INFLATION AND WEALTH DISTRIBUTION

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Abstract

The effect of a permanent change of inflation on the distribution of wealth is analyzed in a general equilibrium OLG model that is calibrated with regard to the characteristics of the US economy. Poor agents accumulate savings predominantly in the form of money, while rich agents participate in the stock market and accumulate equity. Surprisingly, an increase of inflation results in a lower stock market participation rate; in addition, the distribution of wealth becomes more unequal, even though the quantitative effect is economically negligible. Furthermore, we show that the welfare costs of anticipated inflation are considerably lower than in Imrohoroglu (1992).

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1 Introduction

The literature discusses several channels through which inflation may alter income, earnings, or wealth distributions. Among others, these channels include differential indexation of wages across income groups, disproportionally allocated subsidized loans, the tax income bracket effect, and the Tanzi-Olivera effect on taxes and governmental revenues. This paper aims to examine a different channel that stresses the role of capital markets and the portfolio composition of financial wealth between money and equity. It is straightforward to argue that changes in the size and composition of wealth are associated with the domestic rate of inflation of an economy. Given that households from lower income groups do not have access to stock markets, it is only the higher income group that is able to protect itself against inflation by shifting its portfolio from money to indexed assets. This reasoning may be justified on the grounds that a minimum amount of entrance costs is generally required to participate in the stock markets. Many younger and poorer households do not hold equities at all.[1] This fact may be explained if there is a fixed cost of participating in the equity market.

In order to study the effects of inflation on the distribution of wealth, we analyze the households' optimal portfolio allocation over the life cycle in an economy with fixed stock market participation costs.[2] Contrary to studies that examine the popular redistributional wealth effect from (net) creditors to debtors, which is based on unanticipated inflation and average income, we explicitly investigate a permanent, anticipated change of the inflation rate. A further preconception of our analysis is that "any theory of equality must account for the dynamic features of earnings, income, and wealth distributions, i.e., the mobility of individual (heterogeneous) households up and down the economic scale," (cf. Díaz-Giménez et al., 1997, p. 10). For this reason, we introduce (stochastic) heterogenous productivity into our model so that our model is able to match both the Gini coefficient of US labor income and the Gini coefficient of US wealth very closely. As our main result from our computational analysis, we demonstrate that higher inflation increases wealth

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[1]Romer and Romer (1998) note that for US households from the quintile reporting the lowest total income only about one fifth holds a positive amount of financial assets, including non-public stocks (figure based on data from the Federal Reserve's 1995 Survey of Consumer Finances, henceforth SCF).

[2]One of the few papers known to us that studies the inflation effect on wealth inequality is Bhattacharya (2001). The focus of his paper, however, is on the modelling of agency costs.

inequality.

As regards empirical studies, evidence of the relationship between inflation and inequality for the US is quite ambiguous, sometimes even statistically insignificant. A comprehensive survey of the recent literature is given in Galli and van der Hoeven (2001). Accordingly, the vast majority of studies finds a significant progressive effect of inflation on the US income distribution, though mostly small in quantitative terms. About half of the studies, however, does not find a statistically significant effect once additional control variables are considered. For example, Romer and Romer (1998) find statistically insignificant effects of inflation for the US Gini coefficient as well as for the poverty rate as dependent variables in their regressions, in case of controlling for unemployment. However, it should be noted that the focus of this literature is on the US income distribution and therefore abstracts from financial wealth. For the US, wealth is positively correlated with earnings and income, but not strongly (see Díaz-Giménez et al., 1997). All the more, the quantitative, numerical exercise of the present paper aims to shed some light on the relationship between inflation and wealth distribution for the US, in order to fill this gap in the literature.

In addition, a second related problem is studied in this paper. In previous work, İmrohoroğlu (1992) finds a significant welfare effect of inflation. In particular, she demonstrates with the help of numerical computations that an increase of annual inflation from 0% to 10% results in welfare losses equivalent to approximately one percent of GNP. Her model is similar to ours, except that we study two assets, both money and equity, while she only considers money.[3] In this paper, we show that welfare losses are substantially smaller (by a factor of 100) if agents can also accumulate savings in the form of equity.

The remainder of the paper is structured as follows. Section 2 introduces the overlapping-generations model with two assets, money and equity. The model is calibrated with regard to the characteristics of the US economy in section 3. Our numerical results are presented in section 4. Section 5 concludes.

