CESifo Working Papers

SMALL STATES, LARGE UNITARY STATES AND FEDERATIONS

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CESifo Working Paper No. 729 (1)

May 2002

Category 1: Public Finance

Presented at CESifo Area Conference on Public Sector Economics, May 2002

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ISSN 1617-9595

* The author thanks the participants and the discussant Dieter Bös at the CESifo Area Conference for valuable comments. The usual disclaimer applies.

CESifo *Working Paper No. 729*

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Abstract

Employing a political-economics approach, this paper compares small states and unions when the former fail to internalize cross-border externalities of publicly provided goods. It discusses two types of unions: federations with more than one level of government and unitary states. While unitary states are unable to differentiate public spending ac-cording to differing preferences, rents of governments in a federation are higher due to a common-pool problem. The comparison leads to the following results. (1) Citizens prefer small states to large states if spillover effects are weak. (2) They benefit from a multi-level government only if their preferences heavily differ from the median-voter's preferences and if spillovers are strong. Based on this comparison the paper also dis-cusses the creation of unions. Making specific assumption on the distribution of prefer-ences, it analyzes strong Nash equilibria and coalition-proof equilibria at the union formation stage.

JEL Classification: D7, H1, H7.

Keywords: voting theory, electoral accountability, federations, strong Nash equilibria, coalition-proof equilibria.

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1 Introduction

All over the world, the continued existence and the constitution of countries are disputed by centrifugal and centripetal forces. In some countries such as Canada there are strong movements that argue for secession, while other countries such as the member states of the European union are growing together. Moreover, regions, nation states and supra-national institutions argue about the correct distribution of responsibilities. On the one hand, more and more tasks are handed over to central authorities. On the other hand, many regions claim more autonomy. The European Union is a good example for these conflicting demands. The union has a permanently increasing influence on, for example, competition policy and tax policy in the member states. In order to block these centripetal forces, the subsidiarity principle has been set up. Not only in political science, but also in economics the issue of the creation and break up of unions and of the assignment of tasks to the levels of government within a union is hotly debated. This paper contributes to this discussion.

The paper analyzes different structures of countries: small homogeneous states, large unitary states with a heterogeneous population and federations with more than one level of government. The optimum size of countries is a balance between the inability of small states to internalize cross-border externalities of publicly provided goods and the failure of large unitary states to adapt public spending to differing preferences in the various parts of the state. At first sight, federations seem to be superior to both small states and large unitary states since each task can be assigned to a level of government such that the territory approximately matches the users of the publicly provided good. If there were no disadvantage of a multi-level structure, it would be surprising that not all countries in the world are federations with autonomously deciding lower levels of government. This paper discusses an explicit disadvantage of the multi-level structure: the common-pool problem in taxation. This problem arises if governments are not purely benevolent and citizens can only limit the abusive power of governments by promising re-election if public spending and taxation is satisfying. Barro (1973) and Ferejohn (1986) first analyzed performance-oriented voting strategies and showed that voters are able to prevent governments from myopically behaving as a leviathan through retrospective voting strategies. But, since governments have the opportunity to use their power abusively, they have to obtain a rent that equals in present vale terms the maximum instantaneous rent. In a federation, for all levels of government together, voters have to put up with a higher rent than in a unitary state since a common tax base is a common-pool resource for governments and, therefore, runs the risk of overexploitation. In order to take this particular disadvantage of federations into account, this paper sets up a political-economics model that allows for simple voting strategies to control malevolent governments in all types of countries.

The paper borrows from Alesina, Angeloni and Etro (2001 a, b), who discussed similar issues, and Persson, Roland and Tabellini (1997), who first analyzed the common-pool problem for one level of government in the context of the Barro model [for a detailed discussion of the relationship see the concluding section]. However, since this paper simultaneously considers spillovers, heterogeneity of population, and rents of governments, the assessment of federations is not determined a-priori and this paper has the potential to determine a new view of federations.

The purpose of this paper is twofold: First, it compares small states, large unitary states and federations from the viewpoint of citizens when spillovers exist, preferences differ across states and the magnitude of governments' rents depend on the institutional shape. Second, based on this comparison the paper discusses the formation of unions being unitary states or federations. A constitutional stage is considered where citizens decide whether or not to form a union with others.

The results of the paper can be used to evaluate the institutional form of large states and to predict the future of existing unions. The interesting implications for federations such as, e.g., the United States, Canada, Switzerland and Germany, and for con-federations such as the European Union are discussed extensively in the concluding section.

The paper is organized as follows. Section two presents the outline of the model and compares small states, large unitary states and federations for a fixed number of potential members of a union. Afterwards section three analyzes the formation of unions. Section four concludes.

2 A comparison of small states, large unitary states and federations

An infinite series of periods which are independent from each other is considered. Since the conditions in all periods are essentially the same, no time index is used. From the viewpoint of households, the entire allocation task can be considered as a one period problem. A group of N equally sized small states is considered with the population size normalized at one, where N > 1. The economic fundamentals are the same in all states, but preferences possibly differ across states. Each individual has an endowment y and derives utility from a private good and two different public goods that have to be provided by governments. Private consumption of the representative utility in state i is indicated by c_i, and per capita spending in state i for the first and the second public good is s_i and f_i respectively. Later, the first public good will be called the state public good; the second good will be named federal public good. The quasi-linear utility function of individual i is

$$(1) \qquad u_i = c_i + a_i G(s_i) + H\left(f_i + b \sum_{j \in U, j \neq i} f_j \right),$$

where the subutility functions G and H are strictly monotonically increasing and strictly concave. The parameter a_i grasps the relative evaluation of the public good s_i. Without loss of generality the parameters are ordered according to size: $0 < a_1 \leq \cdots \leq a_N$. Public spending for the second public good causes positive external effects, where b captures the size of the spillover effect, with $0 \leq b \leq 1$. Hence, the second publicly provided good is a public good at the state or the union level or something in between. It is assumed that external effects occur only within some type of union U, where U is the set of all indicators of members of the union $U \subseteq \{1, \ldots, N\}$. In contrast to the first publicly provided goods, all individuals have the same preferences with respect to the second public good.[1] Finally, public spending is financed by non-distorting taxes of which magnitude is limited by the endowment of households.[2]

In democracies elected politicians decide upon public goods and taxes. When politicians are unable to commit themselves to political programs, voters can make use of retrospective voting strategies to control politicians. In the following, it will be assumed that politicians are malevolent and that their objective is to maximize rents. When politicians are foresighted, voters can reduce the abusive power of governments by applying retrospective voting strategies. To simplify, the following further assumptions will be made. First, the rent of a politician in power is simply the difference between tax revenue and benevolent public spending. Second, an infinite horizon of politicians is assumed. Third, a politician who is ousted from office by the voters will never be re-elected again. This assumption is a proxy for the absence of barriers to entry to politics. If, in contrast, there were a chance for politicians voted out of office to re-enter sometime in the future, voting power would be less strong. Fourth, only simple retrospective voting strategies that are based on cut-off levels of beneficial public expenditure and on cut-off levels of the tax rate will be considered. Voters guarantee the incumbent re-election if the government provides satisfying quantities of public goods and if the tax rate is not too high in the term. Throughout the paper, the analysis is restricted to voting strategies that condition the decision on re-election on the policies in the preceding term and not on the entire history. Fifth, since it is assumed that voters are able to commit to voting strategies, time consistency issues are neglected.

