Optimal Utilitarian Taxation and Horizontal Equity

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Abstract

We impose a horizontal equity restriction on the problem of finding the optimal utilitarian tax mix. The horizontal equity constraint requires that individuals with the same ability have to pay the same amount of taxes regardless of their preferences for leisure. Contrary to normal findings, we find that a good that is complementary to leisure can be encouraged by the tax system, and that a good that normally should be discouraged by the tax system can be subsidized even if the economy is composed of only two private commodities plus leisure. Also, the marginal effective tax rate can be different from zero at the top (of the ability distribution) when the tax mix obeys the horizontal equity constraint.

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1 Introduction

A well-known problem with income taxes is that they punish hard-working people. Nozick (1974) for example asks why somebody who prefers looking at the sunset should pay less taxes than somebody who has to earn money in order to attain his pleasures. This question is not only important on its own normative ground, but also because tax systems that violate general conceptions of equity will be replaced if enough citizens call them into question.

As a matter of fact, such questions of equity have to a large extent been neglected in the optimal taxation literature, where one of the standard assumptions is that all individuals share the same preferences.[1] At the same time, philosophers and social choice scholars have been investigating redistributive schemes where individuals are held responsible for certain inequalities.[2] In particular, it is often advocated that an individual ought to bear the consequences of the characteristics which he has chosen himself. This line of reasoning, much of which originates from Dworkin (1981a, b), is especially relevant for optimal income taxation if the utility of leisure is heterogeneous across individuals. In such case, the government may not want to compensate people for income differences that are due to differences in tastes. However, since it is generally assumed that the government can only observe the income of an individual, it is impossible to find an income tax scheme that only compensates for differences in abilities. Indeed, in the public debate it is frequently pointed out that transfers to low skilled but hard-working persons are also benefiting more highly skilled but also more epicurean individuals. In the eyes of the government, they are alike since their pre-tax incomes are similar. In this paper we investigate if and how the government can use linear commodity taxation and non-linear income taxation to escape this dilemma.

Related to the principle of responsibility for certain inequalities is the horizontal equity (hereafter HE) principle of equal treatment of equals. Indeed, an interpretation of the HE principle is that if two individuals differ only in tastes, then the government ought to treat them identically. The literature contains several suggestions of the status and definition of HE. Musgrave (1959) argues that in the ability-to-pay approach to taxation,

[1] Possible exceptions are provided by Cuff (2000) and Boadway et al. (2002) for the finite case, while Tarkiainen and Tuomala (1999) develop a computational approach to tackle the problem of two-dimensional population in the continuous case. All the quoted authors neglect the problem of the optimal structure of commodity taxation and work with models where leisure is additive separable from other consumption goods. Sandmo (1993) examines the utilitarian case for a linear income tax under the assumption that differences in earnings are explained by differences in preferences over work and consumption; he also has a brief section in which both market abilities and preferences for leisure are allowed to vary.

[2] See Fleurbaey and Maniquet (2002) for a review.

"horizontal and vertical equity are but different sides of the same coin." However, there are several reasons for taxing people with the same ability differently. Besides the conflicts arising from the government's lack of information, Stiglitz (1982) demonstrates that the HE requirement does not follow from the maximization of a traditional utilitarian or more general social welfare function (which does not consider relations between individual outcomes), and, more strongly, that it may also be inconsistent with Pareto optimality. In a recent contribution, Kaplow and Shavell (2001) prove formally that any non-welfarist method of policy assessment, such as the concern for HE, violates the Pareto principle.[3]

In view of this expected conflict between horizontal and vertical equity, it is often argued that the former should take precedence over the latter. In line with this, Atkinson and Stiglitz (1976, 1980) suggest the imposition of a HE constraint on the maximization of a social welfare function. Feldstein (1976) instead suggests to balance the fulfilment of HE against the utilitarian principle of welfare maximization. Regarding the definition of HE, the proposed measures are as a rule either based on tax payments or on utilities.

In this paper we stay close in spirit to the interpretation of the concept given by Bossert (1995) in terms of "equal transfers for equal circumstances",[4] and require that individuals of the same ability must pay the same amount of taxes irrespective of their preferences.[5] This can be justified by the observation that people with the same ability share the same opportunity set, and while differences in this set can in some moral sense be deemed "irrelevant" and therefore call for compensation, differences in preferences may be regarded as morally "relevant", suggesting that compensation is ruled out for such differences. The individuals are fully responsible for their preferences.

Our approach is to introduce the principle of equal transfers for equal circumstances as a constraint on the maximization of a utilitarian social welfare function. Although we have to admit that the choice of a tax-based rather than a utility-based measure is to some extent arbitrary, it is simple and also sufficient for focusing on the moral difficulties raised by the fact that the government can observe income differences but not differences in abilities or preferences.[6]

[3] The term "non-welfarist" refers to any conception of social welfare that gives weight to factors other than the satisfaction of the individuals' preferences.

[4] This terminology comes from the division of the sources of individual outcomes into wills, resources and circumstances. According to this division, the individual is responsible for his wills, whereas the circumstances are factors outside his control. Differences in circumstances can be compensated by reallocating the resources.

[5] A similar argument was put forward by Allingham (1975).

[6] The policies analyzed in this paper differ from policies that can be derived from conceptions of justice based on equality of opportunity. In contrast to what is suggested by Roemer (1998, 2002) the government in our model does not seek to equalize outcomes for comparable people with different abilities.

The concern for HE modifies the rule for optimal commodity taxation. Contrary to normal findings, a good that is complementary to leisure need not be discouraged by the tax system and, perhaps more peculiar, a good that should be discouraged by the tax system in the absence of the HE condition need not be taxed at a positive rate once this condition is imposed, even if the economy is composed by only two private commodities plus leisure.[7] We also derive effective marginal tax rates for individuals with different characteristics and compare them with the tax rates derived in an ordinary optimal taxation model. Also in this case we find some peculiar results; particularly interesting, the popular end-point result of no distortion at the top (of the skill distribution) can be violated.

The paper is organized as follows. In Section 2, we present the model and give some remarks on how it compares to other related papers. In Section 3 we derive the structure of the optimal tax policy. Section 4 concludes the paper.

2 The Model

We consider an economy where there are three goods (two private consumption goods c and z plus leisure), and three types of individuals. The individuals are characterized by their skill or ability (w^H or w^L) (reflected, by assumption of perfect competition, in the unitary wage rate they are paid) and by their taste for leisure (α^H or α^L), where superscript H (L) denotes a high (low) ability or taste for leisure. There are π^1 low skilled, low taste for leisure individuals (type 1 with w^L and α^L), π^2 high skilled, high taste for leisure individuals (type 2 with w^H and α^H), and π^3 high skilled, low taste for leisure individuals (type 3 with w^H and α^L). Preferences are represented by the utility function $u\left(c, z, \alpha^i l\right)$, where α^i is the preference parameter of an individual of type i and l is the labor supply.

Production is linear and uses labor as the only input; units are chosen to make all producer prices equal to one and good z is chosen as *numéraire* and set untaxed. Consumer prices are represented by the vector $(1+t, 1) = (q, 1)$. In addition to the commodity tax, the individuals have also to pay a non-linear tax $T(Y)$ on income Y. Thus, disposable income B equals $Y - T(Y)$ and the total tax liability is $\tau(Y) = T(Y) + tc$. The indirect utility of an agent of type i is $V^i\left(q, B^i, \frac{\alpha^i}{w^i} Y^i\right)$, where the superscript on the indirect utility function is for notational convenience only.[8] Henceforth

[7] In the standard optimal taxation problem with two private commodities plus leisure, the concept of discouragement (encouragement) becomes equivalent to "being taxed at a positive rate" ("being subsidized").

[8] For given B and Y the conditional indirect utility $V(q, B, Y, w, \alpha)$ is defined as $\max_{c,z} \left\{u\left(c, z, \frac{\alpha}{w} Y\right) \mid qc + z = B\right\}$; optimizing agents will then maximize their own $V(q, B, Y, w, \alpha)$ subject to the link between pre-tax earnings and post-tax earnings avail-

$\left(\frac{\alpha}{w}\right)^{(i)}$ will denote the ratio of the preference parameter to the productivity parameter for an individual of type i. The indirect utility function has the following properties: $V_q < 0$, $V_B > 0$, $V_3 < 0$.[9] In order to satisfy the single-crossing condition (indifference curves cross only once) we will also assume $V_{33} < 0$ (labor is annoying at increasing rates) and $V_{B3} < 0$ (an increase in private consumption is valued more, the less "experienced hours" (αl) the person is working, i.e. normality of private consumption and "experienced" leisure).[10]

Compared with the related models developed by Cuff (2000) and Boadway et al. (2002), the distinguishing feature of our model is the introduction of an additional, taxable commodity. Cuff uses a model with three types of individuals and a two goods economy (private consumption plus leisure), where high skilled individuals have low taste for leisure, while there are low skilled individuals both with high and with low taste for leisure. The individuals' utility functions are quasi-linear and affine in consumption. Boadway et al. (2002) use a model with four types of individuals (and the same two goods economy), in which low skilled individuals with low taste for leisure are indiscernible from high skilled individuals with high taste for leisure. The individuals' utility functions are quasi-linear, but in their case affine in labor.

