income and thus necessitates larger transfers to the poor in order to guarantee them an income above the poverty line.

Ignore for a moment the learning aspect associated with long-term poverty. Assume for example that there is no correlation between a person's present and future earning capacity (i.e. there exists only short-term poverty). Let the proportion of genuinely poor be low. There are, in other words, a lot of potential fraudulent claimants around and it is important to deter non-poor from joining the poverty program. Let c^s be the minimum level of public work that scares

H-individuals o® the poverty program. As we have constructed the problem, the government minimizes costs by imposing a workfare program that requires the poor to work c^s hours in exchange for their bene¯ts.

Assume now that individual earning capacities are correlated over time. This means that the welfare administrator can learn more about peoples' income potential by keeping a record of their past behavior. In fact, since a work requirement of c^s separated the two groups, she correctly infers that those who participated in the workfare program are genuinely poor. If she is free to change policy later on, she will certainly not make individuals work for their bene¯ts in later periods of the poverty program. Now that the screening is done, it is only costly to use workfare. But, and this is the crux of the argument, if H-individuals perceive that welfare will be provided unconditionally at a later stage in the poverty program, they will not be discouraged from participating in a poverty program that requires individuals to work c^s hours in the ¯rst phase of the program.

As this example indicates, in a multi-period framework it becomes essential to specify whether or not policy makers can commit to the design of future poverty alleviation policies. We evaluate the e®ectiveness of di®erent policy programs both with and without commitment.

Optimal policy

When poverty is long-term, and poverty reducing e®ort is of little avail, we ¯nd that work requirements should in general be concentrated to the ¯rst period of the programme. Compared with the cost e±cient policy for eliminating short term poverty, we ¯nd that workfare, as opposed to universal welfare, becomes a more e±cient policy in containing the overall costs when poverty is long term. In some cases though{which we specify in detail later{the concentrated use of work requirements will scare away the poor from the programme. To avoid that, the welfare administrator should allocate work requirements more evenly in time, even though this implies that fewer non-poor people separate.

Once the possibilities to escape poverty become signi¯cant, a new screening problem presents itself in the next period: to screen those that failed to escape poverty from those that didn't. Poverty reducing e®ort thus gives rise to a sequence of screening problems. This sounds like bad news. But in fact, it need not be. The existence of a new screening problem in the future makes it easier for the WA to commit to work requirements in the future. This, in turn, makes it easier to screen the non-poor from the poor in the ¯rst period. To put it di®erently, poverty reducing e®ort allows for some substitution of today's work requirements for future work requirements, and in some cases this lowers the total cost of alleviating poverty. We should note, though, that this substitution in itself reduces the poors' incentive to make an e®ort in the ¯rst period to increase their future income potential. But in terms of overall costs, it is e±cient.

In the ¯nal section of the paper we characterize optimal design of a poverty alleviation program if the welfare administrator can commit to a long term program. If we isolate the screening issue, we ¯nd that the optimal commitment policy coincides with the equilibrium policy under non-commitment. When we in addition take account of how future policy a®ect the poor's' incentive to exert poverty reducing investments, we ¯nd that it is almost always optimal to impose some work requirements on those that receive transfers. More speci¯cally, it is optimal to impose on welfare claimants either a very high work requirement or a low one. This result, di®ers from the conclusion drawn by Besley and Coate (1992); they ¯nd that it is sometimes optimal to commit to a pure welfare program. The reason for this di®erence is that when Besley and Coate (implicitly) assume commitment, they focus solely on the deterrence problem. We study a welfare administrator that has two concerns; in addition to give the poor strong incentives to undertake poverty reducing investments the policy must also be appropriate given the screening problem faced at this stage.

Methodology and related literature

In addition to the light our model sheds on an important policy issue, we believe it has some methodological interest. Formally, we study the design of a dynamic Bayesian game. Our problem is therefore closely related to the literature on dynamic principal agent relationships which emphasize the role that asymmetric information and long-term commitment plays in governance. Our problem of alleviating long-term poverty resembles the basic structure of for example a dynamic regulation problem. Still, the results we derive di®er sharply from those obtained there. A central result in optimal regulation is that a regulator who is able to commit herself to a multi-period contract, ought to repeat the optimal static policy in every period (cf La®ont and Tirole, 1990). This policy is however not time consistent; the regulator will not follow the plan if she is free to re-optimize later on. Lack of commitment is therefore detrimental in a standard dynamic regulation problem.[1] In poverty alleviation it is not always optimal to repeat the static program in each period, and, as a consequence of this, lack of intertemporal commitment is not always a problem. Another notable feature of our model is that if a semi-separating equilibrium exists, it involves randomization from both the agents (welfare recipients) and the principal (the welfare administrator).

Before we dig deeper into the details of our arguments, we should say something about the scope of our perspective, and how it relates to existing literature. The literature on how policy instruments can be used to target transfers to the poor is extensive{see Lipton and Ravallion (1995) for a discussion and for refer-

[1] Weitzman (1980) was the ¯rst to use a principal agent framework to point out the negative e®ects lack of intertemporal commitment has on the agents behaviour. Freixas et al (1985) developed the ¯rst game theoretic analysis of a dynamic principal-agent relationship governed by linear incentive schemes. For other references and for a general discussion of this topic, see chapters 9 and 10 in La®ont and Tirole (1993). Dill¶n and Lundholm (1996) use the framework developed by Freixas et al to discuss optimal income taxation and redistribution in a dynamic model.

ences. Although the possibility of using work requirement to screen the needy from the not-so-needy had been discussed before, Besley and Coate (1992) was the ¯rst paper that gave a detailed analysis of the argument.[2] It is their model we extend to a dynamic environment. We think this is an important extension, both because there is virtually no theoretical work on the dynamics of poverty programs, and because long term poverty is a serious problem: a substantial share of those who live below the poverty line do so persistently.[3]

Admittingly, the \cost e±ciency perspective" on poverty alleviation and the e®ects of workfare that we borrow from Besley and Coate, is narrow. One limitation is that it considers work requirements solely as a stick that scares the non poor from claiming bene¯ts and poor from not doing anything to improve their situation. This is obviously not the whole story. Having a job can also be seen as an essential aspect of life, something that provides people with social recognition and self esteem. Another important point is that making welfare claimants work for their bene¯ts may prevent a deterioration of their working moral and human capital. Furthermore, it is not obvious that individuals are poor{as we assume{ because they are endowed with an insu±cient earning capacity. Alternatively, one may argue that it is the lack of well functioning economic institutions to deal with property rights, information problems, etc. which is the main reason why so many people live in poverty{see Ho® (1996). We also ignore the political legitimacy of di®erent poverty alleviation programs{see Besley (1996). We are not saying that these arguments are unimportant, only that they are irrelevant for the incentive problem we focus on.

Having pointed out the limits of our scope, we should, however, hasten to add that we believe the problem we point at warrants attention. Our arguments should be mentioned in a general debate about how one ought to provide assistance to the long-term poor, which is an important debate, both in developing countries and more modern welfare states. In fact the problem of ¯nding a cost e®ective way to provide assistance to the poor is a highly current topic in many European welfare states where a tightening of public ¯nance constraints has forced welfare administrators to cut their budgets.

The next section presents a formal model of the costs and bene¯ts of using

[2] See also Besely and Coate (1995).

[3] For example, Heady et al (1994) ¯nd that 10 % of the population in Germany are frequently poor or near-poor. Rodgers & Rodgers (1993) conclude that about one third of measured poverty in the US as of 1987 can be regarded as 'chronic', and that over the period they studied, "poverty not only increased, it became more chronic and less transitory in nature" (p 51). Adams & Duncan (1988), in a study of US urban poverty, estimated that of the 13.4% of urban people that where poor in 1979, 34.6% were poor in at least one year between 1974 and 1983, and 5.2% was 'persistently poor'{de¯ned as poor in 8 out of 10 years or 80% of the years covered.

In poor underdeveloped countries the problem of chronic poverty is even more pronounced, Gibson (2001) uses data from a recent household survey in Papua New Guinea to conclude that close to half of those classi¯ed as poor, has a chronic poverty problem.

workfare in targeting the poor. In section 3 we characterize the cost minimizing program in a static framework. In section 4, which is the heart of the paper, we introduce dynamics and study how workfare can be used to minimize the cost of providing transfers to the long term poor. In section 5 we include poverty-reducing investments. Section 6 concludes the paper.

2 A formal model of the costs and benefits of using workfare to target benefits to the poor

As a prerequisite to the dynamic analysis, we analyze poverty alleviation in a static (one period) model. We focus solely on the screening problem. It is natural to postpone the discussion of poverty-reducing investments, since we need a dynamic model to asses how workfare affect the poor's' present effort to escape future poverty.

We follow Besley and Coate (1992) and assume that an administrator of a welfare program, hereafter referred to as the WA, faces a target population of a size normalized to 1. A fraction γ has a very low productivity a_L and a fraction $(1 - \gamma)$ is endowed with a higher productivity a_H. We stress here that the latter also are 'low class', but not as destitute as the former. All people have the same strictly concave utility function defined over disposable income (x) and leisure (ℓ), $u(x, \ell)$, and a time endowment normalized to unity. People choose the amount of private sector labor which maximizes their utility level. Without any welfare program, the L-people (and only L-people) earn a disposable income below the poverty line z. The WA faces the task of designing a cost minimizing welfare program that guarantees everybody at least the minimal income z.

A welfare program consists of the menu $\{(b_L, c_L), (b_H, c_H)\}$, where b is a money transfer and c the number of hours of public work an applicant is required to carry out in order to qualify for the transfer.[4] The menu must guarantee that: (i) all people voluntarily participate in the program, (ii) everybody at least enjoys a disposable income z, (iii) nobody has an incentive to apply for the package intended for somebody with a different productivity, and (iv) the total cost of the program, $\gamma b_L + (1 - \gamma) b_H$, is kept at a minimum (because it will be financed by distortionary taxation on the other people in the economy).

Individual behaviour

An individual with ability a, receiving the package (b, c) decides how much income (y) to earn:

$$\max_{y \geq 0} u(b + y, 1 - c - \frac{y}{a}).$$

[4] As in the Besley-Coate paper, we shall assume that public sector work is unproductive. We discuss the impact of this assumption in footnote 17.

Let us denote the solution by $y(b; c; a)$. Normality of consumption and leisure means that as long as $y(b; c; a) > 0$, the derivatives w.r.t. c and b are negative. Regarding the latter, Moffitt (1992, p 16)) reports on an absolute value of .37 for females, while Sawhill (1988, p 1103) reports on absolute values in the range [.16,.71].

The corresponding maximal utility level is written as $v(b; c; a)$. Note that if the transfer b and/or the work requirement c are very high, it may be optimal to refrain from working privately altogether—the utility level then reduces to $u(b; 1 - c)$. Note also that our concavity assumptions on $u(\cdot)$ implies $v_{bb} < 0$:

The costs of workfare

The aim of the transfer policy is to guarantee L-people a disposable income of at least z. For a given work requirement c_L, let $b_L(c_L)$ be the lowest transfer that accomplishes this. It is defined as

$$b_L(c_L) + y(b_L(c_L); c_L; a_L) = z:$$

Implicit derivation shows that $\frac{db_L(c_L)}{dc_L} = a_L$: a higher work requirement crowds out private sector earnings with a_L, and thus requires an extra a_L Euro to top up disposable income to the poverty line. Imposing a work requirement is thus costly because it necessitates larger transfers to needy people.

We define c^{co} as the work requirement that crowds out private sector earnings completely:[5]

$$c^{co} \stackrel{def}{=} \max\{c : y(b_L(c); c; a_L) \geq 0\}:$$

The necessary transfer $b_L(c)$ thus satisfies

$$\begin{aligned} b_L(c) &= b_L(0) + a_L c \quad \text{if} \quad c \leq c^{co}; \\ &= z \qquad\qquad\qquad\quad\;\; c \geq c^{co}; \end{aligned}$$

and is clearly concave in c.

Another important value is the work requirement that brings L down to his reservation utility level:

$$c^{max} \stackrel{def}{=} \max\{c : v(b_L(c); c; a_L) \geq v(0; 0; a_L)\}:$$

Clearly, c^{max} puts an upper bound on the WA's selection of work requirements.

The benefits of workfare

The WA has to offer appropriate incentives to prevent H-individuals from joining the poverty program. She must make sure that an H-person gets a utility

[5] For a sufficiently high poverty line (compared to Ls earnings capacity a_L), this work requirement may drop to zero: even without work requirement, the transfer necessary to raise L to the poverty line is so large that it crowds out private earnings completely.

level at least as high as the one he gets when pretending to be poor. Pretending to be poor can be easy or difficult, depending on what the WA observes. One possibility is that the WA observes no personal characteristics of the applicants; applying for a welfare package is then a sufficient condition for getting it. But one could also imagine that the WA observes private sector earnings, and that welfare applicants qualify for transfers only when their earnings do not exceed a certain limit. In this paper, we limit ourselves to analyse the first case.[6]

The maximum utility H gets if he receives a transfer b_H in exchange for a work requirement c_H is thus $v(b_H; c_H; a_H)$. On the other hand, when H pretends to be of type L, he attains a welfare level $v(b_L(c_L); c_L; a_H)$. The screening, or no mimicking constraint can thus be written as

$$v(b_H; c_H; a_H) \geq v(b_L(c_L); c_L; a_H).$$

Obviously, it is optimal to choose $c_H = 0$. Supplementing b_H with a positive work requirement implies a higher transfer to H, which increases the total cost of the program. To ease exposition, we drop the subscript on the work requirement since this policy is only relevant for the program intended for the poor.

Let $b_H^s(c)$ be the minimum transfer H must receive in order not to register as poor (the superscript s indicates that we are analyzing a static problem). This is an information rent–resources H receives because the WA cannot observe his earning capacity. Its magnitude is implicitly defined by

$$v(b_H^s(c); 0; a_H) = v(b_L(c); c; a_H). \tag{2.1}$$

Requiring the poor to work for their benefits makes it less attractive for H to mimic L and thus the minimum transfer b_H^s can be reduced. The following lemma informs about the shape of $b_H^s(c)$ (proven in appendix).

Lemma 1 The transfer function $b_H^s(c)$ has the following first and second derivatives:

$$\begin{aligned}\frac{db_H^s(c)}{dc} &= -(a_H - a_L) \text{ if } c < c^{co}\\ &= -a^H \text{ if } c^{co} \le c \le c^{max};\\ \frac{d^2 b_H^s(c)}{dc^2} &= 0.\end{aligned}$$

Moreover $b_H^s(0) = b_L(0)$.

By the last property, universal welfare is equivalent to $c = 0$.

[6] The income observable case is discussed in Besley & Coate (1992) for short term poverty alleviation and in Schroyen & Torsvik (1999) for long term poverty alleviation.

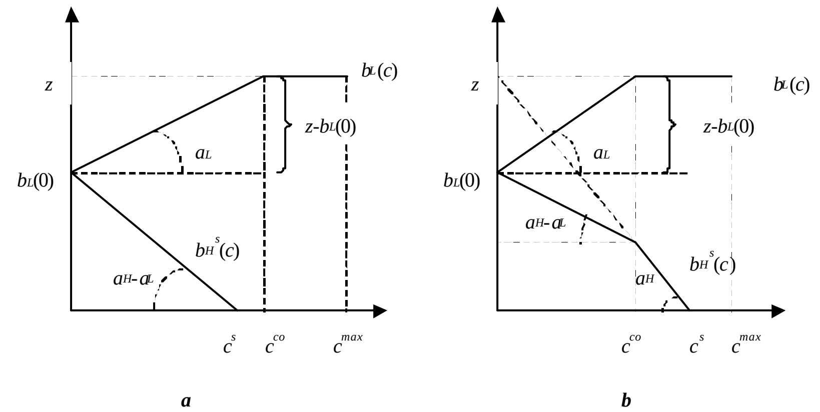

Figure 1. $b_L(c)$ and $b_H^s(c)$ when $c^s < c^{co}$ (a) and $c^s > c^{co}$ (b).