The timing of events in a democratic country will be specified as follows: First, applying the majority rule, voters determine the cut-off levels of the re-appointment rule. Second, governments determine taxes and public spending. Third, elections take

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1 This assumption will later on considerably simplify the analysis since it allows the median voter theorem in a large unitary state to be applied.

2 Implicitly the model assumes constant marginal rates of transformation between all types of goods normalized to one.

place and either the incumbent will be re-elected or an opponent identical in all respects to the incumbent will enter the office. Fourth, the game restarts at stage 2.

2.1 The optimum

As a benchmark case the first-best optimum, not having any necessity to limit the power of governments, is considered first. In order to take advantage of the spillover effects, first-best optima require that the union comprises all countries, i.e., $U = \{1, \ldots, N\}$. Due to equal preferences with respect to the second public good, the set of first-best optima is given by the solutions to

(2) $$\underset{s, \ldots s_N, f}{\text{Max}} \sum_{i=1}^{N} \gamma_i u_i = \sum_{i=1}^{N} \gamma_i \left[y - s_i - f + a_i G(s_i) + H((1 + b(N-1))f) \right],$$

for all non-negative γ_i that satisfy $\sum_{i=1}^{N} \gamma_i = 1$.

The solutions are characterized by the adequately defined Samuelson rules[3]

(3) $$a_i G'(s_i) = 1, \; i = 1, \ldots, N, \quad \text{and} \quad (1 + b(N-1)) H'((1 + b(N-1))f) = 1.$$

The optimum values are denoted by f^o and s_i^o. Due to the properties of the subutility function G, the optimum quantity of the first public good is a strictly monotonically increasing function of the preference parameter a_i: $ds_i^o / da_i = -1/(a_i^2 G'') > 0$. However, an increase in the spillover effect leads to a higher (equal, lower) optimum quantity of the second public good if the degree of relative concavity $-(1 + b(N-1)) f H''/H'$ is at the optimum smaller than (equal to, greater than) one since

(4) $$\frac{df^o}{db(N-1)} = -\frac{H'}{(1 + b(N-1)) H'' b(N-1)} \left(1 + \frac{(1 + b(N-1)) f^o H''}{H'} \right).$$

2.2 Public spending in small states

A small state is a single state that is not a member of any union. As a result, small states cannot take advantage of spillover effects. Public spending in state i is financed by a

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[3] Throughout the paper the focus will be on interior solutions, although, depending on endowment and preferences, corner solutions are possible.

lump sum tax τ_i. Therefore, the government budget restriction in state i is $\tau_i = s_i + f_i + r_i$, where r_i denotes the rent of the incumbent. Private spending of households is therefore determined by $c_i = y - \tau_i$. As mentioned before, voters control politicians with simple retrospective cut-off voting strategies that can be described for a small state as follows. Voters in state i will re-elect the incumbent if and only if $s_i \geq s_i^s$, $f_i \geq f_i^s$, and $\tau_i \leq \tau_i^s$. Given this voting strategy, a rent maximizing politician never provides more than the cut-off levels of public goods s_i^s and f_i^s, and the tax rate is never lower than the critical tax rate τ_i^s. If the government obeys the voting strategy in the current and all following periods, the rent in present value terms is equal to $(\tau_i^s - s_i^s - f_i^s)/(1-\delta)$, where δ is the discount factor, with $0 < \delta < 1$. Clearly, since each incumbent that violates the policy rule determined by the voting strategy will be removed from the office independent of the magnitude of the deviation, a deviating government maximizes instantaneous rents and, therefore, chooses zero public spending and maximum taxes: $s_i = f_i = 0$ and $\tau_i = y$. Since the government can always secure itself a rent y, politicians in state i only follow the policy proposed by voters if and only if[4]

$$(5) \qquad r_i = \tau_i^s - s_i^s - f_i^s \geq (1-\delta)y .$$

Since endowments and discount factors in all states are assumed to be the same, minimum rents of obeying governments are also of the same magnitude in all states. Furthermore, since voters prefer low rents they choose the parameters of the voting strategy in all states so as to minimize rents. Hence, complying governments in all states end up with the same rent per period $r_i = (1-\delta)y$. Therefore, taking into account the household budget restriction and the government restraint, voters in state i choose the parameters of the voting strategy in order to solve

$$(6) \qquad \underset{s_i, f_i}{\text{Max}}\ u_i = \delta y - s_i - f_i + a_i G(s_i) + H(f_i).$$

4 As usual, it is assumed that the government will comply with the proposed policy rule if it is indifferent between compliance and violation.

The first-order conditions are

(7) $a_i G'(s_i) = 1$ and $H'(f_i) = 1$.

In the following, s_i^s and f_i^s indicate the solution values and u_i^s denotes the resultant utility level in the small state i. The optimum tax rate follows from the government budget restriction. Since preferences with respect to the federal public good do not differ across states, public spending for this good is the same in all states: $f_i^s = f^s$. Obviously, voters in small states choose public spending on the state public good according to the first-best rule: $s_i^s = s_i^o$. However, as becomes clear from the discussion at the end of the previous subsection, whether public spending for the federal public good in small states is larger or smaller than the first-best-optimum-spending f^o level depends on the degree of relative concavity.

2.3 Public spending in large unitary states

Large unitary states are defined as states that consist of more than one small state. The number of small states that build the large unitary state is denoted by n, with $1 < n \leq N$. It is assumed that explicit redistribution cannot take place among the member states, i.e., for each state i $z_i \equiv 0$ has to hold. Furthermore, in order to avoid exploitation of a minority by the majority, only uniform policies are feasible, i.e., $s_i = s$, $f_i = f$, and $\tau_i = \tau$. In this model the restriction to uniformity is ad hoc, but Besley and Coate (2000) have shown that centralization harms citizens even if preferences across borders are the same when a majority can exploit the minority through non-uniform tax and spending policies. Voters coordinate on the re-appointment rule: Voters will re-elect the incumbent if and only if $s \geq s^1$, $f \geq f^1$, and $\tau \leq \tau^1$. The government budget restriction for each small state reads $\tau = s + f + r$. By the same reasoning as in the previous subsection, it can be easily obtained that voters have to give up in each state the rent $r = (1-\delta)y$. At the first stage of the game at which the parameters of the re-appointment rule are determined the representative individual in state i prefers a set of policy variables that solve

(8) $$\max_{s,f} u_i = \delta y - s - f + a_i G(s) + H((1 + b(n-1))f),$$

of which solution is characterized by

(9) $$a_i G'(s) = 1 \quad \text{and} \quad (1 + b(n-1))H'((1 + b(n-1))f) = 1.$$

All voters agree on the optimum quantity of the federal public good denoted by f^1. Furthermore, if the large unitary state consisted of all N small states, voters would aim for the first-best-public-good-spending level f°. Again, it depends on the degree of relative concavity whether 'smaller' large unitary states spend more or less on the federal public good f than 'larger' large unitary states. Since preferences of voters of the various states with respect to the state public good are single-peaked, the median-voter theorem can be applied.[5] Using figure 1, this can be easily shown. All solutions of the utility maximization problem lie on the vertical line that crosses the point (f^1, s_m) preferred by the median voter m. The optimum values with respect to the state public good of the states where voters have the lowest and the highest preference parameter a_i are indicated by s_{min} and s_{max} respectively. Furthermore, the indifference curves of the representative voter in some state j with $a_j > a_m$ are also depicted as circles around the preferred bundle (f^1, s_j).[6] Since the marginal rate of substitution of some voter i is equal to

(10) $$\frac{ds}{df} = -\frac{(1 + b(n-1))H'((1 + b(n-1))f) - 1}{a_i G'(s) - 1},$$

the marginal rate of substitution is clearly zero at the vertical line through (f^1, s_m). Hence, any policy proposal that lies below (above) the horizontal line defined by s_m is clearly not preferred to (f^1, s_m) by those voters that have a relatively high (low) preference for the state public good s, i.e., where $a_i > a_m$ $(a_i < a_m)$ holds.