Dealing with a two goods economy, the quoted papers are confined to studying the shape of the optimal income tax schedule and cannot examine the potential role of commodity taxation. Notice however that, even if individual utility functions are not separable between leisure and other goods, when $\frac{\alpha^L}{w^L} = \frac{\alpha^H}{w^H}$ commodity taxes could not be employed in order to screen between low skilled, low taste for leisure individuals and high skilled, high taste for leisure ones. Relaxing this assumption, two cases become possible: i) $\frac{\alpha^L}{w^L} < \frac{\alpha^H}{w^H}$ or ii) $\frac{\alpha^L}{w^L} > \frac{\alpha^H}{w^H}$. Suppose first that $\frac{\alpha^L}{w^L} < \frac{\alpha^H}{w^H}$; then we face the following chain of inequalities: $\frac{\alpha^L}{w^H} < \frac{\alpha^L}{w^L} < \frac{\alpha^H}{w^H}$. This means that at every point in the (Y,B)-space, the slope of the indifference curve of a low skilled, hard working individual is shallower than the one of a high skilled, epicurean individual, and that for this pair of individuals the ordinary ranking of the indifference curves based on their slopes is reversed. If instead it is $\frac{\alpha^L}{w^L} > \frac{\alpha^H}{w^H}$, then the chain of inequalities is $\frac{\alpha^L}{w^H} < \frac{\alpha^H}{w^H} < \frac{\alpha^L}{w^L}$. This case reflects more closely the standard one since there is no individual with high ability that has indifference curves in (Y,B)-space that are steeper than the ones of a low skilled individual. The ordinary ranking of the indifference curves that one gets in a one-dimensional model persists. Since this is probably the more realistic setting, we will only present the solution of the

able for goods expenditure implied by the direct tax schedule.

[9] The subscripts denote partial derivatives; in particular, V_3 denotes the partial derivative with respect to the third argument.

[10] A formal proof of this result can be found in Jordahl and Micheletto (2002).

model under case ii).

One feature of the model should be stressed before going on with the formal analysis. In standard optimal taxation models, the inability to observe the types of the individuals raises a familiar problem. The government wishes to redistribute resources from high skilled to low skilled individuals[11] but, not knowing who is who, all it can do is to tax higher incomes more heavily than lower incomes. This may create an incentive for high skilled individuals to reduce their labor supply and "mimic" the low skilled individuals. Thus, having imposed the single-crossing condition, the binding self-selection constraint thwarting the government in its attempts to redistribute among individuals runs downwards from high skilled (high earning) individuals towards low skilled (low earning) ones. In a finite-class economy this is generalized by saying that an optimal allocation results in a simple monotonic chain to the left (Guesnerie and Seade, 1982), which means that each pair of successive bundles are L-linked[12] by a downwards binding incentive-compatibility constraint. However, as long as individuals differ both according to their market ability and according to their preferences for leisure, this is no longer necessarily true even if (as in our case) the single-crossing condition still holds. We cannot tell *a priori* which one of the pair of self-selection constraints is going to be binding. Both constraints could even be binding at the same time.[13]

3 The Optimal Tax Mix

In this Section we derive the optimal tax mix for a utilitarian government.[14] To get a benchmark case, we will first quickly present the results that are obtained without imposing the HE constraint. Then we present the results of the full model including this constraint.

3.1 Without the HE Constraint

When the (utilitarian) government neglects questions raised by the heterogeneity in tastes, its problem is the following:

[11] This happens because utility normally increases with the wage rate.

[12] Using the terminology of Guesnerie and Seade (1982), a corner (or chosen bundle) is linked to another if they both belong to the optimal set of some individual h, or equivalently if there is an indifference curve of h which passes through both corners and is the highest h can reach on his budget set. Individual h is said to link these corners. A corner C_i is W-linked (W for winner) if some h links C_i to some other corner C_j, and is allocated C_i. A corner C_i is L-linked (L for loser) if some h links C_i to C_j, and is allocated C_j.

[13] Notice that the aforesaid properties are a common feature of all models introducing heterogeneity along more than one dimension (see Balestrino, Cigno and Pettini, 1999, 2002 and Cremer, Pestieau and Rochet, 2001).

[14] In accordance with standard practice in the optimal taxation literature, we will simply assume that a solution exists and characterize the optimal tax mix conditional on this assumption.

$$\max_{B^1,B^2,B^3,Y^1,Y^2,Y^3,t} \pi^1 V^1\left(q, B^1, \tfrac{\alpha^L}{w^L}Y^1\right) + \pi^2 V^2\left(q, B^2, \tfrac{\alpha^H}{w^H}Y^2\right) + \\ +\pi^3 V^3\left(q, B^3, \tfrac{\alpha^L}{w^H}Y^3\right)$$

subject to the budget constraint:[15]

$$\pi^1\left(Y^1 - B^1 + tc^1\right) + \pi^2\left(Y^2 - B^2 + tc^2\right) + \pi^3\left(Y^3 - B^3 + tc^3\right) \geq 0, \qquad (\gamma)$$

and the following self-selection constraints:[16]

$$V^2\left(q, B^2, \tfrac{\alpha^H}{w^H}Y^2\right) \geq \widehat{V^{2(1)}}\left(q, B^1, \tfrac{\alpha^H}{w^H}Y^1\right), \qquad \left(\lambda_2^d\right)$$

$$V^3\left(q, B^3, \tfrac{\alpha^L}{w^H}Y^3\right) \geq \widehat{V^{3(2)}}\left(q, B^2, \tfrac{\alpha^L}{w^H}Y^2\right), \qquad \left(\lambda_3^d\right)$$

where a "hat" above V^i indicates that the indirect utility is evaluated at a point where type i is mimicking another type and we follow the convention to denote by $\widehat{V^{j(i)}}$ the indirect utility of an individual of type j trying to masquerade as an individual of type i. The subscripts on the Lagrange multipliers indicate the type of the potential mimicker, and the superscripts indicate the direction of the incentive-compatibility constraint: "u" for upwards and "d" for downwards (according to the ranking given by the slopes of the indifference curves). Since single-crossing holds, we only need to take the self-selection constraints linking pairs of adjacent individuals into account.

3.1.1 The Indirect Tax Structure

The optimal commodity tax rate t is implicitly given (see Appendix A) by:

$$t\sum_{i=1}^{3}\pi^i\frac{\partial\widetilde{c^i}}{\partial q} = \frac{1}{\gamma}\left[\lambda_2^d\widehat{V_B^{2(1)}}\left(c^1 - \widehat{c^{2(1)}}\right) + \lambda_3^d\widehat{V_B^{3(2)}}\left(\underbrace{c^2 - \widehat{c^{3(2)}}}_{A}\right)\right]. \qquad (1)$$

Notice that in eq. (1) the term A inside brackets (which is referred to agents that are both high skilled) is non-zero as long as c_3^i, the derivative of the demand of agents of type i for commodity c with respect to the third argument in the individual utility function, is different from zero. On the other hand, $c_3 \neq 0$ simply means that the consumption of the taxed commodity is positively related to labor (if $c_3 > 0$) or negatively related to labor (if $c_3 < 0$).

Without the HE constraint the sign of the r.h.s. of (1), and therefore of t, is unambiguously determined once the relation between the taxed commodity and leisure is known.

[15] It is assumed that taxation serves a merely redistributive purpose.

[16] In shaping the self-selection constraints we are implicitly exploiting the circumstance that the utilitarian solution belongs to the family of "normal cases", i.e. entails redistribution from high- to low skilled agents.

3.1.2 The Marginal Effective Tax Rates

Now consider the marginal effective tax rate (METR). Since there are only two commodities and one is chosen as *numéraire* and set untaxed, the effective tax rate is defined as $\tau(Y) = T(Y) + tc\left[q, Y - T(Y), \frac{\alpha}{w}Y\right]$. Differentiating gives the METR:

$$\tau' = T' + t\left[\frac{\partial c}{\partial B}\left(1 - T'\right) + c_3\frac{\alpha}{w}\right]. \tag{2}$$

As usual we can derive an expression for the marginal income tax rate faced by an individual by considering his optimal choice of labor supply. From the first order conditions of the agent's problem max $V\left(q, B, \frac{\alpha}{w}Y\right)$ subject to $B = Y - T(Y)$ it is $T' = 1 + \frac{\alpha}{w}\frac{V_3}{V_B}$. Substituting this value into (2) provides an alternative expression for the METR:

$$\tau' = 1 + tc_3\frac{\alpha}{w} + \frac{\alpha}{w}\frac{V_3}{V_B}\left(1 - t\frac{\partial c}{\partial B}\right). \tag{3}$$

In Appendix B it is shown that, as expected, the METR is positive for agents of type 1 and 2, but zero for agents of type 3.

