A joint Initiative of Ludwig-Maximilians-Universität and Ifo Institute for Economic Research

CESifo Working Papers

A THEORY OF HEALTH INVESTMENT UNDER COMPETING MORTALITY RISKS

Fwu-Ranq Chang*

CESifo Working Paper No. 669 (10)

February 2002

Category 10: Empirical and Theoretical Methods

CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de
ISSN 1617-9595

* I am indebted to Konstantin Tyurin and David Welsch for their comments and suggestions. Naturally, all remaining errors are mine.

CESifo *Working Paper No. 669*
February 2002

A THEORY OF HEALTH INVESTMENT UNDER COMPETING MORTALITY RISKS

Abstract

In this paper we present a theory of health investment when there are multiple causes of death. Since there are several risks “competing“ for one's life, the health investments in avoiding different causes of death are not independent in general. We analyze the optimal investment rules and the comparative statics. In particular, we search for the conditions that make such health investments normal goods, non-Giffen goods, gross complements to one another, and have a positive risk aversion effect. If the proposed conditions fail, then some health investments may become net substitutes, or even gross substitutes to one another.

JEL Classification: I11, I12.

Keywords: competing risks, complementarity, quantity and quality of life, and dominant diagonal matrix.

Fwu-Ranq Chang
Department of Economics
Wylie Hall 301
Indiana University
Bloomington, IN 47405
U.S.A.
changf@indiana.edu

1 Introduction

In this paper we present a theory of health investment under competing mortality risks. The paper is motivated by Dow, Philipson and Sala-i-Martin (1999) in which they rejected the common notion that any cause-specific intervention is wasteful because dying from other causes is "just around the corner." They argued that, since one dies at whichever cause that strikes first, the actual length of life is the Leontief function of cause-specific lengths of life. A cause-specific intervention would be wasteful unless other causes of death are also dealt with. Hence, there is a tendency to equalize the occurrence of the causes, which in turns implies a spillover effect of a cause-specific intervention upon other cause-specific interventions.

Unfortunately, the "equalizing occurrences" argument does not extend to the case when the lifetime is uncertain even though the argument is impeccable in the certainty case. To see it, we consider a simple case in which the risk of dying from each cause is exponentially distributed. In this case, the overall life expectancy is the reciprocal of the sum of cause-specific hazard rates, *not* the Leontief function of cause-specific life expectancies. Two more examples, including Weibull distributions, are given in the text to stress this point. On the other hand, we think it is plausible and intuitively appealing that different types of preventive care are complements. The question is if and when the claim is valid. This is the objective of the paper.

A reduction in the price of a cause-specific preventive care would imply an increase in all types of preventive care if they were complements to one another and satisfied the law of demand. While one is made richer by a lowered price, it is not necessarily true that the increased wealth can afford all these additional preventive cares. If the increased wealth falls short, then the increase in longevity derived from more health investment in all causes is at the expense of the quality of life because the resources available for consumption are reduced. The observation suggests that we would need a strong quantity-of-life effect if the spillover effect were valid.

To this end, let us treat the overall life expectancy as a production function of cause-specific preventive cares. One of the conditions we need is that any two types of preventive care are *complements in production.* We show that, if this complementarity dominates all other second-order effects put together, then different types of preventive care are net complements. Other effects include the complementarity in utility between wealth and longevity, the diminishing marginal utility of longevity, and that of wealth. Interestingly enough, the same set of conditions is sufficient for all types of preventive care to be normal goods and, therefore, gross complements to one another. We also show that, more risk averse people, as measured by Arrow-Pratt's relative risk aversion, will invest more in each cause-specific prevention under the same set of conditions.

On the other hand, if the proposed "strong complementarity" were not true for certain types of preventive care, then some cause-specific preventive care may not be a normal good and the corresponding substitution matrix may have some non-negative entries. In other words, some of them may be net substitutes to one another, which makes gross substitutability among them a possibility.

As for the optimal investment rules, we show that the marginal rate of technical substitution (of prolonging life) between any two types of preventive care must equal their relative price. As an illustration, if each cause-specific risk has a Weibull distribution and if the hazard rate is inversely proportional to the amount of preventive care, then the overall life expectancy is a CES production function of cause-specific preventions. In the special case of exponential distributions, resources are so allocated that the medical expenditure-hazard rate ratio is constant across all causes. It is interesting to note that if the "equilibrium" marginal valuation of life were exogenously given, then the optimal health investment under competing risks also maximizes the net value of life.