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[3]Furthermore, she only considers the state of employment and unemployment, while we replicate the empirical labor income distribution more closely as we allow for multiple productivity types.

2 The model

We study a general equilibrium overlapping generations model with endogenous equity and money distribution. Four sectors can be depicted: households, production, the government, and the central bank. Household maximize discounted life-time utility. Agents can save either with money or with capital. Individuals are heterogeneous with regard to their productivity and cannot insure against idiosyncratic income risk. Firms maximize profits. Output is produced with the help of labor and capital. The government provides unfunded public pensions which are financed by a tax on wage income. The money growth rate is set by the central bank and seignorage is redistributed lump-sum to the households.

2.1 Households

Every year, a generation of equal measure is born. As we only study steady-state behavior, we concentrate on the behavior of an individual born in period 0. Their first period of life is period 1. A subscript j of a variable denotes the age of the generation. The total measure of all households is normalized to one.

Households life a maximum of $T + T^R$ years. Lifetime is stochastic and agents face a probability s_j of surviving up to age j conditional on surviving up to age $j - 1$. During their first T years, agents supply labor l elastically. After T years, retirement is mandatory. Agents maximize life-time utility:

$$E_0 \left[\sum_{j=1}^{T+T^R} \beta^{j-1} \left(\Pi_{i=1}^{j} s_i \right) u(c_j, m_j, 1 - l_j), \right] \tag{1}$$

where β, c_j, and m_j denote the discount factor, consumption at age j, and real money balances at age j, respectively. The instantaneous utility function $u(c, m, 1 - l)$ is the CRRA (constant relative-risk aversion) function:

$$u(c, m, 1 - l) = \begin{cases} \gamma \ln c + (1 - \gamma) \ln m + B \ln(1 - l) & \text{if } \sigma = 1 \\ \frac{\left(c^\gamma m^{1-\gamma} \right)^{1-\sigma}}{1-\sigma} + B \ln(1 - l) & \text{if } \sigma \neq 1 \end{cases} \tag{2}$$

where $\sigma > 0$ denotes the coefficient of relative risk aversion.

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Workers are heterogeneous with regard to their labor earnings per working hour. The worker's labor productivity $e(z, j)$ is stochastic and depends on individual age j and an idiosyncratic labor productivity shock z.

Furthermore, agents are born without wealth, $a_1 = 0$, and cannot borrow, $a_j \geq 0$ for all j. Wealth a is composed of real money m and capital k. Capital or, equally, equity k earns a real interest rate r, but it is costly to enter the stock market. Following Campbell et al. (2001), we assume that once the fixed cost F has been paid, there are no additional costs of adjusting the capital stock k. We further assume a short-sale constraint $k \geq 0$. Parents do not leave altruistic bequests to their children. All accidental bequests are confiscated by the state.

Agents receive income from capital k_j and labor l_j. The budget constraint of the working agent at age $j = 1, \ldots, T$ is given by

$$a_{j+1} = k_{j+1} + m_{j+1} = (1 + r)k_j + \frac{m_j}{1 + \pi} + (f_{j+1} - f_j)F + (1 - \tau)we(z, t)l_j + tr - c_j, \quad (3)$$

where w and $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ denote the wage rate per efficiency unit labor and the inflation rate, respectively. P_t is the price level in period t. In steady state, the wage rate w, the interest rate r, and the inflation rate π are constant so that we omit the time index t. Wage income is taxed at rate τ. In addition, the households receive transfers tr from the central bank. f_j denotes a binary variable that equals zero until the investor pays the fixed cost of entering the stock market and equals one thereafter.

During retirement, agents receive public pensions pen irrespective of their employment history and the budget constraint of the retired agent at age $j = T+1, \ldots, T+T^R$ is given by

$$a_{j+1} = k_{j+1} + m_{j+1} = (1 + r)k_j + \frac{m_j}{1 + \pi} + (f_{j+1} - f_j)F + pen + tr - c_j. \quad (4)$$

2.2 Production

Firms are of measure one and produce output with effective labor N and capital K. Let $l(k, m, z, f, j)$ and $\phi(k, m, z, f, j)$ denote the labor supply and the measure of the j-year

old agent with wealth $a = k + m$, previous equity holdings indicator $f \in \{0, 1\}$, and idiosyncratic productivity z. Effective labor N is given by:

$$N = \sum_{j=1}^{T} \sum_{f=0,1} \int_k \int_m \int_z l(k, m, z, f, j) e(z, j) \phi(k, m, z, f, j) \, dz \, dm \, dk. \tag{5}$$