3.2 With the HE Constraint

Since we defined HE as the requirement that individuals of the same skill level must pay the same amount of taxes, the constraint on the government's problem takes the form:

$$Y^3 = t\left(c^2 - c^3\right) + Y^2 - B^2 + B^3, \tag{4}$$

Substituting (4) into the indirect utility function of type 3 individuals, the government's problem then becomes:

$$\max_{B^1,B^2,B^3,Y^1,Y^2,t} \pi^1 V^1\left(q, B^1, \tfrac{\alpha^L}{w^L}Y^1\right) + \pi^2 V^2\left(q, B^2, \tfrac{\alpha^H}{w^H}Y^2\right) + \\ + \pi^3 V^3\left(q, B^3, \tfrac{\alpha^L}{w^H}\left[t\left(c^2 - c^3\right) + Y^2 - B^2 + B^3\right]\right)$$

subject to the budget constraint:

$$\pi^1\left(Y^1 - B^1 + tc^1\right) + \pi^2\left(Y^2 - B^2 + tc^2\right) + \pi^3\left(Y^2 - B^2 + tc^2\right) \geq 0, \tag{γ}$$

and the following self-selection constraints:[17]

$$V^1\left(q, B^1, \tfrac{\alpha^L}{w^L}Y^1\right) \geq \widehat{V^{1(2)}}\left(q, B^2, \tfrac{\alpha^L}{w^L}Y^2\right), \tag{λ_1^u}$$

[17] Since single-crossing holds, we only need to take the self selection constraints linking pairs of adjacent individuals into account.

$$V^2\left(q,B^2,\tfrac{\alpha^H}{w^H}Y^2\right)\geq\widehat{V^{2(1)}}\left(q,B^1,\tfrac{\alpha^H}{w^H}Y^1\right), \qquad (\lambda_2^d)$$
$$V^2\left(q,B^2,\tfrac{\alpha^H}{w^H}Y^2\right)\geq\widehat{V^{2(3)}}\left(q,B^3,\tfrac{\alpha^H}{w^H}\left[t\left(c^2-c^3\right)+Y^2-B^2+B^3\right]\right), (\lambda_2^u)$$
$$V^3\left(q,B^3,\tfrac{\alpha^L}{w^H}\left[t\left(c^2-c^3\right)+Y^2-B^2+B^3\right]\right)\geq\widehat{V^{3(2)}}\left(q,B^2,\tfrac{\alpha^L}{w^H}Y^2\right). (\lambda_3^d)$$

Notice that every variation in B^2, B^3, Y^2 and t must be accompanied by a proper variation in Y^3, the pre-tax income of type 3 individuals, in order to match the HE requirement. Differentiating the HE constraint (4) we get the following results:

$$\frac{dY^3}{dB^2} = \frac{t\frac{\partial c^2}{\partial B^2}-1}{1+tc_3^3\frac{\alpha^L}{w^H}}, \tag{5}$$
$$\frac{dY^3}{dB^3} = \frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}, \tag{6}$$
$$\frac{dY^3}{dY^2} = \frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}, \tag{7}$$
$$\frac{dY^3}{dq} = \frac{c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)}{1+tc_3^3\frac{\alpha^L}{w^H}}. \tag{8}$$

Considering the "normal" case when redistribution goes from high- to low skilled individuals, we can show that an optimum is not compatible with the Lagrange multiplier λ_1^u being different from zero. The argument is very close to that provided by Brito et al. (1990) to prove the result (stated in their Proposition 1) that, at any efficient allocation, individuals of one type will always view the bundles of individuals of other types who have a larger total tax liability as strictly inferior to their own. To show that a similar result holds also in our model, suppose that there exists a solution to the government's problem such that the constraint λ_1^u is binding. Then the government could improve upon the suggested allocation by simply letting the low skilled agents reach the point intended for those whom they are willing to mimic. Since redistribution is assumed to go from high- to low skilled individuals, this would imply that the low skilled agents switch from a bundle where the total tax liability is negative (they are net receivers) to a bundle where it is positive. Leaving the value of the maximand of the government's problem unaffected (since the low skilled agents were supposed to be indifferent between the two points), such a switch would be socially desirable since it weakens the budget constraint of the government while not tightening the incentive-compatibility constraints. Notice in particular that we couldn't have invoked something similar to Proposition 1 by Brito et al. (1990) if the imposition of the additional (HE) constraint to the standard problem of a utilitarian government had reversed the direction of redistribution among agents. However, it is easy to see that this cannot occur since then, in a purely redistributive context, the objective to maximize the sum of utilities of different agents would be better achieved by an optimal tax policy involving no taxation at all (*laissez-faire* outcome). The HE requirement does not change the fact that if some kind of fiscal policy is in place, then low skilled agents pay a strictly lower amount of taxes than high skilled ones.

On the other hand it is not possible to avoid taking the other self-selection constraints into account. In particular, and contrary to what would have happened hadn't we imposed the HE restriction, we cannot disregard the constraint λ_2^u. This is because individuals belonging to type 2 and 3 are adjacent and, since we require them to pay the same total tax liability, it is possible that either of them would like to mimic the other. Notice however that the HE constraint rules out bunching, which would mean that they earn the same gross income and have the same income tax liability. In order to pay the same total taxes in such a case, they would have to pay the same amount of commodity taxes and this can only happen if $c_3 = 0$. It follows that, apart from this special case, the constraints λ_2^u and λ_3^d cannot be binding at the same time.

3.2.1 The Indirect Tax Structure

The optimal commodity tax rate t is implicitly given (see Appendix C) by:

$$\gamma t \left\{ \left[\pi^1 \frac{\partial \widetilde{c^1}}{\partial q} + \left(\pi^2 + \pi^3\right) \frac{\partial \widetilde{c^2}}{\partial q} \right] + \left(\pi^3 + \lambda_3^d\right) V_3^3 \frac{\alpha^L}{w^H} \frac{\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q}}{1 + t c_3^3 \frac{\alpha^L}{w^H}} \right\} +$$
$$-\lambda_2^u \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \frac{t}{1 + t c_3^3 \frac{\alpha^L}{w^H}} \left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q} \right) = \lambda_2^d \widehat{V_B^{2(1)}} \left(c^1 - \widehat{c^{2(1)}} \right) +$$
$$+\lambda_2^u \widehat{V_B^{2(3)}} \left(c^3 - \widehat{c^{2(3)}} \right) + \lambda_3^d \widehat{V_B^3} \left(c^2 - \widehat{c^{3(2)}} \right). \tag{9}$$

We know that, in a framework where a non-linear income tax is in place, the standard formula for optimal commodity taxation balances the gains of weakening the self-selection constraints against the effects on revenue from a marginal (compensated) change in the commodity tax rate.[18] Comparing (9) and (1) we see that the main differences are confined to the left-hand sides.[19] In the first term on the l.h.s. of (9), $\pi^3 \frac{\partial \widetilde{c^2}}{\partial q}$ appears in place of $\pi^3 \frac{\partial \widetilde{c^3}}{\partial q}$ and this

[18] See for instance Edwards, Keen and Tuomala (1994).

[19] Actually, between the right-hand sides of (9) and (1) there can also be a difference in the pattern of the binding self-selection constraints (but not in the number since, as we previously noticed, the constraints λ_2^u and λ_3^d in (9) cannot be binding at the same time).

happens because in the government's budget constraint the variables referred to individuals of type 3 have been replaced using the HE constraint. The second term on the l.h.s. of (9) provides a social evaluation of the effect of a compensated increase in the commodity tax rate on the indirect utility of individuals of type 3. The last term on the l.h.s. of (9) evaluates a compensated increase in t in terms of the effect on the (eventually) binding incentive-compatibility constraint requiring that agents of type 2 must not be tempted to mimic agents of type 3.

The intuition for the second term on the l.h.s. of (9) is that a tax reform which normally would not affect the utility of individuals of type 3 might do so when we impose the HE constraint. On the other hand, if the reform affects the utility of agents of type 3, it will also affect the well-being that agents belonging to other types can obtain by mimicking agents of type 3. This is relevant since in principle we cannot rule out the possibility that agents of type 2 like to misrepresent their true type and choose the bundle intended for agents of type 3. Therefore, if $\lambda_2^u \neq 0$, the indirect utility of an individual of type 2 who is mimicking an individual of type 3 is not only affected by the different way in which a mimicker spends income across goods ($\widehat{V_B^{2(3)}}\left(c^3 - \widehat{c^{2(3)}}\right)$), but also by the change in labor supply required to be recognized as an agent of type 3.