Figure 1:

Since the transfer function is decreasing and concave in c there exists a critical value for the work requirement on L-persons, c^s, for which the transfer b_H can be set to zero and still secure self-selection, i.e. $b_H^s(c^s) \equiv 0$. It is easy to see that $c^s < c^{max}$. Figures 1a and 1b display $b_L(c)$ and $b_H^s(c)$.

3 The cost minimizing static program

We can now construct the function which maps the work requirement c into the total cost of the welfare program,

$$K^s(c) \stackrel{\text{def}}{=} \gamma b_L(c) + (1 - \gamma) b_H^s(c):$$

By definition, this function gives–for any arbitrary work requirement–the minimal pair of transfer payments which satisfy both the poverty alleviation and incentive compatibility constraints (the poverty alleviation restriction is taken care of by the function $b_L(c)$, while the self-selection constraint for H-agents is verified because they receive a transfer specified by the function $b_H^s(c)$). As H-persons always have the option to stay away from the programme, they cannot be imposed any taxes. This is equivalent to requiring that $b_H(c) \geq 0$ or $c \leq c^s$. The welfare administrator's problem can therefore be written as the following one dimensional optimization problem:

$$\min_{c \leq c^s} K^s(c):$$

Since both transfer functions are piecewise linear but concave in c, there are two possible solutions to the minimization problem: either c^s or 0. Workfare is either used so extensively that H-people do not require any rent in order not to sign up for poverty transfers, or workfare will not be used at all and poverty is alleviated through universal welfare. In the first case the costs of alleviating poverty are $\gamma b_L(c^s)$; in the second, they amount to $b_L(0)$.

That the choice between welfare or workfare depends on the value of γ is not difficult to grasp. An increase in γ, reduces the gain of using workfare: the fewer potential mimickers there are in the population, the lower is the cost of paying them the rent which prevent them from applying for the package meant for the really needy. In the limit, as γ approaches 1, (almost) all individuals are of the L-type and it would be wasteful to distort the behavior of (almost) the whole population in order to eliminate a cost (the rent to the H-people) that is negligible.

Let γ^s be the value of γ for which the administrator is indifferent between choosing no work requirement and the maximal work requirement c^s. It is then easy to check that

$$\gamma^s \overset{\text{def}}{=} \frac{b_H^s(0)}{b_H^s(0) + [b_L(c^s) - b_L(0)]} = \frac{b_L(0)}{b_L(c^s)} = 1 - \frac{a_L}{a_H}\frac{\min\{c^s; c^{co}\}}{c^s}. \tag{3.1}$$

Thus, the WA will opt for a workfare policy when $\gamma < \gamma^s$, and otherwise for universal welfare.

To understand what comes later when we introduce dynamics, it is important to keep in mind that the transfer which H-agents receive (their information rent) is a discontinuous function of γ. It is defined as

$$\rho_H(\gamma) \equiv \begin{cases} b_H^s(0) > 0 & \text{if } \gamma > \gamma^s; \\ 0 & \text{if } \gamma \leq \gamma^s. \end{cases} \tag{3.2}$$

This model contains many interesting insights that we cannot elaborate on here.[7] We just mention the discontinuity of the rent function gives the problem a particular feature that prevents us from translating results from standard dynamic principal agency problems (like the regulation literature) to our setting.

4 Dynamics and the problem of targeting the poor

So far we have followed Besley and Coate (1992) and taken it for granted that the information people reveal by opting for a particular poverty program cannot be utilized by the WA later on. Suppose now that the poverty program runs over

[7] For a detailed description of the static poverty alleviation problem, we refer the reader to Besley and Coate (1992).

several periods, and that the WA can learn something about people's earning capacity as time passes. This obviously adds a new dimension to the problem and new questions pop up: how does lack of intertemporal commitment affect optimal policy? will it make separation of the needy from the non-needy more difficult? will work requirements become a less attractive instrument? Moreover, in a long term setting, the question how poverty alleviation policies affect the poor's' incentive to undertake poverty-reducing investments becomes meaningful. This is a moral hazard aspect that possibly interacts with the adverse selection problem.

For didactical purposes, we first discuss the screening problem in isolation. We start by describing the classes of equilibria that exists when the WA is unable to commit herself to a particular poverty alleviation program in the future. Next, we discuss the optimality of the different equilibria. In section 5, we assume that the poor can exert an effort e in the first period that increases their probability to escape poverty in the future. We characterize how the possibility of using workfare in poverty alleviation affects e, and how this moral hazard problem affects the difficulty of targeting transfers to the poor in the first place.

Finally, we state our assumptions on intertemporal preferences and opportunity sets. Preferences are taken to be additive across periods, with a zero rate of discount. Also the WA uses a zero discount rate to compute intertemporal costs. This choice of discount rate is not crucial to our results, but considerably facilitates the exposition of the arguments. We do not allow individuals to save or borrow money across periods. There are several reasons for constraining individual behavior in this way. First, we want to limit the connection between periods to one stock variable (information). Second, once saving and borrowing is allowed, the definition of the poverty line becomes more fuzzy. Third, it can be regarded as a stylized representation of the poors' imperfect access to capital markets.

4.1 Equilibria: types and existence

The simplest framework to discuss long term poverty alleviation includes two periods and four stages. At this stage, we also assume that individual earning capacities are perfectly correlated over time (in section 5, we investigate how our results change when agents can influence next period's ability).

The structure of the game is as follows.

Period 1

Stage 1: The WA designs a first period poverty program $[(b_L^1; c_L^1); (b_H^1; c_H^1)]$:

Stage 2: Individuals decide which package they want to sign up for.

Period 2

Stage 3: The WA is not committed to any prior announcements. Given her updated information on the basis of what she observed in stage 2, she designs the cost minimizing poverty program $[(b_L^2; c_L^2); (b_H^2; c_H^2)]$:

Stage 4: Individuals decide which packages they want to sign up for.

We can simplify this intertemporal program in several respects. First, because the WA has to alleviate poverty in each period, she will set b_L^1 and b_L^2 equal to $b_L(c_L^1)$ and $b_L(c_L^2)$, respectively. Second, from the static model we know that it is never optimal to impose a work requirement on a high ability person. So at stage 3 the WA will set $c_H^2 = 0$. We also claim here that if the first period transfers given to H-persons are not "too high", an L-person will never want to choose the package intended for H-persons and therefore first period transfers to H will not be made conditional on a work requirement: $c_H^1 = 0$. In the appendix, we give sufficient conditions for this to be verified by the optimal policy. Thus, again, we drop the subscript L on c without any risk of confusion.

Let γ^2 be the updated belief that an agent who opted for bundle $(b_L^1(c^1); c^1)$ in the first period is of type L: An H-person may find it in his interest to apply for this package. If he does, he gets $(b_L^1(c^1); c^1)$ in the first period and $(\beta_H(\gamma^2); 0)$ in period two. On the other hand, should he not register as poor he gets $(b_H^1; 0)$ in the first period and $(0; 0)$ in the second. The values of these two options are $v(b_L(c^1); c^1; a_H) + v(\beta_H(\gamma^2); 0; a_H)$ and $v(b_H^1; 0; a_H) + v(0; 0; a_H)$, respectively. Depending on the magnitude of the transfers, and the work required, there exists three kinds of equilibria.[8] A separating equilibrium in which different types choose different actions in the first period (H-people do not register as poor), a pooling equilibrium in which H-people register as poor, and a semi-separating equilibrium in which H-people randomize between registering as poor or not.

Separating equilibrium

We have a separating equilibrium if an H-person prefers not to register as poor even though the WA believes that all who do are genuinely poor ($\gamma^2 = 1$). That is, if

$$v\left(b_H^1; 0; a_H\right) + v(0; 0; a_H) \geq v\left(b_L(c^1); c^1; a_H\right) + v(b_L(0); 0; a_H):$$

Separation can be induced either by a welfare policy or by a workfare policy. The lower boundary of $(b_H^1; c^1) -$ values giving rise to a separating equilibrium is

[8] The proper equilibrium concept for this game is perfect Bayesian equilibrium. This means that (i) the agents make an optimal choice in period 2 among the packages made available to them by the WA; (ii) the WA's design of the second period's program should be optimal, given her updated beliefs; (iii) the choice of the agents in stage 2 should be optimal given the packages made available by the WA in stage 1 and taking into account the fact that the second period program that is made available to them will depend on the WA's updated beliefs, and therefore on their first period choice; (iv) the WA's choice of program in the first period is optimal given the strategies of the agents and of her own 2nd period strategies; and (v) the WA updates her beliefs by observing the participants' first period behaviour, thus γ^2 =Prob(agent is of type L|agent chose in period 1 the package $[b_L(c^1); c^1]$).

found by letting the inequality above bind. Let $b_H^d(c^1)$ be defined as the minimum transfer that induces separating for a first period work requirement c^1, then

$$v(b_H^d(c^1); 0; a_H) + v(0; 0; a_H) = v(b_L(c^1); c^1; a_H) + v(b_L(0); 0; a_H) \quad (4.1)$$

The following lemma informs about the shape of $b_H^d(c)$ (proven in appendix).

Lemma 2 The transfer function $b_H^d(c)$ has the following first and second derivatives:

$$\frac{db_H^d(c)}{dc} = \frac{v_b^s}{v_b^d}\frac{db_H^s(c)}{dc} < 0$$

$$\frac{d^2 b_H^d(c)}{dc^2} = \frac{(v_b^s)^2}{v_b^d}\left[\frac{v_{bb}^s}{(v_b^s)^2} - \frac{v_{bb}^d}{(v_b^d)^2}\right]\left(\frac{db_H^s(c)}{dc}\right)^2$$

where v_b^s and v_b^d are shorthands for $v_b(b_H^s(c); 0; a_H)$ and $v_b(b_H^d(c); 0; a_H)$, resp., and likewise for the second order income derivatives v_{bb}^s and v_{bb}^d.

That concavity of $b_H^d(c)$ is no longer guaranteed by the assumptions we have invoked so far is easy to see when noting that the rhs of (4.1) can also be written as $v(b_H^s(c^1); 0; a_H) + v(b_H^s(0); 0; a_H)$. Since $b_H^s(c^1)$ is concave in c^1, 1st period (and thus intertemporal) utility when mimicking is strictly concave in c^1. But on the other hand the first period transfer b_H^1 is a strictly convex function of 1st period (and thus intertemporal) utility when being honest. However, if the first mentioned concavity is "strong" compared with the convexity, the term $[\frac{v_{bb}^s}{(v_b^s)^2} - \frac{v_{bb}^d}{(v_b^d)^2}]$ will be negative.[9] This, we assume in the sequel.

With a transfer function that is decreasing and concave in c there exists again a critical value for the work requirement on L-persons, c^d, for which the transfer b_H^d can be reduced to zero while still securing self-selection, i.e. $b_H^d(c^d) \equiv 0$. It is an empirical issue whether c^d exceeds c^{max} or not. If it does, c^d is not implementable, since that would scare away L-people and make the programme meaningless. Then, the best the WA can do is replace it by c^{max} and leave a positive information rent $b_H^d(c^{max})$ to H-people.

Straightforward computation reveals that (i) $b_H^d(0) > 2b_H^s(0)$, (ii) $c^d < 2c^s$, and (iii) $b_H^d(c^s) = b_H^s(0)$. Observation (i) tells us that if the WA decides to alleviate first period poverty by using welfare, she must offer H-people more than twice the amount she needed to give them in the static case. The reson is that v_{bb} is negative. Observation (ii) tells us that if she decides to use workfare to scare

[9] It can be shown that the sign of this term is given by the sign of $\frac{d \log R_a}{d \log m} + R_r$, where R_a and R_r are the coefficients of absolute and relative risk aversion for uncertainty regarding full income m: Decreasing absolute risk aversion and a not too large R_r is thus sufficient for concavity of $b_H^d(c)$:

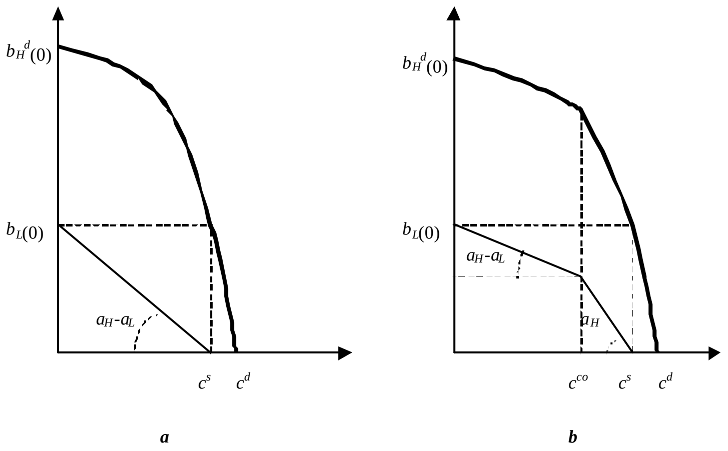

Figure 2. $b_L(c)$ (—) and $b_H^d(c)$ ( ).

Figure 2:

fraudulent H-people off the poverty program, she has to impose a higher work requirement than in the static case, but the number of hours that are sufficient to drive H's rent to zero is less than twice the amount needed in the static case. The reason is again that v_{bb} is negative.[10] Both observations indicate a potential advantage of the workfare instrument in a long term poverty context. Finally, (iii) implies that $b_H^d(c)$ everywhere lies above $b_H^s(c)$. Figures 2a and 2b show the relation of $b_H^d(c)$ to $b_H^s(c)$.

With the two groups successfully separated in the first period, the second period policy reduces to the first best type contingent policy: a cash transfer $b_L(0)$ is offered the poor and nothing to H-individuals.

Pooling equilibrium

Clearly, if b_H^1 and $c^1$are sufficiently low an H-person may prefer to mimic the poor even though the WA knows this (so $\gamma^2 = \gamma^1$) . The condition for a pooling equilibrium is given by the inequality

$$v\left(b_L(c^1); c^1; a_H\right) + v\left(\bar{b}_H(\gamma^1); 0; a_H\right) \geq v\left(b_H^1; 0; a_H\right) + v(0; 0; a_H):$$

The upper boundary for pooling depends on the value γ^1 takes. If $\gamma^1 \geq \gamma^s$ mimicking in the first period generates a welfare policy in the second period and

[10]Evaluating (4.1) at $c^1 = c^d$, noting that $v(0; 0; a_H) = v(b_H^s(c^s); 0; a_H)$ and using the alternative formulation for the rhs, we get that $2v(b_H^s(c^s); 0; a_H) = v(b_H^s(c^d); 0; a_H) + v(b_H^s(0); 0; a_H)$. Since $b_H^s(c)$ is decreasing and concave in c, and $v(b; 0; a_H)$ increasing and strictly concave in b, it follows that $c^d < 2c^s$.

a monetary rent $\beta_H(\gamma^1) = b_H^s(0)$: In this case we can easily see that the upper boundary of the pooling equilibrium coincides with the lower boundary of the separating equilibrium (since by definition $v(b_H^s(0); 0; a_H) = v(b_L(0); 0; a_H)$). If on the other hand $\gamma^1 < \gamma^s$, we know that $\beta_H(\gamma^1) = 0$ and we can see that pooling occurs when $v(b_L(c^1); c^1; a_H) \geq v(b_H^1; 0; a_H)$, which with equality is the equation for separation in the one period static case–eq (2.1).