Effective labor N is paid the wage w. Capital K is hired at rate r and depreciates at rate δ. Production Y is characterized by constant returns to scale and assumed to be Cobb-Douglas:

$$Y = F(K, N) = K^\alpha N^{1-\alpha}. \tag{6}$$

In a factor market equilibrium, factors are rewarded with their marginal product:

$$w = (1 - \alpha) K^\alpha N^{-\alpha}, \tag{7}$$
$$r = \alpha K^{\alpha-1} N^{1-\alpha} - \delta. \tag{8}$$

2.3 Government

The government uses the revenues from taxing labor, $\tau w N$, and from aggregate accidental bequests Beq in order to finance its expenditures on aggregate social security Pen:

$$\tau w N + Beq = Pen. \tag{9}$$

2.4 Monetary Authority

Nominal money grows at the exogenous rate θ:

$$\frac{M_t - M_{t-1}}{M_{t-1}} = \theta. \tag{10}$$

The seignorage is transferred lump-sum to the households:

$$tr_t = \frac{M_t - M_{t-1}}{P_t}. \tag{11}$$

2.5 Stationary Equilibrium

The concept of equilibrium applied in this paper uses a recursive representation of the consumer's problem following Stokey et al. (1989). Let $V_j(k_j, m_j, z_j, f_j)$ be the value of the objective function of the j-year old agent with equity k_j, real money m_j, idiosyncratic productivity level z_j, and either prior stock market participant $f_j = 1$ or not $f_j = 0$. $V_j(k_j, m_j, z_j, f_j)$ is defined as the solution to the dynamic program:

$$V_j(k_j, m_j, z_j, f_j) = \max_{k_{j+1}, m_{j+1}, f_{j+1}, c_j, l_j} \left\{ u\left(c_j, m_j, 1 - l_j\right) + \beta s_{j+1} E\left[V_{j+1}(k_{j+1}, m_{j+1}, z_{j+1}, f_{j+1})\right] \right\} \tag{12}$$

subject to (3) or (4) and $k, m \geq 0$. Optimal decision rules at age j are a function of k_j, m_j, z_j, and f_j, i.e. consumption $c_j(k, m, z, f)$, labor supply $l_j(k, m, z, f)$, next-period capital stock $k_{j+1}(k, m, z, f)$, next-period real money balances $m_{j+1}(k, m, z, f)$, and next-period stock market participation $f_{j+1}(k, m, z, f)$.

We will consider a stationary equilibrium where factor prices and aggregate capital and labor are constant and the distribution of wealth is stationary.

Definition

A stationary equilibrium for a given government policy with pension system *pen* and central bank policy θ is a collection of value functions $V_j(k, m, z, f)$, individual policy rules $c_j(k, m, z, f)$, $l_j(k, m, z, f)$, $k_{j+1}(k, m, z, f)$, $m_{j+1}(k, m, z, f)$, and $f_{j+1}(k, m, z, f)$, relative prices of labor and capital $\{w, r\}$, and distributions $(\phi_1 \equiv \phi(., ., ., ., 1), \dots, \phi_{T+T^R})$, such that:

1. Individual and aggregate behavior are consistent:

$$K = \sum_{j=1}^{T+T^R} \sum_{f_j=0,1} \int_k \int_m \int_z k \, \phi(k, m, z, f, j) dz \, dm \, dk, \tag{13}$$

$$C = \sum_{j=1}^{T+T^R} \sum_{f_j=0,1} \int_k \int_m \int_z c_j(k, m, z, f) \, \phi(k, m, z, f, j) \, dz \, dm \, dk, \tag{14}$$

$$Beq = \sum_{j=1}^{T+T^R} \sum_{f_j=0,1} \int_k \int_m \int_z (1 - s_{j+1}) \, a_{j+1}(k, m, z, f) \phi(k, m, z, f, j) dz \, dm \, dk \tag{15}$$

$$Pen = \sum_{j=T+1}^{T+T^R} \sum_{f_j=0,1} \int_k \int_m \int_z pen \; \phi(k,m,z,f,j) \; dz \; dm \; dk, \qquad (16)$$

$$\frac{M}{P} = \sum_{j=1}^{T+T^R} \sum_{f_j=0,1} \int_k \int_m \int_z m \; \phi(k,m,z,f,j) dz \; dm \; dk, \qquad (17)$$

where $a_{j+1}(k,m,z,f) \equiv k_{j+1}(k,m,z,f) + m_{j+1}(k,m,z,f)$ and aggregate effective labor N is given by (5).