This intuitive explanation can be analytically restated as follows. Consider the effects of a small increase dq in the commodity tax rate accompanied by reductions $dT^i = -c^i dq < 0$, $i = 1, 2, 3$, in the income tax liabilities of the three types of individuals at their original earnings. This reform has no effect on the welfare of individuals of either type 1 or 2 since by use of Roy's identity: $dV^i = V_q^i dq + V_B^i dB^i = -V_B^i (c^i dq + dT^i) = -V_B^i \left(c^i - c^i\right) dq = 0$, $i = 1, 2$. The net effect of this "compensated" reform on the utility of the individuals of type 3 is:

$$dV^3 = V_q^3 dq + V_B^3 dB^3 + V_3^3 \tfrac{\alpha^L}{w^H}\left(1 + tc_3^3 \tfrac{\alpha^L}{w^H}\right)^{-1}\left[c^2 - c^3 + t\left(\tfrac{\partial c^2}{\partial q} - \tfrac{\partial c^3}{\partial q}\right)\right] dq +$$
$$+ V_3^3 \tfrac{\alpha^L}{w^H}\left(1 + tc_3^3 \tfrac{\alpha^L}{w^H}\right)^{-1}\left[\left(1 - t\tfrac{\partial c^3}{\partial B^3}\right) dB^3 + \left(t\tfrac{\partial c^2}{\partial B^2} - 1\right) dB^2\right].$$

Substituting $dB^2 = c^2 dq$ and $dB^3 = c^3 dq$ into the previous equation, and making use of Roy's identity and Slutsky equation, gives:

$$dV^3 = V_3^3 \frac{\alpha^L}{w^H} \underbrace{\left(\frac{\partial \widetilde{c^2}}{\partial q} - \frac{\partial \widetilde{c^3}}{\partial q}\right) \frac{t}{1 + tc_3^3 \frac{\alpha^L}{w^H}}}_{D} dq. \tag{10}$$

This effect on the utility of agents of type 3 is due to the adjustments in gross income Y^3 (therefore in labor supply) required to maintain constraint (4) satisfied when we change, as in the assumed reform, q, B^2 and B^3 (see eqs. (5), (6) and (8)). The total change in Y^3 is measured by term labelled

D in (10). Thus, also an agent of type 2 who tries to mimic an agent of type 3 has now to provide a different amount of effort and the mimicker's evaluation of this change in effort will be given by the product of $\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}$ and term D. We will therefore have:

$$d\widehat{V^{2(3)}} = \widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(\frac{\partial\widetilde{c^2}}{\partial q} - \frac{\partial\widetilde{c^3}}{\partial q}\right)\frac{t}{1+tc_3^3\frac{\alpha^L}{w^H}}dq. \tag{11}$$

Denoting by $d\widetilde{V^3}$ and $d\widetilde{\widehat{V^{2(3)}}}$ (where the "tildes" help to remember that the effects are produced by a "compensated" marginal variation in t) the quantities $\frac{dV^3}{dq}$ and $\frac{d\widehat{V^{2(3)}}}{dq}$ provided by (10) and (11), we can rewrite (9) as:

$$\begin{aligned} &\gamma t\left[\pi^1\frac{\partial\widetilde{c^1}}{\partial q} + \left(\pi^2+\pi^3\right)\frac{\partial\widetilde{c^2}}{\partial q}\right] + \left(\pi^3+\lambda_3^d\right)d\widetilde{V^3} - \lambda_2^u d\widetilde{\widehat{V^{2(3)}}} = \\ &= \lambda_2^d\widehat{V_B^{2(1)}}\left(c^1-\widehat{c^{2(1)}}\right) + \lambda_2^u\widehat{V_B^{2(3)}}\left(c^3-\widehat{c^{2(3)}}\right) + \lambda_3^d\widehat{V_B^{3(2)}}\left(c^2-\widehat{c^{3(2)}}\right). \end{aligned} \tag{12}$$

More insights into this modified commodity taxation rule can be gained looking at the conditions that make (9) collapse into (1). It is immediate to verify that for this purpose we need $\lambda_2^u = 0$ and that one of the two following conditions holds:

$$\frac{\partial\widetilde{c^2}}{\partial q} = \frac{\partial\widetilde{c^3}}{\partial q}, \tag{13}$$

$$\left(\pi^3+\lambda_3^d\right)\left|V_3^3\right|\frac{\alpha^L}{w^H} = \gamma\pi^3\left(1+tc_3^3\frac{\alpha^L}{w^H}\right). \tag{14}$$

According to condition (13), if the value of the derivative of the hicksian demand is the same for agents of type 2 and agents of type 3, then the requirement of HE does not alter the traditional rule governing optimal commodity taxation.

The l.h.s. of (14) represents the costs of raising an additional unit of revenue by increasing the income tax liability of agents of type 3. A marginal increase in the gross income of these agents, holding constant their disposable income, has a total direct negative impact on their indirect utility measured by $\pi^3 V_3^3\frac{\alpha^L}{w^H}$ (since there are π^3 agents of type 3), and as such it negatively affects the objective function of the government. Moreover, since this policy change also tightens the self-selection constraint that prevents agents of type 3 from mimicking agents of type 2, there is another social cost captured by $\lambda_3^d V_3^3\frac{\alpha^L}{w^H}$.

The r.h.s. of (14) represents instead the benefits of this policy measure: the change raises total additional funds and, when evaluated at the shadow price for public funds, the social value of this increase is $\gamma\pi^3\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)$.

Condition (14) therefore says that we are also back to the standard formula for optimal commodity taxation if the social benefits of a marginal increase in the gross income Y^3 are exactly offset by its social costs.

Proposition 1 summarizes the novel properties of the modified rule (12). Following the Proposition, we will discuss some of the most interesting cases that can arise.

Proposition 1 *At the constrained utilitarian optimum with* $\frac{\alpha^H}{w^H}<\frac{\alpha^L}{w^L}$*:*
(a) a commodity that is complementary to leisure could be encouraged whereas a commodity that is complementary to labor could be discouraged;
(b) a commodity that is normally expected to be encouraged could be taxed at a positive rate whereas a commodity that is normally expected to be discouraged could be subsidized.

Proof. *See Appendix D.*

Proposition 1 clearly differs from the popular prescription in the literature on optimal taxation recommending that goods complementary to labor should be encouraged while goods complementary to leisure should be discouraged by the commodity tax system (whereas "encouraged" and "discouraged" are both intended in the Mirrleesian sense[20]).[21] In the standard[22] counterpart of (1) with many types of agents and many commodities, the terms on the r.h.s. provide a social evaluation of the gains, in terms of relaxing the binding incentive compatibility constraints, from a marginal (compensated) increase in one of the commodity tax rates. In that case the prescription to tax (heavier) those commodities which are complementary to leisure is due to the fact that the r.h.s. is negative (positive) when the commodity which price is marginally increased is complementary to leisure (labor).

There are basically two reasons explaining the possibility of deviations from ordinary tax prescriptions. On one hand, we already pointed out that

[20] In a general context where there are n commodities and m individuals, the *index of discouragement* of commodity i is defined by Mirrlees (1976) as $d_i=\sum_{h=1}^{m}\sum_{j=1}^{n}\frac{\partial\widetilde{x_i^h}}{\partial q_j}t_j\left(\sum_{h=1}^{m}x_i^h\right)^{-1}$, where q and t denote consumer prices and commodity tax rates, x_i^h is the demand for commodity i by individual h and a tilde denotes hicksian demand. The index is an approximate measure of the change in compensated demand due to the tax system; positive values of the index mean that the commodity is encouraged by the indirect tax system, while negative values correspond to discouragement.

[21] See again Edwards, Keen and Tuomala (1994).

[22] Standard is here meant to describe a situation where wages are exogenous and individuals differ only with respect to their skills.

we can no longer be sure that the budget set will result in a simple monotonic chain to the left. It can well be the case that $\lambda_2^u > 0$ and $\lambda_3^d = 0$. This accounts for the fact that the r.h.s. of (12) could be positive (negative) even when commodity c is complementary to leisure (labor). On the other hand, whereas in the standard model with two private consumption goods the term by which t is multiplied is always negative (because of the concavity of the expenditure function), here it is not possible to rule out the possibility that, due to the presence of two additional factors ($\widetilde{dV^3}$ and $\widetilde{\widetilde{dV^{2(3)}}}$), it turns out to be positive. If this happens, then we would have the "anomalous" result that a commodity which should be encouraged according to the r.h.s. of (12), should actually be taxed at a positive rate, whereas a commodity which should be discouraged according to the r.h.s. of (12), should actually be subsidized. Let's look more closely at these two "anomalous" outcomes. For this aim, consider the case when $\lambda_2^u = 0$ and one of the two following conditions holds:

i) $\quad t > 0, \quad \left|\frac{\partial \widetilde{c^2}}{\partial q}\right| > \left|\frac{\partial \widetilde{c^3}}{\partial q}\right| \quad$ *and* $\quad 1 + tc_3^3 \frac{\alpha^L}{w^H} > 0;$

ii) $\quad t < 0, \quad \left|\frac{\partial \widetilde{c^2}}{\partial q}\right| > \left|\frac{\partial \widetilde{c^3}}{\partial q}\right| \quad$ *and* $\quad 1 + tc_3^3 \frac{\alpha^L}{w^H} > 0.$

Assuming c to be a commodity complementary to labor, condition *i)* provides an example of the possibility of the former "anomalous" case. In this case a (compensated) decrease in the positive value of the excise would be beneficial in terms of (compensated) revenue and weakening of the self-selection constraints. However, a marginal cut in the commodity tax rate lowers the total tax payment of an agent of type 3 more than it does for an agent of type 2: the HE constraint then requires an additional increase in Y^3 which has the (damaging) effect to lower the utility of agents of type 3. If this cost outweighs the other benefits, the reduction in t is not implemented.

Assuming c to be a commodity complementary to leisure, condition *ii)* provides instead an example of the possibility of the latter "anomalous" case. In this case, a (compensated) reduction of the subsidy would be beneficial in terms of (compensated) revenue and weakening of the self-selection constraints. However, for the same reason as above, the HE constraint requires that this reform should be performed together with an additional increase in Y^3. This increase in income tax payment lowers the utility of agents of type 3 and if this welfare cost more than offsets the other benefits, then the cut in the subsidy will not be implemented.