Semi-Separating equilibrium

The third kind of equilibrium requires the following set of inequalities to be fulfilled:

$$v\left(b_L(c^1); c^1; a_H\right) + v(b_H^s(0); 0; a_H) > v(b_H^1; 0; a_H) + v(0; 0; a_H)$$

$$> v\left(b_L(c^1); c^1; a_H\right) + v\left(\beta_H(\gamma^1); 0; a_H\right):$$

The lhs is H's utility when mimicking as L when the WA believes everybody is of type L ($\gamma^2 = 1$), while the rhs is utility under mimicking when the WA sets $\gamma^2 = \gamma^1$. A necessary condition for this series of inequalities to hold is of course that $\gamma^1 < \gamma^s$, since $\beta_H(\gamma^1) = b_H^s(0)$ if $\gamma^1 \geq \gamma^s$. Suppose then that $\gamma^1 < \gamma^s$. Then we claim that there exists a semi-separating equilibrium in which an H-person chooses the program intended for him (he does not register as poor) with probability

$$\pi^{SS} \stackrel{\text{def}}{=} \frac{\gamma^s - \gamma^1}{(1 - \gamma^1)\gamma^s}; \tag{4.2}$$

and the WA chooses a zero work requirement in the second period (i.e. $c^2 = 0$) with probability

$$q^{SS}(b_H^1; c^1) \stackrel{\text{def}}{=} \frac{[v(b_H^1; 0; a_H) - v(b_L(c^1); c^1; a_H)]}{[v(b_H^s(0); 0; a_H) - v(0; 0; a_H)]}: \tag{4.3}$$

To understand this claim, note that if H mimics with probability π, the WA will rationally believe that among those who opted for poverty transfers in the first period a fraction γ^s are genuinely poor. With such a belief, the WA is indifferent between a workfare and a welfare program in the second period, and therefore willing to randomize between these two policies. A simple computation shows that she must randomize with probability $q^{SS}(b_H^1; c^1)$ in order to make H indifferent between pooling with L-individuals and separating.[11] The semi-separation equilibrium is depicted in the middle part of figure 3 below.

Let us summarize the facts we have established so far.

[11] H's utility when pooling and separating are $v(b_L(c^1); c^1; a_H) + (1 - q)v(b_L(c^s); c^s; a_H) + qv(b_L(0); 0; a_H)$ and $v(b_H^1; 0; a_H) + v(0; 0; a_H)$, respectively. Since $v(b_L(c^s); c^s; a_H) = v(0; 0; a_H)$ and $v(b_L(0); 0; a_H) = v(b_H^s(0); 0; a_H)$, (4.3) follows.

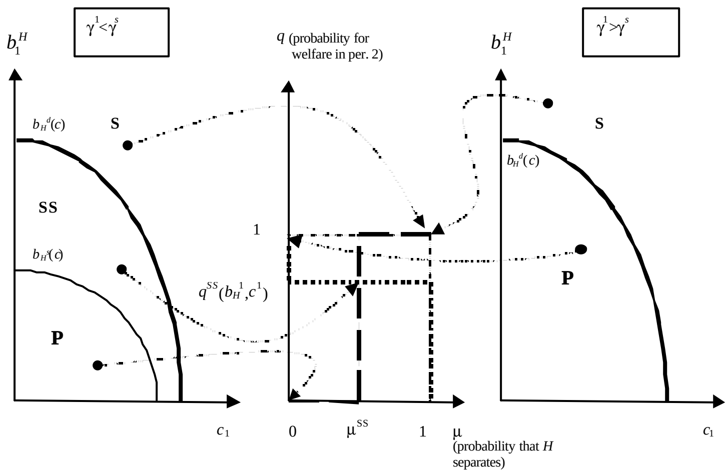

Figure 3. The reaction curves of *WA* ( ) and *H* ( ) and the different continuation equilibria.

Figure 3:

Proposition 1 Depending on the value of γ^1, the following equilibria exist:
For $\gamma^1 < \gamma^s$:
(i) **separating equilibrium.** H and L are separated in the first period, and a type contingent welfare policy is implemented in the second period; (b_H^1, c^1) satisfy $b_H^1 \geq b_H^d(c^1), 0 \leq c^1 \leq \min\{c^d, c^{max}\}$;
(ii) **semi-separating equilibrium.** H and L are partly separated in the first period, and WA chooses randomly between welfare and workfare in the second period; (b_H^1, c^1) satisfy $b_H^s(c^1) \leq b_H^1 < b_H^d(c^1)$, $0 \leq c^1 \leq \min\{c^d, c^{max}\}$; and
(iii) **pooling equilibrium.** H and L are not separated in the first period, and a separating workfare program is offered in the second period; (b_H^1, c^1) satisfy $0 \leq b_H^1 \leq b_H^s(c^1), 0 \leq c^1 \leq c^s$.

For $\gamma^1 \geq \gamma^s$:
(i) **separating equilibrium.** H and L are separated in the first period, and a type contingent welfare policy is implemented in the second period; (b_H^1, c^1) satisfy $b_H^1 \geq b_H^d(c^1), 0 \leq c^1 \leq \min\{c^d, c^{max}\}$; and
(ii) **pooling equilibrium.** H and L are not separated, and universal welfare is offered in the second period; (b_H^1, c^1) satisfies $0 \leq b_H^1 < b_H^d(c^1), 0 \leq c^1 \leq \min\{c^d, c^{max}\}$.

These different equilibria are depicted in figure 3 (for the case where $c^s < c^d < c^{co} < c^{max}$).

4.2 Optimal poverty alleviation programs

Now that we have outlined the continuation equilibrium for each first period program (b^1_H, c^1), we have enough information to characterize the cost minimizing first period program. The first period policy is made up of two instruments: c^1 hours of work requirement on L-persons, and the cash transfer b^1_H to H-persons. In terms of first period resources, it is costly to use both instruments, but on the other hand, an appropriate use of these instruments can make it more efficient to target transfers to the long term poor and to economize on second period transfers. When H-persons separate in the first period with probability μ, the cost of the program in that period is

$$K^1(c^1; b^1_H; \mu) \stackrel{def}{=} [\gamma^1 + (1 - \gamma^1)(1 - \mu)]b_L(c^1) + (1 - \gamma^1)\mu b^1_H. \qquad (4.4)$$

The first square brackets term denotes the number of persons displaying type L behavior: the really needy and the fraction of H-persons pretending to be needy. The second term gives the amount of transfers handed over to those H-persons who reveal themselves as non-needy. Since both instruments c^1 and b^1_H give rise to first period costs, it will be efficient to select them on the lower boundary of each regime. Thus, if separation ($\mu = 1$) is aimed at, the WA should set $b^1_H = b^d_H(c^1)$ and $c^1 \geq \min\{c^d; c^{max}\}$. Likewise, a minimal efficiency requirement for inducing semi-separation is that $b^1_H = b^s_H(c^1)$. And if pooling is intended ($\mu = 0$), costs are minimized when $b^1_H = 0$ and $c^1 = 0$.

We then turn to second period costs. If the WA randomizes and chooses a welfare policy with probability q in the second period, the expected costs are given by

$$\begin{aligned} EK^2(\mu; q) \stackrel{def}{=} \; & \gamma^1[(1 - q)b_L(c^s) + qb_L(0)] \\ & + (1 - \gamma^1)(1 - \mu)[(1 - q) \cdot 0 + qb^s_H(0)]; \end{aligned} \qquad (4.5)$$

where $(\mu; q)$ take on the values (1,1) under separation and type-contingent welfare policy, $(\mu; q^{SS}(b^1_H; c^1))$ under semi-separation and a random policy, $(0; 0)$ under pooling and workfare (if $\gamma^1 < \gamma^s$), and $(0; 1)$ under pooling and welfare (if $\gamma^1 \geq \gamma^s$). In this expression, the first square bracket term is the expected transfer which will be handed over to L-persons, while the second square bracket term is the expected amount of money that will be transferred to every H-person that pooled in the first period with the L -types (those H-persons that revealed themselves in the first period—a fraction $(1 - \gamma^1)\pi$—receive no transfer at all).

With generic cost functions given by (4.4) and (4.5), we can start inquiring about the kind of equilibrium that ought to be established in the first period, and how that equilibrium should be implemented. We first address the latter question for the separating equilibrium. Next, we compare the minimal costs under separation with the minimal cost of a pooling and semi-separation program (should this last program be relevant).

Assume that the WA has decided to induce a separating equilibrium. When is it optimal to rely on a work requirement to screen the two types? Obviously, the answer depends on how large the fraction of potential fraudulent claimants (H-persons) is. Just as for the static case, welfare will be optimal when γ^1 is close to one, and workfare will be optimal when γ^1 is close to zero. To find out when it becomes efficient to switch from workfare to welfare, we equate the first period costs under workfare, $K^1(\min\{c^d, c^{max}\}, 0, 1)$, with those under welfare, $K^1(0, b_H^d(0), 1)$ and solve for the *a priori* belief γ^1:

$$\gamma^d \stackrel{def}{=} \frac{[b_H^d(0) - b_H^d(\min\{c^d, c^{max}\})]}{[b_H^d(0) - b_H^d(\min\{c^d, c^{max}\})] + [b_L(\min\{c^d, c^{max}\}) - b_L(0)]}. \tag{4.6}$$

This leads to

Lemma 3 *A separating equilibrium with a work requirement* $\min\{c^d, c^{max}\}$ *is less costly than a separating equilibrium with welfare if and only if* $\gamma^1 < \gamma^d$.

Since $b_H^d(0) > 2b_H^s(0)$ and $\min\{c^d, c^{max}\} \le c^d < 2c^s$ it follows that $\gamma^d > \gamma^s$ (compare equations (3.1) and (4.6)). Hence, a WA who runs a two-period program strictly prefers a workfare policy if $\gamma^1 = \gamma^s$. This implies that a workfare program is cost effective for a wider range of prior beliefs in a dynamic context. Also note that for a given level of c^d, γ^d is bigger when $c^d < c^{max}$ than when $c^d > c^{max}$ — intuitively, when you have to leave some rent to H when using workfare, you will resort to that instrument 'less often'.

Let us now compare the minimal costs under separating equilibrium with those under the other types of equilibria. First, consider the case where $\gamma^1 < \gamma^s$. The expected second period costs in a semi-separating equilibrium is $\gamma^1 b_L(c^s)$, which is precisely the expected second period cost under pooling (a WA who has learned nothing from the first period implements a workfare program in the second period when $\gamma^1 < \gamma^s$).[12] On the other hand, the minimal first period cost under pooling is $b_L(0)$, while it is $\frac{\gamma^1}{\gamma^s} b_L(c^1) + (1 - \frac{\gamma^1}{\gamma^s}) b_H^s(c^1)$ under semi-separation. Since $b_H^s(0) = b_L(0)$ and $b_H^s(c)$ is decreasing in c, the minimal first period cost under semi-separation is always below the corresponding cost under pooling. This proves

Lemma 4 *Suppose* $\gamma^1 < \gamma^s$. *Then any semi-separation equilibrium with a first period policy* $(c^1, b_H^s(c^1))$, $c^1 \in [0, c^s]$ *is less costly than any first period policy resulting in a pooling equilibrium using the same work requirement.*

Thus, when $\gamma^1 < \gamma^s$ it suffices to compare the most efficient policies yielding semi-separation with the workfare policy leading to full separation. In the appendix we prove

[12] Recall that a semi-separating equilibrium can only occur when $\gamma^1 < \gamma^s$. The expected cost under semi-separation is given by (4.5) with $\mu = \mu^{SS}$ (defined in (4.2)). Making use of (3.1), this reduces to $\gamma^1 b_L(c^s)$, whatever value q takes.

Lemma 5 Suppose $\gamma^1 < \gamma^s$. Then there exists a critical level of γ^1 given by

$$\gamma^{SS} \overset{\text{def}}{=} \frac{b_H^d(\min\{c^d, c^{max}\})}{b_H^d(\min\{c^d, c^{max}\}) + (1+\frac{1}{\gamma^s})b_L(c^s) - z - b_L(0)}$$

such that the cost efficient policy is separation with work requirement $\min\{c^d, c^{max}\}$ iff $\gamma^1 > \gamma^{SS}$, and semi-separation with work requirement c^s otherwise.

For $c^d < c^{max}$ we have $b_H^d(\min\{c^d, c^{max}\}) = 0$ and $\gamma^{SS} = 0$; a separating policy with work requirement c^d costs less than a semi-separating policy for any a priori beliefs $\gamma^1 < \gamma^s$. For $c^d > c^{max}$ we have $b_H^d(\min\{c^d, c^{max}\}) > 0$ and $\gamma^{SS} > 0$; a semi-separating policy with a work requirement c^s in both periods costs less than a separating policy with c^{max} for small values of γ^1.

To explain the last case, note that full separation with maximal use of work requirements implies some rents to the non-poor. This policy is relatively costly if there are many non-poor around (if γ^1 is low). On the other hand, there exists a semi-separating equilibrium where a work requirement c^s is imposed in both periods. To see this, note that if fewer than $(1-\gamma^1)\mu^{SS}$ of the non-poor separate in the first period, it is optimal for the WA to impose a work requirement c^s in the second period, and a first period work requirement c^s is sufficient to make the non-poor indifferent between separation and mimicking. There thus exist an equilibrium with semi-separation that leaves no rents to the non-poor, but imposes a higher total work requirement $(c^s + c^s)$ on the poor. If γ^1 is low the dominant concern becomes to reduce the transfers–the rent–given to the non-poor as much as possible. It is exactly in these circumstances that a semi-separating policy is cost effective.

Let us now consider the optimal first period policy when the WA's prior beliefs belong to the range $[\gamma^s, 1]$. We know that a semi-separation equilibrium can never obtain with such beliefs. We also know that the optimal separation policy is one based on welfare whenever $\gamma^1 \in [\gamma^d, 1]$. This policy gives rise to a total cost of $\gamma^1 b_L(0) + (1-\gamma^1)b_H^d(0) + \gamma^1 b_L(0)$. The total cost of the most efficient pooling policy amounts to $b_L(0) + \gamma^1 b_L(0) + (1-\gamma^1)b_H^s(0)$. Comparing these costs it follows that separation with welfare costs less than pooling if and only if

$$b_H^d(0) - 2b_H^s(0) < b_L(0) - b_H^s(0). \tag{4.7}$$

The lhs of (4.7) can be interpreted as the cost of not being able to smooth out the transfers to H-persons when separating them from the needy. The rhs stands for the static gain when separating with welfare: if H is not separated from L, the former gets $b_L(0)$, while under separation with welfare they get $b_H^s(0)$. So the long term cost of non-smoothing has to fall short of the short term gain of separation for separation to be optimal. But since $b_L(0) = b_H^s(0)$, (4.7) will always be violated, and we can conclude that it will never pay to try to separate the two types with a welfare policy in a long-term poverty model.

