A General Theory of Price and Quantity Aggregation and Welfare Measurement

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Abstract

The paper presents a general theory of the aggregation of prices and quantities that unifies the field and relates topics that in the past have been treated separately and unsatisfactorily, or not at all. The theory does without the common but unrealistic assumptions of homotheticity, or representative agents and is valid with or without an explicit utility maximization assumption. Two different derivations are given, one in continuous time, using Divisia integrals, and one employing more traditional discrete arguments. The unifying concept is the money metric, which is interpreted as a partial welfare indicator, rather than as a comprehensive welfare measure. On this basis, a consistent set of chained price and quantity indexes for a set of additive time series, such as those in the national income and product accounts, is derived. All variants of the theory lead to Törnqvist indexes defined on the appropriate data set. A numerical example confirms that in the non-homothetic case, these indexes are superior both to Fisher's 'ideal' index and to the consumer surplus approximation.

JEL Classification: C43, D61.

Keywords: chain indexes, consumer surplus, cost-of-living, divisia integral, money metric price index, quantity index, real consumption, Törnqvist index.

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1. INTRODUCTION

> Empirical experience is abundant that the Santa Claus hypothesis of homotheticity in tastes and in technical change is quite unrealistic. Therefore, we must not be bemused by the undoubted elegances and richness of the homothetic theory. Nor should we shoot the honest theorist who points out to us the unavoidable truth that in non-homothetic cases of realistic life, one must not expect to be able to make the naive measurements that untutored common sense always longs for; we must accept the sad facts of life, and be grateful for the more complicated procedures economic theory devises. (Samuelson and Swamy, 1974, X. Concluding Warning).

Measurement is at the heart of empirical science because it links theory with empirical phenomena. The most important measures in economics are those that aggregate quantities into measures of real output, or input growth, and those that aggregate prices into measures of inflation. The most important of these measures are: real GDP and its principal component real consumption, the consumer price index and the empirical measure of consumer surplus. Associated with each of these measures are separate and essentially unsatisfactory theories. The consequence has been that measurement as a subject of academic and theoretical research has largely disappeared. Furthermore, those who produce and use these measures are often skeptical, or even hostile, towards the existing theory. This state of affairs is unsatisfactory. Moreover, as this paper demonstrates, the empirical measures that have been constructed have been either sub-optimal, or outright incorrect.

This paper presents a unified theory of price and quantity aggregation based on an elementary, but neglected insight: all of the measures discussed are attempts at restoring the money metric when both prices and quantities vary. This is obvious from the fact that when prices are constant, all of the quantity measures reduce the change in expenditure; similarly, when quantities are constant, all of the price measures reduce to the expenditure change. The problem is thus to decompose the expenditure change into its price and quantity components when both prices and quantities change. Curiously, this elementary insight has never been fully elaborated. Theorists have typically expressed skepticism regarding the money metric and have concentrated their efforts on the elaboration of alternative concepts, such as Pareto optimality or ordinal social welfare functions.

A special form of the money metric, also discussed in this paper, is money metric utility. Samuelson who invented the term in Samuelson (1974) has remained deeply skeptical about the normative significance of the concept. In Samuelson (1990, pp.175-6) reiterated his skepticism. His main objection is that an infinite number of cardinal utility functions can be constructed. This is true; however, every form of measurement involves some arbitrary normalization. The most natural normalization for the money metric is the base period price level. In this paper, I systematically explore the properties of the money metric, both as a theoretical concept and as an approximation derived for empirical measurement.

I deal with three different assumptions regarding the generation of the data. In the most general case considered, no explicit assumption about data generation is made. This is important when applying the theory to the computation of real GDP and its components, since a realistic and computationally feasible model of how all of these data are generated by rational agents is not in sight. The second assumption is that a utility maximizing household generate the quantities. I treat this case without the assumption of homotheticity. The main interest here is as a stepping-stone to assumption 3, that heterogeneous, utility maximizing consumers generate the data.

In going from the assumptions to formulas for measurement, I use two different styles of argumentation, both of which have long traditions. One is to use continuity to define the theoretical measure at a point and then to extend to an interval by integration. This approach is primarily associated with Divisia. The alternative that has dominated the economic theory of price and quantity indexes[1], as well as the axiomatic approach, is to start with the discrete comparison of two price/quantity vectors. Given three alternative assumptions and two alternative styles of argumentation there are in all six variations of the theme. Remarkably, each of the six variations leads to the same result: in each case, the proper measures are a pair of Törnqvist price/quantity indexes defined on the relevant data. Törnqvist indexes turn out to be the uniquely suitable tool for dealing with the general and realistic case of non-homotheticity.

The plan of the paper is as follows: Section 2 reviews the principal theories of price and quantity aggregation. Section 3 develops the without an explicit maximization assumption. Törnqvist indexes are derived as approximations of Divisia integrals as well as from a set of axioms developed by Balk/Diewert. Section 4 shows that the theory remains valid under the assumption of a single utility maximizing consumer and Section 5 extends the analysis to the case of many consumers. Section 6 discusses the chaining of indexes that is essential for the analysis of time series. Section 7 extends the theory of the money metric to sub-aggregates and in particular discusses how the sectors of real GDP should be defined. Section 8 is a numerical example comparing the Fisher and Törnqvist indexes and the consumer surplus approximation. Section 9 is the conclusion.

2. EXISTING THEORIES OF PRICE/QUANTITY AGGREGATION

The first half of the Twentieth Century witnessed an intensive effort to reshape economics and the social sciences in accordance with the natural science paradigm in order to obtain a scientific basis for the solution of social problems. This effort initiated the production of the vast array of social and economic statistics that we have today, including the NIPAs. Many of the most prominent economists and statisticians of this era attempted to provide a theoretical rationale for these measurements. Unfortunately, these

[1] A note on terminology: The designation 'index number theory' for this field of study is as established as it is illogical. The theory is about certain algebraic expressions, or functions, of prices and quantities regarded as variables. It is not about 'numbers' The confusion that this terminology gives rise to can be seen from the following dictionary definitions of'index' in the present context: "A number derived from a formula used to characterize a set of data" (American Heritage). "a numerical scale by means of which levels of the cost of living can be compared with some base number" (Collins). "(A number in) a scale relating (usu. in the form of a percentage) the level of prices, wages, etc., at a particular time to those at a date taken as a base." (Oxford). A correct definition is: A formula that averages the proportional changes of a set of prices, or quantities, between two observations.

efforts failed and largely ceased, following a devastating critique by Samuelson.[2] As an alternative, he embraced the notion of a distributionally sensitive, ordinal and invariant social welfare function. Subsequently there developed a split between researchers at statistical agencies, concerned with the minutia of generating statistical data and academic economists producing abstract theories.

Since the money metric is not a SWF in the Bergson/Samuelson sense, it is desirable to explicate the concept. Economic policy is invariably the result of a mixture of formal and informal considerations. For the formal measures that enter the policy formation process, I propose the term *welfare indicator.* Examples are the level and growth rate of GDP, the CPI, measures of inequality and many other economic and social measures. The price and quantity indexes, based on the money metric, developed in this paper are such indicators. The criticism that the money metric is not distributionally sensitive is irrelevant, since it is not claimed to be a comprehensive SWF.

The split between theory and practice in relation to measurement is so deep and has existed for so long, that by now few economists are aware of it. Therefore, a brief review of the principal existing theories of price/quantity aggregation, with particular emphasis on their underlying assumptions, seems advisable. My aim is not to be comprehensive, but to point to central weaknesses of the different approaches, thereby justifying the need for a comprehensive theory.

2.1 The Money Metric and the Axiomatic Theory of Index Numbers

I argue in this paper that the problem of aggregating prices and quantities should be interpreted as that of restoring the money metric when prices are variable. While I believe that this view is implicit in the theories discussed below, it has hardly ever been made explicit.[3] The fact that the money metric values goods by their prices implies maximizing behavior on the part of agents, and competitive markets, since otherwise prices lose their normative function. However, it does make a difference, whether the maximizing behavior is explicitly modeled, or not. The axiomatic theory of indexes, treated in the present sub-section, leaves the maximization assumption implicit. It is made explicit in the economic theory of index numbers considered in the following sub-section.

What are the key insights of the axiomatic approach? Unfortunately, the views of the principal authors are contradictory. Irving Fisher, who may be regarded as the founder of the axiomatic, or 'test' approach wrote in the introduction to his treatise:

> An index number of prices, then, shows the average percentage change of prices from one point of time to an-other. The percentage change in the price of a single commodity from one time to another

[2] As early as his dissertation, which became the *Foundation of Economic Analysis,* Samuelson (1947) radically rejected any kind of cardinal measure as meaningless and developed his revealed preference approach as an alternative. Samuelson (1942) had shown that there was no meaningful sense in which the marginal utility of income could be constant, a central assumption of traditional consumer surplus analysis. Samuelson (1950) reviewed the attempts at justifying a real national income measure on the basis of compensation measures and showed that all of them were unsatisfactory.

[3] The concept of a money metric is more general than 'money metric utility' introduced by Samuelson (1974). The latter is discussed in Section 4.

is, of course, found by dividing its price at the second time by its price at the first time. The ratio between these two prices is called the price relative of that one particular commodity in relation to those two particular times. An index number of the prices of a number of commodities is an average of their price relatives.

This definition has, for concreteness, been expressed in terms of prices. But in like manner, an Index number can be calculated for wages, for quantities of goods imported or exported, and, in fact, for any subject matter involving divergent changes of a group of magnitudes. (Fisher, 1927, p.3).

In developing his test approach, Fisher did not directly elaborate on this fundamental initial insight. He proposed a set of 'tests' that are more closely related to the 'fixed basket' approach embodied in Laspeyres and Paasche indexes and ultimately led to his 'ideal' index that averages the two, as the only one satisfying the axioms.[4]

Diewert (2001a), in the section on the axiomatic approach, states:

…P is regarded as a function of the two sets of price and quantity vectors, p^0, p^1, q^0, q^1, that are to be aggregated into a single number that summarizes the overall change in the n price ratios, $p_1^1/p_1^0, ..., p_n^1/p_n^0$. (p. 23).

Further:

If n=1, so that there is only one price and quantity to be aggregated, then a natural candidate for P is p_1^1/p_1^0, the single price ratio, and a natural candidate for Q is q_1^1/q_1^0, the single quantity ratio. When the number of commodities or items to be aggregated is greater than one, then what index number theorists have done over the years is propose properties or tests that the price index P should satisfy. These properties are generally multi-dimensional analogues to the one good price index formula p_1^1/p_1^0. (p.24)

Diewert goes on to list 20 axioms that characterize Fisher's ideal index. The axioms are formulated in the levels of the variables, not their relatives.

In a recent paper, Diewert and Balk (2000) offer a set of axioms that are in the spirit of the remarks by Fisher and Diewert cited above, leading to a Törnqvist index. This work is described in Section 3.