The paper is organized as follows. In section 2, we set up the model. In section 3, we employ the standard results from the literature of competing risks to refute the argument of equalizing occurrences. In section 4, we present our theory implications. In section 5, we draw some concluding remarks.

2 The Model

Let τ_i, $i = 1, 2, ..., n$, be the random variable representing the age of death due to cause i if cause i is the only risk present. The hazard rates associated with these random variables are known as the *net* hazard rates. Since one dies only once, the actual length of life is the random variable

$$\tau = \min \{\tau_1, \tau_2, ..., \tau_n\} . \tag{1}$$

To make the model tractable, we assume that both the market interest rate and the subjective discount rate are zero and that the period utility function is stable over time, i.e., $u_t(c) = u(c)$ for all $t \geq 0$. Furthermore, we assume that $u(c)$ is strictly increasing and strictly concave in c, satisfying $u(0) \geq 0$ and $u^{'}(0) = \lim_{x \to 0} u^{'}(x) = \infty$. Given initial wealth W, the consumer maximizes the expected lifetime utility, $E_0 \int_0^{\tau} u(c(t))\, dt$, subject to the lifetime budget constraint

$$E_0 \int_0^{\tau} c(t)\, dt = W. \tag{2}$$

Let $F(t)$ be the distribution function of τ with density $f(t)$, i.e., $F(t)$ represents the probability of dying before or at age t. Then the survival function is

$$S(t) = \Pr(\tau > t) = \Pr\{\tau_1 > t, \tau_2 > t, ..., \tau_n > t\} = 1 - F(t),$$

satisfying $S(\infty) = 0$[1] and, if $S(t)$ is integrable,

$$\int_0^{\infty} S(t)\, dt = \int_0^{\infty} t f(t)\, dt = \bar{\tau}.$$

By definition, $\bar{\tau}$ is the life expectancy. Then the problem is transformed into

$$\max_{c(t)} \int_0^{\infty} u(c(t))\, S(t)\, dt, \text{ s.t. } \int_0^{\infty} c(t)\, S(t)\, dt = W. \tag{3}$$

It is obvious that the optimal consumption is $c(t) = W/\bar{\tau}$, and the indirect utility function of (3) is

$$V(W, \bar{\tau}) = u\left(\frac{W}{\bar{\tau}}\right)\bar{\tau}. \tag{4}$$

[1] We assume that the life-span lies in the interval $[0, \infty)$ instead of $[0, T^*]$ for some maximal life-span T^*. The analysis and the results remain unchanged. We choose ∞ so that statistical distributions such as exponential and Weibull distributions can be directly applied. Furthermore, as pointed out in Chang (1991), $S(t)$ acts like a discount factor in (3) and the budget equation (2) presumes the annuity market is perfect in the sense that the expected (present) value of lifetime consumption is equal to the initial wealth

The strict monotonicity and the strict concavity of $u(c)$ implies

$$V_W = u^{'}\left(\frac{W}{\bar{\tau}}\right) > 0, V_{WW} = u^{''}\left(\frac{W}{\bar{\tau}}\right)\left(\frac{1}{\bar{\tau}}\right) < 0,$$

$$V_{\bar{\tau}} = u\left(\frac{W}{\bar{\tau}}\right) - u^{'}\left(\frac{W}{\bar{\tau}}\right)\left(\frac{W}{\bar{\tau}}\right) > u(0) \geq 0,$$

$$V_{W\bar{\tau}} = -u^{''}\left(\frac{W}{\bar{\tau}}\right)\left(\frac{W}{\bar{\tau}^2}\right) > 0,$$

i.e., wealth and longevity are complements in utility, and,

$$V_{\bar{\tau}\bar{\tau}} = u^{''}\left(\frac{W}{\bar{\tau}}\right)\left(\frac{W^2}{\bar{\tau}^3}\right) < 0,$$

i.e., there is a diminishing marginal utility of longevity.

Now suppose the random age of death due to cause i, τ_i, can be changed through investment x_i. Denote it by $\tau_i(x_i)$, which satisfies $\tau_i^{'}(x_i) > 0$ in the sense that τ_i is increased for any realization. In so doing, we endogenize the hazard rates and hence we may consider this health investment a form of self-protection of Erhlich and Becker (1973). Following Kenkel (1994), we call this type of activities that change the probability of survival *preventive medical care*. It is to be distinguished from the type of health investment that changes health stocks but not survival probabilities. See, for example, Chang (1996).