With (4.7) violated, pooling will dominate separation with welfare for all $\gamma^1 \in [\gamma^d; 1]$. But for $\gamma^1 = \gamma^d$, we know that a separating equilibrium with welfare costs exactly as much as a separating equilibrium with workfare. This means that the latter policy will also be dominated by pooling for some beliefs γ^1 below γ^d. Solving for the belief γ^1 which equates the cost of pooling ($b_L(0) + \gamma^1 b_L(0) + (1 - \gamma^1) b_H^s(0)$) with the total cost of separation with workfare ($\gamma^1 b_L(\min\{c^d; c^{max}\}) + \gamma^1 b_L(0) + (1 - \gamma^1) b_H^d(\min\{c^d; c^{max}\})$) yields

$$\gamma^{dp} \stackrel{def}{=} \frac{2b_L(0) - b_H^d(\min\{c^d; c^{max}\})}{b_L(\min\{c^d; c^{max}\}) + b_L(0) - b_H^d(\min\{c^d; c^{max}\})}; \tag{4.8}$$

which can be shown to be smaller than γ^d but larger than γ^s.[13] We summarize this finding as

Lemma 6 Suppose $\gamma^1 > \gamma^s$. Then, for all $\gamma^1 \in [\gamma^s; \gamma^{dp}]$, the total expected cost of the most efficient workfare policy inducing separation is smaller than the total expected cost of a welfare policy inducing pooling. For all $\gamma^1 \in [\gamma^{dp}; 1]$, the total expected cost of a welfare policy inducing pooling is smaller than the total expected cost of the most efficient policy inducing separation.

Lemma 6 then leads to our second proposition (illustrated in Figure 4):

Proposition 2 If $\gamma^1 > \gamma^{dp}$ the most efficient policy is welfare inducing pooling. If $\gamma^1 < \gamma^{dp}$ and $c^d < c^{max}$, the most efficient policy is workfare c^d inducing separation. However, if $c^d > c^{max}$, for a small range of a priori beliefs $\gamma^1 \in [0; \gamma^{SS}]$ ($\gamma^{SS} < \gamma^{dp}$) the most efficient policy is semi-separation with workfare c^s.

Proposition 2 highlights that workfare should be used 'more often' in the first period of a long term poverty alleviation problem, than under short term poverty alleviation. Once people have been screened, however, workfare has no role to play; second period transfers are made categorical, a cash transfer to the identified L-persons, nothing to the others. The other alternative, which then is used less often, is the universal welfare policy: a welfare grant $b_L(0)$ is handed out unconditionally, to any person who applies for it. In a short term poverty problem, this is the optimal policy for $\gamma^1 > \gamma^s$. In the long term problem, γ^1 must exceed γ^{dp} for this to be the efficient policy. The WA does not learn anything about applicants' types in this case and enters the second period as

[13] For $c^d < c^{max}$, these statements are easy to verify. For $c^d > c^{max}$, they build on the proofs in part 2 of lemma 5.

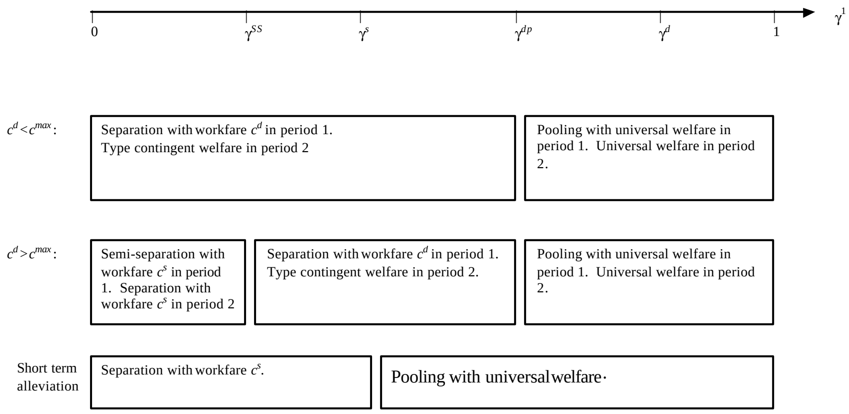

Figure 4 The *WA*'s decision rules for long term and short term poverty alleviation.

Figure 4:

uninformed as she was in the first. Because $\gamma^{dp} > \gamma^{s}$, she continues in the second period to hand out a welfare grant $b_L(0)$ to anybody who asks for it. Put differently, universal welfare is a stationary optimal policy. Finally, there is the possibility that the voluntary participation condition on the poor prevents using a high work requirement ($c^d > c^{max}$). For low values of γ^1, the efficient policy is partial-separating. The WA imposes a work requirement c^s in the first period. Less than $(1 - \gamma^1)\iota^{SS}$ of the non-poor separate which makes it optimal to impose a work requirement of c^s also in the second period. Thus, partial-separation implies a stationary policy with a work requirement c^s in each period.

5 Dynamics with moral hazard: poverty reducing effort

In this section we consider the possibility that a person with an income potential a_L in the first period can escape poverty and achieve productivity a_H in the second period. More specifically, let $\pi \in [0; 1]$ be the probability that an L-person continues to have low productivity in the second period, and $(1 - \pi)$ the probability that he obtains productivity a_H in period two. In what follows, we will refer to these figures respectively as fail probability and escape probability. A poor person can exert effort e in period 1 to reduce the fail probability. Hence, the design of poverty alleviation programs, the choice between a workfare or a welfare program, can affect π in two different ways. First, the amount of public work required in the first period may have a direct effect on π. Second, the amount of public work required in the second period may influence π indirectly, by changing the poor's incentives to exert poverty reducing effort.

We ignore the direct effects, primarily because it is straightforward to understand how they alter benefits and costs of different poverty alleviation programs. If first period work requirements have a direct positive effect on the poor's escape probability, for example by providing on-the-job training, this increases the attractiveness of using workfare in the first period. These effects are straightforward to understand. Instead, we focus on the indirect effect, i.e. on how the choice of second period poverty alleviation program affects a poor person's incentives to exert poverty reducing effort in the first period.

The incentive to undertake poverty reducing effort, PRE for short, in the first period stems from the difference in expected utility levels of having high- and low- ability in the second period. This difference depends on the program that is implemented in the second period. Let $(c, 1-q)$ denote a second period policy in which a workfare program with work requirement c is implemented with probability $1-q$ in the second period. The expected utility difference is given by,

$$\begin{aligned}\Delta(c, 1-q) = {} & [(1-q)\,v(b^s_H(c), 0, a_H) + qv(b^s_H(0), 0, a_H)] - \\ & [(1-q)\,v(b_L(c), c, a_L) + qv((b_L(0), 0, a_L)] > 0 \end{aligned} \tag{5.1}$$

The first term measures expected second period utility for a high ability person if the WA imposes a work requirement c with probability $(1-q)$ and zero work requirement with probability q. The second term gives the expected utility corresponding to a poor person. The fail probability function is given by $\pi(e)$, where $\pi(0) = 1$, $\pi'(e) < 0$, $\pi''(e) > 0$ and $\pi'(0) = -\infty$. To get an interesting problem we assume that the WA cannot observe individual poverty reducing effort—it is therefore impossible to offer effort contingent transfers.

With effort measured in disutility units, the poor exert effort to maximize $(1-\pi(e))\,\Delta(c, 1-q) - e$. Under our assumptions on $\pi(\cdot)$, the optimal effort level, e^*, satisfies the first order condition

$$-\pi'(e^*)\,\Delta(c, 1-q) = 1. \tag{5.2}$$

Slightly abusing notation, we write from now on $\pi(\Delta(c, 1-q))$ as the fail probability when effort is chosen according to (5.2).

Totally differentiating (5.2) gives

$$\frac{de^*}{dq} = \frac{(-\pi'(e^*))^2}{\pi''(e^*)}\,\Delta_q \tag{5.3}$$

where

$$\begin{aligned}\Delta_q \stackrel{\text{def}}{=} \frac{\partial \Delta}{\partial q} = {} & [v(b^s_H(0), 0, a_H) - v(0, 0, a_H)] - \\ & [v(b_L(0), 0, a_L) - v(b_L(c^s), c^s, a_L)]. \end{aligned} \tag{5.4}$$

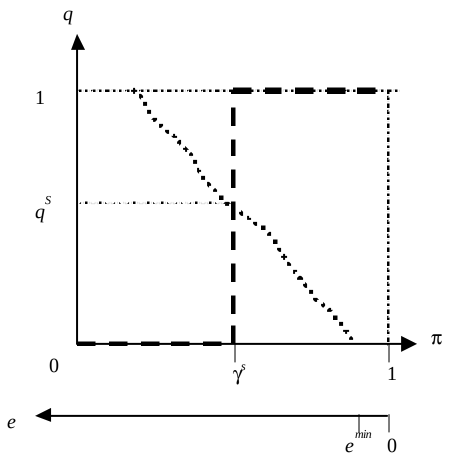

Figure 5. Reaction curves of *L* (•••) and *WA* (⇨ ⇨).

Figure 5:

The first square bracket term in (5.4) is strictly positive: a high ability person strictly prefers a welfare program. What about the second square bracket term? Suppose that with the work requirement c^s (the maximal work requirement which brings H on the reservation utility when posing as L), the L-person still participates in the private labour market. Then by construction, the latter's full income is still equal to z and L is indifferent between workfare or welfare so that the second square bracket term vanishes. We regard this as the benchmark case and formulate it as

Assumption NCO (no crowding out): $c^s < c^{co}$.

Thus under NCO, a higher probability of welfare in period 2 triggers a higher PRE by an L-person in period 1. Welfare in the second period has a carrot effect. The reaction curve of L's effort (or the associated fail probability π) w.r.t. q is thus as in figure 5.

If NCO does not hold, an L-person is strictly worse off under a workfare than under a welfare program. L's disposable income is still z, but the work requirement c^s exceeds c^{co} which is the amount L would choose to work in a welfare program. Put differently, a workfare program becomes a stick that reduces the utility of a poor person if c^s exceeds c^{co}. As long as c^s only slightly exceeds c^{co} the carrot of getting a welfare transfer as non-poor dominates, which means that PRE increases in q: If, however, c^s becomes substantially larger than

c^{co} things change. The prospect of remaining a poor individual on a workfare program in period 2 becomes now so bleak that PRE decreases in q.

To illuminate this argument it is helpful to note that $\Delta(c; 1 - q)$ can be written as $q\Delta(0; 0) + (1 - q)\Delta(c; 1)$ and the derivative of the utility differential is thus simply $\Delta(0; 0) - \Delta(c; 1)$. It is easy to see that $\Delta(c; 1 - q)$ (and therefore PRE) is maximized at $q = 1$ as long as $\Delta(0; 0) > \Delta(c; 1)$. We know that this strict inequality holds for all $c \le c^{co}$. In fact, denoting by $\hat{b}$ the work requirement that solves $\Delta(0; 0) = \Delta(c; 1)$, it is easy to check that $c^{co} < \hat{b} < c^{max}$.[14] We conclude that a welfare program maximizes PRE as long as the workfare program has a work requirement lower than $\hat{b}$. For a work requirement in the interval $[\hat{b}; c^{max}]$ PRE is maximized by choosing $q = 0$; that is, by implementing a workfare program. We return to this deterrence argument in favor of workfare later when comparing our result to that of Besley & Coate (1992), but for the moment we assume that NCO holds. Note, however, that all results in the next section hold true for $c < \hat{b}$.

5.1 Equilibria with PRE

In this section we describe the three types of continuation equilibria when L exerts PRE. The reader is referred to figure 6 below.

Separating equilibrium

Suppose the WA implements a program that separates the poor from the non-poor in the first period. The portion of the poor that stays poor in the second period, is given by $\pi(\Delta(c^s; 1 - q))$. A Bayesian welfare administrator infers that a fraction $\gamma^2 = \pi(\Delta(c^s; 1 - q))$ of those who signed up for the program in the first period are genuinely poor in the second period. Depending on the magnitude of $\pi(\Delta(c^s; 1 - q))$ these beliefs generate three different second period programs:

$\pi(\Delta(c^s; 1 - q)) < \gamma^s \rightarrow$ workfare;
$\pi(\Delta(c^s; 1 - q)) = \gamma^s \rightarrow$ workfare or welfare (the WA is indifferent);
$\pi(\Delta(c^s; 1 - q)) > \gamma^s \rightarrow$ welfare.

We assume now that PRE is neither too productive (A_1) nor too unproductive (A_2):

Assumption A_1 : $\pi(\Delta(c^s; 1)) > \gamma^s$;
Assumption A_2 : $\pi(\Delta(0; 0)) < \gamma^s$.

---

[14] $\Delta_q(c^{max}; 1 - q) = [v(b_L(0); 0; a_H) - v(b_L(0); 0; a_L)] - [v(0; 0; a_H) - v(0; 0; a_L)] = \int_{a_L}^{a_H} \frac{\partial}{\partial a}[v(b_L(0); 0; a) - v(0; 0; a)]da = \int_{a_L}^{a_H} [v_b(b_L(0); 0; a)L(b_L(0); 0; a) - v_b(0; 0; a)L(0; 0; a)]da.$ Decreasing marginal utility of income and normality of leisure guarantees this to be positive. Since $\Delta_q(c; 1 - q)$ is continuous in c, this means that there exists a $\hat{b} \in [0; c^{max}]$ such that $\Delta_q(\hat{b}; 1 - q) = 0$.

Assumption A_1 ensures that the effort level a poor person exerts when the chance of obtaining as non-poor a zero information rent is too low to bring the fail probability below γ^s. A_1 guarantees that L's reaction curve crosses the horizontal axis to the right of γ^s. A_2 ensures that when the odds for obtaining a rent when leaving the poverty status are maximal, the effort level exerted is high enough to bring the fail probability below γ^s. A_2 is invoked to get an interesting situation. If it did not hold there would only exist a separating equilibrium in which the WA imposes welfare with probability one in the second period. We have already analyzed this case in the preceding section.

Assumptions A_1 and A_2 thus imply that there exists a separating policy in the first period if and only if the WA chooses a work requirement c^s in the second period with probability $1 - q^S$, such that

$$\pi(\phi(c^s; 1 - q^S)) = \gamma^s. \tag{5.5}$$

The minimal transfer that for a first period work requirement c^1 induces separation in the first period, $B^d(c^1)$, is defined by[15]

$$\begin{aligned} v(B^d_H(c^1); 0; a_H) + v(0; 0; a_H) = {} & v(b_L(c^1); c^1; a_H) \\ & + q^S v(b_L(0); 0; a_H) + (1 - q^S) v(0; 0; a_H). \end{aligned} \tag{5.6}$$

$B^d_H(c^1)$ is, for the same reason as $b^d_H(c^1)$, decreasing and concave in c^1. Define the work requirement C^d as $B^d_H(C^d) = 0$. Comparing (5.6) with equation (4.1) in the preceding section, we may conclude that $B^d_H(c^1) < b^d_H(c^1)$ for all values of c^1, and hence that $C^d < c^d$.

PRE implies workfare with a positive probability in the second period. This has two implications. The first is that separation in the first period is made easier: PRE allocates some work requirements in the second period; this reduces the rent of H-people in that period and makes it easier to convince H-people to reveal their identity in period 1. A second implication is that the poors' incentives to exert poverty reducing effort is weakened, compared to what their effort would be if the WA could commit to a welfare program with probability 1 in the second period. This is clear from L's reaction curve in figure 5.

Pooling equilibrium

Suppose the welfare program induces pooling in the first period. Rational second period beliefs entail $\gamma^2 = \gamma^1 \pi(\phi(c^s; 1 - q))$. Again, depending of the magnitude of $\pi(\phi(c^s; 1 - q))$, these beliefs generate three different programs for the second period:

$\gamma^1 \pi(\phi(c^s; 1 - q)) < \gamma^s \rightarrow$ workfare;
$\gamma^1 \pi(\phi(c^s; 1 - q)) = \gamma^s \rightarrow$ indifference between workfare or welfare;
$\gamma^1 \pi(\phi(c^s; 1 - q)) > \gamma^s \rightarrow$ welfare.

[15] Capital letters represent variables and functions when the poor can undertake PRE.