2. Relative prices $\{w,r\}$ solve the firm's optimization problem by satisfying (7) and (8).

3. Given relative prices $\{w,r\}$, the government policy pen, the individual policy rules $c_j(.)$, $k_{j+1}(.)$, $m_{j+1}(.)$, $l_j(.)$, and $f_{j+1}(.)$ solve the consumer's dynamic program (12).

4. The government budget (9) is balanced.

5. Money grows at the exogenous rate θ and the seignorage (11) is transferred lump-sum to the households.

6. The goods market clears:

$$K^\alpha N^{1-\alpha} = C + \delta K + \sum_{j=1}^{T+T^R} \int_k \int_m \int_z F \cdot f_{j+1}(k,m,z,0)\phi(k,m,z,f,j) \; dz \; dm \; dk. \quad (18)$$

In particular, stock market participation fees are a social cost.

3   Calibration

Periods correspond to years. We assume that agents are born at real lifetime age 20 which corresponds to $j = 1$. Agents work $T = 40$ years corresponding to a real lifetime age of 60. They life a maximum life of 60 years ($T^R = 20$) so that agents do not become older than real lifetime age 80. The sequence of conditional survival probabilities $\{s_j\}_{j=1}^{59}$ is set equal to the Social Security Administration's survival probabilities for men aged 20-78 for the year 1994.[4] The survival probabilities decrease with age, and s_{60} is set equal to zero.

In our benchmark case, we choose the case of log-linear preferences $\sigma = 1$. For this calibration, money is superneutral in the representative-agent, infinite-lifetime Sidrauski

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[4]We thank Mark Huggett and Gustavo Ventura for providing us with the data.

Table 1: Calibration of parameter values for the US economy

Description	Function	Parameter
utility function	$U = \gamma \ln c + (1 - \gamma) \ln m + B \ln(1 - l)$	$\gamma = 0.984, B = 1.95$
discount factor	β	$\beta = 0.96$
production function	$Y = K^{\alpha} N^{1-\alpha}$	$\alpha = 0.36$
depreciation	δ	$\delta = 0.08$
stock market participation fee	F	$F = 0$
money growth rate	θ	$\theta = 0.05$
pension replacement rate		$\frac{pen}{(1-\tau)wl} = 0.50$
labor endowment process	$z_t = \rho z_{t-1} + \epsilon_t, \; \epsilon_t \sim N(0, \sigma_\epsilon)$ $\ln e(z, 1) \sim N(\bar{y}_1, \sigma_{y_1})$	$\rho = 0.96, \; \sigma_\epsilon = 0.045$ $\sigma_{y1} = 0.38$

model, and we expect the effect of inflation on wealth distribution to be of small magnitude in our OLG model with heterogeneous agents. Furthermore, there is no stock market participation fee in the benchmark, i.e. $F = 0$. The model parameters are presented in table 1.

The calibration of the parameters α, δ, pen, and θ and the Markov process $e(z, j)$ is chosen in accordance with existing general equilibrium studies: Following Prescott (1986), the capital income share α is set equal to 0.36. The annual rate of depreciation is set equal to $\delta = 0.08$. Pensions are distributed lump-sum to the retired agents. The replacement ratio of pensions to net average earnings amounts to 50%. The annual money growth rate θ is set equal to 5% in our benchmark case.

The labor endowment process is given by $e(z, j) = e^{z_j + \bar{y}_j}$, where $\bar{y}_j$ is the mean lognormal income of the j-year old. The mean efficiency index $\bar{y}_j$ of the j-year-old worker is taken from Hansen (1993) and interpolated to in-between years. As a consequence, the model is able to replicate the cross-section age distribution of earnings of the US economy. Following

İmrohoroğlu et al. (1998), we normalize the average efficiency index to one. The age-productivity profile is hump-shaped and earnings peak at age 50.

The idiosyncratic productivity shock z_j follows a Markov process:

$$z_j = \rho z_{j-1} + \epsilon_j, \tag{19}$$

where $\epsilon_j \sim N(0, \sigma_\epsilon)$. Huggett (1996) uses $\rho = 0.96$ and $\sigma_\epsilon = 0.045$. Furthermore, we follow Huggett and choose a lognormal distribution of earnings for the 20-year old with $\sigma_{y_1} = 0.38$ and mean $\overline{y_1}$. As the log endowment of the initial generation of agents is normally distributed, the log efficiency of subsequent agents will continue to be normally distributed. This is a useful property of the earnings process, which has often been described as log normally in the literature.