Under uncertain lifetimes, the actual length of life with preventive care $(x_1, x_2, ..., x_n)$ is defined by

$$\tau(x_1, x_2, ..., x_n) = \min\{\tau_1(x_1), \tau_2(x_2), ..., \tau_n(x_n)\}.$$

Again, $\tau(x_1, x_2, ..., x_n)$ is a random variable. Let

$$\bar{\tau}(x_1, x_2, ..., x_n) = E[\tau(x_1, x_2, ..., x_n)]$$

be the overall life expectancy. We can regard $\bar{\tau}(x_1, x_2, ..., x_n)$ as a production function with input vector $(x_1, x_2, ..., x_n)$ because an increase in x_i delays the cause-specific age of death $\tau_i(x_i)$ for any realization, which in turns raises the overall life expectancy $\bar{\tau}$. In short,

$$\bar{\tau}_i = \frac{\partial \bar{\tau}}{\partial x_i} \geq 0.$$

Given investment $(x_1, x_2, ..., x_n)$, the consumer's wealth is reduced to $W - \sum_{i=1}^{n} p_i x_i$. Hence, the problem of preventive care can be formulated as

$$\max_{\{x_i\}} V\left(W - \sum_{i=1}^{n} p_i x_i, \bar{\tau}(x_1, x_2, ..., x_n)\right). \tag{5}$$

The model is germane to the stochastic model of Dow *et al* (1999). However, we disagree with their claim that, because of its similarity to the certainty case, "the forces towards the equalization of cause-specific lifetimes operate in the more general case as well." (p.1361.)

To ensure the solution to Problem (5) uniquely exists, we assume that the objective function $V(W - \sum_{i=1}^{n} p_i x_i, \bar{\tau}(x_1, ..., x_n))$ is strictly concave in $(x_1, ..., x_n)$. In particular, the Hessian matrix, $H = [h_{ij}]_{n \times n}$, is negative definite, where

$$h_{ij} = V_{WW} p_i p_j - V_{W\bar{\tau}}(p_i \bar{\tau}_j + p_j \bar{\tau}_i) + V_{\bar{\tau}\bar{\tau}} \bar{\tau}_i \bar{\tau}_j + V_{\bar{\tau}} \bar{\tau}_{ij}. \tag{6}$$

To ensure $x_i > 0$ for all i, we further assume $\bar{\tau}_i(x_1, ..., x_n) = \infty$ if $x_i = 0$. Since $u'(0) = \infty$, we have $V_W(0, \cdot) = \infty$. Hence, $W - \sum_{i=1}^{n} p_i x_i > 0$, i.e., the consumer will not spend all her resources on preventive cares. Then the unique solution to problem (5) is interior.

3 Competing Risks

In this section we shall briefly review the theory of competing risks relevant to our problem. The theory is traced back to Daniel Bernoulli who looked into the effect of population mortality if smallpox were eradicated. See, for example, David and Moeschberger (1978, Appendix A) and Elandt-Johnson and Johnson (1980, p. 309). Let $h_i(t)\,dt$ be the probability of dying from cause i in time interval $(t, t+dt)$, in the presence of all risks, conditional on alive at time t. We assume the probability of more than one failure in $(t, t+dt)$ is of order $(dt)^2$. This $h_i(t)$ is known as the (instantaneous) crude hazard rate. Then the overall crude hazard rate satisfies the *additivity property*:

$$h(t) = \sum_{i=1}^{n} h_i(t).$$

It shows that the force of mortality is the *sum* of components.

It is standard in the competing risks literature to assume that all causes of mortality are independent.[2] See, for example, David and Moeschberger (1978, section 4.4) and Elandt-Johnson and Johnson (1980, chs. 9 and 15). If all risks are mutually independent, then $h_i(t)$ is also the net hazard rate. In this case, crude hazard rates and net hazard rates are interchangeable. Then, the survival function for cause i is $S_i(t) = \exp\{-h_i(t)\}$, and the overall survival function is $S(t) = \exp\{-\sum_{i=1}^{n} h_i(t)\}$.