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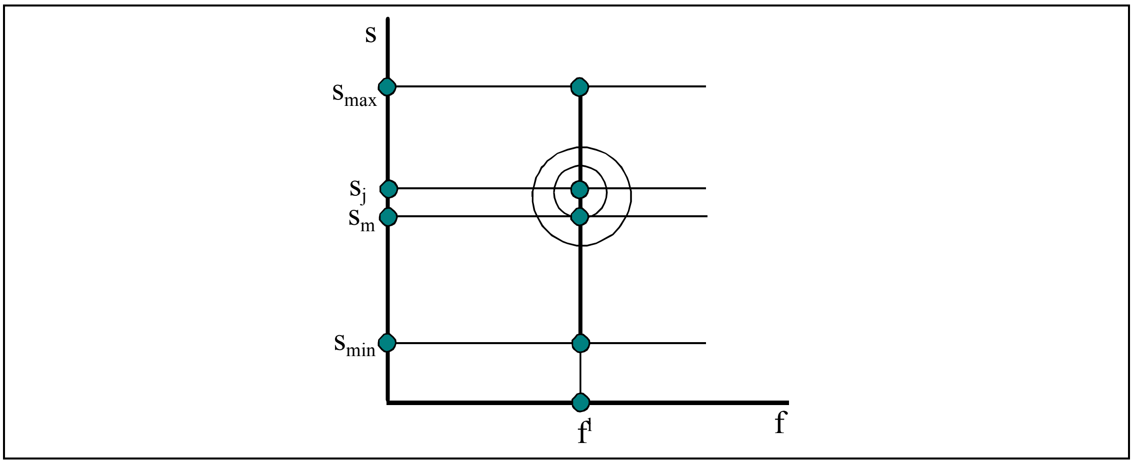
5 If preferences of individuals in different states differed also with respect to the second public good f, the median voter theorem could not have been applied and a more complex voting model ought to have been employed.

6 The indifference curves are not necessarily exact circles.

For the sake of concreteness, it will be assumed in the following that public spending on the state public good would be equal to $(s_{n/2} + s_{n/2+1})/2$ if the number of states n were even, although any value $s \in [s_{n/2}, s_{n/2+1}]$ could be justified by the median-voter theorem.

Figure 1: The median-voter theorem in a large unitary state



According to the median-voter theorem, the solution of the voting-rule determination stage denoted by s^l fulfills: $s^l = s^o_m$. The optimum tax rate is then determined by the government budget restriction. Finally, it should be stressed that, irrespective of the differing preferences, all voters agree to minimize rents. A policy that does not minimize rents can win a majority against the voting rule that maximizes the utility of the median voter within the set of voting rules that minimize rents.

The resultant utility level of the representative individual in state i that belongs to the union U is indicated by $u^l_i(U)$ if the policy is determined at the national level of a large unitary state U.

2.4 Public spending in federations

Federations are also large states that consist of n small states. However, in contrast to the previously discussed large unitary states, governments at two different levels decide on public spending and taxes. Both levels of government levy lump-sum taxes within their territory. The government at the federal level determines the quantity of the federal public good and is restricted to uniform policies due to the same reason as in unitary large states. In each state the lower-level government decides on the 'state' public good.[7] At the state level policies could very well differ across states. Both levels of governments determine public spending and taxes simultaneously and elections at both levels take place at the same time. Although it could be argued that the federal government is a natural Stackelberg leader, simultaneous actions are more plausible than sequential steps if self commitment is sufficiently costly. As in the large unitary state, explicit redistribution through transfers is not allowed.

At each level voters at the respective territory coordinate on the re-appointment rule. In state i voters will re-elect the incumbent if and only if $s_i \geq s_i^f$ and $\tau_i \leq \tau_i^f$, where τ_i denotes the state tax. The federal government will be re-elected if and only if $f \geq f^f$, and $t \leq t^f$, where t denotes the federal tax.[8] It is assumed that voters in all states and at the federal level decide simultaneously on the voting strategies. Furthermore, when they determine the parameters of the re-appointment rule, voters at each level take the parameters of the voting rule(s) at the other level as given.

At the state level, the government of state i faces the budget restriction $\tau_i = s_i + r_i$ The government budget restriction of the federal government reads $nt = nf + r^f$, where r^f denotes the rent at the federal level. If both levels of government obey the different voting rules in the current and all following periods, in present value terms the rent of the state government in state i is equal to $(\tau_i - s_i)/(1-\delta)$ and that of the federal government equals $n(t-f)/(1-\delta)$. Obviously, non-complying governments reduce public spending to zero. Furthermore, they increase the tax rate above the proposed level. It is assumed that governments in the territory of each small state i share the endowment of a household equally, $\tau_i = y/2$ and $t = y/2$, if both tax rates together exceed the endowment of the household in at least one state. Equal tax sharing takes place in all states since overtaxation in one state breaks the basic rule of the entire fed-

7 This assignment of tasks is clearly more worth a discussion than the reverse assignment since individuals' preferences with respect to the state public good differ across states.

8 With just one level of government voters could apply simpler voting rules that just fix the minimum utility level instead of tax rates and expenditure. However, in a multi-level framework such a simple voting rule does not solve the issue of policy selection.

eration.[9,10] Due to the endowment restriction, by deviating from the proposed policy, the state government in state i can achieve an instantaneous rent $\max\{y-t, y/2\}$ when the federal government complies with the voting rule. The abusive power of the federal government is not only limited by the endowment of households, but also by the restriction to uniform policies. Hence, given that all state governments comply with the voting rule, the federal government can obtain an instantaneous rent $n(y-\tau_{max}, y/2)$, where τ_{max} is the largest state tax rate in the union. Since all governments decide simultaneously on public spending and taxes and if voters minimize on rents, overall compliance with the various voting rules can only be an Nash equilibrium at the government level if

$$(11)\qquad \begin{aligned} r_i &= \tau_i - s_i = r = (1-\delta)(y-t) = (1-\delta)\left(y-\left(f + r^f/n\right)\right),\ \forall i \in U, \text{ and} \\ r^f &= n(t-f) = (1-\delta)n(y-\tau_{max}) = (1-\delta)n\left(y-\left(\max\{s_j | j \in U\} + r\right)\right) \end{aligned}$$

hold if $\tau_{max} \leq y/2$ and $t \leq y/2$ are fulfilled. Solving this system of equations for r and r^f, yields rents

$$(12)\qquad r = \frac{(1-\delta)\left(\delta y + (1-\delta)\max\{s_j | j \in U\} - f\right)}{\delta(2-\delta)} \quad \text{and}$$

$$\frac{r^f}{n} = \frac{(1-\delta)\left(\delta y + (1-\delta)f - \max\{s_j | j \in U\}\right)}{\delta(2-\delta)}$$

These rents are not lower than $(1-\delta)y/2$ if

$$(13)\qquad \delta\left(\delta y - 2\max\{s_j | j \in U\}\right) \geq 2\left(f - \max\{s_j | j \in U\}\right) \geq \delta(2f - \delta y)$$

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[9] An alternative assumption would be that rent sharing takes place only in those states where the sum of taxes exceed the endowment. This, however, would considerably complicate the mathematical analysis since different cases have to be considered without changing the qualitative results.