The remaining three parameters β, B, and γ from the utility function are chosen to match the following characteristics of the US economy as closely as possible: i) the capital-output ratio K/Y amounts to 3.0 as found by Auerbach and Kotlikoff (1995), ii) the average labor supply of the working households equals to approximately one third of available time, and iii) the average velocity of money PY/M corresponds to the annual velocity of M1 during 1960-2001, which is equal to 5.18. Our calibration $\beta = 0.96$, $B = 1.95$, and $\gamma = 0.984$ implies a capital-output ratio equal to 2.96, an average labor supply $\bar{l} = 0.326$, and an annual velocity of money equal to 5.23. In our sensitivity analysis, we also study the case $\sigma = 2$. In order to replicate the empirical findings (i)-(iii) from above, we need to choose $\beta = 0.955$, $B = 6.10$, and $\gamma = 0.988$. For this calibration, the capital-output ratio, the velocity of money, and the average labor supply amount to $K/Y = 3.02$, $PY/M = 5.05$, and $\bar{l} = 0.329$, respectively.

4 Results

In this section, we study the effects of a change of the money growth rate θ or, equally, the inflation rate π on the accumulation and distribution of wealth. First, we report findings for the benchmark case as described by the parameterization in table 1. Second, we analyze the effects of a stock market participation fee. Finally, we consider the sensitivity of our results with regard to the assumption of log-linear utility and the replacement ratio of pensions.

Table 2: Inflation rate π, savings, and distribution

π	F	K	M/P	Y	$Gini_k$	$Gini_a$	SMP	Δ_c
5%	0	1.842	0.119	0.622	0.720	0.701	53.6%	0%
20%	0	1.814	0.0529	0.619	0.737	0.729	52.5%	-0.0128%
5%	0.0622	1.823	0.116	0.622	0.732	0.714	48.6%	-0.0150%
20%	0.0616	1.807	0.0528	0.616	0.740	0.732	46.1%	-0.0167%

4.1 The benchmark case

In our benchmark equilibrium, the annual inflation rate is equal to 5% and there are no fixed stock market participation fees, $F = 0$. The endogenous equilibrium values of the benchmark are reported in the first row of table 2. The aggregate capital stock K amounts to 1.84, while effective labor N and the average supply $\bar{l}$ (not reported) are equal to 0.340 and 0.326, respectively, implying a real interest rate r of 4.11%.

In our model, the productivity process is taken as exogenous. As, however, the agent optimizes his lifetime utility by choice of his labor supply, both the distribution of labor income, $(1 - \tau)we(z,t)l$, and the distribution of wealth, equity k plus real money m, are endogenous. The Gini coefficient of the labor income distribution amounts to 0.481 and is close to the values observed empirically: Díaz-Giménez et al. (1997) find a value of 0.51 for households aged 36-50, while Henle and Ryscavage (1980) estimate an average US earnings Gini coefficient for men of 0.42 in the period 1958-77.[5]

Empirically, wealth is distributed much more unequally than income. Greenwood (1983), Wolff (1987), Kessler and Wolff (1992), and Díaz-Giménez et al. (1997) estimate Gini coefficients of the wealth distribution for the US economy in the range of 0.72 (single, without dependents, female household head) to 0.81 (nonworking household head). Our model is able to replicate these findings. In particular, the Gini coefficient of wealth is equal to $Gini_a = 0.701$, while the Gini coefficient of equity is even higher and amounts to $Gini_k = 0.720$. The Lorenz curve for the benchmark case of our model and for the US economy

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[5]Income transfers are excluded in the respective definition of earnings.

are displayed in figure 1.[6] The empirical distribution that is presented by the broken line displays a higher concentration of wealth among the very wealth-rich agents than the model distribution, while the number of households with no wealth is lower in the US economy than in our model. In particular, only 53.6% $(=SMP)$ of all households participate in the stock market in our model.[7] The main reason why our model underestimates the high concentration of wealth among the top 5% of the wealthiest households is the negligence of i) self-employment and ii) bequests.[8]