In the introduction to Theil (1968), that provides one of the foundations to the discrete portion of the present paper, he writes:

The present approach differs from the earlier one to the extent that it works systematically with the logarithms of income and prices rather than these variables themselves. This seems a natural modification, because index numbers are concerned with relative changes in prices and income, and; it leads to rather substantial theoretical improvements as will be shown below.

Working with the logarithms of prices and quantities implies that one is dealing with their relatives, rather then levels. The formulation in terms of logarithms or relatives is also characteristic of Divisia's approach. I believe that the contributions of these authors, as well as the present paper, demonstrate that to view index formulas directly as averages of relatives, rather than as fixed baskets, is the deepest and most productive intuition in this field.

One more property of indexes should be regarded as fundamental. The theory should involve a price and a quantity index pair that satisfy the *product rule*

(2.1) $$PQ = \frac{y^1}{y^0},$$

[4] The complete axiomatic characterization of Fisher's index was given in Diewert (1992b) and is also discussed in Diewert (2001, Section F).

where y^t is the period's expenditure. I refer to such pairs as *dual*, since they exhibit the property of microeconomic duality theory that an expression in prices conveys the same information as one in quantities. The importance of the product rule is that it assures that the expenditure change is fully and exactly decomposed.

2.2 The Economic Theory of Price and Quantity Indexes[5]

The Contribution of Samuelson and Swamy

An influential article by Samuelson and Swamy (1974) is as exemplary for the economic theory of index numbers. The introductory part of their article contains the following statement:

> …we derive here canonical index numbers of price and quantity that do meet the spirit of all of Fisher's criteria in the only case in which a single index number of the price of cost of living makes economic sense-namely, the ("homothetic") case of unitary income elasticities…(p. 566,).

For the case of a homothetic consumer as well as for a linear homogeneous production frontier, the authors derive a number of very elegant results: In the following let $\mathbf{p}^t, \mathbf{x}^t$ be the relevant price quantity vectors and $y^t = \mathbf{p}^t \mathbf{x}^t$ the corresponding total expenditure. In addition to meeting an important set of Fisher's tests, they show that: **a.** The price index P is independent of quantities and takes the form $P\left(\mathbf{p}^0, \mathbf{p}^1\right) = P\left(\mathbf{p}^1\right) / P\left(\mathbf{p}^0\right)$. **b.** The quantity index Q is independent of prices and given by $Q\left(\mathbf{x}^0, \mathbf{x}^1\right) = Q\left(\mathbf{x}^1\right) / Q\left(\mathbf{x}^0\right)$. **c.** The two indexes are dual in the sense that $PQ = y^1 / y^0$. **d.** The theoretical indexes are approximated quadraticly by any symmetric mean of a Laspeyres and a Paasche index, in particular, by Fishers 'ideal' geometric index. These results involved bilateral indexes, i.e., those directly comparing two price/quantity situations. For the continuous Divisia indexes, they prove: **e.** Under homotheticity, the Divisia indexes are independent of path. The paper contains a negative result that is potentially important given the rising popularity of Fisher's ideal index: **f.** The quadratic approximation property of Fisher's index does not carry over to the non-homothetic case.

Regarding the realism of their theory, the authors are under no-illusion:

> From this review of the general case, it will be appreciated how much more simple is the homothetic case. If only it were as realistic as it is elegant! (p.577).

Even more pointed are their concluding remarks, quoted at the beginning of this paper.

Their comment regarding the "more complicated procedures economic theory devises", presumably for the non-homothetic case, is puzzling since no such procedures are described, or referenced in their paper. They appear to have been unaware of Theil's path breaking contribution described below.

One more issue raised by the authors is important. They criticize the fact that the economic theory of indexes has been predominantly a theory of price indexes. Instead, they advocate and practice a symmetric treatment of price and quantity indexes. This symmetry characterizes also the work of Divisia and Theil, discussed below, on which the present paper is based.

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[5] An excellent recent survey of both the economic theory discussed in this subsection as well as of the axiomatic theory discussed in the next one is Diewert (2001a).

Diewert's Contribution

Erwin Diewert has been a prolific contributor to all aspects on index number theory for several decades. He is best known for his theory of 'superlative' indexes. These are broad classes of formulas that approximate a homogeneous aggregator function quadraticly. From the point of view of the present paper, the most important of these are the Fisher and Törnqvist indexes. As can be seen from his recent review (Diewert, 2001) the bulk of the economic theory has remained focused on the price index for the homothetic case and the individual consumer.[6]

The Econometric Approach

The econometric approach is a part of the economic theory and aims at the same kind of inferences, but with different methods. A defining characteristic is that a concrete aggregator function is specified, be it the production function of a firm, or the utility function of a consumer. Inference then proceeds by econometric methods rather than by means of indexes.

Much work, referred to as 'stochastic frontier analysis', has been done in this vein on the measurement of the technical efficiency of firms[7]. This work could serve as a paradigm for the successful integration of economic and econometric theory and practice. Unfortunately, as I argue below, the approach cannot be extended with equal success to the output side of the economy.

A number of economists have contributed to an extension of the econometric approach to the measurement of the welfare of an aggregate of consumers. The contribution of Jorgensen (1990) may be regarded as the culmination of these efforts. He introduces a number of important advances that ought, in one way or another, be adopted by the producers of aggregate statistics. **a.** There is an integrated treatment of the cost-of-living and real consumption. **b.** The aggregate measures are derived from detailed sectoral measures that enable inferences to be made regarding relatively homogeneous groups of consumers. **c.** Welfare measures that take account of household composition are systematically computed for all household classes. These features are not necessarily tied to the econometric approach.

The econometric approach, when applied to consumer behavior involves some fundamental difficulties that have not been adequately discussed by its proponents. **a.** Consumers are assumed to maximize an indirect 'translog' utility function that is identical for all consumers, except for demographic variables. Given that the diversity of tastes in advanced industrialized economies is one of their outstanding characteristics, the adequacy of such a representation has to be demonstrated. **b.** The 'goods' on which the utility functions are defined are five broad categories of consumption expenditure, each being in fact an index defined over millions of individual commodities. These are not the 'goods' of economic theory and their use in this manner needs to be justified.

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[6] There have been some extensions to the non-homothetic case as well as to an aggregate of consumers. See Diewert (2001; Section I).

[7] A comprehensive exposition is Kumbhakar and Lovell (2000).

The econometric approach has not resonated with the statistical agencies producing national economic data. The reason is probably due less to the objections that I have raised, than the inherent complexity of the approach and the fact that it is only applicable to the consumption component of GDP.

2.3 Consumer Surplus

The long history of CS begins with Dupuit (1844), who suggested the area under a demand curve, between two prices as a measure of the 'surplus' utility enjoyed by the consumer. The term 'welfare triangle', or 'Harberger triangle' often used to refer to this area involves the assumption of a linear demand curve.[8] Marshall put his finger on a weakness of Dupuit's argument: the implicit assumption of a constant marginal utility of money, which has bedeviled the subject until modern times.[9]

The rise of the econometric movement and of mathematical economics early in this century motivated efforts at an analytical derivation of the triangular approximation and related measures. Specifically, this was attempted by Hotelling (1938), Hicks (1940-41, 1941-42, 1945-46) and Harberger (1966, 1971). Unfortunately, none of these attempts was satisfactory. They all involve at some point an implicit assumption of constancy of the marginal utility of income, or of some other expressions, homogeneous in $\mathbf{p}, y$.

As mentioned above, Samuelson has been a consistent critic of all cardinal measures including CS. In an extensive survey of the subject (Samuelson 1990, p. 12) he wrote:

> …to this day consumers' surplus discussions involve ambiguities, confusions, approximations, redundancies and Napoleonic pretensions hard to match in other parts of economic theory.

The most thorough critical evaluation of Harberger triangles is due to Diewert (1976b). He showed the following undesirable property: For certain price/quantity combinations, namely those falling within the 'zone of indeterminacy' of revealed preference theory, the measure can be made arbitrarily large positive or negative by a mere rescaling of prices in either period. He concluded that this cannot be a good measure.[10]

In his recent survey on welfare measurement Slesnick (1998) concluded the section on CS by remarking:

> …with multiple price and expenditure changes, consumer's surplus is not single-valued and is of no use in providing an approximation to the Hicksian surplus measures..

Applied economists, committed to CS analysis, ignore the theoretical critique. Instead, they continue attempting to justify the approximation, by repeating the geometric derivation that goes back to Dupuit, even though the unsoundness of this argument has long been known.

The economic theory of index numbers and consumer surplus theory are at opposite ends of a theoretical/empirical spectrum. The former has an elegant theory that is in the main inapplicable because of the narrowness of its assumptions. The latter has a theory that appears to be suitable for its intended application: measuring welfare changes over a

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[8] Auerbach (1985) discusses the history of the welfare triangles.

[9] McKenzie (1983, Ch. 4) discusses both the contribution of Dupuit and the analysis of Marshall. Formally, the marginal utility of money is homogeneous of degree minus one in prices, so that the assumption of constancy makes no sense. Samuelson (1942) provided an exhaustive analysis of this issue.

[10] Diewert proposed to normalize Harberger's measure by deflating prices by expenditure and showed that the resulting measure has better properties.

collection of heterogeneous, non-homothetic households. What has been missing is a derivation that satisfies common mathematical standards. The practitioners of consumer surplus have been unwilling to exchange the latter for the former. Needed is a theory that is both applicable and rigorous.

The subject of CS will be taken up again at the end of this section and in Section 8.

2.4 The NIPA Accounts

GDP and its components are the most important measures of an economy's performance. It is a testimony to the power of measurement that modern economics started when Petty (1691) made the first estimates of national product for Ireland, thereby turning the focus of economic analysis away from the Physiocrats fixation on the treasure of the sovereign, towards general economic activity. The early Twentieth Century saw a great rise in measurement activity, including the construction of the NIPAs and theorizing about their meaning. Unfortunately, theorizing about 'national income' proceeded along a path that led to a dead end. Rather than accepting the money metric as a primitive concept that only needed to be extended to the case of non-constant prices, theorists attempted to reduce it to the Pareto Principle via various compensation rules. After this research terminated, following Samuelson's (1950) critique, the subject essentially disappeared from academic economics and became the responsibility of the statistical agencies. The result, as I argue in Section (7), was that the problem of computing GDP and its components in real terms was never solved.

2.5 The Consumer Price Index[11]

In a recent article, Triplett (2001) reviews a controversy among statisticians and agencies that produce the CPI figures for different countries. Ostensibly, the debate swirls around the question: Is the CPI a cost-of-living index (COLI)? As Triplett notes, the opponents of the COLI concept engage in a rhetoric that is infused with hostility toward economic theory and essentially devoid of meaning. They claim that as they are committed to a "pure price index" an alternative, but this turns out to be simply a fixed base Laspeyres index, hardly a theoretical concept. Moreover, even the advocates of this solution favor changing the base every 5 or 10 years.