[10] Since governments levy lump-sum taxes that are only restricted by the endowment of households, a condition like this is necessary to determine the outcome if governments deviate. Wrede (2000) has analyzed the nature of the common-pool problem in the presence of a distorting wage tax. However, distorting taxes would increase the complexity of the model and would particularly make the comparison of institutions much more difficult.

is fulfilled. Hence, this type of equilibrium occurs only if voters do not want governments to spend too much on public goods at all and if the budgets per state at the two levels of government are of similar size. In the remaining part of the paper it will assumed that this condition holds.[11]

At this equilibrium the total rent per state is $(1-\delta)(2y-f-\max\{s_j|j\in U\})/(2-\delta)$ and the representative individual in state i achieves the utility level

(14) $u_i = y - s_i - f - (1-\delta)(2y-f-\max\{s_j|j\in U\})/(2-\delta) + a_i G(s_i) + H((1+b(n-1))f)$.

Besides this equilibrium other types of equilibria exist. For instance, a disastrous equilibrium where governments completely exploit households, i.e., where $r = r^f = y/2$, always exists.

However, if voters focus on the first type of equilibrium at the first stage of the game at which the parameters of the re-appointment rule are determined, voters in state i choose the policy variable in order to maximize

(15) $\underset{s_i}{\text{Max}} - s_i + (1-\delta)\max\{s_j|j\in U\}/(2-\delta) + a_i G(s_i)$,

taking f and s_j for all $j\in U$ and $j\neq i$ as given. Voters at the federal level solve

(16) $\underset{f}{\text{Max}} - f/(2-\delta) + H((1+b(n-1))f)$.

The first-order conditions are

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[11] The federal government would not even benefit from a (small) deviation if governments share the endowment of a household equally only in that state where both tax rates together exceed the endowment of the household provided that

$$(n-1)(s_{max}-s_{max-1}) \le r^f/(n(1-\delta)) - y/2 = (\delta^2 y - 2(s_{max}-(1-\delta)f))/(2\delta(2-\gamma)).$$

holds, where s_{max} and s_{max-1} the indicate two largest state-public-good spending programs. This condition is fulfilled if n is sufficiently small and if the difference in spending on the state-public good is not too big.

(17) $a_i G'(s_i) = 1$ if $s_i < \max\{s_j | j \in U\}$, $a_i G'(s_i) = 1/(2-\delta)$ if $s_i = \max\{s_j | j \in U\}$, and $(1 + b(n-1))H'((1 + b(n-1))f) = 1/(2-\delta)$.

The optimum values are denoted by s_i^f and f^f, and the resultant utility level is indicated by $u_i^f(U)$. Hence, all voters with a low relative evaluation of the state public good, choose public spending at the state level according to the first-best optimum rule. Those voters, however, who have the strongest preference for this type of public good increase public spending above the first-best level. At the federal level voters also choose a level of public spending that is above the first-best one for a given number n of member states. Hence, there is a tendency to enforce an oversupply of public goods. The intuition for the excess spending is simply that voters reduce the scope for deviation from the proposed policy at the other level of government by increasing public spending at one level. In the remaining part of the paper it will be assumed that the type of equilibrium described above occurs.

2.5 Comparison of institutions

Since public goods can be provided either by small states, by large unitary states or by federations, it is worthwhile to compare the various institutions from the viewpoint of citizens. In particular, it could be argued that citizens, taking into account the nature of the political process under all the different institutional designs, determine by voting whether or not the small state should join a large unitary state or federation. In order to predict the voting result it is necessary to compare the equilibria of the different settings. This section will carry out this comparison for unions U with n member states, where n > 1. In this section the set of members of the union is fixed. Alternative unions with more than one member state are not considered. The median voter in this union is indicated by m, the member states with the lowest and highest evaluation of the state public good are indicated by min and max respectively.

The result of this comparison mainly depends on the magnitude of the spillover effect b and on the distribution, and particularly the variance, of the preference parameters a_i. As a benchmark case, the evaluation starts with a union without spillover effects, i.e. where b = 0. The following proposition states the results (proofs of all propositions are in the appendix).

Proposition 1: In the absence of a spillover effect, a set of small-state governments is Pareto-optimal. Provided that not all voters have the same preferences, a set of small-state governments is the unique Pareto-optimum. #

Small-state governments meet the demands of voters with differing preferences and, therefore, in the absence of spillover effects, are strictly preferred to large unitary states by voters in all states except for the median voter who is indifferent. Furthermore, in federations all voters achieve a lower utility level compared with small states because of two different reasons. First, in federations voters prefer an oversupply of the unionwide public good f in order to reduce the scope for deviation of state governments from the proposed voting rule. In addition, the voters with the highest preference for the state public good also prefer an oversupply of the state public good. Second, the total rent per state in federations $(1-\delta)(2y - f - \max\{s_j | j \in U\})/(2-\delta)$ is higher than the rent in small states $(1-\delta)y$. This is due to the common-pool problem in federations, where both levels of government are able to levy taxes. Politicians at both levels are able to deviate independently, and, believing that the other incumbent sticks to the equilibrium policy, each of them expects a revenue gain from deviation in each state which is more than half of the maximum revenue obtainable by a single state government. In other words: if incumbents at both levels deviated simultaneously, they would be unable to realize those revenue gains. Nevertheless, to avoid a Leviathan policy, voters have to guarantee a rent which is based on those expected revenue gains.

Next, the other extreme case is considered where all voters have the same preferences, i.e., where $a_{max} = a_m = a_{min}$. The results are summarized by the next proposition.[12]

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[12] The ranking in this analysis is only based on preferences of voters, politicians are neglected. If welfare were considered as a weighted sum of voters' utility and politicians' utility, the qualitative results of the paper would still hold provided that the weight attached to politicians were sufficiently low.

Proposition 2: Provided that all voters in the n states have the same preferences, in the presence of spillover effects, a large state is Pareto-superior to small states and a federation. #

Since small states do not internalize spillover effects, since rents in federations are comparatively high, and since decisions are distorted in federations, all voters strictly prefer to live in a large unitary state when they share the same preferences. As a result of the first two propositions, it becomes clear that voters are possibly better off in federations compared with both small states and large unitary states only if preferences differ and spillover effects arise.

When spillover effects occur and preferences differ, usually a Pareto-dominant solution does not exist. The following lemma states a preliminary result concerning the comparison of small states and federations.