If $\gamma^1 < \gamma^s$ the number of genuinely needy in period 2, $\gamma^1\pi(\Delta(c^s, 1-q))$, falls short of γ^s, whatever the level of PRE. The WA will thus rely on workfare with probability 1 in that period, and L-people will exert the minimal effort level e^{min}. On the other hand, if $\gamma^1 > \gamma^s$, the only equilibrium is where the poor choose through their effort level a fail probability γ^s/γ^1, and the WA chooses a welfare programme with probability q^P as defined by

$$\gamma^1\pi(\Delta(c^s, 1-q^P)) = \gamma^s. \tag{5.7}$$

Note that $q^P < q^S$ because $\gamma^s/\gamma^1 > \gamma^s$.

The condition for a pooling equilibrium is given by

$$v(b_H^1, 0, a_H) + v(0, 0, a_H) \geq v(b_L(c^1), c^1, a_H) + q^P v(b_L(0), 0, a_H) + \left(1 - q^P\right) v(0, 0, a_H).$$

The upper boundary for a pooling equilibrium is given by the first period transfer that makes this inequality bind; it is denoted $B_H^{dP}(c^1)$. If $\gamma^1 < \gamma^s$, implying $q^P = 0$, the upper boundary corresponds with the transfer that induces separation in the static case $b_H^s(c^1)$. If $\gamma^1 > \gamma^s$ (and thus $0 < q^P < q^S$) the upper boundary of pooling satisfies the inequalities, $b_H^s(c^1) < B_H^{dP}(c^1) < B_H^d(c^1) (< b_H^d(c^1))$. Note that the upper boundary of pooling always lies strictly below the lower boundary of separation. This indicates that also for $\gamma^1 > \gamma^s$, semi-separation equilibria may occur. (In the preceding section without poverty reducing investments this was not the case.) We now turn to this type of equilibria.

Semi-Separation.

If the non-poor choose to separate from the poor with probability $\mu \in (0, 1)$ in the first period, and the poor exert effort that leads to a fail probability π, Bayesian updating leads to second period beliefs

$$\gamma^2 = \frac{\gamma^1\pi}{\gamma^1 + (1-\gamma^1)(1-\mu)}. \tag{5.8}$$

In a semi-separating equilibrium the non-poor are indifferent between mimicking the poor or separating from them in the first period. To make H-individuals indifferent the WA must choose a welfare program in the second period with probability

$$q^{SS}(b_H^1, c^1) = \frac{v[b_H^1, 0, a_H] - v[b_H^s(c^1), 0, a_H)]}{v[b_H^s, 0, a_H] - v[0, 0, a_H)]}, \tag{5.9}$$

which is the same expression as (4.3). With q being given by q^{SS}, L's optimal effort choice leads to a fail probability π^{SS} given by

$$\pi^{SS} = \pi(\Delta(c^s, 1-q^{SS})). \tag{5.10}$$

We can then solve for the equilibrium value of μ. This is given by the fact that γ^2 must be equal to γ^s to make the WA willing to randomize between workfare and welfare in the second period. Employing (5.8), we get that

$$M^{SS} = \frac{\gamma^s - \gamma^1 \pi^{SS}}{(1 - \gamma^1)\gamma^s}, \tag{5.11}$$

which may be compared with μ^{SS} defined in (4.2).

In order to have a semi-separating equilibrium, that is, in order to have $0 < M^{SS} < 1$, it must be the case that $\gamma^s < \pi^{SS} < \min\{1, \gamma^s/\gamma^1\}$. Two situations may occur. The first is when $\gamma^s/\gamma^1 > 1$ and the relevant interval for π^{SS} is $[\gamma^s, 1]$. If $b^1_H \to b^s_H(c^1)$, $q^{SS} \to 0$ and $\pi^{SS} \to \pi(e_{min})$. In that case, the lower boundary of the SS region corresponds to the upper boundary of the pooling region (when $\gamma^s > \gamma^1$). On the other hand, if $b^1_H \to B^d_H(c^1)$, then from (5.9) it follows that $q^{SS} \to q^S$ (see (5.5)), and $\pi^{SS} \to \gamma^s$. The upper boundary for the semi-separation region is therefore $B^d_H(c^1)$. The other case is where $\gamma^s/\gamma^1 < 1$, so that π^{SS} must belong to the interval $[\gamma^s, \gamma^s/\gamma^1]$. Again it easy to verify that the upper and lower boundary of semi-separation corresponds to the boundaries of separation and pooling respectively.[16]

Summary

Introducing PRE modifies the type and existence of the equilibria in several ways. We emphasize two observations. First, PRE makes it easier to separate the poor from the non-poor at the beginning of the program. With PRE, separation in the first period implies workfare with probability $q^s > 0$ in the second period. This makes it less tempting for H-individuals to mimic L-individuals, implying, $B^d_H(c^1) < b^d_H(c^1)$ for all c^1. Second, PRE implies that there exists a semi-separating equilibrium for all values of γ^1, while, when such effort is to no avail, there exists a semi-separating equilibrium only if $\gamma^1 > \gamma^s$. Without PRE and $\gamma^1 < \gamma^s$, pooling in the first period implies workfare with probability one in the second period. The upper boundary of pooling coincides therefore with the lower boundary of separation and makes no room for semi-separation. With PRE, we have seen that pooling in the first period implies workfare with probability q^P in the second period, while separation in the first period implies workfare with probability q^S in the second. Since q^P is strictly lower than q^S, there is room for a semi-separating equilibrium.

[16] If $b^1_H \to B^{dP}_H(c^1)$, $q^{SS} \to q^P$ and $\pi^{SS} \to \gamma^s/\gamma^1$. The lower boundary of the SS region is thus given by $B^{dP}_H(c^1)$ (the upper boundary of the pooling region (when $\gamma^s < \gamma^1$). That the upper boundary for the semi-separation region is given by $B^d_H(c^1)$, follows from the same argument as in the text.

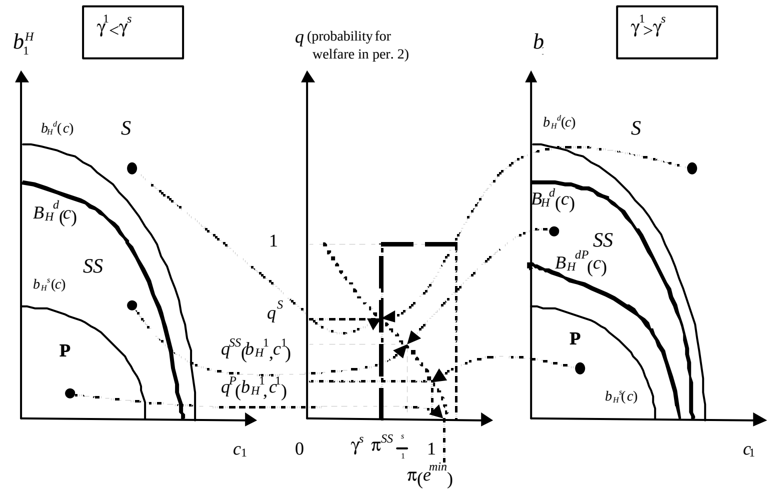

Figure 6. The reaction curves of *WA* (⇨ ⇨) and *L* (•••) and the different continuation equilibria.

Figure 6:

5.2 Selection of programs

How does poverty reducing effort influence the choice among first period programs? We will confine ourselves to discuss the choice among separating programs, and therefore this question narrows down to: does poverty reducing effort increase or reduce the value of using a work requirement to screen the non-poor from the poor?

Whether the WA uses workfare or welfare to separate, second period costs are identical and equal to $\gamma^1 b_L(0)$.[17] First period costs amount to $\gamma^1 b_L(\min\{C^d; c^{max}\}) + (1-\gamma^1)B_H^d(\min\{C^d; c^{max}\})$ under workfare and $\gamma^1 b_L(0) + (1-\gamma^1)B_H^d(0)$ under welfare. The critical value for γ^1 is therefore

$$G^d \stackrel{def}{=} \frac{B_H^d(0) - B_H^d(\min\{C^d; c^{max}\})}{B_H^d(0) - B_H^d(\min\{C^d; c^{max}\}) + b_L(\min\{C^d; c^{max}\}) - b_L(0)}.$$

Since PRE implies workfare with a positive probability in the second period, it becomes easier to separate the poor from the non-poor in the first period, both with workfare and welfare: $C^d < c^d$ and $B^d(0) < b^d(0)$. To assess whether workfare as a screening instrument becomes more costly relative to welfare, we must compare the cost reduction $a_L\left(c^d - C^d\right)$ with $b^d(0) - B^d(0)$. Both magnitudes are determined by the shape of the utility function. Decreasing marginal utility of income makes the difference $b^d(0) - B^d(0)$ relatively large, and the fact that $v(b_L(c); c; a_H)$ utility is decreasing but concave in c makes the difference

[17] Expected second period costs are $\gamma^1\left(\gamma^s[q^S b_L(0) + (1-q^S)b_L(c^s)] + (1-\gamma^s)[q^S b_H^s(0) + (1-q^S)0]\right)$. They can be rearranged as $\gamma^1[q^S b_L(0) + \gamma^s(1-q^S)b_L(c^s)]$, or simply $\gamma^1 b_L(0)$ (since $\gamma^s = \frac{b_L(0)}{b_L(c^s)}$).

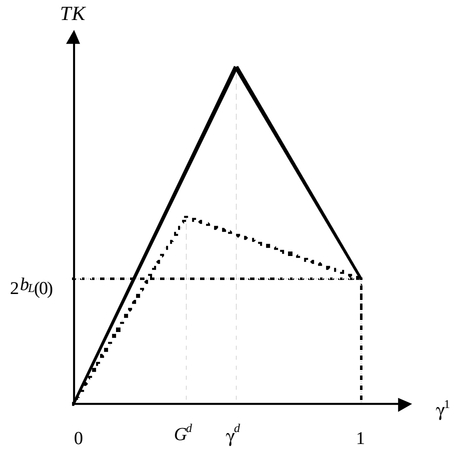

Figure 7. Total cost with efficient separating programmes with (•••) and without (—) the possibility of *PRE*.

Figure 7:

$a_L\left(c^d - C^d\right)$ relatively small. This then implies $b^d(0) - B^d(0) > a_L\left(c^d - C^d\right)$; workfare is used less often when the poor can exert PRE. Formally we have

Proposition 3 $G^d < \gamma^d$.

(The proof is in the appendix.)

Workfare is thus used less often when PRE can be exerted. It is also clear that the total cost of the most efficient separating programme is lower under PRE. As mentioned earlier, second period costs are $\gamma^1 b_L(0)$. This was also the case in section 4. However, we also argued earlier that PRE makes makes it easier for the WA to separate the two types in period 1: when welfare is used, H's carrot is $B_H^d(0)$ rather than $b_H^d(0)$, and when workfare is used, C^d hours suffice (in stead of c^d). An efficient WA's costs thus behave as in figure 7 (for $c^d < c^{max}$).

6 Optimal policy under commitment

So far we have analyzed the costs of different transfer programs assuming that the WA cannot commit to a future program. We have assumed she implements the second period policy that minimizes costs, given the information she has at that stage. What we do next is to characterize the optimal commitment policy and verify how it differs from the time consistent policy when the WA cannot commit. Our discussion is organized as in the no-commitment case; first we address the screening problem, subsequently we increase complexity and include the possibility that the poor can undertake poverty reducing investments.

6.1 Screening and commitment

The "no commitment" assumption prevents a separating policy program from specifying any work requirements or transfers to $H-$individuals in the second period. Formally, separation and sequential rationality imply $c^2 = 0$ and $b_H^2 = 0$. Repeating the static program is therefore impossible for a WA who operates a program that runs over two periods. Does this constraint increase the overall costs of poverty alleviation? Based on what we know about dynamic screening problems in general, we might expect lack of commitment to be a burden–see e.g. Laffont & Tirole (1990).

The fact is, however, that lack of commitment causes no additional screening costs as long as separation by workfare is the cost minimizing policy and $c^d < c^{max}$. If the WA imposes a work requirement c^d in the first period and a zero requirement in the second, we know that she is able to separate the two types. The total cost of separating this way is $\gamma^1[b_L(c^d) + b_L(0)]$. On the other hand, if she implements twice the optimal static workfare policy, she is also able to separate the two types, but at a total cost of $\gamma^1[b_L(c^s) + b_L(c^s)]$. We know that $c^d < 2c^s$ and since $b_L(c)$ is concave in c, it is optimal to impose work requirements only in one period. Hence, even if the WA could commit to a future policy, and therefore could choose $c^2 > 0$, she would be better off choosing $c^1 = c^d$ and $c^2 = 0$.[18]

On the other hand, it is clear that lack of commitment is a potential problem if $c^d > c^{max}$. To see this, suppose a large share of the target population is non-poor (γ^1 is low). In this case it is clearly optimal to use work requirements as much as possible, to constrain the rent of the non-poor. The problem is that even a maximal work requirement in the first period, the maximum being given by the participation constraint of the poor, implies some rents to the non-poor. If the WA could commit to a second period program she would be better off implementing a PAP program with work requirements c^s in both periods, and achieve complete separation without handing out any transfers to the non-poor.

6.2 PRE and commitment

Let us finally focus on the optimal policy for providing the poor with incentives to exert poverty reducing effort. This is what Besley and Coate (1992) do in section IV of their paper. They assume the WA can observe each individual's income potential (ability), an assumption that makes workfare redundant as a screening device, since there is no screening problem when ability is observable.

[18] It is worth mentioning that relaxing the assumption of constant productivity (normalized to zero) in the public sector, may change this conclusion. To see why, assume that the marginal productivity of public work is decreasing in c. This would obviously make an argument for smoothing total work requirement over time. If this effect is strong enough it could counterweigh the concavity of the transfer function and make lack of commitment costly.

Workfare can nevertheless be a useful policy instrument because it can function as a stick to give the poor incentives to undertake poverty reducing effort. Besley and Coate (1992) call this the deterrent argument in favor of workfare. They show that the cost minimizing PAP either imposes no work requirement at all (leaving L with a utility level $v(b_L(0); 0; a_L) > v(0; 0; a_L)$; or the maximal work requirement c^{max} that brings L on his reservation utility level.

It is easy to understand why workfare can be used to stimulate PRE. By imposing the maximal work requirement c^{max} on the poor one reduces their utility as much as possible, given voluntary participation and the constraint that everyone must receive a minimum income z: This policy makes the prospect of staying poor very bleak, and gives those with a low income potential in the first period strong incentives to make an effort to increase their income potential in the second period.

Besley and Coate (1992) do not present an explicit dynamic model to address the incentive problems associated with multi period transfer programs. In the section where they address the deterrent argument in favor of workfare, they include, heuristically, a pre-program stage where L-individuals can exert PRE. They take it for granted that the WA can commit herself–before the poor choose the level of PRE–to very high work requirements on those who are poor in the future. The commitment assumption is not stated explicitly in their paper, but is essential. Ex post, it is obviously not optimal to impose a poverty requirement on the poor. The cost minimizing program ex post, given that there is no screening problem present, is to implement a type contingent welfare policy: a transfer $b_L(0)$ to the L-individuals, and nothing to $H-$ individuals.

Consider then our two period model and assume that the WA has implemented a first period program that separated the poor from the non poor. Abstract from the original H-people – they have already revealed themselves. Assume now, as Besley and Coate (1992) do, that the WA can commit to a second period program before the poor exert poverty reducing effort. But contrary to Besley and Coate, let us hold on to the assumption that the WA cannot observe the income capacity of potential welfare claimants. This difference is important. In section IV of Besley and Coate (1992) the WA has only one objective: to maximize PRE. In our set-up the WA has two concerns. In addition to give the poor strong incentives to exert PRE, she must commit to a policy program that is appropriate given the screening problem she faces in the second period. Two concerns thus need to be balanced. Let us set up the expected cost for period 2:

$$EK^2 = \gamma^1 \{ b_L(c)\pi(\Delta(c;1)) + b^s_H(c)[1 - \pi(\Delta(c;1))] \}$$

where we remind the reader that $\Delta(c;1)$ is equal to $v(b^s_H(c); 0; a_H) - v(b_L(c); c; a_L)$.