Figure 1: Lorenz curve of model and US wealth a
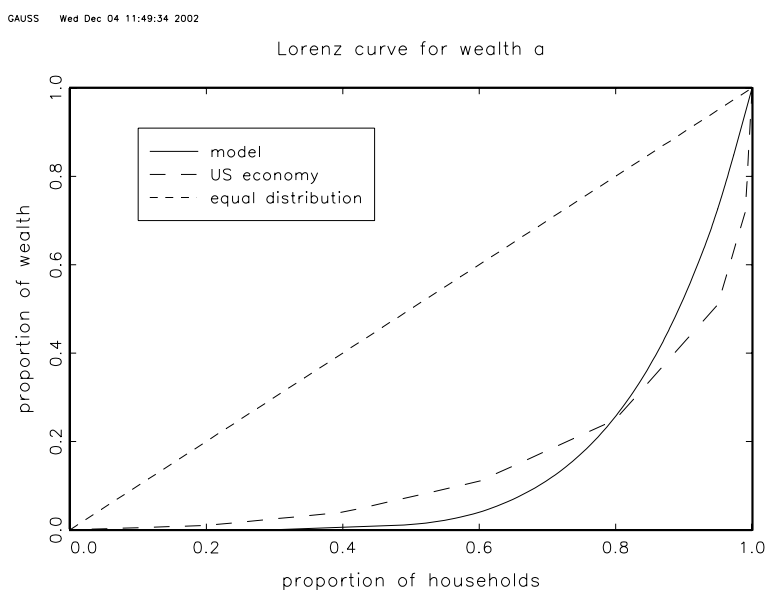
In the Sidrauski model with infinite lifetime and a representative household, money is superneutral for our log-linear functional form of the utility function with $\sigma = 1$. In

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[6]The data for the empirical distribution of wealth are taken from Wolff (1987).

[7]From 1983-98, between 86 and 90% of all US households held interest bearing accounts. In 1998, 38% of households held (tax-deferred) equity. Figures are based on data from SCF (see Poterba and Samwick, 2001). One possible reason that, in our model, the percentage of the households that own stocks is higher than observed empirically is the negligence of investment in housing. In particular, a leveraged position in residential estate may keep younger and poorer households from investing in the stock market (see Cocco, 2001).

[8]For a review of recent studies that explain the wealth distribution in general equilibrium models with, among others, the help of idiosyncratic shocks to labor earnings, business ownership, and changes in health and marital status; see Quadrini and Ríos-Rull (1997). Heer (2001) studies the effects of bequests on the distribution of wealth.

our model, superneutrality does no longer hold. In particular, we assume that seignorage is redistributed lump-sum. As income-rich agents have a higher propensity to save out of income and also hold more money balances than income-poor agents, a higher inflation tax with a corresponding higher seignorage redistributes income to the income-poor agents. As a consequence, not only total money savings M/P but also aggregate equity savings K decline. An increase of the inflation rate π from 5% to 20% decreases aggregate wealth from $K = 1.842$ to 1.814. The distribution of wealth a and capital k becomes more unequal (compare the second row of table 2), but the quantitative effect is of small magnitude. As the lump-sum component of the income is increased, precautionary savings among the wealth-poor and income-poor households decline by a higher percentage than savings among the rich households.

We also analyze steady-state welfare for the different monetary policy regimes θ. In order to compare the welfare effects, we compute the expected discounted lifetime utility of the newborn generation for the different values of θ. To quantitatively assess the effects, we take the benchmark equilibrium as presented in table 1 as our reference economy. The change in welfare Δ_c is computed as the compensation in consumption (relative to the reference economy) required in order to make the average newborn indifferent between the reference economy and the alternative policy regime. As presented in table 2, the steady-state welfare effect of an increase in the anticipated rate of inflation is negligible and amounts to a loss of 0.0128% of total consumption.

4.2   Stock market participation fees

Transaction costs of stock markets have been prominently applied in the explanation of the equity premium puzzle, e.g. by He and Modest (1995), or in the study of the effects of pension reform on retirement wealth, e.g. by Campbell et al. (2001). The motivation to introduce fixed stock market fees into our model is the idea that the adverse effects of inflation on poor households' wealth and on the wealth distribution are exacerbated. Due to the high costs of entering the stock market, most income-poor households hold wealth only in the form of money. As inflation increases, these agents are affected more severely by the inflation tax and might reduce their precautionary money savings. In addition, it is not clear *a priori* if higher inflation also results in higher stock market participation. On

the one hand, the increase of the inflation tax makes capital a more attractive investment than money. On the other hand, agents may have accumulated small wealth in the form of money if their productivity z is low and, in case of a transistory positive productivity shock, they may not have accumulated enough wealth to pay the fixed stock market costs.