To illustrate, we first assume that each cause-specific survival function is

[2] The heuristic argument goes as follows. Assume that one may die from disease 1, disease 2, or disease 1 and 2 simultaneously. Let λ_i be the hazard rate of disease $i, i = 1, 2$, and λ_{12} be the hazard rate of disease 1 and 2 simultaneously. The intuition is this: We can treat dying from disease 1 and 2 simultaneously as if it were another disease independent of the other two. In other words, we are dealing with three independent risks. By induction, the argument applies to the case of multiple causes with dependency.

exponential with parameter $\lambda_i > 0$,

$$S_i(t) = \exp\{-\lambda_i t\}, \lambda_i > 0, t > 0.$$

Then the overall survival function of $\tau = \min\{\tau_1, ..., \tau_n\}$ is also exponential,

$$S(t) = \exp\left\{-\sum_{i=1}^{n} \lambda_i t\right\}, t > 0.$$

See, for example, David and Moeschberger (1978, p.16) or Elandt-Johnson and Johnson (1980, p.65). By construction, the i-th cause life expectancy is

$$E[\tau_i] = \frac{1}{\lambda_i}.$$

(Note that we reserve the notation $\bar{\tau}_i$ for $\partial\bar{\tau}/\partial x_i$.) Then, the overall life expectancy in the presence of competing causes of death is

$$\bar{\tau} = E[\tau] = \frac{1}{\sum_{i=1}^{n} \lambda_i}. \tag{7}$$

Next, we assume each cause-specific risk has a Weibull distribution, i.e., the survival function is

$$S_i(t) = \exp\{-(\lambda_i t)^\rho\}, \lambda_i > 0, t > 0, \rho > 0.$$

Obviously, a Weibull distribution is reduced to an exponential distribution when $\rho = 1$. Then the overall survival function of $\tau = \min\{\tau_1, ..., \tau_n\}$ is also of Weibull form,

$$S(t) = \exp\left\{-\sum_{i=1}^{n} \lambda_i^\rho t^\rho\right\}, t > 0.$$

A direct computation shows that

$$\bar{\tau} = E[\tau] = (1/\rho)\,\Gamma(1/\rho)\left(\sum_{i=1}^{n} \lambda_i^\rho\right)^{-1/\rho}, \tag{8}$$

where

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

is the gamma function.

Third, assume each cause-specific hazard rate is quadratic in time, i.e., $h_i(t) = \lambda_i t + \alpha_i t^2$. Obviously, when $\alpha_i = 0$, for all i, we have the exponential model. Then the overall survival function of $\tau = \min\{\tau_1, ..., \tau_n\}$ is also quadratic in time

$$S(t) = \exp\left\{-\lambda t - \alpha t^2\right\}, t > 0, \text{ where } \lambda = \sum_{i=1}^n \lambda_i, \alpha = \sum_{i=1}^n \alpha_i.$$

A direct computation shows that, if $\alpha > 0$,

$$\bar{\tau} = E[\tau] = \sqrt{\frac{\pi}{\alpha}} \left[1 - N\left(\frac{\lambda}{\sqrt{2\alpha}}\right)\right] \exp\left\{\frac{\lambda^2}{4\alpha}\right\}, \tag{9}$$

where

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$$

is the standard normal function.

These examples present a challenge to Dow *et al*'s "equalizing occurrences" argument. Their intuition is built upon the certainty case of (1),

$$T = \min\{T_1, T_2, ..., T_n\},$$

where T_i is the age of death due to cause $i, i = 1, 2, ..., n$. Since one dies only once, this Leontief function implies that a typical consumer will allocate resources so as to equalize the occurrence of causes. While the stochastic length of life takes the form of a Leontief function as shown in (1), the overall life expectancy as shown in (7), (8) and (9), do not.

More can be said about the overall life expectancy when each cause of death is exponentially distributed. First, equalizing the life expectancies of all causes of death is *not* a necessary condition to achieving a given life expectancy. This can be seen from

$$E[\tau] < \min\{E[\tau_1], E[\tau_2], ..., E[\tau_n]\},$$

using (7). In fact, the Leontief function of cause-specific life expectancies *overestimates* the true life expectancy under competing risks.

Second, the effect of a cause-specific intervention on life expectancy, and hence on lifetime utility, depends on the number of causes and the "weight" of a given cause relative to all causes, $\lambda_i / \sum_{i=1}^{n} \lambda_i$. Specifically, the elasticity of life expectancy with respect to lowering i-th hazard rate is

$$\varepsilon_i = -\frac{\lambda_i}{E[\tau]} \frac{\partial E[\tau]}{\partial \lambda_i} = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i}. \tag{10}$$

An immediate corollary is that, for a given rate of hazard reduction, the cause with a larger share has a greater effect of intervention. In other words, among all causes of death, an intervention on the deadliest cause has the greatest impact on survival. When the number of causes, n, is large, the elasticity tends to be small, other things being equal. The theory thus predicts that, in the absence of an abnormally large hazard rate, the effect of cause-specific intervention in disease-plagued countries would not be very successful at least initially.