Taking the derivative w.r.t. c yields

$$\frac{1}{\gamma_1}\frac{dEK^2}{dc} = \left(\frac{db_L(c)}{dc}\pi + \frac{db_H^s(c)}{dc}(1-\pi)\right) + [b_L(c) - b_H^s(c)]\left(\underbrace{\frac{d\pi}{d\Delta}}_{(-)}\underbrace{\frac{\partial \Delta(c,1)}{\partial c}}_{(-=0)}\right)$$

The first two terms inform about the marginal cost effect of workfare as a screening device. The last term measures its effect as a deterrence device. Regarding the latter, we claim that $\frac{\partial \Delta(c,1)}{\partial c}$

- < 0 for $c \in [0, \min\{c^{co}, c^s\}]$, because H's utility falls;
- > 0 for $c \in [\max\{c^{co}, c^s\}, c^{max}]$, because L's utility falls;
- $= 0$ for $c \in [c^s, c^{co}]$ (if $c^s < c^{co}$);
- < 0 for $c \in [c^{co}, c^s]$ (if $c^s > c^{co}$) because H's utility falls more than L's.[19]

Evaluating $\frac{dEK^2}{dc}$ at $c = 0$, we obtain[20]

$$\frac{1}{\gamma_1}\frac{dEK^2}{dc}\Big|_{c=0} = a_H\left(\pi(\Delta(0,1)) - \gamma^s + \frac{a_L}{a_H}\Big[1 - \frac{\min\{c^s, c^{co}\}}{c^s}\Big]\right).$$

Since $\Delta(0,1) = \Delta(0,0)$, this result shows that if PRE is not too productive in the sense of assumption (A.2) second period costs may fall when setting the work requirement marginally above zero. In fact, they will fall when $c^s < c^{co}$.

At the other extreme, when c approaches c^{max}, the screening effect has disappeared (the derivatives of $b_L(c)$ and $b_H^s(c)$ vanish from resp. c^{co} and c^s onwards) and the negative deterrence effect remains. c^{max} is therefore a local minimum.

Proposition 4 *Suppose that the WA can commit to a second period program before the poor make their effort choice, but that she cannot observe people's ability outcome following that choice. If $c^s < c^{co}$, she should either commit to a program with work requirement c^* implicitly defined as*

$$c^* = \frac{\gamma^s - \pi(\Delta(c^*,1))}{(-\frac{d\pi}{d\Delta})v_b(b_H(c^*),0,a_H)(a_H - a_L)} \in (0, c^s)$$

[19] $\frac{\partial \Delta(c,1)}{\partial c} = \frac{\partial v(b_H^s(c),0,a_H)}{\partial c} - \frac{\partial v(b_L(c),c,a_L)}{\partial c}$. If $c^{co} < c < c^s$, the rhs can also be written as $-v_b(b_H^s(c),0,a_H)a_H + u_\ell(z,1-c)$, or as $u_\ell(z,1-c) - u_\ell(b_H^s(c)+y^*, 1-\frac{y^*}{a_H})$, where y^* is H's optimal choice of private earnings.

Since $z < b_H^s(c) + y^*$ and $c < \frac{y^*}{a_H}$, H's marginal utility of leisure is larger than L's. This then means that $\frac{\partial \Delta(c,1)}{\partial c} < 0$.

[20] $\frac{1}{\gamma_1}\frac{dEK^2}{dc}\Big|_{c=0} = \{a_L\pi(\Delta(0,1)) - (a_H - a_L)[1-\pi(\Delta(0,1))]\} = \gamma_1 a_H\{\frac{a_L}{a_H} - [1-\pi(\Delta(0,1))]\}$ $= \gamma_1 a_H\{\pi(\Delta(0,1)) - \gamma^s + \frac{a_L}{a_H}[1 - \frac{\min\{c^s,c^{co}\}}{c^s}]\}$, where the last equality follows from the definition of γ^s (cf (3.1)).

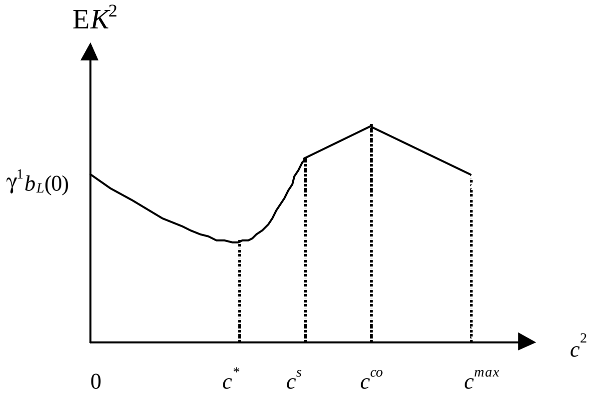

Figure 8. Expected second period cost as a function of the work requirement to which the *WA* commits (for $c^s < c^{co}$).

Figure 8:

or to a program with the maximal work requirement c^{max}. If $c^s > c^{co}$, the same conclusion holds, except when $\frac{dEK^2}{dc}|_{c=0} > 0$; then the choice is between a zero or the maximal work requirement c^{max}.

(The proof is in the appendix.)

We have illustrated the shape of the cost function in the figure below for $c^s < c^{co}$.

In this figure, c^* is the global minimum of the cost function. There exists of course parameter values for which the global minimum is at c^{max}. This depends, among other things, on the shape of the utility function and of $\pi(\cdot)$. Roughly, if c^{max} generates a very high level of PRE, compared to c^*, it is optimal to impose the maximal work requirement.

Note that the commitment assumption bites; the optimal policy is not time consistent without commitment. If the WA could renege on the announced policy, she would implement a work requirement c^s, since this is the optimal screening policy in the second period as long as $\pi < \gamma^s$:

It is well known that altruistic policy makers have a hard time making it credible that they will use the stick and punish the poor unless they make a serious effort to escape poverty. Buchanen (1975) termed this the Samaritan's dilemma.[21] The reason, though, why announcing a work requirement c^{max} for

[21] Later, Bruce and Waldman (1991) and Coate (1995) have argued that the Samaritan's dilemma has important implications for the design of poverty alleviation programs.

the second period is not credible does not re°ect the Samaritan's dilemma. Our policy maker, the WA, is not an altruist; the poor's utility does not enter her welfare function. Her concern is to ensure in the cheapest way that nobody in the society remains under the poverty line z. The reason why the WA reneges on the announced policy is simply that a c^{max} policy is not the cost minimizing poverty alleviation policy in the second period.

7 Concluding remarks

Two fundamental incentive issues arise in the alleviation of long term poverty: (i) how can we prevent non-poor people from claiming bene¯ts meant for the poor? and (ii) how can we encourage the poor to invest in their future income potential? In this paper, we have analysed the usefulness of work requirements both as a screening and deterrence device.

First we looked at the screening problem. The WA can make it less tempting for the non-poor to pose as poor in two di®erent ways. She can increase their utility if they do not join the programme by handing out a carrot{a welfare transfer to those who do not pose as poor. Alternatively, she can reduce their utility when joining the program by threatening with a stick{a work requirement levied on those who claim low ability. A central feature in a dynamic model is that, unless the WA can commit to future policy, separation requires type contingent transfers in the second period. That is, the use of either carrot or stick, necessary to separate the poor from the non-poor, must be concentrated in the ¯rst period. We have shown that this increases the e®ectiveness of workfare as a screening instrument. But there is one proviso to this conclusion: in some cases the concentrated use of the stick in the ¯rst period goes over what the poor can bear, and in order not to scare them away, the WA should spread its use out over time and at the same time present the non-poor with a modest carrot. Though this will no longer result in full separation, it is the best the WA can achieve when the number of initially poor is 'small'.

Both the stick and the carrot are costly sorting instruments. In some cases, in particular when the poor make up a large part of the population, sorting becomes too costly. Then, the WA should just hand out a universal welfare grant in both periods.

We have also added considerations of poverty deterrence to our model. There are two ways to provide the initially poor with incentives to escape poverty, again deserving the metaphor of carrot and stick. The carrot is the promise of a transfer to those who do not pose as poor; this makes it more attractive to become non-poor in the second period. The alternative is to threat with a very high work requirement on the poor as a stick, making it less attractive to stay poor. We have shown that the perfect Bayesian equilibrium in a two period model with both screening and deterrence entails work requirements with a positive probability in the second period. Furthermore we have shown that including the deterrence problem implies that workfare will be used less often as a screening device in the first period. Deterrence considerations thus warrant a substitution of early for later work requirements. This substitution happens in a double sense: a lower amount of work requirement imposed early, but also a lower threshold value (for the a priori belief on the number of real poor) that triggers a welfare programme.

Finally, we discussed the optimal design of programs for alleviating long term poverty when it is possible to commit to future policy. Our most interesting finding here is that it can be optimal to commit to some work requirement in the second period (though a lower level than the one that separates the poor from the non-poor).

Our model can be extended in several directions. One extension we feel worthwhile exploring in future research is to open up for the possibility that initially non-poor persons experience a fall in their income potential below the poverty line. Screening incentives then not only have to be balanced with those for exerting poverty reducing effort, but also with those for poverty avoiding effort.

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A Appendix

Proof of lemma 1

The transfer function $b_H^s(c)$ is implicitly defined as

$$v(b_H^s(c);0;a_H) \equiv v(b_L(c);c;a_H): \tag{A.1}$$

As private earnings of H when mimicking can be freely chosen, equality of utility levels is equivalent to equality of full incomes:

$$b_H^s(c) + a_H = b_L(c) + (1 - c)a_H \tag{A.2}$$

Straightforward differentiation then gives the results. $\blacksquare$

Proof of lemma 2

In the dynamic case, the transfer function is defined by the identity

$$v\left(b_H^d(c^1);0;a_H\right) \equiv v\left(b_L(c^1);c^1;a_H\right) + D \tag{A.3}$$

where $D \stackrel{\text{def}}{=} v(b_L(0);0;a_H) - v(0;0;a_H)$. Using (A.1), implicit differentiation gives

$$\frac{db_H^d(c^1)}{dc^1} = -\frac{v_b(b_H^s(c^1);0;a_H)}{v_b(b_H^d(c^1);0;a_H)}(a_H - a_L): \tag{A.4}$$

Differentiating a second time and rearranging produces

$$\begin{aligned}\frac{d^2b_H^d(c)}{dc^2} = & -\frac{(v_b(b_H^s(c^1);0;a_H))^2}{v_b(b_H^d(c^1);0;a_H)} \times \\ & \left(\frac{v_{bb}(b_H^d(c^1);0;a_H)}{(v_b(b_H^d(c^1);0;a_H))^2} - \frac{v_{bb}(b_H^s(c^1);0;a_H)}{(v_b(b_H^s(c^1);0;a_H))^2}\right)(a_H - a_L);\end{aligned}$$

or simply

$$\frac{d^2b_H^d(c)}{dc^2} = \frac{(v_b^s)^2}{v_b^d}\left[\frac{v_{bb}^s}{(v_b^d)^2} - \frac{v_{bb}^d}{(v_b^d)^2}\right]\frac{db_H^s(c^1)}{dc^1}: \tag{A.5}$$

In signing the term $\frac{v_{bb}^s}{(v_b^d)^2} - \frac{v_{bb}^d}{(v_b^d)^2}$, we may make use of the fact that

$$\frac{d\log\frac{v_{bb}(m)}{(v_b(m))^2}}{d\log m} = \frac{d\log(-\frac{v_{bb}(m)}{v_b(m)})}{d\log m} + \left(-\frac{v_{bb}(m)}{v_b(m)}\right)\cdot m; \tag{A.6}$$

where m is full real income. Since $K > 0$, first period full income is higher when being honest than when mimicking as L: The first rhs term is the logarithmic change in the coefficient of absolute risk aversion, and the second rhs term is the coefficient of relative risk aversion. $\blacksquare$

Proof of lemma 5

The proof is divided up in three parts.

Part 1

Among all efficient policies inducing a semi-separating equilibrium, workfare (c^s) is optimal iff $\gamma^1 < (\gamma^s)^2$.

Proof. Consider a semi-separating equilibrium. The total expected cost under workfare and welfare are respectively given by:

$$\frac{\gamma^1}{\gamma^s} b_L(c^s) + \left(1 - \frac{\gamma^1}{\gamma^s}\right) b_H^s(c^s) + \gamma^1 b_L(c^s) \tag{A.7}$$

and

$$\frac{\gamma^1}{\gamma^s} b_L(0) + \left(1 - \frac{\gamma^1}{\gamma^s}\right) b_H^s(0) + \gamma^1 b_L(c^s): \tag{A.8}$$

As $b_H^s(c^s) = 0$, workfare costs more (less) than welfare iff

$$\frac{\gamma^1}{\gamma^s} > (<) \frac{b_H^s(0)}{b_H^s(0) + [b_L(c^s) - b_L(0)]}: \tag{A.9}$$

Since the rhs is precisely γ^s; the result follows. ∎

Part 2

If $\gamma^1 \in [(\gamma^s)^2; \gamma^s]$, then the total costs under semi-separation with welfare is higher than the total cost under full separation with a work requirement $\min\{c^d; c^{max}\}$.

Proof. A semi-separating equilibrium with welfare costs

$$\frac{\gamma^1}{\gamma^s} b_L(0) + \left(1 - \frac{\gamma^1}{\gamma^s}\right) b_H^s(0) + \gamma^1 b_L(c^s) = b_L(0) + \gamma^1 b_L(c^s): \tag{A.10}$$

Separation with workfare costs

$$\gamma^1 b_L(\min\{c^d; c^{max}\}) + (1 - \gamma^1) b_H^d(\min\{c^d; c^{max}\}) + \gamma^1 b_L(0): \tag{A.11}$$

The latter is cheaper iff

$$(1-\gamma^1)b_L(0)+\gamma^1 b_L(c^s)$$
$$-\gamma^1 b_L(\min\{c^d;c^{max}\})-(1-\gamma^1)b^d_H(\min\{c^d;c^{max}\})>0$$
$$\Updownarrow$$
$$\frac{1-\gamma^1}{\gamma^1}>\frac{b_L(\min\{c^d;c^{max}\})-b_L(c^s)}{b_L(0)-b^d_H(\min\{c^d;c^{max}\})} \tag{A.12}$$

Since $\gamma^1<\gamma^s$, we have that $\frac{1-\gamma^1}{\gamma^1}>\frac{1-\gamma^s}{\gamma^s}=\frac{b_L(c^s)-b_L(0)}{b_L(0)}$, and thus it is sufficient to prove that

$$\frac{b_L(c^s)-b_L(0)}{b_L(0)}>\frac{b_L(\min\{c^d;c^{max}\})-b_L(c^s)}{b_L(0)-b^d_H(\min\{c^d;c^{max}\})} \tag{A.13}$$

If $c^d=\min\{c^d;c^{max}\}; b^d_H(\min\{c^d;c^{max}\})=0$ and the condition reduces to

$$2b_L(c^s)>b_L(0)+b_L(c^d) \tag{A.14}$$

which can easily be verified to be the case.[22]

Let us then consider the case where $c^{max}=\min\{c^d;c^{max}\}$. Then the condition can be written as

$$\frac{b_L(0)-b^d_H(c^{max})}{b_L(0)}>\frac{z-b_L(c^s)}{b_L(c^s)-b_L(0)} \tag{A.15}$$

Clearly, when $c^{co}<c^s<c^{max}$, this is satisfied since the rhs then vanishes. This leaves us with the case where $c^s<c^{co}<c^{max}$.