Triplett takes the economic, or 'COLI' position, but in its present state of development that position is also beset by major difficulties, some of which he explicitly mentions. The COLI concept generally accepted by economists goes back to Konüs (1924) and is given by the ratio of the cost of a given utility level in two periods. The problem begins with the derivation of an observable measure to approximate the theoretical index. Konüs showed that Fisher's ideal price index is a quadratic approximation if the consumer's utility function is a homogeneous quadratic. Diewert (1976a) and in many later publications extended this result. However, as already discussed above, the homotheticity assumption is unrealistic.

The problem of reconciling theory and practice becomes even more severe in going to an aggregate, or group index. The theoretical group COLI is defined as an average over those of households. If the weights are the household expenditure shares, the index

[11] A detailed survey of the theory and applications is Diewert 2000a).

is said to be ‘plutocratic’, if the shares are equal, ‘democratic’. These terms are taken from political theory and I regard them as misleading in the present context. The ‘plutocratic’ index is essentially one for the market as a whole. Any market is influenced more by the big than by the little spenders, to refer to it for that reason as ‘plutocratic’ should appeal to no one, except perhaps a dyed-in-the-wool Marxist. The plutocratic index corresponds to the aggregate COLI defined in Section 5. It measurers the cost of keeping every household at the same utility level when prices change.

In an applied context, the meaning of an aggregate COLI seems unclear even among economists. Triplett mentions that it is often taken to refer to a ‘representative consumer’. This is the position taken in the most extensive recent study of the CPI, the ‘Boskin Report’, (Boskin, et al., 1996). “The CPI is not equipped to account for special characteristics of different consumers or groups of consumers.” (p. 30). “…the aggregate indexes use data reflecting representative consumers.” (p. 72). This concept makes sense if all consumers are faced with the same prices and in addition have either identical incomes and preferences, or identical homothetic preferences.[12]

The operational content of COLI advocacy is often reduced to the advocacy of a symmetric index, to account for intra-period substitutions and chaining to account for inter-period substitutions. This is the way to proceed, but in itself not much of a theory.

In my view, much of the confused and unsatisfactory nature of the CPI/COLI discussions is due to the absence of the dual concept of a real consumption index (RCI). The RCI gives the cost of a changing utility level at base period prices. A constant utility COLI must seem nonsensical, as it does to many statisticians, if one does not understand that deflation by a COLI yields an RCI.

2.6 Immediate Antecedents: The Contributions of Divisia and Theil

Scientific progress in an ill-defined field usually takes a circuitous route. This is true also of the path I followed, leading up to the present paper. My original motivation was to remedy the perceived lack of an economic theory for computing real GDP and its components.

In trying to get a handle on the problem, my attention was drawn to CS and ‘Harberger triangles’ because of their additive nature. This confronted me with the fact, discussed above, that none of the received derivations of the formula was correct. Ultimately, I discovered several valid proofs. This was published in Hillinger (2001a) were I also generalized CS theory to give a symmetric treatment of price and quantity variations.

Having come this far, it became clear to me that I had still not arrived at my goal. The reason is that in the formula for the Harberger triangle, $\frac{1}{2}\left(\mathbf{p}^0+\mathbf{p}^1\right)\left(\mathbf{x}^1-\mathbf{x}^0\right)$, the second price vector should be deflated to the level of the first.[13] In applications to project evaluation this correction has been and can be ignored, since the effect of the project on any general deflator will be negligible. However, my intended application was to the

[12] These conditions can be weakened slightly. Cf. Deaton and Muellbauer (1980. Section 6.1).

[13] The need to do this was the motivation for Weitzman (1988), discussed further in Diewert (1992a, Section III).

NIPAs and chain indexes where the changes in the price level cannot be ignored. I decided that the deflator must come from the same theory as the triangle itself. This led to the Marshall/Edgeworth formula $P=\left(\mathbf{x}^{0}+\mathbf{x}^{1}\right)\mathbf{p}^{1}/\left(\mathbf{x}^{0}+\mathbf{x}^{1}\right)\mathbf{p}^{0}$.[14] In Hillinger (2001b), I described the construction of a consistent set of NIPA accounts on this basis.

Having completed this paper, I began again to feel dissatisfied. One reason for dissatisfaction was that I had started out looking for a measure that could be applied to the entire GDP and its components, but the CS approximation was derived for household and aggregate consumption, not for other components such as investment or government expenditure. An explicit and realistic maximizing model applicable to all of GDP appeared out of reach. Could there be a theoretical measure in the absence of an explicit maximization assumption? The only candidate that I could think of was a Divisia index. Continuity rather than maximization is here the theoretical assumption. With the Divisia index, I faced a similar problem, as in the case of CS in that none of the existing approximation results was satisfactory. I believe that I was able to obtain a satisfactory solution that will be discussed in Section 3.

About this time, my conceptualization of the entire problem began to change. From the beginning, I had made an implicit assumption that real magnitudes should be computed directly by the relevant quantity measure. This assumption seems to be held universally by NIPA statisticians also. Only, were they use quantity indexes, i.e. ratios, to compute real NIPA components, I used Harberger triangles, i.e. differences. It became clear to me that rather than deflating differences, I could deflate the levels of the nominal expenditures directly. Moreover, this is the only method that takes account of what I regard as a fundamental feature of the money metric: that it must respect relative prices. All of these insights were incorporated in a first version of the present paper. That version was based entirely on Divisia integrals and their approximation by Törnqvist indexes.

For a second version, I had mainly intended to improve the exposition. While looking through some articles on the subject that I had collected in the past, I looked again at Theil (1968), the significance of which had unfortunately escaped me earlier. Now I realized that his contribution was path breaking and relevant for my own work. Theil gives a complete dual theory of household real consumption and cost-of-living for the general non-homothetic case. The theory leads to Törnqvist indexes as quadratic approximations. A more recent paper by Balk and Diewert (2001a) obtained a similar result in the context of the axiomatic approach, without an explicit maximization assumption. These results suggested to me the two-pronged approach of the present paper. I now derive every result twice: once assuming continuity and using the Divisia index, secondly starting from the discrete results of Balk/Diewert and Theil. The second route required an extension of Theil's result to an aggregate of consumers.

My motivation for proceeding in this manner was threefold: **a.** Since the two approaches have different strengths and weaknesses, the fact that they lead to the same results should strengthen our confidence in the results. **b.** Since most index theory is for the discrete case, it seemed desirable to include it. **c.** I hope that I can contribute to rescuing Theil's important work from oblivion.

[14] The derivation is given in Section 8.

2.7 Numerical Aspects

A final important step in the evolution of my own understanding occurred as I was finishing this paper. My motivation had been to provide a theory for the general non-homothetic case. I did not think that the implied Törnqvist index would be superior to other symmetric measures from a numerical point of view. The origins of this view were: **a.** Diewert (1978) had shown that all superlative indexes, of which the Fisher and Törnqvist are the most prominent members, approximate each other quadraticly. This suggests that they should give similar results, a conjecture supported by various numerical computations.[15] **b.** In Hillinger (2001a), I proved that the CS approximation is quadratic for a non-homothetic utility function. In spite of these expectations, I decided to test the three measures against data generated by a non-homothetic utility function. The result was a startling superiority of the Törnqvist index. This forced me to reconsider the existing results on these indexes. The numerical results and their interpretation are discussed in Section 8.

3. DIVISIA INTEGRALS AND TÖRNQVIST INDEXES

3.1 The Bennet and Divisia Differentials

Bennet (1920) laid the foundation of the continuous approach by assuming that total expenditure could be regarded as the product of a price and a quantity index:[16]

(3.1) $$P(t)Q(t) = \mathbf{p}(t)\mathbf{x}(t).$$

He also defined the corresponding differentials

(3.2) $$QdP = \mathbf{x}d\mathbf{p}, \quad PdQ = \mathbf{p}d\mathbf{x}.$$

Based on these differentials, Bennet suggested the following discrete approximations to the indexes, which for simplicity I define for the time periods 0, 1:

(3.3) $$P^1 - P^0 = (1/2)(\mathbf{q}^1 + \mathbf{q}^0)(\mathbf{p}^1 - \mathbf{p}^0)$$

(3.4) $$Q^1 - Q^0 = (1/2)(\mathbf{p}^0 + \mathbf{p}^1)(\mathbf{q}^1 - \mathbf{q}^0).$$

It is easy to check that the two expressions are dual, i.e., they add exactly to the total expenditure change. Dupuit (1844) first proposed the right hand side of (3.3) as an approximation to consumer surplus. Bennet did not attempt to prove any approximation property for these expressions.

Divisia converted the Bennet differentials to proportional form, which makes them independent of units of measurement. The Divisia price differential is derived as follows:

(3.5) $$\frac{PQ\dfrac{dP}{P}}{PQ} = d\ln P = \frac{\sum p_i x_i \dfrac{dp_i}{p_i}}{y} = \sum \alpha_i \, d\ln p_i, \quad \alpha_i = \frac{p_i x_i}{y}.$$

Similarly, the Divisia quantity differential is

(3.6) $$d\ln Q = \sum \alpha_i \, d\ln q_i$$

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[15] Diewert (2001, Appendix 3.2) is the most complete and up to date discussion of these results.

[16] A brief discussion of the related work of Bennet and Divisia is found in Diewert (1998).

The two differentials decompose the change in expenditure according to

(3.7) $$d \ln y = d \ln P + d \ln Q.$$

The decomposition of the nominal change into a price change and a real change is unique and compelling. It is also testimony to the superiority of the continuous approach, since none of the theoretical indexes arrived at by discrete arguments, described below, has this fundamental property.

The decomposition has two paramount features:

a. The real growth rate is a weighted average of the quantity changes and the inflation rate is a weighted average of the price changes, the weights being the expenditures shares.

b. Real growth and inflation rates are dual so that real growth can be computed directly, or indirectly via deflation, yield the same value.

3.2 The Divisia Approach Over an Interval

The problems associated with the Divisia approach arise when taking the continuous approach from a point to an interval. In fact, this problem has thus far not found a satisfactory solution. The root of the problem is that the changes of the theoretical indexes P, Q are not functions of $\mathbf{p}, \mathbf{q}$, but are dependent on the path of these variables over the interval. This is not simply a consequence of the continuity assumption, but rather an inescapable aspect of the money metric that manifests itself also anytime a chain index is constructed.

Divisia pointed out that the standard discrete approximation of his differential leads to a Laspeyres index if the initial slope is used and to a Paasche index if the final slope is used. These indexes are linear approximations. I show in the next section that this result can be improved by taking the average slope. This leads to a Törnqvist index and a quadratic approximation.

Törnqvist (1936) arrived at his index differently. He integrated the Divisia differential over an interval and showed that the value of the integral is given by his index *if value shares are constant.* While this argument was a nice heuristic, leading to the discovery of the index, it does not establish any approximation property for the realistic case of non-constant shares.

Considerable effort has been devoted to finding exact expressions for the Divisia integrals by postulating a specific, ingeniously constructed path for the variables. Depending on the path, it could be shown that the integral equals various well-known or previously unknown index formulas. The problem with this approach is that an actual path will generally be far from the postulated one.[17]

In the following section, I show that the Törnqvist index is a quadratic approximation to an arbitrary path. In section 3.3 I investigate restrictions on the path that assure that both the value of the theoretical index and the approximation to it are reasonable.