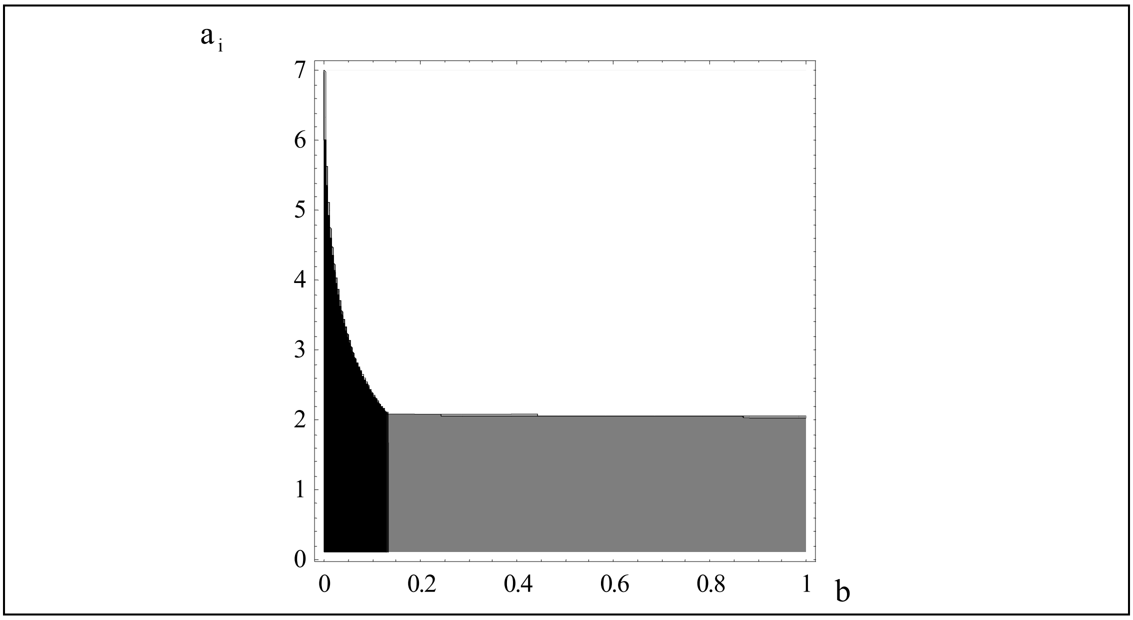
Lemma: With the exception of some of those voters that have the highest preference for the state public good, all voters of the n states necessarily agree on the choice between small-state governments and a federation. If the voters that have the highest preference for the state public good prefer a state-government to the federation, all other voters share this opinion. #

Since state governments in small states and federations maximize the utility of voters in all states, from the viewpoint of voters who do not choose the highest level of the state public good federations differ from small states only with respect to total rents and the federal public good. However, since voters agree on rents and federal public goods, there are no differences in opinions among those voters. Compared with these voters, voters who choose the highest level of the state public good spending have an influence on the rent of the federal governments and, therefore, have a slightly higher preference for federations. Using the previous proposition and this lemma, the next proposition can be stated.

Proposition 3: (a) In the presence of spillover effects, the large-state environment is never Pareto-dominated by either small states or a federation. (b) When the spillover effect is sufficiently small, the federation is strictly Pareto-dominated by small states. (c) If at all, federations Pareto-dominate small states only if the spillover effect is sufficiently high. #

Since in large unitary states spillover effects are perfectly internalized and the median voter can have his/her way, he/she clearly prefers the large unitary state to a small state. Furthermore, the higher the spillover effect is, the more voters benefit from internalization of spillover effects in federations. Hence, voters benefit from a federation only when spillover effects are large, and, clearly, the median voter would oppose a transformation of a large unitary state into a federation.

Figure 2: Spillover effect, preference parameter and the preferred institution



	small state		federation		large unitary state

Next, the comparison of institutions will be illustrated by means of an example: subutility functions with constant relative concavity

$$(18) \qquad G(x) = H(x) = \begin{cases} x^{(1-\gamma)}/(1-\gamma) & \text{if } \gamma \neq 1, \gamma > 0 \\ \ln x & \text{if } \gamma = 1, \end{cases}$$

where γ is the degree of relative concavity. If the degree of relative concavity γ is smaller than (equal to, greater than) one, a large state spends more (the same, less) on the federal public good in comparison to a small state when spillover effects arise.

Figure 2 shows the preferred institution depending on the size of the spillover effect and the preference parameter (n = 10, y = 20, δ = 0.95, γ = 1.8, a_{max} = 14, a_m = 7).

3 The creation of a union

Whether or not small states join a union is ultimately also determined by citizens. This section will discuss the creation of large unitary states and federations. In this connection, unions that consist of all potential member states N will be in the center of the discussion. An additional stage is added at the beginning of the game: (0) Voters in small states decide whether or not to build unions. To simplify the notation, a union U is a subset $U \subseteq \{1, \ldots, N\}$ that can be either a federation or a large unitary state or even a small state if it consists of just one small state. In a larger union voters have to decide whether to give up the right to decide at the state level on the state public good. It is assumed that voters decide sequentially on the issues of building a union and the institutional shape and that the member states choose commonly the institutional framework. Stage 0 is therefore broken up into the following two stages. (0a) Voters in small states decide whether or not to build unions. (0b) Applying the majority rule, voters of a union determine whether the union is organized as a large unitary state or as a federation if the size of the union is larger than one. At all the stages voters take the decisions at the subsequent stages into account.

Explicit redistribution among states through a transfer system has already been excluded from the discussion. Throughout the rest of the paper it will consistently be assumed that side payments at the union formation stage are also not available. A ban on interstate transfers and side payments at the union formation stage is in the interest of citizens if a transfer system reduces the transparency of politics and decreases the power of voters to control politicians and to limit their discretionary power.

If the set of member states of a union is denoted by U and if the set of voters within the union that prefer a federation is denoted by U_f (where $u_i^f(U) \geq u_i^l(U)$ for all

$i \in U_f$), if $U_l = U \backslash U_f$, and if the resultant utility of the voter in member state i is denoted by $u_i^u(U)$, therefore[13]

$$\begin{aligned} &\text{U is a large unitary state and } u_i^u(U) = u_i^l(U) \ \forall i \in U \\ &\qquad\qquad \text{if and only if } |U_f| < |U_l| \text{ and } |U| > 1, \\ (19) \quad &\text{U is a federation and } u_i^u(U) = u_i^f(U) \ \forall i \in U \\ &\qquad\qquad \text{if and only if } |U_f| \geq |U_l| \text{ and } |U| > 1, \\ &\text{U is a small state and } u_i^u(U) = u_i^s \text{ if and only if } U = \{i\} \end{aligned}$$

holds. Since group decisions are also considered, the notion of a strong Nash equilibrium developed by Aumann (1959) is helpful. This concept defines an equilibrium by the requirement that no player would benefit from a unilateral or a multilateral deviation. At a strong Nash equilibrium no state would benefit either from exiting a union by itself or from building a different union together with other member states of any union. Formally, a strong Nash equilibrium is defined as follows.

Definition: A strong Nash equilibrium (SNE) with utility u_i^{SNE} for the voter in state i is a set of unions $\{U_1, \ldots, U_M\}$, where $U_i \cap U_j = \emptyset$ for all i, j with $i \neq j$ and $\bigcup_i U_i$ = $\{1, \ldots, N\}$, such that no union $S \subseteq \{1, \ldots, N\}$ exists, where $u_i^u(S) > u_i^{SNE}$ for all $i \in S$. #

Since a strong Nash equilibrium does often not exist, a weaker refinement, the coalition-proof equilibrium analyzed by Bernheim, Peleg and Whinston (1987), will also be considered. This concept defines an equilibrium by the requirement that no player would benefit from a unilateral deviation or from building a deviating coalition together with others that no member would like to leave. In order to apply this approach, the following definition is useful.

Definition: A union U is break-up-resistant if and only if no break-up resistant union $S \subset U$ exists, such that $u_i^u(S) > u_i^u(U)$ for all $i \in S$. #

[13] Without loss of generality it will be assumed that a federation is built in case of a tie.