Because $\frac{z-b_L(c^s)}{b_L(c^s)-b_L(0)}=\frac{c^{co}}{c^s}-1$ and $b_L(0)=b^d_H(c^s)$, we need to prove that

$$1-\frac{b^d_H(c^{max})}{b^d_H(c^s)}>\frac{c^{co}}{c^s}-1 \tag{A.16}$$

or

$$2-\frac{c^{co}}{c^s}>\frac{b^d_H(c^{max})}{b^d_H(c^s)}. \tag{A.17}$$

Since $b^d_H(c^{co})>b^d_H(c^{max})$, it suffices to show that

$$2-\frac{c^{co}}{c^s}>\frac{b^d_H(c^{co})}{b^d_H(c^s)}. \tag{A.18}$$

[22] If $c^{co}<c^s<c^d$, the rhs is zero. If $c^s<c^d<c^{co}$, the inequality reduces to $2c^s>c^d$. If $c^s<c^{co}<c^d$, the inequality reduces to $b_L(c^s)>\frac{z+b_L(0)}{2}$ which is also the case since $c^s>c^{co}/2$.

This we claim is to be the case because

$$\left(2 - \frac{c^{co}}{c^s}\right) > 1 - \frac{v_b(0;0;a_H)}{v_b(b_L(0);0;a_H)}\left(\frac{c^{co}}{c^s} - 1\right) > \frac{b_H^d(c^{co})}{b_H^d(c^s)}. \tag{A.19}$$

The first inequality follows from the decreasing marginal utility of income. The second from the fact that $b_H^d(c)$ is concave in c and the second order Taylor expansion of $b_H^d(c)$ around c^s yields for $b_H^d(c^{co})$:

$$b_H^d(c^{co}) = b_H^d(c^s) - \frac{v_b(0;0;a_H)}{v_b(b_L(0);0;a_H)}(a_H - a_L)(c^{co} - c^s) + (-). \tag{A.20}$$

(Since $(a_H - a_L) = \frac{b_L(0)}{c^s} = \frac{b_H^d(c^s)}{c^s}$.) $\blacksquare$

Part 3

If $\gamma^1 \in [0;(\gamma^s)^2]$, the total costs under semi-separation with workfare c^s is higher than the total cost under full separation with a work requirement $\min\{c^d;c^{max}\}$; unless $c^{max} = \min\{c^d;c^{max}\}$ and $\gamma^1 < \gamma^{SS}$: then the opposite is the case.

Proof.

If $\gamma^1 < (\gamma^s)^2$, we know that the cheapest semi-separation policy is a work requirement c^s. The cheapest separation policy has a work requirement $\min\{c^d;c^{max}\}$. The latter is cheaper if and only if

$$\frac{\gamma^1}{\gamma^s}b_L(c^s) + \left(1 - \frac{\gamma^1}{\gamma^s}\right)b_H^s(c^s) + \gamma^1 b_L(c^s) >$$
$$\gamma^1 b_L(\min\{c^d;c^{max}\}) + (1 - \gamma^1)b_H^d(\min\{c^d;c^{max}\}) + \gamma^1 b_L(0)$$

Using the fact that $b_H^s(c^s) = 0$, we get

$$\gamma^1 > \frac{b_H^d(\min\{c^d;c^{max}\})}{b_H^d(\min\{c^d;c^{max}\}) + (1 + \frac{1}{\gamma^s})b_L(c^s) + b_L(\min\{c^d;c^{max}\}) + b_L(0)}.$$

If $c^d < c^{max}$, $b_H^d(\min\{c^d;c^{max}\})$ vanishes and the inequality is trivially verified. On the other hand, if $c^d > c^{max}$, $b_H^d(\min\{c^d;c^{max}\})$ remains positive, viz. $b_H^d(c^{max}) > 0$. Since $b_L(c^{max}) = z$, a necessary and sufficient condition for separating with work requirement c^{max} to be the cheapest is that

$$\gamma^1 > \frac{b_H^d(c^{max})}{b_H^d(c^{max}) + (1 + \frac{1}{\gamma^s})b_L(c^s) + z + b_L(0)}. \tag{A.21}$$

The rhs of this inequality was in the text defined as γ^{SS}. $\blacksquare$

Proof of proposition 3

We need to prove that $G^d < \gamma^d$ or alternatively that $\frac{1}{G^d} > \frac{1}{\gamma^d}$. Using the definitions for the critical values, this is equivalent to proving that

$$\frac{b_L(\min\{C^d, c^{max}\}) - b_L(0)}{B^d_H(0) - B^d_H(\min\{C^d, c^{max}\})} > \frac{b_L(\min\{c^d, c^{max}\}) - b_L(0)}{b^d_H(0) - b^d_H(\min\{c^d, c^{max}\})}. \tag{A.22}$$

Suppose first that $C^d > c^{max}$. Then also $c^d > c^{max}$, and the above inequality reduces to

$$b^d_H(0) - B^d_H(0) > b^d_H(c^{max}) - B^d_H(c^{max}). \tag{A.23}$$

Since $b^d_H(c)$ is defined as

$$v(b^d_H(c), 0, a_H) + v(0, 0, a_H) = v(b_L(c), c, a_H) + v(b_L(0), 0, a_H), \tag{A.24}$$

and $B^d_H(c)$ is defined as

$$\begin{aligned} v(B^d_H(c), 0, a_H) + v(0, 0, a_H) = \\ v(b_L(c), c, a_H) + q^S v(b_L(0), 0, a_H) + \left(1 - q^S\right) v(0, 0, a_H), \end{aligned}$$

we also have that

$$v(b^d_H(c), 0, a_H) - v(B^d_H(c), 0, a_H) = \left(1 - q^S\right) \cdot D \tag{A.25}$$

where $D \stackrel{def}{=} v(b^s_H(0), 0, a_H) - v(0, 0, a_H)$.

Since (A.25) holds for any c, differentiation gives

$$v_b(b^d_H(c), 0, a_H)\frac{db^d_H(c)}{dc} = v_b(B^d_H(c), 0, a_H)\frac{dB^d_H(c)}{dc},$$

or

$$\frac{dB^d_H(c)}{dc} = \frac{v_b(b^d_H(c), 0, a_H)}{v_b(B^d_H(c), 0, a_H)}\frac{db^d_H(c)}{dc}.$$

As $B^d_H(c) < b^d_H(c)$, $\frac{v_b(b^d_H(c),0,a_H)}{v_b(B^d_H(c),0,a_H)} < 1$, and $\frac{d[b^d_H(c) - B^d_H(c)]}{dc} < 0$. This then implies (A.23).

The remainder of the proof concerns the case where $C^d < c^{max}$. It is divided into four parts:

Part 1

C^d is defined as

$$v(0;0;a_H) + v(0;0;a_H) = v(b_L(C^d);C^d;a_H) + q^S v(b_L(0);0;a_H) + \left(1 - q^S\right) v(0;0;a_H).$$

Since $v(0;0;a_H) = v(b_L(c^s);c^s;a_H)$, this identity may also be written as

$$v(b_L(C^d);C^d;a_H) = v(b_L(c^s);c^s;a_H) - q \cdot D\,|_{q=q^S}. \tag{A.26}$$

Similarly, $B_H^d(0)$ must satisfy the identity

$$v(B_H^d(0);c^1;a_H) + v(0;0;a_H) = v(b_L(0);0;a_H) + q^S v(b_L(0);0;a_H) + \left(1 - q^S\right) v(0;0;a_H);$$

which can be rearranged as

$$\begin{aligned} v(B_H^d(0);0;a_H) &= v(b_L(0;0;a_H) + q \cdot D\,|_{q=q^S} \\ &= v(b_H^s(0);0;a_H) + q \cdot D\,|_{q=q^S}. \end{aligned} \tag{A.27}$$

Part 2

Suppose now that q where to change for some reason. How will $B_H^d(0)$ and C^d be affected? From (A.27) we get that

$$\frac{\partial v(B_H^d(0);0;a_H)}{\partial B_H^d(0)} dB_H^d(0) = D \cdot dq \tag{A.28}$$

$$\Updownarrow$$

$$v_b(B_H^{dS}(0);0;a_H)\,dB_H^{dS}(0) = D \cdot dq. \tag{A.29}$$

And likewise, from (A.26):

$$\frac{\partial v(b_H^s(C^d);0;a_H)}{\partial C^d} dC^d = -D \cdot dq$$

$$\Updownarrow \tag{A.30}$$

$$v_b(b_H^s(C^d);0;a_H)\frac{db_H^s(C^d)}{dC^d} dC^d = -D \cdot dq^S. \tag{A.31}$$

Therefore,

$$\frac{dB_H^d(0)/dq}{dC^d/dq} = -\frac{v_b(b_H^s(C^d);0;a_H)}{v_b(B_H^d(0);0;a_H)} \frac{db_H^s(C^d)}{dC^d}. \tag{A.32}$$

Part 3

The defining equation for $B_H^d(c^1)$ is

$$\begin{aligned} v(B_H^d(c^1), 0, a_H) + v(0, 0, a_H) = \\ v(b_L(c^1), c^1, a_H) + q^S v(b_L(0), 0, a_H) + \left(1 - q^S\right) v(0, 0, a_H), \end{aligned}$$

which can be rearranged as

$$v(B_H^d(c^1), 0, a_H) = v(b_H^s(c^1), 0, a_H) + q \cdot D \,|_{q=q^S}. \tag{A.33}$$

Whence,

$$\frac{dB_H^d(c^1)}{dc^1} = \frac{v_b(b_H^s(c^1), 0, a_H)}{v_b(B_H^d(c^1), 0, a_H)} \frac{db_H^s(c^1)}{dc^1}. \tag{A.34}$$

Evaluating this derivative at $c^1 = C^d$, the rhs denominator becomes $v_b(0, 0, a_H)$ (because $C^d < c^{max}$).

Therefore, (A.32) may also be written as

$$\frac{dB_H^d(0)/dq}{dC^d/dq} = \frac{v_b(0, 0, a_H)}{v_b(B_H^d(0), 0, a_H)} \left[- \frac{dB_H^d(c^1)}{dc^1} \Big|_{c^1=C^d} \right]. \tag{A.35}$$

Because the ratio of marginal utilities multiplying the rhs square bracket term is larger than one, we have

$$\frac{dB_H^d(0)/dq^S}{dC^d/dq^S} > \left[- \frac{dB_H^d(c^1)}{dc^1} \Big|_{c^1=C^d} \right] \tag{A.36}$$

Part 4

Recall that $B_H^d(c^1)$ is a decreasing and strictly concave function in c^1. Thus the absolute slope of this function at C^d is larger than $\frac{B_H^d(0)}{C^d}$, the slope of the cord connecting $(0, B_H^d(0))$ and $(C^d, 0)$. We may therefore write that

$$\frac{dB_H^d(0)/dq}{dC^d/dq} > \frac{B_H^d(0)}{C^d}. \tag{A.37}$$

Because

$$\begin{aligned} \frac{d}{dq}\left(\frac{B_H^d(0)}{C^d}\right) &= \frac{1}{C^d}\left(\frac{dB_H^d(0)}{dq} - \frac{B_H^d(0)}{C^d}\frac{dC^d}{dq}\right) \\ &= \frac{dC^d/dq}{C^d}\left(\frac{dB_H^d(0)/dq}{dC^d/dq} - \frac{B_H^d(0)}{C^d}\right), \end{aligned} \tag{A.38}$$

we have just shown that $\frac{d}{dq}\left(\frac{B_H^d(0)}{C^d}\right) > 0$. But since $q = q^S < 1$ in the case of poverty reducing effort, and $q = 1$ in the case without poverty reducing effort, we also have shown that $\frac{B_H^d(0)}{C^d} < \frac{b_H^d(0)}{c^d}$. $\blacksquare$

Proof of proposition 5

Case 1: $c^s < c^{co}$

(a) $0 < c < c^s$:

$$\frac{1}{\gamma_1}\frac{dEK^2}{dc} = a_H\{\pi(\Delta(c;1)) - \gamma^s\} + \left([b_L(c) - b_H^s(c)]\underbrace{\frac{d\pi}{d\Delta}}_{(-)}\underbrace{\frac{\partial\Delta(c;1)}{\partial c}}_{(-)}\right) \tag{A.39}$$

Since $\Delta(c;1) < \Delta(0;1)$; π will take a higher value than as for $c = 0$. The derivative will therefore become less negative as c is raised above 0.

Let us investigate the derivative as c approaches c^s from the left:

$$\frac{1}{\gamma_1}\frac{dEK^2}{dc}\Big|_{c\to c^s-} = a_H(\pi(\Delta(c^s;1)) - \gamma^s) + \left(a_L c^s\underbrace{\frac{d\pi}{d\Delta}}_{(-)}\underbrace{\frac{\partial\Delta(c;1)}{\partial c}\Big|_{c\to c^s-}}_{(-)}\right) \tag{A.40}$$

From the analysis of the separation equilibrium, we know that $\pi(\Delta(c^s;1 - q^S)) = \gamma^s$. Thus, if we choose a work requirement c^s with probability 1, L will not put in more than the minimal effort level (referred to as e^{min} in the middle panel of figure 6) and we get $\pi(\Delta(c^s;1)) > \gamma^s$. It then follows that the first round bracket term in (A.40) is also positive. Thus, as c approaches c^s from the left, second period costs will increase. Since $EK^2(\cdot)$ is continuous in c, this means that there exists a $c^* \in (0;c^s)$ where $EK^2(\cdot)$ reaches a local minimum:

$$a_L - a_H(1 - \pi(\Delta(c^*;1)) = a_H c^*(-\frac{d\pi}{d\Delta})\frac{\partial\Delta(c^*;1)}{\partial c} \tag{A.41}$$

or

$$\frac{\gamma^s - \pi(\Delta(c^*;1))}{(-\frac{d\pi}{d\Delta})v_b(b_H(c^*);0;a_H)(a_H - a_L)} = c^* \tag{A.42}$$

(b) $c^s < c < c^{co}$:

$$\frac{1}{\gamma_1}\frac{\partial K}{\partial c} = a_L\pi + b_L(c)\underbrace{\frac{d\pi}{d\Delta}}_{(-)}\underbrace{\frac{\partial\Delta(c;1)}{\partial c}}_{(0)} < 0 \tag{A.43}$$

(c) $c^{co} < c < c^{max}$:

$$\frac{1}{\gamma_1}\frac{\partial K}{\partial c} = z\underbrace{\frac{d\pi}{d\Delta}}_{(-)}\underbrace{\frac{\partial\Delta(c;1)}{\partial c}}_{(+)} < 0 \tag{A.44}$$

We can therefore conclude that $K(c)$ reaches a local minimum at $c^* \in (0;c^s)$ and c^{max} (and local maximum at 0 and c^{co}):

Case 2: $c^s > c^{co}$

(a) $c < c^{co}$:

$$\begin{aligned}\frac{1}{\gamma_1}\frac{dEK^2}{dc} &= a_H\left\{\pi(\theta(c,1)) - \gamma^s + \frac{a_L}{a_H}\left(1-\frac{c^{co}}{c^s}\right)\right\} \\ &+ \left([b_L(c) - b_H^s(c)]\frac{d\pi}{d\theta}\frac{\partial\theta(c,1)}{\partial c}\right) \qquad \text{(A.45)} \\ &= a_H\left\{\pi(\theta(c,1)) - \gamma^s + \frac{a_L}{a_H}\left(1-\frac{c^{co}}{c^s}\right)\right\} \\ &+ a_H c \underbrace{\frac{d\pi}{d\theta}}_{(-)}\underbrace{\frac{\partial\theta(c,1)}{\partial c}}_{(-)}\end{aligned}$$

Since $\theta(c,1) < \theta(0,1)$, π will take a higher value than for $c = 0$, and the first round bracket term increases. Thus, if the marginal cost of a work requirement was already positive for $c = 0$, it it becomes even more positive, and this is reinforced by the second term.