As a counterpart of condition (13) above, notice that standard commodity tax rules are more likely reversed the greater the difference between the derivatives of the hicksian demands $\frac{\partial \widetilde{c^2}}{\partial q}$ and $\frac{\partial \widetilde{c^3}}{\partial q}$.

3.2.2 The Marginal Effective Tax Rates

We now turn to the problem of evaluating the METR faced by the individuals at the optimal allocation. Using (3) we can characterize the METR in the following Proposition.

Proposition 2 *Under the assumption $\frac{\alpha^H}{w^H} < \frac{\alpha^L}{w^L}$, the constrained utilitarian optimum with redistribution from high- to low skilled individuals is characterized by:*
(a) a positive METR faced by agents of type 1 (low skilled, low taste for leisure);
(b) a zero METR faced by either agents of type 2 (high skilled, high taste for leisure) or type 3 (high skilled, low taste for leisure), and a METR different from zero for the other type.

Proof. *See Appendix E.*

In Appendix E we show that the METR faced by type 2 and type 3 are respectively:

$$\tau_2' = \frac{\lambda_3^d \widehat{V_B^{3(2)}} \left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}} \alpha^L - \frac{V_3^2}{V_B^2} \alpha^H \right)}{\gamma w^H \left(\pi^2 + \pi^3\right) + \frac{1}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \left[\left(\pi^3 + \lambda_3^d\right) V_3^3 \alpha^L - \lambda_2^u \widehat{V_3^{2(3)}} \alpha^H \right]}, \quad (15)$$

$$\tau_3' = \frac{\lambda_2^u \widehat{V_B^{2(3)}}}{\left(\pi^3 + \lambda_3^d\right) V_B^3} \left(1 + \frac{\widehat{V_3^{2(3)}}}{\widehat{V_B^{2(3)}}} \frac{\alpha^H}{w^H} \frac{1 - t \frac{\partial c^3}{\partial B^3}}{1 + tc_3^3 \frac{\alpha^L}{w^H}} \right) \left(1 + tc_3^3 \frac{\alpha^L}{w^H} \right). \quad (16)$$

The positive METR faced by low skilled individuals is not surprising if we notice that the corner intended for them is L-linked with another corner (the one intended for agents of type 2) by a downwards incentive compatibility constraint. In such a circumstance this distortion is the standard one which makes it possible to relax the binding constraint.

Now consider the high skilled agents. Why will either type 2 or type 3 be distorted at the margin?[23] We have already noticed that λ_2^u and λ_3^d cannot be binding at the same time. Suppose first that $\lambda_2^u \neq 0$ while the other self-selection constraint is slack ($\lambda_3^d = 0$) From (15) we get $\tau_2' = 0$: the corner intended for agents of type 2 is not L-linked to any other corner and they should therefore be "on average" undistorted at the margin. From (16), instead, we get $\tau_3' \neq 0$, in accordance with the rule prescribing that an individual should be "on average" distorted at the margin when the bundle

[23] This result somewhat resembles Balcer and Sadka (1986) who find that the marginal tax rate is positive for one-member families and negative for two-member families.

intended for him is L-linked to another bundle by individuals of a different type. However, whereas we would have expected a marginal subsidy since the self-selection constraint is binding upwards, the sign of the METR is ambiguous. Denoting by $\frac{d\widehat{V^{2(3)}}}{dB^3}$ the term $\widehat{V_B^{2(3)}}\left(1+\frac{\widehat{V_3^{2(3)}}}{\widehat{V_B^{2(3)}}}\frac{\alpha^H}{w^H}\frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}\right)$, we have that the sign of the METR is determined by the sign of the product $\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)\frac{d\widehat{V^{2(3)}}}{dB^3}$. Therefore, $\tau_3'>0$ when one of the following conditions holds:

iii) $\quad 1+tc_3^3\frac{\alpha^L}{w^H}>0 \quad$ *and* $\quad \frac{d\widehat{V^{2(3)}}}{dB^3}>0$;

iv) $\quad 1+tc_3^3\frac{\alpha^L}{w^H}<0 \quad$ *and* $\quad \frac{d\widehat{V^{2(3)}}}{dB^3}<0$.

Differentiating the budget constraint of an agent of type 3 ($qc^3+z^3=B^3$) with respect to disposable income B, it can be shown that normality of commodities c and z implies $1-t\frac{\partial c^3}{\partial B^3}>0$. Then, condition *iii)* implies $\frac{dB^3}{dY^3}=\frac{1+tc_3^3\frac{\alpha^L}{w^H}}{1-t\frac{\partial c^3}{\partial B^3}}>0$. This means that a marginal increase in the labor supply of type 3 individuals (in this case the ones being potentially mimicked) would be accomplished with an increase in their disposable income in order to satisfy the HE constraint. However, we also have that $\frac{d\widehat{V^{2(3)}}}{dB^3}>0$, which means that the mimickers would profit by the change. To prevent this and weaken the binding self-selection constraint it is useful to have agents of type 3 facing a positive distortion at the margin, discouraging them from increasing labor supply.

On the other hand, condition *iv)* implies $\frac{dB^3}{dY^3}=\frac{1+tc_3^3\frac{\alpha^L}{w^H}}{1-t\frac{\partial c^3}{\partial B^3}}<0$: a marginal increase in the labor supply of agents of type 3 would be accomplished with a reduction in their disposable income in order to keep the HE constraint satisfied. Since $\frac{d\widehat{V^{2(3)}}}{dB^3}<0$, the mimickers would also in this case profit by the change. Once again, to prevent this and weaken the binding self-selection constraint, a positive METR, discouraging agents of type 3 from increasing labor supply, is recommended.

Suppose now that $\lambda_3^d\neq 0$ whereas $\lambda_2^u=0$. From (16) we get $\tau_3'=0$: the corner intended for type 3 agents is not L-linked to any other corner and they are "on average" undistorted at the margin. From (15), instead, we get $\tau_2'\neq 0$. Considering the pattern of the binding self-selection constraints (only downwards), we would expect that the sign of τ_2' were positive since this is the standard prescription for weakening the binding constraints in such a case. However, with $\lambda_2^u=0$, the standard formula for the METR faced by agents of type 2 is amended by the presence at the denominator of the additional term $\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)^{-1}\left(\pi^3+\lambda_3^d\right)V_3^3\alpha^L$. Since when $\lambda_2^u=0$ a necessary condition for the existence of an optimum is $1+tc_3^3\frac{\alpha^L}{w^H}>0$ (see Appendix C, eq. (C3)), this additional term will be negative. Thus, in

(15) the sign of the denominator becomes ambiguous and the fact that the numerator is positive is not sufficient to decide the sign of τ_2'. A sufficiently high absolute value of $\left(1 + tc_3^3 \frac{\alpha^L}{w^H}\right)^{-1} \left(\pi^3 + \lambda_3^d\right) V_3^3 \alpha^L$ would entail $\tau_2' < 0$.

The reason is once again related to the requirement of HE. The sign of the METR faced by agents of type 2 determines the sign of the variation in their total tax liability as they increase the labor supply. This in turn means that, if agents of type 2 should marginally increase their labor supply, it would be necessary to adjust Y^3 to restore the condition of equal total tax payments: this would obviously affect the indirect utility of agents of type 3 in a way that the government must take into account. Analytically, suppose that agents of type 2 choose to marginally increase their gross income Y^2. In order to be induced to do so, their disposable income has to be increased by their marginal valuation of foregone leisure $-\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}$. From eq. (7) and (5) the variation in Y^3 needed to keep constraint (4) satisfied is:

$$dY^3 = \frac{-\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left(t\frac{\partial c^2}{\partial B^2} - 1\right) + 1 + tc_3^2\frac{\alpha^H}{w^H}}{1 + tc_3^3\frac{\alpha^L}{w^H}} = \frac{\tau_2'}{1 + tc_3^3\frac{\alpha^L}{w^H}}. \tag{17}$$

If $1 + tc_3^3 \frac{\alpha^L}{w^H} > 0$, $\tau_2' < (>)\, 0$ becomes a more (less) attractive policy option since it implies $dY^3 < (>)\, 0$, which is welfare improving (damaging) for agents of type 3 since it means more (less) leisure.

4 Concluding Remarks

Our aim with this paper has not been to make an ethical case for the horizontal equity principle. As argued by Kaplow (1995), although horizontal equity is intuitively appealing, there is need for studies that try both to justify this principle and to derive a precise measure of equity from the justification. Our intentions have rather been to investigate how the preferred tax mix might change if we were to take horizontal equity seriously.

The investigation has made clear that the horizontal equity principle may seriously affect the incentives for income and commodity taxation. The basic intuition and also the more well-established policy implications from models with heterogeneity in ability can turn out to be very misleading if we consider a more realistic setting where heterogeneity in preferences is allowed for together with heterogeneity in abilities. Contrary to normal findings, our results indicate that a good that is complementary to leisure need not be discouraged by the tax system, and a good that is normally expected to be discouraged need not be taxed even if the economy is composed by only two private goods and leisure. As expected, the direction of redistribution is a crucial factor for the marginal effective tax rates, but the introduction of the horizontal equity restriction complicates matters here as well. It is for instance possible to have a marginal tax instead of a subsidy "on average" for the high ability, hard working-type even though the self-selection constraint relating them to the high ability, epicurean type is binding upwards.