A union is break-up resistant if no member of this union would benefit either from exiting the union by itself or from exiting together with other members and building a new smaller break-up resistant union. If spillover effects exist, the latter requirement might be stronger than the first since commonly leaving states could build a union and internalize spillover effects. Based on this definition, a coalition-proof equilibrium could be defined.

Definition: A coalition-proof equilibrium (CPE) with utility u_i^{CPE} for the voter in state i is a set of unions $\{U_1, \ldots, U_M\}$, where $U_i \cap U_j = \varnothing$ for all i, j with $i \neq j$ and $\bigcup_i U_i$ = $\{1, \ldots, N\}$, such that no break-up resistant union $S \subseteq \{1, \ldots, N\}$ exists, where $u_i^u(S) > u_i^{CPE}$ for all $i \in S$. #

At a coalition-proof equilibrium no state would benefit either from exiting a union by itself or from building a break-up resistant union together with other member states of some union. Each SNE is a CPE, but not vice versa.

Although it is impossible to make general statements on the properties of strong Nash equilibria in this game, for certain distributions of preferences strong Nash equilibria could be described. Clearly, if no spillover effects arise, a SNE would be a set of small states since no state can gain anything from becoming a member of a larger union that is either a federation or a large unitary state (see the first proposition). This SNE is unique if no two or more voters have identical preferences. Furthermore, if in the presence of spillover effects all voters share the same preferences, the unique SNE is a large unitary state that consists of all N states since all voters would be clearly worse off in a federation of any size, in a smaller large state or in a small state (see the second proposition). In the remaining part of this section the case in between will be considered, where spillover effects arise and preferences differ to some extent. The analysis is restricted to specific distributions of preferences. The first type of distribution under consideration is defined as follows.

Definition: A symmetric bimodal distribution of preferences with equally sized homogeneous subgroups is a bimodal distribution for an even number of states N given by $a_1 = \cdots = a_{N/2} < a_{N/2+1} = \cdots = a_N$, where $S_s = \{1, \ldots, N/2\}$ and $S_l = \{N/2+1, \ldots, N\}$. #

States are divided into two groups of equal size, where the member states within a group share the same preferences. For this type of distribution of preferences the next proposition describes the properties of a SNE.

Proposition 4: For a symmetric bimodal distribution of preferences with equally sized homogeneous subgroups (I) a federation that consists of all states, $F = \{1, \ldots, N\}$, is a SNE if (a) spillover effects arise (b > 0), (b) $u_i^f(F) \geq u_i^l(F)$ for all $i \in F$, and (c) $u_i^f(F) \geq u_i^l(S_j)$ for all $i \in S_j$, j = s, l. (II) This equilibrium is the unique CPE if the inequality of condition (c) holds in a strict form and if in addition (d) $u_i^s > u_i^l(\{1, \ldots, N-1\})$ for $i \in S_l$ and $u_i^s > u_i^l(\{2, \ldots, N\})$ for $i \in S_s$ hold. #

The conditions state that (a) spillovers effects arise, (b) voters weakly prefer a federation to a unitary state when all states belong to the union and (c) all voters are not worse off in a large federation than in a unitary state built just by the subgroup they belong to. Condition (d) requires that voters in a asymmetric subset of the entire group of states prefer a small state to a large unitary state when they are in the minority.

Provided that states are divided into two homogeneous groups of equal size, a large federation, which consists of all N states, is a strong Nash equilibrium if spillover effects are sufficiently strong and differences in preferences are sufficiently large. If spillover effects were weak, citizens would prefer to be a member of a homogenous subgroup of size N/2 that build a large unitary state. In case of similar preferences citizens would benefit from building a large unitary state that covers the entire territory. For a large federation to be a unique coalition-proof equilibrium, the degree of heterogeneity has to be very high.

The qualitative results stated by this proposition would still hold if preferences within each subgroup were slightly different. However, the result is very sensitive to the assumption of equally sized subgroups. In case of an uneven number N of states, the median voter with respect to preferences on the state public good would belong to the larger subgroup that would, therefore, vote for a large unitary state. A federation with N member states could never be an equilibrium. Hence, the analysis is extended to trimodal distributions with homogenous subgroups specified as follows.

Definition: A symmetric trimodal distribution of preferences with homogeneous subgroups is a distribution given by $a_1 = \cdots = a_j < a_{j+1} = \cdots = a_{N-j} < a_{N-j+1} = \cdots = a_N$, where $S_s = \{1, \ldots, j\}$, $S_m = \{j+1, \ldots, N-j\}$ and $S_l = \{N-j+1, \ldots, N\}$, $1 < j < N/2$. #

The group of states is divided into three homogeneous subgroups were the subgroups with the extreme preferences are equally sized. Equilibria are characterized in the next two propositions.

Proposition 5: For a symmetric trimodal distribution of preferences with homogeneous subgroups a federation that consists of all states, $F = \{1, \ldots, N\}$, is a SNE if (a) spillover effects arise (b > 0), (b) $|S_s \cup S_l| \geq |S_m|$, (c) $u_i^f(F) \geq u_i^l(F)$ for all $i \in S_s \cup S_l$, (d) $u_i^f(F) \geq u_i^l(S_j)$ for all $i \in S_j$, j = s, m, l, and (e) for j = s, l holds: for all S_m^s, S_j^s, where $S_k^s \subset S_k$, $k = m, j$, $\exists i$ $u_i^f(F) \geq u_i^l(S_m^s \cup S_j^s)$. #

The proposition requires that (a) spillovers exist, (b) the middle subgroup is in the minority in the entire group, (c) at least the members of the subgroups with extreme preferences are not worse off in a federation than in a unitary state if the union covers the entire territory, (d) all voters weakly prefer the large federation to a unitary state built just by the subgroup they belong to, and (e) when some voters of the middle subgroup and some of one of the border subgroups build a large unitary state, at least one voter is not better off than in the large federation.

Provided that states are divided into three homogeneous groups, a large federation, which consists of all N states, might be a strong Nash equilibrium if spillover effects are strong and differences in preferences are large and if the subgroups with the extreme preferences are in the majority. However, the requirements are strong. In particular, condition (e) strictly limits the space of preferences. Strong differences between the middle and the border groups are necessary. What makes this condition particularly strong is that in some cases it has to be a member of the middle group that prefers to live in a large federation although this group has a strong preference for a large unitary state since its members have the median-voter preferences. For a coalition-proof equilibrium with just one large federation to exist, the next proposition states weaker conditions than the previous one.

Proposition 6: For a symmetric trimodal distribution of preferences with homogeneous subgroups a large federation that consists of all states, $F = \{1, \ldots, N\}$, is a CPE if conditions (a) through (d) of the previous proposition are fulfilled and if (e) for j = s, l holds: for all S_m^s, S_j^s, where $S_k^s \subset S_k$, k = m, j, $\exists k \in \{m, j\}$ such that $u_i^l(S_k^s) > u_i^l(S_m^s \cup S_j^s)$ for all $i \in S_k^s$ whenever the median voter of $S_m^s \cup S_j^s$ with respect to preferences is not a member of S_m^s and when $u_i^l(S_m^s \cup S_j^s) > u_i^f(F)$ for all $i \in S_m^s \cup S_j^s$. #

The additional condition in this proposition requires the following: (e) when some voters of the middle subgroup and some of one of the border subgroups build a large unitary state because they are better off than in the large federation that covers the entire territory, at least the members of one subgroup would benefit from forming a separate unitary state. This condition is still strong but since it does not mix with the federation it is in some respects weaker than condition (e) of the previous proposition.