4 Theory Implications

4.1 Resource Allocation

The first-order conditions of (5) are

$$-V_W p_i + V_{\bar{\tau}} \bar{\tau}_i = 0, i = 1, 2, ..., n. \tag{11}$$

This is the "marginal benefit equals marginal cost" equation that shows the trade-off between the quantity and quality of life. Specifically, an additional unit of preventive care, x_i, increases the life expectancy by $\bar{\tau}_i$, and hence increases the utility by $V_{\bar{\tau}} \bar{\tau}_i$ units. This is the marginal benefit of preventive care derived from increased quantity of life. In contrast, an additional units of x_i costs p_i dollars, which in turns loses $V_W p_i$ units of utility. This is the marginal cost of preventive care derived from decreased quality of life.

Rewrite (11) as

$$\left(\frac{V_{\bar{\tau}}}{V_W} \right) \bar{\tau}_i = p_i, i = 1, 2, ..., n. \tag{12}$$

It follows that

$$\frac{\bar{\tau}_i (x_1, x_2, ..., x_n)}{\bar{\tau}_j (x_1, x_2, ..., x_n)} = \frac{p_i}{p_j}, \forall i, j. \tag{13}$$

Since $\bar{\tau} (x_1, ..., x_n)$ is a production function with input vector $(x_1, ..., x_n)$, equation (13) is the familiar formula that the marginal rate of technical substitution equals the relative price. It shows that $\bar{\tau} (x_1, ..., x_n)$ is, in general, not a Leontief function of different types of preventive care. For example, if each cause-specific risk has a Weibull distribution and if the hazard rate is inversely proportional to its investment, i.e., $\lambda_i (x_i) = a_i / x_i$ for some a_i, then, from (8),

$$\bar{\tau} (x_1, x_2, ..., x_n) = (1/\rho) \Gamma (1/\rho) \left(\sum_{i=1}^{n} a_i^{\rho} x_i^{-\rho} \right)^{-1/\rho} \tag{14}$$

is a CES production function with the elasticity of substitution $1/(1+\rho) \leq 1$. Moreover, from (13), the optimal investment rules are

$$\frac{p_i x_i}{p_j x_j} = \left[\frac{\lambda_i(x_i)}{\lambda_j(x_j)}\right]^\rho, \forall i, j,$$

i.e., the cause-specific medical expenditure is positively related to its hazard rate in equilibrium. In particular, when $\rho = 1$, (when the distribution is reduced to exponential), resources are so allocated that the ratio of medical expenditure, $p_i x_i$, to hazard rate, $\lambda_i(x_i)$, is constant across all causes.

Equation (12) has an interesting interpretation. Let

$$\theta(x_1, ..., x_n) = \frac{V_{\bar{\tau}}(x_1, ..., x_n)}{V_W(x_1, ..., x_n)}.$$

By definition, θ is the marginal rate of substitution between longevity (the quantity of life) $\bar{\tau}$ and the quality of life W. It measures the willingness to pay for an additional year of statistical life and hence is the marginal valuation of life. See Rosen (1994) for a discussion on this subject. Then the left-hand-side of (12) is the value of marginal product of preventive care, and the right-hand-side is the marginal cost of preventive care. At equilibrium $(x_1^*, ..., x_n^*)$, $\theta^* = \theta(x_1^*, ..., x_n^*)$ is *the* marginal valuation of life. In fact, (12) is the first order condition to

$$\max_{x_1,\dots,x_n} \left\{\theta^* \bar{\tau}(x_1, x_2, ..., x_n) - \sum_{i=1}^{n} p_i x_i\right\}.$$

In other words, *given* the equilibrium marginal valuation of life, optimal health investment under mortality risks also maximizes the net value of life.

4.2 Wealth Effect

From (11), the wealth effect is given by

$$\begin{bmatrix} \partial x_1/\partial W \\ \cdot \\ \cdot \\ \partial x_n/\partial W \end{bmatrix} = [h_{ij}]^{-1} \begin{bmatrix} V_{WW}p_1 - V_{\bar{\tau}W}\bar{\tau}_1 \\ \cdot \\ \cdot \\ V_{WW}p_n - V_{\bar{\tau}W}\bar{\tau}_n \end{bmatrix}. \quad (15)$$

As shown earlier, $V_{WW} < 0$ and $V_{W\bar{\tau}} > 0$. It follows that $V_{WW}p_i - V_{\bar{\tau}W}\bar{\tau}_i < 0$.