Before concluding, we note some possible *caveats*. Since the individuals are held responsible for their preferences, a higher taste for leisure can in our model be interpreted as laziness. Obviously there are several alternative interpretations. Cuff (2000) discusses the alternative to interpret a high taste for leisure as a kind of disability. Possibly, her interpretation leads to very different implications. Whereas it is intuitive to argue that compensation for laziness should be ruled out, it is—at least in the framework of responsibility and compensation—less obvious that people who are for some other reason unable to work as hard as others, should not receive any compensation for this disability. The discussion clearly touches upon the concept of free will and whether preferences are to be treated as given or as acquired.[24] It would be presumptuous to try to answer such questions in passing—we confine ourselves to saying that besides the benefits associated with a focus on one of the polar cases, our findings are also relevant as long as the taste for leisure among some individuals is to some extent interpreted more as laziness than as a disability.

Moreover, the economic analysis of the family (Becker, 1991; Cigno, 1991) can be used to explain why certain individuals are less prone to work longer hours. If production of certain goods and services can take place at home, individuals who are relatively more productive at home than at work will act as if they had a greater taste for leisure. Home production could mean the producing of substitutes to services available on the market as in Kleven, Richter and Sørensen (2000), or it could be child rearing as in Balestrino, Cigno and Pettini (2002). Apart from explaining why the labor market has a greater appeal to certain individuals, these studies also narrow down the set of goods that are candidates for relatively higher taxes. Almost fifty years ago, Corlett and Hague (1953) suggested that efficiency could be improved by taxing more heavily goods that are complementary to leisure. Yet their rule has not come to much use since the relation between most goods and leisure is wrapped in mystery. This lack of information is of course just as problematic in models like ours. Therefore models with home production provide promising inputs to extensions of our model which aim at more practically oriented policy implications.

Although our model is very simple and stylized, we hope that we have called attention to the relevance and some potential consequences for tax policy of a concern for horizontal equity which can emerge once we relax the traditional assumption of homogeneity in individuals' preferences, and especially in preference for leisure. Without doubt there are prospects for

[24] Roemer (1998) argues that the range of expended effort levels differs between groups of people with different backgrounds.

more research in this relatively unexplored area of tax theory.

5 Appendix

5.1 Appendix A: Derivation of eq. (1)

The f.o.c. for B^1, B^2, B^3, and t are respectively given by:

$$-\pi^1 V_B^1 = \pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1}-1\right)-\lambda_2^d\widehat{V_B^{2(1)}}; \tag{A1}$$

$$-\left(\pi^2+\lambda_2^d\right)V_B^2 = \gamma\pi^2\left(t\frac{\partial c^2}{\partial B^2}-1\right)-\lambda_3^d\widehat{V_B^{3(2)}}; \tag{A2}$$

$$-\left(\pi^3+\lambda_3^d\right)V_B^3 = \pi^3\gamma\left(t\frac{\partial c^3}{\partial B^3}-1\right); \tag{A3}$$

$$\pi^1 V_q^1+\pi^2 V_q^2+\pi^3 V_q^3+\lambda_2^d\left(V_q^2-\widehat{V_q^{2(1)}}\right)+\lambda_3^d\left(V_q^3-\widehat{V_q^{3(2)}}\right)+\gamma\left(\pi^1 c^1+\pi^2 c^2\right)+$$
$$\gamma\left[\pi^3 c^3+t\left(\frac{\partial c^1}{\partial q}\pi^1+\frac{\partial c^2}{\partial q}\pi^2+\frac{\partial c^3}{\partial q}\pi^3\right)\right]=0. \tag{A4}$$

Applying Roy's identity, eq. (A4) becomes:

$$-\pi^1 c^1 V_B^1-\pi^2 c^2 V_B^2-\pi^3 c^3 V_B^3-\lambda_2^d c^2 V_B^2-\lambda_3^d c^3 V_B^3+\lambda_2^d\widehat{c^{2(1)}}\widehat{V_B^{2(1)}}+\lambda_3^d\widehat{c^{3(2)}}\widehat{V_B^{3(2)}}+$$
$$\gamma\left[\pi^1 c^1+\pi^2 c^2+\pi^3 c^3+t\left(\frac{\partial c^1}{\partial q}\pi^1+\frac{\partial c^2}{\partial q}\pi^2+\frac{\partial c^3}{\partial q}\pi^3\right)\right]=0. \tag{A5}$$

Using (A1), (A2) and (A3), we can rewrite (A5) as:

$$\gamma\pi^1 c^1\left(t\frac{\partial c^1}{\partial B^1}-1\right)+\gamma\left[c^2\pi^2\left(t\frac{\partial c^2}{\partial B^2}-1\right)+c^3\pi^3\left(t\frac{\partial c^3}{\partial B^3}-1\right)\right]-c^1\lambda_2^d\widehat{V_B^{2(1)}}-$$
$$c^2\lambda_3^d\widehat{V_B^{3(2)}}+\lambda_2^d\widehat{c^{2(1)}}\widehat{V_B^{2(1)}}+\lambda_3^d\widehat{c^{3(2)}}\widehat{V_B^{3(2)}}+\gamma\left(\pi^1 c^1+\pi^2 c^2+\pi^3 c^3+t\frac{\partial c^1}{\partial q}\pi^1\right)+$$
$$\gamma t\left(\frac{\partial c^2}{\partial q}\pi^2+\frac{\partial c^3}{\partial q}\pi^3\right)=0. \tag{A6}$$

Eq. (1) is obtained using the Slutsky decomposition in (A6) and simplifying terms.

5.2 Appendix B: The Marginal Effective Tax Rates

The f.o.c. for Y^1, Y^2 and Y^3 are respectively given by:

$$\pi^1 V_3^1\frac{\alpha^L}{w^L}=\lambda_2^d\widehat{V_3^{2(1)}}\frac{\alpha^H}{w^H}-\pi^1\gamma\left(1+tc_3^1\frac{\alpha^L}{w^L}\right); \tag{B1}$$

$$\left(\pi^2+\lambda_2^d\right)V_3^2\frac{\alpha^H}{w^H}=\lambda_3^d\widehat{V_3^{3(2)}}\frac{\alpha^L}{w^H}-\pi^2\gamma\left(1+tc_3^2\frac{\alpha^H}{w^H}\right); \tag{B2}$$

$$\left(\pi^3+\lambda_3^d\right)V_3^3\tfrac{\alpha^L}{w^H}=-\pi^3\gamma\left(1+tc_3^3\tfrac{\alpha^L}{w^H}\right). \tag{B3}$$

To find the METR faced by agents of type 1, divide (B1) by (A1) and multiply the result by $\pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1}-1\right)-\lambda_2^d\widehat{V_B^{2(1)}}$. This gives

$$\tfrac{\alpha^L}{w^L}\tfrac{V_3^1}{V_B^1}\left[\lambda_2^d\widehat{V_B^{2(1)}}-\pi^1\gamma\left(t\tfrac{\partial c^1}{\partial B^1}-1\right)\right]=\lambda_2^d\widehat{V_3^{2(1)}}\tfrac{\alpha^H}{w^H}-\pi^1\gamma\left(1+tc_3^1\tfrac{\alpha^L}{w^L}\right).$$

Using notation $\overline{w^1}=\frac{w^L}{\alpha^L}$, $\overline{w^2}=\frac{w^H}{\alpha^H}$, $\overline{w^3}=\frac{w^H}{\alpha^L}$ and $\overline{\Omega^{1,2}}=\frac{\overline{w^1}}{\overline{w^2}}$ we get

$$\tau_1'=\frac{\lambda_2^d\widehat{V_B^{2(1)}}}{\gamma\pi^1}\frac{1}{\overline{w^1}}\left(\frac{\widehat{V_3^{2(1)}}}{\widehat{V_B^{2(1)}}}\overline{\Omega^{1,2}}-\frac{V_3^1}{V_B^1}\right).$$

Since single-crossing holds and assumption 2 implies that $\overline{\Omega^{1,2}}<1$, the METR faced by type 1 is positive.

Similarly, for the METR faced by agents of type 2, divide (B2) by (A2) and multiply the result by $\gamma\pi^2\left(t\frac{\partial c^2}{\partial B^2}-1\right)-\lambda_3^d\widehat{V_B^{3(2)}}$. This gives

$$\tfrac{\alpha^H}{w^H}\tfrac{V_3^2}{V_B^2}\left[-\gamma\pi^2\left(t\tfrac{\partial c^2}{\partial B^2}-1\right)+\lambda_3^d\widehat{V_B^{3(2)}}\right]=-\gamma\pi^2\left(1+tc_3^2\tfrac{\alpha^H}{w^H}\right)+\lambda_3^d\widehat{V_3^{3(2)}}\tfrac{\alpha^L}{w^H},$$

which can be rewritten as

$$\gamma\pi^2\left[1+tc_3^2\tfrac{\alpha^H}{w^H}+\tfrac{\alpha^H}{w^H}\tfrac{V_3^2}{V_B^2}\left(1-t\tfrac{\partial c^2}{\partial B^2}\right)\right]=\lambda_3^d\widehat{V_B^{3(2)}}\left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}}\frac{\alpha^L}{w^H}-\frac{V_3^2}{V_B^2}\frac{\alpha^H}{w^H}\right).$$

Using the definition of METR ($\tau'=1+tc_3\frac{\alpha}{w}+\frac{\alpha}{w}\frac{V_3}{V_B}\left(1-t\frac{\partial c}{\partial B}\right)$) and rearranging, we get

$$\tau_2'=\frac{\lambda_3^d\widehat{V_B^{3(2)}}}{\gamma\pi^2w^H}\left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}}\alpha^L-\frac{V_3^2}{V_B^2}\alpha^H\right),$$

which again gives a positive value for τ_2'.