Our results for the model with transaction costs are presented in the third and fourth row of table 2. Like Campbell et al. (2001), we choose a fixed stock market fee F equal to 10% of average annual income as the upper bound. As a consequence, the stock market participation rate drops by five percentage points in comparison to the benchmark case, from 53.6% to 48.6%. The capital stock K only declines by a small fraction from 1.842 to 1.823 as the income-rich agents continue to hold wealth in the form of equity. Furthermore, the wealth inequality increases slightly because the very poor cannot afford to participate in the stock market.

As in the case without transaction costs, $F = 0$, an increase of inflation from 5% to 20% results in only quantitatively negligible welfare losses equivalent to 0.0017% of total consumption. Our quantitative welfare effects of inflation are in sharp contrast with the findings of İmrorohoğlu (1992). She finds that reducing steady-state inflation from 10% to 0% results in a considerable welfare gain equivalent to an increase of income equal to 1.07%. Similar to the model in the present paper, she considers a heterogeneous-agent economy with imperfect insurance. The distribution of wealth is endogenous and agents are also subject to a productivity shock that can only take the value zero (unemployment) or one (employment). Different from our economy, however, agents can accumulate savings only in the form of non-interest bearing money. In our model, some households enter the stock market and also use equity in order to smooth consumption intertemporally, while the other agents do not pay the stock market participation fee so that they accumulate savings, if any, in the form of money. For this, as we believe, more realistic description of the savings behavior of individual agents, we find that welfare losses from higher anticipated inflation are negligible.

4.3 Sensitivity analysis

In this subsection, we examine the sensitivity of our results with regard to the choice of the utility parameter and the pension replacement ratio. First, we will analyze the case $\sigma = 2$.

Table 3: Inflation rate π, savings, and distribution for $\sigma = 2$

π	F	K	M/P	Y	$Gini_k$	$Gini_a$	SMP	Δ_c
5%	0	1.507	0.0980	0.4950	0.716	0.697	54.1%	0%
20%	0	1.496	0.0463	0.4933	0.731	0.721	48.5%	-0.0058%
5%	0.0494	1.465	0.0952	0.4944	0.734	0.715	48.5%	-0.0249%
20%	0.0493	1.457	0.0468	0.4928	0.742	0.731	46.7%	-0.0342%

For this calibration, money is no longer superneutral even in the model with homogeneous agents.[9] However, as in the representative-agent case, the quantitative effects of a change in the inflation rate are demonstrated to be of small magnitude. Second, we study a change in the replacement ratio of public pensions from 50% to 30%. In this case, workers increase their savings and more income-poor agents participate in the stock market. Again, fixed stock market participation fees are found to be of minor importance for the inflation effects.

Utility function

Table 3 presents our numerical results for a change in the utility parameter σ. For higher risk aversion $\sigma = 2$, agents increase their precautionary savings and their labor supply. For this reason, we recalibrated our model in order to have the same average labor supply and the same capital-output ratio as in our benchmark case. In particular, the relative weight of the utility from leisure B has been increased and the discount factor β has been decreased, as has been already pointed out in section 3. With higher risk aversion, the labor supply of the low-income group increases relative to the one of the high-income group resulting in a drop of the labor income heterogeneity. The Gini coefficient of labor income decreases from 0.481 in the benchmark case with $\sigma = 1$ to 0.374 in the case with $\sigma = 2$. Notice that this strong decrease in income inequality is not accompanied by a proportional change in the wealth inequality, which, according to the Gini coefficient of wealth a, only drops from 0.701 to 0.697 (compare table 2 and table 3 for the two cases $\sigma = 1$ and $\sigma = 2$).

All our numerical results are analogous to those in the case of $\sigma = 1$. In particular, an

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[9]See, e.g., Walsh (1998), Ch. 2.3.