The nature of the equilibrium changes when the middle group is in the majority. A federation that covers the entire territory is clearly not an equilibrium since the median voter belongs to the middle group of which members therefore prefer a large unitary state to a federation and are able to enforce it. Hence, if at all, a large unitary state is an equilibrium candidate.

Proposition 7: For a symmetric trimodal distribution of preferences with homogeneous subgroups a large unitary state that consists of all small states, $L = \{1, \ldots, N\}$, is a CPE if (a) spillover effects arise (b > 0), (b) $|S_s \cup S_l| < |S_m|$, (c) $u_i^l(L) \geq u_i^l(S_j)$ for all $i \in S_j$, j = s, l, and (d) for all S_s^s and S_l^s with $|S_s^s| = |S_l^s|$, where $S_j^s \subset S_j$, j = s, l, holds: (d1) if the majority of $S_s^s \cup S_l^s$ prefers a federation to a large state and $u_i^f(S_s^s \cup S_l^s) > u_i^l(L)$ for all $i \in S_s^s \cup S_l^s$, $\exists j \in \{s, l\}$ such that $u_i^l(S_j^s) > u_i^f(S_s^s \cup S_l^s)$ for all $i \in S_j^s$ and (d2) if the majority of $S_s^s \cup S_l^s$ prefers a large state to a federation, $\exists j \in \{s, l\}$ such that $u_i^l(L) \geq u_i^l(S_s^s \cup S_l^s)$ for all $i \in S_j^s$. #

The requirements of this proposition are that (a) spillovers exist, (b) the middle subgroup is now in the majority in the entire group, (c) the members of the subgroups with extreme preferences are not worse off in a large unitary state that consists of all states than in a unitary state built just by the subgroup they belong to and (d) when equal sized parts of the border subgroups build a union, depending on its institutional shape either this union is not break-up resistant or at least one member of this union is not better off than in the equilibrium.

When a large center exists and differences in preferences are relatively small, a unitary state that is a compound of all small states is a coalition-proof equilibrium if by specific requirements it is ensured that the two minor groups do not want to form a union on their own. The higher rents are in a federation and the larger the middle group is, the easier these conditions can be fulfilled.

4 Concluding remarks

The aim of this paper was to compare small states, unitary states and federations from the viewpoint of voters when spillovers exist, preferences differ across states and the magnitude of governments' rents depend on the institutional shape. At first glance, federations seem to be a superior institution since, in contrast to small states, they do not fail to internalize cross-border externalities of publicly provided goods, and, differently from unitary states, they are able to differentiate public spending according to different preferences in different regions. However, a complete positive analysis of policies also requires a discussion of the citizens' ability to control governments. The starting point of this paper in this respect was, based on Barro (1973), that voters can limit the discretionary abusive power of governments by using retrospective voting strategies. Employing this approach and following Persson, Roland and Tabellini (1997), it was shown that due to the common-pool problem rents of governments in a federation are higher than in small or large unitary states. Hence, a comparative analysis has to weigh the size of spillover effects, the variance in the distribution of preferences and the magnitude of governments' rents.

The comparison led to the following results. First, and not surprising, if citizens were identical, the size of the state's territory should exactly match the users of the publicly provided good such that all spillover effects are internalized. Under the circumstances, adding a second level of government only increases costs. Second, if preferences differ across states, large unitary states are never Pareto-dominated by small states or federations since the choice of the median voter is clearly a unitary state. Third, citizens benefit from a multi-level government only if their preferences heavily differ from the median-voter's preferences and if spillover effects are strong.

Based on this comparison the paper also discussed the creation of unions. Making specific assumption on the distribution of preferences, it analyzed strong Nash equilibria and coalition-proof equilibria at the union formation stage. The results were as follows. First, provided that states are divided into two homogeneous groups of equal size, a large federation, which consists of all small states, is a strong Nash equilibrium if spillover effects are sufficiently strong and differences in preferences are sufficiently large. Second, under further strong assumptions this result still holds if a third group of states is added when citizens in these states have moderate preferences with respect to that public good that is provided by the lower-level of government. Weaker conditions for a coalition-proof equilibrium to exist were also given. Third, when a large center exists and differences in preferences are relatively small, a unitary state that is a compound of all small states is a coalition-proof equilibrium if it is ensured that the two minor groups do not want to form a union by their own.

On the one hand, this paper has something in common with Alesina, Angeloni and Etro (2001 a, b), but there are also several important differences. First, Alesina, Angeloni and Etro (2001 a, b) considered benevolent governments that maximize the utility of the median voter. This paper explicitly took into account the necessity to control politicians with adequately adjusted voting rules. Second, while Alesina, Angeloni and Etro (2001 b) discussed a sequential choice of public spending at the different levels of government, this paper assumed that voters at the different levels simultaneously choose the parameters of the voting strategies. Third, and most important, Alesina, Angeloni and Etro (2001 b) found out that depending on the timing of policy choices all members of a unitary state or at least the majority of members benefit from a federal structure. This paper discussed an explicit disadvantage of the multi-level structure: the common-pool problem in taxation. Hence, the majority of citizens can well be among the losers of a transformation from a unitary state to a federation. Fourth, Alesina, Angeloni and Etro (2001 a) also discussed coalition-proof equilibria at the union formation stage. However, in contrast to this paper, while discussing the equilibrium size of a union they excluded by assumption that members of a union that exit together could build a new union.

On the other hand, the discussion of the common-pool problem in this paper heavily relied on Persson, Roland and Tabellini (1997). However, in contrast to them, the purpose of this paper was not to discuss the separation of powers at one particular level of government but to analyze the assignment of the power to tax to more than one level of government [see also Persson and Tabellini (2000)]. Therefore, unlike Persson, Roland and Tabellini (1997) this paper explicitly allowed for differences among voters.

Leaving behind the strict model inherent discussion, the results could be used to explain the existence of federations and unitary states and to predict the future development of existing unions like the European Union. According to the model, countries with two different relatively homogeneous large groups such as the French and German spoken cantons in Switzerland will be organized as federations with tax autonomy at both levels of government. Even countries with more than two groups like the United States, where attitudes towards state intervention differ heavily across states, could well be stable federations if spillover effects at the national level are strong. If the United States were considered as a federation of three groups of states - the east coast states, the west coast states and the central states – the model predicts that the autonomy of states will not be abolished as long as the group that contains the median is not too large. In contrast, one should not be surprised that lower level governments are less autonomous in setting taxes in countries such as Germany where preferences are relatively similar across states. In those countries, the interest in low rents outweigh the necessity to adapt policies to differing preferences. The model also makes some statements concerning the development of the European Union possible. In order to internalize spillover effects certain common tasks should be assigned to the union level. However, since the variance in the preferences' distribution is, particularly after the enlargement, large, national governments should still be responsible for a lot of tasks. In order to reduce rents of the politicians, additional levels of government like the German laender will likely become less autonomous if voters are powerful enough to enforce it.