Finally, to obtain the METR faced by type 3, divide (B3) by (A3) and multiply the result by $\pi^3\gamma\left(t\frac{\partial c^3}{\partial B^3}-1\right)$. This gives $\frac{\alpha^L}{w^H}\frac{V_3^3}{V_B^3}\left[\pi^3\gamma\left(1-t\frac{\partial c^3}{\partial B^3}\right)\right]=-\pi^3\gamma\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)$. Using the definition of METR (eq. (3)) and rearranging terms, we get $\tau_3'=0$.

5.3 Appendix C: Derivation of eq. (9)

The f.o.c. for B^1, B^2, B^3 and t are respectively:

$$-\pi^1 V_B^1 = \pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1}-1\right)-\lambda_2^d\widehat{V_B^{2(1)}}; \tag{C1}$$

$$-\left(\pi^2+\lambda_2^d+\lambda_2^u\right)V_B^2 = \left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}-\gamma\left(\pi^2+\pi^3\right)+$$
$$\gamma\left(\pi^2+\pi^3\right)t\frac{\partial c^2}{\partial B^2}-\lambda_3^d\widehat{V_B^{3(2)}}-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}; \tag{C2}$$

$$-\left(\pi^3+\lambda_3^d\right)V_B^3 = -\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}t\frac{\partial c^3}{\partial B^3}\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$
$$\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}t\frac{\partial c^3}{\partial B^3}\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}-\lambda_2^u\left(\widehat{V_B^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right); \tag{C3}$$

$$\pi^1V_q^1+\pi^2V_q^2+\pi^3\left\{V_q^3+V_3^3\frac{\alpha^L}{w^H}\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)^{-1}\left[c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)\right]\right\}+$$
$$\gamma\left[\pi^1c^1+\left(\pi^2+\pi^3\right)c^2+t\left(\frac{\partial c^1}{\partial q}\pi^1+\frac{\partial c^2}{\partial q}\left(\pi^2+\pi^3\right)\right)\right]+\lambda_2^d\left(V_q^2-\widehat{V_q^{2(1)}}\right)-\lambda_2^uV_q^2+$$
$$\lambda_2^u\left\{\widehat{V_q^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)^{-1}\left[c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)\right]\right\}+\lambda_3^dV_q^3-\lambda_3^d\widehat{V_q^{3(2)}}+$$
$$\lambda_3^dV_3^3\frac{\alpha^L}{w^H}\left(1+tc_3^3\frac{\alpha^L}{w^H}\right)^{-1}\left[c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)\right]=0. \tag{C4}$$

Applying Roy's identity, f.o.c. t becomes:

$$-\pi^1c^1V_B^1-\pi^2c^2V_B^2-\pi^3c^3V_B^3-\lambda_2^dc^2V_B^2-\lambda_2^uc^2V_B^2-\lambda_3^dc^3V_B^3+\gamma\left(\pi^1c^1+\pi^2c^2\right)+$$
$$\gamma t\left[\frac{\partial c^1}{\partial q}\pi^1+\frac{\partial c^2}{\partial q}\left(\pi^2+\pi^3\right)\right]+\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left[c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$
$$\lambda_2^d\widehat{c^{2(1)}}\widehat{V_B^{2(1)}}+\lambda_2^u\widehat{c^{2(3)}}\widehat{V_B^{2(3)}}-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left[c^2-c^3+t\left(\frac{\partial c^2}{\partial q}-\frac{\partial c^3}{\partial q}\right)\right]\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$
$$\lambda_3^d\widehat{c^{3(2)}}\widehat{V_B^{3(2)}}+\gamma\pi^3c^2=0. \tag{C5}$$

Using (C1), (C2) and (C3), we can rewrite (C5) as:

$$\gamma\pi^1c^1\left(t\frac{\partial c^1}{\partial B^1}-1\right)-c^1\lambda_2^d\widehat{V_B^{2(1)}}+\left(\pi^3+\lambda_3^d\right)c^2V_3^3\frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$
$$\gamma c^2\left(\pi^2+\pi^3\right)\left(t\frac{\partial c^2}{\partial B^2}-1\right)-c^2\lambda_3^d\widehat{V_B^{3(2)}}-c^2\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+$$
$$\left(\pi^3+\lambda_3^d\right)c^3V_3^3\frac{\alpha^L}{w^H}\left(1-t\frac{\partial c^3}{\partial B^3}\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}-c^3\lambda_2^u\left(\widehat{V_B^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right)+$$
$$c^3\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}t\frac{\partial c^3}{\partial B^3}\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+\gamma\left[\pi^1c^1+\left(\pi^2+\pi^3\right)c^2+t\left(\frac{\partial c^1}{\partial q}\pi^1+\frac{\partial c^2}{\partial q}\left(\pi^2+\pi^3\right)\right)\right]+$$

$$\lambda_2^d \widehat{c^{2(1)}} \widehat{V_B^{2(1)}} + \lambda_2^u \widehat{c^{2(3)}} \widehat{V_B^{2(3)}} - \lambda_2^u \widehat{V_3^{2(3)}} \frac{\alpha^H}{w^H} \left[c^2 - c^3 + t\left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + tc_3^3 \frac{\alpha^L}{w^H}} +$$
$$\lambda_3^d \widehat{c^{3(2)}} \widehat{V_B^{3(2)}} + \left(\pi^3 + \lambda_3^d\right) V_3^3 \frac{\alpha^L}{w^H} \left[c^2 - c^3 + t\left(\frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + tc_3^3 \frac{\alpha^L}{w^H}} = 0.$$

Eq. (9) is obtained using the Slutsky decomposition and simplifying terms.

5.4 Appendix D: Proof of Proposition 1

In eq. (9), denoting the right hand side by RHS, four cases are possible:

Case 1: $RHS > 0, \quad t > 0, \quad \gamma\pi^1 \frac{\partial \tilde{c^1}}{\partial q} + \gamma \frac{\partial \tilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} > 0;$

Case 2: $RHS > 0, \quad t < 0, \quad \gamma\pi^1 \frac{\partial \tilde{c^1}}{\partial q} + \gamma \frac{\partial \tilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} < 0;$

Case 3: $RHS < 0, \quad t > 0, \quad \gamma\pi^1 \frac{\partial \tilde{c^1}}{\partial q} + \gamma \frac{\partial \tilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} < 0;$

Case 4: $RHS < 0, \quad t < 0, \quad \gamma\pi^1 \frac{\partial \tilde{c^1}}{\partial q} + \gamma \frac{\partial \tilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i + \left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t} > 0.$

For (a), note that the sign of the right hand side is not determined *a priori* by the relation of the taxed commodity to labor or leisure. If the taxed commodity is complementary to labor we have that $c^1 > \widehat{c^{2(1)}}$, $c^3 < \widehat{c^{2(3)}}$, and $c^2 > \widehat{c^{3(2)}}$. Thus, if $\lambda_3^d = 0$ and $\lambda_2^u > 0$, case 3 encompasses a "non-ordinary" sub-case with a commodity complementary to labor which in spite of this is taxed at a positive rate, besides the standard case of subsidizing a commodity which is complementary to labor. If the taxed commodity is instead complementary to leisure we have that $c^1 < \widehat{c^{2(1)}}$, $c^3 > \widehat{c^{2(3)}}$, and $c^2 < \widehat{c^{3(2)}}$. Thus, case 2 encompasses a "non-ordinary" sub-case with a commodity complementary to leisure which in spite of this is subsidized. Those two "non-ordinary" sub-cases demonstrate that a good that is normally expected to be discouraged (encouraged) in order to loosen the self-selection constraints, may actually be encouraged (discouraged) in a model where the agents differ along more than one dimension.