Table 4: Inflation rate π, savings, and distribution for $rep = 0.3$

π	F	K	M/P	Y	$Gini_k$	$Gini_a$	SMP	Δ_c
5%	0	1.954	0.1232	0.6464	0.705	0.688	54.8%	0%
20%	0	1.943	0.0572	0.6432	0.720	0.711	50.7%	-0.0072%
5%	0.0643	1.942	0.1211	0.6434	0.715	0.697	51.5%	-0.0086%
20%	0.0641	1.930	0.0585	0.6414	0.721	0.714	49.9%	-0.0108%

increase of inflation by 15 percentage points from 5% to 20% results in i) a decrease of aggregate capital K by less than one percent, ii) a small increase of wealth inequality as measured by the Gini coefficient, and iii) a welfare loss in the magnitude of approximately 0.01% of total consumption. Again, the consideration of fixed stock market participation fees does not alter our qualitative and quantitative results. Compared to the case with no stock market entry costs $F = 0$, savings K are smaller and wealth inequality is higher.

Pensions

Due to the high lump-sum pensions, low-income households do not save for retirement, neither in the form of money nor in the form of equity. Reducing the pension replacement ratio from 50% to 30% results in higher savings. In the benchmark case with $\sigma = 1$, aggregate equity increases from $K = 1.842$ to $K = 1.991$. The change of the other equilibrium values is also presented in table 4. Notice that the effects of higher inflation on aggregate capital K, wealth inequality, and welfare are insensitive with regard to the choice of the pension replacement ratio. Following a rise of the inflation π, K decreases, Δ_c is negative, but small, and the Gini coefficients of capital and wealth increase.

5 Conclusion

Empirical results on the effects of inflation on the distribution of wealth are scarce. While there is empirical support to the hypothesis that anticipated inflation increases income inequality, no related evidence is available for the effects of inflation on wealth inequality.

This paper offers a contribution to fill this gap in the empirical literature by analyzing a computable general equilibrium model of the wealth distribution. As our main result, we find that higher inflation increases wealth inequality; however, quantitative effects are small even in the presence of substantial stock market participation fees.

In this model, we also study the welfare costs of inflation. In our economy, agents face idiosyncratic risk and try to smooth consumption intertemporally as in İmrohoroğlu (1992). Different from her, agents cannot only accumulate savings in the form of money but also in the form of equity. From her results, A. İmrohoroğlu concludes that the area under the empirical money demand curve is a poor measure of the welfare costs of inflation. In her model, welfare costs are higher by the factor 3-4. We come to the opposite conclusion. In our study, the area under the empirical money demand curve overestimates the true costs of inflation by a factor of approximately 25-40, once we allow for general equilibrium effects, heterogeneous agents, and consumption smoothing with the help of equity.

6   Appendix: Computation

The solution algorithm is described by the following steps:

1. Parameterize the model.

2. Make initial guesses of the aggregate capital stock K, aggregate effective labor N, aggregate real money M/P, and the wage income tax rate τ.

3. Compute the values of w and r that solve the firm's Euler equations. Compute the pension pen so that the replacement rate of average pensions is equal to the empirical value. Compute the transfers tr.

4. Compute the household's decision functions by backwards iteration.

5. Compute the steady-state distribution of the state variable $\{k, m, f, z, t\}$ by forward induction.

6. Compute the aggregate capital stock K, aggregate real money balances M/P, and aggregate accidental bequests Beq and pensions Pen. Update K, M/P, N, and τ and return to step 2 until convergence.

We discretize the state space (k, m, z, f) using an equispaced grid over the capital stock k, the money balances m, and the individual productivity z. The upper grid points $k_{max} = 10.0$ and $m_{max} = 0.3$ are found to be non-binding. For the productivity z, the grid ranges from $-2\sigma_{y_1}$ to $2\sigma_{y_1}$. The probability of having productivity shock z_1 in the first period of life is computed by integrating the area under the normal distribution. The transition probabilities are computed using the method of Tauchen (1986). As a consequence, the efficiency index $e(z, j)$ follows a finite Markov chain.

In step 4, a finite-time dynamic programming problem is to be solved. We use standard value function iteration over the state space grid $\{k, m, z, f\}$ to find the optimal next-period values k', m', and f'. For the working agent, we also have to compute the optimal labor supply from his budget constraint and his first-order condition:

$$u_{1-l}(c, m, 1 - l) = u_c(c, m, 1 - l)(1 - \tau)e(z, j)w. \tag{20}$$

As the household is born without any assets, his first-period wealth and his real money balances are zero. As a consequence, the value function would take the value $-\infty$ as $m_1 = 0$. For computational purposes, therefore, we slightly change the utility function and introduce a small constant ψ into (2), $\tilde{u} = u(c, m + \psi, 1 - l)$.

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