Proposition 1 *All $x_i's$ are normal goods if $h_{ij} \geq 0, \forall i \neq j$.*

The proposition follows directly from a well-known theorem in the literature of dominant diagonal matrices. Specifically, if $h_{ij} \geq 0$, then a negative definite matrix has an inverse matrix with only negative entries. See, for example, Takayama (1985, Theorem 4.D.3). Since all entries in $[h_{ij}]^{-1}$ are negative, we have $\partial x_i/\partial W > 0, \forall i$. In other words, if different types of preventive care are *complements in utility*, then they are normal goods.[3]

This mathematical result on the inverse matrix will become useful later. To make use of it, we note that if C_{ij} is the (i,j) −cofactor of $[h_{ij}]$, then

$$[h_{ij}]^{-1} = \frac{1}{\det [h_{ij}]} [C_{ji}]_{n\times n}.$$

Hence, for all i and j, C_{ji} and $\det [h_{ij}]$ are opposite in sign.

Notice that every term in (6), except $V_{\bar{\tau}}\bar{\tau}_{ij}$, is negative. A necessary condition for $h_{ij} \geq 0$ is that $\bar{\tau}_{ij} > 0$, i.e., inputs x_i and x_j are complements in

[3]It should be mentioned that comparative statics results can also be obtained using supermodularity approach. See, Topkis (1998, Theorem 2.8.2). Monotone comparative statics ($\partial x_i/\partial W \geq 0, \forall i$) holds if the objective function of (5) is supermodular in $(x_1, x_2, ..., x_n, W)$, which translates into $h_{ij} \geq 0, \forall i \neq j$, and $-V_{WW}p_i + V_{\bar{\tau}W}\bar{\tau}_i \geq 0, , \forall i$.

production. For example, if each cause-specific risk has a Weibull distribution and if the hazard rate is inversely proportional to its investment, then, from (14), we have $\bar{\tau}_i > 0$ and $\bar{\tau}_{ij} > 0$. A sufficient condition for $h_{ij} \geq 0$ is that the term $V_{\bar{\tau}}\bar{\tau}_{ij}$ must dominate all other negative terms in h_{ij}. The strength of this quantity-of-life effect and the feasibility of $h_{ij} \geq 0, \forall i \neq j$, will be discussed later.

4.3 Price Effect

From (11), the price effect is given by

$$\begin{bmatrix} \partial x_1/\partial p_j \\ \cdot \\ \cdot \\ \partial x_n/\partial p_j \end{bmatrix} = [h_{ij}]^{-1} \left\{ V_W \mathbf{e}_j - x_j \begin{bmatrix} V_{WW}p_1 - V_{\bar{\tau}W}\bar{\tau}_1 \\ \cdot \\ \cdot \\ V_{WW}p_n - V_{\bar{\tau}W}\bar{\tau}_n \end{bmatrix} \right\},$$

where $\mathbf{e}_j$ is the column vector with 1 in the j-th row and 0 elsewhere. Then the Slutsky equations are given by

$$\frac{\partial x_i}{\partial p_j} = \frac{C_{ji}}{\det[h_{ij}]} V_W - x_j \frac{\partial x_i}{\partial W}, \forall i, j. \tag{16}$$

The first term on the right-hand-side of (16) is the substitution effect and the second term is the wealth effect.

Proposition 2 *If $h_{ij} \geq 0, \forall i \neq j$, then all $x_i's$ obey the law of demand ($\partial x_i/\partial p_i < 0, \forall i$) and any two $x_i's$ are gross complements ($\partial x_i/\partial p_j < 0, \forall i \neq j$).*

The proposition is again a corollary of the theory of dominant diagonal matrices. As mentioned before, for all i and j, C_{ji} and $\det[h_{ij}]$ are opposite in sign if $h_{ij} \geq 0, \forall i \neq j$. Hence, all entries of the substitution matrix are negative. When $i = j$, the substitution effect of (16) is negative and the law of demand is satisfied. When $i \neq j$, different types of preventive care are net complements. Since the set of conditions for a negative substitution effect is identical to the one for a positive wealth effect, different types of preventive care are also gross complements.

The spillover effect of competing risks and the trade-off between the quantity and quality of life can now be better understood. Since $\bar{\tau}_{ij} > 0$ is the only positive term that makes $h_{ij} \geq 0$, it tells us that spillovers are not implied by the complementarity in utility between wealth and longevity nor by the diminishing marginal utility of longevity. A surprising result is that the diminishing marginal utility of wealth actually works against $h_{ij} \geq 0$. Instead, it is the strong complementarity among health investments that ensures the quantity-of-life effect would dominate all other effects and produce the spillover effects.

On the other hand, some types of preventive care may be substitutes to one another if the conditions "$h_{ij} \geq 0$ for all $i \neq j$" fail. In this case, it is possible that not all entries of $[h_{ij}]^{-1}$ are negative. Assume there exist some i and j such that C_{ji} and $\det[h_{ij}]$ have the same sign. Then, from (16), x_i is a *net substitute* to x_j and, from (15), the wealth effect is ambiguous. Even if x_i remains a normal good, it is still possible that the substitution effect dominates the wealth effect and, consequently, x_i becomes a *gross substitute* to x_j. In short, if the quantity-of-life effect as represented by $\bar{\tau}_{ij} > 0$ is not large enough for some i and j, then the claim of a spillover effect could be false.

4.4 Risk Aversion

Consider $u(x) = x^{1-\alpha}, 0 < \alpha < 1$. The parameter α is Arrow-Pratt's relative risk aversion. Then

$$V\left(W - \sum_{i=1}^{n} p_i x_i, \bar{\tau}(x_1, ..., x_n)\right) = \left(W - \sum_{i=1}^{n} p_i x_i\right)^{1-\alpha} [\bar{\tau}(x_1, ..., x_n)]^{\alpha}. \tag{17}$$

The first-order conditions are

$$\left(\frac{\bar{\tau}}{W - \sum_{i=1}^{n} p_i x_i}\right)^{\alpha} \left[\frac{\alpha \left(W - \sum_{i=1}^{n} p_i x_i\right) \bar{\tau}_i}{\bar{\tau}} - (1-\alpha) p_i\right] = 0, \forall i.$$

Then

$$\begin{bmatrix} \partial x_1 / \partial \alpha \\ \cdot \\ \cdot \\ \partial x_n / \partial \alpha \end{bmatrix} = [h_{ij}]^{-1} \left(\frac{\bar{\tau}}{W - \sum_{i=1}^{n} p_i x_i}\right)^{\alpha} \begin{bmatrix} -p_1 - \frac{\left(W - \sum_{i=1}^{n} p_i x_i\right) \bar{\tau}_1}{\bar{\tau}} \\ \cdot \\ \cdot \\ -p_n - \frac{\left(W - \sum_{i=1}^{n} p_i x_i\right) \bar{\tau}_n}{\bar{\tau}} \end{bmatrix}.$$

Proposition 3 *If $h_{ij} \geq 0, \forall i \neq j$, then $\partial x_i / \partial \alpha > 0$ for all i.*

Again, the proposition follows immediately from the fact that $[h_{ij}]^{-1}$ has only negative entries. We thus conclude that more risk averse people will invest more in preventive cares.

4.5 On Feasibility

As mentioned before, the cross effect on longevity, $V_{\bar{\tau}} \bar{\tau}_{ij}$, must be large enough to ensure $h_{ij} \geq 0, \forall i \neq j$. To illustrate the strength of this effect, we assume that $u(x) = x^{1-\alpha}, 0 < \alpha < 1$ and that each τ_i is exponentially distributed. Then

$$\bar{\tau}(x_1, x_2, ..., x_n) = \frac{1}{\sum_{i=1}^{n} \lambda_i(x_i)}.$$

Any health investment that prolongs life is the health investment that reduces the corresponding hazard rate, i.e., $\lambda_i^{'}(x_i) < 0$. [Note that we did not assume the special functional form $\lambda_i(x_i) = a_i/x_i$ here.] If each $\lambda_i(x_i)$ is twice continuously differentiable in x_i, then $\bar{\tau}(x_1, x_2, ..., x_n)$ is twice continuously differentiable in $(x_1, x_2, ..., x_n)$. In particular, we have

$$\bar{\tau}_i = -\bar{\tau}^2 \lambda_i^{'}(x_i) > 0,$$

and

$$\bar{\tau}_{ij} = \begin{cases} 2\bar{\tau}^3 \lambda_i^{'}(x_i)\, \lambda_j^{'}(x_j), \text{ if } j \neq i. \\ 2\bar{\tau}^3 \left[\lambda_i^{'}(x_i)\right]^2 - \bar{\tau}^2 \lambda_i^{''}(x_i), \text{ if } j = i. \end{cases} \tag{18}$$