For (b), consider in case 1 the sub-case of a commodity complementary to labor that should be encouraged, according to the right hand side of eq. (9); nevertheless, due to a positive value of $\left(\pi^3 + \lambda_3^d\right) \frac{d\widetilde{V^3}}{t} - \lambda_2^u \frac{d\widetilde{\widehat{V^{2(3)}}}}{t}$ which is greater than the absolute value of $\gamma\pi^1 \frac{\partial \tilde{c^1}}{\partial q} + \gamma \frac{\partial \tilde{c^2}}{\partial q} \sum_{i=2}^{3} \pi^i$, the commodity is taxed. Similarly, consider in case 4 the sub-case of a commodity complementary to leisure that should be discouraged, according to the right hand side of eq. (9). The commodity is nonetheless subsidized since also in this instance the requirement to uphold horizontal equity implies that the term that multiplies t on the left hand side of eq. (9) turns out to be positive. However, whilst in the former sub-case this requires a high and positive value of $\left(\pi^3+\lambda_3^d\right)\widetilde{dV^3}-\lambda_2^u\widetilde{dV^{2(3)}}$ (since we are looking conditions compatible with a tax), in the latter sub-case this requirement means a high and negative value of the aforesaid term (since we are looking for conditions compatible with a subsidy). Apart from this difference, from eq. (9) we get that $\left|\frac{\partial \widetilde{c^2}}{\partial q}\right|>\left|\frac{\partial \widetilde{c^3}}{\partial q}\right|$ must be satisfied in both of these sub-cases.

■

5.5 Appendix E: Proof of Proposition 2

Considering the "normal" case when redistribution is directed towards the low skilled individuals and $\lambda_1^u=0$, the f.o.c. of the planner's problem w.r.t. gross incomes Y^1 and Y^2 are respectively:

$$\pi^1 V_3^1 \tfrac{\alpha^L}{w^L}=\lambda_2^d \widehat{V_3^{2(1)}}\tfrac{\alpha^H}{w^H}-\pi^1\gamma\left(1+tc_3^1\tfrac{\alpha^L}{w^L}\right); \tag{E1}$$

$$\pi^2 V_3^2\tfrac{\alpha^H}{w^H}+\pi^3 V_3^3\tfrac{\alpha^L}{w^H}\tfrac{dY^3}{dY^2}+\gamma\left(\pi^2+\pi^3\right)\left(1+tc_3^2\tfrac{\alpha^H}{w^H}\right)+\lambda_2^d V_3^2\tfrac{\alpha^H}{w^H}+\lambda_2^u V_3^2\tfrac{\alpha^H}{w^H}-\lambda_2^u\widehat{V_3^{2(3)}}\tfrac{\alpha^H}{w^H}\tfrac{dY^3}{dY^2}+\lambda_3^d V_3^3\tfrac{\alpha^L}{w^H}\tfrac{dY^3}{dY^2}-\lambda_3^d\widehat{V_3^{3(2)}}\tfrac{\alpha^L}{w^H}=0. \tag{E2}$$

Making use of eq. (7), eq. (E2) becomes:

$$\left(\pi^2+\lambda_2^u+\lambda_2^d\right)V_3^2\tfrac{\alpha^H}{w^H}=-\left(\pi^3+\lambda_3^d\right)V_3^3\tfrac{\alpha^L}{w^H}\frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}-\gamma\left(\pi^2+\pi^3\right)tc_3^2\tfrac{\alpha^H}{w^H}-\gamma\left(\pi^2+\pi^3\right)+\lambda_2^u\widehat{V_3^{2(3)}}\tfrac{\alpha^H}{w^H}\frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}+\lambda_3^d\widehat{V_3^{3(2)}}\tfrac{\alpha^L}{w^H}. \tag{E3}$$

For (a), divide (E1) by (C1) and multiply the result by $\pi^1\gamma\left(t\frac{\partial c^1}{\partial B^1}-1\right)-\lambda_2^d\widehat{V_B^{2(1)}}$. This gives

$$\tfrac{\alpha^L}{w^L}\tfrac{V_3^1}{V_B^1}\left[\lambda_2^d\widehat{V_B^{2(1)}}-\pi^1\gamma\left(t\tfrac{\partial c^1}{\partial B^1}-1\right)\right]=\lambda_2^d\widehat{V_3^{2(1)}}\tfrac{\alpha^H}{w^H}-\pi^1\gamma\left(1+tc_3^1\tfrac{\alpha^L}{w^L}\right).$$

Using notation $\overline{w^1}=\frac{w^L}{\alpha^L}$, $\overline{w^2}=\frac{w^H}{\alpha^H}$, $\overline{w^3}=\frac{w^H}{\alpha^L}$ and $\overline{\Omega^{1,2}}=\frac{\overline{w^1}}{\overline{w^2}}$ we get

$$\frac{\lambda_2^d\widehat{V_B^{2(1)}}}{\gamma\pi^1}\frac{1}{\overline{w^1}}\left(\frac{\widehat{V_3^{2(1)}}}{\widehat{V_B^{2(1)}}}\overline{\Omega^{1,2}}-\frac{V_3^1}{V_B^1}\right)=\tau_1'.$$

Since single-crossing holds and assumption 2 implies that $\overline{\Omega^{1,2}}<1$, the METR faced by type 1 is positive.

For (b), we first need expressions for the METR faced by type 2 and 3. Starting with type 2, we divide (E3) by (C2) and multiply the result by $\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}+\gamma\left(\pi^2+\pi^3\right)\left(t\frac{\partial c^2}{\partial B^2}-1\right)-\lambda_3^d\widehat{V_B^{3(2)}}-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\left(t\frac{\partial c^2}{\partial B^2}-1\right)\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}$. This gives

$$\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left[-\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\frac{t\frac{\partial c^2}{\partial B^2}-1}{1+tc_3^3\frac{\alpha^L}{w^H}}+\gamma\left(\pi^2+\pi^3\right)+\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{t\frac{\partial c^2}{\partial B^2}-1}{1+tc_3^3\frac{\alpha^L}{w^H}}\right]+$$
$$\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\lambda_3^d\widehat{V_B^{3(2)}}-\gamma\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left(\pi^2+\pi^3\right)t\frac{\partial c^2}{\partial B^2}=-\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}\frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}-\gamma\pi^2-$$
$$\gamma\pi^3+\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{1+tc_3^2\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}+\lambda_3^d\widehat{V_3^{3(2)}}\frac{\alpha^L}{w^H}-\gamma\left(\pi^2+\pi^3\right)tc_3^2\frac{\alpha^H}{w^H}.$$

Manipulating this expression we get

$$\left[1+tc_3^2\frac{\alpha^H}{w^H}+\frac{\alpha^H}{w^H}\frac{V_3^2}{V_B^2}\left(1-t\frac{\partial c^2}{\partial B^2}\right)\right]\left[\frac{\left(\pi^3+\lambda_3^d\right)V_3^3\frac{\alpha^L}{w^H}-\lambda_2^u\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}}{1+tc_3^3\frac{\alpha^L}{w^H}}+\gamma\left(\pi^2+\pi^3\right)\right]=$$
$$\lambda_3^d\widehat{V_B^{3(2)}}\left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}}\frac{\alpha^L}{w^H}-\frac{V_3^2}{V_B^2}\frac{\alpha^H}{w^H}\right).$$

Using eq. (3) and rearranging, we get

$$\tau_2'=\frac{\lambda_3^d\widehat{V_B^{3(2)}}\left(\frac{\widehat{V_3^{3(2)}}}{\widehat{V_B^{3(2)}}}\alpha^L-\frac{V_3^2}{V_B^2}\alpha^H\right)}{\gamma w^H\left(\pi^2+\pi^3\right)+\frac{1}{1+tc_3^3\frac{\alpha^L}{w^H}}\left[\left(\pi^3+\lambda_3^d\right)V_3^3\alpha^L-\lambda_2^u\widehat{V_3^{2(3)}}\alpha^H\right]}.\tag{E4}$$

To obtain the METR faced by type 3, note that from eq. (C3) we have that

$$-1=\frac{V_3^3}{V_B^3}\frac{\alpha^L}{w^H}\frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}-\frac{\lambda_2^u}{\left(\pi^3+\lambda_3^d\right)V_B^3}\left(\widehat{V_B^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}\right).$$

Multiplying by $1+tc_3^3\frac{\alpha^L}{w^H}$ and rearranging terms gives

$$\frac{V_3^3}{V_B^3}\frac{\alpha^L}{w^H}\left(1-t\frac{\partial c^3}{\partial B^3}\right)+1+tc_3^3\frac{\alpha^L}{w^H}=\frac{\lambda_2^u}{\left(\pi^3+\lambda_3^d\right)V_B^3}\left(\widehat{V_B^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}\right)+$$
$$tc_3^3\frac{\alpha^L}{w^H}\frac{\lambda_2^u}{\left(\pi^3+\lambda_3^d\right)V_B^3}\left(\widehat{V_B^{2(3)}}+\widehat{V_3^{2(3)}}\frac{\alpha^H}{w^H}\frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}\right).\tag{E5}$$

Using eq. (3), eq. (E5) can be written

$$\tau_3' = \frac{\lambda_2^u \widehat{V_B^{2(3)}}}{(\pi^3+\lambda_3^d)V_B^3} \left(1 + \frac{\widehat{V_3^{2(3)}}}{V_B^{2(3)}} \frac{\alpha^H}{w^H} \frac{1-t\frac{\partial c^3}{\partial B^3}}{1+tc_3^3\frac{\alpha^L}{w^H}}\right)\left(1+tc_3^3\frac{\alpha^L}{w^H}\right). \tag{E6}$$

Since we already noticed that it is not possible that λ_3^d and λ_2^u, which enter the expressions for τ_2' and τ_3' multiplicatively, are both binding or slack at the same time, part (b) of the Proposition follows from eq. (E4) and (E6).

■

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