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Proof of proposition 1

Obviously $u_i^s > u_i^l$, for all $i \neq m$ and $u_m^s = u_m^l$. Furthermore, $u_i^s - u_i^f = (1-\delta)(\delta y - f^f - \max\{s_j^f | j \in U\})/(2-\delta) + H(f^s) - f^s - (H(f^f) - f^f) > 0$ is fulfilled if $s_i < \max\{s_j | j \in U\}$ since f^s maximizes $H(f^s) - f^s$, but f^f does not maximize $H(f^f) - f^f$. Moreover, $u_i^s - u_i^f = (1-\delta)(\delta y - f^f - \max\{s_j^f | j \in U\})/(2-\delta) + H(f^s) - f^s - (H(f^f) - f^f) + a_i G(s_i^s) - s_i^s - (a_i G(s_i^f) - s_i^f) > 0$ if $s_i = \max\{s_j | j \in U\}$ since s_i^f also does not maximize $a_i G(s_i^f) - s_i^f$ in that case. QED

Proof of proposition 2

Obviously due to the spillover effect $u_i^l > u_i^s$, for all i. Furthermore, $u_i^l - u_i^f = (1-\delta)(\delta y - f^f - \max\{s^f | j \in U\})/(2-\delta) + H((1+b(n-1))f^l) - f^l - (H((1+b(n-1))f^f) - f^f) + a_m G(s_m^l) - s_m^l - (a_m G(s_i^f) - s_i^f) > 0$ for all i since rents in federations are higher, f^f does not maximize $H((1+b(n-1))f^f) - f^f$, and s_i^f does not maximize $a_m G(s_i^f) - s_i^f$ if $s_i = \max\{s_j | j \in U\}$. QED

Proof of lemma

First, s_i necessarily increases in both types of a union as the preference parameter a_i increases if $s_i < \max\{s_j | j \in U\}$. Furthermore, for $s_i < \max\{s_j | j \in U\}$ $u_i^s - u_i^f = \Delta_i$, $\Delta_i = (1-\delta)(\delta y - f^f - \max\{s_j^f | j \in U\})/(2-\delta) + H(f^s) - f^s - (H((1+b(n-1))f^f) - f^f)$, is independent of a_i. However, $u_i^s - u_i^f = (1-\delta)(\delta y - f^f - \max\{s_j^f | j \in U\})/(2-\delta) + H(f^s) - f^s - (H((1+b(n-1))f^f) - f^f) + a_i G(s_i^s) - s_i^s - (a_i G(s_i^f) - s_i^f)$ is smaller than Δ_i for $s_i = \max\{s_j | j \in U\}$ since this state would choose s_i as to maximize $a_i G(s_i) - s_i + (1-\delta)\max\{s_j | j \in U\}/(2-\delta)$ if it were a member of the federation. QED

Proof of proposition 3

(a) Due to the spillover effect, $u_m^l > u_m^s$ holds, and according to the proof of the previous proposition $u_m^l > u_m^f$ is also fulfilled. (b/c) Using the envelope theorem, $d(u_i^s - u_i^f)/db = -(n-1)f^f H'((1+b(n-1))f^f) < 0$ for all i. Hence, (b) follows from the first proposition. Due to high rents and strong disincentive effects with respect to deviations by the federal government, the statement (c) is slightly weaker than (b). QED

Proof of proposition 4

(I) (1) Due to (b) a union $\{1,\ldots,N\}$ is a federation and, therefore, $u_i^u(F)=u_i^f(F)$ for all i. (2) Because of (a) voters are worse off in a smaller federation since $dH(((1+b(n-1))f^f)-f^f/(2-\delta))/dn = bf^f H'((1+b(n-1))f^f)$ is clearly positive. Furthermore, $\max\{s_j|j\in U\}$ certainly does not decrease when the federation becomes larger. (3) From (c) follows that voters are worse off in a homogenous large unitary state S_j, j = s, l. Due to (a) this is all the more the case in 'smaller' homogenous large unitary states and in a small state. (4) In a heterogeneous large unitary state L with the same median as in F, voters are not as well off as in F because of (a) and (b). (5) In an asymmetric heterogeneous large unitary state L, where the median voter is either a member of S_s or of S_l, a member i of the minority achieves a lower utility level than $u_i^l(F)$ due to (a) and the unsatisfactory median-voter program in comparison to the median-voter program in F if F were a large unitary state. Therefore, because of (b) voters are worse off than in the federation F. (6) Hence, neither single members nor all members of some coalition could gain from deviating and F is a SNE.

(II) (1') Provided that (c) holds in strict form, from (2) – (4) follows that the only alternative CPE could be a set of unions where at least one union is a asymmetric heterogeneous large unitary state since in cases (2) – (4) all states would benefit from building the federation F, which is indeed break-up resistant. (2') If the size of the largest union were smaller than N/2, not only the members of the minority in a asymmetric large unitary state but also the members of the majority in this state would benefit from building a federation F because of (a) and (c). Members of other relatively small unions also benefit. (3') However, if the size of the asymmetric heterogeneous large unitary state were larger than N/2, by (a) till (c) it is not ensured that the majority also benefits from building the federation F. (d) guarantees that in this case the members of the minority would benefit from staying alone. (4') Hence, no CPE apart from F exists. QED

Proof of proposition 5

(1) Due to (b) and (c) a union $\{1,\ldots,N\}$ is a federation and, therefore, $u_i^u(F)=u_i^f(F)$ for all i. (2) Because of (a) and possibly higher rents voters are worse off in a smaller federation. (3) From (d) follows that voters are worse off in a homogenous large unitary state S_j, j = s, m, l. Due to (a) this is all the more the case in 'smaller' homogenous large unitary states and in a small state. (4) In a heterogeneous large unitary state L with $s^l(L)\leq s^l(F)$ ($s^l(L)\geq s^l(F)$), members of S_l (S_s) are not as well off as in a federation F because of (a) and (c) and the unsatisfactory median-voter program. Hence, a deviating coalition which consists of members of S_l and S_s (and possibly S_m) that build a large state does not exist. By similar reasoning it becomes clear that a deviating coalition composed of members from S_l (or alternatively S_s) and S_m that build a large state would also not exist if the median m were a member of S_m. (5) Since in a heterogeneous large unitary state L that consists of members of S_m and S_l (or alternatively S_s), because of (e) members of S_l (S_s) or of S_m are not better off than in the federation F, a deviating coalition composed of members from S_l (or alternatively S_s) and S_m that build a large state would not exist even if the median m were not a member of S_m. QED

Proof of proposition 6

(1) – (4) see the proof of the previous proposition. (5) Because of (e) a deviating break-up-resistant coalition composed of members from S_l (or alternatively S_s) and S_m that build a large state would not exist even if the median m were not a member of S_m.

QED

Proof of proposition 7

(1) Due to (b) a union $\{1,\ldots,N\}$ is a large unitary state and, therefore, $u_i^u(L) = u_i^l(L)$ for all i. (2) Therefore, due to (a) - and the higher rents in federations - members of S_m will never deviate, neither alone nor with others. (3) From (c) follows that voters are worse off in a homogenous large unitary state S_j, j = s, l. Due to (a) it is all the more the case in 'smaller' homogenous large unitary states and in a small state. (4) A asymmetric union U that consists of members of S_l and S_s will be large unitary state. Since either $s^l(U) < s^l(L)$ or $s^l(U) > s^l(L)$ holds and because of (a), members of S_l or S_s are worse off than in L. (4) Next a symmetric union that consists of members of S_l and S_s is considered. In case of (d1) at least one group would prefer to stay alone, and such union is, therefore, not break-up resistant. In case of (d2) some members of this coalition would be not better off than in the large state L. QED