[17] A very comprehensive review of work that has been done on Divisia indexes and their approximation is Balk (2000).

3.3 Divisia Integrals

The point decomposition (3.7) can only be a theoretical starting point since in an empirical context we will always be interested in comparing two or more distinct observations. A necessary step in that direction is to define the integrals corresponding to the Divisia differentials.

The Divisia price and quantity integral are

(3.8) $$V_P = \ln \frac{P^1}{P^0} = \int_0^1 \sum \alpha_i(\tau) \frac{p_i'(\tau)}{p_i(\tau)} d\tau$$

(3.9) $$V_Q = \ln \frac{Q^1}{Q^0} = \int_0^1 \sum \alpha_i(\tau) \frac{x_i'(\tau)}{x_i(\tau)} d\tau.$$

Individually these integrals are path-dependent. Their sum is the integral of the total differential of logarithmic expenditure and thus path-independent:

(3.10) $$V_P + V_Q = \int_0^1 d \ln y(\tau) = \ln \frac{y^1}{y^0}.$$

The Divisia indexes corresponding to the integrals are

(3.11) $$P_D = \frac{P^1}{P^0} = \exp V_P, \quad Q_D = \frac{Q^1}{Q^0} = \exp V_Q.$$

The integrals provide an unambiguous decomposition of nominal expenditure over an interval, with properties that are analogous to the differentials. The difference is that the expenditure shares, which serve as weights, are now variable over the path of integration.

These important positive features notwithstanding, there remains the stark fact that the value of the integral is not defined unless a path for the variables of integration is specified. Even more seriously, the path dependence property means for any given endpoints a path can be constructed that is arbitrarily large, either in the positive or in the negative direction. Usually, such a path involves loops in the N-dimensional space of the variables. This suggests that we limit ourselves to considering monotone paths. The insight that only monotonous paths should be considered, when there is path dependence, has been recognized in connection with the computation of chain indexes. Most importantly, this precludes the use of chain indexes with seasonal data.[18]

Since integration and addition commute, we can write the ith component of V_P as

(3.12) $$V_{iP} = \int \alpha_i d \ln p_i.$$

Let $\alpha_i^{\min}, \alpha_i^{\max}$ be the largest and smallest values respectively of α_i at the end points of the path of integration. Then:

(3.13) $$\alpha_i^{\min} \Delta \ln p_i \le V_{iP} \le \alpha_i^{\max} \Delta \ln p_i.$$

The integral is now bounded and the geometry suggests

(3.14) $$V_{iP} \approx \bar{\alpha} \Delta \ln p_i, \quad \bar{\alpha} = \frac{1}{2}\left(\alpha^0 + \alpha^1\right).$$

[18] See Diewert (2001, Section E).

as the most reasonable estimate in the absence of additional information. This approximation leads directly to the Törnqvist index P_T.

(3.15) $$\ln \frac{P^1}{P^0} \approx \sum \bar{\alpha}_i \Delta \ln p_i = \sum \ln \left(\frac{p_i^1}{p_i^0} \right)^{\bar{\alpha}_i} = \ln \prod \left(\frac{p_i^1}{p_i^0} \right)^{\bar{\alpha}_i}.$$

(3.16) $$P_D = \frac{P^1}{P^0} \approx \prod \left(\frac{p_i^1}{p_i^0} \right)^{\bar{\alpha}_i} = P_T.$$

By an analogous argument

(3.17) $$Q_D = \frac{Q^1}{Q^0} \approx \prod \left(\frac{x_i^1}{x_i^0} \right)^{\bar{\alpha}_i} = Q_T.$$

3.4 Quadratic Approximation Property of Törnqvist Indexes

I give two slightly different proofs of the quadratic approximation property of Törnqvist indexes. The first assumes that all variables grow at constant rates. This is the most reasonable assumption one can make if one assumes a specific path. This path can also be given a normative interpretation: If the actual path is unknown, than the integral should be given the value associated with the most regular path. The second proof only requires the assumption of monotone paths. Both proofs are based on the

Trapezoid Rule: [19]

(3.18) $$\int_a^b f(x)\,dx = \frac{b-a}{2}\left[f(a) + f(b) \right] - \frac{(b-a)^3}{12} f''(\zeta), \quad \zeta \in [a,b].$$

The first term on the right is the trapezoidal approximation to the area above (or below) the interval *b-a*, based on the height of the function at the endpoints. The second term is the residual, which is cubic in Δx.

The theorem will be discussed in relation to the price index, the case of the quantity index being analogous. In order to employ the scalar form of the trapezoid rule we write the ith component as

(3.19) $$V_{iP} = \int_0^1 \alpha_i(\tau) \frac{p_i'(\tau)}{p_i(\tau)} d\tau.$$

First Törnqvist Approximation Theorem: Assume that prices and quantities grow at constant rates. Then.

(3.20) $$\exp V_p = P_T + O_3, \quad \exp V_Q = Q_T + O_3.$$

Proof:

[19] For a discussion of the rule and related results see Judd (1998, Section 7.1). The trapezoid rule is closely related to the quadratic approximation lemma given in Diewert (1976a) and used there for a different derivation of the Törnqvist index in the context of the economic theory of indexes. For an exhaustive treatment of the lemma and its applications in index theory see Diewert (2000b).

Letting r_i be the rate for the ith price, it is determined by

(3.21) $$p_i^1 = p_i^0 \exp r_i, \quad \Rightarrow r_i = \ln \frac{p_i^1}{p_i^0}.$$

Then

(3.22) $$V_P = \int_0^1 \sum \alpha_i(\tau) \ln \frac{p_i^1}{p_i^0} d\tau.$$

The ith component

(3.23) $$V_{iP} = \int_0^1 \alpha_i(\tau) \ln \frac{p_i^1}{p_i^0} d\tau$$

is of the standard form given in (3.18), so that

(3.24) $$V_{iP} = \bar{\alpha}_i \ln \frac{p_i^1}{p_i^0} + O_3(\Delta\tau), \quad \bar{\alpha}_i = \frac{1}{2}(\alpha_i^0 + \alpha_i^1).$$

As shown above, this is the approximation that leads to the Törnqvist price index.

Putting all of the results together, we can write the approximation of the theoretical Divisia price index by the Törnqvist price indexes as

(3.25) $$P_D = \exp V_P = P_T + O_3(\Delta\tau).$$

An analogous derivation for the Törnqvist quantity index gives

(3.26) $$Q_D = \exp V_Q = Q_T + O_3(\Delta\tau).$$

The assumption of constant growth rates is the most natural and simplest assumption that can be made in order to prove the quadratic approximation property of Törnqvist indexes. Nevertheless, it is interesting to ask if the result holds under more general conditions. This is the subject of the

Second Törnqvist Approximation Theorem: Assume that: prices and quantities grow monotonically in the interval (0, 1*). Then* (3.25) *and* (3.26) *hold.*

Proof:

Given the monotonicity assumption, there is a unique correspondence between each price or quantity and the corresponding value share. We can therefore write

(3.27) $$V_{iP} = \int_{\ln p_i^0}^{\ln p_i^1} \alpha_i\left(\ln p_i\right) d \ln p_i.$$

This expression is of the form given in(3.18) so that

(3.28) $$V_{iP} = \bar{\alpha}_i(\ln p_i^1 - \ln p_i^0) + O_3(\Delta \ln p_i).$$

Comment:

The approximation property of the Törnqvist index is actually more general. The index computes the exact change of a quadratic with initial and final slopes equal to those of the integrand. In this sense, the index approximates an arbitrary path quadraticly. The theorems assure the meaningfulness of the integrals themselves as well as of the approximation.

3.5 Discrete Derivation of Törnqvist Indexes

I believe that the derivation of the Törnqvist index from the Divisia index is a powerful argument in its favor. Nevertheless, it must be recognized that the bulk of index number theory is devoted to the analysis of bilateral indexes, i.e., those defined on two sets of price, quantity vectors, without invoking a continuity assumption. Bilateral index theory also involves a strong economic intuition: that the comparison of two points should be independent of the path between them. Fortunately, Balk and Diewert (2001) recently gave a derivation of the Törnqvist index as a bilateral index. While they focus on the price index, their analysis applies equally to the quantity index.

Their starting point is to postulate the following form for the price index:

$$\ln P = \sum m_i\left(\alpha_i^0, \alpha_i^1\right) \ln \frac{p_i^1}{p_i^0}, \tag{3.29}$$

where m_i is an averaging function for the ith pair of expenditure functions.

I believe that this formulation embodies the most basic intuition that we have about an index: The proportional change of an index is a weighted average of the proportional changes of its constituent elements, the weights being average market shares that reflect the economic importance of the different commodities. The obvious choice for the weight is the arithmetic average. Balk and Diewert also show that it is the only choice that satisfies certain basic axioms of homogeneity and symmetry.

I mention at this point two basic advantages of Törnqvist indexes from both pragmatic and theoretical points of view. They can be explained to the public as averages of proportional changes.[20] A great advantage of the Törnqvist indexes is their role in theoretical analysis. The approximation of the Divisia index was the first example of this, others will follow.

3.6 The Modified Törnqvist Index

While this entire paper is a strong argument for the Törnqvist index, it does have a minor blemish: duality is approximate, not exact. From a statistical point of view, the Törnqvist index is an estimate of an exact Divisia index that does not incorporate all of our prior knowledge. It is interesting to investigate if this blemish can be removed and the estimate thereby improved. The answer turns out to be affirmative.

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[20] The Fisher index can be expressed in terms of relatives also, but the expression is more complex. This can be seen from the fact that while the relative expression for the Laspeyres price index takes the simple form $P_L = \sum \left(\frac{p_i^1}{p_i^0}\right)\alpha_i^0$, the Paasche price index takes the more complex harmonic form $P_P = \left[\sum \left(\frac{p_i^1}{p_i^0}\right)^{-1} \alpha_i^1\right]^{-1}$.