On the other hand, if the marginal cost of a work requirement was negative for $c = 0$, the increase in c makes it less negative.

(b) For all $c^{co} < c < c^s$:

Let us investigate the derivative as c approaches c^s from the left:

$$\begin{aligned}\frac{1}{\gamma_1}\frac{\partial K}{\partial c}\Big|_{c\to c^s-} &= a_H\left(\pi(\theta(c^s,1)) - \gamma^s + \frac{a_L}{a_H}\left(1-\frac{c^{co}}{c^s}\right)\right) \\ &+ \left((a_L + a_H)c^s \underbrace{\frac{d\pi}{d\theta}}_{(-)}\underbrace{\frac{\partial\theta(c,1)}{\partial c}\Big|_{c\to c^s-}}_{(-)}\right) \qquad \text{(A.46)}\end{aligned}$$

For the same reasons as in case 1 we have $\pi(\theta(c^s,1)) > \gamma^s$, and therefore that second period costs increase as c approaches c^s.

Since $EK^2(\cdot)$ is continuous in c, the results of (a) and (b) imply that there exists a $c^* \in (0, c^s)$ where $EK^2(\cdot)$ reaches a local minimum if and only if $\frac{\partial K}{\partial c}|_{c=0} < 0$.

(c) For all $c^s < c < c^{max}$:

$$\frac{1}{\gamma_1}\frac{dEK^2}{dc} = z\underbrace{\frac{d\pi}{d\theta}}_{(-)}\underbrace{\frac{\partial\theta(c,1)}{\partial c}}_{(+)} < 0 \qquad \text{(A.47)}$$

We can therefore conclude case 2 by saying that we have local minima at 0 and c^{max} (if $\frac{\partial K}{\partial c}|_{c=0} > 0$), and local minima at $c^* \in (0; c^s)$ and c^{max} (if $\frac{\partial K}{\partial c}|_{c=0} < 0$). ■

Sufficient conditions for L-people not to take-the-money-and-run

Consider a first period program $\left\{[b_L(c^1); c^1]; \left[b^d_H(c^1); 0\right]\right\}$ intended to separate the two types. An L-person will not choose $\left[b^d_H(c^1); 0\right]$ iff

$$v\left(b^d_H(c^1); 0; a_L\right) + v(0; 0; a_L) \leq v\left(b_L(c^1); c^1; a_L\right) + v\left(b_L(0); 0; a_L\right). \tag{A.48}$$

We will first give sufficient conditions for this to hold when $c^1 = 0$, and then show that if it holds for $c^1 = 0$, it will also hold for any $c^1 \in (0; c^{max}]$.

Part 1

Lemma 7 $v_b(b; 0; a) \cdot L(b; 0; a)$ sufficiently convex in b guarantees that a low ability person does not to take the money and run (t-m-r) when $c^1 = 0$. Sufficient conditions for convexity of $v_b L$ are (taken together): decreasing absolute risk aversion regarding consumption, normality of leisure, a labour supply function that is convex in lump sum income.

Proof.
By the definition of $b^d_H(c^1)$, we have that

$$v\left(b^d_H(0); 0; a_H\right) + v(0; 0; a_H) = 2v\left(b_L(0); 0; a_H\right); \tag{A.49}$$

since $b^s_H(0) = b_L(0)$.
We would like to show that

$$v\left(b^d_H(0); 0; a_L\right) + v(0; 0; a_L) < 2v\left(b_L(0); 0; a_L\right). \tag{A.50}$$

Define

$$RHS(c^1; a) = v\left(b_L(c^1); c^1; a\right) + v(b_L(0); 0; a) \tag{A.51}$$

and

$$LHS(c^1; a) = v\left(b^d_H(c^1); 0; a\right) + v(0; 0; a). \tag{A.52}$$

Then (A.50) follows from (A.49) when $\frac{d[RHS(0;a) - LHS(0;a)]}{da} < 0$.
Since

$$v(b; 0; a) = u(b + aL^*; 1 - L^*); \tag{A.53}$$

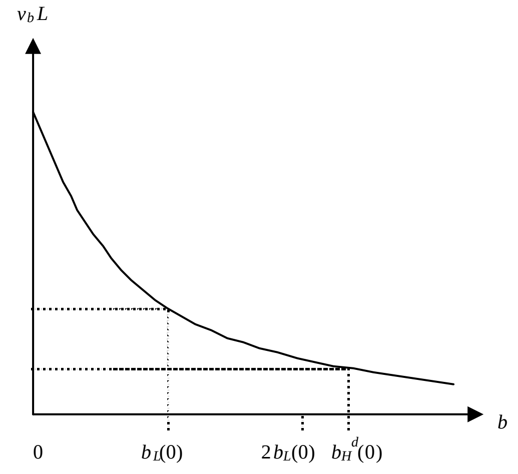

Figure A. $v_b(b,0,a)\cdot L(b,0,a)$ sufficiently convex in b.

Figure 9:

where L^* is the optimal labour supply satisfying the foc $u_x a - u_\ell$, we have that

$$\frac{\partial v(b;0;a)}{\partial a} = v_b(b;0;a)\cdot L(b;0;a) = (v_b L)_{(b;0;a)}. \tag{A.54}$$

Therefore,

$$\begin{aligned}\frac{d[RHS(0;a) - LHS(0;a)]}{da} = {} & \{(v_b L)_{(b_L(0);0;a)} - (v_b L)_{(b_H^d(0);0;a)}\} \\ & - \{(v_b L)_{(0;0;a)} - (v_b L)_{(b_L(0);0;a)}\}.\end{aligned} \tag{A.55}$$

With decreasing marginal utility of income and normality of leisure, $v_b L$ is decreasing in b, and both curly bracket terms are positive. Consider then the figure below.

If $v_b L$ is sufficiently convex in b, the above expression is negative.

The second derivative of $v_b L$ w.r.t. b is given by

$$\frac{\partial^2(v_b L)}{\partial b^2} = u_{xxx} L + 2u_{xx}\frac{\partial L}{\partial b} + u_x\frac{\partial^2 L}{\partial b^2} \tag{A.56}$$

Decreasing absolute risk aversion implies that $u_{xxx} > 0$. The second term is positive since leisure is assumed to be a normal good. Utility maximisation does not impose restrictions on the sign of $\frac{\partial^2 L}{\partial b^2}$. It can go either way. With Cobb-Douglas preferences, for example, labour supply is linear in lump sum income.

The above argument is valid for $b_H^d(0)$ slightly above $2b_L(0)$. But, as we have argued in the text, decreasing marginal utility of income is the reason why $b_H^d(0) > 2b_L(0)$. The faster marginal utility in income is falling, the more will $b_H^d(0)$ exceed $2b_L(0)$. But while the extent to which $b_H^d(0)$ exceeds $2b_L(0)$ is dependent on the degree of absolute risk aversion, the convexity of $v_b L$ depends on the sensitivity of absolute risk aversion to income and on the curvature properties of the labour supply function. The two aspects are therefore not at odds with one another. $\blacksquare$

Part 2

Lemma 8 If an L-person does not have an incentive to t-m-r when $c^1 = 0$, he will not have it either for any $c^1 \in (0; c^{max}]$.

Proof.

Suppose that the low ability person does not have an incentive to t-m-r when the work requirement is zero, i.e.

$$v\left(b_H^d(c^1); 0; a_L\right) + v(0; 0; a_L) \leq v\left(b_L(c^1); c^1; a_L\right) + v(b_L(0); 0; a_L) \qquad (A.57)$$

for $c^1 = 0$.

Since $b_H^d(c^1)$ is decreasing in c^1, the utility when dissembling as H, will certainly decrease. On the other hand, for any $c^1 \in [0; c^{co}]$, $v(b_L(c^1); 0; a_L) = v(b_L(0); 0; a_L)$, so that the intertemporal utility when behaving honest remains the same. We may thus conclude that for any $c^1 \in (0; c^{co}]$, the low ability person will not t-m-r if such incentive is absent for $c^1 = 0$.

It then remains to check whether t-m-r may become lucrative for $c^1 \in (c^{co}; c^{max}]$.

Let us for that purpose analyse $\frac{d[RHS(c^1;a_L) - LHS(c^1;a_L)]}{dc^1}$ for $c^1 \in (c^{co}; c^{max}]$: If this expression is always negative, we can conclude that the incentives to t-m-r only become weaker.

Substitution gives us

$$\begin{aligned}\frac{d[RHS(c^1; a_L) - LHS(c^1; a_L)]}{dc^1} &= \\ &\frac{\partial v(b_L(c^1); c^1; a_L)}{\partial c^1} - v_b(b_H^d(c^1); 0; a_L)\frac{db_H^d(c^1)}{dc^1} = \\ &\frac{\partial u(z; 1 - c^1)}{\partial c^1} + v_b(b_H^d(c^1); 0; a_L)\frac{v_b(b_H^s(c^1); 0; a_H)}{v_b(b_H^d(c^1); 0; a_H)}a_H;\end{aligned}$$

where we have made use of lemma 2 and the fact that for the fact $c^1 \geq c^{co}; b_H^s(c^1) = z$.

Since H is unconstrained, the foc w.r.t his optimal earnings (y^*) allows us to write $v_b(b^s_H(c^1);0;a_H)a_H$ as $u_\ell(z+y^*;1-c^1-\frac{y^*}{a_H})$. We then get

$$
\begin{aligned}
&\frac{d[RHS(c^1;a_L)-LHS(c^1;a_L)]}{dc^1} = \\
&\qquad -u_\ell(z;1-c^1)+\frac{v_b(b^d_H(c^1);0;a_L)}{v_b(b^d_H(c^1);0;a_H)}u_\ell(z+y^*;1-c^1-\frac{y^*}{a_H}) > \\
&\qquad\qquad -u_\ell(z;1-c^1)+u_\ell(z+y^*;1-c^1-\frac{y^*}{a_H})
\end{aligned}
$$

where the inequality follows from $\frac{v_b(b^d_H(c^1);0;a_L)}{v_b(b^d_H(c^1);0;a_H)} > 1$. Because consumption is a normal good, the last expression is positive, and we can conclude that the incentive to t-m-r continues to deteriorate for values of $c^1 \geq c^{co}$. $\blacksquare$

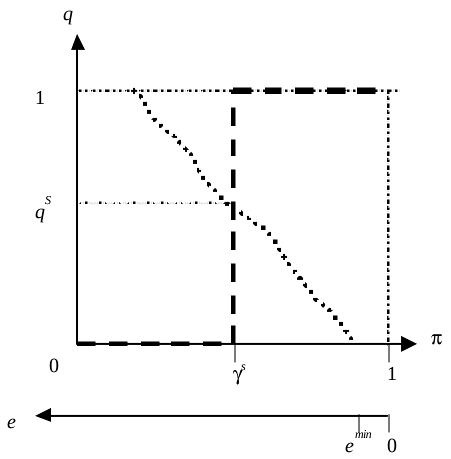

Figure 5. Reaction curves of L (•••) and WA (⇨ ⇨).

Figure 10:

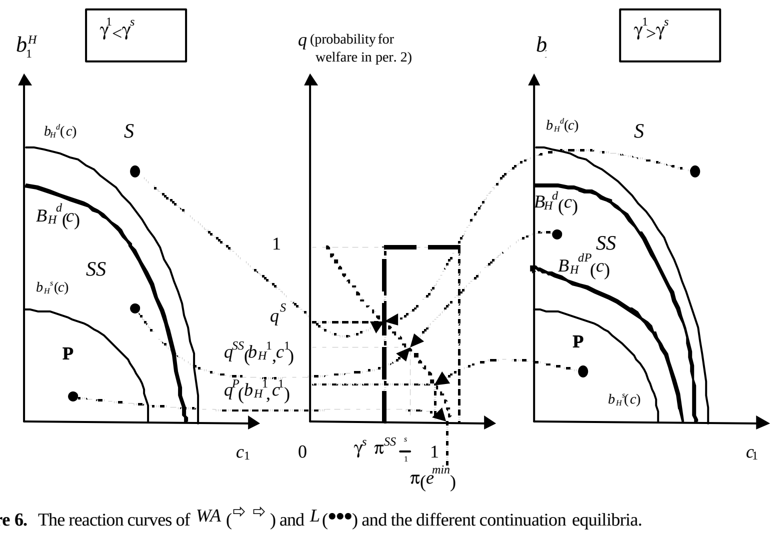

Figure 6. The reaction curves of WA (⇨ ⇨) and L (•••) and the different continuation equilibria.

Figure 11:

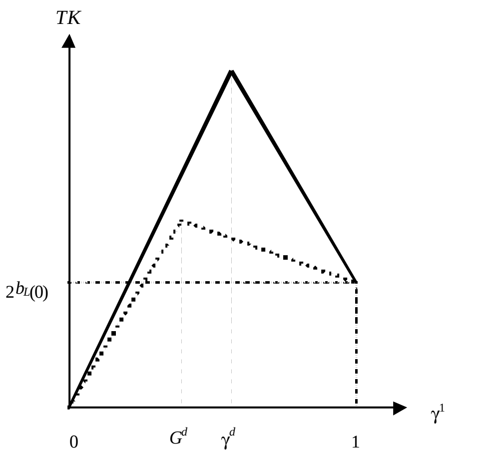

Figure 7. Total cost with efficient separating programmes with (•••) and without (♉) the possibility of *PRE*.

Figure 12:

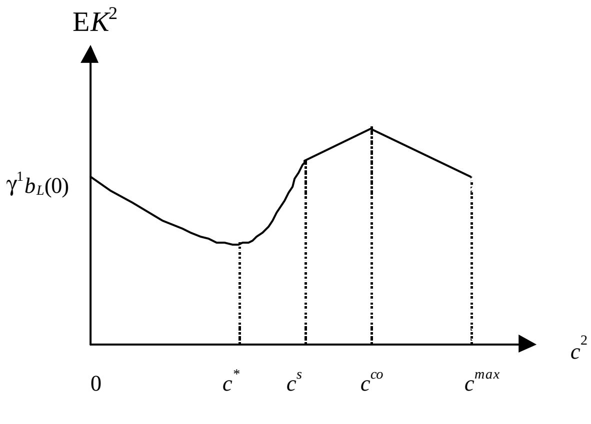

Figure 8. Expected second period cost as a function of the work requirement to which the *WA* commits (for $c^s < c^{co}$).

Figure 13:

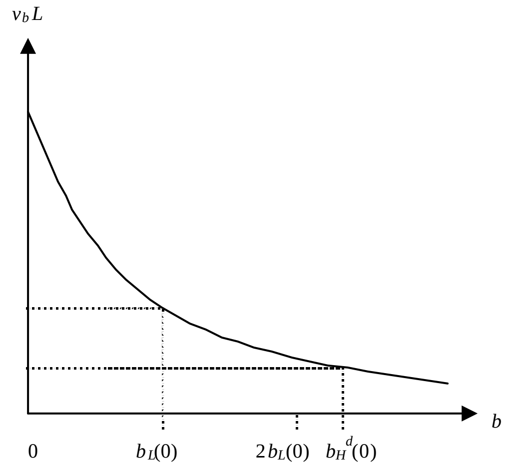

Figure A. $v_b(b,0,a) \cdot L(b,0,a)$ sufficiently convex in b.

Figure 14: