A joint Initiative of Ludwig-Maximilians-Universität and Ifo Institute for Economic Research

CESifo Working Papers

INCREASES IN RISK AND THE WELFARE STATE

Thomas Eichner
Andreas Wagener

CESifo Working Paper No. 685 (1)

March 2002

Category 1: Public Finance

CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224-1410 - Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de
ISSN 1617-9595

An electronic version of the paper may be downloaded
• from the SSRN website: www.SSRN.com
• from the CESifo website: www.CESifo.de

CESifo *Working Paper No. 685*
March 2002

INCREASES IN RISK AND THE WELFARE STATE

Abstract

According to many observers, the world is currently getting riskier along many of its dimensions. In this paper we analyse how the welfare state, i.e., social insurance that works through redistributive taxation, should deal with this trend. We distinguish between risks that can be insured by the welfare state and such than cannot (background risks). Insurable risks can be reduced either by individual self-insurance or, through pooling, by social insurance. Both ways are costly in terms of income foregone. We show: (i) Self-insurance will be higher the more costly is the welfare state and the larger are background or insured risks. (ii) Full risk coverage by the welfare state can only be optimal in a costless welfare state. (iii) The optimal size of the welfare state is larger the higher are the risks that it cannot insure. The impact of the size of risks that can be insured is, however, unclear.

JEL Classification: H53, D63, D8.

Keywords: welfare state, background risks, social insurance.

Thomas Eichner
VWL IV, FB 5
University of Siegen
Hölderlinstr. 3
57068 Siegen
Germany

Andreas Wagener
VWL IV, FB 5
University of Siegen
Hölderlinstr. 3
57068 Siegen
Germany
wagener@vwl.wiwi.uni-siegen.de

1 Introduction

Observers from various angles have it that "the world" is currently getting less secure along many of its dimensions (see, e.g., Beck, 1992; Adam et al., 2000): Sociologists argue that individual biographies and careers get more diverse and less predictable, ecologists warn that climate change and environmental deterioration create unprecedented environmental hazards and frequent catastrophes, globalization critics claim (and quite many economists agree) that globalization increases economic volatility and income variability to an unprecedented degree (Holzmann and Jørgensen, 2001), and political experts foresee, both nationally and internationally, growing instabilities, social unrest and political as well as military risks. Risk-averse individuals dislike such changes; they will increasedly seek for opportunities to insure against and hedge such risks. Indeed, according to Beck (1992), societal risk management has to be regarded as the prime task and challenge for modern policy-making.

Naturally, what is a "hedge" varies with the risk at stake. Increased defence spending might insure against military threats, foreign aid can stabilize shaky regions of the world, and building dams limits the adverse consequences of floods. If hazards can be measured in monetary equivalents, private insurance markets are good, but not quite perfect, at handling them. Imperfectness may result from problems of adverse selection, moral hazard, missing commitments or excessive costs. In such cases, a positive role may be left for insurance through the state (Barr, 1998; Sandmo, 1998). This is particular true for risks associated with the contingencies in individual lifes, such as the market valuation of one's talents, one's health status, or one's occupational career. In that realm, Varian (1980) and, in particular, Sinn (1995, 1996) have vividly and convincingly emphasized an efficiency enhancing role of a redistributive welfare state. As Sinn (1995, 1996) shows, welfare improvements through social insurance originate from *two* sources: from the provision of insurance (risk-pooling) and from the stimulation of individual risk-taking which increases income and sets free productive forces. While these positive effects may partially be undone by moral hazard-problems (when transfers cannot be made contingent on individual choices), they nevertheless provide a strong case for social insurance or, from an *ex-post* view, redistributive taxation.

However, these arguments tacitly presuppose that the risk at stake can actually be insured. This requires (among others) that the individual risks are not or, at least, not strongly positively correlated across insureds. Then, by pooling the risks faced by a large number of individuals, insurers – private companies or a welfare state – can exploit the Law of Large Numbers to essentially eliminate aggregate risk. For example, in private car insurance or in a social insurance (in Sinn's sense) the insurance company and the welfare authority can pool the largely independent risks of accidents or bad health from a large number of individuals to construct predictable, (almost) risk-free aggregate damage payouts.

Many of the risks mentioned at the outset are genuinely not distributed independently across individuals but rather covariant: Natural catastrophes such as hurricanes, earthquakes, floods or drought affect a large segment of a population; risks from global or regional economic fluctuations fall on everybody, and the hazards of wars, civil strife, social unrest and political instabilities cannot be decomposed into independent small lotteries. To be sure, there might exist opportunities to reduce the exposure to such positively associated risks. E.g., the state can accumulate and hold large reserves or use deficit finance for reasons of intertemporal risk-sharing.[1] Alternatively governments could insure against fluctuations in national incomes by entering into risk-sharing arrangements with other countries. Still, many of these opportunities are unused[2] – and even if they were implemented could they not fully insure against global risks or worldwide recessions.[3] The bottom line of all this is that, though many risks can be efficiently insured by private markets or state provisions, quite a number of sizeable risks remain – necessarily or deliberately – uninsured (Holzmann and Jørgensen, 2001).

In this paper we investigate the performance of the welfare state in a riskier world. Increases in riskiness can take two forms: those risks might grow which the welfare state does insure – or those risks increase that are not or cannot be insured. As argued before, this type of "background risk" might become more and more severe in the near future. We are interested in the individual reactions on changes in the risk structure within and outside the welfare system, and in the optimal size of the welfare state, measured by the fraction of insurable risks that is covered by the system. *Prima facie* one would suspect that higher risks call for a larger welfare state. However, this is not quite true: While an increase in background risks – i.e., in those risks which are *uninsured* by the welfare state – indeed optimally implies a higher rate of social insurance, things might be different for an increase in those risks that the welfare state can and does insure. A rough intuition for this asymmetry is as follows: An increase in a background risk makes individuals — under the assumption that their preferences are risk-vulnerable — behave in a more risk-averse way. Hence, more insurance against the insurable risk is sought (also see Dionne and Eeckhoudt, 1985) and the exposure to the insurable risk will be decreased. This can be achieved by a raise in the social insurance rate which a society of more risk-averse individuals will always be willing to finance. On the contrary, if the insured risk itself gets more

[1] In fact, taxes and transfers in place today provide (partial) insurance against income fluctuations: With an income tax, tax payments decrease when individual incomes fall, and with unemployment insurance and welfare payments the net transfers to an individual (weakly) increase when individual income declines.

[2] International risk-sharing is virtually non-existing. This even holds for economically and politically integrated systems such as the EU. Shiller (1993) reports estimates that a one-euro shock to gross national products in any EU-country reduces net tax payments to the EU by only half a cent.

[3] Shiller (1993) suggests a market solution to this problem: He proposes to design financial contracts in the form of perpetual claims on national incomes and to establish (futures) markets where these assets are traded. This, he argues, will provide (worldwide) insurance against fluctuations in living standards. However, Shiller admits, such macro markets are currently non-existent.

pronounced, a marginal increase in the social insurance rate will typically not suffice to push back the exposure to the insurable risk to its initial level; the change of the tax rate must be large enough to more than compensate for the increased riskiness. It is not clear whether the increase in the willingness-to-pay for more insurance will be sufficiently large to render such a "large" (and thus more costly) expansion of the welfare state worthwile.

The perspective of this paper is a constitutional, *ex-ante* one: Policies are assessed from behind a veil of ignorance, i.e., without any knowledge about the specific conditions in which a society will find itself in front of the veil, but with a complete understanding of how certain policies work in principle. In particular, it is assumed that individuals anticipate that tax revenues will, except for some operating cost of the welfare state (see below), be redistributed to the public. While probably too demanding for everyday politics, such an assumption might be justified under the specific veil-of-ignorance perspective which we have in mind.[4]

Welfare states do not operate costlessly. Therefore we assume that a certain percentage of tax revenues is lost for society. There are several explanations for this: The losses might represent administrative and operative cost of the welfare state, they might capture the marginal costs of public funds (excess burden) which would incur with distortionary taxation, they also might be regarded as a proxy for the welfare losses resulting from (unmodelled) moral hazard problems, or they might simply reflect governmental waste and inefficiencies. It is sometimes argued (see, e.g., Agell, 2000) that globalization and economic integration increases the cost of taxation or social insurance. Our model is able to capture that effect. Moreover, with a costly welfare state it can never be socially optimal to entirely wash out the insurable risks (or to run an egalitarian regime) by setting the tax rate to 100%.

Our paper also makes a methodological contribution: Following Sinn (1995, 1996) we employ the two-parameter, mean-standard deviation approach for modelling preferences over lotteries. In our comparative static analysis we will then resort to properties of risk preferences such as absolute or relative risk aversion and risk vulnerability that, though familiar from the expected-utility framework, have only rarely or (to our knowledge) not at all been used in a two-parameter framework.

The remainder of this paper is organized as follows: Section 2 describes a model economy with insurable and uninsurable risks. Insurable risks can be hedged either through individual self-insurance or through social insurance by redistributive taxation. Both methods are costly. In Section 3 we discuss the comparative statics of self-insurance with respect to changes in the cost of the welfare state, the magnitudes of the insured and the uninsured risks, and the social insurance tax rate. Section 4 derives the optimal size of the welfare (measured by the social insurance tax rate) and derives its comparative statics. Section 5 concludes.

[4]We leave it for future research to examine the implications of adding moral hazard problems to our framework. First experiments suggest, however, that this task has no trivial solution.

2 The Model

We consider a model of a welfare state *à la* Sinn (1995, 1996). To keep the exposition brief, we refer the reader to these papers for a thorough discussion of all assumptions underlying the model.

Income and shocks. Individuals are interested in the expected utility of their final income (or consumption) y. This is given by

$$Y = \tilde{N} + \tilde{M} - e - T + P \tag{1}$$

where $\tilde{M}$, $\tilde{N}$, e, T and P denote market income, other income components, expenditures for self-insurance, taxes, and transfers, respectively. Market income amounts to

$$\tilde{M} = m - \lambda(e)\theta, \tag{2}$$

where m is a deterministic maximal income and $\lambda(e)\theta$ is an adverse random shock. The random variable θ describes the unalleviated impact of the shock; θ has continuous support on $(0, m)$. The individual can dampen the full effects by undertaking costly self-insurance activities e. The damage-reducing effect of this expenditure is captured in the function $\lambda : \mathbb{R}_+ \to [0, 1]$ which is assumed to satisfy $\lambda(0) = 1$, $\lambda'(e) < 0$, and $\lambda''(e) \geq 0$.

By $\tilde{N}$ we denote income that is not feasible for, or subject to, redistributive taxation. In a broader understanding $\tilde{N}$ may be an income equivalent of any other variable which individuals find desirable.[5] We will assume that $\tilde{N}$ is stochastic; this is different from Sinn's model where θ is the only random impact. $\tilde{N}$ is continuously distributed with support in $\mathbb{R}$. We allow $\tilde{N}$ to take negative values, and even its expectation, $N := \mathbf{E}_{\tilde{N}}\tilde{N}$, might be non-positive. For obvious reasons, we will, however, assume that the distributions of θ and of $\tilde{N}$ are such that negative values for final income Y can, with probability one, not occur. For simplicity, we assume that θ and n are independent random variables.

The important difference between the risks θ and $\tilde{N}$ is that the latter cannot be hedged, neither by individual self-insurance nor by social insurance. With respect to the individuals' choice variable e, $\tilde{N}$ is an undiversifiable background risk. We assume that the individual cannot reduce his exposure to the risk to non-market income. The assumption that $\tilde{N}$ cannot at all be (socially) insured is overly strict; what is needed is that there remains a positive amount of risk even in the aggregate. In that sense, $\tilde{N}$ might represent fluctuations in income streams or living standards that hit the population as a whole and that are positively correlated across individuals. $\tilde{N}$ might also represent the undesirable outcomes of catastrophic events; think of $\tilde{N}$ as being the value of (living in) one's own house, and that the risk to $\tilde{N}$ is the danger that

[5]Sinn (1995, 1996) denotes $\tilde{N}$ as "non-market income" and sometimes interprets it as leisure.

the house might be destroyed in a hurricane or a war. Finally, think of $\tilde{N}$ income components (from self-employment, say) that cannot – or, for whatever reason, are not – be subjected to redistributive taxation.

Taxes and transfers. Taxes T in (1) are levied as a constant proportion τ of after-shock market income:

$$T = \tau(\tilde{M} - e) = \tau\left[m - \lambda(e) \cdot \theta - e\right]. \tag{3}$$

We assume that the government redistributes its tax revenues to the tax payers on a per-capita basis. Unlike Sinn, we allow for a costly welfare state. In a simplistic manner we assume that a certain fraction $k \in [0, 1]$ of tax revenues are "lost" in the welfare state, i.e., they cannot be redistributed back to taxpayers. This may reflect shadow costs of public funds, organizational and administrative cost of the welfare system, leakages in the system or governmental waste of resources. Assuming that the law of large numbers applies to θ, expected (average) tax rebates per-capita thus amount to

$$P = (1-k)\mathbf{E}_\theta T = (1-k)\tau\mathbf{E}_\theta(\tilde{M} - e) = (1-k)\tau\left[m - \lambda(e)\mu_\theta - e\right] \tag{4}$$

where $\mu_\theta = \mathbf{E}_\theta \theta > 0$ is the expected or mean market shock.
No taxes are levied on $\tilde{N}$ in our model. This can, but need not, mean that $\tilde{N}$ is a non-taxable good (leisure, say). It is, however, not really important that $\tilde{N}$ is tax-free; what matters is that $\tilde{N}$ cannot be subjected to the same sort of redistributive taxation as m.

The model in terms of mean and standard deviation. For sake of comparability with Sinn's result we will put our model in a two-parameter framework where agents' preferences only depend on the mean and the standard deviation of income. Given that individuals are identical in all respects, we can identify mean income as average income and the standard deviation of income as a measure for income inequality in our model economy.
The pre-tax standard deviation of market income is given by

$$\sigma_G := \lambda(e)\sigma_\theta. \tag{5}$$

This can be used to eliminate the effort variable and to depict mean market income as a function of income inequality:

$$\bar{\mu}(\sigma_G) := \mathbf{E}_\theta \tilde{M} - e = m - \sigma_G \frac{\mu_\theta}{\sigma_\theta} - \lambda^{-1}\left(\frac{\sigma_G}{\sigma_\theta}\right) \tag{6}$$

Sinn calls $\bar{\mu}(\sigma_G)$ the self-insurance function; due to $\lambda''(e) \geq 0$ it is concave in σ_G. With the help of (6) the mean post-tax income and the variance of post-tax income can be written as

$$\mu_Y = N + (1-\tau k)\bar{\mu}(\sigma_G) \quad \text{and} \quad \sigma_Y^2 = \sigma_N^2 + (1-\tau)^2\sigma_G^2 \tag{7}$$

where $N = \mathbf{E}\tilde{N}$ and $\sigma_N^2 = \mathrm{Var}\tilde{N}$ denote the mean and the variance of non-market income, respectively.

For simplicity we will sometimes assume that the self-insurance function is linear. This results from a linear self-insurance technology, $\lambda(e) = 1 - e/b$, where b is a positive constant, representing the amount of income that an individual must spend on self-insurance to entirely eliminate his income risk. It is plausible to assume that b is large and, in particular, exceeds the income level m the individuals can maximally obtain. Using the linear technology, the self-insurance locus, too, is linear and has negative intercept and positive slope:

$$\bar{\mu}(\sigma_G) = (m - b) + \frac{b - \mu_\theta}{\sigma_\theta} \cdot \sigma_G. \tag{8}$$

This setting (which is quite special; see the Appendix) is also discussed in Sinn (1996, Section 5) and Bird (2001).

Preferences. As mentioned before, individual preferences over income distributions (or lotteries) are assumed to be representable by a function of the mean and standard deviation of incomes only, i.e.: $U = U(\mu_Y, \sigma_Y)$. In the present framework such two-parameter functionals are a perfect substitute for the expected utility (EU) approach where preferences over income lotteries are represented as the expectation of the von-Neumann/Morgenstern utility of income,

$$\mathbf{E}u(y) = \int_a^b u(\mu_Y + \sigma_Y x) \mathrm{d}F(x) =: U(\mu_Y, \sigma_Y). \tag{9}$$

The substitutability of the two approaches results from the location-scale property of our model (see Meyer, 1987; Sinn, 1983). Clearly, properties of u and U mirror each other. Given the importance of some of these properties for the results we are going to derive, we will briefly sketch some of the analogies between the two-parameter and the EU-framework right here.

Suppose that the von-Neumann/Morgenstern utility function $u(y)$ is smooth, increasing and concave (risk aversion). This is equivalent to[6]

$$U_\mu(\mu_Y, \sigma_Y) > 0 > U_\sigma(\mu_Y, \sigma_Y) \tag{10}$$

for all (μ_Y, σ_Y). Furthermore, indifference curves for U in the (μ_Y, σ_Y)-space are convex. Define the marginal rate of substitution between μ_Y and σ_Y in U as

$$\frac{\mathrm{d}\mu_Y}{\mathrm{d}\sigma_Y} = -\frac{U_\sigma(\mu_Y, \sigma_Y)}{U_\mu(\mu_Y, \sigma_Y)} =: \alpha(\mu_Y, \sigma_Y) > 0. \tag{11}$$

The following lemma collects those correspondences between the two-parameter and the EU approach that are relevant for our results and their interpretation:

[6]Subscripts μ and σ to U or other functions of (μ_Y, σ_Y) denote partial derivatives with respect to their first and second argument.

Lemma 1 *1. (Meyer, 1987, Property 5) $u(y)$ exhibits decreasing [increasing] absolute risk aversion (i.e., $(-u''(y)/u'(y))' < [>]0$) if and only if $\alpha_\mu(\mu_Y, \sigma_Y) < [>]0$.*

2. *(Meyer, 1987, Property 7) $u(y)$ exhibits decreasing [increasing] relative risk aversion (i.e., $(-yu''(y)/u'(y))' < [>]0$) if and only if $\mu_Y \cdot \alpha_\mu + \sigma_Y \cdot \alpha_\sigma < [>]0$.*

3. *(Sinn, 1990) If u exhibits decreasing absolute risk aversion, then $\alpha_\sigma(\mu_Y, \sigma_Y) > 0$; the converse does not hold.*

4. *(Eichner and Wagener, 2001) The degree of relative risk aversion $-yu''(y)/u(y)$ in u is greater [smaller] than unity if and only if $\mu_Y U_{\mu\mu}(\mu_Y, \sigma_Y) + \sigma_Y U_{\mu\sigma}(\mu_Y, \sigma_Y) + U_\mu(\mu_Y, \sigma_Y) < [>]0$.*

Individually optimal choices. The individual chooses the self-insurance effort e such as to maximize his utility. Via (6) we rephrase the decision problem such that the pre-tax standard deviation of income is the action variable:

$$\max_{\sigma_G} U(\mu_Y, \sigma_Y) \quad \text{where} \quad \mu_Y = N + (1 - \tau k)\bar{\mu}(\sigma_G), \;\; \sigma_Y = \sqrt{\sigma_N^2 + (1-\tau)^2 \sigma_G^2}. \tag{12}$$

The first-order condition for an optimal level σ_G^* of pre-tax income risk is given by

$$\frac{\bar{\mu}'(\sigma_G)(1 - \tau k)\sigma_Y}{(1-\tau)^2 \sigma_G} - \alpha(\mu_Y, \sigma_Y) = 0. \tag{13}$$

The second-order condition is always satisfied due to the concavity of U, the concavity of $\bar{\mu}$ and the independence of θ and $\tilde{N}$. In a (μ, σ)-diagram eq. (13) graphically requires the slope of the indifference curve (the marginal rate of substitution α between mean income and income inequality) to equal the slope of the self-insurance line. With $k = 0$ (costless welfare state) and $\sigma_Y = (1-\tau)\sigma_G$ (no background risk; $\sigma_N = 0$) eq. (13) naturally coincides with the FOC derived in Sinn (1995, eq. (17)). Note that (13) requires that optimal average income always falls short of the maximal average income: $\bar{\mu}'(\sigma_G^*) > 0$. This is an implication of risk aversion (see Sinn, 1995, Proposition 1).

3 Comparative Statics

In this section we investigate the impact of changes in parameters of the model on pre-tax income risks (which is inversely related to individual self-insurance efforts), on post-tax income inequality, and on average income. All results emerge from implicit differentiation of (13), details are available upon request.

3.1 Increases in the cost of the welfare state

Let us first consider the comparative static effects of changes in the cost of redistribution. Such changes can occur for various reasons: Reduces in costs might be due to technical progress and organizational advances. Globalization is often held to raise the marginal cost of taxation and redistributive policies (see, e.g., Agell, 2000) such that, if k is interpreted as the shadow cost of public funds, deeper international integration might cause k to increase. Calculate:[7]

$$\operatorname{sgn} \frac{\partial \sigma_G^*}{\partial k} = \operatorname{sgn} \left[\alpha_\mu \cdot (\mu_Y - N) - \alpha \right], \tag{14a}$$

$$\frac{\partial \sigma_Y}{\partial k} = (1-\tau)^2 \cdot \frac{\sigma_G}{\sigma_Y} \cdot \frac{\partial \sigma_G^*}{\partial k}, \tag{14b}$$

$$\frac{\partial \mu_Y}{\partial k} = -\tau \bar{\mu}(\sigma_G) + (1 - \tau k) \cdot \bar{\mu}'(\sigma_G) \cdot \frac{\partial \sigma_G^*}{\partial k}. \tag{14c}$$

Given Lemma 1.1, we thus obtain

Proposition 1 *An increase in the costs of the welfare state leads to a decrease in the inequality both of pre-tax and of post-tax incomes, to a higher degree of self-insurance, and to a reduction in average post-tax income if, but not only if, preferences exhibit non-increasing absolute risk aversion.*

The more costly redistribution, the smaller is *ceteris paribus* the expected value of tax rebates, while the income inequality is not affected. From the individual perspective, this has two consequences: First, the reduction in the volume of redistribution has an income effect, captured by α_μ in (14a). Getting less wealthy, individuals with decreasing absolute risk aversion will thus seek for less variability in their incomes or, equivalently, increase their efforts to self-insure. Second, an increase in k makes self-insurance *less* costly: In a costless welfare state a reduction of pre-tax income inequality (i.e., an increase in self-insurance) comes at a marginal loss in expected income of $\bar{\mu}'(\sigma_G) > 0$. In a costly welfare state, the corresponding income loss is only $(1 - \tau k)\bar{\mu}'(\sigma_G)$; at the margin, a costly welfare state effectively subsidizes self-insurance. This "price effect" is captured in the MRS α on the RHS of (14a). It strenghthens the income effect, leaving the total effect of an increase in k on both pre- and post-tax inequality unambiguously negative. As self-insurance is costly (but less so than before), average income decreases; see (14c).

Eq. (14a) obviously leaves scope for non-DARA preferences, and one might thus ask for other conditions on preferences that trigger the effects identified in Proposition 1. In principle, the magnitude of relative risk aversion governs the comparative static effects. However, due to the possible non-linearity of the self-insurance function $\bar{\mu}(\sigma_G)$ and due to the presence of a background risk, these effects get blurred. One can show, however, that relative risk aversion

[7] For notational convenience we suppress the arguments of the function $\alpha(\mu_Y, \sigma_Y)$ and its derivatives.

being smaller than one suffices to generate the effects identified in Proposition 1 if the self-insurance line is linear (see (8) and there is no background risk.[8] This parallels results derived by, e.g., Meyer and Ormiston (1995) and Cheng et al. (1987), for (formally) similar comparative static problems with uncertainty and opportunity sets with linear frontiers.

3.2 Increases in the background risk

Next we consider an increase in the uninsurable background risk. The effects are formally captured in the following equations:

$$\operatorname{sgn}\frac{\partial\sigma_G^*}{\partial\sigma_N} = \operatorname{sgn}\left[\alpha-\sigma_Y\cdot\alpha_\sigma\right], \tag{15a}$$

$$\frac{\partial\mu_Y}{\partial\sigma_N} = (1-\tau k)\bar{\mu}'(\sigma_G)\frac{\partial\sigma_G^*}{\partial\sigma_N}, \tag{15b}$$

$$\frac{\partial\sigma_Y}{\partial\sigma_N} = \frac{1}{\sigma_Y}\cdot\left[\sigma_N+(1-\tau)^2\sigma_G\cdot\frac{\partial\sigma_G^*}{\partial\sigma_N}\right]. \tag{15c}$$

These give rise to

Proposition 2 • *An increase in the background risk leads to a decrease in the inequality of pre-tax incomes, to higher self-insurance efforts, and to a reduction in average income if and only if preferences satisfy $\alpha_{\sigma\sigma}>0$.*

- *The effects of an increase in the background risk on post-tax inequality are ambiguous.*

Proof: Verify that $\alpha(\mu_Y,0)=0$ for all $\mu_Y>0$. Hence $\alpha_\sigma\geq\alpha/\sigma_Y$ is equivalent to $\alpha_{\sigma\sigma}\geq 0$. Use this in (15a) and (15b) to obtain the first item of the proposition. If $\alpha_{\sigma\sigma}>0$, then (15c) contains two expressions of opposite signs. ■

The first item of this result identifies $\alpha_{\sigma\sigma}$ as the relevant property of preferences that determine the effects of changes in background risk. Its geometric translation, the slope of (μ,σ)-indifference curves being convex or concave in σ, is of limited help for an interpretation. However, $\alpha_{\sigma\sigma}$ is related to the idea of risk vulnerability that is well-known from the EU-framework (Gollier and Pratt, 1996): An agent's preferences are said to be risk-vulnerable whenever the introduction of a mean-zero background risk makes that agent behave in a more risk-averse manner. Risk-vulnerability is intuitively appealing and supported by empirical evidence (see, e.g., Guiso et al., 1996; Guiso and Jappelli, 1998). Yet it is rather complex to analyse formally (see Gollier, 2001). For the EU-framework, two independent sets of sufficient conditions for risk-vulnerability

[8] *Proof:* For $\sigma_N=0$ the FOC (13) reduces to $\alpha=\bar{\mu}'(\sigma_G)(1-\tau k)/(1-\tau)>(1-\tau k)\bar{\mu}_G/\sigma_Y$ where we assume a linear self-insurance technology. Hence, the square-bracketed terms in (14a) are smaller than $(\mu_Y-N)/\sigma_Y\cdot[-1+\sigma_Y\alpha_\mu]$. The square-bracketed expression itself is smaller than $-1+[\mu_Y U_{\mu\mu}+\sigma_Y U_{\mu\sigma}]/U_\mu$. For relative risk aversion not larger than unity, this expression is non-positive according to Lemma 1.4.

have been identified: (i) the Arrow-Pratt index of absolute risk aversion being decreasing and convex in income; and (ii) standardness (i.e., the combination of decreasing absolute risk aversion and decreasing absolute prudence). Eichner and Wagener (2001) argue that, while these sets of conditions are valid in the (μ, σ)-approach when stochastics are Gaussian, they generally do not apply in that approach, the reason being that increases in background risks in that approach are only representable through increases in the variance of a background variable – which, according to the classical result of Rothschild and Stiglitz (1970), is *not* equivalent to the standard notion of a general increase in risk (i.e., a change in the stochastics which every risk-averse person would disapprove). However, Eichner and Wagener (2001) also show that increases in the variance of a background risk make individuals with (μ, σ)-preferences behave in a more risk-averse way if and only if $\alpha_{\sigma\sigma} > 0$. *In that sense*, we can (and will) will say that two-parameter preferences are *risk-vulnerable* whenever $\alpha_{\sigma\sigma}(\mu, \sigma) > 0$. Hence, a less formal version of Proposition 2 reads:

Proposition 2' *An increase in the background risk leads to a decrease in the inequality of pre-tax incomes, to a higher self-insurance efforts, and to a reduction in average income if and only if preferences are risk-vulnerable. The effects on post-tax inequality are ambiguous.*

In that version, Proposition 2 has a simple interpretation: Risk-vulnerability, meaning that agents act in a more risk-averse manner when background uncertainties prevail, leads to more self-insurance (i.e., less risk-taking) if uninsurable risks become more volatile (see Dionne and Eeckhoudt, 1985).

3.3 Increase in the insurable risk

Now consider an increase of the insurable risk, represented by a raise in σ_θ. Pre-tax income inequality σ_G being itself a function of σ_θ, (see (5)), correct comparative static results respect to σ_θ can only be derived via the detour of phrasing the decision problem (12) in terms of self-insurance activities e. In the Appendix we derive the following comparative static results:

$$\operatorname{sgn} \frac{\partial e^*}{\partial \sigma_\theta} = -\operatorname{sgn} \frac{\partial \mu_Y}{\partial \sigma_\theta} = \operatorname{sgn} \left[\alpha \cdot (\sigma_Y^2 + \sigma_N^2) + \alpha_\sigma \cdot (\sigma_Y^2 - \sigma_N^2) \right], \tag{16a}$$

$$\operatorname{sgn} \frac{\partial \sigma_G^*}{\partial \sigma_\theta} = \operatorname{sgn} \frac{\partial \sigma_Y}{\partial \sigma_\theta} = \operatorname{sgn} \left[\lambda(e) + \lambda'(e) \sigma_\theta \cdot \frac{\partial e^*}{\partial \sigma_\theta} \right]. \tag{16b}$$

From this we obtain

Proposition 3 • *An increase in the insurable risk leads to a higher degree of self-insurance and to a reduction in average post-tax income if, but not only if, preferences exhibit non-increasing absolute risk aversion.*

- *Post-tax inequality varies in the same direction as pre-tax inequality, but these changes are ambiguous in sign.*

Similar to the case of a background risk, an increase in the insured risk raises the variance of income. Unlike the background risk, individuals can, however, now reduce the riskiness of pre-tax income through expanding self-insurance. That is what risk-averse agents will indeed do. Obviously, decreasing the variance of income comes at the cost of decreasing average incomes which is the second message of Proposition 3. It remains unclear whether the expansion of self-insurance more than compensates for the increase in σ_θ such that the overall variance (or, for that reason, inequality) of income indeed declines.

3.4 Increases in the tax rate

Next we focus on the comparative static effects of the tax rate. They are characterized by the following equations (for details see the Appendix):

$$\operatorname{sgn}\frac{\partial \sigma_G^*}{\partial \tau} = \operatorname{sgn}\left[\alpha \cdot \left(\frac{\sigma_N^2}{\sigma_Y^2} + \frac{1-k}{1-\tau k}\right) + \alpha_\sigma \cdot \frac{\sigma_Y^2 - \sigma_N^2}{\sigma_Y} + \alpha_\mu \cdot k(1-\tau)(\mu_Y - N)\right], \tag{17a}$$

$$\frac{\partial \mu_Y}{\partial \tau} = -k\bar{\mu}(\sigma_G) + (1-\tau k)\bar{\mu}'(\sigma_G)\frac{\partial \sigma_G^*}{\partial \tau}, \tag{17b}$$

$$\operatorname{sgn}\frac{\partial \sigma_Y}{\partial \tau} = \operatorname{sgn}\left[-\sigma_G + (1-\tau)\cdot\frac{\partial \sigma_G^*}{\partial \tau}\right]. \tag{17c}$$

To better understand these expressions we go through some special cases.

Zero cost of the welfare state, no background risk. This is the original case from Sinn (1995, 1996). Putting $k = 0$ and $\sigma_N = 0$ (and, thus, $\sigma_Y = (1-\tau)\sigma_G$) above, we obtain:

$$\operatorname{sgn}\frac{\partial \mu_Y}{\partial \tau} = \operatorname{sgn}\frac{\partial \sigma_G^*}{\partial \tau} = \operatorname{sgn}\left[\alpha + \sigma_Y \alpha_\sigma\right] > 0, \tag{18a}$$

$$\operatorname{sgn}\frac{\partial \sigma_Y}{\partial \tau} = \operatorname{sgn}\left[-\sigma_G + (1-\tau)\cdot\frac{\partial \sigma_G^*}{\partial \tau}\right]. \tag{18b}$$

Eq. (18a) decomposes the effect of a tax increase into two parts: First, increases in τ reduce the standard deviation of income at any given level of average income, thereby increasing the marginal return to risk-taking: The slope of the redistribution line (which in the optimum equals α) gets steeper. Let us call this the *return effect*. Second, the risk reduction caused by increases in τ itself makes the individual less risk-averse and reduces their willingness-to-pay for self-insurance. This is the essence of $\alpha_\sigma > 0$ which itself will always be satisfied when preferences are DARA (see Lemma 1.1). Let us call this the *risk-aversion effect* of a tax increase. Both effects work into the same direction.

This reproduces Proposition 2 in Sinn (1995) which states that, in a costless welfare state without background uncertainties, increases in the volume of redistribution raise the pre-tax inequality of incomes, crowd out self-insurance activities, and raise average income whenever preferences exhibit decreasing absolute risk aversion. The effect on post-tax inequality is unclear.

Zero cost of the welfare state, background risk. Allowing for background uncertainty ($\sigma_N > 0$) in a still costless welfare state ($k = 0$) changes (18a) into

$$\operatorname{sgn} \frac{\partial \mu_Y}{\partial \tau} = \operatorname{sgn} \frac{\partial \sigma_G^*}{\partial \tau} = \operatorname{sgn} \left[\alpha \cdot \left(\frac{\sigma_N^2}{\sigma_Y^2} + 1 \right) + \alpha_\sigma \cdot \frac{\sigma_Y^2 - \sigma_N^2}{\sigma_Y} \right], \tag{19}$$

while leaving the appearance of (18b) unchanged. Unsurprisingly, the crowding-out effect of the welfare state on self-insurance also prevails in the presence of background risks. Comparing (19) and (18a) one can, however, discover two differences. Both originate from the fact that a background risk weakens the (relative) risk reduction caused by an increase in the tax rate: only a part of income variability, namely the insurable risk is diminished. With a relatively smaller risk reduction, the decline in risk aversion (captured in α_σ) turns out to be smaller, which weakens the risk-aversion effect relative to the case of no background risk. However, the return effect is strengthened since the redistribution line gets steeper to an extent that surpasses the corresponding increase in steepness in the case of no background risk.[9]

Positive cost of the welfare state, no background risk. Now let $k > 0$ and $\sigma_N = 0$. Then (17a) becomes:

$$\operatorname{sgn} \frac{\partial \sigma_G^*}{\partial \tau} = \operatorname{sgn} \left[\alpha \cdot \frac{1-k}{1-\tau k} + \alpha_\sigma \cdot \sigma_Y + \alpha_\mu \cdot \frac{k(1-\tau)}{1-\tau k} \cdot (\mu_Y - N) \right]. \tag{20}$$

Compared to the previous cases, a negative *income effect* ($\alpha_\mu < 0$) enters the comparative statics: With a costly welfare state, tax rate increases involve a loss in expected income since tax rebates do not fully meet expected tax liabilities. With DARA-preferences such a reduction in income makes individuals more risk averse, thereby encouraging their self-insurance activities and (*ceteris paribus*) reducing pre-tax income risk. Further note that the return effect (the one associated with α) is weakened, as compared to the zero-cost scenario; the cofactor $(1 - k)/(1 - \tau k)$ is smaller than one: A tax increase leads to a smaller increase in the slope of the

[9] To see this formally, suppose that σ_G is an individual optimum with a corresponding $\bar{\mu}(\sigma_G)$. For $k = 0$, the slope of the redistribution line through σ_G is $\bar{\mu}'(\sigma_G)\sigma_Y/((1-\tau)^2\sigma_G)$ from (13). Now consider a change in τ. That will change the slope of the redistribution line at σ_G by

$$\bar{\mu}'(\sigma_G) \frac{\sigma_Y}{\sigma_G(1-\tau)^3} \cdot \left(\frac{\sigma_N^2}{\sigma_Y^2} + 1 \right) > 0.$$

As σ_Y is strictly increasing σ_N, the whole expression is strictly increasing in σ_N. Hence, the increase in the slope of the redistribution line that is caused by an increase in τ is larger the greater is σ_N. This also holds for a comparison of $\sigma_N > 0$ and $\sigma_N = 0$.

redistribution line (which measures the marginal returns of risk-taking in terms of expected income) the larger is k.[10]

With DARA (20) consists of two non-negative and one non-positive terms, the overall effect thus being *a priori* unclear. To get a handle on this let us first suppose that k approaches one — which means that no redistribution takes place since the government "wastes" all its revenues or spends them on (unmodelled) purposes other than redistribution. Then:

$$\operatorname{sgn} \frac{\partial \sigma_G^*}{\partial \tau} = \operatorname{sgn} \left[\alpha_\sigma \cdot \sigma_Y + \alpha_\mu \cdot (\mu_Y - N) \right]. \tag{21}$$

Suppose $N = 0$. Following Lemma 1.2, (21) is positive [negative] if and only if preferences satisfy increasing [decreasing] relative risk aversion. If $N < 0$ [$N > 0$], (21) will be negative [positive] whenever preferences exhibit constant or decreasing [constant or increasing] relative risk aversion. For $N > 0$, decreasing relative risk aversion (which necessitates that absolute risk aversion decreases sufficiently fast) is a necessary condition for (21) to be negative.[11] These observations are (in a formal sense) similar to findings by Sandmo (1971) and Briys and Eeckhoudt (1985) who investigate the impact of a profit tax rate on output decisions of a competitive firm. For an entrepreneur, the profit tax is a costly way to reduce the volatility of his net income; it thus exerts a similar effect as a social security tax τ in our model. In both cases, income and risk-aversion effects run into opposite directions, and the weight the decision maker attaches to them is determined by the degree of relative risk aversion.

For $k < 1$ the income effect, which outweighs the risk-aversion effect whenever preferences exhibit decreasing relative risk aversion, gets less important since parts of the taxes are rebated. For lower values of k eventually even individuals with decreasing relative risk aversion will adjust their self-insurance efforts negatively to an increase in τ. However, and we keep this as the main message of this paragraph, in a costly welfare state it cannot be taken for granted that a higher degree of redistributive taxation always goes along with a reduction of self-insurance activities by the taxed individuals.

Positive cost of the welfare state plus background risk. We now combine our observations to analyse (17a) in total. Comparing Sinn's scenario ($k = \sigma_N = 0$) with the case $k, \sigma_N > 0$ we identify

- a new income effect (represented by α_μ) which, in isolation, tends to boost self-insurance;
- a smaller risk-aversion effect (represented by α_σ) which, in isolation, lowers the extent to which social insurance crowds out self-insurance; and

[10] The formal argument is similar to the one in footnote 9: For $k > 0 = \sigma_N$, the slope of the redistribution line through σ_G is $\bar{\mu}'(\sigma_G)(1 - \tau k)/(1 - \tau)$ from (13). A marginal increase in τ increases that slope by magnitude of $\bar{\mu}'(\sigma_G)(1 - k)/(1 - \tau)^2$ — which is decreasing in k.

[11] Employing a measure of *partial* relative risk aversion in the sense of Menezes and Hanson (1970) one can obtain sufficient conditions for $\partial \sigma_G^* / \partial \tau < 0$ too. See Eichner and Wagener (2001) for such a reasoning.

- an ambiguous change in the return effect (represented by α) with no clearcut impact on tax-induced crowding-out.

Obviously, the sum of these effects is unclear both in sign and in size; both will, among others, depend on the magnitudes of k and σ_N. While our previous arguments suggest that a reversal of Sinn's finding that social insurance crowds out private insurance seems unlikely (though not impossible), the scale of crowding-out might well become smaller. The following result sums up the main aspects of our previous discussion:

Proposition 4

- *In a costless welfare state ($k = 0$), a higher redistributive tax rate always goes along with less self-insurance and higher average incomes, regardless of the existence or non-existence of a background risk.*
- *In a costly welfare state ($k > 0$) and in the absence of a background risk, increases in the tax rate might encourage self-insurance and thus lead to a decreases in both pre- and post-tax income inequality as well as in average income. A necessary condition for this to happen is that preferences satisfy decreasing relative risk aversion. Typically, the effect of a decreasing income inequality is more likely the higher are the cost of the welfare state, as measured by k.*

4 The preferred size of the welfare state

4.1 General results

In this section we derive the optimal size of the welfare state, measured in terms of the magnitude of τ, the percentage of market incomes that is subject to redistribution (or, for that reason, social insurance).

Denote indirect utility by

$$U^* = U(\mu_Y^*, \sigma_Y^*) = U\left(N + (1-\tau k)\bar{\mu}(\sigma_G^*), \sqrt{\sigma_N^2 + (1-\tau)^2(\sigma_G^*)^2}\right) \tag{22}$$

where stars indicate that the individuals' optimal choices of pre-tax income risk are considered. Invoking the envelope theorem, one gets

$$\frac{\partial U^*}{\partial \tau} = -k\bar{\mu}(\sigma_G^*) \cdot U_\mu(\mu_Y^*, \sigma_Y^*) - \frac{(1-\tau)\sigma_G^{*2}}{\sigma_Y^*} \cdot U_\sigma(\mu_Y^*, \sigma_Y^*). \tag{23}$$

For a costless welfare state with $k = 0$ this obviously becomes

$$\frac{\partial U^*}{\partial \tau} = -\frac{(1-\tau)\sigma_G^{*2}}{\sigma_Y^*} \cdot U_\sigma(\mu_Y^*, \sigma_Y^*) > 0$$

for all $0 \leq \tau < 1$. Hence, if redistribution is costless, then full redistribution $\tau^* = 1$ is optimal. This result has already been obtained by Sinn (1995, p. 520).

We will next show that for a costly welfare state ($k > 0$), full redistribution ($\tau = 1$) can *never* be optimal. To see this we evaluate (23) at $\tau = 1$:

$$\left.\frac{\partial U^*}{\partial \tau}\right|_{\tau=1} = -k\bar{\mu}(\sigma_G^*) \cdot U_\mu(\mu_Y^*, \sigma_Y^*)$$

which is negative for all $k > 0$. The finding that full insurance is optimal when insurance is costless and thus can be organized in an actuarially fair way ($k = 0$ implies $\tau = 1$) while it is never optimal whenever insurance is costly and actuarially less than fair ($k > 0$ implies $\tau < 1$) was some decades ago made by Mossin (1968) and Smith (1968) in the context of private insurance demand. Here it is transferred to the public sphere.

We next discuss whether or not *some* social insurance is desirable if the welfare state is costly. Evaluate (23) at $\tau = 0$ and use the FOC (13) to obtain:

$$\begin{aligned}\left.\frac{\partial U^*}{\partial \tau}\right|_{\tau=0} &= -k\bar{\mu} \cdot U_\mu - \frac{\sigma_G^{*2}}{\sigma_Y^*} U_\sigma \\ &= U_\mu \cdot \left(\sigma_G^* \bar{\mu}'(\sigma_G^*) - k\bar{\mu}(\sigma_G^*)\right) \geq U_\mu \cdot \left(\sigma_G^* \bar{\mu}'(\sigma_G^*) - \bar{\mu}(\sigma_G^*)\right).\end{aligned} \tag{24}$$

From the first expression, this expression is positive whenever k is equal or close to zero. It is furthermore positive for *all* $k \in [0, 1]$ if the self-insurance locus is linear; cf. (8). Furthermore, we obtain $\sigma_G^* \bar{\mu}'(\sigma_G^*) - \bar{\mu}(\sigma_G^*) > 0$ as a sufficient condition. Since $\bar{\mu}(\sigma_G)$ is concave this condition can neither be ensured nor rejected globally. It will be satisfied, however, if the self-insurance line is sufficiently steep, i.e., self-insurance is, in terms of income foregone, highly costly for the individual. To sum up:

Proposition 5 *1. Full redistribution ($\tau^* = 1$) is optimal if and only if the welfare state operates without cost. Otherwise less than full redistribution ($\tau^* < 1$) will be optimal.*

2. Some insurance ($\tau^* > 0$) is always optimal whenever (i) welfare cost k are small, or (ii) the self-insurance locus is linear, or (iii) self-insurance is sufficiently costly.

Note that Proposition 5 holds irrespective of the magnitude of the insurable risk and irrespective of whether a background risk prevails or not. Together, for $k > 0$ the two items of Proposition 5 imply that there will typically (i.e., if (24) is positive) exist an optimal tax rate τ^* in the interior of the unit interval: $0 < \tau^* < 1$.

4.2 Comparative statics for τ^*

The optimal tax rate τ^* depends on the exogenous parameters of our model. In particular, we are interested in the properties of τ^* as a function of the cost k of the welfare state, the size of the background risk σ_N, and the magnitude of the insurable risk.

Proposition 6 *1. Let $k = 0$. Then a small increase in k optimally leads to a smaller welfare state while neither changes in the insurable risk nor in the background background risk affect the optimality of full redistribution.*

2. Let $k > 0$. Suppose that the self-insurance function is strictly concave and that pre-tax income inequality increases in the tax rate: $\partial\sigma_G/\partial\tau > 0$. Then, under plausible conditions (see below):

- *The effect of an increase in the cost of redistribution on the optimal size of the welfare state is generally ambiguous.*
- *The optimal size of the welfare state is larger the higher are background risks if and only if preferences are risk-vulnerable.*
- *The optimal size of the welfare state is optimally larger the greater is the risk that it insures if and only if an increase in the insured risk lowers pre-tax inequality.*

Proof: The case $k = 0$ is immediate from Proposition 5, first item. Next suppose that $k > 0$. If $\tau^* \in (0, 1)$ solves $\partial U^*/\partial\tau = 0$ then it satisfies

$$\begin{aligned} 0 &= -k\bar{\mu}(\sigma_G^*) \cdot U_\mu(\mu_Y^*, \sigma_Y^*) - \frac{(1-\tau)\sigma_G^{*2}}{\sigma_Y^*} \cdot U_\sigma(\mu_Y^*, \sigma_Y^*) \\ &= U_\mu(\mu_Y^*, \sigma_Y^*) \cdot \left[-k\bar{\mu}(\sigma_G^*) + \frac{\sigma_G^*(1-\tau k)}{1-\tau} \cdot \bar{\mu}'(\sigma_G^*)\right], \end{aligned} \tag{25}$$

from (23) and the FOC (13). As $U_\mu > 0$, this is equivalent to:

$$\psi(\tau, k, \sigma_N, \sigma_\theta) := -(1-\tau)k\bar{\mu}(\sigma_G^*) + \sigma_G^*(1-\tau k)\bar{\mu}'(\sigma_G^*) = 0.$$

We have[12] $\partial\tau^*/\partial k = -\psi_k/\psi_\tau$, $\partial\tau^*/\partial\sigma_N = -\psi_k/\psi_{\sigma_N}$, and $\partial\tau^*/\partial\sigma_\theta = -\psi_k/\psi_{\sigma_\theta}$ where:

$$\psi_\tau(\tau, k, \sigma_N, \sigma_\theta) = k \cdot (\bar{\mu} - \sigma_G^*\bar{\mu}') + \frac{\partial\sigma_G^*}{\partial\tau} \cdot \left[(1-k)\bar{\mu}' + \sigma_G^*(1-\tau k)\bar{\mu}''\right], \tag{26a}$$

$$\psi_k(\tau, k, \sigma_N, \sigma_\theta) = -(1-\tau)\bar{\mu} - \tau\sigma_G^*\bar{\mu}' + \frac{\partial\sigma_G^*}{\partial k} \cdot \left[(1-k)\bar{\mu}' + \sigma_G^*(1-\tau k)\bar{\mu}''\right], \tag{26b}$$

$$\psi_{\sigma_N}(\tau, k, \sigma_N, \sigma_\theta) = \frac{\partial\sigma_G^*}{\partial\sigma_N} \cdot \left[(1-k)\bar{\mu}' + \sigma_G^*(1-\tau k)\bar{\mu}''\right], \tag{26c}$$

$$\psi_{\sigma_\theta}(\tau, k, \sigma_N, \sigma_\theta) = \frac{\partial\sigma_G^*}{\partial\sigma_\theta} \cdot \left[(1-k)\bar{\mu}' + \sigma_G^*(1-\tau k)\bar{\mu}''\right]. \tag{26d}$$

As τ^* is a maximizer, we must (locally) have $\psi_\tau < 0$.[13] As we assume that $\partial\sigma_G^*/\partial\tau > 0$, this requires, for k sufficiently small, that $[(1-k)\bar{\mu}' + \sigma_G^*(1-\tau k)\bar{\mu}'']$ is negative. The same

[12] The reason why we distinguish the cases $k = 0$ and $k > 0$ is that for $k = 0$ (and, thus, $\tau^* = 1$) the function ψ is not well-defined. In particular, the FOC (13) cannot be invoked as individuals will optimally choose the corner solution $e = 0$.

[13] It does not matter for this argument that we only consider the function ψ rather than the "full" FOC (25) which is a multiple of ψ. Generally the function $U^* = U^*(\tau, k, \sigma_N)$ need not be monotonic in τ. In particular, several local maxima and minima might exist (for given k, σ_N, and $sigma_\theta$). One of the local maxima is, however, the global one.

holds for arbitrary k whenever $\bar{\mu}(\sigma_G^*) \geq \sigma_G^* \bar{\mu}'(\sigma_G^*)$. Finally, if k is large (i.e., close to one) then $[(1-k)\bar{\mu}' + \sigma_G^*(1-\tau k)\bar{\mu}'']$ will always be negative due to the strict concavity of $\bar{\mu}(\sigma_G)$. This provided, we get

- from the fact that $\partial\sigma_G^*/\partial k < 0$ (see Proposition 1) that (26b) cannot be signed unambiguously;
- that $\text{sgn}\,(\partial\tau^*/\partial\sigma_N) = -\text{sgn}\,(\partial\sigma_G^*/\partial\sigma_N)$ which is positive if and only if preferences are risk-vulnerable (see Proposition 2);
- that $\text{sgn}\,(\partial\tau^*/\partial\sigma_\theta) = -\text{sgn}\,(\partial\sigma_G^*/\partial\sigma_\theta)$. ■

While Proposition 6.1 does not require an explanation, the three cases of the second item are of greater interest.

An increase in the cost parameter k has two welfare effects: First, it renders redistribution more costly to society, calling for a reduction in the volume of redistribution and, hence, for lowering τ^*. Second, however, we know from Proposition 1 that individuals increase their self-insurance activities as a response to a more costly welfare state. Self-insurance reduces pre-tax income volatility but also has a negative wealth impact and thus increases risk aversion. This in turn calls for a higher degree of redistribution by the state. It is unclear which of the two welfare effects dominates.

An increase in the background risk calls, given that preferences are risk-vulnerable, for an increase in the size of the welfare state. First recall that risk-vulnerable individuals react with an increase of self-insurance on a higher background volatility (see Proposition 2). This decreases the volatility σ_G of pre-tax incomes. By the desire for a higher tax rate, this effect is partially offset since, following an increase in τ, individuals will reduce self-insurance and let the volatility of pre-tax incomes rise again (recall that $\partial\sigma_G^*/\partial\tau > 0$). Hence, the call for a higher τ springs off the wish to substitute individual insurance for social insurance. This motive also can be seen from the FOC for an optimal tax rate τ^* in (25): This condition requires that, in terms of average income, the marginal cost of social insurance, $k\bar{m}u_G$, have to equal the marginal benefits, $\bar{\mu}'\sigma_G(1-\tau k)/1-\tau)$. These benefits result from making private insurance marginally cheaper. Now consider again a change in the background risk σ_N. As a response, individuals lower σ_G, thereby rendering self-insurance marginally more expensive (the self-insurance function is strictly concave). Given that, the marginal benefits from social insurance also rise, leading to a higher social insurance rate τ.

The same argument that basically also drives the comparative statics with respect to the insurable risk. However, here the story might well go into the opposite direction since it is unclear whether an increase in σ_θ increases or lowers pre-tax income inequality σ_G (see Proposition 3).

If σ_G decreases, the previous argument fully applies. If $\partial\sigma_G/\partial\sigma_\theta > 0$, individual reactions effectively cause marginal cost of self-insurance to decrease and thereby make social insurance less beneficial. Hence the desire to reduce τ^*. As we discussed in Section 3.3, whether σ_G will in- or decrease upon a change in the insurable risk depends on whether the marginal willlingness-to-pay for self-insurance is sufficiently large.

Proposition 6.2 depends on several assumptions: First it is required that pre-tax income volatility is an increasing function of the tax rate. Our discussion in Section 3.4 showed that this is indeed the dominant case which will, in particular, always prevail when the cost of the welfare state are not too large. Yet, it deserves mention that the comparative statics of the optimal τ will look quite different if this assumption is not met.

Next, in the proposition we loosely refer to additional "plausible" assumptions. By this we mean that the term $(1-k)\bar{\mu}' + \sigma_G^*(1-\tau k)\bar{\mu}''$, which plays an important role in the proof, is negative. As is explained in the proof, this condition is likely to be met. In particular, it holds if k is close to zero or one, or if self-insurance is sufficiently costly. The latter condition also appears as condition (iii) in Proposition 5.2 where it ensures that *some* social insurance is desirable.

Proposition 6.2 indeed only holds for strictly concave self-insurance functions ($\bar{\mu}''(\sigma_G) < 0$). The linear case is more complicated, and we relegate its discussion to the appendix. The most striking oberservation there is that the welfare state optimally crowds-out private self-insurance entirely: $e^*(\tau^*) = 0$; all insurance is provided by the state (see Proposition 6'). This is excluded, however, with strictly concave self-insurance functions.

Finally note that Proposition 6 reveals a remarkable asymmetry: If an uninsurable (or uninsured) risk increases, the welfare state should unambiguously expand, given that preferences exhibit the plausible property of risk-vulnerability. If, however, the insured risk becomes larger, the recommendation for the welfare state is less clear. It might well happen that the welfare state should be cut back. This will be the case when individuals respond to changes in insurable risks with increases in their self-insurance effort that are large enough to more than compensate for the increase in the volatility of their pre-tax incomes.

5 Conclusions

We analyzed the changing role of costly social insurance (or, what is the same here, redistributive taxation) in a riskier world. Some catchy phrases summarize our findings:

- A more costly or less efficient welfare state makes the society poorer, but more equal.
- A higher background risk does not necessarily make societies more unequal.
- Social insurance might, but typcially does not, crowd-in private insurance in a costly welfare state.

- The welfare state is not necessarily optimally larger the less costly it is.
- The welfare state need not optimaly expand with the magnitude of the risks that it does insure.
- Yet, the welfare state should optimally expand with the magnitude of those risk (the "background risk") that it does *not* insure.

Similar to Sinn (1995), we thus find that the welfare state might possess features that conventional wisdom would not ascribe to it. However, to achieve a more complete account of the welfare state, our model requires – and allows for – extensions into various directions: First, one might give up the assumption that individuals fully see through the redistributive process; similar to Sinn (1995, 1996), moral hazard should be included. Second, one might wish to endogenize the insurability property of certain risks rather than taking for granted that some risks are insurable while others are not. Third, one might try to test our predictions empirically. Bird (2001) shows that the idea by Sinn and others that welfare state induces risk-taking is corroborated by cross-national data. It would now be interesting to see whether and how the role of the welfare state changes between periods of different riskiness.

Appendix

Derivation of (16a) and (16b)

In terms of self-insurance effort e the decision problem (12) reads as:

$$\max_e U\left(N + (1-\tau k)\left[m - \lambda(e)\cdot\mu_\theta - e\right], \sqrt{\sigma_N^2 + (1-\tau)^2\lambda(e)^2\sigma_\theta^2}\right). \tag{27}$$

The first-order condition for an optimal level e^* is:

$$\frac{(1-\tau k)(\lambda'(e)\cdot\mu_\theta + 1)\sigma_Y}{(1-\tau)^2\lambda(e)\lambda'(e)\sigma_\theta^2} + \alpha(\mu_Y,\sigma_Y) = 0. \tag{28}$$

Implicit differentiation of (28) yields:

$$\begin{aligned}
\operatorname{sgn}\frac{\partial e^*}{\partial\sigma_\theta} &= \operatorname{sgn}\left[\alpha\cdot(\sigma_Y^2+\sigma_N^2) + \alpha_\sigma\cdot(\sigma_Y^2-\sigma_N^2)\right], \\
\operatorname{sgn}\frac{\partial\mu_Y}{\partial\sigma_\theta} &= -(1-\tau k)\left[\lambda'(e)\cdot\mu_\theta + 1\right]\frac{\partial e^*}{\partial\sigma_\theta}, \\
\operatorname{sgn}\frac{\partial\sigma_G^*}{\partial\sigma_\theta} &= \lambda(e) + \lambda'(e)\sigma_\theta\cdot\frac{\partial e^*}{\partial\sigma_\theta}, \\
\operatorname{sgn}\frac{\partial\sigma_Y}{\partial\sigma_\theta} &= \frac{(1-\tau)^2\lambda(e)\sigma_\theta}{\sigma_Y}\left[\lambda(e) + \lambda'(e)\sigma_\theta\cdot\frac{\partial e^*}{\partial\sigma_\theta}\right].
\end{aligned}$$

Observe that (28) implies $\lambda'(e)\cdot\mu_\theta + 1 > 0$. Then (16a) and (16b) are straightforward. ■

Derivation of (17a)

Denote the LHS of (13) by ϕ. Then, $\partial\sigma_G^*/\partial\tau = -\phi_\tau/\phi_{\sigma_G}$. The denominator is negative due to the SOC. Thus the sign of $\partial\sigma_G^*/\partial\tau$ is equal to that of ϕ_τ. Calculate that:

$$\begin{aligned}
\frac{\partial}{\partial\tau}\left(\frac{\bar\mu'(\sigma_G)(1-\tau k)\sigma_Y}{(1-\tau)^2\sigma_G}\right) &= \frac{\bar\mu'(\sigma_G)}{\sigma_G(1-\tau)^3}\left[-(1-\tau)k\sigma_Y - \frac{(1-\tau k)(1-\tau)^2\sigma_G^2}{\sigma_Y} + 2(1-\tau k)\sigma_Y\right] \\
&= \frac{\bar\mu'(\sigma_G)\sigma_Y}{\sigma_G(1-\tau)^3}\left[1-k+(1-\tau k)\cdot\frac{\sigma_N^2}{\sigma_Y^2}\right] \\
&= \frac{\alpha}{1-\tau}\cdot\left[\frac{1-k}{1-\tau k}+\frac{\sigma_N^2}{\sigma_Y^2}\right] \qquad (29)
\end{aligned}$$

where we used (7) and (13). Furthermore,

$$\frac{\partial\alpha}{\partial\tau} = -k\bar\mu\cdot\alpha_\mu - \frac{(1-\tau)\sigma_G^2}{\sigma_Y}\cdot\alpha_\sigma. \qquad (30)$$

Subtracting (30) from (29) yields ϕ_τ, and (17a) follows from multiplication by $(1-\tau)$. ■

The optimal welfare state when the self-insurance technology is linear

In Proposition 6 we assume that the self-insurance locus $\bar\mu(\sigma_G)$ is globally concave. We also mention that the linear case is indeed different. Here is a brief discussion of this issue. Throughout we assume $k > 0$.

The concavity assumption ensures that individuals will always engage in *some* self-insurance whenever $\tau < 1$: $e^*(\tau) > 0$. As Sinn (1995, p. 503) shows, the self-insurance locus $\bar\mu(\sigma_G)$ is strictly concave whenever the self-insurance technology $\lambda(e)$ is strictly convex. Furthermore it has an interior maximum and the point (μ_Y^0, σ_Y^0) associated with $e = 0$ is always in the decreasing part of the self-insurance locus: $\bar\mu'(\sigma_Y^0) < 0$. This ensures that $e = 0$ can never be an optimal choice. For $\tau = 1$, however, the optimal individual choice for self-insurance obviously is $e^* = 0$, as the insurable risk is entirely washed out by the state.

With a linear self-insurance technology, the case $e^* = 0$ can also occur for $\tau < 1$. This is due to the fact that linear self-insurance loci attain their maxima in the (σ_Y, μ_Y)-diagram at $e = 0$. In this case the FOC (13) does not hold such that the technique used in the proof of Proposition 6 is not applicable. Instead the following approach is viable (details are available from the authors): First, one can show that there exists τ_0, strictly smaller than unity, such that $e^*(\tau) = 0$ for all $\tau \geq \tau_0$ while $e^*(\tau) > 0$ else.[14] One can also show that the utility level at τ_0 is higher than at all smaller tax rates: $U^*(\tau_0, \cdot) > U^*(\tau, \cdot)$ for all $\tau < \tau_0$. Hence, the most preferred tax rate τ^* must lie in the interval $[\tau_0, 1)$; the case $\tau^* = 1$ can be excluded from Proposition 5. Next define

$$C := \left\{(\sigma_Y, \mu_Y)\,\middle|\,\sigma_Y = \sqrt{\sigma_N^2 + (1-\tau)^2\sigma_\theta^2},\ \mu_Y = N + (1-\tau k)(m-\mu_\theta),\ 0 < \tau < 1\right\}$$

[14]The value of τ_0 depends on the other model parameters which we suppress here.

as the locus of all (σ_Y, μ_Y)-pairs that obtain for $e = 0$ when we let the tax rate vary (also cf. (27) and recall that $\lambda(0) = 1$). In a (σ, μ)-diagram, the locus C is increasing and strictly concave [linear] for $\sigma_N > 0$ [$\sigma_N = 0$]. Then the optimal tax rate τ^* can be found by solving $\max_\tau \{U(\mu_Y, \sigma_Y) \,|(\sigma_Y, \mu_Y) \in C\}$. The corresponding FOC is given by:

$$\alpha - \frac{\sigma_Y k(m - \mu_\theta)}{(1-\tau)\sigma_\theta^2} = 0.$$

Implicit differentiation yields, after some manipulations:

$$\begin{aligned}
\operatorname{sgn} \frac{\partial \tau^*}{\partial k} &= \operatorname{sgn} \left[-\alpha - \tau k(m - \mu_\theta)\alpha_\mu\right]; \\
\operatorname{sgn} \frac{\partial \tau^*}{\partial \sigma_\theta} &= \operatorname{sgn} \left[\alpha \cdot \left(1 + \frac{2\sigma_N^2}{(1-\tau)^2\sigma_\theta^2}\right) + \sigma_Y \cdot \alpha_\sigma\right]; \\
\operatorname{sgn} \frac{\partial \tau^*}{\partial \sigma_N} &= \operatorname{sgn} \left[-\alpha + \sigma_Y \cdot \alpha_\sigma\right].
\end{aligned}$$

Invoking familiar arguments, this yields

Proposition 6' *Suppose that the self-insurance technology $\lambda(e)$ is linear.*

- *The welfare state optimally crowds out private self-insurance entirely: $e^*(\tau^*) = 0$.*
- *The optimal size of the welfare state*
  - *increasesin the size of the insurable risk: $\partial\tau^*/\partial\sigma_\theta > 0$;*
  - *increases in the size of the background risk if and only if preferences are risk-vulnerable: $\partial\tau^*/\partial\sigma_N > 0$ iff $\alpha_{\sigma\sigma} > 0$.*
  - *varies ambiguously with an increase in the cost parameter k.*

References

Adam, B., Beck, U., van Loon, J., 2000. The risk society and beyond. Sage Publications, London.

Agell, J., 2000. On the determinants of labour market institutions: Rent-sharing versus social insurance. CESifo Working Paper No. 384, Munich.

Barr, N., 1998. The economics of the welfare state, 3rd Edition. Oxford University Press, Oxford/New York.

Beck, U., 1992. Risk society. Sage Publications, London.

Bird, E.J., 2001. Does the welfare state induce risk-taking? Journal of Public Economics 80, 357–383.

Briys, E., Eeckhoudt, L., 1985. Relative risk aversion in comparative statics: Comment. American Economic Review 75, 281–283.

Cheng, H.-C., Magill, M. J., Shafer, W. J., 1987. Some results on comparative statics under uncertainty. International Economic Review 28, 493–507.

Dionne, G., Eeckhoudt, L., 1985. Self-Insurance, Self-Protection and Increased Risk Aversion. Economics Letters 17, 39-42.

Eichner, T., Wagener, A., 2001. Concepts of risk aversion, background risks, and comparative statics with mean-variance preferences. Mimeo, University of Siegen.

Gollier, C., 2001. The economics of risk and time. MIT Press, Cambridge, Massachusetts.

Gollier, C., Pratt, J. W., 1996. Risk vulnerability and the tempering effect of background risk. Econometrica 64, 1109–1123.

Guiso, L., Jappelli, T., 1998. Background uncertainty and the demand for insurance against insurable risks. Geneva Papers on Risk and Insurance Theory 23, 2–27.

Guiso, L., Jappelli, T., Terlizzese, D., 1996. Income risk, borrowing constraints and portfolio choice. American Economic Review 86, 158–172.

Holzmann R., Jørgensen, S., 2001. Social Risk Management: A New Conceptual Framework for Social Protection, and Beyond. International Tax and Public Finance 8, 529-556.

Menezes, C. F., Hanson, D. L., 1970. On the theory of risk aversion. International Economic Review 11, 481–487.

Meyer, J., 1987. Two-moment decision models and expected utility maximization. American Economic Review 77, 421–430.

Meyer, J., Ormiston, M. B., 1995. Demand for insurance in a portfolio setting. Geneva Papers on Risk and Insurance Theory 20, 203–211.

Mossin, J., 1968. Aspects of rational insurance purchasing. Journal of Political Economy 76, 553–568.

Rothschild, M., Stiglitz, J. E., 1970. Increasing risk I: A definition. Journal of Economic Theory 2, 225–243.

Sandmo, A., 1971. On the theory of the competitive firm under price uncertainty. American Economic Review 61, 65–73.

Sandmo, A., 1998. The welfare state: A theoretical framework for justification and criticism. Swedish Economic Policy Review 5, 11–33.

Shiller, R. J., 1993. Macro markets. Creating institutions for managing society's largest economic risks. Oxford University Press, Oxford.

Sinn, H.-W., 1983. Economic decisions under uncertainty. North-Holland, Amsterdam.

Sinn, H.-W., 1990. Expected utility, $\mu - \sigma$ preferences and linear distribution classes: A further result. Journal of Risk and Uncertainty 3, 277–281.

Sinn, H.-W., 1995. A theory of the welfare state. Scandinavian Journal of Economics 97, 495–426.

Sinn, H.-W., 1996. Social insurance, incentives and risk-taking. International Tax and Public Finance 3, 259–280.

Smith, V. L., 1968. Optimal insurance coverage. Journal of Political Economy 76, 68–77.

Varian, H., 1980. Redistributive taxation as social insurance. Journal of Public Economics 14, 49–68.