We define a pair of modified Törnqvist indexes as geometric averages of the direct and indirect indexes:

$$\tilde{P}_T = \left(P_T \frac{y^1}{y_0} Q_T^{-1} \right)^{\frac{1}{2}}, \quad \tilde{Q}_T = \left(Q_T \frac{y^1}{y^0} P_T^{-1} \right)^{\frac{1}{2}}. \tag{3.30}$$

It is easy to check that

$$\tilde{P}_T \tilde{Q}_T = \frac{y^1}{y^0}. \tag{3.31}$$

Each modified index is based on both price and quantity relatives and thus uses all the information in the data as well as satisfying the prior restriction of the product rule. We can expect the modified indexes to be better approximations to underlying theoretical indexes than the standard Törnqvist indexes. To make this conjecture more precise, assume that P, Q are the relevant theoretical indexes. These could for, for example, be defined as Divisia indexes with constant growth paths for the variables from their initial to their final values. The deviations of the computed from the theoretical indexes can be expressed as

$$P_T = P\left(1+\delta\right), \quad Q_T = Q\left(1+\varepsilon\right). \tag{3.32}$$

Then,

$$\begin{aligned} \ln \tilde{P}_T &= \ln P + \frac{1}{2}\ln\left(1+\delta\right) - \frac{1}{2}\ln\left(1+\varepsilon\right), \\ \ln \tilde{Q}_T &= \ln Q + \frac{1}{2}\ln\left(1+\varepsilon\right) - \frac{1}{2}\ln\left(1+\delta\right). \end{aligned} \tag{3.33}$$

Using the fact that for a small $x, \ln\left(1+x\right) \approx x,$ we have

$$\begin{aligned} \ln \tilde{P}_T - \ln P &\approx \frac{1}{2}\left(\delta - \varepsilon\right) = u, \\ \ln \tilde{Q}_T - \ln Q &\approx \frac{1}{2}\left(\varepsilon - \delta\right) = -u. \end{aligned} \tag{3.34}$$

Finally, on the further assumptions

$$E\delta = E\varepsilon = E\delta\varepsilon = 0, \quad E\delta^2 = E\varepsilon^2 = \sigma^2, \mathbf{u} = (u_1, \ldots, u_K) \tag{3.35}$$

it follows that $Eu^2 = \sigma^2 / 2.$

Empirically, the differences between the two pairs of indexes is likely to be small, but given that the cost of computing the adjusted index is negligible, it may be worthwhile to reduce the errors. In addition, it should be remembered that in constructing chain indexes, the errors cumulate.

The following sections are formulated in relation to standard Törnqvist indexes, but the modified indexes could also be used in each case. A numerical comparison is given in Section 8.

4. THE RATIONALITY ASSUMPTION

4.1 The Continuous Approach

The existing literature on the application of the Divisia index to the problem of the utility maximizing consumer has focused on the assumption of homotheticity. This leads to an elegant theory that avoids the path dependency of the usual Divisia index.[21] In this section, I present the Divisia theory for the non-homothetic, but rational consumer (household)[22].

The following definitions will be used: Let $\mathbf{x}$ be the household consumption vector, $\mathbf{p}$ the corresponding price vector, $y = \mathbf{px}$ the household expenditure and $u(\mathbf{x})$ a utility function, assumed twice continuously differentiable and strictly quasi-concave. The corresponding expenditure function

(4.1) $$e(\mathbf{p},u) = \min_{\mathbf{x}} \ \mathbf{px}\colon u(\mathbf{x}) \geq u$$

specifies the minimum expenditure required to reach the utility level u at prices $\mathbf{p}$. The expenditure function is the fundamental tool for aggregating prices and quantities in this context. How this is to be done in the general non-homothetic case has not been clarified in the received theory. I propose to do this analogously to the preceding sections by using continuity in order to arrive at unambiguous parameterizations. I also adopt a terminology appropriate for the consumer sector: the price measure will now be referred to as the *cost-of-living* (*C*) and the quantity measure as *real consumption* (*R*).

(4.2) $$C(t)R(t) = e(\mathbf{p}(t),u(t)) = y(t).$$

The increment in the cost-of living is given by

(4.3) $$RdC{=}\nabla_{\mathbf{p}}\mathrm{e}(\mathbf{p},u)\,\mathrm{d}\mathbf{p}$$

and the increment of real consumption by

(4.4) $$CdR = \nabla_{\mathbf{x}} e(\mathbf{p},u(\mathbf{x}))\,d\mathbf{x}.$$

These increments decompose the expenditure so that

(4.5) $$de = dy = RdC + CdR.$$

Further progress requires the following

Lemmas on duality of the expenditure function:

Let $\mathbf{h}(\mathbf{p},u)$ be the Hicksian (compensated) demand function.

(4.6) $$\nabla_{\mathbf{p}} e(\mathbf{p},u) = \mathbf{h}(\mathbf{p},u) = \mathbf{x}.$$

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[21] This theory is reviewed in Balk (2000, Section 8) and in Diewert (2001, Section D.1)

[22] The conditions under which a household, as opposed to an individual consumer, can be assumed to be utility maximizing are the subject of a literature that began with Samuelson (1956) and was elaborated further by Pollak (1980). More recent contributions are Apps and Rees (1997) and Chiuri and Simmons (1997).

(4.7)(Balk) $$\nabla_{\mathbf{x}} e\left(\mathbf{p}, u\left(\mathbf{x}\right)\right) = \mathbf{p}\,.$$[23]

Both Lemmas are an immediate consequence of the identity

(4.8) $$e(\mathbf{p}, u(\mathbf{x})) \equiv \mathbf{px}.$$

Converting(4.3) to logarithmic form and using(4.6) gives

(4.9) $$\frac{RC\dfrac{dC}{C}}{RC} = d\ln C = \frac{\sum p_i \dfrac{\partial}{p_i} e(\mathbf{p}, u)\dfrac{dp_i}{p_i}}{e(\mathbf{p}, u)} = \frac{\sum p_i x_i \dfrac{dp_i}{p_i}}{y} = \sum \alpha_i d\ln p_i .$$

Similarly,

(4.10) $$\frac{CR\dfrac{dR}{R}}{CR} = d\ln R = \frac{\sum x_i \dfrac{\partial}{x_i} e\left(\mathbf{p}, u\right)\dfrac{dx_i}{x_i}}{e\left(\mathbf{p}, u\right)} = \frac{\sum x_i p_i \dfrac{dx_i}{x_i}}{y} = \sum \alpha_i d\ln x_i .$$

The logarithmic differentials of C and R are precisely those obtained earlier in the case of the Divisia price and quantity integrals.

Equation (4.9) gives the differential of the generalized Marshallian surplus, the only difference to the usual formulation being that the price changes are expressed in logarithms. This has the advantage that the computed change is independent of the price level, or the units of measurement. This interpretation establishes the connectedness of the present approach with the traditional literatures on consumer surplus, general equilibrium analysis and welfare economics.

The differential in (4.10) gives the change in the consumer's well being as his consumption changes because of both price and income change. It defines the change in money metric utility in the general non-homothetic case and under the continuity assumption.

In the theory presented here, the Divisia differentials apply to the individual utility maximizing consumer, without the homotheticity assumption. The rest of the theory is therefore completely analogous to that given in Section 3 and involves the Törnqvist approximation to the Divisia integrals, prices and quantities being those relevant to the household under investigation.

4.2 The Discrete Approach

The fixation of index number theory on the assumption of a homogeneous aggregator function is the more surprising as Theil (1968), in a brilliant but neglected contribution, developed the theory of the general case for the individual utility maximizing consumer. Only his assumptions and results are given here, the reader is referred to the original paper for the proofs.

Theil begins his analysis by defining the theoretical index of the cost-of-living

(4.11) $$C\left(\mathbf{p}^1, \mathbf{p}^0; u^*\right) = \frac{e\left(\mathbf{p}^1, u^*\right)}{e\left(\mathbf{p}^0, u^*\right)},$$

[23] Balk (1989, Section 9) derived the result in the more general setting of varying preferences. Cf. Balk (2000, equation 120).

where the reference utility level u^* remains to be determined.

The real consumption index is defined as

$$(4.12) \qquad R\left(u^1, u^0; \mathbf{p}^*\right) = \frac{e\left(u^1, \mathbf{p}^*\right)}{e\left(u^0, \mathbf{p}^*\right)}$$

with the reference price vector $\mathbf{p}^*$ to be determined.

Theil explicitly points out the consequence on non-homotheticity: C is not independent of u^* and R is not independent of $\mathbf{p}^*$.

In order to determine $\mathbf{p}^*, u^*$ Theil assumes. that $\mathbf{p}^*$ is an average of $\mathbf{p}^0, \mathbf{p}^1$ and that u^* is determined by the indirect utility function $u^* = u\left(y^*, \mathbf{p}^*\right)$, where y^* is the same average of y^0, y^1, as $\mathbf{p}^*$ is of $\mathbf{p}^0, \mathbf{p}^1$. There follow five elementary conditions of symmetry and homogeneity for the averaging function that narrow it down to the geometric one. Specifically, we must have $p_i^* = \left(p_i^0 p_i^1\right)^{\frac{1}{2}}$, $y_i^* = \left(y_i^0 y_i^1\right)^{\frac{1}{2}}$.

Having obtained unique expressions for the theoretical indexes, Theil turns to the question of their approximation. I state here only the results:

$$(4.13) \qquad C = P_T + O_3, \; R = Q_T + O_3.$$

The theoretical indexes are approximated quadraticly by the corresponding Törnqvist indexes.

Diewert (1976a, Theorem 2.16) obtained a similar result for the Törnqvist price index via a different route. He showed that on the assumption that the consumer maximizes a general, quadratic, non-homothetic, translog utility function

$$(4.14) \qquad C\left(\mathbf{p}^1, \mathbf{p}^0; u*\right) \equiv P_T, \quad u* = \left(u^1 u^0\right)^{\frac{1}{2}}.$$

The results of this section can be summed up as follows: The change in household expenditure can be decomposed into two parts. One is the change in real consumption, or money metric utility, the other the change in the cost of living. The theoretical magnitudes can be defined by means of continuously changing parameters, or by means of discrete parameters that are averages of values taken at the endpoints of the interval. In either case, quadratic approximations are given by the appropriate Törnqvist indexes. It should be mentioned that the continuous theory described in this paper is considerably simpler.

5. AGGREGATION OVER AGENTS AND SECTORS

5.1 Continuous Aggregation

Up to this point we considered Divisia and Törnqvist indexes as aggregators of prices and quantities pertaining to a single unit, be it a household or a market. This section considers aggregation over such units. Unless we are dealing specifically with aggregation over households, we will use the term 'sector'. The method of aggregation is essentially the same, only that there are now three different kinds of expenditure shares to be considered: The share of the ith good in the kth sector α_{ik}, the share of the ith good in the total α_i, the share of the kth sector's expenditure in the total β_k. These are related by

$$\alpha_i = \alpha_{ik}\beta_k, \quad i \in (1,\cdots,I), \quad k \in (1,\cdots,K). \tag{5.1}$$

The following discussion concentrates on price indexes, but the aggregation of quantity indexes is analogous. The logarithmic Divisia price index for the aggregate is

$$\begin{aligned} V_P = \ln\frac{P^1}{P^0} &= \int_0^1 \sum_i \alpha_i(\tau)\frac{p_i'(\tau)}{p_i(\tau)}d\tau \\ &= \int_0^1 \sum_k \beta_k(\tau)\sum_i \alpha_{ik}(\tau)\frac{p_i'(\tau)}{p_i(\tau)}d\tau \\ &= \sum_k \int_0^1 \beta_k(\tau)\sum_i \alpha_{ik}(\tau)\frac{p_i'(\tau)}{p_i(\tau)}d\tau. \end{aligned} \tag{5.2}$$

The final equation shows clearly that the aggregate integral is a weighted average of the sectoral integral, the weights being the instantaneous market shares of the sectors. The result immediately extends to the Divisia and Törnqvist price indexes.