Clearly, $\bar{\tau}_{ij} > 0$ if $j \neq i$. Since $\bar{\tau}(x_1, x_2, ..., x_n)$ is treated as a production function, we shall assume it is a concave function and, in particular, $\bar{\tau}_{ii} < 0$. Then equation (13) can be written as

$$b = \frac{p_i}{\lambda_i^{'}(x_i)} < 0, \forall i. \tag{19}$$

From (17),

$$h_{ij} = \alpha(1+\alpha)\left(W - \sum_{i=1}^{n} p_i x_i\right)^{-\alpha-1} \bar{\tau}^{\alpha} p_i p_j g_{ij},$$

where

$$\begin{aligned} g_{ij} &= -\frac{1-\alpha}{1+\alpha} + \frac{2(1-\alpha)}{1+\alpha}\left(W - \sum_{i=1}^{n} p_i x_i\right)\left(\frac{\bar{\tau}}{b}\right) + \left(W - \sum_{i=1}^{n} p_i x_i\right)^2 \left(\frac{\bar{\tau}}{b}\right)^2 \\ &= \left[\left(W - \sum_{i=1}^{n} p_i x_i\right)\left(\frac{\bar{\tau}}{b}\right) + \frac{1-\alpha}{1+\alpha}\right]^2 - \frac{2(1-\alpha)}{(1+\alpha)^2}. \end{aligned}$$

Then $g_{ij} > 0$ is feasible. For example, if $\left|\left(W - \sum_{i=1}^{n} p_i x_i\right)(\bar{\tau}/b)\right| \geq 1 + \sqrt{2}$, then $g_{ij} > 0$, since $0 < (1-\alpha)/(1+\alpha) < 1$.

The condition $\left|\left(W-\sum_{i=1}^{n} p_i x_i\right)(\bar{\tau}/b)\right| > 1+\sqrt{2}$ is quite feasible. Since $W-\sum_{i=1}^{n} p_i x_i > 0$ represents the wealth for lifetime consumption, which is not small. The same is true for life expectancy $\bar{\tau}$. The absolute value of $\left(W-\sum_{i=1}^{n} p_i x_i\right)(\bar{\tau}/b) < 0$ will be large if $|b|$ is relatively small. To ensure $|b|$ is bounded from above, we need $\left|\lambda_i^{'}(x_i)\right|$ is bounded away from zero for all i. Given $\bar{\tau}_{ii} < 0$, it is necessary that, from (18), $\lambda_i^{''}(x_i) > 0$. That is, $\lambda_i(x_i)$ is downward sloping and convex to the origin in the (x_i, λ_i)-plane. Since the consumer spends only a portion of her resources in preventive care, x_i is bounded and, hence, $\left|\lambda_i^{'}(x_i)\right|$ is bounded away from zero.

5 Concluding Remarks

In this paper we show that, if cause-specific preventive cares are complements in utility, then they are normal goods, gross complements to one another, and have a positive risk aversion effect. We point out that, with uncertain lifetimes, the driving force behind the spillover effect is the strong complementarity in production that generates life expectancy, not the argument of equalizing the occurrence of different causes of death. Without this strong complementarity in production, we show that net or even gross substitutes among some of them, and hence the failure of the spillover effect, are quite likely. In this sense our theory extends Dow, Philipson and Sala-i-Martin's theory and complements their empirical findings.

References

[1] Chang, Fwu-Ranq (1991), "Uncertain Lifetime, Retirement and Economic Welfare," *Economica 58*: 215-232.

[2] Chang, Fwu-Ranq (1996), "Uncertainty and Investment in Health," *Journal of Health Economics* 15(3): 369-376.

[3] David, H. A. and M. L. Moeschberger (1978), *The Theory of Competing Risks*, Macmillan Publishing Co. Inc., New York.

[4] Dow, William, Tomas Phillipson, and Xavier Sala-i-Martin (1999), "Longevity Complementarities under Competing Risks," *American Economic Review* 99(5): 1358-1371.

[5] Ehrlich, Isaac and Gary Becker (1972), "Market Insurance, Self-Insurance, and Self-Protection," *Journal of Political Economy,* 80(4), 623-648.

[6] Elandt-Johnson, Regina and Norman Johnson (1980) *Survival Models and Data Analysis*, John Wiley & Sons, Inc., New York.

[7] Kenkel, Donald (1994), "The Demand for Preventive Medical Care," *Applied Economics* 26: 313-325.

[8] Rosen, Sherwin (1994), "The Quantity and Quality of Life: A Conceptual Framework," in *Valuing Health for Policy: An Economic Approach*, Ed. by George Tolley, Donald Kenkel, and Robert Fabian, University of Chicago Press, Chicago.

[9] Takayama, A. (1985), *Mathematical Economics*, 2nd edition, Cambridge University Press, Cambridge.

[10] Topkis, Donald, (1998), *Supermodularity and Complementarity*, Princeton University Press, Princeton, New Jersey.