Starting from the aggregate Bennett differential, an analogous result for the Divisia quantity index can be obtained,

$$\begin{aligned} PdQ = \mathbf{p}d\mathbf{x} = \sum_i p_i dx_i &= \sum_i p_i \sum_k dx_{ik} \\ &= \sum_i p_i \sum_k x_{ik}\frac{dx_{ik}}{x_{ik}} = \sum_i \sum_k p_i x_{ik} d\ln x_{ik}. \end{aligned} \tag{5.3}$$

Then,

$$\begin{aligned} \frac{PQ\frac{dQ}{Q}}{y} = \frac{dQ}{Q} &= \sum_i \sum_k \frac{y_k}{y}\frac{p_i x_{ik}}{y_k} d\ln x_{ik} \\ &= \sum_i \sum_k \beta_k \alpha_{ik} d\ln x_{ik}. \end{aligned} \tag{5.4}$$

Finally,

$$\begin{aligned} V_Q = \ln\frac{Q_1}{Q_0} &= \int_0^1 \sum_i \sum_k \beta_{ik} d\ln x_{ik}(\tau)d\tau \\ &= \sum_k \int_0^1 \sum_i \beta_{ik} d\ln x_{ik}(\tau)d\tau = \sum_k V_{Q_k}. \end{aligned} \tag{5.5}$$

We have obtained the aggregate Divisia integrals as a weighted sum of the sectoral integrals, with the weights again being the sectoral expenditure shares. The result applies immediately to the corresponding indexes.

5.2 Aggregation of Törnqvist Indexes

The summation of proportional changes along an interval generally requires an integral, since the shares that serve as weights vary continuously. It is a remarkable property of Törnqvist indexes is that they can be aggregated exactly, using only the initial and final shares. For this purpose, the Törnqvist price index is written as the product of two geometric price indexes, one involving the initial and the other the final period.

(5.6)
$$P_T = \left[\prod\left(\frac{p_i^1}{p_i^0}\right)^{\alpha_i^0}\right]^{\frac{1}{2}}\left[\prod\left(\frac{p_i^1}{p_i^0}\right)^{\alpha_i^1}\right]^{\frac{1}{2}}$$
$$= \left(P_G^0\right)^{\frac{1}{2}}\left(P_G^1\right)^{\frac{1}{2}}.$$

Similarly, for the kth sectoral index

(5.7)
$$P_{Tk} = \left[\prod\left(\frac{p_{ik}^1}{p_{ik}^0}\right)^{\alpha_{ik}^0}\right]^{\frac{1}{2}}\left[\prod\left(\frac{p_{ik}^1}{p_{ik}^0}\right)^{\alpha_{ik}^1}\right]^{\frac{1}{2}}$$
$$= \left(P_{Gk}^0\right)^{\frac{1}{2}}\left(P_{Gk}^1\right)^{\frac{1}{2}}$$

The aggregating equation is

(5.8)
$$P_T = \prod\left[\left(P_{Gk}^0\right)^{\beta_k^0}\left(P_{Gk}^1\right)^{\beta_k^1}\right]^{\frac{1}{2}}.$$

Similarly,

(5.9)
$$Q_T = \prod\left[\left(Q_{Gk}^0\right)^{\beta_k^0}\left(Q_{Gk}^1\right)^{\beta_k^1}\right]^{\frac{1}{2}}.$$

Unlike the literature on approximate aggregation of indexes, this aggregation is exact. In addition to its theoretical interest, it can also be used for efficient computation, for example in the context of the NIPAs. Once a set of indexes have been computed at a given level of aggregation, the raw data used for these computations is no longer required in computing the indexes of the next higher level.

5.3 Divisia Aggregation over rational households

As developed thus far, the theory refers to the individual household, but it is easily extended to an aggregate of households. The kth consumer has expenditure y_k and faces market prices $\mathbf{p}$. The aggregate consumption vector is $\mathbf{x} = \sum \mathbf{x}_k$. The collection of utilities is $\mathbf{u} = (u_1, ..., u_K)$. Aggregate expenditure is: $y = \Sigma y_k = \mathbf{p}\Sigma\mathbf{x}_k = \mathbf{px}$. Define $\mathbf{X} = (\mathbf{x}_1 ... \mathbf{x}_K)$ and the aggregate expenditure function $e(\mathbf{p}, \mathbf{u}(\mathbf{X})) = \sum e_k(\mathbf{p}, u(\mathbf{x}_k))$. The gradient of *e*() wrt $\mathbf{p}$ is given by

(5.10)
$$\nabla_{\mathbf{p}} e(\mathbf{p}, \mathbf{u}) = \nabla_{\mathbf{p}} \sum e_k(\mathbf{p}, u_j) = \sum \mathbf{x}_k = \mathbf{x}$$

It would be nice if we could have an analogous gradient wrt $\mathbf{x}$ of the form

(5.11)
$$\nabla_{\mathbf{x}} e(\mathbf{p}, \mathbf{u}(\mathbf{X})) = \mathbf{p}.$$

This seems at first sight nonsensical since $\mathbf{x}$ is not an argument of $e(\ )$. The expression would make sense if we could show that

(5.12) $$\nabla_{\mathbf{X}} e\left(\mathbf{p}, \mathbf{u}\left(\mathbf{X}\right)\right)\Delta\mathbf{X} = \mathbf{p}\Delta\mathbf{x}$$

because (5.11) could then be viewed as an instruction to compute $\Delta e(\ )$ according to the formula

(5.13) $$e(\mathbf{X}^1, \mathbf{p}) - e(\mathbf{X}^0, \mathbf{p}) = \mathbf{p}\Delta\mathbf{x} + O_2(\Delta\mathbf{X}).$$

The validity of (5.13) follows from

(5.14) $$\nabla_{\mathbf{X}} e\left(\mathbf{p}, \mathbf{u}\left(\mathbf{X}\right)\right)\Delta\mathbf{X} = \sum \nabla_{\mathbf{x}_k} e_k\left(\mathbf{p}, u_k\left(\mathbf{x}_k\right)\right)\Delta\mathbf{x}_k = \sum \mathbf{p}\Delta\mathbf{x}_k = \mathbf{p}\Delta\mathbf{x}.$$

The interpretation of (5.11) is that, when the variations of the $\Delta\mathbf{x}_k$ are small and their sum is given, their distribution is immaterial for the determination of $\Delta_{\mathbf{X}} e(\ )$. An alternative derivation of (5.11) is to regard it as an implication of (5.10) , given duality.

With these preliminaries, we are in a position to define the logarithmic differentials of the *Aggregate Cost-of-Living C* and of *Aggregate Real Consumption R*.

(5.15) $$d\ln C = \frac{\nabla_{\mathbf{p}} e(\mathbf{p}, \mathbf{u})\, d\,\mathbf{p}}{e(\mathbf{p}, \mathbf{u})} = \frac{\mathbf{x}\, d\,\mathbf{p}}{y} = \boldsymbol{\alpha}\, d\,\mathbf{ln}\,\mathbf{p}$$

(5.16) $$d\ln R = \frac{\nabla_{\mathbf{x}}\, e(\mathbf{p}, \mathbf{u}(\mathbf{X}))\, d\,\mathbf{x}}{e(\mathbf{p}, \mathbf{u}(\mathbf{X}))} = \frac{\mathbf{p}\, d\,\mathbf{x}}{\mathbf{y}} = \boldsymbol{\alpha}\, d\,\mathbf{ln}\,\mathbf{x}.$$

Again, the differentials are those of Divisia integrals, this time defined on the vectors of aggregate consumption quantities and their prices. The appropriate indexes therefore again have the Törnqvist form.

The interpretation of the theoretical indexes is that C^1 / C^0 gives the change in aggregate expenditure required to keep each consumer at an average level of utility, while R^1 / R^0 is a measure of the increase in aggregate expenditure that would be required at an average price level to give to each consumer the change in utility that he actually experienced.

5.4 Bilateral Aggregation over Rational Consumers

In the following, I restate the results of Theil for bilateral indexes, this time in a notation that refers to the kth consumer out of N:

Define

(5.17) $$\mathbf{p}^* = \left(\left(p_1^0 p_1^1\right)^{\frac{1}{2}}, \cdots, \left(p_I^0 p_I^1\right)^{\frac{1}{2}}\right), \quad y_k^* = \left(y_k^0 y_k^1\right)^{\frac{1}{2}}, \quad u_k^* = u_k\left(\mathbf{p}^*, y_k^*\right).$$

(5.18) $$C_k\left(\mathbf{p}_1, \mathbf{p}_0; u_k^*\right) = \frac{e_k\left(\mathbf{p}_1, u_k^*\right)}{e_k\left(\mathbf{p}_0, u_k^*\right)} = P_{Tk} + O_3,$$

(5.19) $$R_k\left(u_k^0, u_k^1; \mathbf{p}^*\right) = \frac{e_k\left(u_1, \mathbf{p}^*\right)}{e_k\left(u_0, \mathbf{p}^*\right)} = Q_{Tk} + O_3.$$

Since the theoretical expenditure changes of the kth household are approximated to the second order by the corresponding Törnqvist indexes we can use the aggregation formulas (5.8) and (5.9). Given that the Törnqvist index aggregates exactly, the result is:

$$\begin{aligned} &C\left(\mathbf{p}_1,\mathbf{p}_0;\mathbf{u}^*\right)=P_T+O_3,\\ &R\left(\mathbf{u}^0,\mathbf{u}^1;\mathbf{p}^*\right)=Q_T+O_3 \quad \mathbf{u}=\left(u_1,\ldots,u_K\right). \end{aligned} \tag{5.20}$$

6. CHAINING

Thus far, we analyzed bilateral comparisons based on the implicit assumption that the price and quantity vectors being compared are not too different, so that a reasonable approximation of the empirical to the theoretical measures will result. In a time series context, a bilateral index is suitable for year-to-year comparisons. It has long been recognized that a fixed index base cannot be maintained for too long, because it becomes irrelevant. The alternative is some form of chaining. It has also been recognized that chaining introduces path dependence, usually referred to as violation of Fisher's circularity axiom. This has left practitioners in a quandary. The past practice in the context of the NIPAs has been to keep the base constant for 5 or 10 years and then to do some kind of rebasing to establish comparability of the different segments. The problems involved in this will be discussed further in the next section. Currently opinion has shifted towards the use of annually chained indexes. Furthermore, the view, at least of theoreticians, is that a symmetric index, not the usual Laspeyres formula should be used.

From a theoretical point of view, a sequence of chain indexes may be regarded as an approximation to a Divisia index over the entire interval. If year-to-year Törnqvist indexes are used, a sequence of quadratic approximations to the continuous path is obtained. From a numerical point of view, using more points of interpolation and thus more information, increases accuracy. In the present context, a limit to this improvement is set by annual data. Quarterly or monthly data introduce additional drift due to seasonal fluctuations. In addition, the accuracy of the data declines sharply. At the other end, the traditional method of holding the base constant over longer periods is seen to be pointless. The underlying continuous index is not changed thereby, only the approximation to it is worse.

For completeness, I state here how chain indexes are used to compute levels. This is done by means of the usual convention that identifies the initial real magnitude with the nominal expenditure. The initial price level is, by implication, one. In terms of the theoretical indexes

$$P^t=1\bullet\frac{P^1}{P^0}\bullet\bullet\bullet\frac{P^t}{P^{t-1}},\quad Q^t=y^0\bullet\frac{Q^1}{Q^0}\bullet\bullet\bullet\frac{Q^t}{Q^{t-1}}. \tag{6.1}$$

Denote by P_T^t the price level at time *t*, estimated with a Törnqvist price index, by Q_T^t the real expenditure of time t, estimated by means of a Törnqvist quantity index and by $\tilde{Q}_T^t$ the real expenditure of time t, estimated by means of the indirect Törnqvist quantity index. The Törnqvist indexes between periods $t-1,t$ are designated by P_T^t,Q_T^t. Since each theoretical ratio is approximated by the corresponding Törnqvist index, we have

$$
\begin{aligned}
&P_T^t = 1 \bullet P_T^1 \bullet\bullet\bullet P_T^t = P^t + O_3 \\
(6.2) \qquad &Q_T^t = y^0 \bullet Q_T^1 \bullet\bullet\bullet Q_T^t = Q^t + O_3 \\
&\tilde{Q}_T^t = y^t / P_T^t = Q^t + O_3.
\end{aligned}
$$

The remainder terms contain third and higher order differences of the variables of each period and must therefore be expected to grow with time. For this and other reasons, such as changing tastes and specification errors, the meaningfulness of the formal measures will decline as comparisons are made over longer periods. This is an inevitable limitation of the money metric.

I conclude this section by pointing to some open problems. The problem posed by seasonal fluctuations, in particular that they cause undesirable drifts in price and quantity indexes, has been well recognized, nevertheless, it does not appear to have been solved. In a recent book, Alterman, Diewert and Feenstra (1999, Ch. 5) ague that seasonally adjusted price date cannot be interpreted as giving the actual short run price movements, but must be interpreted as estimates of the trend. This interpretation, however, ignores the existence of business cycles as well as of singular deviations from the trend. Personally, I have preferred in econometric work to use annual data. As far as I know, the problems caused by non-seasonal deviations from trend have not been discussed in the literature at all. These cause drifts in price and quantity indexes in the same way as seasonal fluctuations. In computing secular trends in real expenditures and price levels, it may be preferable to first detrend the raw data.

7. THE DEFINITION OF REAL MAGNITUDES IN THE NIPAS

7.1 The Problem

The most elementary property of any set of accounts is that subtotals must add to totals. Economists have long accepted without question, and taught their students, that this holds also for the NIPAs, both in nominal and real terms. In relation to the methods by which the NIPAs in real terms have been constructed in the past this belief is false. The fallacy has been abetted by the statistical agencies that restored additivity to their data by essentially arbitrary adjustments. Only relatively recently have intensive discussions of the problem within the community of NIPA statisticians begun. The most recent comprehensive guide to the international system of NIPA accounts, Eurostat et al. (1993) contains a chapter by Hill (1993) dealing with the computation of index numbers including a detailed discussion of the additivity problem, but without coming to any resolution.

While there are differences between national agencies, the tendency is one of skepticism regarding the meaningfulness of real measures of NIPA components, going so far that even the abandonment of such measures is being discussed. In the following I discus first at a theoretical level the nature of the problem and its solution. Then I discuss how the issues are being dealt with in practice in relation to the US NIPAs.

Given that the NIPAs and their components are the most important statistics that exist, it is remarkable that there is no economic theory of how the components of the NIPAs, or of any other aggregate for that matter, should be computed in real terms.

The question of how sub-aggregates of an economy, or of some sector, should be defined cannot be discussed separately from their use. I assume that the use is economic analysis. This involves quite generally the question of how goods can be substituted in order to best achieve some aim. For an optimum, it must generally be the case that goods can be substituted in production or consumption in the inverse ratio of their market prices. In order for this condition to be implemented in an empirical model, the real variables entering it must all have the same deflator, so that the relative prices of goods are the same after deflation as before. Once stated, this position may seem obvious, but it conflicts with the past consensus of the NIPA statisticians, who have argued that sectoral data should be sector specific.[24]

A complication is that the definition of the market is itself a function of the problem being investigated. Different agents have a different reach across markets. For general macroeconomic analysis, the national economy is the usual analytical unit. If the problem is the construction of a deflator for transfer payments to consumers, then a deflator based on the prices of consumer goods only, such as the CPI, is more relevant. Deflators that are even more specific may be needed if the aim is to maintain the standard of living of specific demographic groups, for example of a low income, urban household with children.

An implication of this analysis is that sectoral measures are not unique, but rather dependent on the purpose at hand. This distinction has not been recognized in the literature, so that there is no corresponding terminology. I propose the terms *additively consistent* for sectoral measures obtained by deflating with the common aggregate deflator and *sector specific* for measures that depend only on the data of the sector referred to. Additively consistent real measures depend not only on weighted quantity measures within the sector, but also on changes of relative prices between the sectors. Sector specific real measures are independent of intersectoral changes in relative prices. The computation of these measures is elaborated in the next section.

7.2 Deflating the NIPA Sectors

I turn to a description of what seems to me to be the optimal design for the NIPAs. Assuming that the nominal accounts, with their sectoral breakdowns, will be published as before, I suggest that there be a parallel set of price indexes published, one for each sector of the nominal accounts. The set of additively consistent real expenditures is obtained by deflating all expenditures by the same aggregate price index:

(7.1)
$$\tilde{Q}^t_{TK} = \frac{y^t_K}{P^t_T}, \quad \tilde{Q}^t_T = \frac{y^t}{P^t_T}.$$

Figures on real GDP will surely continue to be published explicitly, but users could easily compute real sectoral magnitudes themselves. If a user needs a sector specific quantity measure, he can also compute it himself, using the formula

(7.2)
$$\tilde{Q}^t_{TK}(SEC) = \frac{y^t_J}{P^t_{TK}}.$$

[24] The point has been made also by Triplett (1983).

Any desired information can be obtained readily and efficiently in this system of accounts.

It may be useful to clarify at this point the meaning of the base year when chain indexes are used. The current practice is to keep a base year constant for 5-10 years and to re-compute earlier data when the base is changed. As mentioned in Section 6, these base changes are necessary, since the relative prices of the base year become more and more dated. The problem is that there is no reasonable method for making the re-computations and they are essentially arbitrary. Under the method proposed here, the problem does not arise. The base year here represents a constant *price level* to which the deflator remains attached, the change in *relative prices* is taken into account with each link of the chain. Empirical work with long time series thereby becomes not only conceptually clean, but also much easier from a practical point of view.

I turn to a description of what seems to me to be the optimal design for the NIPAs. Assuming that the nominal accounts, with their sectoral breakdowns, will be published as before, I suggest that there be a parallel set of price indexes published, one for each sector of the nominal accounts. The set of additively consistent real expenditures is obtained by deflating all expenditures by the same aggregate price index:

$$\tilde{Q}_{TJ}^t = \frac{y_J^t}{P_T^t} \text{ and } \tilde{Q}_T^t = \frac{y^t}{P_T^t}. \tag{7.3}$$

Figures on real GDP will surely continue to be published explicitly, but users could easily compute real sectoral magnitudes themselves. If a user needs a purely sectoral quantity measure, he can also compute it himself, using the formula

$$\tilde{Q}_{TJ}^t(SEC) = \frac{y_J^t}{P_{TJ}^t}. \tag{7.4}$$

Any information that may be desired can be obtained readily and efficiently in this system of accounts.

The important issue of how best to define a general measure of inflation is beyond the scope of this paper. It is discussed in detail by Diewert (2001b)

7.3 Real Magnitudes in the US NIPAs

I will now discuss the construction of real magnitudes in the US NIPA accounts.[25] The basic innovation introduced with the 1966 major revision of the accounts was that the Bureau of Economic Analysis (BEA) switched from constant price to annually chained index measures of real activity. Three sets of measures are now being reported: **a.** *Quantity indexes* with the base for every index set at 100 in the base year 1966. **b.** *Chained-dollar measures* which differ from **a.** only in that the base is set at the nominal expenditure of the relevant sector in 1966. **c.** *Contributions to growth* that will be explained below.

The publication of the quantity indexes published under **a.** appears to me as pointless. They contain the same information as those under **b.** except that all size differences have been eliminated. I can think of no possible motivation for these figures

[25] An official description of the accounts and the statistical methods used is U: S: Department of Commerce (2001).

except a desire to avoid criticism regarding non-additivity by producing figures that cannot be expected to add up. For the chained-dollar estimates, the BEA provides a residual line giving the difference between GDP and the sum of its parts.

The BEA is itself skeptical about the indexes reported under **a.** and **b.**:

> "For most analyses, the current-dollar, or "nominal," estimates provide more appropriate measures of the relative importance of GDP components, and the contributions tables…present the appropriate measures of contributions to real growth."

I turn to a more detailed analysis of the "contribution to real growth" measure, the principal novelty introduced. The formula given for the contribution of the ith quantity to the percentage change of the aggregate is

$$C_{i,t}^{Q} = 100\frac{\left(p_{i,t-1} + \left(p_{i,t} / P_t^F\right)\right)\left(x_{i,t} - x_{i,t-1}\right)}{\sum_j \left(p_{j,t-1} + p_{j,t} / P_t^F\right) x_{j,t-1}}, \tag{7.5}$$

where P_t^F is the Fisher price index between periods $t-1, t$.

BEA does not discuss the formula, but its logic can be explained as follows: The change in the quantity of the ith good is valued at an average price, with the second period's price being deflated to the level of the first. The change may therefore be regarded as a real change at period $t-1$ prices. This change is then converted to a percentage by dividing with the aggregate valued at prices averaged in the same manner. The BEA views these measures as superior to its level estimates, but it must be emphasized that the method is implicitly based on the construction of levels, only these are not being reported. The base period level is in any case arbitrary, if it is take to be the nominal expenditure, then the computed percentage changes give the level at base period price for each subsequent period[26]. These levels are additively consistent.

Unfortunately, the formula is flawed. BEA needlessly multiplies both quantity vectors by an average price that is not exact for either. The consequence is that when the changes are used to compute levels, past quantity vectors doe not cancel out. This has the consequence that real expenditure of a period is a function of the quantities of past periods, which is absurd. As I have argued, it is correct to define real expenditure simply as nominal expenditure divided by the aggregate price level. When that is done, the formula for the proportional effect of a quantity change on the aggregate becomes:

$$C_{i,t}^{Q} = 100\frac{\dfrac{p_{i,t}}{P_t^F} x_{i,t} - p_{i,t-1} x_{i,t-1}}{\sum_j \left(p_{j,t-1}\right) x_{j,t-1}}. \tag{7.6}$$

In this formula, each quantity is multiplied by the correct price.

There are further reasons why it is better to stay with the established method of reporting levels rather than contributions: **a.** It is simpler and easier to explain. **b.** Economic and econometric models are formulated in terms of levels, or differences, or

[26] It may be thought that this procedure would not work for multiple periods since the deflator in (7.5) changes from period to period. However, the introduction of a fixed base index to deflate both prices would only result in the multiplication of nominator and denominator by the same constant.

growth rates, not in terms of the ‘contributions’ defined by the BEA. **c.** Given levels, the user can decide for himself how to process them further. For, example, he may wish to construct growth rates using the average quantities as base, rather than the initial quantities, as done by BEA. The latter procedure leads to an overstatement of a continuous exponential rate of growth.

The good news is that given an aggregate chained price index, such as the BEA now provides, the user can easily compute the required levels himself from the nominal figures.

8. A NUMERICAL EXAMPLE

As described in section 2, the received opinion is that there is little difference in the numerical performance of different symmetric measures. However, the numerical examples, involving superlative indexes, that have been computed have not explicitly considered the problem of non-homotheticity. A related problem is that the numerical examples only compare the various measures to each other, not to an exact theoretical measure. To the extent that the empirical measures deviate from each other, it was impossible to tell which was more correct.

This section presents a numerical comparison of the Fisher and Törnqvist indexes as well as the CS approximation in the presence of non-homotheticity. The data are assumed to be generated by a consumer with a Stone/Geary utility function, the simplest form allowing non-homotheticity. A problem connected with the construction of theoretical measures is that these are generally not unique, but dependent on the choice of a parametrization in the discrete case, or the choice of a path in the continuous case. I avoided this problem by constructing a comparison of two periods with the same utility level in both periods. In this case the theoretical indexes are unique and given by

(8.1) $$RCI = Q = 1, \quad CLI = P = \frac{y^1}{y^0}.$$

I used the utility function

(8.2) $$u = \left(x_1 - 10\right)^{\frac{1}{2}} \left(x_2\right)^{\frac{1}{2}}.$$

The solution is

(8.3) $$p_1\bar{x}_1 = p_2 x_2 = \frac{1}{2}\bar{y}, \quad \bar{x}_1 = x_1 - 10, \quad \bar{y} = y - 10p_1$$

The two situations being compared are described in Table 1.

Table 1

Period	p_1	$\bar{x}_1$	x_1	p_2	x_2	α_1	α_2	y_1	y_2	$\bar{y}$	y
1	1	10	20	10	1	0.667	0.333	20	10	20	30
2	5	1	11	0.5	10	0.917	0.083	55	5	10	60

Table 2 contains the values and errors relative to the theoretical indexes of the Laspeyres, Paasche and Fisher price and quantity indexes defined by

$$Q_L = \frac{\mathbf{p}^0\mathbf{x}^1}{\mathbf{p}^0\mathbf{x}^0}, \quad Q_P = \frac{\mathbf{p}^1\mathbf{x}^1}{\mathbf{p}^1\mathbf{x}^0}, \quad Q_F = \left(Q_L Q_P\right)^{\frac{1}{2}}$$

(8.4)

$$P_L = \frac{\mathbf{x}^0\mathbf{p}^1}{\mathbf{x}^0\mathbf{p}^0}, \quad P_P = \frac{\mathbf{x}^1\mathbf{p}^1}{\mathbf{x}^1\mathbf{p}^0}, \quad P_F = \left(P_L P_P\right)^{\frac{1}{2}}.$$

Table 2

Q_L	Q_P	Q_F	P_L	P_P	P_F
3.667 (266)	0.597 (-40)	1.48 (48)	3.35 (68)	0.541 (-73)	1.346 (-33)

The Laspeyres and Paasche indexes are seen to be quite poor and the Fisher indexes are only moderate improvements.

Next, I turn to the geometric Laspeyres and Paasche indexes as well as the Törnqvist and modified Törnqvist indexes constructed from them. These are defined as

$$Q_{GL} = \prod \left(\frac{x_i^1}{x_i^0} \right)^{\alpha_i^0}, \quad Q_{GP} = \prod \left(\frac{x_i^1}{x_i^0} \right)^{\alpha_i^1}, \quad Q_T = \left(Q_{GL} Q_{GP}\right)^{\frac{1}{2}}$$

(8.5)

$$P_{GL} = \prod \left(\frac{p_i^1}{p_i^0} \right)^{\alpha_i^0}, \quad P_{GP} = \prod \left(\frac{p_i^1}{p_i^0} \right)^{\alpha_i^1}, \quad P_T = \left(P_{GL} P_{GP}\right)^{\frac{1}{2}}$$

$$\tilde{Q}_T = \left[Q_T \frac{y^1}{y^0} P_T^{-1} \right]^{\frac{1}{2}}, \quad \tilde{P}_T = \left[P_T \frac{y^1}{y^0} Q_T^{-1} \right].$$

Table 3

Q_{GL}	Q_{GP}	Q_T	$\tilde{Q}_T$	P_{GL}	P_{GP}	P_T	$\tilde{P}_T$
1.445 (45)	0.700 (-30)	1.006 (0.6)	1.024 (2.4)	1.079 (-46)	3.412 (71)	1.919 (-4)	1.953 (-2)

A comparison with the previous table shows that the geometric indexes have less variability around the theoretical values. The Törnqvist indexes are remarkably accurate, not leaving much room for improvement. Nevertheless, the maximum error of the modified pair is less than half the maximum error of the standard pair.

The final computations involve the CS approximations. The basic measures are the quantity and price variations defined by

(8.6) $$Q_V = \bar{\mathbf{p}}\Delta\mathbf{x}, \quad P_V = \bar{\mathbf{x}}\Delta\mathbf{p}, \quad \bar{\mathbf{p}} = \frac{1}{2}\left(\mathbf{p}^0 + \mathbf{p}^1\right), \quad \bar{\mathbf{x}} = \frac{1}{2}\left(\mathbf{x}^0 + \mathbf{x}^1\right).$$

The theoretical quantity variation in this example must be zero, and the theoretical price variation must be 30, the change in nominal expenditure. As I argued in Hillinger (2001b), a related price index P_{VI} should have the following property:

(8.7) $$\bar{\mathbf{x}}\left[\left(\mathbf{p}^1 / P_{VI}\right) - \mathbf{p}^0\right] = 0 \rightarrow P_{VI} = \frac{\bar{\mathbf{x}}\mathbf{p}^1}{\bar{\mathbf{x}}\mathbf{p}^0} = P_{JM}.$$

The requirement that the price variation must be zero when the second price vector is deflated implies the Jevons/ Marshall price index. Using this deflator, we can also define a modified quantity variation at the initial period price level:

(8.8) $$\tilde{Q}_V = \frac{1}{2}\left[\left(\mathbf{p}^1 / P_{VI}\right) + \mathbf{p}^0\right]\Delta\mathbf{x}.$$

The numerical results are given in

Table4

Q_V	P_V	P_{JM}	$\tilde{Q}_V$
30.25 (67)	9.75 (-45)	1.138 (-43)	22.703 (76)

Relative errors for the variations were obtained by dividing with the average expenditure of the two periods, except for the deflated quantity variation where the initial expenditure was used as divisor. The error magnitudes are of the same order as for the Fisher indexes.

How should these results be interpreted? I confess to being at first startled by them. On reflection, I realized that they are fully in line with theory. The problem is that the implications of existing theory have not been widely appreciated.

Regarding Fisher's index, I have mentioned that Samuelson and Swamy (1974) have shown that it is not a quadratic approximation to the true index in the general non-homothetic case.

Regarding the CS approximation, I discussed the long history of theoretical skepticism in Section 2. The present paper was motivated by the fact that I was dissatisfied with this approximation even after my own contributions to it. The numerical results above are just one more item of evidence regarding the relative inferiority of the CS approximation.

Another reason why I was initially surprised by the fact that the Törnqvist and Fisher indexes differed substantially was that Diewert (1978) had shown that all superlative indexes approximate each other quadraticly. The Fisher and Törnqvist indexes are the most prominent members of this class. On reexamining the proof, I realized the significance of the fact that Diewert had proved his result at a point, i.e., as $\left(\mathbf{p}^1, \mathbf{x}^1\right) \to \left(\mathbf{p}^0, \mathbf{x}^0\right)$. This means that superlative indexes are likely to approximate each other well over small intervals, but not over large ones. The numerical results in Diewert (2001, App. 3.3) confirm this: Törnqvist chain indexes are close, but fixed base indexes taken over longer periods with larger changes are not.

Finally, it may be agued that Törnqvist indexes are superior because they are the conclusion of a superior argument. First, as Theil (1968) had argued, one should start with proportional changes, not levels of the variables. Secondly, the continuous approach is preferable because it avoids arbitrary parametrizations and involves a great simplification relative to discrete approaches. Taking both of these propositions into account leads directly to the Törnqvist indexes.

9. CONCLUSION

The current state of economic measurement is a scandal. The derivation of the formulas that are used is either incorrect, as in the case of consumer surplus, or it is based on the unrealistic assumptions of homothetic preferences and representative agents, as in most of the economic theory of index numbers. This paper provides a theory of the money metric that is both rigorous and general in the sense that it can be applied to a collection of heterogeneous consumers with non-homothetic preferences as well as when no explicit maximization assumption is used. The theory is also general in that it can be based either on the continuous approach as pioneered by Divisia or on a discrete approach that starts from contributions of Balk/Diewert and Theil. A novelty of the paper is that the idea of the money metric is extended to the definition of real sub-aggregates. I argue that relative prices must be respected and that the methods of the NIPA statisticians fail this elementary test.

A unifying feature is that in every case the appropriate measure is a Törnqvist index defined on the relevant data set. The Törnqvist index is the fundamental tool for the analysis of the non-homogeneous case.

The paper demonstrates that a single rigorous and realistic theory is applicable to such diverse and hitherto largely separate fields as consumer surplus, NIPAs in real terms and the CPI. Moreover, using the approach I advocate involves no additional costs relative to the methods currently employed. In the case of the NIPAs, savings could actually be realized by discarding useless series and avoiding the permanent revision of past data.

The subject of this paper has been the ex-post measurement of price and quantity changes and the derivation of their welfare implications. I believe that the money metric has an equal potential of reforming general equilibrium and welfare theory in the direction of greater empirical relevance. Adam Smith had described the working of the invisible had as resulting from the efforts of entrepreneurs to increase their profits at prevailing prices, whenever they found an opportunity to do so. Each such step increases the aggregate money metric. At a Walrasian equilibrium the money metric at the equilibrium price vector is a maximum. This property allows the easiest proofs of the first and second theorems of welfare economics.[27] The greatest open challenge appears to be the formulation of applied general equilibrium models that start from the assumption of heterogeneous agents and allow econometric predictions of the changes in aggregate price and quantity indexes in response to policy actions.

We are now living in a world that has acquired an abundance of data as well as the computing capacity and the statistical tools for analyzing them. Empirical economic research is sharply on the rise. The time is over-ripe for economists to be concerned about the theoretical foundation of the data they use.

[27] Cf. Varian (1992, Section18.6)

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